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ANALYSIS OF INSTRUMENTATION ERROR EFFECTS ON THE IDENTIFICATION ACCURACY OF AIRCRAFT PARAMETERS

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FORWARD

This report was prepared under Contract NAS 1-10791 by Systems Control, Inc., Palo Alto, California, for Langley Research Center, National Aeronautics and Space Administration, Hampton, Virginia. NASA project monitors were W. H. Bryant and W. F. Hodge. At SCI, the project manager was J. S. Tyler, Jr. Project engineers were J. D. Powell and J. A. Sorensen. Project programmers were G. Plevyak and N. Taniguchi.

This report contains the technical details of analytical techniques and a preliminary study which makes use of these techniques. An accompanying document, NASA CR-112122, presents the details of the computer program developed as part of the effort.

ABSTRACT

This report presents an analytical investigation of the effect of unmodeled measurement system errors on the accuracy of aircraft stability and control derivatives identified from flight test data. Such error sources include biases, scale factor errors, instrument position errors, misalignments, and instrument dynamics. Output error identification algorithms that tend to minimize quadratic functions of the difference between actual and modeled aircraft trajectory measurements are studied.

Two techniques - ensemble analysis and simulated data analysis - are formulated to determine the quantitative variations to the identified parameters resulting from the unmodeled instrumentation errors. The parameter accuracy that would result from flight tests of the F-4C aircraft with typical quality instrumentation is determined using these techniques.

It is shown that unmodeled instrument errors can greatly increase the uncertainty in the value of the identified parameters. Some improvement can be made to the identification accuracy by treating the error sources as unknown parameters and identifying them along with the stability and control derivatives. Additional accuracy improvement can be obtained by choosing elements of the identification cost algorithm's function weighting matrix so that the sensitivity to the dominant error sources is reduced.

Computation of the sensitivity matrix of aircraft parameter deviations to individual instrumentation error sources is made to enable determining what statistical variations the identified parameters will have due to each of the error sources. This sensitivity matrix is also used to specify instrumentation quality necessary for obtaining aircraft parameters to a desired level of accuracy.

General recommendations are made of procedures to be followed to insure that the measurement system associated with identifying stabilty and control derivatives from flight test provides sufficient accuracy.

CONTENTS

FIGURES

Page

TABLES

Page

SYMBOLS

B	vector of biases affecting measurement of y
B_c	vector of biases affecting measurement of u
b_θ, b_q, b_α, b_u, b_{n_x}, b_{n_z}, $b_{\dot{q}}$	biases in the longitudinal output measurements
b_β, b_p, b_r, b_ϕ, b_{ny}, $b_{\dot{p}}$, $b_{\dot{r}}$	biases in the lateral output measurements
C	matrix which accounts for cross-product of inertia term in the lateral equations of motion
c.g.	center-of-gravity
D, $D(p)$	control measurement matrix relating u to y
$D(\hat{p})$	model of D containing parameter estimates $\hat{p}$
E_{noise}	covariance of parameter estimate errors due to the output noise w_i
e	measurement errors
e_{ij}	elements of the T matrix
e_M	mean measurement errors
e_R	random constant measurement errors
e_θ, e_q, e_α, e_u, e_{nx}, e_{nz}, $e_{\dot{q}}$	scale factor errors in the longitudinal output measurements

e_β, e_p, e_r, e_ϕ, e_{ny}, $e_{\dot{p}}$, $e_{\dot{r}}$	scale factor errors in the lateral output measurements
F, F(p)	system dynamics matrix containing some of the unknown parameters p
$F(\hat{p})$	model of system dynamics matrix containing parameter estimates $\hat{p}$
F_c	diagonal matrix representing inverse of control measurements' time constants
F_m	diagonal matrix representing inverse of output instruments' time constants
G, G(p)	control distribution matrix containing some of the unknown parameters p
$G(\hat{p})$	model of control distribtuion matrix containing parameter estimates $\hat{p}$
g	acceleration due to gravity
H, $H(\hat{p})$	model of H containing parameter estimates $\hat{p}$
I	identity matrix
Ixx, Izz	moments of inertia about the aircraft longitudinal and vertical axes
Ixz	product of inertia of aircraft in the longitudinal plane
J	performance index used to estimate parameters in the identification process
K	conversion from radians to degrees
Lβ, Lp, Lr, Lδa, Lδr	roll moments due to lateral velocity, roll rate, yaw rate, aileron deflection, and rudder deflection perturbations

M_q, M_w, M_u, $M_{\delta e}$	pitching moment due to pitch angle, vertical speed, longitudinal speed and elevator deflection perturbations
N_β, N_p, N_r, $N_{\delta a}$, $N_{\delta r}$	yaw moments due to lateral velocity, roll rate, yaw rate, aileron deflection, and rudder deflection perturbations
n	number of points collected in the measurement sequence
$n(k)$	quantity representing error due to signal quantization
n_x, n_y, n_z	longitudinal, lateral, and vertical aircraft accelerations
n_{xc}, n_{yc}, n_{zc}	corrections to accelerometer readings
p	vector of parameters to be estimated
$\hat{p}$	estimated value of the vector p
p, q, r	roll, pitch and yaw attitude rates
p_b	unknown biases to be estimated
p_p	unknown stability and control derivatives to be estimated
p_{IC}	unknown state initial conditions to be estimated
Q	quantization level
R	covariance matrix of measurement noise w_i
R_c	covariance matrix of w_{ci}
T	matrix representing scale factor errors and cross-coupling errors in the measurement of y
T_c	diagonal matrix representing scale factor errors in the measurement of u
u	control input vector
u, v, w	forward, lateral, and vertical velocity perturbations
u_I	indicated value of u due to scaling and bias errors

u_L	value of u_I due to measurement system lags
u_m	measurement value of u_L corrupted by noise
V	nominal airspeed
w_c	control measurement noise
w_i	contaminating output measurement noise assumed to be white
w_θ, w_q, w_{nx}, w_{nz}	standard deviation of noise in the longitudinal output measurements
X_{vcg}	angle-of-attack vane distance from the aircraft center of gravity
Xw, Xu	longitudinal force due to vertical and longitudinal speed perturbations
x, x_o	aircraft state vector
x_{cg}, y_{cg}, z_{cg}	components of the accelerometer position from the aircraft c.g.
Yβ, Yδa, Yδr	side force on aircraft due to lateral velocity, aileron deflection, and rudder deflection perturbations
y	measurement vector of aircraft state and its derivatives
y_I	indicated value of y_T due to scaling, cross-coupling, and bias errors
y_L	value of y_I due to measurement system lags
y_T	true value of y
Zw, Zu, Zδe	vertical force due to vertical speed, longitudinal speed, and elevator deflection perturbations
z(k)	sampled signal

α_o , α	nominal angle-of-attack
β	nominal sideslip angle
γ_{nx}, γ_{nz}	misalignments of the longitudinal and vertical accelerometers about the aircraft lateral axis
γ_p, γ_r	misalignments of the roll and yaw gyros about the aircraft lateral axis
$\gamma_{\dot{p}}$, $\gamma_{\dot{r}}$	misalignments of the roll and yaw angular accelerometers about the aircraft lateral axis
Δp	error in p obtained from an individual collection of measurement data
$\overline{\Delta}p$	mean value of Δp
Δt_L	time step of the numerical integration method
Δt_s	sample time step
δe	elevator deflection
δ_{ij}	Kroneker delta
δp	the difference between p and $\hat{p}$
ε_{ax}, ε_{az}	errors in the accelerometer location
ε_{cgx}, ε_{cgz}	error in e.g. location
ε_{vx}	error in α-vane location
θ_o	nominal pitch angle
ψ	nominal yaw angle
ρ	correlation of consecutive sampled terms
σ_n	standard deviation of n(k)
τ	time delay in sampling control input
ϕ	nominal roll angle

NOTATION

$\dot{( )}$	time derivative of variable
$( )_i$	sample of variable at i^{th} instant
$\hat{( )}$	estimated value of a parameter
$E\{ \}$	expected value of a variable
$( )^T$	matrix transpose
$( )^{-1}$	matrix inverse
$\frac{\partial( )}{\partial p}$	gradient of a constant with respect to the vector p. For a constant, this is taken to be a row vector. For a vector, this is taken to be a matrix with the number of columns equal to the order of p and the number of rows equal to the order of the vector.
$\sum_{i=1}^{n} ( )$	summation of points from 1 through n
$\Delta( )$	perturbation of a quantity about the nominal value or trim position
$( )^*$	modification of parameter due to cross-product of inertia terms
$( )_m$	measured value of aircraft states and their derivatives

I

INTRODUCTION

The process of determining stability and control derivatives of an aircraft from flight test data is called aircraft parameter identification. There are several reasons why this process has developed into a very important field of endeavor. These include:

1. Many instances where the prototype aircraft do not have the same characteristics as predicted by their wind tunnel models. The cost to the United States government due to out-of-control aircraft losses has been subtantial[1]. Major cost and safety considerations motivate determining ways of obtaining better knowledge of the aircraft parameters;

2. Requirements for better understanding and calibration of wind tunnel testing and its relationship to actual flight vehicle performance;

3. The potential of allowing the deeper understanding of aerodynamic phenomena and the relationship to vehicle stability;

4. Requirements for ground-based simulators which are more accurate representations of the aircraft in all flight regimes;

5. Requirements for superior stability augmentation and adaptive flight control systems.

There are three essential elements in the development of more adequate methods for identifying aircraft parameters from flight data:

1. Improved algorithms and computer programs to identify the derivatives, their confidence levels (variances), and related parameters such as sensor errors and wind gusts;

2. The determination of proper sequences of flight control inputs (surface deflections) which will excite all the aircraft response modes from which parameters are to be extracted, and methods of displaying this information to the test pilot so that he is aware of when a suitable maneuver has been executed; and

3. Adequate instrumentation (the right kind of sensors with necessary accuracy) and recording equipment with which to collect the flight data.

This study is concerned with this last point, namely, the establishment of what constitutes instrumentation accuracy to enable the collection of flight data which is of adequate quality for identifying the aircraft parameters to the accuracy desired.

In general, flight instrumentation is not specified today for the direct intention of identifying stability and control derivatives. Rather, its intended purpose is for checking aircraft handling qualities and general measures of performance. If instrumentation specification is made, it is typically based on what is known to be available. Part of the reason for this status is that estimating stability derivatives from flight test data has only been a secondary activity of companies building aircraft. If a problem arises in the handling qualities, the manufacturer may attempt to determine the derivatives responsible for the undesirable characteristic as an aid to the best design fix; however, generally no full identification program is undertaken. Flight simulators are built using wind tunnel estimates of stability derivatives, and only corrections for gross discrepancies are made.

There have been two notable exceptions[2,3] to the lack of attention given to specifying instrumentation for the direct purpose of extracting stability and control derivatives from flight test data. The Technological University at Delft, the Netherlands[2] has developed instrumentation systems with digital data acquisition, precision temperature controlled electronics for uniform instrument dynamics, and inertial instruments in a temperature controlled housing. However, data of individual instrument contributions to identification errors have not been collected, nor have the individual error effects on particular stability derivatives been determined.

LTV Aerospace Corporation[3] has studied instrument error effects on VTOL parameter identification accuracy. The LTV work involved repeated simulation of the identification process and included random noise error sources. A least-squares identification algorithm was used. The large parameter estimate errors which are characteristic of least-squares methods in the presence of random measurement noise were avoided by including "pre-filters" in the data processing procedure. These analog pre-filters were implemented on the aircraft to prevent aliasing in the sampling process of digital data acquisition. No individual parameter sensitivities to particular error sources were reported in their work, so that instrument tradeoff judgements couldn't be made. Rather, one instrument set and its accuracy level were defined which met the requirements of a particular VTOL testing program.

The purpose of this present study has been threefold:

1. The development of techniques, algorithms, and a computer program with which to assess the uncertainty due to instrumentation errors in the accuracy of the aircraft parameters identified from flight test data;

2. The application of these techniques to examine the variation of parameters obtained from typical flight tests with typical instrumentation errors; and

3. The determination of the general effects of instrumentation quality variations, the type of instruments used, and other quantities governing the data collection and identification process on the identified parameter accuracy.

This study is a first step in the overall task of specifying and providing adequate flight instrumentation for parameter identification. The results determine important factors which must be considered and procedures which should be followed to insure the measurement system is sufficient.

II

DEVELOPMENT OF ERROR ANALYSIS TECHNIQUES

Techniques are developed in this section to determine quantitatively the parameter variations which would result from using an output error identification algorithm in the presence of unmodeled instrument errors. It is assumed that the identification algorithm is convergent and that it tends to minimize a quadratic function of the difference between actual and modeled aircraft trajectory measurements. The modified Newton-Raphson identification algorithm is specifically used. It is further assumed that a single application of this algorithm can determine the major portion of the variation in the identified parameter value due to the instrumentation errors.

2.1 Modified Newton-Raphson Parameter Identification Process

The modified Newton-Raphson algorithm[4] is essentially one of several output error identification methods which are used. This basic identification process is illustrated in Fig. 1. The algorithm's objective is to choose parameters $\hat{p}$ of a mathematical model of the aircraft so that the difference between the output measurements of the model and the actual aircraft are minimized. With no measurement errors, external disturbances, or model structure inaccuracies, the output errors are minimized when the model parameters equal those of the aircraft. Output error identification methods have the following characteristics:

1. They require good initial estimates of the aircraft states and the parameters;
2. They give unbiased estimates in the presence of zero mean white measurement noise;
3. They can be used for identifying the parameters of aircraft with both linear and non-linear equations of motion; and
4. They do not work well in the presence of random disturbances to the dynamics (process noise).

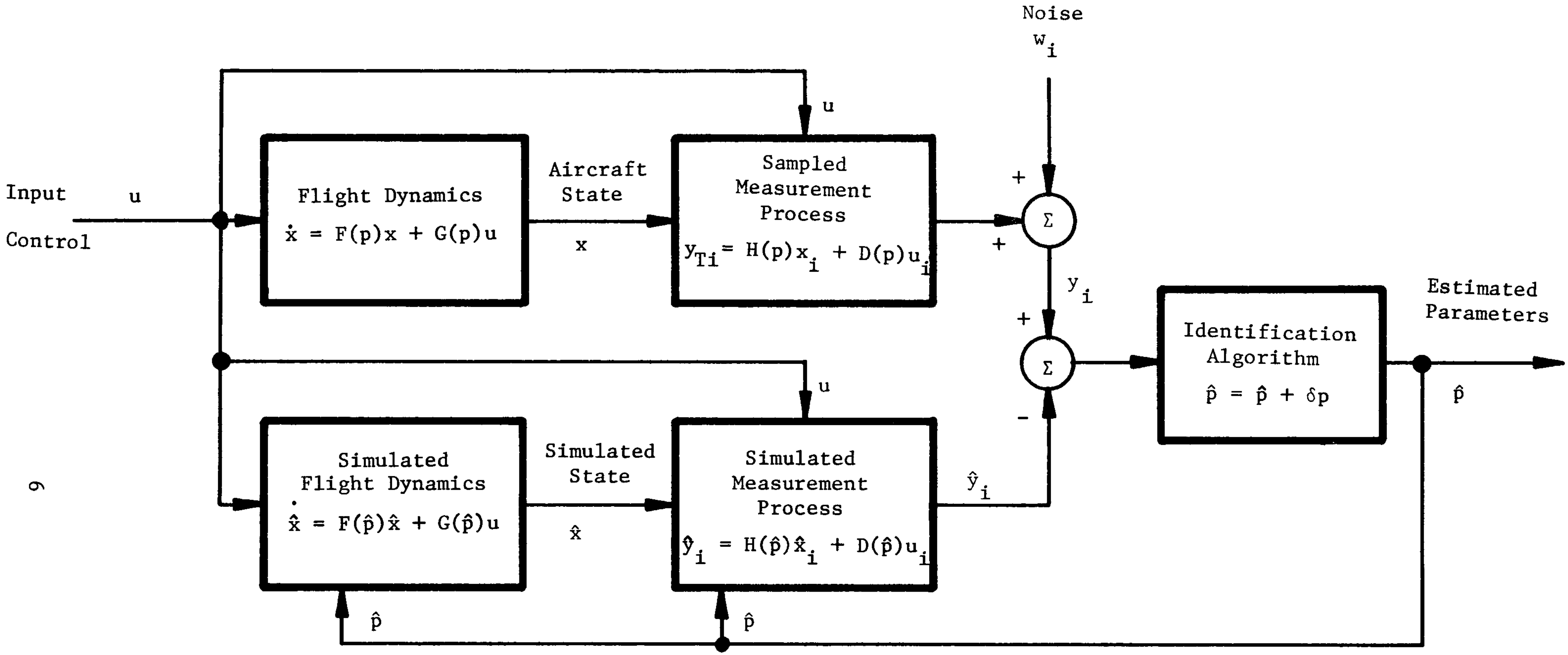


FIGURE 1. THE OUTPUT ERROR IDENTIFICATION PROCESS

In this study, the aircraft equations of motion as perturbed from the nominal flight path are assumed to be linear with constant coefficients, and of the form

$$\dot{x} = F(p)x + G(p)u \quad ; \quad x(0) = x_o \tag{2.1}$$

where

$x \triangleq$ aircraft state vector

$u \triangleq$ control input vector

$F(p) \triangleq$ system dynamics matrix containing some of the unknown parameters p

$G(p) \triangleq$ control distribution matrix containing the other unknown parameters

The identification process identifies the parameters of F and G.

The output y of this system consists of measurements of the elements of x and $\dot{x}$. It is modeled as a sampled process by the equation

$$y_i = H(p)x_i + D(p)u_i + w_i \tag{2.2}$$

where H and D are other constant-coefficient matrices also containing elements of p. The vector w_i is contaminating noise. The subscript i indicates that the output is sampled at time i and processed by a digital computer.

The modeled equations of aircraft motion are of the same order and form as Eqs. (2.1) and (2.2), and they are represented by

$$\dot{\hat{x}} = F(\hat{p})\hat{x} + G(\hat{p})u \quad ; \quad \hat{x}(0) = \hat{x}_o \tag{2.3}$$

Here, $F(\hat{p})$ and $G(\hat{p})$ are formed by using the estimated parameters $\hat{p}$. The simulated output equation is

$$\hat{y}_i = H(\hat{p})\ \hat{x} + D(\hat{p})u \tag{2.4}$$

If $\hat{p}$ equals p and $\hat{x}_o = x_o$, the only difference between y_i and $\hat{y}_i$ is due to the measurement noise w_i. In the Newton-Raphson identification scheme, it is assumed that w_i is a sequence of zero mean white noise vectors with the covariance matrix

$$E\{w_i w_j^T\} = R\delta_{ij} \tag{2.5}$$

Furthermore, it is assumed that the elements of w_i are independent so that R is a diagonal matrix.

The Newton-Raphson identification technique chooses parameters $\hat{p}$ which minimize the performance index or cost function

$$J = \sum_{i=1}^{n} (y_i - \hat{y}_i)^T R^{-1} (y_i - \hat{y}_i) \tag{2.6}$$

where n is the number of points collected in the measurement sequence. This is done by iteratively applying the equation

$$\hat{p}_{k+1} = \hat{p}_k - \left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \frac{\partial J}{\partial p}^T * \tag{2.7}$$

The first partial of J with respect to p is, from Eq. (2.6)

$$\frac{\partial J}{\partial p} = -2\sum_{i=1}^{n} (y_i - \hat{y}_i)^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \tag{2.8}$$

The second partial is

$$\frac{\partial^2 J}{\partial p^2} = 2 \sum_{i=1}^{n} \left[\frac{\partial \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} - (y_i - \hat{y}_i)^T R^{-1} \frac{\partial^2 \hat{y}_i}{\partial p^2}\right] \tag{2.9}$$

*The notation $\frac{\partial(\ )}{\partial p}$ refers to taking the partial with respect to the estimated parameter $\hat{p}$.

This is often approximated by

$$\frac{\partial^2 J}{\partial p^2} = 2 \sum_{i=1}^{n} \left[\left(\frac{\partial \hat{y}_i}{\partial p} \right)^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \right] \tag{2.10}$$

Equations (2.8) and (2.10) are substituted into Eq. (2.7) to yield

$$\hat{p}_{k+1} = \hat{p}_k + \left[\sum_{i=1}^{n} \left(\frac{\partial \hat{y}_i}{\partial p} \right)^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \right]^{-1} \left[\sum_{i=1}^{n} \left(\frac{\partial \hat{y}_i}{\partial p} \right)^T R^{-1} (y_i - \hat{y}_i) \right] \tag{2.11}$$

Equation (2.11) is the "modified" Newton-Raphson optimization technique. It is applied repeatedly to update $\hat{p}$ until Eq. (2.8) approaches a zero value.

The variance of the estimated parameter vector due to noise is

$$E \left\{ \delta p \; \delta p^T \right\}_{\text{Noise}} = \left[\frac{\partial^2 J}{\partial p^2} \right]^{-1} \tag{2.12}$$

where

$$\delta p \triangleq p - \hat{p}$$

Equation (2.12) is obtained by assuming that the errors due to w_i are small so that J is a quadratic surface in the vicinity of $\hat{p}$ and p. Then, one can write

$$\delta p = - \left\{ \left[\frac{\partial^2 J}{\partial p^2} \right]^{-1} \left(\frac{\partial J}{\partial p} \right)^T \right\} \tag{2.13}$$

where $y_i - \hat{y}_i$ is w_i and y_i is generated using the correct parameter p. Thus, from Eqs. (2.12) and (2.13),

$$E\left\{\delta p \quad \delta p^T\right\} = E\left\{\left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \left(\frac{\partial J}{\partial p}\right)^T \frac{\partial J}{\partial p} \left[\frac{\partial^2 J}{\partial p^2}\right]^{-1}\right\} \tag{2.14}$$

Because $\left[\frac{\partial^2 J}{\partial p^2}\right]$ has no noise dependence, this becomes

$$E\left\{\delta p \quad \delta p^T\right\} = \left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} E\left\{\left(\frac{\partial J}{\partial p}\right)^T \frac{\partial J}{\partial p}\right\}\left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \tag{2.15}$$

The inner term is expanded to yield

$$E\left\{\left(\frac{\partial J}{\partial p}\right)^T \left(\frac{\partial J}{\partial p}\right)\right\} = E\left\{\left[\sum_{i=1}^{n}\left(\frac{\partial y_i}{\partial p}\right)^T R^{-1} (y_i - \hat{y}_i)\right]\left[\sum_{j=1}^{n} (y_j - \hat{y}_j)^T R^{-1} \frac{\partial y_i}{\partial p}\right]\right\} \tag{2.16}$$

Because the measurement noise is assumed to be white,

$$E\left\{(y_i - \hat{y}_i)\,(y_j - \hat{y}_j)^T\right\} = R\,\delta_{ij}$$

The double summation reduces to a single summation, and the expectation is replaced by R yielding

$$E\left\{\left(\frac{\partial J}{\partial p}\right)^T \left(\frac{\partial J}{\partial p}\right)\right\} = \sum_{i=1}^{n}\left[\left(\frac{\partial y_i}{\partial p}\right)^T R^{-1} R\, R^{-1} \frac{\partial y_i}{\partial p}\right] \tag{2.17}$$

which, is exactly equal to $\frac{\partial^2 J}{\partial p^2}$ from Eq. (2.10). By substituting this result in Eq. (2.15), the desired relation (Eq. (2.12)) is established.

2.2 Linearized Aircraft Equations of Motion and the Measurements

It is assumed that the aircraft begins a maneuver from a quasi-steady flight condition with a constant airspeed V, angle-of-attack α_0, and pitch angle θ_0. The roll angle ϕ, yaw angle ψ, sideslip angle β, and the attitude rates p, q, and r are all assumed to be initially zero. The equations of motion of small perturbations of the aircraft in the longitudinal plane are [6,7]

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{q} \\ \Delta\dot{w} \\ \Delta\dot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & Mq & Mw & Mu \\ -g\sin\theta_0/K & V\cos\alpha_0/K & Zw & Zu \\ -g\cos\theta_0/K & -V\sin\alpha_0/K & Xw & Xu \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta q \\ \Delta w \\ \Delta u \end{bmatrix} + \begin{bmatrix} 0 \\ M\delta e \\ Z\delta e \\ 0 \end{bmatrix} \begin{bmatrix} \Delta\delta e \end{bmatrix}$$

Here (2.18)

$$x^T \triangleq \begin{bmatrix} \Delta\theta & \Delta q & \Delta w & \Delta u \end{bmatrix}$$

and consists of perturbations in pitch, pitch rate, the normal component of relative velocity, and the longitudinal component of relative velocity. The control $\Delta\delta e$ is the deflection of the elevator surface about the trim position. The constant K is the conversion from radians to degrees.

If only the short-period motion is to be studied, the equations

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{q} \\ \Delta\dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & Mq & Mw \\ -g\sin\theta_0/K & V\cos\alpha_0/K & Zw \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta q \\ \Delta w \end{bmatrix} + \begin{bmatrix} 0 \\ M\delta e \\ Z\delta e \end{bmatrix} \begin{bmatrix} \Delta\delta e \end{bmatrix}$$

are used. (2.19)

In Eq. (2.18), the unknown parameters to be identified consist of

$$p^T \triangleq \begin{bmatrix} Mq & Mw & Mu & M\delta e & Zw & Zu & Z\delta e & Xw & Xu \end{bmatrix} \tag{2.20}$$

In Eq. (2.19), this reduces to

$$p^T = \begin{bmatrix} Mq & Mw & M\delta e & Zw & Z\delta e \end{bmatrix} \tag{2.21}$$

The lateral equations are in the form

$$C\dot{x} = F'x + G'u \tag{2.22}$$

where C is a matrix which accounts for the cross-product of inertia term Ixz. The state

$$x \triangleq \begin{bmatrix} \Delta\beta & \Delta p & \Delta r & \Delta\phi \end{bmatrix}$$

consists of perturbations in the angle-of-sideslip, roll rate, yaw rate, and roll angle. Then C has the form

$$C \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & Ixz/Ixx & 0 \\ 0 & Ixz/Izz & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.23}$$

By letting $F = C^{-1}F'$ and $G = C^{-1}G'$, Eq. (2.22) can be modified to the more standard form

$$\dot{x} = C^{-1}F'x + C^{-1}G'u = Fx + Gu \tag{2.24}$$

or in full form,

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y\beta & \sin\alpha_o & -\cos\alpha_o & g\cos\theta_o/V \\ L\beta^* & Lp^* & Lr^* & 0 \\ N\beta^* & Np^* & Nr^* & 0 \\ 0 & 1 & \tan\theta_o & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} Y\delta a & Y\delta r \\ L\delta a^* & L\delta r^* \\ N\delta a^* & N\delta r^* \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta a \\ \Delta\delta r \end{bmatrix} \tag{2.25}$$

The starred (*) quantities are modified from their normal values due to C^{-1} in Eq. (2.24). The control deflections $\Delta\delta a$ and $\Delta\delta r$ are those of the ailerons and rudder, respectively. In Eq. (2.25), the unknown parameters are

$$p^T \triangleq \left[Y\beta\ Y\delta\alpha\ Y\delta r\ L\beta^*\ Lp^*\ Lr^*\ L\delta\alpha^*\ L\delta r^*\ N\beta^*\ Np^*\ Nr^*\ N\delta\alpha^*\ N\delta r^* \right] \tag{2.26}$$

The seven instruments which are assumed to be available for longitudinal measurements are:

1. pitch attitude gyro (θ)
2. pitch rate gyro (q)
3. angle-of-attack vane (α)
4. longitudinal pitot tube or air speed indicator (u)
5. longitudinal accelerometer (n_x)
6. normal accelerometer (n_z)
7. pitch angular accelerometer ($\dot{q}$)

For the short period equations, the pitot tube and longitudinal accelerometer are omitted.

The lateral instruments are assumed to be

1. angle-of-sideslip vane (β)
2. roll rate gyro (p)
3. yaw rate gyro (r)
4. roll attitude gyro (ϕ)

5. lateral accelerometer (n_y)

6. roll angular accelerometer ($\dot{p}$)

7. yaw angular accelerometer ($\dot{r}$)

The relation between the instrument measurements and the equations of motion are obvious except for the accelerations, which are:[8]

$$\Delta n_x = \frac{1}{g}\left(\Delta \dot{u} + w_o \Delta q\right) + \cos\theta_o \quad \Delta\theta \tag{2.27}$$

$$\Delta n_y = \frac{1}{g}\left(\Delta \dot{v} + u_o \Delta r - w_o \Delta p\right) - \cos\theta_o \quad \Delta\phi$$

$$\Delta n_z = \frac{1}{g}\left(\Delta \dot{w} - u_o \Delta q\right) + \sin\theta_o \quad \Delta\theta$$

where

$$w_o = V \sin\alpha_o$$
$$u_o = V \cos\alpha_o$$
$$\Delta\dot{v} \cong V\,\Delta\dot{\beta}$$
$$\Delta\dot{w} \cong V\,\Delta\dot{\alpha}$$

Making the substitutions and fitting the longitudinal measurements into the form of Eq. (2.4) yields (for linear accelerations measured in g's)

$$\begin{bmatrix} \Delta\theta_m \\ \Delta q_m \\ \Delta\alpha_m \\ \Delta u \\ \Delta n_{xm} \\ \Delta n_{zm} \\ \Delta\dot{q}_m \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{K\cos\alpha_o}{V} & -\dfrac{K\sin\alpha_o}{V} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{Xw}{g} & \dfrac{Xu}{g} \\ 0 & 0 & \dfrac{Zw}{g} & \dfrac{Zu}{g} \\ 0 & Mq & Mw & Mu \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta q \\ \Delta w \\ \Delta u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{Z\delta e}{g} \\ M\delta e \end{bmatrix} \left[\Delta\delta e\right] \tag{2.28}$$

The lateral measurements are

$$\begin{bmatrix} \Delta\beta_m \\ \Delta p_m \\ \Delta r_m \\ \Delta\phi_m \\ \Delta n_{ym} \\ \Delta\dot{p}_m \\ \Delta\dot{r}_m \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{VY\beta}{gK} & 0 & 0 & 0 \\ L\beta^* & Lp^* & Lr^* & 0 \\ N\beta^* & Np^* & Nr^* & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{Y\delta a}{gK} & \frac{Y\delta r}{gK} \\ L\delta a^* & L\delta r^* \\ N\delta a^* & N\delta r^* \end{bmatrix} \begin{bmatrix} \Delta\delta a \\ \Delta\delta r \end{bmatrix} \tag{2.29}$$

Equations (2.28) and (2.29) assume perfect measurements of the aircraft state x and the control input u.

2.3 Effect of Measurement Errors on the Identification Process.

Often, no other measurement errors except for the white noise indicated earlier are assumed to be present in the flight data used for identifying aircraft derivatives. Sometimes biases are assumed to affect the measurements and these terms are identified along with the equation parameters and state initial conditions. However, there are many other types of errors which do affect the estimation accuracy as will be seen. In this discussion, the emphasis is placed on those error sources whose effect can be determined by linear analysis.

2.3.1 General Instrument Error Models

First, consider the measurement of the aircraft state. For constant value of these outputs the actual indicated readings would be of the form

$$y_I = Ty_T + B \tag{2.30}$$

where

$$T = \begin{bmatrix} 1 + e_{11} & e_{12} & \cdots & \\ e_{21} & 1 + e_{22} & \vdots & \\ \vdots & \cdots & & 1 + e_{77} \end{bmatrix} \tag{2.31}$$

The diagonal terms in the T matrix represent scaling errors while offdiagonal terms represent cross-coupling errors. The vector B represents the bias errors.

The measurements are also affected by the dynamic characteristics of the instruments and the recording equipment. The slowest instrument/smoothing filter combination encountered (9) has a natural frequency of 1 cps which is about a factor of 2 higher than the aircraft dynamics. Therefore, the important aspects of the dynamic errors are the phase lag and amplitude attenuation of the instruments at frequencies below their natural frequencies. These characteristics can be approximately simulated by a first order lag regardless of the order of the instrument dynamics. The matrix equation representing this is

$$\dot{y}_L = -F_m y_L + F_m y_I \quad ; \quad y_L(0) = y_I(0) \tag{2.32}$$

where

y_L = "lagged" measurement

F_m = diagonal matrix of elements representing one over the instruments' time constants.

The addition of the random noise for each instrument yields the final measurement equation

$$y_i = y_{Li} + w_i \tag{2.33}$$

where y_i is the output measurement vector with all errors sampled at time i, and w_i is the random output noise vector with

$$E\ \{w_i\} = 0; \quad E\ \{w_i w_j^T\} \triangleq R\delta_{ij}$$

In this study, it is always assumed that the random noise is correctly modeled; that is, the covariance matrix R is known and is correctly used in the cost function J of Eq. (2.6).

The other source of measurement error is in the recording of the control input u by either surface deflection potentiometer or servo measurements. These control measurements are also subject to scale factor errors and biases which can be represented by the equation

$$u_I = T_c\ u + B_c \tag{2.34}$$

The measurement of u_I is also subject to dynamic effects which are again approximated as first order lags by the equation

$$\dot{u}_L = -F_c u_L + F_c u_I \quad ; \quad u_L(0) = u_I(0) \tag{2.35}$$

Here,

u_L = "lagged" control

F_c = diagonal matrix of one over the time constants of the control measurements.

The actual recorded control input is sampled and is subject to noise. It is represented by the equation

$$u_{mi} = u_{Li} + w_{ci} \tag{2.36}$$

where

u_{mi} = control measurement vector with errors sampled at time i

w_{ci} = random control noise vector; $E\{w_{ci}\} = 0$; $E\{w_{ci} w_{cj}\} \triangleq R_c \delta_{ij}$

The overall identification process flow diagram changes from that depicted in Fig. 1 to that depicted in Fig. 2. In the linear analysis which follows, the control measurement noise w_{ci} is ignored. This noise acts as a random disturbance to the system dynamics (process noise) and cannot be analyzed with linear methods.

2.3.2 Particular Errors Studied

Before proceeding to the analysis, a description is first presented of some of the error sources which can be studied by the preceding equations. The diagonal elements of T, T_c, F_m, and F_c have been explained. B and B_c are bias vectors.

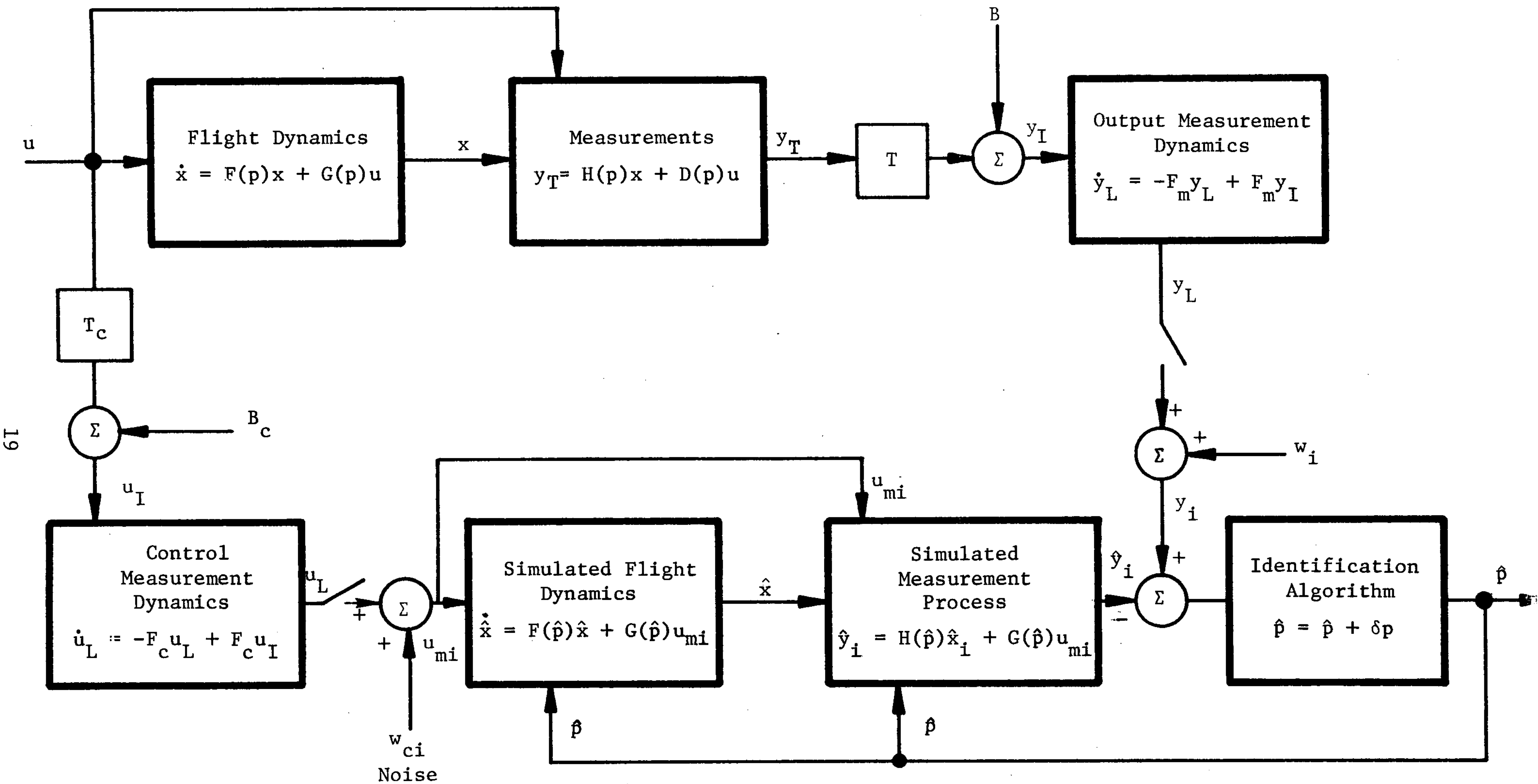


FIGURE 2. EFFECT OF MEASUREMENT ERRORS ON THE IDENTIFICATION PROCESS

Some of the specific errors which are included in the off-diagonal terms of the T matrix include:

1) α and β boom corrections
2) accelerometer location corrections
3) misalignments (accelerometers & gyros)

A simplified α boom correction equation is [8]

$$\alpha_I = \alpha - \frac{X_{vcg}}{V} q_T \tag{2.37}$$

where V is aircraft total velocity and X_{vcg} is the angle-of-attack vane distance from the aircraft center-of-gravity (c.g.). If both vane location and c.g. location are precisely known and accounted for, there is no error. However, if the actual value of X_{vcg} is different from that used in the correction, or if no correction is made, an error in the measurement results. The error in X_{vcg} is thus divided into two parts, the error in vane location (ε_{vx}) and the error in c.g. location (ε_{cgx}). The separation of the contributions is made because vane location uncertainty only affects the α correction, while c.g. location uncertainties affect accelerometer corrections as well. If all seven longitudinal instruments are being used as in Eq. (2.28), introducing the error Eq. (2.37) into the T matrix Eq. (2.30) yields:

$$e_{32} = - \frac{\varepsilon_{vx} + \varepsilon_{xcg}}{V} \tag{2.38}$$

Similar capability is provided for the β vane correction errors. Other errors which can affect α and β readings are due to upwash and boom bending.

Linear accelerometer corrections are necessary when these instruments are not mounted at the aircraft center-of-gravity. If $[x_{cg}\, y_{cg}\, z_{cg}]$ are the components of the accelerometer position from the c.g. in aircraft fixed coordinates, then the corrections should be [8]

$$n_{xc} = (r^2 + q^2)x_{cg} + (\dot{r} - pq)y_{cg} - (\dot{q} + pr)z_{cg} \tag{2.39}$$

$$n_{yc} = -(\dot{r} + pq)x_{cg} + (p^2 + r^2)y_{cg} + (\dot{p} - qr)z_{cg}$$

$$n_{zc} = (\dot{q} - rp)x_{cg} - (\dot{p} + qr)y_{cg} + (p^2 + q^2)z_{cg}$$

These equations can be decoupled into lateral and longitudinal parts. If the nonlinear terms are neglected, (valid for p, q, r, (expressed in radians/second) which are << 1) , the corrections to the longitudinal instruments are

$$n_{xc} \cong -\dot{q}\, z_{cg} \tag{2.40}$$

$$n_{zc} \cong \dot{q}\, x_{cg}$$

If the value of x_{cg} and z_{cg} are in error because of the uncertainty in the c.g. position or the c.g. offset of the accelerometers is neglected, then the error terms

$$e_{57} = (\varepsilon_{az} + \varepsilon_{cgz})\ /Kg \tag{2.41}$$

$$e_{67} = (\varepsilon_{ax} + \varepsilon_{cgx})\ /Kg$$

appear in the T matrix. In Eq. (2.41), the term

$\varepsilon_{ax,\ az}$ = errors in the accelerometer location when a correction is made.

= distance from c.g. to the accelerometer when a correction is not made.

$\varepsilon_{cgx,\ cgz}$ = errors in the knowledge of the c.g. location

Similarly, the lateral accelerometer has the two errors

$$e_{56} = -(\varepsilon_{az} + \varepsilon_{cgz})\ /Kg \qquad (2.42)$$

$$e_{57} = (\varepsilon_{ax} + \varepsilon_{cgx})\ /Kg$$

Other elements in the T matrix are due to mounting misalignments of the gyros and accelerometers. In the longitudinal equations, the terms

$$e_{56} = -\gamma_{nx}/K \qquad (2.43)$$

$$e_{65} = \gamma_{nz}/K$$

appear, where γ_{nx} and γ_{nz} are the small misalignment angles. In the lateral equations, the T matrix contains the terms

$$e_{23} = -\gamma_p/K \qquad (2.44)$$

$$e_{32} = \gamma_r/K$$

$$e_{67} = -\gamma_{\dot{p}}/K$$

$$e_{76} = \gamma_{\dot{r}}/K$$

which are similarly defined.

Effects of the above mentioned off-diagonal terms of the T matrices of the longitudinal and lateral measurements are presented in Section 3. Other errors which could be contained in T include angular accelerometer sensitivity to linear acceleration and rate gyro mass unbalance.

Another error source is introduced into the measurements shown in Fig. 2 due to the sampling and quantization. This is illustrated in Fig. 3a. The errors introduced by this process can be duplicated by the addition of a noise source to the sampled signal as illustrated in Fig. 3 b. Given the quantization level Q and the statistics of the sampled signal z (k), Widrow [(10)] has developed expressions for the statistics of n(k).

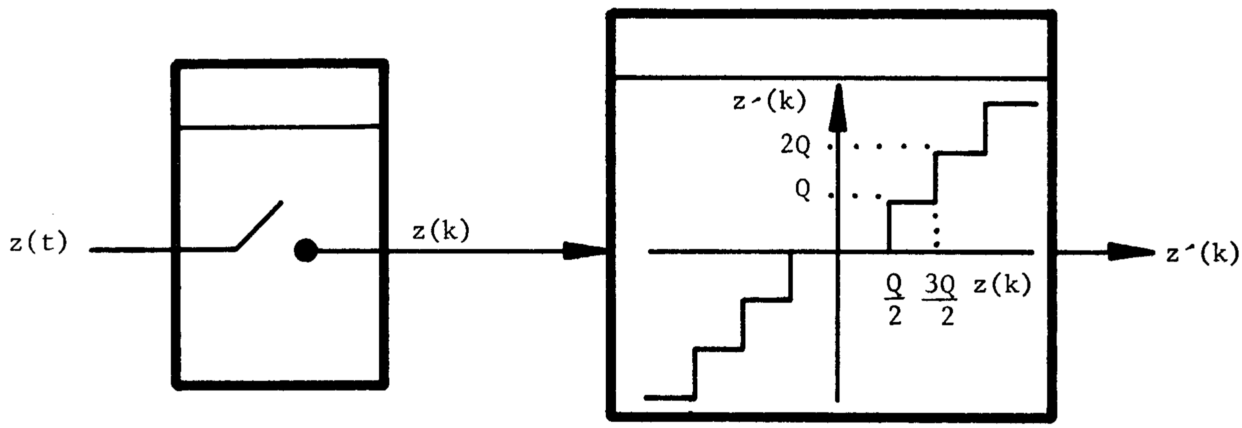
For all but very course quantization, the distribution of n(k) is uniform between -Q/2 and Q/2, and

$$E\{n(k)\} = 0 \qquad E\{n^2(k)\} = Q^2/12 \triangleq \sigma_n^2 \tag{2.45}$$

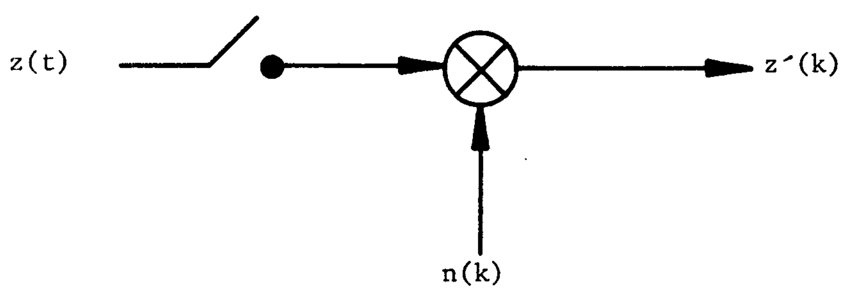
The error in this approximation is computed based on the relative magnitudes of Q and the standard deviation of z(k), ($\triangleq \sigma_z$), where z(k) is Gaussian. When $Q > \sigma_z$ (an extremely course quantization level for any airplane measurement system), the error in assuming that

$$E\{n^2(k)\} = Q^2/12 \tag{2.46}$$

is $2.6 \times 10^{-10}\, Q^2$, which is very small.



a. SAMPLING AND QUANTIZATION



b. EQUIVALENT MODEL

FIGURE 3. MODEL OF SAMPLING AND QUANTIZATION EFFECTS ON MEASUREMENT SIGNALS.

Perhaps of more interest is the whiteness of the noise sequence $n(k)$, (i.e., is $E\{n(k)\ n(k+1)\} = 0$?). Widrow also gave an expression for this quantity which is

$$E\{n(k)\ n(k+1)\} = \sigma_n^2\ e^{-(1-\rho)4\pi^2\sigma_z^2/Q^2} \tag{2.47}$$

where

$$\rho = \frac{E\{z(k)\ \ z(k+1)\}}{\sigma_z^2} \tag{2.48}$$

For frequent sampling, $z(k)$ will be highly correlated with $z(k+1)$, i.e. $\rho \simeq 1$ so that $1 - \rho$ is small ($\ll 1$). However, most aircraft measurement systems will have a fine quantization level, where σ_z/Q is large ($\gg 1$). The net result is that it is not clear whether n is white or not.

As an example, assume that typical numbers for these quantities are

$$\rho \simeq .99$$

$$\sigma_z/Q = 10$$

These yield

$$e^{-(1-\rho)4\pi^2\ \sigma_z^2/Q^2} \cong e^{-40}$$

which is essentially zero. The assumption that the quantization adds white noise to the sampled measurements seems reasonable. Thus, for the preceding example no special modeling procedure needs to be added to include the effect of quantization.

In summary, there are three types of errors which affect the accuracy of instrument measurement of the aircraft motion - random noise, random constants such as biases and scale factor errors, and mean errors. Mean errors are those terms which are known to produce measurement errors but are neglected because they are assumed to have negligible effects. (like accelerometer offset from the c.g.) Instrument lags can be thought of as mean errors with random variations about the nominal mean value.

2.4 Ensemble Analysis of Measurement Error Effects

As explained in Section 2.1, the modified Newton-Raphson identification scheme minimizes the cost function J of Eq.(2.6) by repeated application of Eq.(2.11). Convergence on the minimum is achieved when $\frac{\partial J}{\partial p} \simeq 0$. In this study, it is assumed that the true value of p is known. It is further assumed that the instrumentation errors cause the minimum point on the cost function surface J to shift a small amount from the true p. If the small error assumption is correct, only one application of Eq. (2.6) (with $\hat{p}$ set to p) can determine the shift due to the measurement error on the estimate of p. This is the key assumption of the linear analysis which is used in this study. The resulting perturbation to the parameter vector is:

$$\delta p = \hat{p} - p = - \left[\frac{\partial^2 J}{\partial p^2} \right]^{-1} \frac{\partial J^T}{\partial p} \tag{2.49}$$

where

$\hat{p}$ = perturbed parameter estimate due to measurement errors

p = true value of the parameter.

From Eq. (2.11), this can be written as

$$\delta p = + \left[\sum_{i=1}^{n} \left\{ \frac{\delta \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \right\} \right]^{-1} \sum_{i=1}^{n} \frac{\partial \hat{y}_i}{\partial p}^T R^{-1} (y_i - \hat{y}_i) \tag{2.50}$$

The y_i are the sampled output measurements taken from the aircraft (Eqs. (2.1), (2.2) (2.30) - (2.33)) and the $\hat{y}_i$ are the simulated output values (Eqs. (2.3), (2.4)) obtained using the measured control input. The sensitivity term $\frac{\partial \hat{y}_i}{\partial p}$ is computed by the identification algorithm about the latest estimates of p. Again, for the linear error analysis, this is the correct value of p.

To compute $\frac{\partial \hat{y}_i}{\partial p}$ in Eq. (2.50) requires integration of

$$\dot{\hat{x}} = F\hat{x} + Gu \ ; \quad \hat{x}(0) = \hat{x}_o \tag{2.51}$$

This assumes perfect measurement of the control input u. Also, the sensitivities of the states to parameter changes are found by integrating

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial p_p}\right) = F\left(\frac{\partial \hat{x}}{\partial p}\right) + \frac{\partial F}{\partial p_p} \hat{x} + \frac{\partial G}{\partial p_p} u \ ; \quad \frac{\partial \hat{x}}{\partial p_p} = 0 \tag{2.52}$$

where p_p is the parameter vector containing the elements presented in Eqs. (2.19), (2.20) or (2.26). If state initial conditions are also estimated, the identification process integrates

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial p_{IC}}\right) = F\left(\frac{\partial \hat{x}}{\partial p_{IC}}\right); \quad \frac{\partial \hat{x}}{\partial p_{IC}}(0) = I \tag{2.53}$$

Then, from Eq. (2.4), the output sensitivity matrix for the parameters p_p is

$$\frac{\partial \hat{y}}{\partial p_p} = H \frac{\partial \hat{x}}{\partial p_p} + \frac{\partial H}{\partial p_p} \hat{x} + \frac{\partial D}{\partial p_p} u \tag{2.54}$$

For the initial conditions, this becomes

$$\frac{\partial \hat{y}}{\partial p_{IC}} = H \frac{\partial \hat{x}}{\partial p_{IC}} \tag{2.55}$$

If output measurement biases are also estimated, the sensitivities

$$\frac{\partial \hat{y}}{\partial p_b} = I \tag{2.56}$$

must also be included. The total sensitivity used in Eq. (2.50) is then

$$\frac{\partial \hat{y}_i}{\partial p} \triangleq \left[\frac{\partial \hat{y}_i}{\partial p_p} \quad \frac{\partial \hat{y}_i}{\partial p_{IC}} \quad \frac{\partial \hat{y}_i}{\partial p_b} \right] \tag{2.57}$$

Output measurement errors affect the value of y_i in Eq. (2.50). From Eqs. (2.30) - (2.33), the measurement equations can be written as

$$\dot{y}_L = - F_m y_L + F_m (T[Hx + Du] + B) \tag{2.58}$$

By neglecting the measurement noise temporarily, the output y_i is the sampled value of y_L. The sensitivity of the error δp in Eq. (2.50) to an error source e is

$$\frac{\partial}{\partial e} (\delta p) = \left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \sum_{i=1}^{n} \frac{\partial \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial y_i}{\partial e} \tag{2.59}$$

This requires computation of the sensitivity matrix $\frac{\partial y_i}{\partial e}$

The sensitivity of y_i to a bias element of B is approximately

$$\frac{\partial y_i}{\partial e} = \frac{\partial B}{\partial e} \tag{2.60}$$

For an element in T, this is

$$\frac{\partial y_i}{\partial e} = \frac{\partial T}{\partial e} (Hx + Du) \tag{2.61}$$

For an unestimated initial condition treated as an error source, the sensitivity of y_i is

$$\frac{\partial y_i}{\partial e} = H \frac{\partial x_i}{\partial e} \tag{2.62}$$

where $\frac{\partial x}{\partial e}$ comes from integrating

$$\frac{d}{dt}\left(\frac{\partial x}{\partial e}\right) = F\,\frac{\partial x}{\partial e} \quad ; \quad \frac{\partial x}{\partial e}(0) = I \tag{2.63}$$

For an unknown time constant in the matrix F_m, the sensitivity must be determined by integrating

$$\frac{d}{dt}\left(\frac{\partial y_L}{\partial e}\right) = \frac{\partial F_m}{\partial e}\left[-\,y_L + (Hx + Du)\right] - F_m\,\frac{\partial y_L}{\partial e} \;;\; \frac{\partial y_L}{de}(0) = 0 \tag{2.64}$$

The results of Eqs. (2.60) - (2.64) are combined into a general vector $\frac{\partial y_i}{\partial e}$ for each error e which affects the output measurements.

The sensitivity of parameter estimates due to control input measurements errors is of the form

$$\frac{\partial}{\partial e}(\delta p) = -\left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \sum_{i=1}^{n} \frac{\partial \hat{y}_i}{\partial p}^{T} R^{-1} \frac{\partial \hat{y}_i}{\partial e} \tag{2.65}$$

This requires knowing the sensitivity of the simulated output $\hat{y}_i$ to control measurement errors e.

For control measurement biases, the sensitivity of the control input is

$$\frac{\partial u_{mi}}{\partial e} = \frac{\partial B_c}{\partial e} \tag{2.66}$$

For scale factor errors, this is

$$\frac{\partial u_{mi}}{\partial e} = \frac{\partial T_c}{\partial e} u \tag{2.67}$$

The effect on the states is found by integrating

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial e}\right) = F \frac{\partial \hat{x}}{\partial e} + G \frac{\partial u_{mi}}{\partial e} \tag{2.68}$$

from Eqs. (2.3). The resulting effect on the simulated output $\hat{y}_i$ is

$$\frac{\partial \hat{y}_i}{\partial e} = H \frac{\partial \hat{x}_i}{\partial e} + D \frac{\partial u_{mi}}{\partial e} \tag{2.69}$$

Evaluation of the sensitivity to control measurement lags requires integrating

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial e}\right) = F \frac{\partial \hat{x}}{\partial e} + G \frac{\partial u_L}{\partial e} \quad ; \qquad \frac{\partial \hat{x}}{\partial e}(0) = 0 \tag{2.70}$$

$$\frac{d}{dt}\left(\frac{\partial u_L}{\partial e}\right) = -F_c \frac{\partial u_L}{\partial e} + \frac{\partial F_c}{\partial e}\left[-u_L + u\right] \quad ; \qquad \frac{\partial u_L}{\partial e}(0) = 0 \tag{2.71}$$

The resulting output sensitivity is again found from Eq. (2.69).

For random constant measurement errors, e_R, the total covariance of the individual parameters being identified is

$$E\left\{\delta p \delta p^T\right\}_{total} = E\left\{\delta p \delta p^T\right\}_{noise} + \frac{\partial(\delta p)}{\partial e_R} E\left\{e_R e_R^T\right\} \frac{\partial(\delta p)}{\partial e_R}^T \tag{2.72}$$

where $E\left\{e_R e_R^T\right\}$ is the covariance matrix of the random measurement errors not including measurement noise. The sensitivities $\frac{\partial(\delta p)}{\partial e_R}$ come from Eqs. (2.59) and (2.65). The covariance due to noise comes from Eq. (2.12). For mean errors, e_M, the expected error in the parameter is

$$E\left\{\delta p\right\} = \frac{\partial(\delta p)}{\partial e_M} E\left\{e_M\right\} \tag{2.73}$$

The above error analysis is referred to here as the ensemble error analysis. It is valid for small errors which affect the measurements linearly.

2.5. Simulated Data Analysis of Measurement Error Effects

Sometimes it is useful to determine the effects of instrument errors by actually simulating the identification process and the measurement data contaminated by errors. If the errors are large or if nonlinear errors are to be studied, the one step assumptions and linearization which were used in the ensemble analysis method may not be valid. Therefore, the simulated data analysis method complements the ensemble analysis.

2.5.1. Linear Analysis

There are two common ways in which error analysis is conducted by simulation. The first simulates the effect of each random error individually, assuming they are independent, and uses the error's standard deviation for the magnitude used in the simulation. The results of simulating each of the random errors are root-sum-squared to find the approximate total error effect. For investigating the effect of measurement errors on parameter estimation accuracy, this method is not limited to a single-step application. For small errors which linearly affect the output measurements, the results of this simulated data analysis match those of the ensemble analysis.

The simulated data analysis method is related to the ensemble analysis in that the basic equation utilized is Eq. (2.50). The matrix $\frac{\partial^2 J}{\partial p^2}$ is unchanged for any one step. What is different is that rather than computing the sensitivities $\frac{\partial y_i}{\partial e}$ or $\frac{\partial \hat{y}_i}{\partial e}$, the analysis computes the residual $(y_i - \hat{y}_i)$ in $\frac{\partial J}{\partial p}$.

For output measurement biases, the residual is simply

$$(y_i - \hat{y}_i) = y_{Ii} - \hat{y}_i = B \tag{2.74}$$

where y_{Ii} is the i^{th} sample of y_I defined by Equation (2.30).

For transformation errors due to the T matrix,

$$(y_i - \hat{y}_i) = T(H\hat{x}_i + Du_i) - (H\hat{x}_i + Du_i) = (T - I)\ (H\hat{x}_i + Du_i) \tag{2.75}$$

If initial conditions are unestimated, both x and $\hat{x}$ in Eqs. (2.1) and (2.3) are integrated, and

$$(y_i - \hat{y}_i) = (y_{T_i} - \hat{y}_i) = H(x_i - \hat{x}_i) \tag{2.76}$$

For biases to the control measurements, Eqs. (2.3) and (2.4) are reevaluated using

$$u_{mi} = u_i + B_c \tag{2.77}$$

For scale factor errors, the control

$$u_{mi} = T_c u_i \tag{2.78}$$

is used.

For random errors, the total parameter covariance is

$$E\,\{\delta p \delta p^T\} = E_{noise} + \sum_{j=1}^{r} (\delta p_j \delta p_j^T) \tag{2.79}$$

where r is the number of random error sources. E_{noise} is the covariance of the parameter estimate errors due to the output noise w_i. This again comes from Eq. (2.12).

Mean errors include some elements in the T matrix and all elements in F_m and F_c (Eqs. (2.32) and (2.35). If there are no lag errors, the evaluation is just like Eq. (2.75).

If there are control measurement lag errors, Eq. (2.35) needs to be evaluated with $u_I = u$. If there are output lags, Eq. (2.32) needs evaluation. This requires reintegration of Eq. (2.3) and Eq. (2.35) in the form

$$\dot{y}_L = -F_m \, y_L + F_m \, ([Hx + Du]) \tag{2.80}$$

Equations (2.51) - (2.57) need to be reevaluated in case of control lags. The total effect due to lags on Eq. (2.50) is

$$y_i - \hat{y}_i = y_{Li} - (H\hat{x}_i + Du_{mi}) \tag{2.81}$$

For mean errors, the total effect on the parameter values is found from

$$E \left\{\delta p\right\} = \sum_{j=1}^{m} (\delta p_j) \tag{2.82}$$

where m is the number of mean error sources, and δp_j are the mean errors due to each source computed from Eq. (2.50).

F

2.5.2 Nonlinear Analysis

For large or non-linear measurement errors and the presence of process noise, a Monte Carlo simulated data analysis technique should be used. In this method, several different data sequences are simulated and used sequentially in the identification process.

The random errors contained in B, part of T, B_c, T_c, and x_o are generated at the beginning of each simulation using the errors' standard deviations and a random number generator. These errors are held constant during each single Monte Carlo run, but are changed from run to run. The random noise w_i and w_{ci} are regenerated at each sample point during each run. Each of the mean errors in T plus elements of F_m and F_c are set equal to the constant mean values and are not changed during any of the runs.

For output measurement errors in T, B and w_i only, the residual $(y_i - \hat{y}_i)$ in Eq. (2.50) is computed by

$$y_i - \hat{y}_i = T(H\hat{x}_i + Du_i) + B + w_i - (H\hat{x}_i + Du_i) \tag{2.83}$$

For random initial conditions, Eq. (2.1) must be integrated each time and Eq. (2.83) gets changed to

$$y_i - \hat{y}_i = T(Hx_i + Du_i) + B + w_i - (H\hat{x}_i + Du_i) \tag{2.84}$$

For non-lag control measurement errors, Eqs. (2.34) and (2.36) get combined so that at each sample point

$$u_{mi} = T_c u + B_c + w_{ci} \tag{2.85}$$

where w_{ci} is randomly generated each time point. Because of this change, Eqs. (2.51) - (2.53) require integration each pass through, and Eqs. (2.54) - (2.57) require re-evaluation each pass through. With these changes, Eq. (2.85) becomes

$$y_i - \hat{y}_i = T(Hx_i + Du_i) + B + w_i - (H\hat{x}_i + Du_{mi}) \tag{2.86}$$

The error Δp_j in the parameter vector obtained from each run is saved. For m Monte Carlo runs, the mean error in p is

$$\overline{\Delta p} \triangleq E\{\Delta p\} = \frac{1}{m} \sum_{j=1}^{m} \Delta p_j \tag{2.87}$$

The standard deviation about this mean is

$$E_{\_}\{\delta p \delta p^T\} = \frac{1}{m} \sum_{j=1}^{m} (\Delta p_j - \overline{\Delta p})(\Delta p_j - \overline{\Delta p})^T \tag{2.88}$$

2.6. Implementation of the Analysis Techniques

The ensemble analysis and simulated data analyses techniques described in Sections 2.4 and 2.5 were coded into a digital computer program. This program enables the assessment of uncertainty (due to instrumentation errors) in the accuracy of the aircraft parameters identified from flight test data. A summary of the equations contained within this computer program is presented in the Appendix of this report. The longitudinal equations including the short period mode and the lateral equations are both contained in the program.

This program has been exercised using stability and control derivatives from the DC-8, the F-4C, the Cessna 172, and the HL-10 lifting body. In all cases, an input sequence is first found such that the recorded output has an appropriate amount of information to allow the identification process to take place.

In the next section of this report, the results of exercising the ensemble analysis option of the program using the F-4C as an example aircraft are presented. In addition to output measurement noise, the measurement errors which are studied include output biases and elements in the T matrix.

III

STUDY OF THE EFFECT OF MEASUREMENT ERRORS ON PARAMETER IDENTIFICATION ACCURACY

In this section, the results of a study using the previously described error analysis program that determines the effects of measurement errors on parameter identification accuracy are presented. Both longitudinal and lateral motion of an F-4C aircraft with typical control surface deflections are studied. A range of measurement and recording errors, representing the current state of flight instrumentation is investigated.

3.1 Current Flight Instrumentation Accuracy

The general measurement accuracy range of instruments for sensing aircraft longitudinal state outputs which is typical of current flight tests is presented in Table 1. A similar table representing lateral instrumentation accuracy is presented later as Table 10. The error sources contained in these tables are a result of examining product literature and the specifications used by flight testing organizations. Although cases were found where the standard deviations of instrument errors exceeded the minimum and maximum table values, these values are judged to be reasonable ones for this study.

The "max" values for noise, bias, and scale factor error shown in Table 1 are basically 0.5% of the highest dynamic range typically used in flight testing. Most instruments are considerably better than this; however, analog data acquisition systems have an average accuracy of about 0.5%, so it was selected as the worst case. Noise and bias are related because data trace values taken before a maneuver is executed are used as the null points. The ability to determine these zero values is a direct function of both the noise and bias present.

For a digital data acquisition system, the 0.5% error is too large. Common error values in the measurements due to a 10-bit data acquisition system will be about 0.05% and correspondingly lower for more bits. With this recording accuracy, the instrument errors start replacing the data acquisition errors as the important

Table 1

LONGITUDINAL INSTRUMENT ERRORS

STANDARD DEVIATIONS

Instrument	Units	Full Scale Deflection*	Random Noise			Random Biases			Random Scale Factors		
			Minimum	Base	Maximum	Minimum	Base	Maximum	Minimum	Base	Maximum
Pitch attitude gyro	deg	30-90	.015	.15	.45	.015	.15	.45	.05%	.5%	.5%
Pitch rate gyro	deg/sec	30-60	.015	.10	.30	.015	.10	.30	.05%	.5%	.5%
Angle-of-attack vane	deg	20	.01	.10	.10	.010	.10	.10	.05%	.5%	.5%
Pitot tube	ft/sec	1000	.50	1.00	2.50	.50	1.00	2.50	.05%	.5%	.5%
Forward accelerometer	g's	1	.0005	.005	.005	.0005	.005	.005	.05%	.5%	.5%
Vertical accelerometer	g's	2.5	.0025	.005	.025	.0025	.005	.025	.05%	.5%	.5%
Pitch accelerometer	deg/sec^2	30-60	.015	.10	.30	.015	.10	.30	.05%	.5%	.5%

Other Types of Instrument Errors	Minimum	Base	Maximum	Instrument Lags	Bandwidth (sec^{-1})	Standard Deviation (%)
Linear accelerometer misalignment (deg)	.15	.60	.90	Gyros	150	5
Pitch angular acceleration sensitivity to linear acceleration (deg/sec^2·g)	.10		.60	Linear accelerometers	600	5
Pitch gyro mass unbalance (deg/sec·g)	.025		.60	Angular accelerometers	180	10
Center-of-gravity uncertainty (ft)	.25	.50	1.0	Tape recorder	6	5

*When a range of deflection values is given, the lower number is associated with the minimum random errors. The larger number is associated with the maximum random errors.

error sources. The "min" values presented in Table 1 are based on the lower values of References 11-13. Manufacturers' guarantees were interpreted as 2σ values, although in no case was the data given with any statistical information.

Neither the instrument manufacturers nor the flight test agencies compile statistical data of instrument accuracies in the form required for this error analysis. The validity of the range of accuracies available, such as those presented in Table 1 must be questioned without supporting laboratory test data.

As a means of having a reference set of instruments with which to conduct the study, a "baseline" set of instrument accuracies was chosen within the range of Table 1. This set of accuracies is listed in the "base" columns, and is assumed to represent values of a typical flight test program.

The effects of instrument lags, control surface deflection measurement errors, angular accelerometer sensitivity to linear accelerations, and gyro mass unbalance were not studied in this preliminary investigation. Other unknown meaurement errors might exist because of voltage supply fluctuations, temperature effects, aircraft body bending, and nonlinear errors. Additional errors exist in comparing parameters obtained from wind tunnel and flight tests because of the uncertainty in the aircraft inertia terms which can be in error up to about 5% of actual values.[14]

3.2 Model of the F-4C Aircraft

To conduct the study, the F-4C aircraft was chosen with a level flight path and an air speed of 827 ft./sec. The linearized perturbation equations about this condition, as represented by Eqs. (2.16) and (2.25) for longitudinal and lateral motion, were utilized. The corresponding stability and control derivatives of the F-4 are presented in Table 2.[15]

Table 2

REFERENCE STABILITY AND CONTROL DERIVATIVE PARAMETERS FOR THE F - 4C AIRCRAFT

Longitudinal Motion		Lateral Motion		Reference Flight Path	
Parameter	Nominal Value	Parameter	Nominal Value	Parameter	Nominal Value
Mq	$-.719\ sec^{-1}$	$Y\beta$	$-.157\ sec^{-1}$	V	827.ft/sec
Mw	$-.591^{\circ}$ /ft.sec	$Y\delta a$	$-.00338\ sec^{-1}$	α_o	2.6°
Mu	$-.0295^{\circ}$ /ft.sec	$Y\delta r$	$.0246\ sec^{-1}$	Θ_o	2.6°
$M\delta e$	$-16.2\ sec^{-2}$	$L\beta$	$-15.98\ sec^{-2}$		
Zw	$-.762\ sec^{-1}$	Lp	$-1.608\ sec^{-1}$		
Zu	$-.0617\ sec^{-1}$	Lr	$.384\ sec^{-1}$		
$Z\delta e$	-1.24ft/deg.sec^{2}	$L\delta a$	$10.92\ sec^{-2}$		
Xw	$.0273\ sec^{-1}$	$L_{\delta r}$	$2.54\ sec^{-2}$		
Xu	$.00701\ sec^{-1}$	$N\beta$	$6.563\ sec^{-2}$		
		Np	$-.0997\ sec^{-1}$		
		Nr	$-.343\ sec^{-1}$		
		$N_{\delta a}$	$.707\ sec^{-2}$		
		$N_{\delta r}$	$-3.902\ sec^{-2}$		

The longitudinal equations of motion have the characteristic equation

$$s^4 + 1.488s^3 + 9.091s^2 + .05997s + .03284 = 0 \tag{3.1}$$

Factoring this equation results in a short-period frequency of 3.01 rad/sec with a damping ratio of 0.246. The phugoid frequency is 0.0190 rad/sec with a damping ratio of 0.158.

The lateral equations of motion have the characteristic equation

$$s^4 + 2.108s^3 + 7.458s^2 + 12.86s + .1153 = 0 \tag{3.2}$$

This produces a Dutch roll frequency of 2.63 rad/sec with a damping term of 0.0519. The roll subsidence has a time constant of 0.548 sec. The spiral convergence time constant is stable with a value of 111. sec.[16]

3.3 Effect of Longitudinal Measurement Errors

The reference maneuver used to identify the F-4's longitudinal stability and control derivative is shown in Fig. 4. The measurement data sequence consisted of 300 points taken every 0.05 sec for a 15 sec. time span. The elevator deflection consisted of a doublet of $\pm 2.50^\circ$ followed by step inputs of -0.5° and 0.5°. Figure 4 shows the resulting trajectories for pitch angle, pitch rate, angle-of-attack, and longitudinal speed perturbations about the reference flight path. This sequence was selected because it provides adequate information for the identification process.

3.3.1 Basic Instrumentation Error Effects

In studying the effect of instrument errors, two different identification cases were used. In the first, it was assumed that only the stability and control derivatives were identified, so that all bias errors affected the total estimation uncertainty. In the second, it was assumed that state initial conditions and instrument biases were estimated so that their contributions were essentially eliminated. In both cases, initial conditions were not used as error sources.

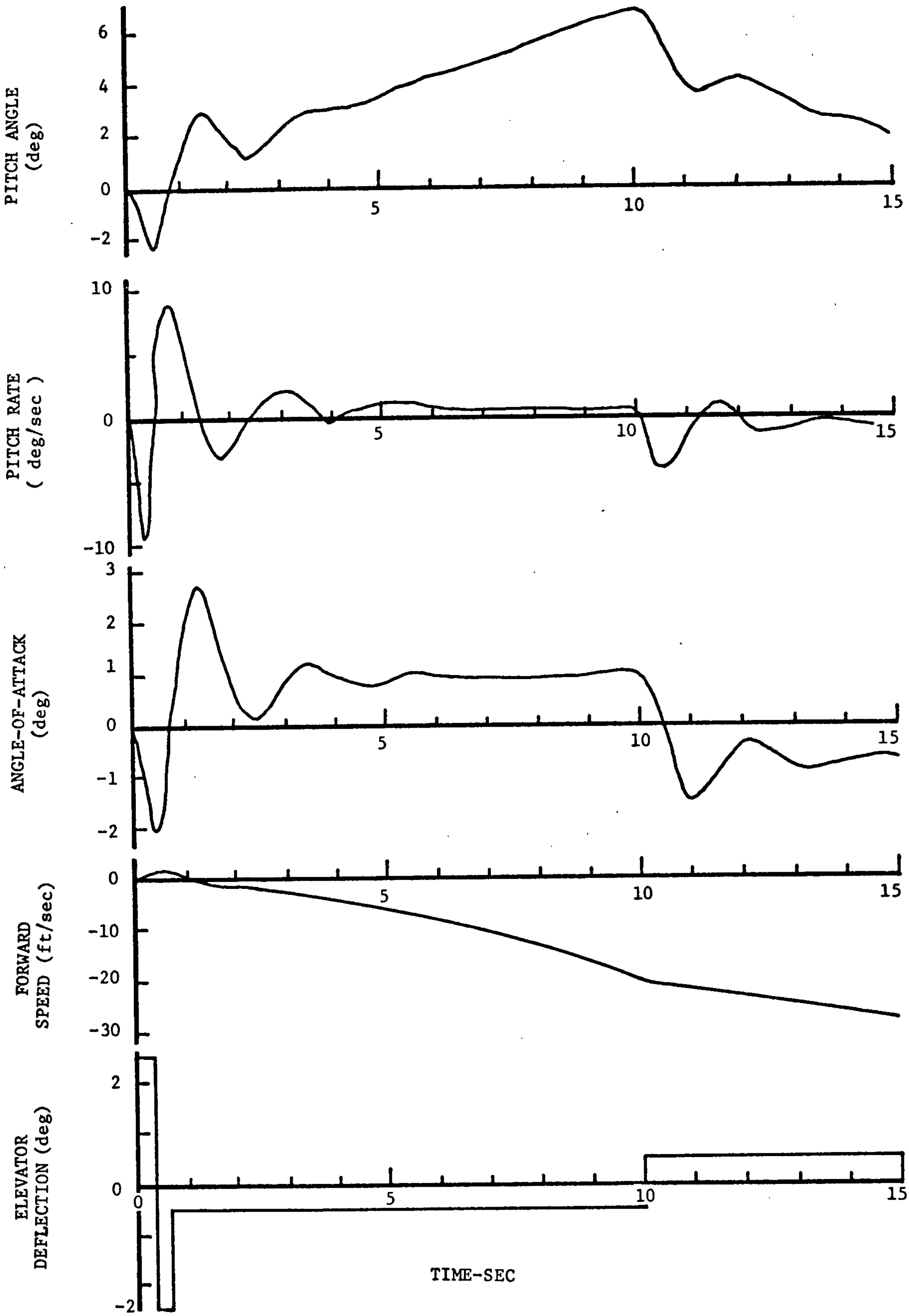


FIGURE 4 LONGITUDINAL REFERENCE TRAJECTORY AND ELEVATOR DEFLECTION INPUT.

For the full longitudinal mode, the system equations are so structured that the longitudinal speed perturbation's initial value appears to have almost the same effect on the output equations as does a bias in the pitot tube or air speed indicator. In other words, both of these parameters are not simultaneously observable from a data sequence over a short time span. Thus, for the second identification problem described above, the pitot tube bias is not identified, and it enters an error source. Most of this error is identified as a forward speed initial condition.

Table 3 presents the results of using the ensemble analysis program to compute the standard deviations of the longitudinal parameters as identified from the trajectory in Fig. 4. These results are for the baseline, minimum, and maximum values of the random error sources listed in Table 1. The resulting parameter deviations are those due to white noise only and those due to the sum of white noise and the rest of the instrumentation errors assumed present. The important quantities that can be obtained from this table are the ratio of parameter deviation to the nominal parameter value, the increase in the deviation size due to error sources which aren't noise, and the effect that estimating biases has on the total deviations.

From Table 3, the following conclusions can be made:

1. Addition of non-noise error sources has a substantial effect on the standard deviation of the parameter estimate accuracy. As seen in Table 3, the errors in accuracy of parameters Mu, Zu, Xw, and Xu are increased by over an order of magnitude by the non-modeled instrument errors. This is true for the minimum, baseline , and maximum error values. For example, the deviation of Mu goes from 8.7% to 119.6% (.00257 to .0353) of the parameter value for the baseline error set without biases being estimated. This growth in the standard deviations is illustrated more distinctly in the bar graph in Fig. 5. It must be pointed out that the largest errors are in the parameters associated with the phugoid mode. This is

Table 3

STANDARD DEVIATIONS OF PARAMETER ESTIMATES DUE TO INSTRUMENT ERRORS

LONGITUDINAL EQUATIONS OF MOTION

Parameter	Nominal** Value	Baseline Errors				Minimum Errors				Maximum Errors			
		Biases Not Estimated		Biases* Estimated		Biases Not Estimated		Biases* Estimated		Biases Not Estimated		Biases* Estimated	
		Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors
Mq	$-.719\ s^{-1}$	.182-2 #	.704-2	.189-2	.683-2	.420-3	.271-2	.431-3	.265-2	.623-2	.162-1	.649-2	.128-1
Mw	$-.591 d/f{\cdot}s$	.359-3	.130-2	.455-3	.495-3	.677-4	.125-3	.789-4	.105-3	.110-2	.187-2	.132-2	.161-2
Mu	$-.0295 d/f{\cdot}s$	.257-2	.353-1	.442-2	.588-2	.294-3	.409-2	.522-3	.636-3	.298-2	.413-1	.534-2	.627-2
$M\delta e$	$-16.2\ s^{-2}$	.106-1	.788-1	.117-1	.778-1	.166-2	.832-2	.183-2	.783-2	.306-1	.118	.346-1	.810-1
Zw	$-.762\ s^{-1}$	.914-3	.790-2	.103-2	.717-2	.322-3	.285-2	.339-3	.265-2	.372-2	.184-1	.407-2	.120-1
Zu	$-.0617\ s^{-1}$	.345-2	.484-1	.617-2	.753-2	.459-3	.633-2	.800-3	.109-2	.478-2	.669-1	.852-2	.971-2
$Z\delta e$	$-1.24 f/d{\cdot}s^2$	.167-1	.150	.183-1	.144	.775-2	.734-1	.795-2	.715-1	.800-1	.365	.843-1	.297
Xw	$.0273\ s^{-1}$	.494-3	.675-2	.769-3	.852-2	.605-4	.222-2	.777-4	.272-2	.588-3	.121-1	.779-3	.140-1
Xu	$.00701\ s^{-1}$	.478-3	.596-2	.116-2	.376-2	.596-4	.885-3	.121-3	.180-2	.574-3	.818-2	.119-2	.727-2

* Biases of six instruments estimated. Pitot tube bias is not directly estimated.

** Dimensions are: s-sec; d-deg; f-ft.

Deviations are in the form .182-2 which means $.182 \times 10^{-2}$

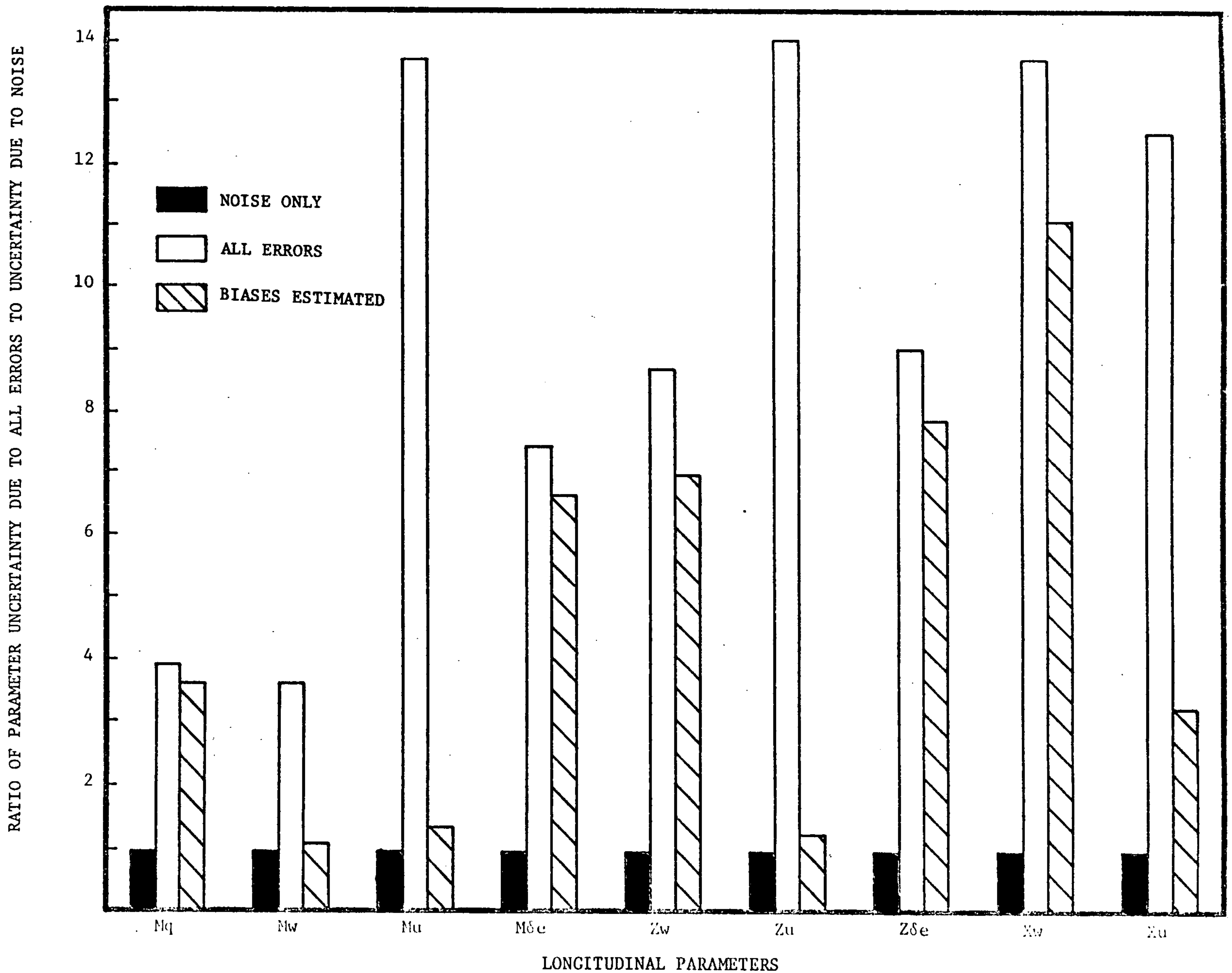


FIGURE 5 COMPARISON OF STANDARD DEVIATIONS OF LONGITUDINAL MOTION OF PARAMETER ESTIMATES FOR BASELINE INSTRUMENT ERRORS.

because the phugoid period is 330 sec, so the 15 sec data span doesn't have as much information content to obtain better accuracies for the parameters which govern the phugoid motion.

2. The estimation of biases increases the parameter deviation due to noise only, but generally reduces the total deviation of each parameter. For the baseline case, the deviations of Mw, Mu, and Zu are reduced more than 60%. However, by estimating biases, the deviation of Xw increases 26%, in the baseline case. This is because the sensitivity of Xw to the misalignment in the longitudinal accelerometer increases when biases are estimated, and for Xw, this misalignment is the dominant error source. This points out that it might be better to structure the identification scheme so that other errors, such as the accelerometer misalignments, are also estimated, in addition to the biases.

3. The general effects of error sources other than noise, and the effects of estimating instrument biases are the same for the minimum, baseline, and maximum error values. Thus, the trends exhibited by the baseline error magnitude study can be used as general results.

From Eq.(2.75), the effect of any small random instrument error e on any parameter's variance can be written as

$$E\{\delta p^2\}_{total} = E\{\delta p^2\}_{noise} + E\{\delta p^2\}_{\substack{other\\errors}} + \left(\frac{\partial(\delta p)}{\partial e}\right)^2 E\{e^2\} \qquad (3.3)$$

Thus, to provide high quality parameter estimates, it is necessary to keep the errors small or to keep the sensitivity of a parameter's deviation $\frac{\partial(\delta p)}{\partial e}$ to an error source small.

The sensitivities of the longitudinal parameter estimates to random and mean instrument errors for the baseline case when biases aren't estimated are presented in Table 4. Corresponding sensitivities of these parameters for the

Table 4

LONGITUDINAL PARAMETER SENSITIVITY TO INSTRUMENT ERRORS
WHEN NO BIASES ARE ESTIMATED

Parameter	Random Biases						
	b_θ	b_q	b_α	b_u	b_{nx}	b_{nz}	$b_{\dot{q}}$
Mq	.249-02	.295-02	.622-02	.258-05	.559-02	.674-01	-.141-02
Mw	-.321-02	-.171-02	-.103-01	-.305-04	-.294-01	-.852-01	-.886-04
Zw	-.149-01	-.336-02	-.121-02	.339-03	-.151-00	.299-00	.277-03
Mu	-.938-01	-.334-01	-.298-00	.899-03	-.966-00	-.202+01	-.787-03
Zu	-.108+00	-.359-01	-.395-00	.103-03	-.129+01	-.415+01	.150-03
Xu	-.413-03	.229-03	.828-02	-.250-02	-.967-00	.183-00	-.228-04
Xw	.416-02	.131-02	.368-02	.375-02	.125+00	-.687-01	-.955-05
Mδe	-.653-01	.529-02	-.911-01	.410-02	-.775-00	.758-00	-.371-02
Zδe	.138-00	.166-01	-.189-00	-.564-02	.131+01	-.628+01	-.618-02

Parameter	Random Scale Factors						
	e_θ	e_q	e_α	e_u	e_{nx}	e_{nz}	$e_{\dot{q}}$
Mq	.878-02	.190-01	-.196-01	.201-02	-.441-03	.610-00	-.620-00
Mw	-.121-01	.314-02	.812-02	-.639-03	-.223-03	-.167-01	.184-01
Zw	-.634-01	.363-01	.172-01	-.930-02	-.103-02	-.722-00	.742-00
Mu	-.382-00	-.481-02	.327-01	-.344-01	-.110-01	.509-00	-.109+00
Zu	-.445-00	-.714-02	.434-01	-.210-01	-.153-01	.546-00	-.101+00
Xu	-.229-02	-.315-03	-.165-02	.632-01	.778-03	-.539-01	-.578-02
Xw	.172-01	-.527-03	-.192-02	-.588-01	.166-01	.376-01	-.102-01
Mδe	-.313-00	-.586-00	-.232-00	-.633-01	-.109-01	.285-00	-.153+02
Zδe	.619-00	.104+00	-.795-01	.122+00	.109-01	-.224+01	.218-00

Parameter	Random Misalignments and Center-of-Gravity Position Errors			
	γ_{nx}	γ_{nz}	ε_{cgx}	ε_{cgz}
Mq	-.309-03	-.372-03	-.103-01	-.121-03
Mw	-.302-04	.605-05	.207-05	.700-04
Zw	-.124-03	.432-03	.103-01	.154-03
Mu	-.325-02	-.462-03	.809-06	.160-02
Zu	-.465-02	-.678-03	.458-03	.223-02
Xu	.327-02	.463-04	.505-05	-.505-03
Xw	.882-02	-.264-04	.252-04	-.287-02
Mδe	-.389-02	-.203-03	.229-03	.899-03
Zδe	.209-02	.756-03	.284-00	-.242-02

Parameter	Mean Errors--Accelerometer Positions and Angle-of-Attack Vane Position		
	ε_{ax}	ε_{az}	ε_{vx}
Mq	-.103-01	-.121-03	.154-04
Mw	.624-05	.700-04	-.417-05
Zw	.103-01	.154-03	-.294-05
Mu	.144-03	.160-02	-.143-03
Zu	.648-03	.223-02	-.189-03
Xu	.687-06	-.505-03	.436-05
Xw	.224-04	-.287-02	.276-05
Mδe	.675-04	.899-03	.162-03
Zδe	.284-00	-.242-02	.115-03

baseline case in which biases are estimated are presented in Table 5. By using Eq. (3.3) and the range of non-noise errors of Table 1, one can determine which error sources have the major effect on the accuracy of each parameter estimated. This was done, and the results are delineated in Table 6.

It is noted from Tables 4, 5, and 6 that adding the capability of estimating initial conditions and biases tends to restructure the values of the elements in the sensitivity matrix. As shown in Table 6, directly estimating all the instrument biases except the pitot tube bias b_u causes b_u to emerge as a dominant error in the estimation of Mw, Mu, and Zu. But, the same estimation scheme removes b_u as a major error source of Xw and Xu.

Sensitivity tables such as Tables 4 and 5 are useful in specifying the accuracy required of the instruments or other aircraft parameters which affect the accuracy of the estimated stability and control derivatives. As an example of this application, Fig. 6 illustrates the deviation of the parameter $Z\delta e$ due to the uncertainty in the longitudinal position of the aircraft center-of-gravity. For the $Z\delta e$ uncertainty to be less than 10% of the nominal value, the position of the center-of-gravity must be known to within 0.4 ft.

3.3.2 Effect of Changed Input and Data Span

There are several other effects which must be considered in drawing general conclusions of the importance of instrumentation errors on flight identification. Some of these include:

1. The type and configuration of the aircraft being studied
2. The control input sequence used to excite the aircraft
3. The sampling rate and time span of the data collected
4. The types of instruments available

Doubling the amplitude of elevator deflection cuts the effect of bias errors in half. All other error effects are unchanged by the increased amplitude. This

Table 5

LONGITUDINAL PARAMETER SENSITIVITY TO INSTRUMENT ERRORS WHEN INITIAL CONDITIONS AND BIASES EXCEPT b_u ARE ESTIMATED

Random Scale Factors							
Parameter	e_θ	e_q	e_α	e_u	e_{nx}	e_{nz}	$e_{\dot{q}}$
Mq	.486-02	-.107-02	-.254-01	-.248-02	-.113-02	.585-00	-.560-00
Mw	-.469-03	.441-02	.229-01	.195-02	.165-03	-.290-02	-.261-01
Zw	-.198-01	.456-01	.229-01	.220-02	.734-03	-.712-00	.661-00
Mu	-.134-00	-.454-01	.385-00	.480-01	.211-02	.260-00	-.515-00
Zu	-.142-00	-.811-01	.529-00	.899-01	.143-02	.171-00	-.569-00
Xu	.124-02	.326-02	-.184-01	.933-02	-.871-02	-.101-01	.234-01
Xw	.563-03	-.187-02	-.691-03	-.171-02	.266-01	.223-02	-.252-01
Mδe	-.204-00	-.510-00	-.179-00	.228-02	.135-02	.486-01	-.154+02
Zδe	.186-00	.741-01	.787-01	.259-01	-.440-02	-.239+01	.783-00

Random Bias, Misalignments, and Center-of-Gravity Position Errors					
Parameter	b_u	γ_{nx}	γ_{nz}	ε_{cgx}	ε_{cgz}
Mq	-.137-03	-.545-03	-.352-03	-.103-01	.172-03
Mw	.557-04	.891-04	-.296-05	.109-04	.646-04
Zw	.249-03	.453-03	.423-03	.103-01	.503-04
Mu	.137-02	.120-02	-.281-03	-.212-04	-.409-03
Zu	.129-02	.783-03	-.384-03	.438-03	-.387-03
Xu	-.607-04	-.715-03	.125-04	.150-04	.710-02
Xw	-.132-04	.127-01	-.516-07	.190-04	-.739-02
Mδe	.784-03	.779-03	-.749-04	.613-04	-.164-02
Zδe	-.196-02	-.308-02	.893-03	.284-00	-.242-02

Mean Errors--Accelerometer Positions and Angle-of-Attack Vane Position			
Parameter	ε_{ax}	ε_{az}	ε_{vx}
Mq	-.103-01	.172-03	.172-04
Mw	.354-04	.646-04	-.245-04
Zw	.103-01	.503-04	-.146-04
Mu	.502-03	-.409-03	-.524-03
Zu	.116-02	-.387-03	-.722-03
Xu	-.971-05	.710-02	.247-04
Xw	.193-04	-.739-02	-.293-06
Mδe	-.130-03	-.164-02	.191-03
Zδe	.284-00	-.242-02	.506-04

Table 6

DOMINANT RANDOM ERROR SOURCES FOR LONGITUDINAL PARAMETER IDENTIFICATION

Parameter	Biases Not Estimated	All Biases but bu Estimated
Mq	e_{nz}, $e_{\dot{q}}$, ε_{cgx}	e_{nz}, $e_{\dot{q}}$, ε_{cgx}
Mw	b_{α}	b_u, e_{α}, $e_{\dot{q}}$
Mu	$b_{\dot{\alpha}}$	noise, b_u
Mδe	$e_{\dot{q}}$	$e_{\dot{q}}$
Zw	b_{θ}, ε_{cgx}	e_{nz}, ε_{cgx}
Zu	b_{θ}, b_{α}	noise, b_u
Zδe	ε_{xcg}	ε_{xcg}
Xw	b_u, γ_{nx}	γ_{nx}
Xu	b_u	ε_{cgz}

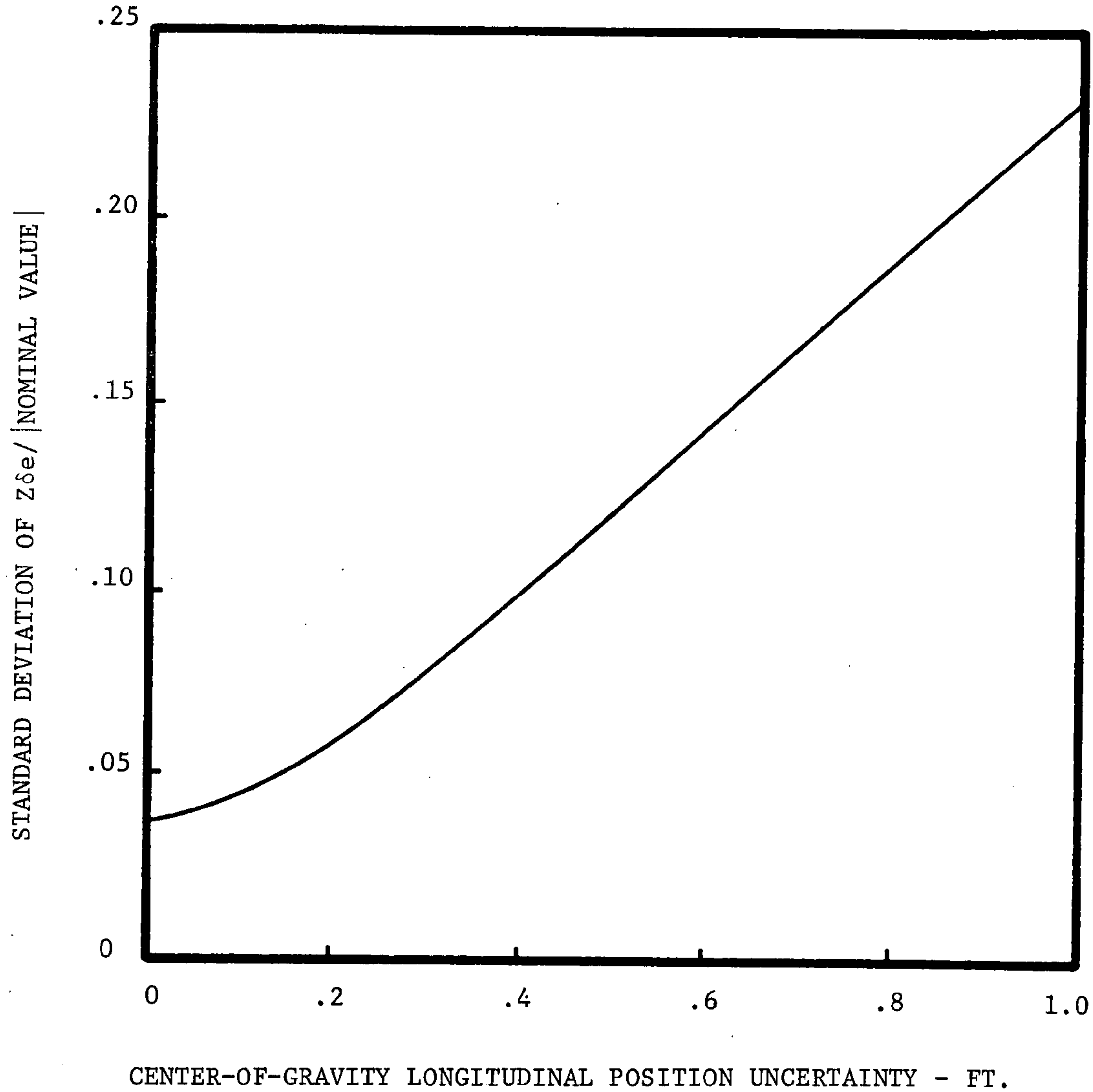


FIGURE 6 EFFECT OF CENTER-OF-GRAVITY POSITION UNCERTAINTY ON THE ESTIMATION ACCURACY OF THE PARAMETER Zδe.

can be directly obtained from Eq. (2.59).

Another effect of input changes was found by doubling the pulse length of the elevator deflections shown in Fig. 4 but halving the pulse amplitude. The data sequence was doubled to 30 sec with samples taken 0.10 sec. The resulting input had the same number of sample points and same area under the input deflection curve. The results of the input time history as compared to the baseline case are illustrated in Table 7. When biases are not estimated, doubling the time span increased the noise-only and total errors for 4 parameters; it decreases the noise-only and total errors for three parameters; and it increases the noise effect but reduces the total errors for two of the parameters. Notice that phugoid mode parameter accuracy is generally improved by increasing the time span, and the accuracy of the short period mode parameters is generally reduced.

As shown in Table 7, when biases are estimated, doubling the time span and halving the input amplitude decreases the accuracy of all parameter estimates. For this case, the combined effect of the reduced signal-to-noise ratio and the sensitivity changes due to the addition of more parameters being estimated resulted in no improved accuracies. Table 7 points out that any definite conclusions on instrument accuracy effects are dependent upon the maneuvers flown and the parameters being estimated.

3.3.3 Effect of Fewer Instruments

The above results were obtained assuming that the aircraft has seven instruments for obtaining longitudinal information. It was assumed that these instruments, discussed in Section 2.3, had errors as modeled in Eqs.(2.60)-(2.61). Parameter identification can be conducted with fewer instruments, and so it is desirable to know what reducing the number of instruments has on the overall parameter accuracy. It is known that reduced instrumentation increases the parameter uncertainty due to white noise only. But it is conceivable that removing an instrument also removes a major unmodeled and unestimated error source.

To test this idea, a set of runs was made in which different instruments

Table 7

EFFECT OF DOUBLING THE TIME SPAN OF THE MEASUREMENT SEQUENCE WITH CONSTANT INPUT ENERGY

Parameter	Nominal Value	Reference Trajectory				Doubled Time Span			
		Biases Not Estimated		Biases Estimated		Biases Not Estimated		Biases Estimated	
		Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors
Mq	$-.719\ s^{-1}$	.182-2	.704-2	.189-2	.683-2	.282-2	.787-2	.333-2	.724-2
Mw	$-.591 d/f \cdot s$	.359-3	.130-2	.455-3	.495-3	.432-3	.784-3	.123-2	.128-2
Mu	$-.0295 d/f \cdot s$	.257-2	.353-1	.442-2	.588-2	.158-2	.214-1	.255-1	.270-1
Mδe	$-16.2 s^{-2}$	.106-1	.788-1	.117-1	.778-1	.221-1	.837-1	.307-1	.798-1
Zw	$-.762 s^{-1}$	.914-3	.790-2	.103-2	.717-2	.142-2	.784-2	.211-2	.718-2
Zu	$-.0617 s^{-1}$	.345-2	.484-1	.617-2	.753-2	.213-2	.295-1	.336-1	.353-1
Zδe	$-1.24 f/d \cdot s^{2}$	.167-1	.150	.183-1	.144	.342-1	.191	.482-1	.153
Xw	$.0273 s^{-1}$	.494-3	.675-2	.769-3	.852-2	.698-3	.739-2	.128-2	.938-2
Xu	$.00701\ s^{-1}$	.478-3	.596-2	.116-2	.376-2	.238-3	.249-2	.581-2	.597-2

were individually and collectively removed from use. The resulting deviation of the parameters is shown in Table 8 for cases with bias estimated and not estimated. The comments which can be made from Table 8 are:

1. Removal of the angle-of-attack vane approximately doubles the deviation of Mu and Zu for both cases where biases are and are not estimated. This is substantial in terms of the nominal values of these parameters. The percentage change in the deviation of Mw is large, but the values of the deviations are small with respect to the nominal value (.591).

2. Removal of the pitch angular accelerometer alone does not substantially affect any of the parameter deviations. However, flight test personnel have commented that they only correct for center-of-gravity position errors in other inertial instruments when measurements of the angular accelerations are directly available. So, in a practical sense, no pitch accelerometer also means additional errors elsewhere.

3. Removal of the pitot tube alone when biases are not estimated has a large effect on the total deviations of Xw and Xu. The removal increases the deviation percentages from 24.7% and 85.1% to 32.8% and 125.8% for Xw and Xu, respectively. Removal of the pitot tube when biases are estimated has the largest effect on Mu, Zu, and Xu. However, the deviations of these parameters with biases estimated are all smaller than when the pitot tube data are used without estimating the biases.

4. Removal of the angle-of-attack vane, the pitot tube, and the pitch angular accelerometer simultaneously makes it highly desirable to estimate biases. For this situation, the parameters Mu, Zu, and Xu again were most highly affected by the removal of the three instruments. The deviations of all parameters but Zw and $Z\delta e$ were increased by the instrument removal.

The data shown in Tables 3, 4, 5, and 8 are quite useful in specifying what the best set of instruments are and what the corresponding instrument accuracies must be to obtain parameter accuracies within some acceptable level. Figure 7 illustrates the effect of instrument bias variations on the accuracy of Mu for

Table 8

EFFECT OF INSTRUMENT REMOVAL ON PARAMETER DEVIATIONS FOR BASELINE SET OF ERRORS AND LONGITUDINAL EQUATIONS OF MOTION

Parameter	Nominal Value**	All Instruments		No Angle-of-Attack Vane (α)		No Pitch Angle Accelerometer ($\dot{q}$)		No Pitot Tube ($\dot{u}$)		No α, $\dot{q}$, or u	
		Biases*	bu*	Biases	bu	Biases	bu	Biases	No Biases	Biases	No Biases
Mq	$-.719s^{-1}$	.704-2	.683-2	.766-2	.679-2	.120-1	.811-2	.705-2	.676-2	.108-1	.825-2
Mw	$-.591d/f \cdot s$	.130-2	.495-3	.291-2	.690-3	.124-2	.669-3	.130-2	.869-3	.290-2	.171-2
Mu	$-.0295d/f \cdot s$	.353-1	.588-2	.679-1	.138-1	.361-1	.589-2	.354-1	.182-1	.716-1	.400-1
Mδe	$-16.2s^{-2}$	.788-1	.778-1	.792-1	.785-1	.181	.837-1	.788-1	.784-1	.151	.953-1
Zw	$-.762s^{-1}$	.790-2	.717-2	.815-2	.717-2	.926-2	.738-2	.792-2	.712-2	.780-2	.755-2
Zu	$-.0617s^{-1}$	.484-1	.753-2	.921-1	.181-1	.490-1	.763-2	.484-1	.318-1	.960-1	.527-1
Zδe	$-1.24f/d \cdot s^2$	.150	.144	.150	.143	.148	.144	.150	.144	.148	.144
Xw	$.0273s^{-1}$	.675-2	.852-2	.681-2	.852-2	.674-2	.852-2	.896-2	.878-2	.897-2	.878-2
Xu	$.00701s^{-1}$	.596-2	.376-2	.608-2	.384-2	.596-2	.376-2	.882-2	.505-2	.937-2	.658-2

*"Biases" means that all biases are error sources; "bu" means all biases except b_u are directly estimated.

**Dimensions are: s = second; d = degrees; f = feet.

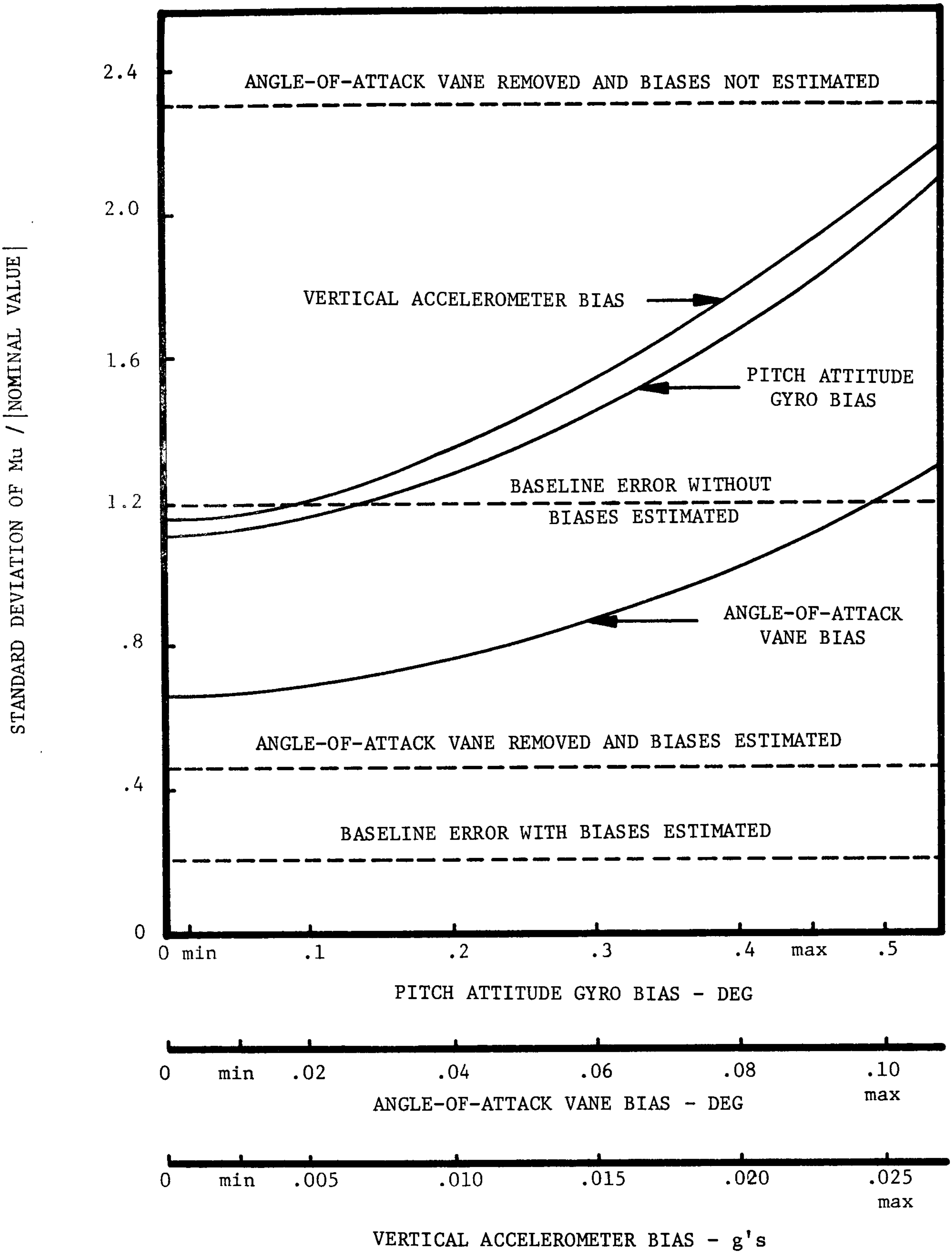


FIGURE 7 EFFECT OF VARIOUS INSTRUMENT BIASES ON THE ESTIMATION ACCURACY OF THE PARAMETER Mu. ALSO SHOWN IS THE EFFECT OF NOT USING THE ANGLE-OF-ATTACK VANE DATA.

the baseline set of errors. The standard deviation of the vertical accelerometer, pitch attitude gyro, and angle-of-attack vane biases are indicated by the values where their curves cross the reference error line. Reduction of the angle-of-attack vane bias can reduce the ratio of the standard deviation of Mu to its nominal value from about 1.2 to 0.65. On the other hand, an increase in either the vertical accelerometer or pitch attitude gyro bias from the baseline values can cause large increases in the error in Mu. With baseline error values, the removal of the angle-of-attack vane causes the standard deviation of Mu to almost double (ratio goes from 1.2 to 2.3). If biases are estimated, the standard deviation of Mu is reduced about 85%. Removing the use of the α-vane when biases are estimated still is considerably better than the reference baseline case.

For some stability and control derivatives, it is possible to improve their accuracy by not using an instrument's data. Figure 8 illustrates such a case. Here, the ratio of the standard deviation of Xw to its nominal value is shown as a function of the standard deviation of the pitot tube bias. For the baseline reference case, the pitot tube's deviation is one (1) ft/sec and it is better to use the pitot tube data. If the maximum value of the pitot tube bias is expected, however, (Refer to Table 1), it is better not to use this data in estimating Xw. The cross-over deviation of b_u beyond which the pitot tube shouldn't be used is about 1.8 ft/sec.

3.3.4 Effect of Changing the Algorithm Weighting Matrix

Referring back to Table 8 again, to the case where the three instruments are removed and the biases are estimated (last column), it can be seen that the deviations of Mq, Mw, Mδe, and Zw are acceptably small. The main contributions to errors in Mu and Zu are due to noise. The chief error sources affecting Zδe, Xw, and Xu are the center-of-gravity uncertainty and the misalignment of the forward accelerometer. A flight test requirement might be to improve the accuracies of the two gyros and two linear accelerometers so these error sources are acceptably reduced. This may also include better calibration of the center-of-gravity position.

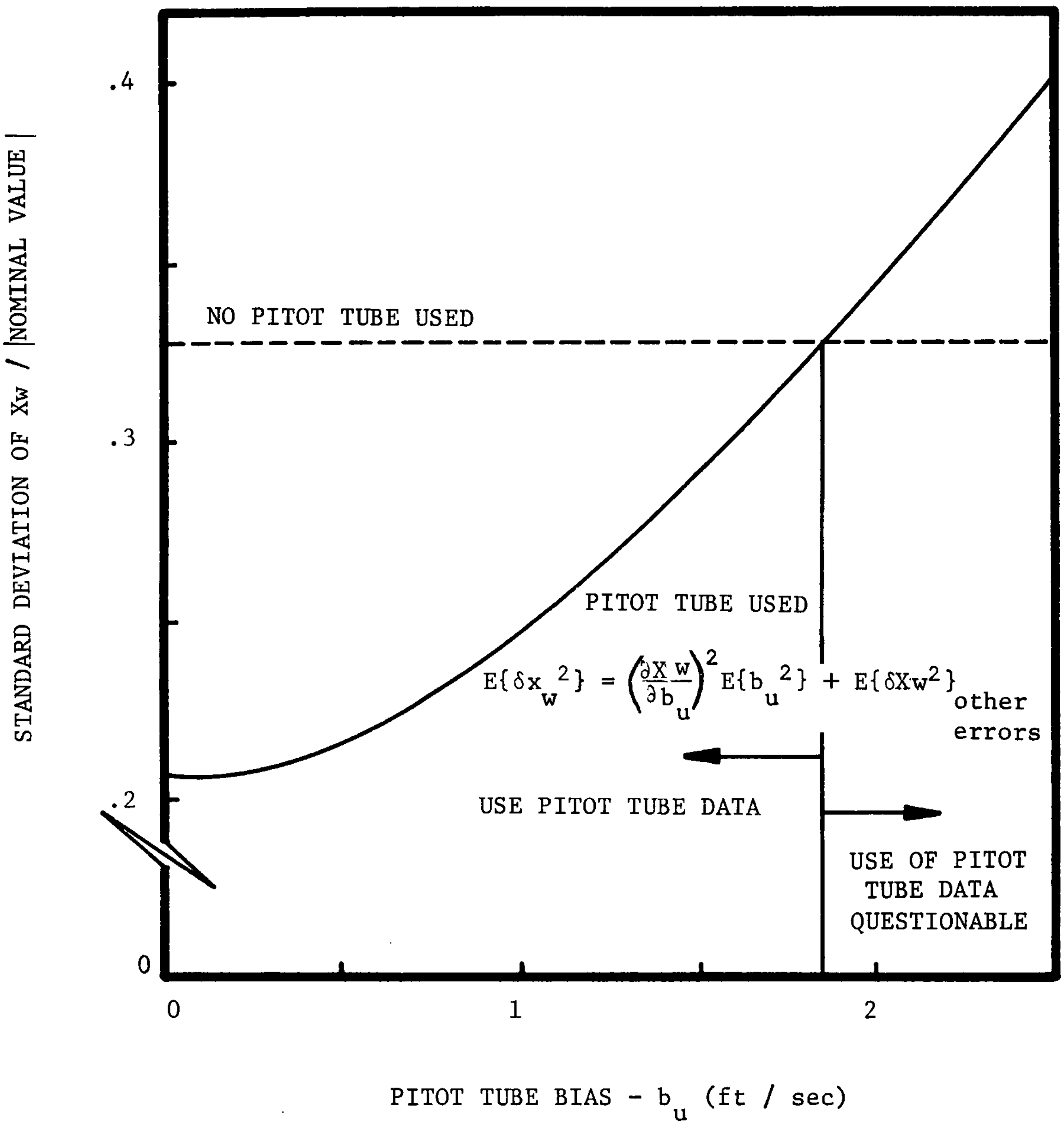


FIGURE 8 EFFECT OF PITOT TUBE BIAS ON THE ESTIMATION ACCURACY OF THE PARAMETER Xw

More computer runs were made to determine the effect of reducing instrument error values on the overall parameter accuracy when only four instruments are used. The results are presented in Table 9. Concentration was placed on the five parameters (Mu, Zu, $Z\delta e$, Xw, and Xu) with sizeable deviations. Case (a) is the reference case which is the same as the last column of Table 8. In Table 9, the parameter deviations due to noise only, to other errors, and their root-sum-square total is shown.

In Case (b) , the standard deviations of the white noise errors were halved. (The other error statistics were held constant.) This reduced the diagonal elements of the matrix R used in the cost function J (see Eq.(2.6)) to one-fourth their reference value. This caused the anticipated result - the errors in the five parameters due to noise were cut in half, and this significantly affected the deviations of Mu and Zu.

In Case (c), the standard deviations of the four noise sources and other errors were set to the minimum values shown in Table 1. As was expected, the deviations of the parameters due to noise only were all reduced. But surprise-ingly, the error in Xu due to other errors increases. This is due to the fact that in reducing the noise values, the ratios of the elements of the weighting matrix R are changed. This causes the elements of the sensitivity matrix of parameter deviations to error sources to change. In the case of Xu, the sensitivity of Xu to the error in the vertical position of the center-of-gravity increased from 0.0113/ft·sec to 0.0318/ft·sec. Thus, the deviation of Xu also grew.

The best set of noise parameters (and cost function weighting terms) was found to be setting the longitudinal accelerometer noise to the baseline value and the other three terms to the minimum value. Results of this situation are shown as Case (d).

The above four cases illustrate that the effect of instrumentation error sources on stability and control derivative estimate accuracies is also highly

Table 9

EFFECT OF CHANGED NOISE VARIANCES ON TOTAL PARAMETER ESTIMATE DEVIATIONS

Parameter	Nominal Value	Reference Case (a)*			Noise Parameters Halved (b)*			Minimum Noise With Reduced Errors (c)*			Special Case (d)*		
		Parameter Deviations			Parameter Deviations			Parameter Deviations			Parameter Deviations		
		Noise Only	Other Errors	Total	Noise Only	Other Errors	Total	Noise Only	Other Errors	Total	Noise Only	Other Errors	Total
Mu	$-.0295 d/f\cdot s$	.369-1	.153-1	.401-1	.185-1	.153-1	.240-1	.528-2	.140-1	.149-1	.529-2	.136-1	.146-1
Zu	$-.0617 s^{-1}$	.483-1	.212-1	.527-1	.242-1	.212-1	.321-1	.744-2	.235-1	.246-1	.811-2	.146-1	.167-1
$Z\delta e$	$-1.24 f/d\cdot s^{2}$	.217-1	.142	.144	.109-1	.142	.142	.928-2	.720-1	.726-1	.953-2	.717-1	.723-1
Xw	$.0273 s^{-1}$	.783-3	.875-2	.878-2	.392-3	.875-2	.876-2	.793-4	.278-2	.278-2	.776-3	.273-2	.284-2
Xu	$.00701 s^{-1}$	.326-2	.571-2	.658-2	.163-2	.571-2	.594-2	.679-3	.804-2	.807-2	.164-2	.358-2	.393-2

*(a) $w_\theta = .15°$; $w_q = .10°/sec$; $w_{nx} = .005$ g's; $w_{nz} = .005$ g's; $\gamma_{nx,z} = .6°$; $\varepsilon_{cgx,z} = .5$ ft

(b) $w_\theta = .075°$; $w_q = .05°/sec$; $w_{nx} = .0025$ g's; $w_{nz} = .0025$ g's; $\gamma_{nx,z} = .6°$; $\varepsilon_{cgx,z} = .5$ ft

(c) $w_\theta = .015°$; $w_q = .015°/sec$; $w_{nx} = .0005$ g's; $w_{nz} = .0025$ g's; $\gamma_{nx,z} = .15°$; $\varepsilon_{cgx,z} = .25$ ft

(d) $w_\theta = .015°$; $w_q = .015°/sec$; $w_{nx} = .005$ g's; $w_{nz} = .0025$ g's; $\gamma_{nx,z} = .15°$; $\varepsilon_{cgx,z} = .25$ ft.

dependent upon the weighting matrix used by the Newton-Raphson identification algorithm. In fact, an important problem which should be solved is how to choose the weighting matrix for the identification process so that the total parameter deviations due to all types of measurement errors are minimized.

3.4 Effect of Lateral Measurement Errors

Similar program runs to those discussed in Section 3.3 were made for the lateral equations of motion of the F-4 aircraft. Again, seven instruments were modeled which are described by Eqs.(2.60)-(2.61). The range of instrument and calibration errors investigated are similar to those of the longitudinal instruments, and they are presented in Table 10.

The aileron and rudder deflections used to excite the lateral motion and the resulting aircraft trajectory are depicted in Fig. 9. Again, a 15 sec time span was simulated, and 300 data points accuring every 0.05 sec. were processed. The trajectory shown in Fig. 9 was used as the reference case for the lateral study.

A study was made of the identification process applied to the reference trajectory using the baseline, minimum, and maximum sets of instrument errors listed in Table 10. The results are presented in Table 11. The conclusions which can be made from the lateral study with all seven instruments are:

1. The addition of non-noise error sources to the output measurements causes a substantial effect on the accuracy of the estimated parameters. The standard deviations of $Y\beta$, $Y\delta a$, $Y\delta r$, Lp, and Np increased by over an order of magnitude for the baseline case. The standard deviation of $Y\delta a$ increased from about 3% to 164% of the nominal value. These increases are depicted more clearly in Fig. 10. The trends are the same for the minimum and maximum error value cases.
2. Unlike the longitudinal equations of motion, the addition of bias estimation was more detrimental than good. Of the 13 parameters

Table 10

LATERAL INSTRUMENT ERRORS

STANDARD DEVIATIONS

Instrument	Units	Full Scale Deflection*	Random Noise			Random Biases			Random Scale Factors		
			Minimum	Base	Maximum	Minimum	Base	Maximum	Minimum	Base	Maximum
Angle-of-sideslip vane	deg	10	.005	.05	.05	.005	.05	.05	.05%	.5%	.5%
Roll rate gyro	deg/sec	30-150	.015	.10	.75	.015	.10	.75	.05%	.5%	.5%
Yaw rate gyro	deg/sec	30-60	.015	.10	.30	.015	.10	.30	.05%	.5%	.5%
Roll attitude gyro	deg	360	.09	.50	.90	.09	.50	.90	.05%	.5%	.5%
Lateral accelerometer	g's	.2	.0001	.0005	.001	.0001	.0005	.001	.05%	.5%	.5%
Roll accelerometer	deg/sec^2	30-60	.015	.10	.30	.015	.10	.30	.05%	.5%	.5%
Yaw accelerometer	deg/sec^2	30-60	.015	.10	.30	.015	.10	.30	.05%	.5%	.5%

Other Types of Instrument Errors	Minimum	Base	Maximum	Instrument Lags	Bandwidth (sec^{-1})	Standard Deviation (%)
Rate gyro misalignment (deg)	.15	.60	.90	Gyros	150	5
Rate gyro mass unbalance (deg/sec·g)	.025		.60	Linear accelerometers	600	5
Angular accelerometer misalignment (deg)	.15	.60	.90	Angular accelerometers	180	10
Angular accelerometer sensitivity to linear acceleration ($deg/sec^2 \cdot g$)	.10		.60	Tape recorder	6	5
Center-of-gravity uncertainty (ft)	.25	.5	1.0			

*When a range of deflection values is given, the lower number is associated with the minimum random errors. The larger number is associated with the maximum random errors.

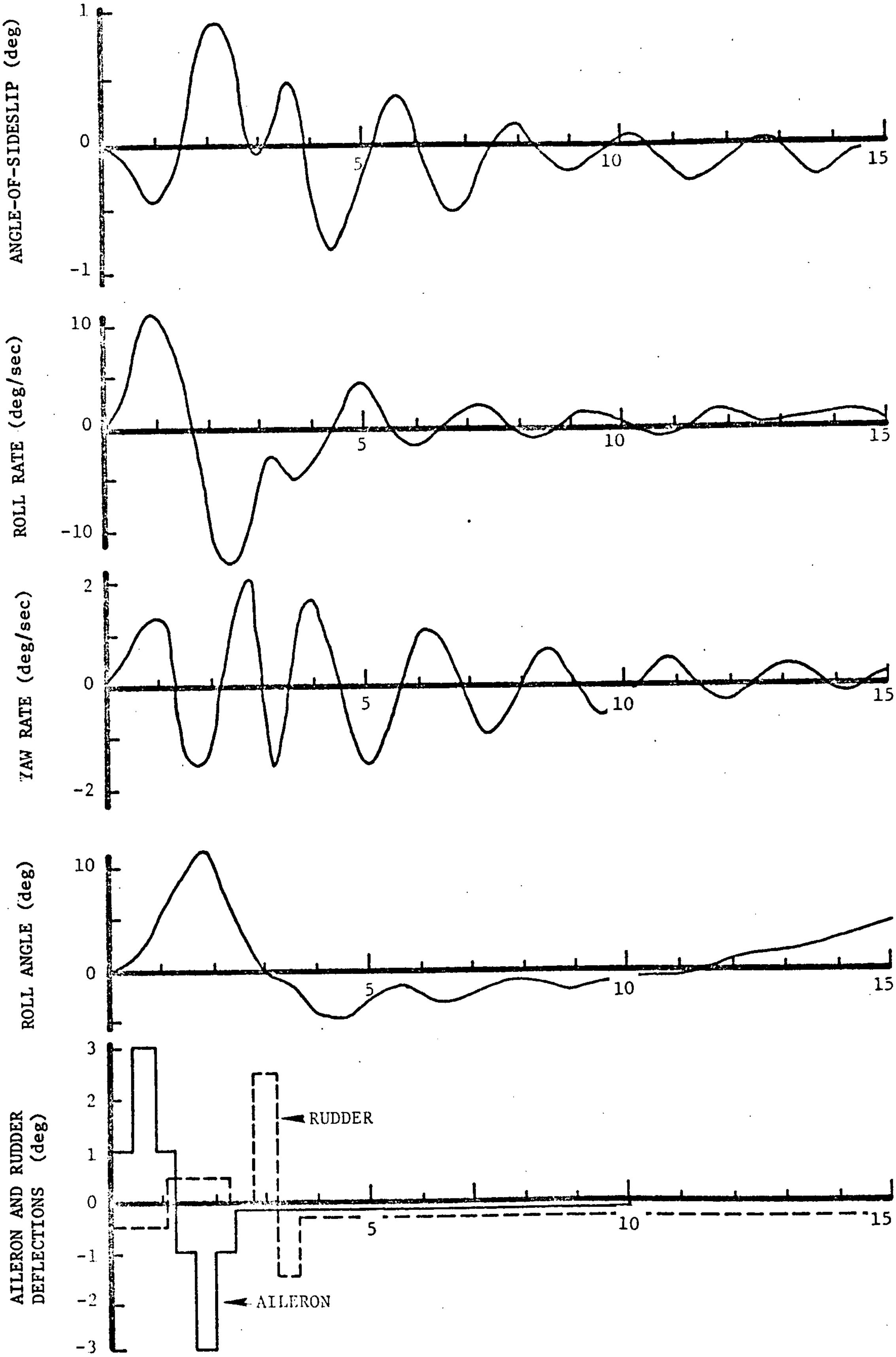


FIGURE 9 LATERAL REFERENCE TRAJECTORY AND INPUT OF AILERON AND RUDDER DEFLECTIONS

Table 11

STANDARD DEVIATIONS OF PARAMETER ESTIMATES DUE TO INSTRUMENT ERRORS
LATERAL EQUATIONS OF MOTION

Parameter	Nominal Value**	Baseline Errors				Minimum Errors				Maximum Errors			
		Biases Not Estimated		Biases* Estimated		Biases Not Estimated		Biases* Estimated		Biases Not Estimated		Biases* Estimated	
		Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors	Noise Only	Total Errors
Yβ	$-.157s^{-1}$	.347-3	.683-2	.356-3	.636-2	.581-4	.310-2	.599-4	.296-2	.820-3	.133-1	.845-3	.117-1
Yδa	$-.00338s^{-1}$	.102-3	.554-2	.136-3	.694-2	.199-4	.278-2	.246-4	.327-2	.211-3	.110-1	.297-3	.141-1
Yδr	$.0246s^{-1}$	.117-3	.295-2	.124-3	.305-2	.230-4	.150-2	.242-4	.156-2	.239-3	.574-2	.254-3	.604-2
Lβ	$-15.98s^{-2}$	.388-1	.247	.401-1	.271	.584-2	.572-1	.611-2	.127	.108	.537	.112	.536
Lp	$-1.608s^{-1}$	.298-2	.375-1	.317-2	.585-1	.464-3	.143-1	.510-3	.237-1	.923-2	.107	.945-2	.131
Lr	$.384s^{-1}$	.114-1	.389-1	.115-1	.595-1	.175-2	.132-1	.179-2	.303-1	.339-1	.106	.341-1	.110
Lδa	$10.92s^{-2}$	.130-1	.941-1	.152-1	.764-1	.201-2	.256-1	.235-2	.179-1	.386-1	.243	.431-1	.950-1
Lδr	$2.54s^{-2}$	.118-1	.862-1	.123-1	.557-1	.180-2	.217-1	.188-2	.608-1	.345-1	.217	.356-1	.144
Nβ	$6.563s^{-2}$	.341-2	.232-1	.370-2	.206-1	.553-3	.536-2	.598-3	.741-2	.872-2	.681-1	.964-2	.244-1
Np	$-.0997s^{-1}$	.364-3	.490-2	.588-3	.132-1	.602-4	.231-2	.978-4	.546-2	.961-3	.899-2	.150-2	.280-1
Nr	$-.343s^{-1}$	.143-2	.819-2	.166-2	.206-1	.234-3	.331-2	.271-3	.842-2	.371-2	.162-1	.432-2	.484-1
Nδa	$.707s^{-2}$	.144-2	.833-2	.234-2	.280-1	.232-3	.159-2	.383-3	.115-1	.379-2	.282-1	.598-2	.624-1
Nδr	$-3.902s^{-2}$	.705-2	.497-1	.713-2	.434-1	.104-2	.113-1	.105-2	.956-2	.183-1	.752-1	.187-1	.647-1

* Biases of six instruments estimated. Roll angle gyro bias is not directly estimated.

** Dimension is: s = second.

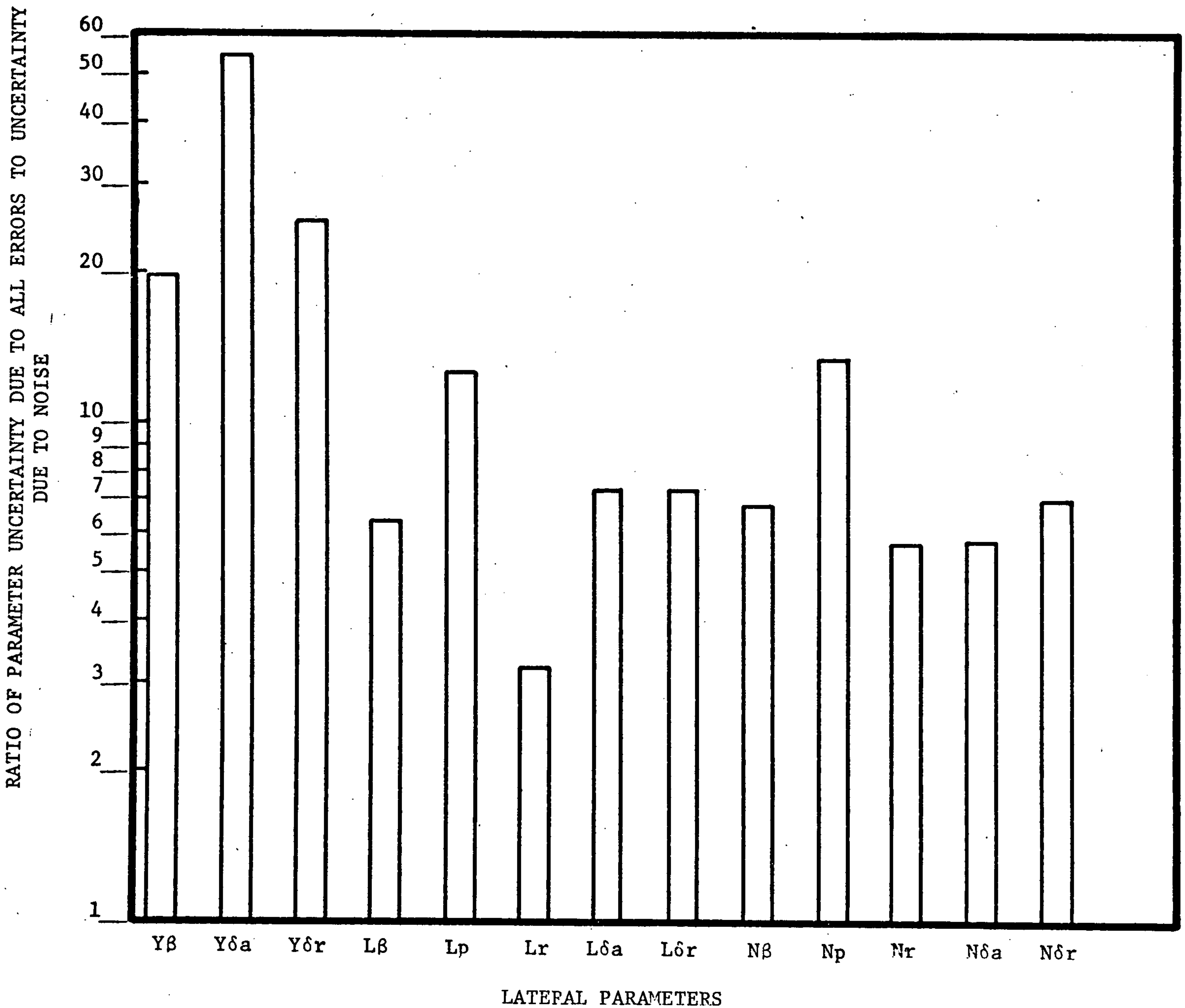


FIGURE 10 COMPARISON OF STANDARD DEVIATIONS OF LATERAL MOTION PARAMETER ESTIMATES FOR BASELINE INSTRUMENT ERRORS.

estimated, only 4, 2, and 6 total standard deviations were lowered by estimating biases for the baseline, minimum, and maximum error sets, respectively. This is because the predominant error sources are not due to biases for the lateral equations. By estimating biases, the sensitivity of the parameters to the other error sources is generally increased. Thus, the resulting estimate error increases. It is concluded that for the range of instrument errors given and the trajectory flown it is better not to estimate biases. However, the dependence on the aircraft parameters and trajectory should be taken into account before any general statement can be made concerning whether or not biases should be estimated.

Tables 12 and 13 present the aircraft lateral parameter sensitivities to the random and mean error sources which affect the accuracy of the system output measurements. As with the longitudinal sensitivities presented in Tables 4 and 5, these sensitivities are useful for determining what the primary sources of error are, which affect the estimation accuracy of the lateral parameters. The sensitivities are also useful for specifying instrumentation and calibration accuracy required to provide a given level of identification accuracy.

Table 14 presents the dominant random error sources which affect the lateral parameters' variances. These sources were determined from use of Tables 10, 12, and 13. To improve the estimation accuracy of any one given parameter requires concentrating on lowering the magnitude of the chief error sources affecting that parameter. For example, for a more accurate parameter Np, the uncertainty in the center-of-gravity position of the aircraft should be reduced.

The effect of reducing the number of instruments used to measure lateral motion was also studied. Table 15 contains the results of simulating the identification process while removing individually and collectively the angle-of-sideslip vane, the roll angle accelerometer, and the yaw angle accelerometer. Conclusions which can be made from Table 15 are:

Table 12

LATERAL PARAMETER SENSITIVITY TO INSTRUMENT ERRORS
WHEN NO BIASES ARE ESTIMATED

	Random Biases						
Parameter	b_β	b_p	b_r	b_ϕ	b_{ny}	$b_{\dot p}$	$b_{\dot r}$
Yβ	-.560-02	-.160-02	-.420-03	-.265-03	-.231+01	-.142-03	-.342-04
Lβ	-.580-00	-.959-00	-.179-00	-.143-00	-.499+01	-.306-01	-.306-01
Nβ	-.994-02	-.810-01	-.163-01	-.140-01	-.150+01	-.889-02	.201-02
Lp	.112-02	-.294-01	-.486-02	-.352-02	.275+01	.355-02	-.504-03
Np	.204-02	-.835-02	-.189-02	-.147-02	-.111+01	-.599-03	-.215-03
Lr	-.396-03	-.475-02	.128-01	.153-01	.141+02	.163-01	-.289-02
Nr	.698-02	-.269-01	-.525-02	-.322-02	-.136+01	-.470-02	.159-04
Yδa	-.166-03	-.838-03	-.241-03	-.148-03	-.910-00	-.779-04	-.445-04
Lδa	-.257-01	-.255-00	-.633-01	-.532-01	-.240+02	-.295-01	-.106-01
Nδa	-.790-02	.923-02	.182-02	.916-03	.168+01	.306-04	.192-03
Yδr	-.295-03	-.509-03	-.183-03	-.979-04	-.120+01	.201-05	-.720-04
Lδr	-.522-01	-.394-00	-.769-01	-.611-01	-.438+01	-.300-01	-.205-01
Nδr	.137-00	-.421-01	-.106-01	-.622-02	-.263+01	-.812-02	-.195-02

	Random Scale Factors						
Parameter	e_β	e_p	e_r	e_ϕ	e_{ny}	$e_{\dot p}$	$e_{\dot r}$
Yβ	.720-02	.990-03	.947-02	-.258-03	-.157-00	.106-01	.129-00
Lβ	.697-00	-.240+01	.957-00	-.144-00	-.277-01	-.122+02	.131+02
Nβ	.214-01	-.779-01	.431-01	-.549-02	-.362-01	-.549-00	.604-00
Lp	-.359-02	.121+00	-.886-03	.949-02	-.241-02	-.130-00	.671-02
Np	-.213-02	.249-01	-.361-02	.434-03	-.658-02	.529-01	-.659-01
Lr	-.105-01	-.172-00	-.159-01	-.104-01	-.110+00	.570-00	-.251-00
Nr	-.398-02	-.248-01	-.514-02	-.495-02	.152-00	.340-01	-.147-00
Yδa	.168-03	-.283-03	.197-03	-.980-04	-.378-02	.326-04	.382-03
Lδa	.272-01	.176+01	.215-01	.524-02	.638-02	.908+01	.237-01
Nδa	.934-02	.726-01	.109-01	.302-03	.313-02	.410-00	.201-00
Yδr	.262-03	.327-03	.530-04	-.766-04	.246-01	-.159-03	-.423-03
Lδr	.358-01	-.864-00	.857-02	-.105+00	-.151-01	.353+01	-.461-01
Nδr	-.171-00	-.120+00	-.230-00	-.985-02	-.736-05	-.285-01	-.334+01

	Random Misalignments and Center-of-Gravity Position Errors					
Parameter	γ_p	γ_r	$\gamma_{\dot p}$	$\gamma_{\dot r}$	ϵ_{cgx}	ϵ_{cgz}
Yβ	.582-05	-.293-03	.102-03	-.249-02	-.967-02	.859-02
Lβ	-.759-02	-.396-01	-.970-01	-.283-00	.109+00	.109-01
Nβ	-.377-03	-.221-02	-.425-02	-.187-01	.319-01	.119-02
Lp	.401-03	-.130-02	.133-02	-.685-02	.738-01	.393-02
Np	.584-05	.224-03	.526-05	.179-02	-.913-02	-.224-03
Lr	-.162-02	-.214-02	-.201-01	.111-02	.603-01	.287-01
Nr	-.198-03	.282-03	-.234-02	.545-02	-975-02	-.812-02
Yδa	.129-05	.108-04	-.104-05	.541-04	.110-01	.580-03
Lδa	-.238-02	.490-02	-.103-01	.268-01	-.136-00	-.125-01
Nδa	-.233-03	-.181-03	-.136-03	-.146-02	-.154-01	-.944-03
Yδr	-.241-05	.104-04	.142-05	.116-03	.339-02	-.467-02
Lδr	.248-02	-.958-03	.652-01	.131-01	-.106+00	-.529-02
Nδr	-.358-04	.758-02	.297-03	.696-01	.347-01	.195-02

	Mean Errors--Accelerometer Positions and Angle-of-Sideslip Vane Position		
Parameter	ϵ_{ax}	ϵ_{az}	ϵ_{vx}
Yβ	-.967-02	-.859-02	.199-05
Lβ	.109+00	-.109-01	.121-03
Nβ	.318-01	-.119-02	.443-04
Lp	.738-01	-.393-02	-.826-05
Np	-.913-02	.224-03	.726-06
Lr	.604-01	-.287-01	-.850-04
Nr	-.974-02	.812-02	-.102-04
Yδa	.110-01	-.580-03	-.721-08
Lδa	-.136-00	.125-01	.318-04
Nδa	-.154-01	.944-03	.842-05
Yδr	.339-02	.467-02	.260-06
Lδr	-.106+00	.529-02	-.272-04
Nδr	.347-01	-.195-02	-.689-04

Table 13

LATERAL PARAMETER SENSITIVITY TO INSTRUMENT ERRORS WHEN INITIAL CONDITIONS AND BIASES EXCEPT b_ϕ ARE ESTIMATED

Random Scale Factors							
Parameter	e_β	e_p	e_r	e_ϕ	e_{ny}	$e_{\dot{p}}$	$e_{\dot{r}}$
Yβ	.671-02	.126-02	.959-02	-.950-04	-.158-00	.102-01	.130-00
Lβ	.626-00	-.197+01	.958-00	-.322-01	-.472-01	-.130+02	.134+02
Nβ	.285-01	-.185-01	.460-01	-.489-02	-.358-01	-.562-00	.547-00
Lp	-.674-02	.187-00	-.346-02	.177-01	-.271-02	-.237-00	.457-01
Np	.766-03	.433-01	-.294-02	-.111-02	-.607-02	.526-01	-.865-01
Lr	-.119-01	.184-02	-.224-01	-.107-01	-.106+00	.361-00	-.212-00
Nr	.159-02	.147-01	-.392-02	-.743-02	.153-00	.262-01	-.184-00
Yδa	-.352-03	-.505-02	.256-03	.266-03	-.407-02	.186-02	.371-02
Lδa	-.194-01	.132+01	.337-01	.514-01	-.195-01	.926+01	.292-00
Nδa	.492-02	.168-00	.480-02	.111-01	.414-02	.255-00	.259-00
Yδr	.299-03	.152-02	.656-04	.618-05	.245-01	-.127-02	-.551-03
Lδr	.342-01	-.764-00	.977-02	-.679-01	-.209-01	.337+01	-.227-01
Nδr	-.156-00	-.172-00	-.227-00	-.127-01	-.158-02	.514-01	-.338+01

Random Bias, Misalignments, and Center-of-Gravity Position Errors							
Parameter	b_ϕ	γ_p	γ_r	$\gamma_{\dot{p}}$	$\gamma_{\dot{r}}$	ϵ_{cgx}	ϵ_{cgz}
Yβ	-.957-05	.333-05	-.278-03	.105-03	-.239-02	-.862-02	.863-02
Lβ	-.109-01	-.979-02	-.368-01	-.935-01	-.233-00	.397-00	.287-01
Nβ	-.546-03	-.547-03	-.323-02	-.406-02	-.295-01	-.165-01	-.179-02
Lp	-.658-03	.235-03	-.103-02	.156-02	.686-04	.117+00	.669-02
Np	-.127-04	-.195-04	-.152-03	.161-04	-.210-02	-.261-01	-.127-02
Lr	-.894-03	-.188-02	-.261-02	-.199-01	.627-02	.109+00	.321-01
Nr	-.162-03	-.269-03	-.406-03	-.229-02	-.150-02	-.396-01	-.994-02
Yδa	-.171-04	.487-05	.103-03	-.709-06	.772-03	.138-01	.736-03
Lδa	-.291-02	-.233-02	.135-01	-.973-02	.859-01	.509-01	-.236-02
Nδa	-.833-04	-.401-03	-.193-03	.104-03	.805-02	.546-01	.364-02
Yδr	-.512-06	-.466-05	.485-05	.333-05	.115-03	.394-02	-.464-02
Lδr	-.351-02	.174-02	-.143-02	.663-01	.176-01	-.616-01	-.246-02
Nδr	-.255-03	.410-06	.768-02	.250-03	.651-01	-.120-02	-.439-03

Mean Errors--Accelerometer Positions and Angle-of-Sideslip Vane Position			
Parameter	ϵ_{ax}	ϵ_{az}	ϵ_{vx}
Yβ	-.862-02	-.863-02	.158-05
Lβ	.397-00	-.287-01	.297-04
Nβ	-.166-01	.179-02	.614-04
Lp	.117+00	-.669-02	-.156-04
Np	-.261-01	.127-02	.722-05
Lr	.109+00	-.321-01	-.860-04
Nr	-.396-01	.994-02	.175-05
Yδa	.138-01	-.736-03	-.131-05
Lδa	.510-01	.236-02	-.863-04
Nδa	.546-01	-.364-02	.878-06
Yδa	.394-02	.464-02	.368-06
Lδa	-.616-01	.246-02	-.269-04
Nδa	-.115-02	.439-03	-.573-04

Table 14

DOMINANT RANDOM ERROR SOURCES FOR LATERAL PARAMETER IDENTIFICATION

Parameter	Biases Not Estimated	All Biases But ϕ Estimated
$Y\beta$	b_{ny}, ε_{cgx}, ε_{cgz}	$\gamma_{\dot{r}}$, ε_{cgx}, ε_{cgz}
$Y\delta a$	ε_{cgx}	$\gamma_{\dot{r}}$, ε_{cgx}, ε_{cgz}
$Y\delta r$	b_{ny}, ε_{cgx}, ε_{cgz}	ε_{cgx}, ε_{cgz}
$L\beta$	b_p, $\gamma_{\dot{r}}$, ε_{cgx}	$\gamma_{\dot{r}}$, ε_{cgx}
Lp	ε_{cgx}	ε_{cgx}
Lr	$\gamma_{\dot{p}}$, ε_{cgx}, ε_{cgz}	$\gamma_{\dot{p}}$, ε_{cgx}, ε_{cgz}
$L\delta a$	b_p, ε_{cgx}	$e_{\dot{p}}$, $\gamma_{\dot{r}}$, ε_{cgx}
$L\delta r$	b_p, ε_{cgx}	$e_{\dot{p}}$, $\gamma_{\dot{p}}$, $\gamma_{\dot{r}}$, ε_{cgx}
$N\beta$	b_p, b_ϕ, $\gamma_{\dot{r}}$, ε_{cgx}	$\gamma_{\dot{r}}$, ε_{cgx}
Np	ε_{cgx}	ε_{cgx}
Nr	b_p, $\gamma_{\dot{r}}$, ε_{cgx}, ε_{cgz}	ε_{cgx}, ε_{cgz}
$N\delta a$	ε_{cgx}	ε_{cgx}
$N\delta r$	$e_{\dot{r}}$, $\gamma_{\dot{r}}$, ε_{cgx}	$e_{\dot{r}}$, $\gamma_{\dot{r}}$

Table 15

EFFECT OF INSTRUMENT REMOVAL ON PARAMETER DEVIATIONS FOR BASELINE SET OF ERRORS AND LATERAL EQUATIONS OF MOTION

Parameter	Nominal Value**	All Instruments		No Angle of Sideslip Vane (β)		No Roll Angle Accelerometer ($\dot{p}$)		No Yaw Angle Accelerometer ($\dot{r}$)		No β, $\dot{p}$, or $\dot{r}$	
		Biases*	bϕ*	Biases	bϕ	Biases	bϕ	Biases	bϕ	Biases	bϕ
Yβ	$-.157s^{-1}$	.683-2	.636-2	.692-2	.647-2	.720-2	.654-2	.792-2	.598-2	.951-2	.694-2
Yδa	$-.00338s^{-1}$	.554-2	.694-2	.553-2	.700-2	.548-2	.706-2	.553-2	.748-2	.544-2	.803-2
Yδr	$.0246s^{-1}$	.295-2	.305-2	.295-2	.304-2	.283-2	.295-2	.292-2	.306-2	.275-2	.298-2
Lβ	$-15.98s^{-2}$	.247	.271	.250	.270	.270	.264	.339	.397	.363	.416
Lp	$-1.608s^{-1}$	.375-1	.585-1	.380-1	.597-1	.495-1	.671-1	.407-1	.657-1	.557-1	.739-1
Lr	$.384s^{-1}$	.389-1	.595-1	.407-1	.609-1	.143	.105	.418-1	.567-1	.150	.855-1
Lδa	$10.92s^{-2}$	.941-1	.764-1	.967-1	.786-1	.204	.148	.101	.915-1	.215	.796-1
Lδr	$2.54s^{-2}$	.862-1	.557-1	.887-1	.561-1	.227	.126	.830-1	.327-1	.246	.119
Nβ	$6.563s^{-2}$	.232-1	.206-1	.240-1	.214-1	.367-1	.249-1	.250-1	.134-1	.374-1	.202-1
Np	$-.0997s^{-1}$	.490-2	.132-1	.491-2	.134-1	.510-2	.146-1	.476-2	.156-1	.460-2	.178-1
Nr	$-.343s^{-1}$	.819-2	.206-1	.864-2	.210-1	.896-2	.211-1	.899-2	.257-1	.851-2	.284-1
Nδa	$.707s^{-2}$	.833-2	.280-1	.864-2	.282-1	.168-1	.242-1	.125-1	.358-1	.243-1	.342-1
Nδr	$-3.902s^{-2}$	.497-1	.434-1	.520-1	.455-1	.513-1	.431-1	.905-1	.569-1	.115	.790-1

* "Biases" means that all biases are error sources; "bϕ" means all biases except b_ϕ are directly estimated.

** Dimension is: s = seconds

1. Removal of the angle-of-sideslip vane has very little effect on the accuracy of the parameters.

2. Removal of the roll angular accelerometer has a sizeable effect on the accuracy of L_p, L_r, $L_{\delta a}$, $L_{\delta r}$, N_β, and $N_{\delta a}$. The largest effect is on L_r with the standard deviation increasing from about 10% to 37% of the nominal parameter value.

3. Removal of the yaw angular accelerometer has a marked effect on the accuracies of L_β, $N_{\delta a}$, and $N_{\delta r}$, with $N_{\delta r}$'s standard deviation increasing by 82%.

4. Removal of the sideslip vane and the roll and yaw angular accelerometers simultaneously caused standard deviation of $Y_{\delta a}$, $Y_{\delta r}$, and N_p to decrease. The maximum increase was for L_r.

5. Estimating biases caused little improvement in the identification accuracy. More standard deviation magnitudes were increased than decreased, which again points out the fact that biases are not the prime source of error for lateral parameters.

IV

SUMMARY AND CONCLUSIONS

Two techniques - ensemble analysis and simulated data analysis - have been developed for determining the effects of instrumentation errors on identified aircraft parameter accuracy. The instrument errors contaminate the measured data which is used in the identification process. The analysis techniques are based on the assumption that an output error identification technique such as the modified Newton - Raphson algorithm is used.

The two statistical analysis techniques have been coded into a digital computer program which allows rapid assessment of the error effects. The uses which can be made of this program include the following:

1. The determination of the effect of instrumentation errors on the statistical accuracy of the stability and control derivatives and other parameters identified from flight test data can be made. This includes the mean error and standard deviation of each of the parameters identified. The contribution of each error source on each parameter is determined.
2. The effects of such variables as aircraft type and flight condition, control input sequence, and data sampling rate on the accuracy of the identified parameters can be determined.
3. Tradeoff studies can be made between instrument quality and identification accuracy.
4. Different combinations of instruments can be studied for use in collecting the flight data.
5. Tradeoff studies between fewer instruments with greater quality and more instruments with larger errors can be made.
6. The necessary instrument accuracy required in a flight test program to allow identifying aircraft parameters to a desired level of certainty can be specified.

To exercise this program and determine the effect of some of the instrument errors on the variation of the identified stability and control derivatives, a study was conducted using the McDonnell F-4C aircraft. Both longitudinal and lateral equations of motion were utilized. Error sources assumed present were output measurement biases, scale factor errors, and correlated errors due to instrument misalignment, etc. Some of the general conclusions which can be made from the results of this study include:

1. The instrumentation errors which are not due to noise can have very large effects on the identification accuracy. In the tests run, a growth in the standard deviations of many of the identified parameters exceeded an order of magnitude.

2. Improvement can be made in the accuracy of the parameters by identifying the values of the dominant bias errors affecting the data. For the longitudinal equations of the F-4, identifying measurement biases generally improved the parameter accuracy obtained. However, identifying biases for the lateral equations was of little value because the dominant errors were from other sources. Adding the capability of identifying error sources which are of minor importance generally reduces the overall accuracy of the identified parameters because the same information is used to determine more quantities.

3. The ratio of elements in the weighting matrix R of the cost function used by the Newton - Raphson scheme has an important roll in the overall accuracy of the identified parameters. Changing the ratio of these elements changes the sensitivity of the identified parameter variations due to each error source. The effect of an error source which has a large variation can be minimized with the proper choice of the weighting matrix elements.

4. The key output of the analysis program is the sensitivity matrix of the stability and control derivative errors to each of the instrumentation error sources. With this matrix and an estimate of the instrument quality available, the test engineer can determine what the

accuracy of the identified aircraft parameters will be. He can also specify what instrumentation quality is necessary in order to identify the parameters to a desired level of accuracy.

5. The control input sequence is a very important part of minimizing the effect of instrumentation errors and the uncertainty in the accuracy of the identified parameters. Also important to the parameter accuracy is the data sampling rate and the length of the data span.

The above conclusions were based on linear analysis. Further conclusions can be expected if dynamic errors, control measurement errors, and known non-linear errors would also be included.

Finally, it must be emphasized that no extensive laboratory or inflight study has been made of the kinds of errors that are prevalent in most flight instrumentation, including a statistical description of the error magnitudes. The range of instrument errors used in the study of this report were based on conversations with flight test personnel and reviewing instrument company literature. The information obtained from these sources was not in the statistical form necessary for the analysis, so assumptions had to be made.

V

RECOMMENDATIONS

In order to make general conclusions concerning the effect of instrumentation errors on aircraft parameter identification, the following extensions to the study presented in Section III should be made:

1. The effects of all known instrument errors not studied in this report should be determined. This includes instrument dynamics, control measurement errors, and non-linear error sources.
2. Different types of aircraft and different flight regimes should be studied to determine their effect on the overall conclusions.
3. A method of finding the "best" control input sequence corresponding to the available instruments used to collect flight data should be found. Always using the best control sequence in each study would remove the dependence of the accuracies on the control input. If the same control input is used when studying different sets of instruments, it is conceivable that one instrument set will produce better results solely because the input sequence is more favorable to that set.
4. Similarly, a method should be obtained for finding the most favorable weighting matrix used in the cost function of the Newton - Raphson identification algorithm. This matrix should tend to minimize the effect of the most prominant unidentified error sources.
5. It should be established whether other error sources in addition to output biases can be identified. In cases where these other error sources are more dominant, identification accuracy can possibly be improved by identifying them as parameters and removing their effect.
6. In order that instrumentation quality can be specified to meet the flight test objectives, a method must be established to define what constitutes an acceptable level of aircraft parameter uncertainty.

Based on the results of studying the F-4C aircraft, the following recommendations are offered to insure that adequate instrumentation is provided for identification purposes:

1. It was shown that instrumentation errors can produce significant increases in the identified parameter uncertainty. Therefore, the instrumentation quality required, including basic instrument accuracies, mounting accuracies, and recording equipment accuracies, should be specified prior to any flight test program.
2. To insure that these specifications are met, laboratory and flight test studies should be made of all aircraft instrumentation used to collect data for identification. Statistical measurements of the instrument accuracy should be obtained to provide compatibility with the analysis methods.
3. In addition to using instruments of necessary quality, care should be taken to align the instruments' sensitive axes, to calibrate the instruments, and to measure the instruments' and c.g. locations within the tolerances specified.

This study has also shown that the specification of flight instrumentation may be sensitive to the particular flight control input sequence. Therefore, it is recommended that a means be developed to display the desired input to the pilot or to generate this input directly into the control system. Also, a method should be provided to tell the pilot if sufficient information is contained in the data collected during each maneuver which will enable identification of the parameters to the level of accuracy desired.

APPENDIX A
SUMMARY OF EQUATIONS CODED IN ANALYSIS PROGRAM

A.1 Explanation of Time Increments Used by Program

The three time steps which are used in the program are:

Δt_L - This is the time step of the Runge-Kutta integration package. It is set small enough so the effect of the highest frequency dynamics is correctly simulated. This term is usually governed by the control or output measurement lag with the smallest time constant.

Δt_s - This is the sample time increment used by the identification process. When measurement lags are ignored, it is also used as the integration step.

τ - This is the time delay that governs when sampling of the control input is made after time zero. Normally, $0 \leq \tau \leq \Delta t_s$. Use of τ enables sampling the control input at different time points than the output measurements.

A.2 Simulated Data Analysis Subprogram - Mode 1

A.2.1 Simulated Airplane Equations

The equations governing the simulated airplane are:

$$\dot{x} = Fx + Gu_{input}; \quad x(0) = \hat{x}_0 \tag{A.1}$$

$$y_T = Hx + Du_{input} \tag{A.2}$$

(true output)

$$y_I = Ty_T + B \tag{A.3}$$

(indicated output - no lags)

$$\dot{y}_L = -F_m y_L + F_m y_I; \quad y_L(0) = y_I(0) \tag{A.4}$$

(effect of lags)

$$y_i = y_{Li} + w_i \tag{A.5}$$

(recorded output. If lags are present, this samples y_L every Δt_s seconds starting at time Δt_s.)

A.2.2 Control Input Equations

The equations governing the recorded control input are:

$$u = u_{input} \tag{A.6}$$

(true control which is input to program)

$$u_I = T_c u + B_c \tag{A.7}$$

(indicated control with no lags)

$$\dot{u}_L = -F_c u_L + F_c u_I; \quad u_L(0) = u_I(0) \tag{A.8}$$

(effect of lags)

$$u_{mi} = u_{Li} + w_{ci}^{*} \tag{A.9}$$

(recorded input. If lags are present, this samples u_L every Δt_s seconds starting at time τ.)

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* Note The control measurement noise is only used in the Monte Carlo Mode (Mode 2) of the Simulated Data Analysis. It is set to zero otherwise (Mode 1).

A.2.3 Ensemble Analysis Subprogram Equations

The random errors are first analyzed. To do this, the following equations are first integrated and evaluated in the ensemble analysis subprogram:

$$\dot{\hat{x}} = F\hat{x} + Gu \; ; \quad \hat{x}(0) = \hat{x}_0 \tag{A.10}$$

(Δt_s integration step)

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial p_p}\right) = F\left(\frac{\partial \hat{x}}{\partial p_p}\right) + \frac{\partial F}{\partial p_p}\hat{x} + \frac{\partial G}{\partial p_p}u \; ; \quad \frac{\partial \hat{x}}{\partial p_p} = 0 \tag{A.11}$$

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial p_{IC}}\right) = F\left(\frac{\partial \hat{x}}{\partial p_{IC}}\right) ; \quad \frac{\partial \hat{x}}{\partial p_{IC}}(0) = 1 \tag{A.12}$$

(for each initial condition estimated)

$$\frac{\partial \hat{y}}{\partial p_p} = H\frac{\partial \hat{x}}{\partial p_p} + \frac{\partial H}{\partial p_p}\hat{x} + \frac{\partial D}{\partial p_p}u \tag{A.13}$$

(from Eq. A.2)

$$\frac{\partial \hat{y}}{\partial p_{IC}} = H\frac{\partial \hat{x}}{\partial p_{IC}} \tag{A.14}$$

(for each initial condition estimated)

$$\frac{\partial \hat{y}}{\partial p_b} = 1 \tag{A.15}$$

(for each bias term estimated)

$$\frac{\partial \hat{y}_i}{\partial p} \triangleq \left[\frac{\partial \hat{y}_i}{\partial p_p} \quad \frac{\partial \hat{y}_i}{\partial p_{IC}} \quad \frac{\partial \hat{y}_i}{\partial p_b}\right] \tag{A.16}$$

$$\frac{\partial^2 J}{\partial p^2} = \sum_{i=1}^{n} \left[\frac{\partial \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \right] \tag{A.17}$$

$$\text{Invert } \frac{\partial^2 J}{\partial p^2} \triangleq \left[\frac{\partial^2 J}{\partial p^2} \right]^{-1} \tag{A.18}$$

A.2.4 Simulated Data Analysis Subprogram Equations for Random Errors

Next, each of the random output measurement errors is sequenced. The object is to evaluate:

$$\frac{\partial J}{\partial p} = - \sum_{i=1}^{n} (y_i - \hat{y}_i)^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \tag{A.19}$$

and

$$\delta p = - \left[\frac{\partial^2 J}{\partial p^2} \right]^{-1} \left[\frac{\partial J}{\partial p} \right]^T \tag{A.20}$$

First, the effect of each unestimated output measurement bias is computed. This affects only the residual of Eq. (A.19) as:

$$(y_i - \hat{y}_i) = y_{Ii} - \hat{y}_i = B \tag{A.21}$$

Then, each of the output transformation errors of T is sequenced. Again, the effect on Eq. (A.19) is:

$$(y_i - \hat{y}_i) = T(H\hat{x}_i + Du_i) - (H\hat{x}_i + Du_i) = (T - I)\ (H\hat{x}_i + Du_i) \tag{A.22}$$

Then, each of the unestimated initial condition errors is sequenced. This requires the integration of Eq. (A.1) with $x(0) = x_0$. Then, from Eq.(A.2),

$$(y_i - \hat{y}_i) = (y_{T_i} - \hat{y}_i) = H(x_i - \hat{x}_i) \tag{A.23}$$

Next, the control measurement errors due to T_c and B_c are sequenced. To do this, the previous results of integrating Eq. (A.10) are first stored. These state values represent the true airplane equations in this case. So

$$x_i = \hat{x}_i \tag{A.24}$$

Then, Eqs. (A.10), (A.11), and (A.12) are reintegrated using the Δt_s integration step and modified u_{mi}. Equations (A.12) - (A.18) are then reevaluated. For bias errors

$$u_{mi} = u_i + B_c \tag{A.25}$$

For scale factor errors

$$u_{mi} = T_c u_i \tag{A.26}$$

For the random errors, the total parameter covariance is

$$E\,\{\delta p \delta p^T\} = E_{noise} + \sum_{j=1}^{r} (\delta p_j \delta p_j^T) \tag{A.27}$$

where r is the number of random error sources. E_{noise} is the covariance of the parameter estimate errors due to the output noise w_i. E_{noise} is either set to Eq. (A.18) or computed using the Monte Carlo option with

$$(y_i - \hat{y}_i) = w_i \tag{A.28}$$

at each time point.

A.2.5 Mean Errors

The mean errors are also sequenced. These include some elements in the T matrix (Eq. (A.3) and all elements in F_m and F_c (Eqs. (A.4) and (A.8)). If there are no lag errors input, the evaluation is just like Eq. (A.22).

If there are control measurement lag errors, Eq. (A.8) needs to be evaluated with $u_I = u$, over each interval Δt_s. The integration step is Δt_L. The stored u_{mi} is sampled every Δt_s but beginning τ seconds after time zero.

If there are output lags, Eq. (A.4) needs evaluation. This requires reintegration of Eq. (A.1), with $x(0) = \hat{x}_0$, and Eq. (A.4) of the form

$$\dot{y}_L = -F_m\, y_L + F_m\, (Hx + Du) \tag{A.29}$$

The step size is Δt_L. The output of Eq. (A.29) is sampled every Δt_s seconds with

$$y_i = y_{Li} \tag{A.30}$$

Again, Eqs. (A.10) - (A.18) need to be reevaluated in case of control lags. Equations (A.10) - (A.12) use an integration step size of Δt_s, however. The effect on Eq. (A.19) is

$$y_i - \hat{y}_i = y_{Li} - (H\hat{x}_i + Du_{mi}) \tag{A.31}$$

A.3 Simulated Data Analysis Subprogram - Mode 2

The random errors contained in B, part of T, B_c T_c, and x_o are generated at the beginning of each run using the input standard deviations and the random number generator. They are held constant during each single Monte Carlo run. The random noise w_i and w_{ci} are regenerated each sample point during each run. Each of the mean errors in T plus elements of F_m and F_c are set equal to the values input and are not changed during any of the runs.

For output measurement errors in T, B and w_i only, the residual $(y_i - \hat{y}_i)$ in Eq. (A.19) is computed by

$$y_i - \hat{y}_i = T(H\hat{x}_i + Du_i) + B + w_i - (H\hat{x}_i + Du_i) \tag{A.32}$$

For random initial conditions added, Eq. (A.1) must be integrated each time and Eq. (A.32) gets changed to

$$y_i - \hat{y}_i = T(Hx_i + Du_i) + B + w_i - (H\hat{x}_i + Du_i) \tag{A.33}$$

For non-lag control measurement errors, Eqs. (A.7) and (A.9) get combined so that at each sample point

$$u_{mi} = T_c u + B_c + w_{ci} \tag{A.34}$$

where w_{ci} is randomly generated each time point. Because of this change, Eqs. (A.10) - (A.12) require integration each pass through, and Eqs. (A.13) - (A.18) require re-evaluation each pass through. With these changes, Eq. (A.33) becomes

$$y_i - \hat{y}_i = T(Hx_i + Du_i) + B + w_i - (H\hat{x}_i + Du_{mi}) \tag{A.35}$$

For lags in the control input measurements and output measurements, two integration step sizes have to be used. First, Eqs. (A.1) - (A.4) are evaluated using the Δt_L step. The resulting y_L is sampled every Δt_s seconds starting at time Δt_s. Then, Eq. (A.5) is evaluated and the resulting y_L stored. Next, Eqs. (A.7) - (A.8) are evaluated using the Δt_L step. The resulting u_L is sampled every Δt_s seconds starting at time τ. Then, Eq. (A.9) is evaluated and the resulting u_{mi} stored. Next, Eqs. (A.10) - (A.12) are evaluated using the Δt_s step and the stored u_{mi}. Then, Eqs. (A.13) - (A.18) are evaluated. Then, the residual for Eq. (A.19) is computed using

$$y_i - \hat{y}_i = y_i - (H\hat{x} + Du_{mi}) \tag{A.36}$$

In Mode 2, w_{ci} is used just like w_i.

In Mode 2 , several Monte Carlo runs are made. In each run, the values of the random elements in T, B, and $\hat{x}_o$ are randomly generated at the beginning of the run. For comparison, the standard deviations used to generate these values are the same as those values used one at a time in Mode 1. These random values remain constant throughout a run, but change from run to run. The white noise vector w_i is randomly generated, and it changes every sample point throughout the run. Those mean error sources in T, F_m, and F_c which aren't random are held constant throughout all the runs. The error Δp_j in the parameter vector obtained from each run is saved. For m Monte Carlo runs, the mean error in p is

$$\overline{\Delta p} \triangleq E\{\Delta p\} = \frac{1}{m} \sum_{j=1}^{m} \Delta p_j \tag{A.37}$$

The standard deviation about this mean is

$$E\{\delta p \delta p^T\} = \frac{1}{m} \sum_{j=1}^{m} (\Delta p_j - \overline{\Delta p})(\Delta p_j - \overline{\Delta p})^T \tag{A.38}$$

For small values of the error sources, the results of Eq. (A.38) should match those of Eq. (A.27). Also, Eq. (A.37) should match the value of the mean obtained in Mode 1.

A.4 Ensemble Error Analysis Subprogram

A.4.1 Output Errors

The basic equation which is used to compute changes in the parameters is

$$\delta p = - \left[\frac{\partial^2 J}{\partial p^2} \right]^{-1} \frac{\partial J}{\partial p}^T$$

$$= + \left[\sum_{i=1}^{n} \left\{ \frac{\partial \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial \hat{y}_i}{\partial p} \right\} \right]^{-1} \sum_{i=1}^{n} \frac{\partial \hat{y}_i}{\partial p}^T R^{-1} (y_i - \hat{y}_i)^T \qquad \text{(A.20)}$$

The y_i are the sampled measurements taken from the aircraft (Eqs. (A.1) - (A.5)) and the $\hat{y}_i$ are the simulated values obtained from the measured control input (Eqs. (A.6) - (A.9) and Eq. (A.10)). The term $\frac{\partial \hat{y}_i}{\partial p}$ comes from Eqs. (A.11) - (A.16).

It is assumed that if lag errors exist, lag terms are present in all control input and system output measurement equations. If the measurements of y_i have lags, the dynamic equations can be written as

$$\dot{x} = Fx + Gu \qquad \text{(A.1)}$$

$$\dot{y}_L = -F_m y_L + F_m (T\ [H\ x + D\ u] + B\ ) \qquad \text{(A.39)}$$

Otherwise, the output equations are of the form

$$y = T(Hx + Du) + B \qquad \text{(A.40)}$$

The sensitivity of y_i to each error e is computed. For biases,

$$\frac{\partial y}{\partial e} = 1 \tag{A.41}$$

For errors due to the matrix T, the sensitivity can be found as

$$\frac{\partial y}{\partial e} = \frac{\partial T}{\partial e} (Hx + Du) \tag{A.42}$$

For initial condition errors,

$$\frac{\partial y}{\partial e} = H\frac{\partial x}{\partial e} \tag{A.43}$$

where the term $\frac{\partial x}{\partial e}$ comes from integrating

$$\frac{d}{dt}\left(\frac{\partial x}{\partial e}\right) = F\frac{\partial x}{\partial e} \;;\; \frac{\partial x}{\partial e}(0) = I \tag{A.44}$$

When output lags are present, the sensitivity differential equation comes from Eq. (A.39), and is

$$\frac{d}{dt}\left(\frac{\partial y_L}{\partial e}\right) = \frac{\partial F_m}{\partial e}\left[- y_L + Hx + Du\right] - F_m \frac{\partial y_L}{\partial e} \tag{A.45}$$

The results of Eqs. (A.41) through (A.45) are combined into a general $\frac{\partial y_i}{\partial e}$ for each error e which affects the output measurements.

A.4.2 Control Input Errors

With respect to Eq. (A.20), the error effect on the term $\hat{y}_i$ is computed. The differential equations of the simulated aircraft are

$$\dot{u}_L = -F_c u_L + F_c (T_c u + B_c) \; ; \; u_L(0) = u_I(0) \tag{A.46}$$

$$\dot{\hat{x}} = F\hat{x} + Gu_{Li} \tag{A.47}$$

$$\hat{y}_i = H\hat{x}_i + Du_{Li} \tag{A.48}$$

For control measurement lags the following equations, that come from Eqs. (A.46 and (A.47), are first integrated.

$$\frac{d}{dt}\left(\frac{\partial \hat{x}}{\partial e}\right) = F\frac{\partial \hat{x}}{\partial e} + G\frac{\partial u_L}{\partial e} \; ; \quad \frac{\partial \hat{x}}{\partial e}(0) = 0 \tag{A.49}$$

$$\frac{d}{dt}\left(\frac{\partial u_L}{\partial e}\right) = -F_c\frac{\partial u_L}{\partial e} + \frac{\partial F_c}{\partial e}(-u_L + u) \; ; \quad \frac{\partial u_L}{\partial e}(0) = \frac{\partial u_i}{\partial e}(0) \tag{A.50}$$

Then, from Eq. (A.48)

$$\frac{\partial \hat{y}_i}{\partial e} = H\frac{\partial \hat{x}_i}{\partial e} + D\frac{\partial u_{Li}}{\partial e} \tag{A.51}$$

For control scale factor errors, the program first integrates

$$\frac{d}{dt}\frac{\partial \hat{x}}{\partial e} = F\frac{\partial \hat{x}}{\partial e} + G\frac{\partial T_c}{\partial e}u \; ; \quad \frac{d\hat{x}}{de}(0) = 0 \tag{A.52}$$

Then, the sensitivity is computed as

$$\frac{\partial \hat{y}_i}{\partial e} = H\frac{\partial \hat{x}_i}{\partial e} + D\frac{\partial T_c}{\partial e}u_i \tag{A.53}$$

For control biases, the program integrates

$$\frac{\partial}{\partial t}\left(\frac{\partial \hat{x}}{\partial e}\right) = F\frac{\partial \hat{x}}{\partial e} + G\frac{\partial B_c}{\partial e}\ ; \quad \frac{\partial \hat{x}}{\partial e}(0) = 0 \tag{A.54}$$

Then, the sensitivity becomes

$$\frac{\partial \hat{y}}{\partial e} = H\frac{\partial \hat{x}}{\partial e} + D\frac{\partial B_c}{\partial e} \tag{A.55}$$

A.4.3 Effect on Parameter Estimates

The sensitivity of parameter estimates due to output measurement is of the form

$$\frac{\partial}{\partial e}(\delta p) = \left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \sum_{i=1}^{n} \frac{\partial \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial y_i}{\partial e} \tag{A.56}$$

where $\frac{\partial y_i}{\partial e}$ comes from evaluating Eqs. (A.41) through (A.45).

The sensitivity of parameter estimates due to control input errors is of the form

$$\frac{\partial}{\partial e}(\delta p) = -\left[\frac{\partial^2 J}{\partial p^2}\right]^{-1} \sum_{i=1}^{n} \frac{\partial \hat{y}_i}{\partial p}^T R^{-1} \frac{\partial \hat{y}_i}{\partial e} \tag{A.57}$$

where $\frac{\partial \hat{y}_i}{\partial e}$ comes from evaluating Eqs. (A.51), (A.53), and (A.55).

For random errors, the total parameter covariance is

$$E\{\delta p\ \delta p^T\} = E_{noise} + \frac{\partial}{\partial e}(\delta p)E\{e\ e^T\}\frac{\partial}{\partial e}(\delta p)^T \tag{A.58}$$

For mean errors, the mean parameter error is

$$E\{\delta p\} = \frac{\partial(\delta p)}{\partial e}E\{e\} \tag{A.59}$$

REFERENCES

1. "ad hoc Team Report on F-111 Stall/Post Stall/Spin Presentation Program," Aeronautical Systems Division, Air Force Systems Command, U.S.A.F., 28 August, 1970.

2. Gerlach, O. H., "Determination of Performance and Stability Parameters from Nonsteady Flight Test Maneuvers," SAE Paper. No. 700236, March, 1970.

3. Hill, R. W., Clinkenbeard, I. L., and Bolling, N.F., "V/STOL Flight Test Instrumentation Requirements for Extraction of Aerodynamic Coefficients," Air Force Flight Dynamics Laboratory Technical Report AFFDL-TR-68-154, Dayton, Ohio, December 1968.

4. Taylor, L. W., Jr., Iliff, K. W., and Powers, B. G., "A Comparison of Newton-Raphson and Other Methods for Determining Stability Derivatives from Flight Data," AIAA Third Flight Test, Simulation and Support Conference, Houston, Texas, March 10-12, 1969.

5. Taylor, L. W. Jr., and Iliff, K. W., "A Modified Newton-Raphson Method for Determining Stability Derivatives from Flight Data," 2nd International Conference on Computing Methods in Optimization Problems, Sanremo, Italy, September 9-13, 1968.

6. Etkin, B., Dynamics of Flight: Stability and Control, John Wiley and Sons, New York, 1959.

7. Seckel, E., Stability and Control of Airplanes and Helicopters, Academic Press, New York, 1964.

8. Wolowicz, C., "Considerations in the Determination of Stability and Control Derivatives and Dynamic Characteristics from Flight Data," AGARD Report 549, Part I, 1966.

9. Gerlach, O. H., "High Accuracy Instrumentation Techniques for Non-Steady Flight Measurements," Flight Test Instrumentation, Vol. III. (M.A. Perry, editor). Pergamon Press, London, 1964.

10. Widrow, B., "Statistical Analysis of Amplitude-Quantized Sample-Data Systems," AIEE Transactions, Paper No. 60-1240, Vol. 79, Part II, January, 1961.

11. "Product Guide to Servo Accelerometers/Inertial Devices/Flight Systems," Systron Donner Corporation, 1967.

12. "Rate Gyroscope Technical Handbook," Timex Corporation.

13. Bryant, W. H., NASA Langley Research Center, Private communication.

14. Wolowicz, C., NASA Flight Research Center, Private communication.

15. "Aerodynamic Derivatives of F/RF-4B-C, "McDonnell Aircraft Corporation MAC 9842, February 1964.

16. Blakelock, J. H., Automatic Control of Aircraft and Missiles, John Wiley & Sons, Inc., New York, 1965.