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DEVELOPMENT OF EQUILIBRIUM AIR COMPUTER PROGRAMS SUITABLE FOR NUMERICAL COMPUTATION USING TIME-DEPENDENT OR SHOCK-CAPTURING METHODS

by John C. Tannehill and Robert A. Mohling

Prepared by
IOWA STATE UNIVERSITY
Ames, Iowa
for Ames Research Center

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16. Abstract

Computer programs were developed which compute the thermodynamic properties of equilibrium air for use in either the "time-dependent" or "shock-capturing" computational methods. For the "time-dependent" method, the NASA-ARC RGAS computer program was modified to allow internal energy and density to be used as the independent variables. In addition, simplified curve fits for $p = p(e,\rho)$, $a = a(e,\rho)$, and $T - T(p,\rho)$ were devised to reduce computer time. For the "shock-capturing" method a simplified curve fit for $h = h(p,\rho)$ was made. These approximate curve fits may be particularly useful when employed on advanced computers such as the Burrough's ILLIAC IV or the CDC STAR since they avoid the cumbersome table-lookup feature of the RGAS program.

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DEVELOPMENT OF EQUILIBRIUM AIR COMPUTER PROGRAMS SUITABLE FOR NUMERICAL COMPUTATION USING TIME-DEPENDENT OR SHOCK-CAPTURING METHODS

John C. Tannehill and Robert A. Mohling

INTRODUCTION

This report summarizes the research accomplished under NASA Grant NGR 16-002-038 for the period November 15, 1971 through March 15, 1972. During this period, computer programs were developed which compute the thermodynamic properties of equilibrium air for use in either the "time-dependent" or "shock-capturing" computational methods. For the "time-dependent" method, the NASA-ARC RGAS computer program was modified to allow internal energy and density to be used as the independent variables. In addition, simplified curve fits for $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(p, \rho)$ were devised to reduce computer time. For the "shock-capturing" method a simplified curve fit for $h = h(p, \rho)$ was made. These approximate curve fits may be particularly useful when employed on advanced computers such as the Burrough's ILLIAC IV or the CDC STAR since they avoid the cumbersome table-lookup feature of the RGAS program.

MODIFICATION OF THE NASA RGAS SUBROUTINE

The NASA Ames Research Center real gas computer program has been modified to give the user the option of entering with new independent variables, internal energy e and density ρ. Although the calling sequence has been altered in order to transfer e to the modified RGAS subroutine, the logic and other features of the original RGAS subroutine have been retained. The modified subroutine requires new cubic coefficients for e and ρ entry. A short program was written to generate and store the coefficients on tape for air. In order to understand the method of generating the new coefficients and their use in the modified subroutine, a brief discussion of the original RGAS subroutine follows.

Original RGAS Subroutine

Version I of the original RGAS program for real gas calculations is based on the gas properties determined by Bailey[1] for temperatures up to 45,000 ^oK and densities from 10^{-7} to 10^3 amagats. These properties were used to generate 13 files of information on a tape for use in determining the thermodynamic properties (a,h,T, and s) of 13 different gas mixtures. Each file on the tape contains the cubic spline fit coefficients along with the lowest value of the independent variable F, for each interva on the 11 constant density lines ($R_j = \log_{10} \rho/\rho_o = -7.0, -6.0, \ldots\ldots,$ $+3.0$; $j = 1, 11$). F varies between zero and FM (the maximum value of F) and is defined by

$$F = \frac{\log_{10} (p/p_o) - \log_{10} (\rho/\rho_o) - B}{1 + E \log_{10} (\rho/\rho_o) + D [\log_{10} (\rho/\rho_o)]^2} \qquad (1)$$

where B, D, E, FM, p_o, and ρ_o are known constants for each gas.

Subroutine RGAS calls subroutine SERCH which uses F to locate the cubic coefficients required for the calculation of the thermodynamic properties. For example, if h is to be determined with p and ρ known, then R is first calculated ($R = \log_{10} \rho/\rho_o$) to determine the two adjacent R_j lines. Once F is found from Eq. (1), the two sets of cubic coefficients can be located for each R_j line. With these coefficients, the values of h on the two R_j lines are calculated from

$$h_1 = a_1 + b_1F + c_1F^2 + d_1F^3$$
$$h_2 = a_2 + b_2F + c_2F^2 + d_2F^3 \qquad (2)$$

allowing h to be found by linear interpolation:

$$h = h_1 + (h_2 - h_1)(R - R_j) \qquad (3)$$

The method is illustrated in Fig. 1.

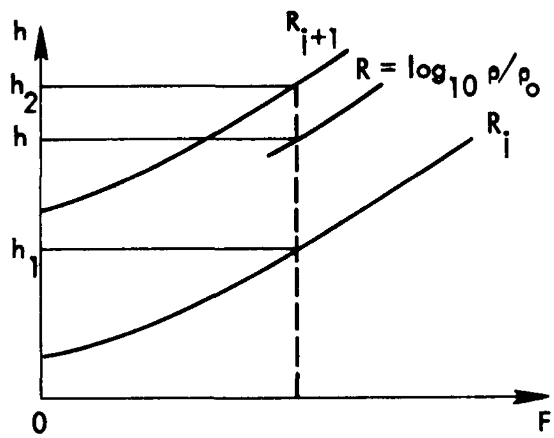

Fig. 1. Calculation of enthalpy.

Generation of Cubic Coefficients for Modified RGAS

In order to use e and ρ for the independent variables, it was necessary to generate new cubic coefficients for every e interval such that

$$F = a + be + ce^2 + de^3 \quad . \tag{4}$$

This was accomplished by subdividing each F interval of the enthalpy curve into three equal parts as shown in Fig. 2 where $F_i = F_1 + i\Delta F/3$, $i = 0, 1, 2, 3$.

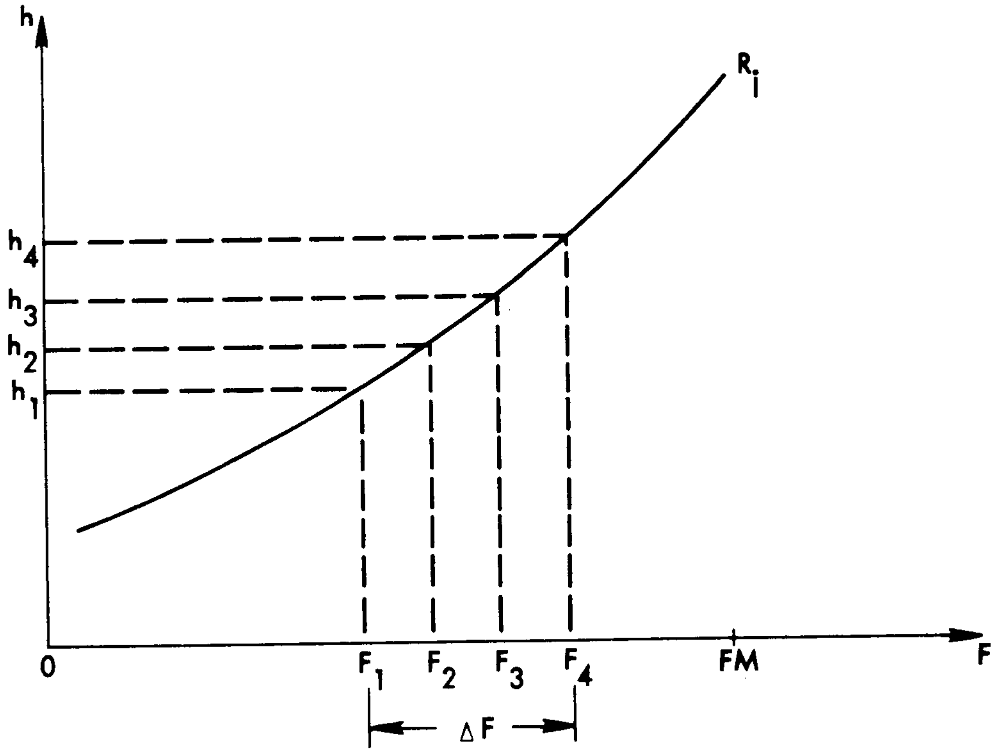

Fig. 2. Subdivision of F interval on enthalpy curve.

Equation (1) was then solved for the four values of p.

$$p_i = p_o(10)^{n_i} \qquad i = 1,2,3,4 \qquad (5)$$

where

$$n_i = F_i \left\{1 + E \log_{10}(\rho/\rho_o) + D[\log_{10}(\rho/\rho_o)]^2\right\} + \log_{10}(\rho/\rho_o) + B$$

The internal energy e_i was found using the definition of enthalpy

$$e_i = h_i - p_i/\rho \qquad i = 1,2,3,4 \qquad (6)$$

with h_i determined using the original RGAS subroutine knowing p_i and ρ. Each value of F can be written as

$$F_i = a + be_i + ce_i^{\,2} + de_i^{\,3} \qquad i = 1,2,3,4 \qquad (7)$$

or in matrix notation

$$\overline{F} = M\overline{x}, \text{ where } \overline{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}, \; M = \begin{bmatrix} 1 & e_1 & e_1^{\,2} & e_1^{\,3} \\ 1 & e_2 & e_2^{\,2} & e_2^{\,3} \\ 1 & e_3 & e_3^{\,2} & e_3^{\,3} \\ 1 & e_4 & e_4^{\,2} & e_4^{\,3} \end{bmatrix}, \; \overline{x} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \qquad (8)$$

An IBM library subroutine GELG was used to solve the system of four equations for a, b, c, and d for each F interval and all eleven constant density lines. These coefficients along with the initial values of the e intervals (e_1's) were stored on the tape in File 14. A listing of the program which generates the coefficients appears in Appendix A.

The curve generated for $F = F(e, \rho)$ is shown in Fig. 3. The maximum values of e allowed for each of the density ratios are given in Table 1 and plotted in Fig. 4.

Table 1. Maximum values of e for each density ratio

ρ/ρ_o	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}
$e_{max}(10)^{-8}\left[\frac{ft^2}{sec^2}\right]$	43.27	42.66	39.39	32.72	25.40	19.79	17.15	13.40	9.864	7.440	5.878

Modified RGAS Subroutine

The argument list of the calling sequence for the modified RGAS subroutine is located in Appendix B. An e, ρ entry is accomplished by setting the calling argument NTEST equal to 1, which signals a tape read of the newly calculated cubic coefficients. NTEST equal to -1 and 0 are still reserved for the original RGAS subroutine to make real gas and perfect gas calculations, respectively. For the known ρ, subroutine SERCH uses e to locate the two sets of cubic coefficients needed to calculate

$$\begin{aligned} F_1 &= a_1 + b_1 e + c_1 e^2 + d_1 e^3 \\ F_2 &= a_2 + b_2 e + c_2 e^2 + d_2 e^3 \end{aligned} \tag{9}$$

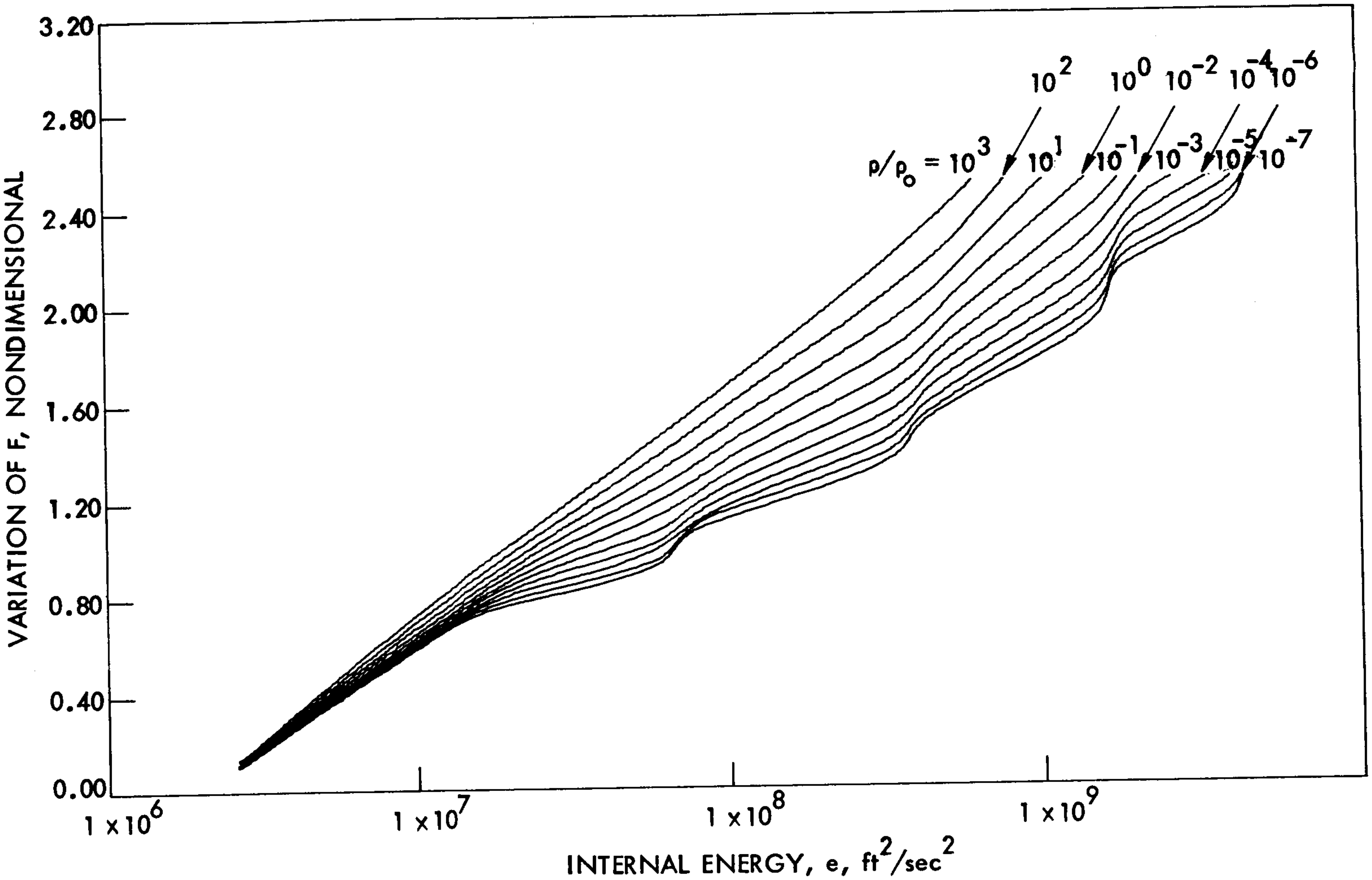

Fig. 3. Curve fit for F = F(e, ρ).

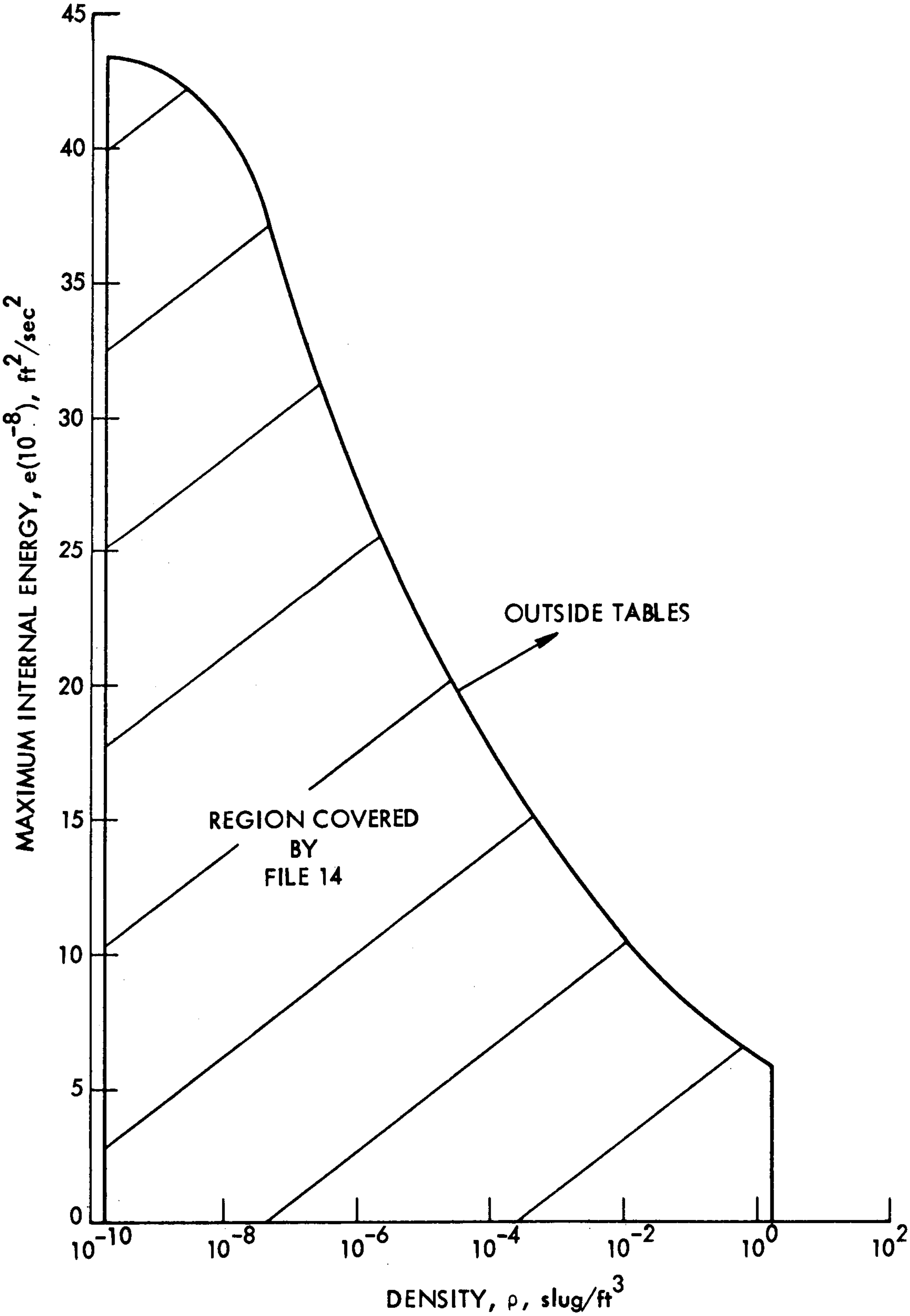

Fig. 4. Allowable range on e.

As illustrated in Fig. 5, a linear interpolation similar to Eq. (3) gives

$$F = F_1 + (F_2 - F_1)(R - R_j) \quad (10)$$

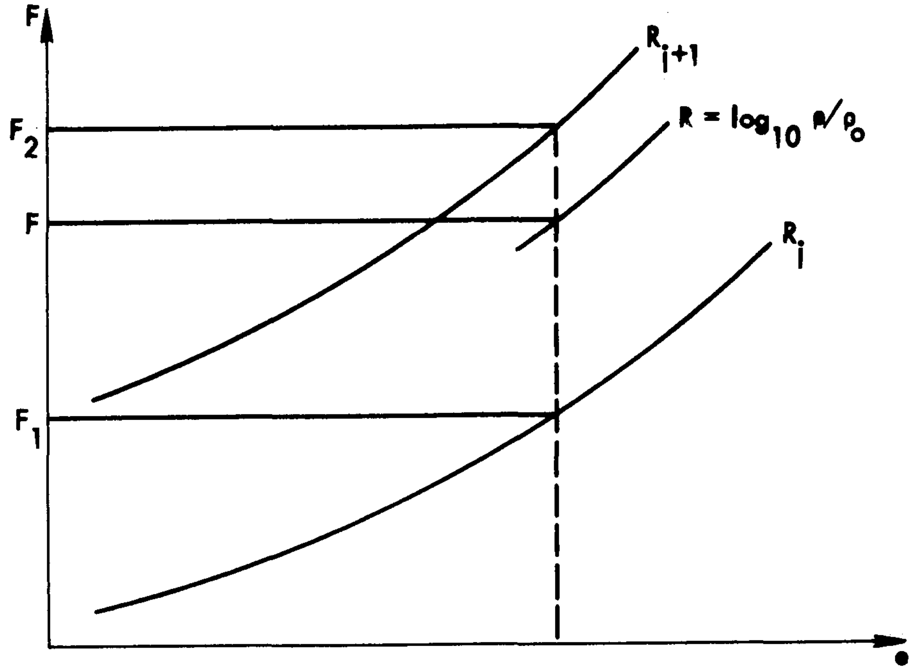

Fig. 5. Calculation of F.

With F known, p can be determined from Eq. (5). The logic of the original RGAS subroutine is then used to determine the other thermodynamic properties requested through the argument code NUMX. A listing of the modified RGAS subroutine is given in Appendix C.

Estimates of the accuracy of the modified RGAS subroutine were obtained by entering with e and ρ data to determine p and subsequently h from

$$h = e + p/\rho \quad (11)$$

For comparison, h was also calculated from the original RGAS subroutine using the newly determined p and ρ from above. 1083 comparisons were made of these two values of h. The numbers in the first line of Table 2 are percentages of compared values with errors greater than the given value. The second line contains percentages of the 2624 comparisons made by Lomax and Inouye[2] between the original RGAS subroutine and the data by Bailey[1].

Table 2. Accuracy of Modified RGAS.

Error	0.5%	1%	2%	3%	4%	5%	10%
Modified RGAS	13.30	4.80	0.37	0.09	0	0	0
Original RGAS	5.04	0.45	0	0	0	0	0

SIMPLIFIED CURVE FITS FOR EQUILIBRIUM AIR

Simplified curve fits for the thermodynamic properties of equilibrium air have been constructed for use in either the "time-dependent" or "shock-capturing" computational methods. For the "time-dependent" method, correlations were developed for $p = p(e,\rho)$, $a = a(e,\rho)$, and $T = T(p,\rho)$ while for the "shock-capturing" method a correlation was made for $h = h(p,\rho)$. The ranges of validity for these correlation formulas are the same as the NASA RGAS subroutine, namely, temperatures up to 45,000 ^{o}K and densities from 10^{-7} to 10^{3} amagats.

The simplified curve fits developed here allow the user to reduce computer time and storage while maintaining reasonable accuracy. This may be particularly true in the "time-dependent" method since the simplified curve fits could be used until near the end of a calculation when the steady-state solution is approached. Then, the modified RGAS subroutine could be used to give more accurate thermodynamic properties for the final steps. Substantial savings in computer time may also result in the "shock-capturing" method since an iterative procedure involving $h = h(p, \rho)$ is required for equilibrium calculations.

The curve fits for $p = p(e, \rho)$ and $h = h(p, \rho)$ were constructed using Grabau-type transistion functions[3] in a manner similar to Lewis and Burgess[4] and Barnwell[5]. A transistion function of this type can be used to smoothly connect two surfaces $f_1(x,y)$ and $f_2(x,y)$. For y = constant, the Grabau-type transistion function (with an inflection point) becomes

$$z = f_1(x) + \frac{f_2(x) - f_1(x)}{1 + \exp\left[K(x - x_o)\right]} \tag{12}$$

where K is the parameter which determines the rate at which the curve moves from $f_1(x)$ to $f_2(x)$ and x_o is the location of the inflection point as shown in Fig. 6.

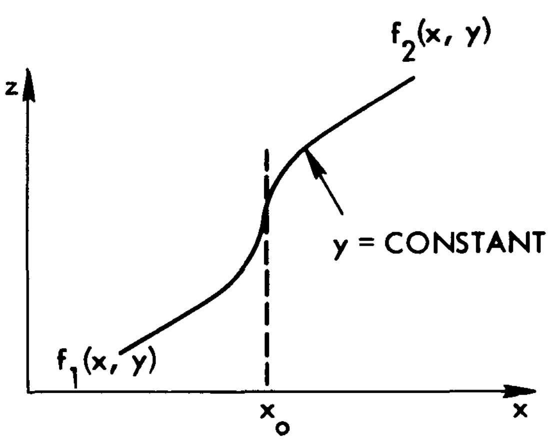

Fig. 6. Grabau-type transistion with inflection point.

For the present curve fits, two Grabau-type transistion functions were joined with the equation for a perfect gas as shown in Fig. 7.

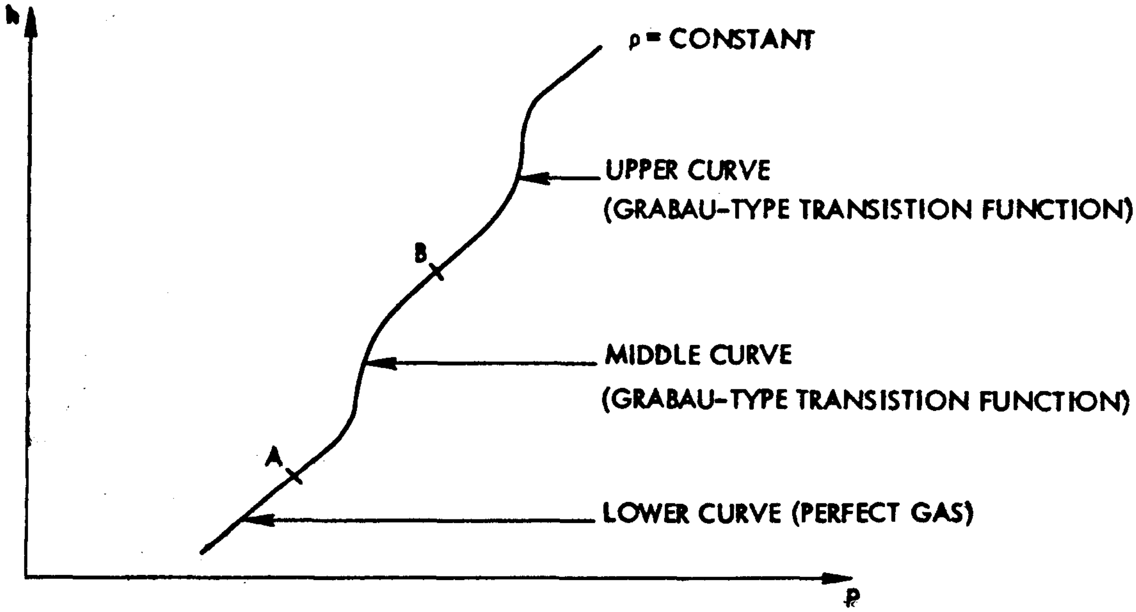

Fig. 7. Correlation curves for $h = h(p, \rho)$.

In addition, the range of the independent variable ρ was subdivided into three separate regions with different coefficients being used in the curve fits for each region (See Fig. 8.) The division lines are located at $\rho/\rho_o = 5 \times 10^{-5}$ and $\rho/\rho_o = 0.5$.

The coefficients for $f_1(x,y)$ and $f_2(x,y)$ were determined using a least squares best fit computer program in conjunction with the original NASA RGAS subroutine. The selection of the form of the equations for $f_1(x,y)$ and $f_2(x,y)$ was largely a trial-and-error process. By including more terms, a better curve fit could be achieved. In fact, if a sufficient number of terms are retained in $f_1(x,y)$ and $f_2(x,y)$, the accuracy of these curve fits can be made to approach that of the RGAS program but without any savings in computer time.

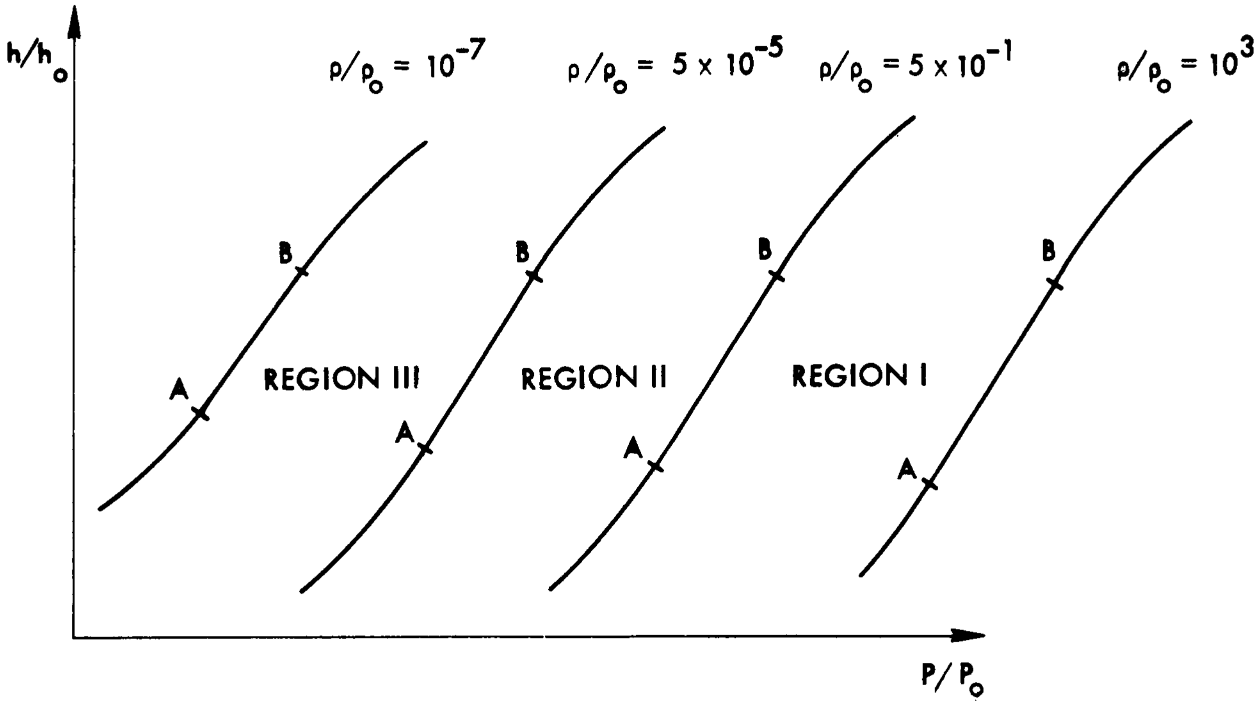

Fig. 8. Division of curve fit range by density.

For the correlation of $p = p(e,\rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted as a function of e and ρ so that p could be found from

$$p = \rho e(\tilde{\gamma} - 1) \tag{13}$$

This equation is derived directly from the definition of enthalpy.

Barnwell[5] has derived the following expression for the speed of sound:

$$a = \left[e \left\{ (\tilde{\gamma} - 1) \left[\tilde{\gamma} + \left(\frac{\partial \tilde{\gamma}}{\partial \log_e e} \right)_\rho \right] + \left(\frac{\partial \tilde{\gamma}}{\partial \log_e \rho} \right)_e \right\} \right]^{1/2} \tag{14}$$

In the present study, it was found that a much better correlation for $a = a(e, \rho)$ could be obtained from

$$a = \left[e \left\{ K_1 + (\tilde{\gamma} - 1) \left[\tilde{\gamma} + K_2 \left(\frac{\partial \tilde{\gamma}}{\partial \log_e e} \right)_\rho \right] + K_3 \left(\frac{\partial \tilde{\gamma}}{\partial \log_e \rho} \right)_e \right\} \right]^{1/2} \tag{15}$$

where the coefficients K_1, K_2, and K_3 were determined using the least squares best fit program in conjunction with the NASA RGAS program. The curve fit for $T = T(p, \rho)$ uses a Grabau-type transistion function without an inflection point[4] for the upper curve and a non-Grabau-type function for the middle curve.

For the correlation of $h = h(p, \rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted as a function of p and ρ so that h could be found from

$$h = (p/\rho) \left(\frac{\tilde{\gamma}}{\tilde{\gamma} - 1} \right) \tag{16}$$

This equation was derived from the definition of enthalpy.

The equations for all the present curve fits appear in Appendix D. The calling sequence and listing of the subroutine TGAS to find $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(p, \rho)$ and the subroutine TGAS to find $h = h(p, \rho)$ appear in Appendix E and F, respectively.

Comparisons of the relative accuracy of the curve fits for $p = p(e, \rho)$, $a = a(e, \rho)$ and $T = T(p, \rho)$ with the original RGAS subroutine are shown in Figs. 9, 10, and 11. Included on these graphs are the curve fits by Barnwell[5]. It should be pointed out that the limits of applicability of the Barnwell curve fits for $p = p(e,\rho)$ and $a = a(e,\rho)$ are $10^{-4} \le \rho/\rho_o \le 10$ and $e/e_o \le 565$. The limits of applicability for $T = T(p,\rho)$ are $6 \times 10^{-2} \le p/p_o \le 20$ and $10^{-3} \le \rho/\rho_o \le 10^{-1}$. A comparison between the present curve fit for $h = h(p,\rho)$ and the original RGAS subroutine is shown in Fig. 12. The deviations between the curve fits for $p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(p,\rho)$, and $h = h(p,\rho)$; and the original RGAS subroutine are plotted in Figs. 13, 14, 15, and 16. In order to make the comparison for $p = p(e,\rho)$ the following procedure was used. First, e and ρ input data was supplied which allowed TGAS to compute $p = p(e,\rho)$. Then, this p and the original ρ were inputed into RGAS to give h and subsequently e from the definition of enthalpy. This e was compared with the original e to determine the accuracy of the curve fit for $p = p(e,\rho)$.

A comparison of the relative computer times required for the TGAS subroutines and the NASA RGAS programs are given in Table 3. The TGAS subroutine for finding $p = p(e,\rho)$, $a = a(e,\rho)$, and $T = T(p,\rho)$ is approximately

Table 3. Comparison of computer time.

Curve Fit	Number of Data Points	TGAS	RGAS
$p = p(e,\rho)$ $a = a(e,\rho)$ $T = T(p,\rho)$	508	1.434 sec	3.535 sec (0-1) includes tape read 2.367 sec (2-508) 5.902 sec
$h = h(p,\rho)$	464	1.035 sec	2.434 sec (0-1) includes tape read 1.765 sec (2-464) 4.199 sec

65% faster than the modified NASA RGAS subroutine. This comparison does not include the time spent by the RGAS subroutine in reading the tape. The TGAS subroutine for finding $h = h(p, \rho)$ is approximately 71% faster than the original RGAS program, again excluding the tape read time. If there are not more than a few hundred calls made to these real gas subroutines, then the TGAS subroutines are substantially faster than the RGAS subroutines when the tape read time is included. For instance, if there are 500 calls made to the subroutines to find $p = p(e,\rho)$, $a = a(e,\rho)$ and $T = T(p,\rho)$, then TGAS is 317% faster than the modified RGAS subroutine.

Comparisons of the values obtained using the curve fits at the juncture points A and B (See Fig. 7) are shown in Table 4. The maximum deviation between the curves at the juncture points for the primary variables $p = p(e,\rho)$ and $h = h(p,\rho)$ is 1.1%.

Additional work is being conducted to improve the accuracy and computation times of the simplified curve fits developed in this study. In particular, it has been found that a substantial improvement in accuracy without much increase in computer time can be achieved if four instead of two Grabau-type transistion functions are joined with the equation for a perfect gas as in Fig. 7.

Table 4. Comparison of variables at juncture points.

Curve Fit	Density Ratio ρ/ρ_o	Point A Lower Curve	Point A Middle Curve	Point B Middle Curve	Point B Upper Curve
$p = p(e,\rho)$	10^3	1.400	1.399	1.244	1.243
	10^2	1.400	1.400	1.228	1.225
	10^1	1.400	1.401	1.209	1.207
	10^0	1.400	1.401	1.189	1.190
	10^{-1}	1.400	1.403	1.168	1.172
	10^{-2}	1.400	1.407	1.160	1.158
	10^{-3}	1.400	1.408	1.151	1.143
	10^{-4}	1.400	1.407	1.139	1.127
	10^{-5}	1.400	1.407	1.150	1.163
	10^{-6}	1.400	1.411	1.151	1.156
	10^{-7}	1.400	1.410	1.149	1.149
$a = a(e,\rho)$	10^3	442.9	438.3	326.8	324.7
	10^2	442.9	440.1	313.7	309.2
	10^1	442.9	440.9	298.4	293.6
	10^0	442.9	440.9	281.1	277.8
	10^{-1}	442.9	442.3	261.7	261.7
	10^{-2}	442.9	445.0	256.0	251.2
	10^{-3}	442.9	446.1	248.6	239.6
	10^{-4}	442.9	445.6	239.4	226.8
	10^{-5}	442.9	447.0	241.4	251.5
	10^{-6}	442.9	450.5	244.9	244.9
	10^{-7}	442.9	450.6	244.8	238.1
$T = T(p,\rho)$	10^3	1870.0	1885.0	17039.0	16384.0
	10^2	1871.0	1868.0	13772.0	13818.0
	10^1	1862.0	1838.0	11450.0	11600.0
	10^0	1843.0	1797.0	9727.0	9697.0
	10^{-1}	1849.0	1863.0	8503.0	8629.0
	10^{-2}	1823.0	1815.0	7615.0	7850.0
	10^{-3}	1783.0	1752.0	6815.0	7029.0
	10^{-4}	1730.0	1677.0	6030.0	6162.0
	10^{-5}	1721.0	1718.0	5435.0	5628.0
	10^{-6}	1689.0	1650.0	4965.0	5143.0
	10^{-7}	1620.0	1543.0	4723.0	4699.0
$h = h(p,\rho)$	10^3	1.400	1.396	1.267	1.266
	10^2	1.400	1.396	1.251	1.251
	10^1	1.400	1.396	1.234	1.236
	10^0	1.400	1.395	1.217	1.218
	10^{-1}	1.400	1.395	1.191	1.197
	10^{-2}	1.400	1.396	1.171	1.183
	10^{-3}	1.400	1.397	1.150	1.158
	10^{-4}	1.400	1.399	1.130	1.130
	10^{-5}	1.400	1.401	1.116	1.119
	10^{-6}	1.400	1.404	1.105	1.105
	10^{-7}	1.400	1.406	1.093	1.091

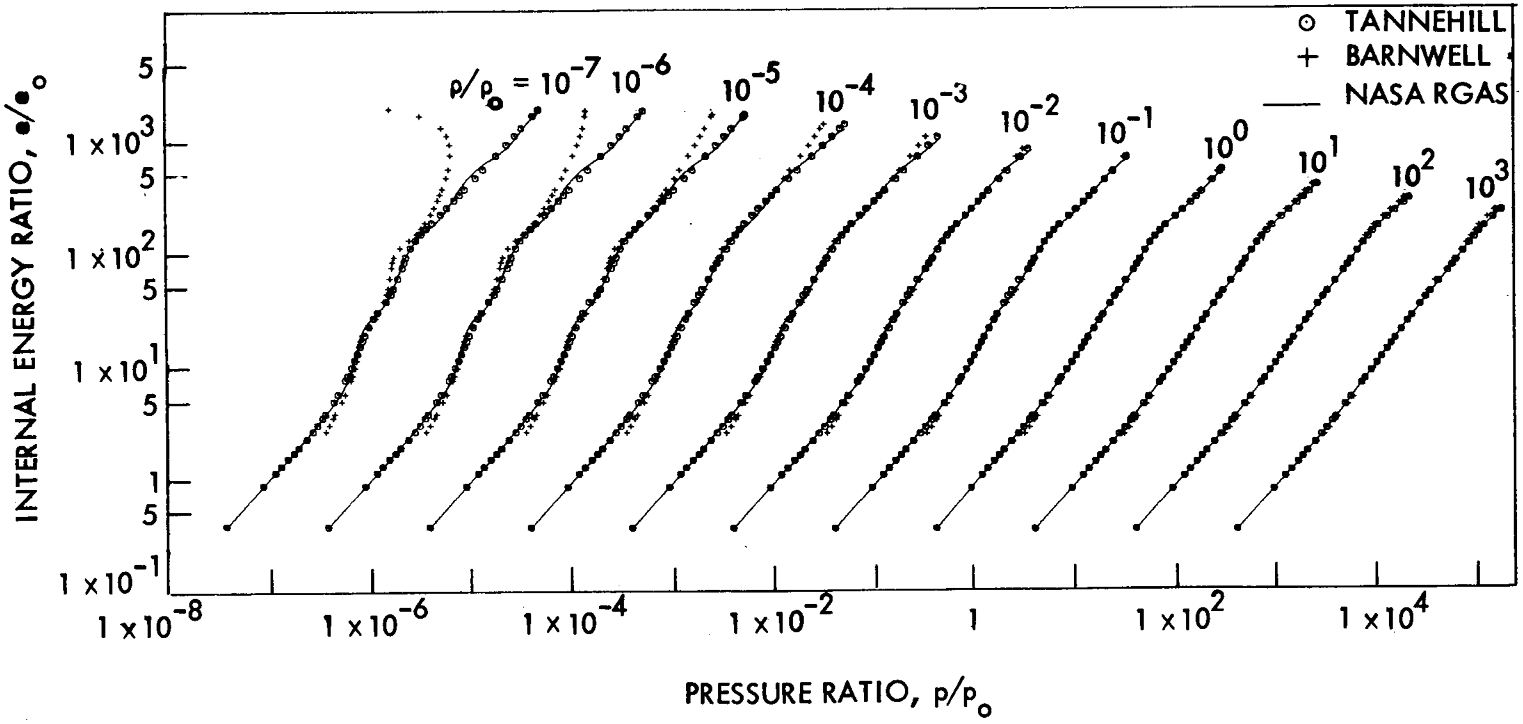

Fig. 9. Comparison of curve fits for $p = p(e, \rho)$.

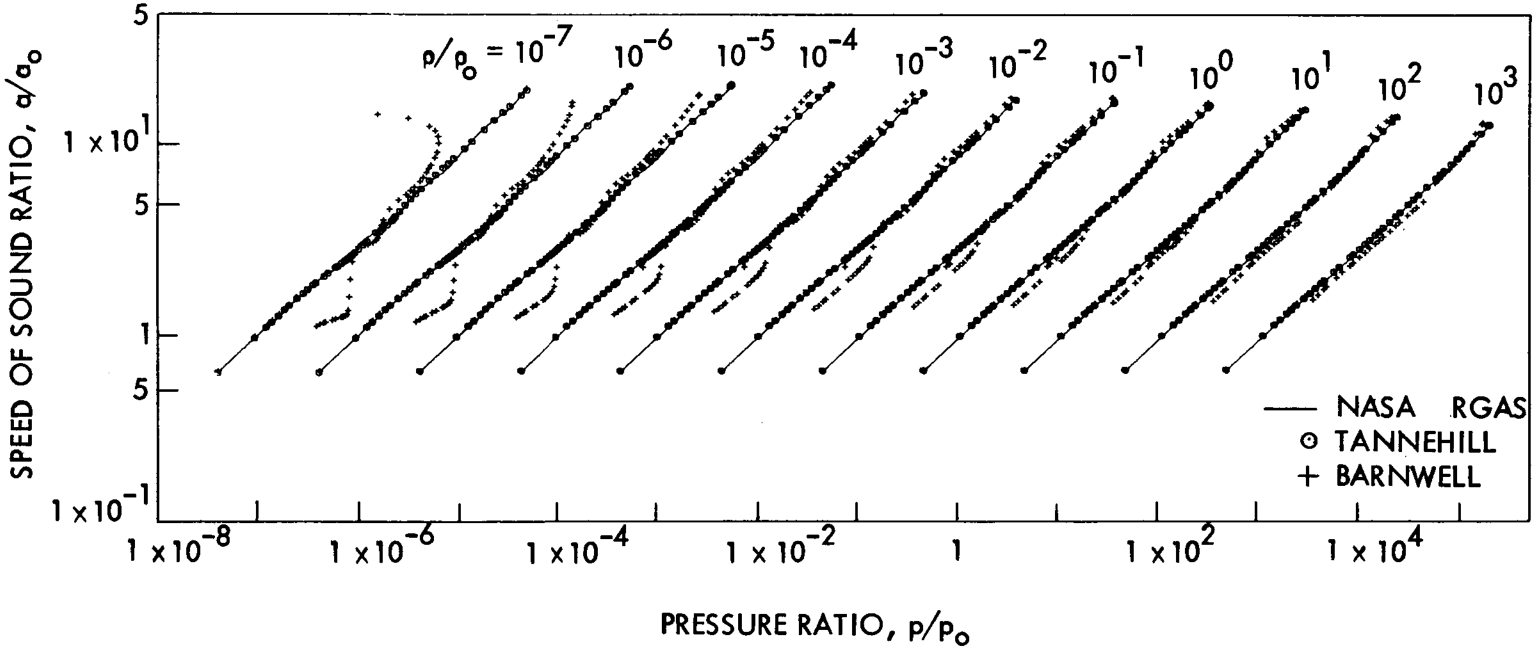

Fig. 10. Comparison of curve fits for $a = a(e,\rho)$.

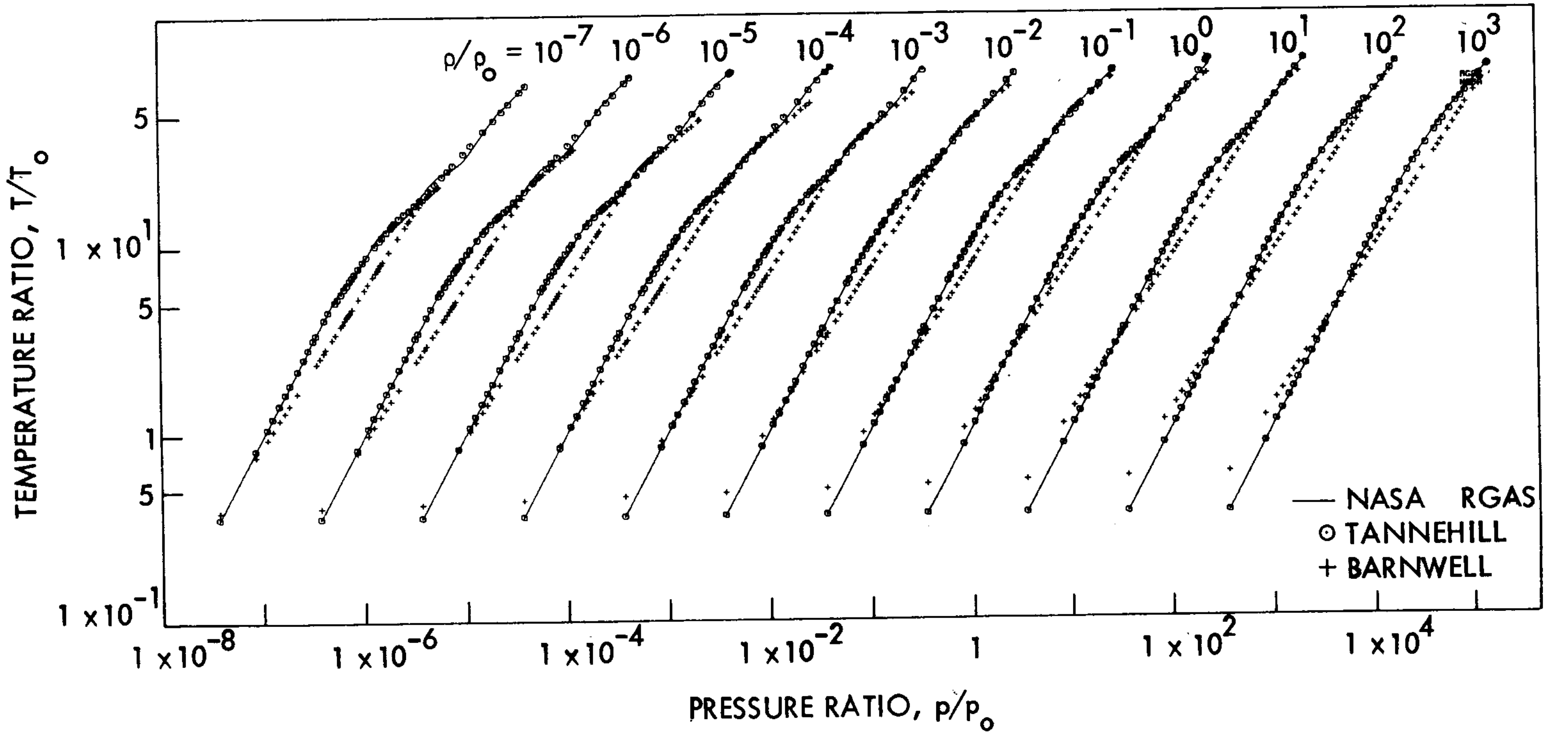

Fig. 11. Comparison of curve fits for $T = T(p,\rho)$.

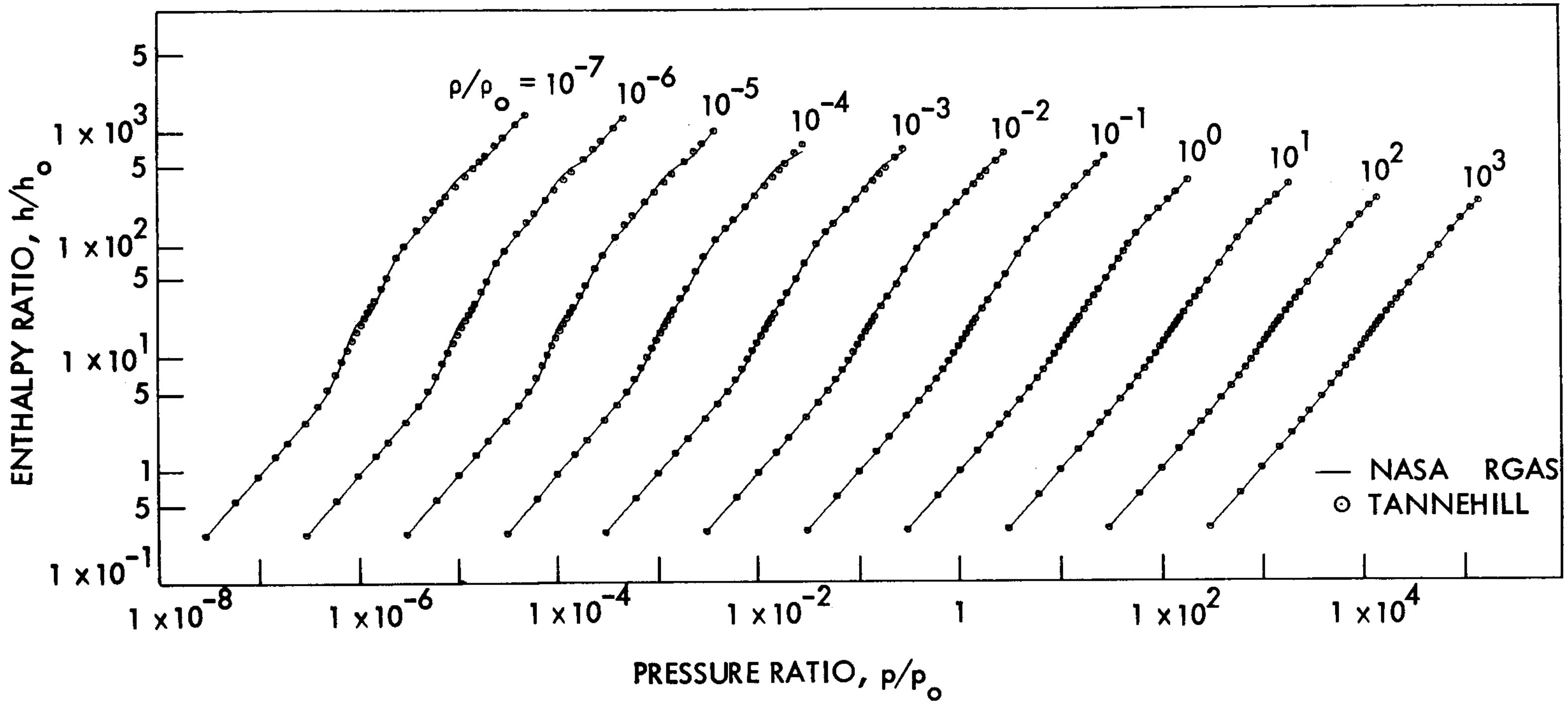

Fig. 12. Comparison of curve fits for $h = h(p,\rho)$.

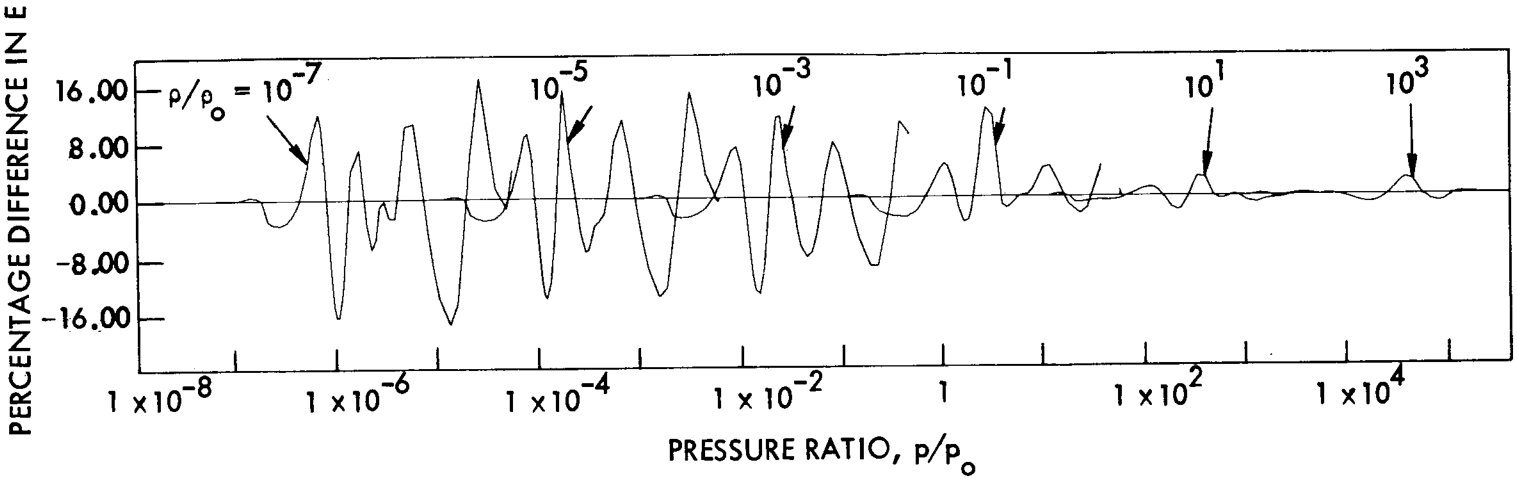

Fig. 13. Percentage difference between RGAS and TGAS for $p = p(e,\rho)$.

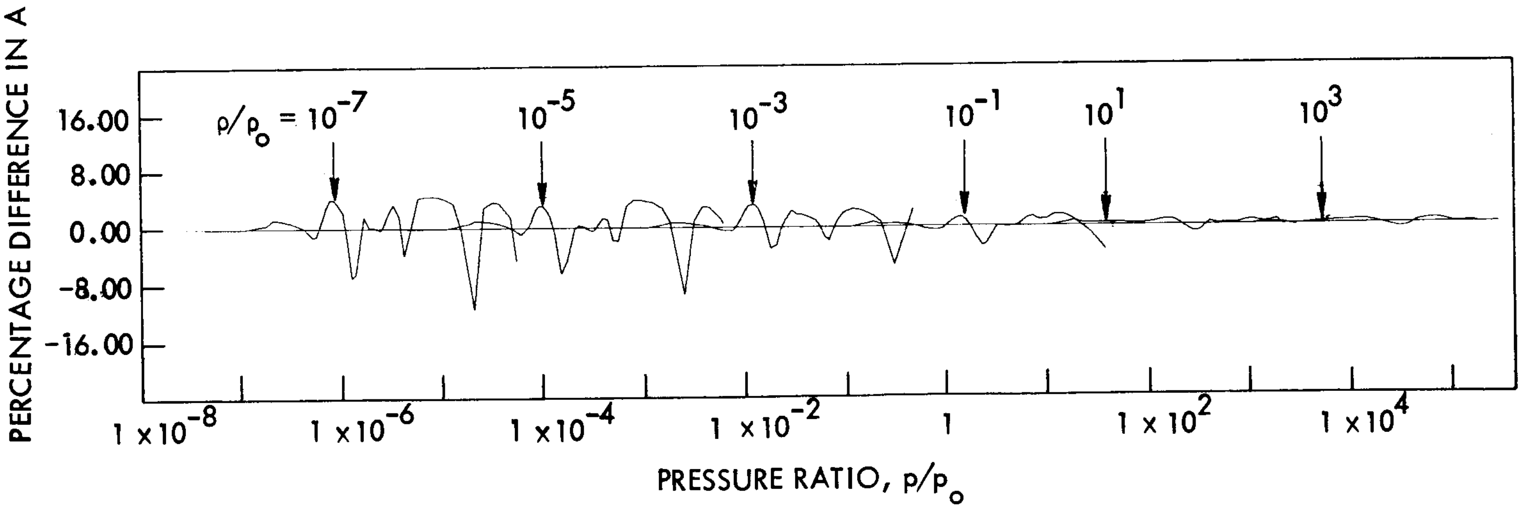

Fig. 14. Percentage difference between RGAS and TGAS for $a = a(e,\rho)$.

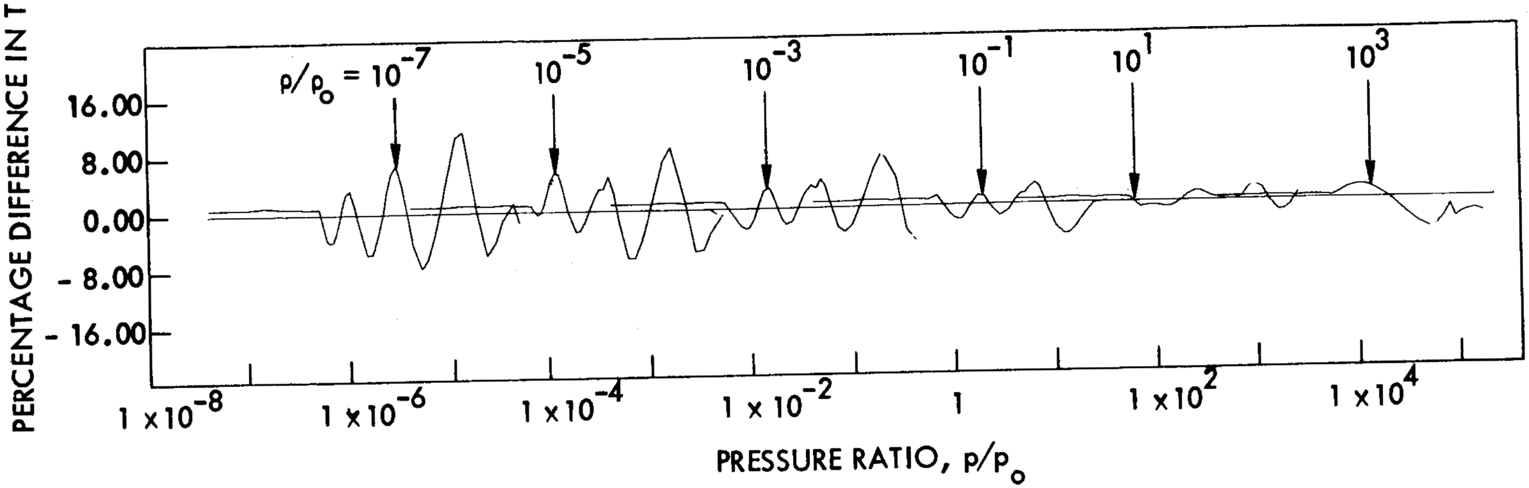

Fig. 15. Percentage difference between RGAS and TGAS for T = T(p, ρ).

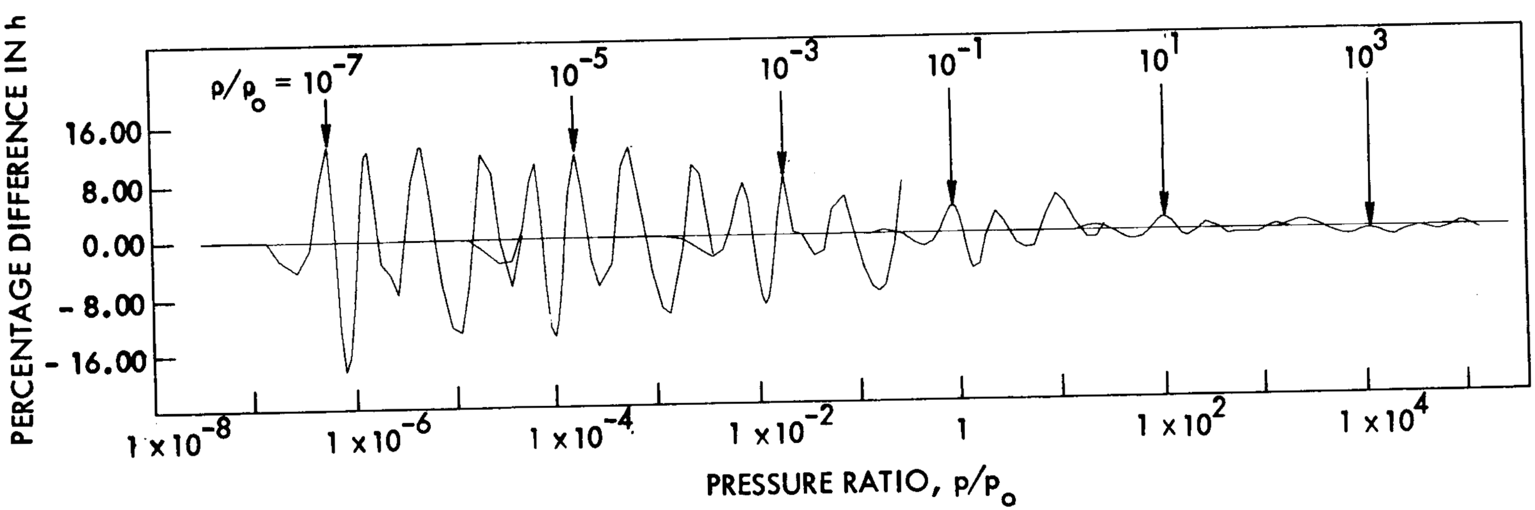

Fig. 16. Percentage difference between RGAS and TGAS for h = h(p, ρ).

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APPENDICES

APPENDIX A

Program for Generating Cubic Coefficients

```
      DIMENSION F(4),EE(4),V(4,4),U(4,4),FP(4),P(4),NTT(11),EXT(11,2)
      DIMENSION  A0(11,22),A1(11,22),A2(11,22),A3(11,22),EEE(11,22)
      DIMENSION TZ(3000),C(7),NDZ(89),NDL(4,11),NDU(4,11)
      EQUIVALENCE (NDZ,NDL),(NDZ(45),NDU)
      READ(9,201) WTMIX,(C(N),N=1,7)
      READ(9,200) (NDZ(N),N=1,89)
      NMM=NDZ(89)
      READ(9,201) (TZ(N),N=1,NMM)
      REWIND 9
      WRITE(6,2030)
      DO 120 N=1,88
  120 NDZ(N) = 5*NDZ(N)
      CONC = WTMIX/28.966
      P0 = 2116.
      R0 = .002498*CONC
      B = TZ(NMM-2)
      E = TZ(NMM-1)
      D = TZ(NMM)
      FM = 2.1632 + .3463*CONC
      DO 1000 NR =1,11
      NL = NDL(2,NR)
      NU = NDU(2,NR)
      NT = ((NU-NL)/5) + 1
      NTT(NR)= NT
      WRITE(6,2011) NR,NTT(NR)
      RHO = R0*10.**(NR-8)
      DO 900 I =1,NT
      ND = NL + 5*(I-1)
      FMAX = TZ(ND + 5)
      IF(I .EQ. NT) FMAX = FM - .00001
      F(1) = TZ(ND)
      F(2) = ((FMAX - TZ(ND))/3.) + TZ(ND)
      F(3) = (FMAX - TZ(ND))*(2./3.) + TZ(ND)
      F(4) = FMAX
      DO 800 J=1,4
      P(J) =P0*10.**(F(J)*(1.+E*ALOG10(RHO/R0)+ D*(ALOG10(RHO/R0))**2 )
     1 +ALOG10(RHO/R0) + B)
      CALL RGAS(P(J),RHO,A,H,T,S,RR,G,-1,2,2)
      EE(J)= H - P(J)/RHO
  800 CONTINUE
      WRITE(6,2010)  (F(K),K=1,4),(EE(K),K=1,4)
      DO 20 J=1,4
      DO 10 K=1,4
      V(K,J) =EE(K)**(J-1)
   10 U(K,J) = V(K,J)
   20 CONTINUE
      EPS = .000001
      CALL GELG(F,V,4,1,EPS,IER)
      IF(IER)50,60,50
   50 WRITE(6,2020) IER
      DO 70 J=1,4
      SUM = 0.
      DO 80 K=1,4
   80 SUM = SUM + U(J,K)*  F(K)
   70 FP(J)= SUM
      WRITE(6,2022) FP(1),FP(2),FP(3),FP(4)
   60 WRITE(6,2023) F(1),F(2),F(3),F(4),EE(1)
       A0(NR,I)  = F(1)
       A1(NR,I)  =  F(2)
       A2(NR,I)  =  F(3)
       A3(NR,I)  =  F(4)
```

```
      EEE(NR,I)  = FE(1)
  900 CONTINUE
      EXT(NR,1) = EEE(1,1)
      EXT(NR,2) = EE(4)
 1000 CONTINUE
      REWIND 9
      WRITE(6,44) (EXT(NR,1),NR=1,11),(EXT(NR,2),NR=1,11),B,E,D,WTMIX
      WRITE(10,44)(EXT(NR,1),NR=1,11),(EXT(NR,2),NR=1,11),B,E,D,WTMIX
      NDZ(1) = 1
      NSAVE = 0
      DO 22 II=2,11
      NDZ(II) = NTT(II-1) + 1 + NSAVE
      NDZ(II+10) = NTT(II-1) + NSAVE
      NSAVE = NDZ(II+10)
   22 CONTINLE
      NDZ(22) = NTT(11) +     NSAVE
      NDZ(23) = 5*NDZ(22)
      WRITE (6,2) (NDZ(N),N=1,23)
      WRITE(10,2) (NDZ(N),N=1,23)
      DO 42 I=1,11
      NT  = NTT(I)
      DO 41 J=1,NT
      WRITE(10,43) AO(I,J),A1(I,J),A2(I,J),A3(I,J),EEE(I,J)
   41 WRITE (6,43) AO(I,J),A1(I,J),A2(I,J),A3(I,J),EEE(I,J)
   42 CONTINLE
    2 FORMAT(16I8)
   43 FORMAT(5E16.8)
   44 FORMAT(6E16.8)
  200 FORMAT (16I8)
  201 FORMAT (8E16.8)
 2010 FORMAT(8F15.7)
 2011 FORMAT(1H ,5X,'NR=',I5,15X,'NT=',I5)
 2020 FORMAT(1H ,3X,'IER=',I4)
 2022 FORMAT(1H ,4X,'FP(1)=',E15.7,5X,'FP(2)=',E15.7,5X,'FP(3)=',E15.7,
     1     5X,'FP(4)=',E15.7)
 2023 FORMAT(1H ,4X,'AC=',F15.7,5X,'A1=',E14.7,5X,'A2=',E15.7,5X,'A3=',
     1     E15.7,5X,'E(1)=',E15.7)
 2030 FORMAT(1H1,4X,'F(1)',13X,'F(2)',11X,'F(3)',11X,'F(4)',11X,'E(1)',
     1 11X,'E(2)',11X,'E(3)',11X,'E(4)')
      STCP
      ENC
//GO.FT09F001  DC  UNIT=TAPE7,VOLUME=SER=T2145,DISP=(OLD,KEEP),        X
//             DCB=(DEN=1,TRTCH=ET,RECFM=U,BLKSIZE=132),LABEL=(2,NL)
```

APPENDIX B

CALLING SEQUENCE FOR MODIFIED RGAS

The modified RGAS subroutine is called by the following statement

```
CALL RGAS(P, R, A, H, T, S, RR, G, NTEST, NUMX, NGAS, E) with
```

P = Pressure lb/ft^2

R = Density, $slugs/ft^3$

A = Speed of sound, ft/sec

H = Enthalpy, ft^2/sec^2

T = Temperature, ^oR

S = Entropy, $ft^2/sec^2 - {}^oR$

RR = Gas constant, $ft^2/sec^2 - {}^oR$

G = Ratio of specific heats

E = Internal energy in ft^2/sec^2

NTEST = - 1 enter with P,R or P,S data for real gas

= 0 enter with P,R, or P,S data for perfect gas

= 1 enter with E,R data for real gas

		NTEST = - 1 or 0	NTEST = 1
NUMX = 1	Input =	P, R, RR, G	E, R, RR, G
	Output =	A	P, A
= 2	Input =	Same as NUMX = 1	Same as NUMX = 1
	Output =	A, H	P, A, H
= 3	Input =	Same as NUMX = 1	Same as NUMX = 1
	Output =	A, H, T	P, A, H, T
= 4	Input =	Same as NUMX = 1	Same as NUMX = 1
	Output =	A, H, T, S	P, A, H, T, S
= 5	Input =	P, S, RR, G	
	Output =	R, A, H, T	

NGAS = File number on permanent tape which contains desired gas.

APPENDIX C

Listing of Modified RGAS

```
      SUBROUTINE RGAS(PX,RX,AX,HX,TX,SX,RRX,GX,NTEST,NUMX,NGAS,EX)
      DIMENSION NLL(10),JXX(10),DZZ(10),TZ(3000),NDZ(89)
      DIMENSION TH(5,600),NDL(4,11),NDU(4,11),AN(4),C(7),ANR(17),BN(4)
      DIMENSION        TTZ(750),NNDZ(23),DH(5,150),EXT(11,2)
      EQUIVALENCE(TZ,TH),(NDZ,NDL),(NDZ(45),NDU),(TTZ,DH)
      FIRST = 1.0
      DATA KEY,NTIMES/0,0/
      DATA WORD1,WORD2/4HNUMH,4HNUML/
      DATA NFIRST/0/
      DATA NTAPE/9/
      DATA MFIRST,MTIMES/0,0/
  164 KEY=KEY+1
      P=PX
      S = SX
      R=RX
      NUM=NUMX
      IF(NUM) 1,1,2
    2 IF(NUM-5) 3,3,4
    4 WORD=WORD1
    6 WRITE(6,5) WORD
    5 FORMAT(12H0 ER IN RGAS,3X,A4)
   25 CALL EXIT
    1 WORD=WORD2
      GO TO 6
3     IF(NTEST) 7,8,500
  500 E = EX
      IF(MFIRST.GT.0) GO TO 531
      READ(10,499) (EXT(NR,1),NR=1,11),(EXT(NR,2),NR=1,11),B,E1,D,WTMIX
  499 FORMAT(6E16.8)
      READ(10,501)(NNDZ(N),N=1,23)
  501 FORMAT(16I8)
      NNMM=NNDZ(23)
      READ(10,502)(TTZ(N),N=1,NNMM)
  502 FORMAT(5E16.8)
      END FILE 10
      REWIND 10
      DO 503 N=1,22
  503 NNDZ(N)=5*NNDZ(N)
      CONC = WTMIX/28.966
      PO = 2116.
      RO = .002498*CONC
  531 R = ALOG10(R/RO)
      IF(R) 512,512,513
  512 NR = R-1.
      IF(NR+7)516,516,515
  516 NR = -7
      GO TO 515
  513 NR = R
      IF(NR-3) 515,514,514
  514 NR = 2
  515 DX = R - FLOAT(NR)
      NR = NR + 8
      IF(E.LT.EXT(NR,1)) GO TO 521
      IF(E.GT.EXT(NR,2)) GO TO 519
  525 NL =NNDZ(NR)
        IF(NLL(9)-NL) 601,602,601
  602 J = JXX(9)
      DIFF2 = E - DH(5,J)
      IF(DIFF2) 601,608,608
  608 IF(DZZ(9)-ABS(DIFF2)) 601,601,603
  601 NU = NNDZ(NR+11)
```

```
      CALL SERCH(E,DH,NL,NU,5,J,NER)
      J = J/5
      DZZ(9) = ABS(DH(5,J+1) - DH(5,J))
      JXX(9)=J
      NLL(9) = NL
  603 NL = NNDZ(NR+1)
      IF(NLL(10)-NL) 605,606,605
  606 K = JXX(10)
      DIFF2 = E - DH(5,K)
      IF(DIFF2) 605,609,609
  609 IF(DZZ(10)-ABS(DIFF2)) 605,605,607
  605 NU = NNDZ(NR + 12)
      CALL SERCH(E,DH,NL,NU,5,K,NER)
      K = K/5
      DZZ(10) = ABS(DH(5,K+1) - DH(5,K))
      JXX(10) = K
      NLL(10) = NL
  607 F1 = DH(1,J) + E*(DH(2,J) + E*(DH(3,J) +E*DH(4,J)))
      F2 = DH(1,K) + E*(DH(2,K) + E*(DH(3,K) +E*DH(4,K)))
      F = F1 + DX*(F2-F1)
      PX= PO*10.**(F*(1.+E1*R +D*R*R) +R +B)
  521 IF(E.LT.EXT(NR,1)) PX = RX*EX*.4
      P = PX
      R = RX
      MFIRST = 1
      GO TO 7
  519 MTIMES = MTIMES + 1
      WRITE(6,520) E
  520 FORMAT(1H0,10X,'OUTSIDE TABLES IN RGAS ENTERING WITH E=',E16.8)
      WRITE(6,523) RX
  523 FORMAT(48X,'R=',E16.8)
      IF(MTIMES-10) 525,525,25
    7 IF (NFIRST-NGAS) 10,9,10
   10 NFIRST=NGAS
C     CALL LOCATE(NGAS,NTAPE)
C
C                    FOR TAPE WRITTEN BY FORTRAN 2
C
      READ(9,201) WTMIX,(C(N),N=1,7)
C     READ (NTAPE) (NDZ(N),N=1,89)
      READ(9,200) (NDZ(N),N=1,89)
  200 FORMAT(16I8)
      NMM=NDZ(89)
C
C     READ (NTAPE) (TZ(N),N=1,NMM),WTMIX,(C(N),N=1,7)
      READ(9,201) (TZ(N),N=1,NMM),X,X,X,X,X,X,X,X
  201 FORMAT(8E16.8)
 1190 READ(9,201,END=1200) X,X,X,X,X,X,X,X
      GO TO 1190
 1200 CONTINUE
      REWIND 9
      DO 120 N=1,88
  120 NDZ(N)=5*NDZ(N)
      CONC=WTMIX/28.966
      PO=2116.
      RO=.002498*CONC
      RRR=1716./CONC
      RRX=RRR
      RTC=RRR*493.635
      SQPORO=SQRT (RO/PO)
      B=TZ(NMM-2)
```

```
      D=TZ(NMM)
      FM=2.1632+.3468*CONC
      AA=D*FM
      BB=E*FM+1.
      CCC=B+FM
    9 P=ALOG10(P/P0)
      E=TZ(NMM-1)
      GO TO (40,40,40,40,31,4,4,4),NUM
   31 REAL=S/RRR
      GG=(REAL-C(1)-C(2)*P)/(C(3)+P*(C(4)+P*C(5)))
  110 R=C(6)*GG+C(7)*P
      RL=P-B
      CC=CCC-P
      RH=-CC*(1.+AA*CC/(BB*BB))/BB+.005
      IF(RH+7.)183,185,185
  183 RH=-7.
  185 IF(R-RH) 180,181,181
  180 R=RH
  181 IF(3.-RL) 184,186,186
  184 RL=3.
  186 IF(RL-R) 182,163,163
  182 R=RL
  163 NUMB=0
      NIMX=0
   35 NUMM=5
      NBCT=9-NUM
      NUP=NBOT
      GO TO 42
40    R=ALOG10(R/R0)
      NUMM=5
      NBCT=1
      NUP=NUM
   42 CONTINUE
      IF(R) 12,12,13
12    NR=R-1.
      IF(NR+7) 16,16,15
16    NR=-7
      GO TO 15
13    NR=R
      IF(NR-3) 15,14,14
14    NR=2
   15 DX=R-FLOAT (NR)
      NR=NR+8
      F=(P-R-B)/(1.+R*(E+D*R))
      IF(NUMM-9+NUM) 22,162,22
  162 IF(F-.000001) 27,161,161
  161 IF(FM-F) 44,22,22
   22 DO 17 N1=NBOT,NUP
      IF(N1-NUMM) 36,81,36
   36 NER1=N1
      NER2=N1+4
      NL=NDL(N1,NR)
      IF(NLL(NER1)-NL) 301,302,301
  302 J=JXX(NER1)
      DIFF2=F-TH(5,J)
      IF(DIFF2) 301,308,308
  308 IF(DZZ(NER1)-ABS (DIFF2)) 301,301,303
301   NU=NDU(N1,NR)
      CALL SERCH(F,TH,NL,NU,5,J,NER)
      J=J/5
      DZZ(NER1)=ABS (TH(5,J+1)-TH(5,J))
```

```
      JXX(NER1)=J
      NLL(NER1)=NL
  303 XYZ=XYZ
      NL=NDL(N1,NR+1)
      IF(NLL(NER2)-NL) 305,306,305
  306 K=JXX(NER2)
      DIFF2=F-TH(5,K)
      IF(DIFF2) 305,309,309
 309  IF(DZZ(NER2)-ABS(DIFF2)) 305,305,307
  305 NU=NDU(N1,NR+1)
      CALL SERCH(F,TH,NL,NU,5,K,NER)
      K=K/5
      DZZ(NER2)=ABS (TH(5,K+1)-TH(5,K))
      JXX(NER2)=K
      NLL(NER2)=NL
  307 Y1=TH(1,J)+F*(TH(2,J)+F*(TH(3,J)+F*TH(4,J)))
  128 Y2=TH(1,K)+F*(TH(2,K)+F*(TH(3,K)+F*TH(4,K)))
      AN(N1)=Y1+DX*(Y2-Y1)
      GO TO 17
   81 AN(N1)=REAL
 17   CONTINUE
      IF(NUM-5) 51,52,52
   51 GO TO (121,122,123,124,124,124,124,124),NUM
  124 SX=AN(4)*RRR
 123  TX= AN(3)*1.8
  122 HX=AN(2)*RTO
  121 AX=AN(1)/SQPORO
      GO TO 109
   52 IF(NUMM-9+NUM) 39,108,39
  108 RX=RC*10.**R
      GO TO 51
   39 DIFF=ABS ((REAL-AN(NUP))/REAL)
      IF(DIFF-.C001)  37,37,38
   37 NUMM=9-NUM
      NBOT=1
      NUP=4
      GO TO 42
   38 NUMB=NUMB+1
      NIMX=NIMX+1
      IF(NIMX-20) 43,43,44
   43 IF(NUMB-2)82,83,84
   82 IF(REAL-AN(NUP)) 85,37,86
   85 R1=R
  141 S1=AN(NUP)
      R=R+.3
      IF(RL-R)150,99,99
  150 R=RL
   99 R2=R
  151 L=C
      GO TO 42
   86 R2=R
  153 S2=AN(NUP)
      R=R-.3
      IF(R-RH)142,102,102
  142 R=RH
  102 R1=R
  143 L=1
      GO TO 42
   83 IF(L) 91,90,91
   90 S2=AN(NUP)
  126 R=R2-(S2     -REAL)/(S2-S1)*(R2-R1)
```

```
      IF(RL-R) 187,93,93
  187 R=RL
      GO TO 93
   91 S1=AN(NUP)
  127 R=(REAL-S1      )/(S2-S1)*(R2-R1)+R1
      IF(R-RH) 188,93,93
  188 R=RH
   93 IF(R2-R) 104,37 ,105
  104 NUMB=1
      R1=R2
      S1=S2
      L=0
      IF(R2+.3-RL) 210,211,211
  211 R2=RL
      R=R2
      GO TO 42
  210 R2=R2+.3
      R=R2
      GO TO 42
  105 IF(R-R1) 106,37,42
  106 NUMB=1
      R2=R1
      S2=S1
      L=1
      IF(RH-R1+.3) 212,213,213
  213 R1=RH
      R=R1
      GO TO 42
  212 R1=R1-.3
      R=R1
      GO TO 42
   84 IF(REAL-AN(NUP)) 87,87,88
   87 R1=R
      GO TO 91
   88 R2=R
      GO TO 90
   44 IF(F-.000001) 27,444,444
  444 NTIMES=NTIMES+1
      WRITE(6,190)
  190 FORMAT(1H0,10X,36HOUTSIDE TABLES IN RGAS ENTERING WITH)
      WRITE(6,191) PX
  191 FORMAT(11X,2HP=,E13.6)
      IF(NUM-5) 192,193,193
  192 WRITE(6,194) RX
194   FORMAT(11X,2HR=E14.6)
      GO TO 196
  193 WRITE(6,195) SX
  195 FORMAT(11X,2HS=,E13.6)
  196 IF(NTIMES-10) 109,197,197
  197 WRITE(6,198)
  198 FORMAT(20X,28HEXIT CALLED ON TENTH FAILURE)
      GO TO 25
    8 L=0
      IF(GTEST-GX) 64,441,64
64    GTEST= GX
      L1=2
      ANR(1)=RRX
      ANR(2)=GX
      ANR(3)=ANR(1)/(ANR(2)-1.)
      ANR(4)=ANR(1)+ANR(3)
      ANR(8)=49008.609-ANR(3)*ALOG(171.6/.0001 **ANR(2))
```

```
  26 ANR(L+5)=1./ANR(L+2)
     ANR(L+6)=ANR(L+4)/ANR(L+1)
     ANR(L+7)=ANR(L+6)/ANR(L+2)
441  GO TO (440,440,440,440,69,70,71,72),NUM
 440 QUOD=P/R**ANR(L+2)
     QUOT=P/R
     GO TO(65,66,67,68,69,70,71,72),NUM
  68 S=ANR(L+8)+ANR(L+3)*ALOG(QUOD)
  67 T=QUOT/ANR(L+1)
  66 H=QUOT*ANR(L+6)
  65 LL=L+L1
     A=SQRT (ANR(LL  )*QUOT)
     GO TO 30
  69 EX=S-ANR(L+8)
     EX=EXP (EX/ANR(L+3))
     R=(P/EX)**ANR(L+5)
     QUOD=P/R**ANR(L+2)
     QUOT=P/R
     GO TO 67
  70 R=P/(T*ANR(L+1))
     QUOD=P/R**ANR(L+2)
     QUOT=P/R
     S=ANR(L+8)+ANR(L+3)*ALOG(QUOD)
     GO TO 66
  71 ASSIGN 65 TO NJUMP
  73 T=H/ANR(L+4)
     R=P/(T*ANR(L+1))
     QUOD=P/R**ANR(L+2)
     QUOT=P/R
     S=ANR(L+8)+ANR(L+3)*ALOG(QUOD)
     GO TO NJUMP,(65,30)
  72 ASSIGN 30 TO NJUMP
     H=ANR(L+7)*A**2
     GO TO 73
30   AX=A
     HX=H
     TX= T
     SX=S
     RX=R
 109 RETURN
  27 L=8
     P=PX
     R=RX
     IF(GTESTR-GX) 24,441,24
24   GTESTR=GX
     L1=9
     Z2=RO/10.**7
     PR=-7.+B
     PR=PC*10.**PR
     Z1=PR
     DO 21 N1=1,4
     NL=NDL(N1,1)
     NU=NDU(N1,1)
     F=0.
     CALL SERCH(F,TH,NL,NU,5,J,NER)
     J=J/5
  21 BN(N1)=TH(1,J)
     BN(1)=BN(1)/SQPORO
     BN(2)=BN(2)*RTO
     BN(3)=BN(3)*1.8
     BN(4)=BN(4)*RRR
```

```
      ANR(9)=PR/(Z2 *BN(3))
      RRX=ANR(9)
      ANR(12)=BN(2)/BN (3)
      ANR(10)=1.+ANR(9)/(ANR(12)-ANR(9))
      ANR(11)=ANR(12)/ANR(10)
      ANR(17)=BN(1)*BN(1)*Z2/Z1
      ANR(16)=BN(4)-ANR(11)*ALOG(Z1/Z2**ANR(10))
      LAST = 1
      GO TO 26
      END
//GO.FT09F001  DD  UNIT=TAPE7,VOLUME=SER=T2145,DISP=(OLD,PASS),          X
//             DCB=(DEN=1,TRTCH=ET,RECFM=U,BLKSIZE=132),LABEL=(2,NL)
//GO.FT10F001  DD  UNIT=TAPE7,VOLUME=SER=T2145,DISP=(NEW,KEEP),          X
//             DCB=(DEN=1,TRTCH=ET,RECFM=U,BLKSIZE=132),LABEL=(14,NL)
```

APPENDIX D

EQUATIONS FOR APPROXIMATE CURVE FITS

$\underline{p = p(e, \rho)}$

For the correlation of $p = p(e, \rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted using the following equation:

$$\tilde{\gamma} = a_1 + a_2 Y + a_3 Z + a_4 (Y)(Z) + \frac{a_5 + a_6 Y + a_7 Z + a_8 (Y)(Z)}{1 + \exp(a_9 + a_{10} Y + a_{11} Z)} \qquad \text{(D1)}$$

where $Y = \log_{10}(\rho/1.292)$ and $Z = \log_{10}(e/78408.4)$. The units of ρ are kg/m^3 and the units of e are m^2/sec^2. Once $\tilde{\gamma}$ is determined, $p(N/m^2)$ is found from

$$p = \rho e(\tilde{\gamma} - 1) \qquad \text{(D2)}$$

The coefficients a_1, a_2 .... a_{11} are given in Table 5.

$\underline{a = a(e, \rho)}$

The correlation for $a = a(e, \rho)$ is given by Eq. (15). The coefficients K_1, K_2, and K_3 are listed in Table 5.

$\underline{T = T(p, \rho)}$

The temperature is given by the following equations:

$$T = p/(287 \times \rho) \qquad Z \leq 1.30 \qquad \text{(D3)}$$

Table 5. Coefficients for curve fits $p = p(e,\rho)$ and $a = (e,\rho)$.

Density range	Curve range	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	κ_1	κ_2	κ_3
$Y > -0.50$	upper curve $Z > 2.3$	1.53746	-0.0411865	-0.151113	0.0256504	-0.305574	-0.0488379	0.135442	0.014528	27.08	0.320	-10.0	-0.0016115	0.198341	0.216819
	middle curve $0.65 < Z \le 2.3$	1.8379	-0.0140959	0.532783	-0.0347996	-3.81752	0.0571566	0.274508	0.0425175	1.943	0.032	-1.0	-0.0033884	0.150553	0.189652
	lower curve $Z \le 0.65$	1.40000	0	0	0	0	0	0	0	0	0	0	0	0	0
$-4.5 < Y \le -0.50$	upper curve $Z > 2.3$	1.58945	0.0956959	-0.174938	-0.0354141	-0.390197	-0.0455076	0.166996	0.0242439	27.08	0.320	-10.0	0.014277	0.259574	-0.192579
	middle curve $0.65 < Z \le 2.3$	2.10606	-0.052556	1.00142	-0.0302711	-5.83208	0.132134	0.276381	0.0043623	1.80	0.032	-1.0	-0.00605	0.147174	0.428416
	lower curve $Z \le 0.65$	1.40000	0	0	0	0	0	0	0	0	0	0	0	0	0
$-7 \le Y \le -4.5$	upper curve $Z > 2.02$	1.38221	-0.0069761	-0.0916976	0.0068208	-0.322099	0.0054549	0.118999	-0.0037667	27.08	-0.320	-10.0	0.007608	0.342076	0.023652
	middle curve $0.65 < Z \le 2.02$	1.35397	-0.0049197	0.0314908	-0.0733289	-0.390859	0.257944	-0.0637166	-0.0377134	3.40	0.064	-2.0	-0.0015942	0.11552	0.135562
	lower curve $Z \le 0.65$	1.40000	0	0	0	0	0	0	0	0	0	0	0	0	0

$$\log_{10}T = b_1 + b_2(Y - RO) + b_3 \frac{\exp[2.(RO - Y) - 3.4]}{RO + 10.}$$

$$+ b_4 \; |RO|^{2(Z) - 2.18} \qquad 1.30 < Z \le 2.6 \qquad \text{(D4)}$$

where $RO = \log_{10}(p/1.013 \times 10^5) + 2.42$ and

$$\log_{10}T = c_1 + c_2(RO) + c_3(Y - RO) + c_4 \frac{(Y - RO)}{1. - \exp[-50.25(Y - RO)]} \qquad \text{(D5)}$$

where $RO = 0.941 \log_{10}(p/1.013 \times 10^5) - 1.97.$ $\qquad Z > 2.6$

The units of T are ^{o}K. The coefficients b_1, b_2, b_3, b_4 and c_1, c_2, c_3, c_4 are given in Table 6.

Table 6. Coefficients for curve fit $T = T(p, \rho)$.

Density range	b_1	b_2	b_3	b_4	c_1	c_2	c_3	c_4
$Y > -0.50$	-0.450363	-1.16372	-1.37558	0.000355	4.17976	0.083386	-0.673668	0.071504
$-4.5 < Y \le -0.50$	-0.022868	-1.03203	-1.02635	0.000441	4.17471	0.06272	-0.537628	-0.052594
$-7 \le Y \le -4.5$	0.113084	-0.994532	-0.970778	0.001902	4.12619	0.05437	-0.551836	0.051818

Table 7. Coefficients for curve fit h = h(p,ρ).

Density range	Curve range	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}
$Y > -0.50$	upper curve $X > 1.40$	1.36839	-0.0032924	-0.103499	0.0124481	-0.341674	-0.0228219	0.171891	0.0089432	17.0	11.0	-10.0
	middle curve $0.25 < X \le 1.40$	1.40715	0.0020473	-0.0472791	-0.0068224	0.0414595	-0.0325854	-0.118581	0.040486	9.0	10.31	-10.0
	lower curve $X \le 0.25$	1.40000	0	0	0	0	0	0	0	0	0	0
$-4.5 < Y \le -0.50$	upper curve $X > 1.40$	1.26133	-0.0884646	-0.0387273	0.0633959	-0.183174	0.0874683	0.078411	-0.056993	17.0	11.0	-10.0
	middle curve $0.25 < X \le 1.40$	1.40668	0.0036616	-0.053836	-0.203309	0.0675256	0.0121155	-0.134637	0.0234357	9.0	10.31	-10.0
	lower curve $X \le 0.25$	1.40000	0	0	0	0	0	0	0	0	0	0
$-7 \le Y \le -4.5$	upper curve $X > 1.40$	2.04727	0.115191	-0.631801	-0.91035	-1.00382	-0.114356	0.677464	0.0942628	8.5	5.5	-5.0
	middle curve $0.25 < X \le 1.40$	1.40168	0.002569	-0.0644343	-0.0229381	0.0746109	0.0148297	-0.151756	0.0187665	9.0	10.31	-10.0
	lower curve $X \le 0.25$	1.40000	0	0	0	0	0	0	0	0	0	0

h = h(p, ρ)

For the correlation $h = h(p, \rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted using the following equation:

$$\tilde{\gamma} = d_1 + d_2Y + d_3X + d_4(X)(Y) + \frac{d_5 + d_6Y + d_7X + d_8(X)(Y)}{1 + \exp(d_9 + d_{10}Y + d_{11}X')}$$

where $X' = \log_{10}(p/1.013 \times 10^5)$ and $X = X' - Y$. The units for h are m^2/sec^2. Once $\tilde{\gamma}$ is determined, h is found from

$$h = (p/\rho)(\tilde{\gamma}/\tilde{\gamma} - 1) \qquad \text{(D6)}$$

The coefficients d_1, d_2, ... d_{11} are listed in Table 7.

APPENDIX E:

SUBROUTINE TGAS FOR $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(p, \rho)$

The calling statement for this subroutine is

```
CALL TGAS(E, RHO, P, T, A)
```

with

E - Internal energy in m^2/sec^2

RHO - Density in kg/m^3

P - Pressure in $newtons/m^2$

T - Temperature in ^oK

A - Speed of sound in m/sec

The following logic can be employed when the English system of units is used:

```
  EI = E * 0.0929
RHO1 = RHO * 515.4
CALL TGAS (E1, RHO1, P1, T1, A1)
P = P1 * 0.02088
T = T1 * 1.80
A = A1 * 3.281
```

with

E = Internal energy, ft^2/sec^2

RHO = Density, $slugs/ft^3$

P = Pressure, lbs/ft^2

T = Temperature, ^oR

A = Speed of sound, ft/sec

Listing of TGAS for $p = p\ (e,\rho)$, $a = a(e,\rho)$, and $T = T(p,\rho)$

```
      SUBROUTINE TGAS(E,RHO,P,T,A)
      Y2= ALOG10(RHO/1.292)
      Z2= ALOG10(E/78408.4)
      IF(Y2 .GT. -.50) GO TO 14
      IF(Y2 .GT. -4.50) GO TO 7
      IF(Z2.GT..650)GO TO 1
      GAMM=1.400
      SNDSQ= E*.560
      GO TO 4
    1 IF(Z2.GT.2.02) GO TO 2
      GAS1= 1.35397-.0049197*Y2
      GAS2= -.0314908+.0733289*Y2
      GAS3= .390859-.257944*Y2
      GAS4= -.0637166-.0377134*Y2
      GAS5= .0049197+.0733289*Z2
      GAS6= .257944-.0377134*Z2
      GAS7= EXP(-2.*Z2+.064*Y2+3.40)
      GAS8= -2.0
      GAS9= .064
      GAS12= .115520
      GAS13= .135562
      GAS14= -.00159424
      GO TO 3
    2 GAS1= 1.38221-.0069761*Y2
      GAS2= .0916976-.0068208*Y2
      GAS3= .322099-.0054549*Y2
      GAS4= .118599-.0037667*Y2
      GAS5= .0069761-.0068208*Z2
      GAS6= .0054549-.0037667*Z2
      GAS7= EXP(-10.*Z2+.320*Y2+27.08)
      GAS8= -10.0
      GAS9= .320
      GAS12= .342076
      GAS13= .0236520
      GAS14= .007608
    3 GAS10=1./(1.+GAS7)
      GAS11=(GAS3-GAS4*Z2)*GAS7*GAS10**2
      GAMM=GAS1-GAS2*Z2-(GAS3-GAS4*Z2)*GAS10
      GAME=2.304*(-GAS2+GAS4*GAS10+GAS11*GAS8)
      GAME= GAME*GAS12
      GAMR=2.304*(-GAS5+GAS6*GAS10+GAS11*GAS9)
      GAMR= GAMR*GAS13
      SNDSQ= E*((GAMM-1.)*(GAMM+GAME)+GAMR+GAS14)
    4 A= SQRT(ABS(SNDSQ))
      P= RHO*E*(GAMM-1.0)
      IF(Z2 .GT. 1.30) GO TO 5
      T= P/(287.*RHO)
      GO TO 102
    5 IF(Z2 .GT. 2.6) GO TO 6
      RO= ALOG10(P/1.013E+05)+2.42
      T= .113084-.994532*(Y2-RO)-.970778*(1./(RO+10.))*EXP(2.*((RO-Y2)
     1-3.40))+.001902*(ABS(RO))**(2.*Z2-2.18)
      GO TO 101
    6 RO= 0.941*ALOG10(P/1.013E+05)-1.970
      IF (Y2 .EQ. RO) GO TO 100
      T= 4.126190+.054370*RO-.551836*(Y2-RO)+.051818*(Y2-RO)/(1.0-
     1EXP(-50.25*(Y2-RO)))
      GO TO 101
    7 IF(Z2.GT..650)GO TO 8
      GAMM=1.400
      SNDSQ= E*.560
```

```
      GO TO 11
    8 IF(Z2.GT.2.3)GO TO 9
      GAS1= 2.10606-.0525560*Y2
      GAS2= -1.00142+.0302711*Y2
      GAS3= 5.83208-.132134*Y2
      GAS4= .276381+.0043623*Y2
      GAS5= .0525560+.0302711*Z2
      GAS6= .132134+.0043623*Z2
      GAS7= EXP(-Z2+.032*Y2+1.80)
      GAS8= -1.0
      GAS9= .032
      GAS12= .147174
      GAS13= .428416
      GAS14= -.00605
      GO TO 10
    9 GAS1= 1.58945+.0956959*Y2
      GAS2= .174938+.0354141*Y2
      GAS3= .390197+.0455076*Y2
      GAS4= .166996+.0242439*Y2
      GAS5= -.0956959+.0354141*Z2
      GAS6= -.0455076+.0242439*Z2
      GAS7= EXP(-10.0*Z2+.320*Y2+27.08)
      GAS8= -10.0
      GAS9= .320
      GAS12= .259574
      GAS13= -.192579
      GAS14= .014277
   10 GAS10=1./(1.+GAS7)
      GAS11=(GAS3-GAS4*Z2)*GAS7*GAS10**2
      GAMM=GAS1-GAS2*Z2-(GAS3-GAS4*Z2)*GAS10
      GAME=2.304*(-GAS2+GAS4*GAS10+GAS11*GAS8)
      GAME= GAME*GAS12
      GAMR=2.304*(-GAS5+GAS6*GAS10+GAS11*GAS9)
      GAMR= GAMR*GAS13
      SNDSQ= E*((GAMM-1.)*(GAMM+GAME)+GAMR+GAS14)
   11 A= SQRT(ABS(SNDSQ))
      P= RHO*E*(GAMM-1.0)
      IF(Z2 .GT. 1.30) GO TO 12
      T= P/(287.*RHO)
      GO TO 102
   12 IF(Z2 .GT. 2.6) GO TO 13
      RO= ALOG10(P/1.013E+05)+2.42
      T=-.022868-1.03203*(Y2-RO)-1.02635*(1./(RO+10.))*EXP(2.*((RO-Y2)
     1-3.40))+.000441*(ABS(RO))**(2.*Z2-2.18)
      GO TO 101
   13 RO= 0.941*ALOG10(P/1.013E+05)-1.970
      IF (Y2 .EQ. RO) GO TO 100
      T= 4.17471+.062720*RO-.537628*(Y2-RO)-.052594*(Y2-RO)/(1.0-
     1EXP(-50.25*(Y2-RO)))
      GO TO 101
   14 IF(Z2.GT..650)GO TO 15
      GAMM=1.400
      SNDSQ= E*.560
      GO TO 18
   15 IF(Z2.GT.2.3)GO TO 16
      GAS1= 1.8379-.0140959*Y2
      GAS2= -.532783+.0347996*Y2
      GAS3= 3.81752-.0571566*Y2
      GAS4= .274508+.0425175*Y2
      GAS5= .0140959+.0347996*Z2
      GAS6= .0571566+.0425175*Z2
```

```
      GAS7= EXP(-1.0*Z2+.032*Y2+1.943)
      GAS8= -1.0
      GAS9= .032
      GAS12= .150553
      GAS13= .189652
      GAS14= -.0033884
      GO TO 17
   16 GAS1= 1.53746-.0411865*Y2
      GAS2= .151113-.0256504*Y2
      GAS3= .305574+.0488379*Y2
      GAS4= .135442+.0145258*Y2
      GAS5= .0411865-.0256504*Z2
      GAS6= -.0488379+.0145258*Z2
      GAS7= EXP(-10.0*Z2+.320*Y2+27.08)
      GAS8= -10.0
      GAS9= .320
      GAS12= .198341
      GAS13= .216819
      GAS14= -.0016115
   17 GAS10=1./(1.+GAS7)
      GAS11=(GAS3-GAS4*Z2)*GAS7*GAS10**2
      GAMM=GAS1-GAS2*Z2-(GAS3-GAS4*Z2)*GAS10
      GAME=2.304*(-GAS2+GAS4*GAS10+GAS11*GAS8)
      GAME= GAME*GAS12
      GAMR=2.304*(-GAS5+GAS6*GAS10+GAS11*GAS9)
      GAMR= GAMR*GAS13
      SNDSQ= E*((GAMM-1.)*(GAMM+GAME)+GAMR+GAS14)
   18 A= SQRT(ABS(SNDSQ))
      P= RHO*E*(GAMM-1.0)
      IF(Z2 .GT. 1.30) GO TO 19
      T= P/(287.*RHO)
      GO TO 102
   19 IF(Z2 .GT. 2.6) GO TO 20
      RO= ALOG10(P/1.013E+05)+2.42
      T=-.450363-1.16372*(Y2-RO)-1.37558*(1./(RO+10.))*EXP(2.*((RO-Y2)
     1-3.40))+.000355*(ABS(RO))**(2.*Z2-2.18)
      GO TO 101
   20 RO= 0.941*ALOG10(P/1.013E+05)-1.970
      IF (Y2 .EQ. RO) GO TO 100
      T= 4.17976+.083386*RO-.673668*(Y2-RO)+.071504*(Y2-RO)/(1.0-
     1EXP(-50.25*(Y2-RO)))
      GO TO 101
  100 T= 4.174+.063*Y2
  101 T= 10.**T
  102 RETURN
      END
```

APPENDIX F

SUBROUTINE TGAS FOR $h = h(p,\rho)$

The calling statement for this subroutine TGAS is

```
CALL TGAS(P, RHO, H)
```

with

P = Pressure, newtons/m^2

RHO = Density, kg/m^3

H = Enthalpy, m^2/sec^2

The following logic can be employed when the English system of units is used:

```
  P1 = P/0.02088
RHO1 = RHO * 515.4
CALL TGAS(P1, RHO1, H1)
   H = H1/0.0929
```

with

P - Pressure in lbs/ft^2

RHO - Density in slugs/ft^3

H - Enthalpy in ft^2/sec^2

Listing of TGAS for $h = h(p,\rho)$

```
      SUBROUTINE TGAS(P,RHO,H)
      Y2= ALOG10(RHO/1.292)
      X2= ALOG10(P/1.013E+05)
      Z3= X2-Y2
      IF (Y2 .GT. -.50) GO TO 6
      IF (Y2 .GT. -4.50) GO TO 3
      IF(Z3 .GT. .250) GO TO 1
      H= (P/RHO)*3.50
      RETURN
    1 IF (Z3 .GT. 1.40) GO TO 2
      GAS1= 1.40168+.0025690*Y2
      GAS2= .0644343+.0229381*Y2
      GAS3= -.0746109-.0148297*Y2
      GAS4= -.151756+.0187665*Y2
      GAS5=EXP(-10.0*X2+10.31*Y2+9.0)
      GO TO 9
    2 GAS1= 2.04727+.115191*Y2
      GAS2= .631801+.0910350*Y2
      GAS3= 1.00382+.114356*Y2
      GAS4= .677464+.0942628*Y2
      GAS5=EXP(-5.0*X2+5.5*Y2+8.5)
      GO TO 9
    3 IF(Z3 .GT. .250) GO TO 4
      H= (P/RHO)*3.50
      RETURN
    4 IF (Z3 .GT. 1.40) GO TO 5
      GAS1= 1.40668+.0036616*Y2
      GAS2= .053836+.0203309*Y2
      GAS3= -.0675256-.0121155*Y2
      GAS4= -.134637+.0235357*Y2
      GAS5=EXP(-10.0*X2+10.31*Y2+9.0)
      GO TO 9
    5 GAS1= 1.26133-.0884646*Y2
      GAS2= .0387273-.0633959*Y2
      GAS3= .183174-.0874683*Y2
      GAS4= .0784110-.0569930*Y2
      GAS5=EXP(-10.0*X2+11.0*Y2+17.0)
      GO TO 9
    6 IF(Z3 .GT. .250) GO TO 7
      H= (P/RHO)*3.50
      RETURN
    7 IF (Z3 .GT. 1.40) GO TO 8
      GAS1= 1.40715+.0020473*Y2
      GAS2= .0472791+.0068224*Y2
      GAS3= -.0414595+.0325854*Y2
      GAS4= -.118581+.0404860*Y2
      GAS5=EXP(-10.0*X2+10.31*Y2+9.0)
      GO TO 9
    8 GAS1= 1.36839-.0032924*Y2
      GAS2= .103499-.0124481*Y2
      GAS3= .341674+.0228219*Y2
      GAS4= .171891+.0089432*Y2
      GAS5=EXP(-10.0*X2+11.0*Y2+17.0)
    9 GAMM=GAS1-GAS2*Z3-(GAS3-GAS4*Z3)/(1.+GAS5)
   10 H= (P/RHO)*(GAMM/(GAMM-1.))
      RETURN
      END
```