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THERMAL EFFECTS ON THE CLEARANCE AND STIFFNESS OF FOIL JOURNAL BEARINGS FOR A BRAYTON CYCLE TURBOALTERNATOR

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16. Abstract

An analysis of foil journal bearings for a NASA Brayton Cycle Unit (BRU) is presented. The study represents an extension of previous work in that it includes the effects of thermal expansion of foil-bearing components, as well as an improved model of the influence of foil flexure. The results presented give the bearing film thickness, the bearing stiffness, and the foil tension as functions of the operating temperatures and the elasto-hydrodynamic and geometrical parameters pertinent to the design of BRU foil bearings. A computer program for the evaluation of design data and for parametric studies is included.

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TABLE OF CONTENTS

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SUMMARY

An analysis of foil journal bearings for a NASA Brayton Cycle Unit (BRU) is presented. The study represents an extension of previous work in that it includes the effects of thermal expansion of foil-bearing components, as well as an improved model of the influence of foil flexure.

The BRU foil-bearing design, based on parameters previously established through a simplified thermal calculation, has been tested by the computer program developed in this work. The results presented herein estimate the range of the bearing film thickness, the bearing stiffness and the foil tension as functions of the expected operating temperatures and the elastohydrodynamic and geometrical parameters selected in this design. The parameter values chosen in the design to meet the requirements of stiffness and gap are confirmed. Results for alternative foil material and thickness combinations are also given to illustrate off-design conditions.

It is shown that the foil material is a sensitive variable, which the designer has at his disposal for controlling the bearing characteristics when other parameters have been firmly established.

1.0 INTRODUCTION

The three principal attributes of foil bearings -- (a) stability, (b) ability to accommodate thermal distortions, and (c) tolerance of foreign particles in the gas film -- have been demonstrated in the course of previous investigations [1] to [7]. The development has progressed to the point, at which pivoted-shoe bearings in one experimental Brayton Cycle Unit (Fig. 1) are to be replaced with foil bearings of the present design.

The objective of the analysis and data presented herewith is to provide a design tool and means of estimating the performance of similar types of foil bearings in smaller, or larger turbomachines.

Because of the complexities of foil bearing analysis, it has not been feasible to combine the time-dependence of the fluid film with a full treatment of the thermal, elastic and geometric aspects of the problem. In reference [5], however, it is shown for the case of a similar but idealized configuration, that damping assumes no negative values and that the foil bearing is, therefore, stable. At the same time, the stiffness does not differ appreciably from the quasi-static value in a wide band of frequencies of excitation. Therefore, without degrading the results, the more tractable quasi-static approach permits the inclusion of realistic thermal, elastic and geometric aspects of foil bearings in the analysis.

The present work represents an addition and a refinement of analyses included in references [1] and [6], in that a more realistic model is used in accounting for thermal effects, and refinements are introduced in estimating the influence of foil flexure. For continuity of presentation, certain portions of these analyses are included here with the added improvements.

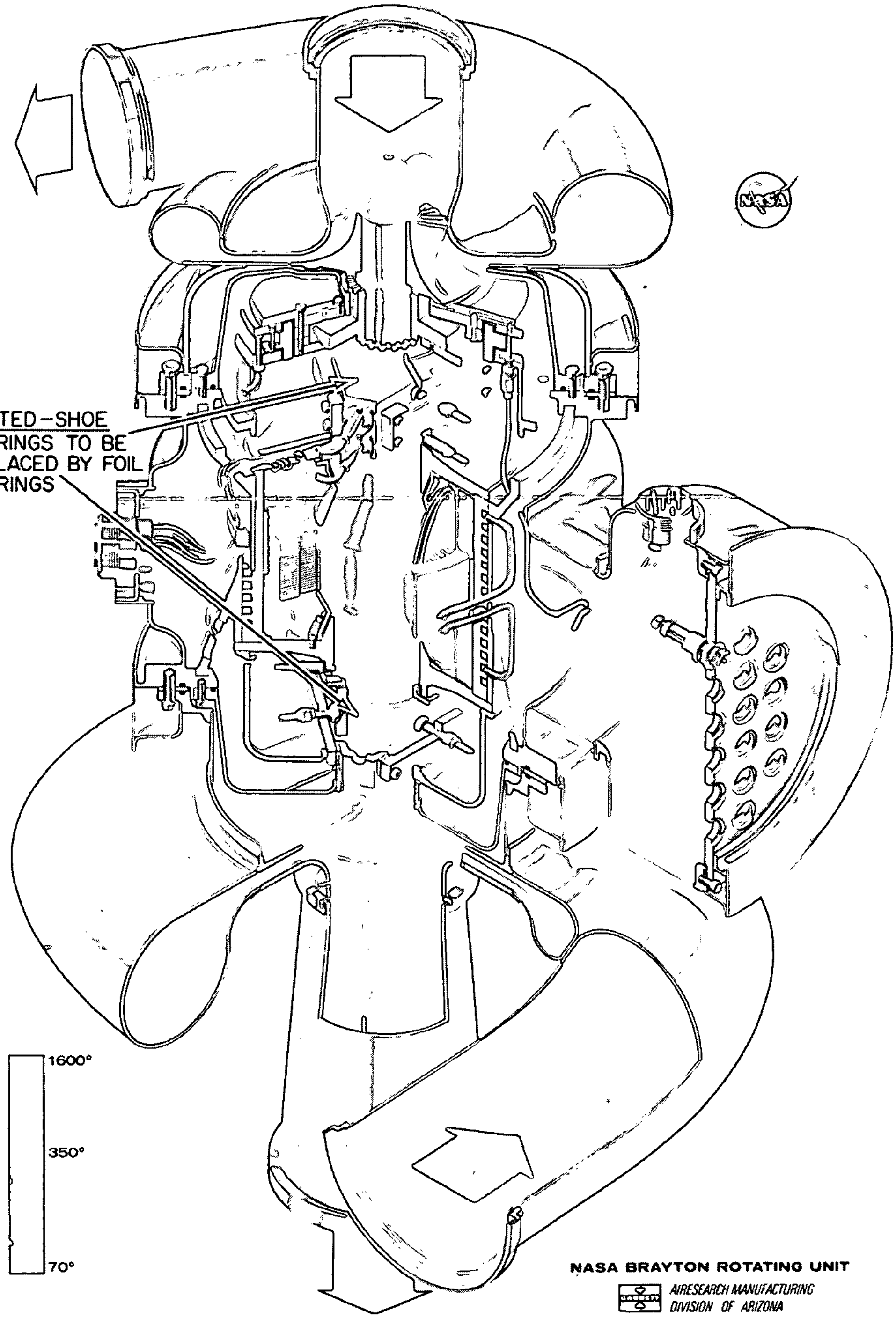

Fig. 1 NASA Brayton Cycle Unit with Pivoted Bearings. (Manufactured by AiResearch Co.) The pivoted-shoe bearings in one unit are being replaced by foil bearings designed and manufactured by the Ampex Corporation.

2.0 FORMULATION

Consider three "infinitely wide" foil sectors spaced at equal angular intervals and wrapped around a journal as shown schematically in Fig. 2. Each foil sector is supported by a kidney-shaped "shield-damper" and wrapped around the cylindrically milled ends, referred to as "guides". The shield-dampers are rigidly connected to plates parallel to the plane of the paper.

In the absence of radial load and fluid film and with a uniform temperature τ_0 and preload tension per unit width T_0 in each foil sector, the journal center is located at the point O, which will be taken as the origin. This state will be termed the "reference state". When rotation ensues, fluid films build up between the foil sectors and the journal, but under steady-state conditions, thermal axial symmetry and no radial load, the rotor center remains at O. Although the results of this report are limited to these conditions, the evaluation of stiffness requires consideration of rotor displacements from O. Consequently, the formulation starts with a general rotor position and is later specialized for axial symmetry.

Assume that at some instant, the journal is located at a point (x,y) in terms of a fixed Cartesian system of coordinates with origin at O. Consider, further, a set of three auxiliary coordinate systems x_k, y_k (k = 1, 2, 3) with origins at O, such that y_k is directed along the bisector of the kth foil arc, positive away from the foil and making an angle γ_k with the y axis. For the given shaft displacement (x,y), the coordinates (x_k, y_k) are given by the transformation

$$x_k = x \cos \gamma_k + y \sin \gamma_k \tag{1a}$$

$$y_k = -x \sin \gamma_k + y \cos \gamma_k \tag{1b}$$

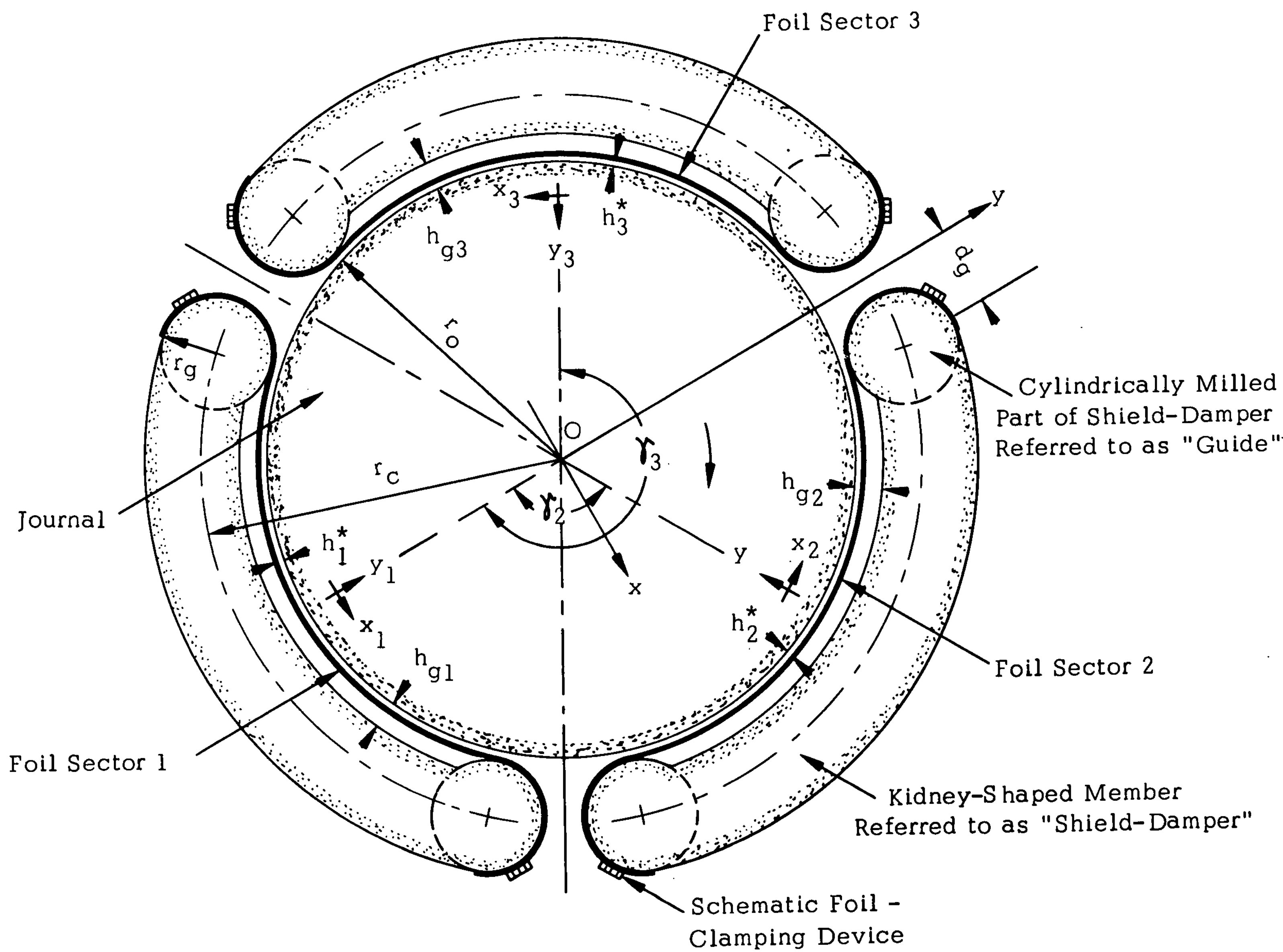

Fig. 2 Schematic Diagram of Foil Bearing Configuration

In Refs. [1] , [5] it was shown that for Θ the order of unity, the force of each foil sector on the rotor is virtually normal since the tangential force component is negligible. Furthermore, the frictional torque and its effect on tension variations may also be neglected. With these approximations, the resultant forces on the rotor are (Fig. 3)

$$F_x = -\sum_{k=1}^{3} T_k \left(\sin \alpha_{ik} + \sin \alpha_{ek}\right) \sin \gamma_k \tag{2a}$$

$$F_y = \sum_{k=1}^{3} T_k \left(\sin \alpha_{ik} + \sin \alpha_{ek}\right) \cos \gamma_k \tag{2b}$$

The subscripts i and e denote in the above " inlet" and "exit" respectively.

The foil length is considered to be comprised of (a) two nearly straight sections, (b) of a circular arc wrapping the rotor, and (c) of two circular arcs wrapping the guides. In addition, correction terms σ_{ik} and σ_{ek} are added to account for slack due to foil flexure (see Appendix A). Mathematically

$$\ell_k = b_{ik} + b_{ek} + (r_o + h_k^* + \delta r_o)(\alpha_{ik} + \alpha_{ek}) + (r_g + \delta r_{gk})(\beta_{ik} + \beta_{ek}) + \sigma_{ik} + \sigma_{ek} \tag{3}$$

In Eq. (3) δr_o, δr_{gk} represent the thermal* increments in r_o and r_{gk} relative to the reference state.

For evaluation of the thermal expansion of the structure and the elongation of the foil, the following thermal model will be assumed. The journal will be lumped at one temperature τ_r . Each shield-damper will be considered to be at a single temperature τ_{gk} . The free length of the foil will be assumed to be at a temperature τ_{fk} , whereas the

* δr_o may also include the centrifugal growth of the rotor.

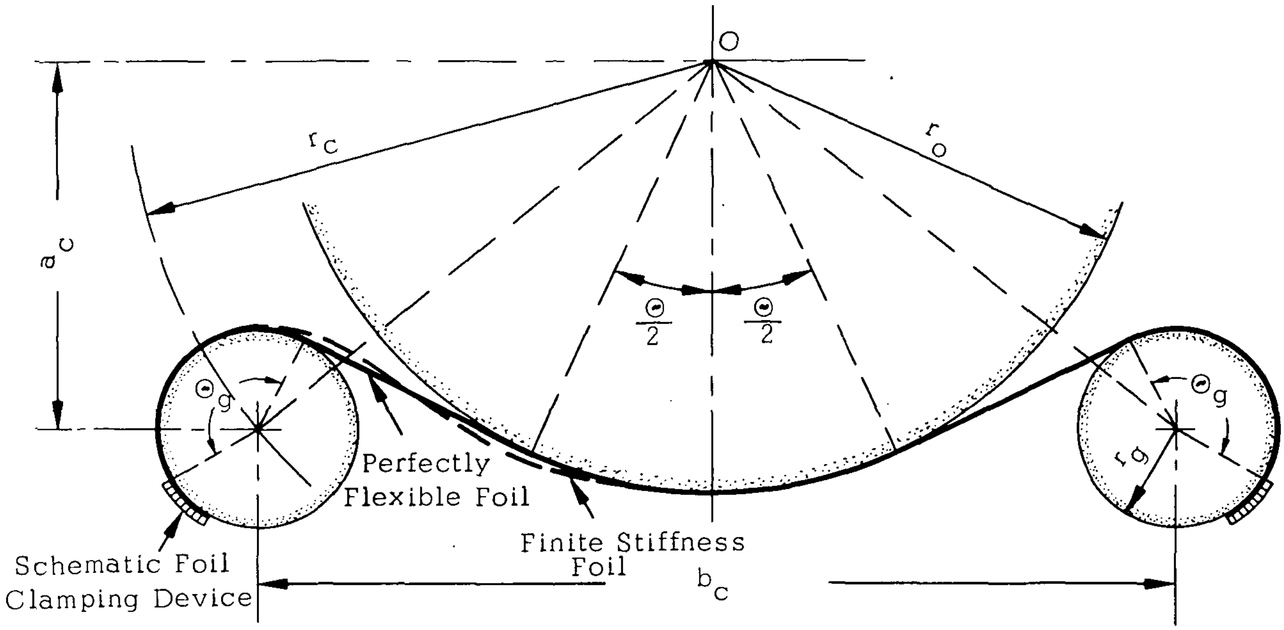

Fig. 3(a) Schematic Diagram of a Single Foil Sector at the Reference State (Tension T_o, No Rotation, Room Temperature τ_o, No Radial Load)

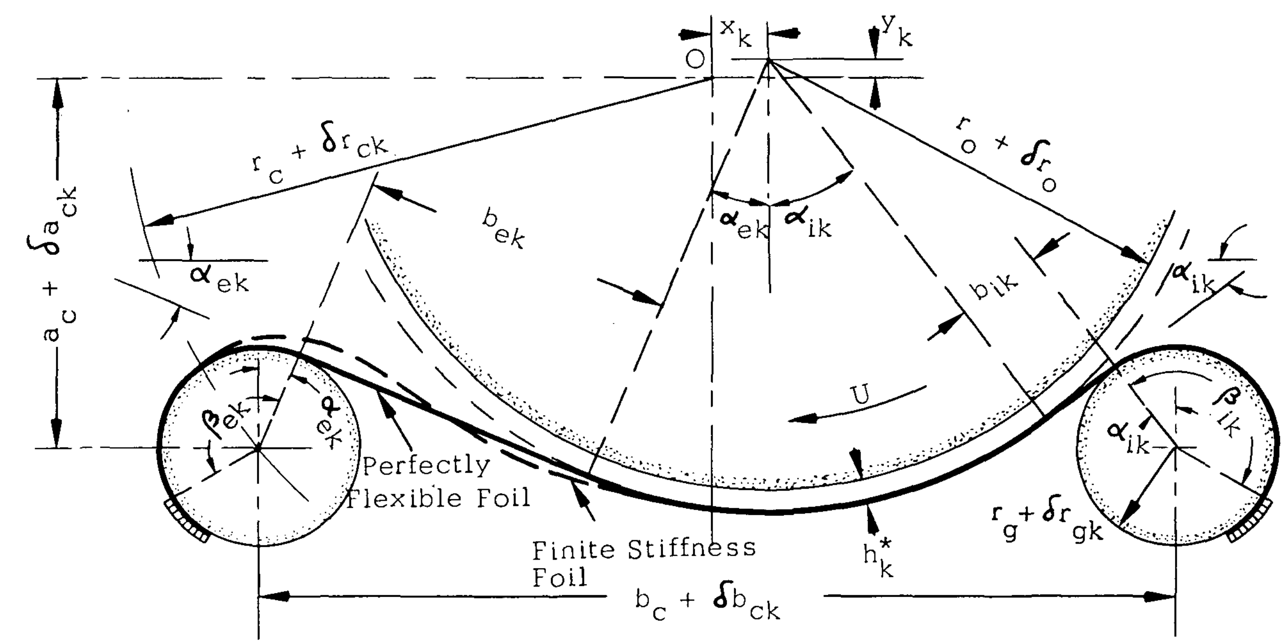

Fig. 3(b) Schematic Diagram of a Single Foil Sector at Operating Conditions the portion of the foil which is in contact with the guides will be assumed to be at the temperature τ_{gk}. The excess of these temperatures over τ_o, the temperature at the reference state, will be denoted respectively by $\delta\tau_r$, $\delta\tau_{gk}$, $\delta\tau_{fk}$. With the above assumptions, the elongation of the foil is:

$$\delta l_k = \frac{T_k - T_o}{Ed}\left(l_o - 2r_g\theta_g\right) + 2\frac{T_k - T_o}{Ed}\frac{r_g}{f}\left(1 - e^{-f\theta_g}\right) + \tag{4}$$

$$+ \alpha_f\left(l_o - 2r_g\theta_g\right)\delta\tau_{fk} + 2\alpha_f r_g\theta_g\,\delta\tau_{gk}$$

The following geometric relations complete the formulation:

$$\frac{b_c + \delta b_{ck}}{2} - x_k = \left(r_o + h_k^* + \delta r_o + r_g + \delta r_{gk}\right)\sin\alpha_{ik} + b_{ik}\cos\alpha_{ik} \tag{5a}$$

$$\frac{b_c + \delta b_{ck}}{2} + x_k = \left(r_o + h_k^* + \delta r_o + r_g + \delta r_{gk}\right)\sin\alpha_{ek} + b_{ek}\cos\alpha_{ek} \tag{5b}$$

$$a_c + \delta a_{ck} + y_k = \left(r_o + h_k^* + \delta r_o + r_g + \delta r_{gk}\right)\cos\alpha_{ik} - b_{ik}\sin\alpha_{ik} \tag{5c}$$

$$a_c + \delta a_{ck} + y_k = \left(r_o + h_k^* + \delta r_o + r_g + \delta r_{gk}\right)\cos\alpha_{ek} - b_{ek}\sin\alpha_{ek} \tag{5d}$$

To summarize, given the instantaneous journal position x,y and the temperature distribution (dictating δa_{ck}, δb_{ck}, δr_{gk}) and given the clearance h_k^*, the above set of 2+(3x8) equations can be solved for the unknowns F_x, F_y and x_k, y_k, T_k, α_{ik}, α_{ek}, b_{ik}, b_{ek}, l_k where k = 1, 2, 3.

3.0 SIMPLIFICATIONS

Considerable simplification without loss of generality is obtained if consideration is restricted to journal displacements and thermal expansions such that:

$$x_k\,,\ y_k\,,\ \delta r_o\,,\ \delta r_{gk}\,,\ \delta b_{ck}\,,\ \delta a_{ck}\,,\ h_k^* \ll r_o$$

$$\delta\alpha_{ik}\,,\ \delta\alpha_{ek}\,,\ \delta\beta_{ik}\,,\ \delta\beta_{ek} \ll \Theta$$

Neglecting products of perturbations from the reference state, the perturbation in foil length can be found from Eq. (3),

$$\delta\ell_k = \delta b_{ik} + \delta b_{ek} + (r_o + r_g)(\delta\alpha_{ik} + \delta\alpha_{ek}) + \\ + (h_k^* + \delta r_o)\Theta \quad + 2\delta r_{gk}\Theta_g \quad + \delta\sigma_{ik} + \delta\sigma_{ek} \tag{6}$$

The unknown differentials in Eq. (6) can be found by taking differentials of Eqs.(5).

$$x_k - \frac{\delta b_{ck}}{2} + (h^* + \delta r_o + \delta r_{gk}) \sin\frac{\Theta}{2} \quad + \delta b_{ik} \cos\frac{\Theta}{2} \; + \\ + \left[(r_o + r_g)\cos\frac{\Theta}{2} - \frac{\ell_o - r_o\Theta - 2r_g\Theta_g}{2} \sin\frac{\Theta}{2} \right] \delta\alpha_{ik} = 0 \tag{7a}$$

$$-x_k - \frac{\delta b_{ck}}{2} + (h_k^* + \delta r_o + \delta r_{gk}) \sin\frac{\Theta}{2} \quad + \delta b_{ek} \cos\frac{\Theta}{2} \; + \\ + \left[(r_o + r_g)\cos\frac{\Theta}{2} - \frac{\ell_o - r_o\Theta - 2r_g\Theta_g}{2} \sin\frac{\Theta}{2} \right] \delta\alpha_{ek} = 0 \tag{7b}$$

$$-y_k + \delta a_{ck} + (h_k^* + \delta r_o + \delta r_{gk}) \cos\frac{\Theta}{2} - \delta b_{ik} \sin\frac{\Theta}{2} \; - \\ - \left[(r_o + r_g)\sin\frac{\Theta}{2} + \frac{\ell_o - r_o\Theta - 2r_g\Theta_g}{2} \cos\frac{\Theta}{2} \right] \delta\alpha_{ik} = 0 \tag{7c}$$

$$-y_k + \delta a_{ck} + (h_k^* + \delta r_o + \delta r_{gk}) \cos\frac{\Theta}{2} - \delta b_{ek} \sin\frac{\Theta}{2} \; - \\ - \left[(r_o + r_g)\sin\frac{\Theta}{2} + \frac{\ell_o - r_o\Theta - 2r_g\Theta_g}{2} \cos\frac{\Theta}{2} \right] \delta\alpha_{ek} = 0 \tag{7d}$$

Using Eqs.(7), one can deduce the relations

$$\delta\alpha_{ik} + \delta\alpha_{ek} = \frac{2(h_k^* + \delta r_o + \delta r_{gk}) - 2(y_k + \delta a_{ck})\cos\frac{\Theta}{2} - \delta b_{ck}\sin\frac{\Theta}{2}}{\frac{l_o - r_o\Theta - 2r_g\Theta_g}{2}} \tag{8a}$$

$$\delta b_{ik} + \delta b_{ek} = \delta b_{ck}\cos\frac{\Theta}{2} - 2(y_k + \delta a_{ck})\sin\frac{\Theta}{2} - (r_o + r_g)(\delta\alpha_{ik} + \delta\alpha_{ek}) \tag{8b}$$

Substituting Eq. (8) into Eq. (6), and Eq. (6) into Eq. (4) yields:

$$\frac{T_k - T_o}{Ed} = \frac{-2y_k\sin\frac{\Theta}{2} + h_k^*\Theta + \delta\sigma_{ik} + \delta\sigma_{ek} - \delta l_{\tau k}}{l_{eff}} \tag{9}$$

where

$$\delta l_{\tau k} = \alpha_f(l_o - 2r_g\Theta_g)\delta\tau_{fk} + (2\alpha_f r_g\Theta_g - 2\alpha_g r_g\Theta_g)\delta\tau_{gk} - \alpha_r r_o\Theta\delta\tau_r - \delta b_{ck}\cos\frac{\Theta}{2} + 2\delta a_{ck}\sin\frac{\Theta}{2} \tag{10}$$

$$l_{eff} = l_o - 2r_g\Theta_g + \frac{2}{f}r_g(1 - e^{-f\Theta_g}) \tag{11}$$

$$h_k^* = r_o H_k^* \left(\frac{6\mu_o U}{T_k}\right)^{2/3} \quad [1] \tag{12a}$$

$$H_k^* \cong 0.643 + 0.286\left(\frac{\frac{1}{2}\rho_a U^2}{T_k/r_o}\right) + 1.905\left(\frac{\frac{1}{2}\rho_a U}{T_k/r_o}\right)^2 - 0.183\frac{T_k}{p_a r_o} \tag{12b}$$

[8]

$$\frac{\delta r_c}{r_c}(l_o - r_o\Theta - 2r_g\Theta_g) = \delta b_{ck}\cos\frac{\Theta}{2} - 2\delta a_{ck}\sin\frac{\Theta}{2} \tag{13a}$$

$$\alpha_g r_c \delta\tau_g = \delta r_c \tag{13b}$$

The above equations deserve a few comments. The variable ℓ_{eff} given by Eq. (11) describes the effective foil length, taking into account the fact that extension of part of the foil is restrained due to frictional contact with the guides. Equation (12b) gives an approximate curve fit to account for the effects of inertia and compressibility of the fluid.[8] Finally, Eqs. (13) state the assumption that the shield-dampers and their supporting structure are at sensibly uniform temperatures and the fact that their coefficients of thermal expansion are equal.

Using simplifications consistent with the derivation of the foregoing equations, the bearing forces on the rotor can be reduced from Eq.(2) to the form:

$$F_x = \sqrt{3}\,\sin\frac{\Theta}{2}\,(T_3 - T_2) \tag{14a}$$

$$F_y = 2\,\sin\frac{\Theta}{2}\left(T_1 - \frac{T_2}{2} - \frac{T_3}{2}\right) \tag{14b}$$

Since axial symmetry is assumed for the present bearing, the subscripts k may be omitted in Eqs. (9) - (13). An iterative solution for T and h* is incorporated in the program given in Appendix B.

4.0 STIFFNESS DETERMINATION

The evaluation of the equivalent spring constants of the foil-bearing system, requires differentiation of Eqs. (14), for which the derivatives $\frac{\partial T_k}{\partial x}, \frac{\partial T_k}{\partial y}$ are needed. The evaluation of these expressions from Eq.(9) necessitates, in turn, finding the magnitudes of

$$\frac{\partial \sigma_{ik}}{\partial x_k}, \frac{\partial \sigma_{ik}}{\partial y_k}, \frac{\partial \sigma_{ek}}{\partial x_k}, \frac{\partial \sigma_{ek}}{\partial y_k}$$

For this purpose, use will be made of the following functional forms derived from Eqs. (5) and Eq. (A26) of Appendix A respectively:

$$b_{ik} = f(x_k, y_k, r_o) \tag{15a}$$

$$\sigma_{ik} = f(b_{ik}, r_o, T_k) \tag{15b}$$

with similar expressions for b_{ek}, σ_{ek}. For simplicity, the dependence upon additional parameters which are not affected by rotor displacement has not been shown in Eqs. (15). It should also be noted that the shown dependence upon r_o stems from the fact that for the purpose considered in this section the radius extends to the foil and includes h*, which is an implicit function of x and y.

Differentiation, therefore, gives:

$$\left(\frac{\partial \sigma_{ik}}{\partial x_k}\right)_{y_k} = \left(\frac{\partial \sigma_{ik}}{\partial b_{ik}}\right)_{\substack{r_0\\T_k}} \left[\left(\frac{\partial b_{ik}}{\partial x_k}\right)_{\substack{y_k\\r_0}} + \left(\frac{\partial b_{ik}}{\partial r_0}\right)_{\substack{x_k\\y_k}} \left(\frac{\partial h_k^*}{\partial x_k}\right)_{y_k}\right] +$$

$$+ \left(\frac{\partial \sigma_{ik}}{\partial r_0}\right)_{\substack{b_{ik}\\T_k}} \left(\frac{\partial h_k^*}{\partial x_k}\right)_{y_k} + \left(\frac{\partial \sigma_{ik}}{\partial T_k}\right)_{\substack{b_{ik}\\r_0}} \left(\frac{\partial T_k}{\partial x_k}\right)_{y_k} \tag{16}$$

Similar expressions are obtained by replacing i by e and x_k by y_k. Differentiating Eq. (9), with respect to x_k, y_k and collecting terms gives respectively;

$$\left(\frac{\partial T_k}{\partial x_k}\right)_{y_k} = \tag{17}$$

$$= \frac{\left(\frac{\partial \sigma_{ik}}{\partial b_{ik}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ik}}{\partial x_k}\right)_{\substack{y_k\\r_0}} + \left(\frac{\partial \sigma_{ek}}{\partial b_{ek}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ek}}{\partial x_k}\right)_{\substack{y_k\\r_0}}}{\frac{l_{eff}}{Ed} - r_0\left[\frac{6\mu_a U}{T_k}\right]^{\frac{2}{3}} \left[\frac{\partial H_k^*}{\partial T_k} - \frac{2}{3}\frac{H_k^*}{T_k}\right]\left[\theta + \left(\frac{\partial \sigma_{ik}}{\partial r_0}\right)_{\substack{b_{ik}\\T_k}} + \left(\frac{\partial \sigma_{ek}}{\partial r_0}\right)_{\substack{b_{ek}\\T_k}} + \left(\frac{\partial \sigma_{ik}}{\partial b_{ik}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ik}}{\partial r_0}\right)_{\substack{x_k\\y_k}} + \left(\frac{\partial \sigma_{ek}}{\partial b_{ek}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ek}}{\partial r_0}\right)_{\substack{x_k\\y_k}}\right] - \left[\frac{\partial \sigma_{ik}}{\partial T_k}\right]_{\substack{b_{ik}\\r_0}} - \left[\frac{\partial \sigma_{ek}}{\partial T_k}\right]_{\substack{b_{ek}\\r_0}}}$$

$$\left(\frac{\partial T_k}{\partial y_k}\right)_{x_k} = \tag{18}$$

$$= \frac{-2\sin\frac{\theta}{2} + \left(\frac{\partial \sigma_{ik}}{\partial b_{ik}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ik}}{\partial y_k}\right)_{\substack{x_k\\r_0}} + \left(\frac{\partial \sigma_{ek}}{\partial b_{ek}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ek}}{\partial y_k}\right)_{\substack{x_k\\r_0}}}{\frac{l_{eff}}{Ed} - r_0\left[\frac{6\mu_a U}{T_k}\right]^{\frac{2}{3}} \left[\frac{\partial H_k^*}{\partial T_k} - \frac{2}{3}\frac{H_k^*}{T_k}\right]\left[\theta + \left(\frac{\partial \sigma_{ik}}{\partial r_0}\right)_{\substack{b_{ik}\\T_k}} + \left(\frac{\partial \sigma_{ek}}{\partial r_0}\right)_{\substack{b_{ek}\\T_k}} + \left(\frac{\partial \sigma_{ik}}{\partial b_{ik}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ik}}{\partial r_0}\right)_{\substack{x_k\\y_k}} + \left(\frac{\partial \sigma_{ek}}{\partial b_{ek}}\right)_{\substack{r_0\\T_k}} \left(\frac{\partial b_{ek}}{\partial r_0}\right)_{\substack{x_k\\y_k}}\right] - \left[\frac{\partial \sigma_{ik}}{\partial T_k}\right]_{\substack{b_{ik}\\r_0}} - \left[\frac{\partial \sigma_{ek}}{\partial T_k}\right]_{\substack{b_{ek}\\r_0}}}$$

For the particular case of no static load, the equilibrium position is $x = y = 0$ and consequently, at this position

$$T_1 = T_2 = T_3 = T \tag{19a}$$

$$\sigma_{ik} = \sigma_{ek} = \sigma \tag{19b}$$

$$b_{ik} = b_{ek} = b \tag{19c}$$

$$h_1^* = h_2^* = h_3^* = h^* \tag{19d}$$

$$\frac{\partial b_{ik}}{\partial x_k} = -\frac{\partial b_{ek}}{\partial x_k} \tag{19e}$$

$$\frac{\partial b_{ik}}{\partial y_k} = \frac{\partial b_{ek}}{\partial y_k} \tag{19f}$$

$$\frac{\partial \sigma_{ik}}{\partial b_{ik}} = \frac{\partial \sigma_{ek}}{\partial b_{ek}} \tag{19g}$$

It follows from Eqs. (17) and (18) that under these conditions

$$\frac{\partial T_k}{\partial x_k} = 0 \tag{20a}$$

$$\frac{\partial T_k}{\partial y_k} = -A \tag{20b}$$

where

$$A = \frac{2\sin\frac{\Theta}{2} - 2\left(\frac{\partial\sigma}{\partial b}\right)_{\substack{r_0\\T}}\left(\frac{\partial b}{\partial y_k}\right)_{\substack{x_k\\r_0}}}{\frac{l_{eff}}{Ed} - r_0\left[\frac{6\mu_a U}{T}\right]^{2/3}\left[\frac{\partial H^*}{\partial T} - \frac{2}{3}\frac{H^*}{T}\right]\left[\Theta + 2\left(\frac{\partial\sigma}{\partial r_0}\right)_{\substack{b\\T}} + 2\left(\frac{\partial\sigma}{\partial b}\right)_{\substack{r_0\\T}}\left(\frac{\partial b}{\partial r_0}\right)_{\substack{x_k\\y_k}}\right] - 2\left[\frac{\partial\sigma}{\partial T}\right]_{\substack{b\\r_0}}}$$

This allows evaluation of $\frac{\partial T_k}{\partial x}, \frac{\partial T_k}{\partial y}$ by means of formulae of the form:

$$\frac{\partial T_k}{\partial x} = \frac{\partial T_k}{\partial x_k}\frac{\partial x_k}{\partial x} + \frac{\partial T_k}{\partial y_k}\frac{\partial y_k}{\partial x} \tag{21}$$

Thus,

$$\frac{\partial T_k}{\partial x} = A \sin\gamma_k \qquad \begin{cases} \sin\gamma_1 = 0 \\ \sin\gamma_2 = \frac{\sqrt{3}}{2} \\ \sin\gamma_3 = -\frac{\sqrt{3}}{2} \end{cases} \tag{22a}$$

$$\frac{\partial T_k}{\partial y} = -A \cos\gamma_k \qquad \begin{cases} \cos\gamma_1 = 1 \\ \cos\gamma_2 = -\frac{1}{2} \\ \cos\gamma_3 = -\frac{1}{2} \end{cases} \tag{22b}$$

It follows from Eq. (14) that the stiffness matrix of the bearing at the point O is composed of the terms:

$$k_{xx} = -\frac{\partial F_x}{\partial x} = 3 \sin\frac{\Theta}{2}\ A \tag{23a}$$

$$k_{yy} = -\frac{\partial F_y}{\partial y} = 3 \sin\frac{\Theta}{2}\ A \tag{23b}$$

$$k_{xy} = -\frac{\partial F_x}{\partial y} = 0 \tag{23c}$$

$$k_{yx} = -\frac{\partial F_y}{\partial x} = 0 \tag{23d}$$

5.0 THERMAL ANALYSIS

5.1 Introductory Remarks

It was shown how the operating tension, the fluid film thickness and the bearing stiffness could be determined, provided that lumped temperatures of the foil, the shield-damper-guide and the rotor were known. The present section deals with the determination of foil temperature and gives formulas for the evaluation of temperature-dependent fluid properties.

In principle, one could construct a mathematical model of the entire machine by judiciously lumping parts of it and simulating mathematically the variation of temperatures of the lump-regions as functions of time. This approach, however, would necessitate a very extensive effort. A more tractable analysis was to prescribe upper and lower bounds on the temperatures τ_g, τ_r on the basis of available information and to compute the corresponding bounds on h*, k, T, τ_f. The results will be limited to the axisymmetrical case.

5.2 Determination of Foil Temperature τ_f

The steady state, planar model shown in Fig. 4 is considered. It is presumed that Couette flow prevails between the foil and the rotor. The foil-temperature variations are neglected and the temperature distribution in the rather stagnant gas between the foil and the "shield-dampers" is taken to be linear. With the assumption of Couette flow, the temperature distribution across the fluid film is [9] :

$$\frac{\tau(z)-\tau_f}{\tau_r-\tau_f} = \frac{z}{h^*} + \frac{\mu_a U^2}{2\lambda_a(\tau_r-\tau_f)}\frac{z}{h^*}\left(1-\frac{z}{h^*}\right) \tag{24}$$

where λ_a is the thermal conductivity of the fluid. Assuming a quasi-steady state (or, equivalently, neglecting the thermal inertia of the foil) the net heat flow into the foil is zero so that

$$-\lambda_a\frac{\tau_f-\tau_r}{h^*} + \frac{\mu_a U^2}{2h^*} - \lambda_a\frac{\tau_f-\tau_g}{h_g-h^*-d} = 0 \tag{25}$$

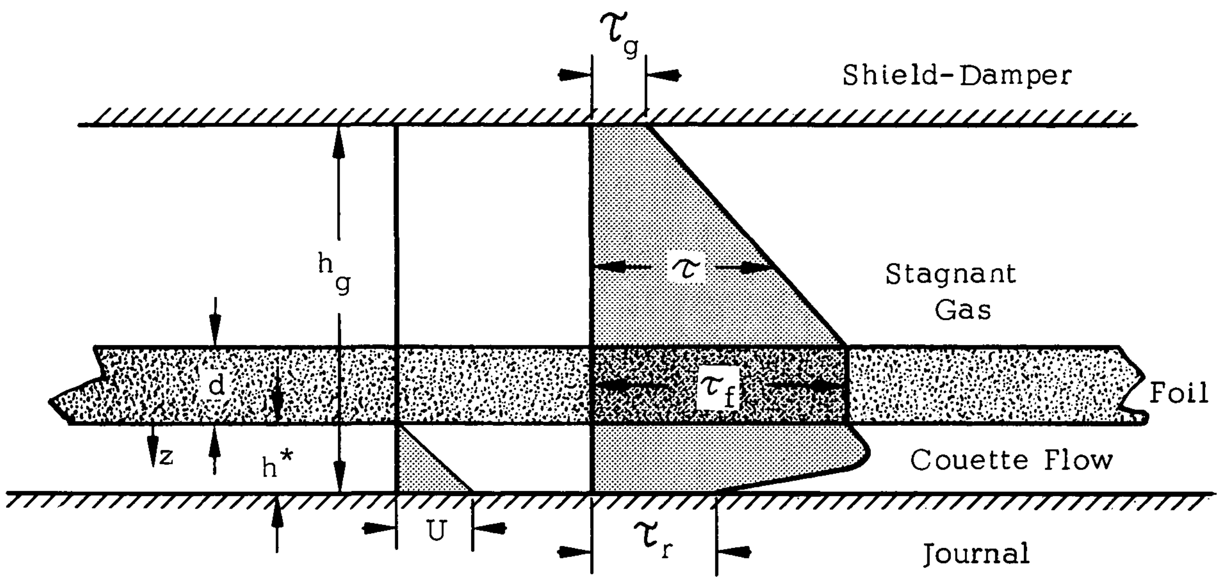

Fig. 4 Schematic Diagram of Temperature and Velocity Fields Between Rotor and Shield-Damper

It follows that

$$\tau_f = \frac{\mu_a U^2}{2\lambda_a} \frac{h_g - h^* - d}{h_g - d} + \tau_g \frac{h^*}{h_g - d} + \tau_r \frac{h_g - h^* - d}{h_g - d} \tag{26}$$

where the spacing h_g is a function of the thermal expansion of the rotor and the shield-damper , expressed by the equation:

$$h_g = h_{g_o} + (\tau_g - \tau_o)\alpha_g (r_c - r_g) - (\tau_r - \tau_o)\alpha_r r_o \tag{27}$$

The spacing between the rotor and the shield-damper in the reference state is denoted in Eq. (27) by h_{go}.

5.3 Variation of Fluid Properties

The properties of the lubricating fluid are evaluated in this report at the temperature $\tau_a = (\tau_f + \tau_r)/2$, and at the pressure p_a of the fluid entering the bearing. The fluid density is found from the perfect gas equation:

$$\rho_a = \frac{M\, p_a}{\bar{R}\, \tau_{a(abs)}} \tag{28}$$

The viscosity μ_a and thermal conductivity λ_a are found from the Sutherland formulae [10, 11]

$$\mu_a = \mu_b \left(\frac{\tau_{b(abs)} + C_\mu}{\tau_{a(abs)} + C_\mu} \right) \left(\frac{\tau_{a(abs)}}{\tau_{b(abs)}} \right)^{3/2} \tag{29}$$

$$\lambda_a = \lambda_b \left(\frac{\tau_{b(abs)} + C_\lambda}{\tau_{a(abs)} + C_\lambda} \right) \left(\frac{\tau_{a(abs)}}{\tau_{b(abs)}} \right)^{3/2} \tag{30}$$

In Eqs. (29) and (30) the base viscosity μ_b and conductivity λ_b are given at the temperature τ_b (using the absolute scale) and the constants C_μ, C_λ are known from results of curve fitting over a range of temperatures.

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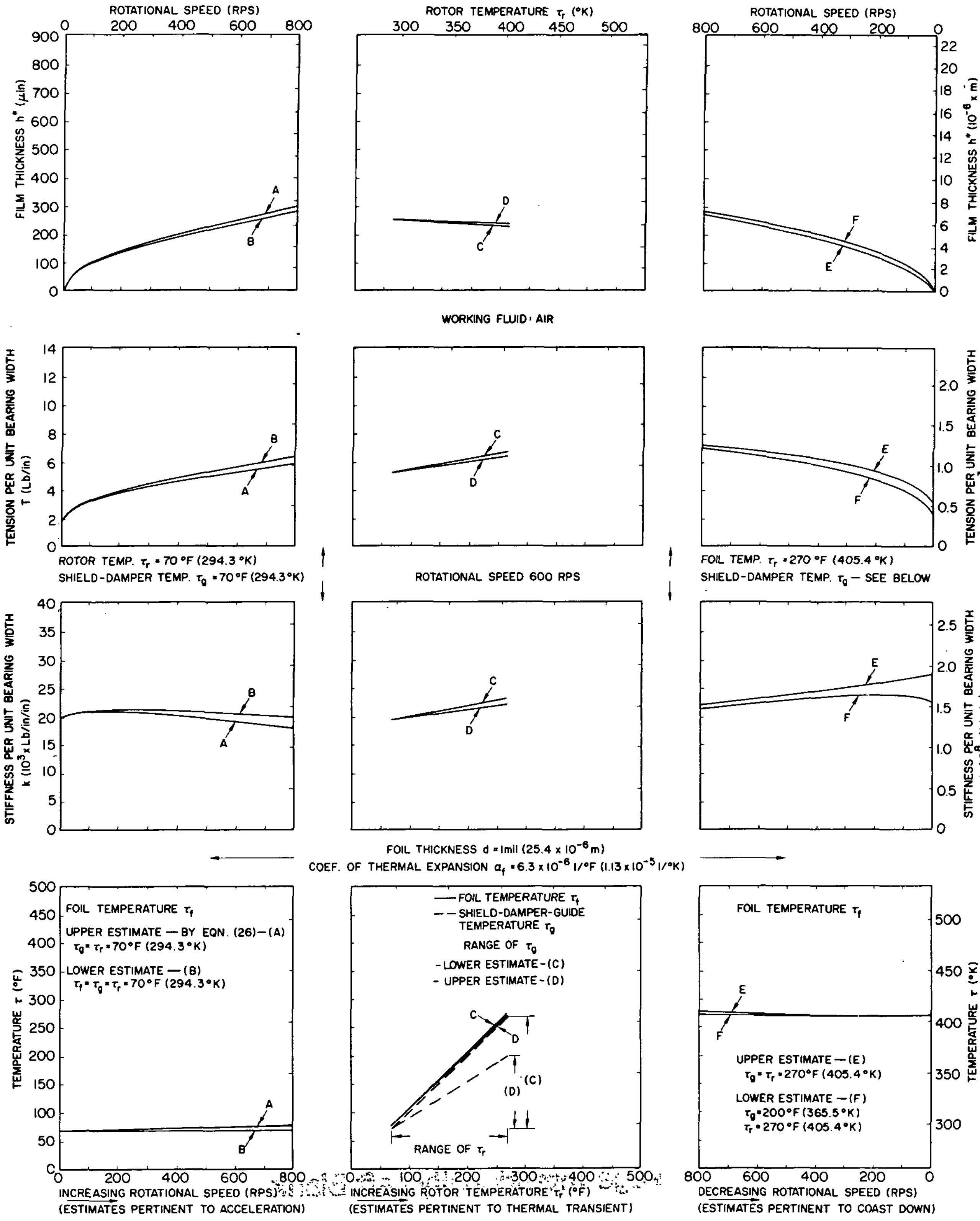


ROTATIONAL SPEED (RPS)
ROTOR TEMPERATURE τ_r (°K)
FILM THICKNESS h^* (μin)
FILM THICKNESS h^* (10^{-6} x m)
WORKING FLUID: AIR
TENSION PER UNIT BEARING WIDTH T (Lb/in)
TENSION PER UNIT BEARING WIDTH T (10^3 x N/m)
ROTOR TEMP. τ_r = 70 °F (294.3 °K)
SHIELD-DAMPER TEMP. τ_g = 70 °F (294.3 °K)
ROTATIONAL SPEED 600 RPS
FOIL TEMP. τ_f = 270 °F (405.4 °K)
SHIELD-DAMPER TEMP. τ_g — SEE BELOW
STIFFNESS PER UNIT BEARING WIDTH k (10^3 x Lb/in/in)
STIFFNESS PER UNIT BEARING WIDTH k (10^8 x N/m/m)
FOIL THICKNESS d = 1mil (25.4 x 10^{-6}m)
COEF. OF THERMAL EXPANSION α_f = 6.3 x 10^{-6} 1/°F (1.13 x 10^{-5} 1/°K)
FOIL TEMPERATURE τ_f
UPPER ESTIMATE — BY EQN. (26) — (A)
$\tau_g = \tau_r$ = 70°F (294.3°K)
LOWER ESTIMATE — (B)
$\tau_f = \tau_g = \tau_r$ = 70°F (294.3°K)
— FOIL TEMPERATURE τ_f
– – SHIELD-DAMPER-GUIDE TEMPERATURE τ_g
RANGE OF τ_g
- LOWER ESTIMATE - (C)
- UPPER ESTIMATE - (D)
RANGE OF τ_r
UPPER ESTIMATE — (E)
$\tau_g = \tau_r$ = 270°F (405.4°K)
LOWER ESTIMATE — (F)
τ_g = 200°F (365.5°K)
τ_r = 270°F (405.4°K)
TEMPERATURE τ (°F)
TEMPERATURE τ (°K)
INCREASING ROTATIONAL SPEED (RPS)
(ESTIMATES PERTINENT TO ACCELERATION)
(a)
INCREASING ROTOR TEMPERATURE τ_r (°F)
(ESTIMATES PERTINENT TO THERMAL TRANSIENT)
(b)
DECREASING ROTATIONAL SPEED (RPS)
(ESTIMATES PERTINENT TO COAST DOWN)
(c)

Geometrical Parameters

r_o = 0.8750 in (0.02222 m)
r_g = 0.1225 in (0.00311 m)
r_c = 1.0096 in (0.02564 m)
h_{go} = 0.0053 in (0.00013 m)

b = 0.1493 in (0.00379 m)
d_g = 0.1250 in (0.00317 m)
Θ = 88.8°
Θ_g = 133.5°

Elastic Characteristics

T_o = 2.0 lbf/in (350. N/m)
E = 30×10^{6} psi (20.7×10^{10} N/m^2)

f = 0.15
ν = 0.3

Fluid and Thermal Characteristics

Working Fluid - Air
p_a = 14.7 psi (10.1×10^{4} N/m^2)
α_g = 5.6×10^{-6} $^{\circ}F^{-1}$ (1.01×10^{-5} $^{\circ}K^{-1}$)

M = 28.97
τ_{room} = $70^{\circ}F$ ($294.3^{\circ}K$)
α_r = 7.3×10^{-6} $^{\circ}F^{-1}$ (1.31×10^{-5} $^{\circ}K^{-1}$)

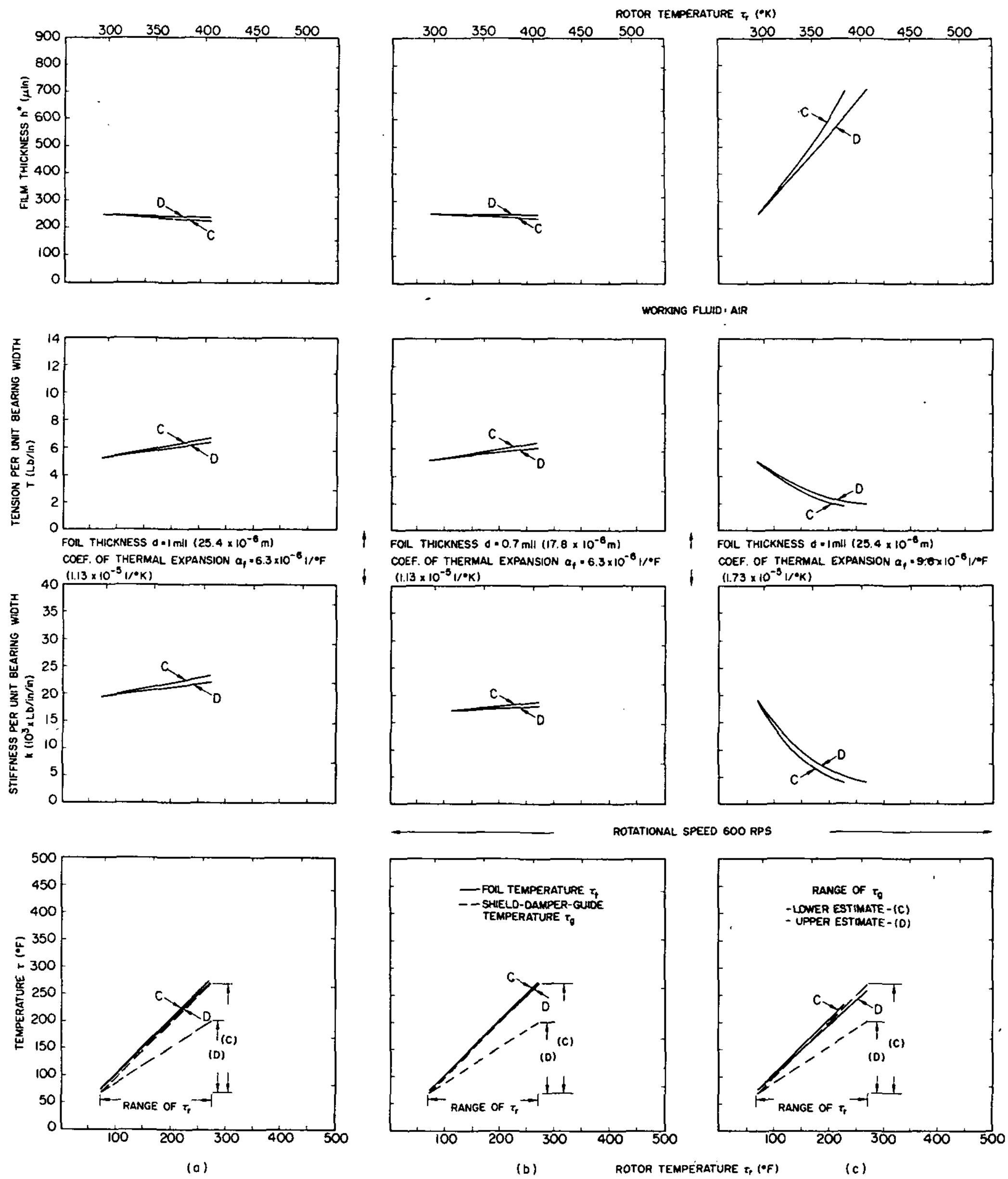
Viscosity Parameters (Eq. 29)

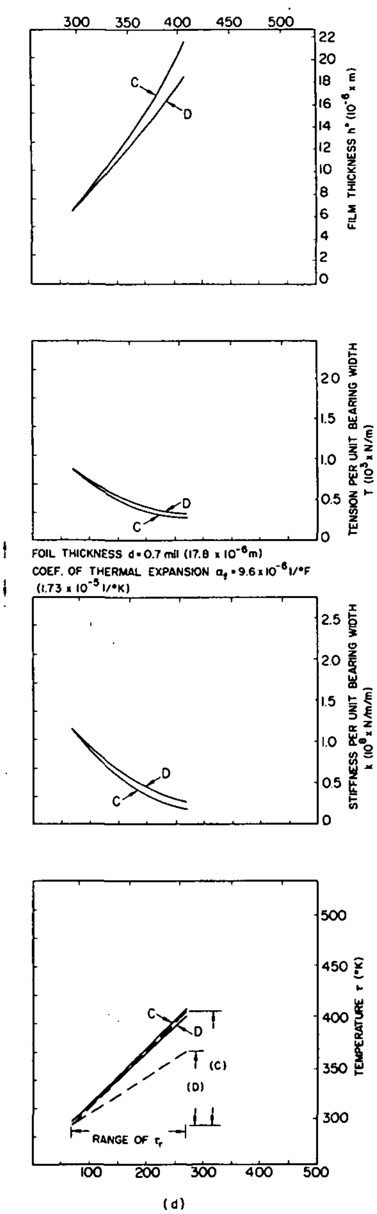
μ_b = 0.262×10^{-8} lbf·sec/in^2 (1.806×10^{-5} N·s/m^2)
τ_b = $68^{\circ}F$ ($293^{\circ}K$)
C_{μ} = $216\,^{\circ}F$ ($120^{\circ}K$)

Thermal Conductivity Parameters (Eq. 30)

λ_b = 0.0149 BTU/(hr·ft $^{\circ}F$)(1.706 J/(m.s $^{\circ}K$))
τ_b = $68^{\circ}F$ ($293^{\circ}K$)
C_{λ} = $225^{\circ}F$ ($125^{\circ}K$)

Fig. 5 Estimated Range of h^*, T, k, τ_f Pertinent to operation with Air (Acceleration, Thermal Transient at Nominal Speed, and Coast Down for One Combination of Foil Thickness d and Foil Thermal Expansion Coefficient α_f.)


ROTOR TEMPERATURE τ_r (°K)
FILM THICKNESS h^* (μin)
WORKING FLUID: AIR
TENSION PER UNIT BEARING WIDTH T (Lb/in)
FOIL THICKNESS d = 1 mil (25.4 x 10^{-6} m)
COEF. OF THERMAL EXPANSION α_f = 6.3 x 10^{-6} 1/°F (1.13 x 10^{-5} 1/°K)
FOIL THICKNESS d = 0.7 mil (17.8 x 10^{-6} m)
COEF. OF THERMAL EXPANSION α_f = 6.3 x 10^{-6} 1/°F (1.13 x 10^{-5} 1/°K)
FOIL THICKNESS d = 1 mil (25.4 x 10^{-6} m)
COEF. OF THERMAL EXPANSION α_f = 9.6 x 10^{-6} 1/°F (1.73 x 10^{-5} 1/°K)
STIFFNESS PER UNIT BEARING WIDTH k (10^3 x Lb/in/in)
ROTATIONAL SPEED 600 RPS
TEMPERATURE τ (°F)
FOIL TEMPERATURE τ_f
SHIELD-DAMPER-GUIDE TEMPERATURE τ_g
RANGE OF τ_g
LOWER ESTIMATE - (C)
UPPER ESTIMATE - (D)
RANGE OF τ_r
ROTOR TEMPERATURE τ_r (°F)
(a)
(b)
(c)



Geometrical Parameters

r_o = 0.8750 in (0.02222 m)
r_g = 0.1225 in (0.00311 m)
r_c = 1.0096 in (0.02564 m)
h_{go} = 0.0053 in (0.00013 m)

b = 0.1493 in (0.00379 m)
d_g = 0.1250 in (0.00317 m)
Θ = 88.8°
Θ_g = 133.5°

Elastic Characteristics

T_o = 2.0 lbf/in (350. N/m)
E = 30×10^6 psi (20.7×10^{10} N/m^2)
f = 0.15
ν = 0.3

Fluid and Thermal Characteristics

Working Fluid - Air
p_a = 14.7 psi (10.1×10^4 N/m^2)
α_g = 5.6×10^{-6} °F^{-1} (1.01×10^{-5} °K^{-1})

M = 28.97
τ_{room} = 70°F (294.3°K)
α_r = 7.3×10^{-6} °F^{-1} (1.31×10^{-5} °K^{-1})

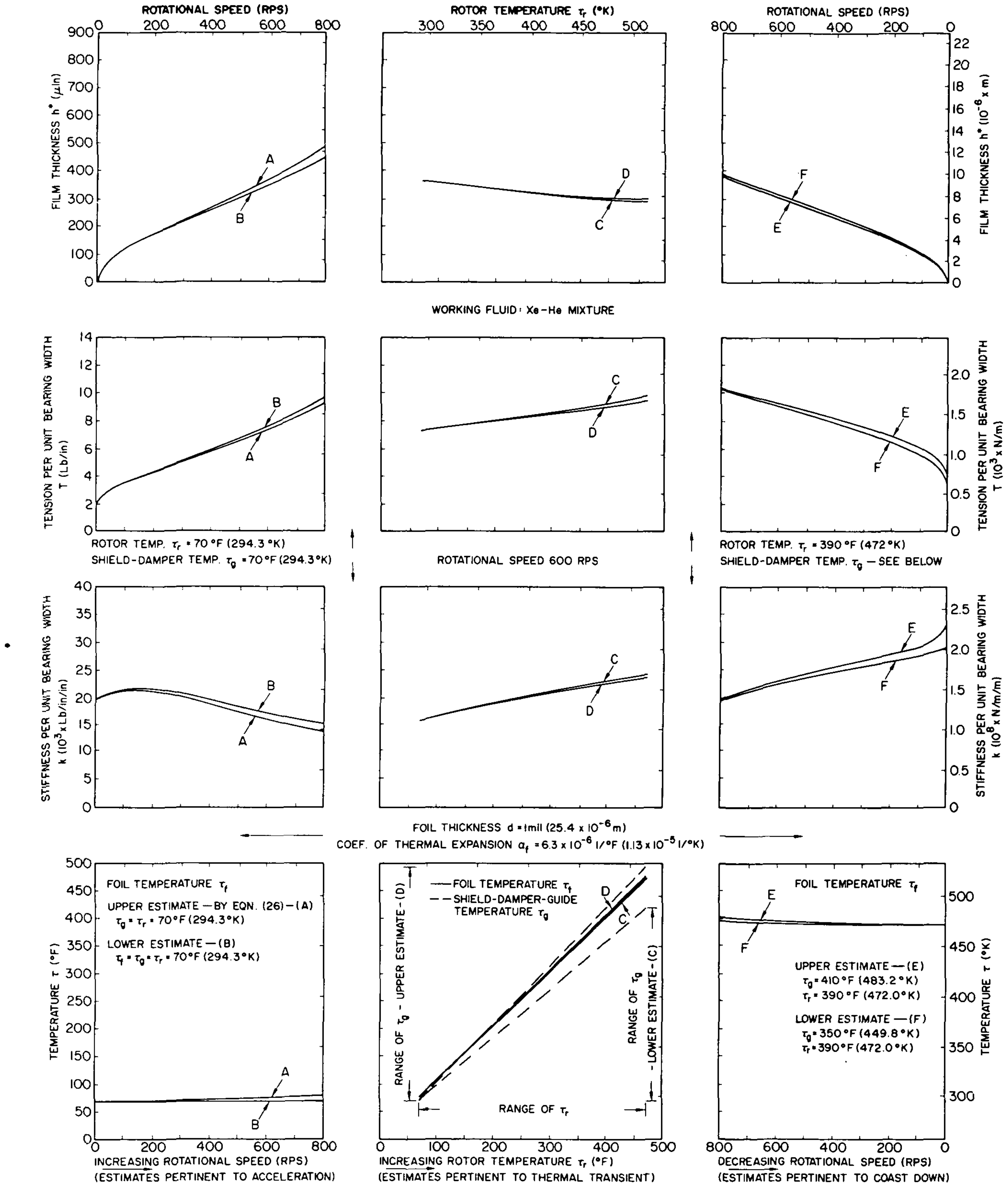
Viscosity Parameters (Eq. 29)

μ_b = 0.262×10^{-8} lbf·sec/in^2 (1.806×10^{-5} N·s/m^2)
τ_b = 68°F (293°K)
C_μ = 216 °F (120°K)

Thermal Conductivity Parameters (Eq. 30)

λ_b = 0.0149 BTU/(hr·ft °F)(1.706 J/(m.s °K))
τ_b = 68°F (293°K)
C_λ = 225°F (125°K)

Fig. 6 Estimated Range of h^*, T, k, τ_f Pertinent to Operation with Air (Thermal Transient at Nominal Speed, for Four Combinations of Foil Thickness d and Foil Thermal Expansion Coefficient α_f.)



ROTATIONAL SPEED (RPS)
ROTOR TEMPERATURE τr (°K)
ROTATIONAL SPEED (RPS)
FILM THICKNESS h* (μin)
FILM THICKNESS h* (10⁻⁶ x m)
WORKING FLUID: Xe-He MIXTURE
TENSION PER UNIT BEARING WIDTH T (Lb/in)
TENSION PER UNIT BEARING WIDTH T (10³ x N/m)
ROTOR TEMP. τr = 70 °F (294.3 °K)
SHIELD-DAMPER TEMP. τg = 70 °F (294.3 °K)
ROTATIONAL SPEED 600 RPS
ROTOR TEMP. τr = 390 °F (472 °K)
SHIELD-DAMPER TEMP. τg — SEE BELOW
STIFFNESS PER UNIT BEARING WIDTH k (10³ x Lb/in/in)
STIFFNESS PER UNIT BEARING WIDTH k (10⁸ x N/m/m)
FOIL THICKNESS d = 1mil (25.4 x 10⁻⁶ m)
COEF. OF THERMAL EXPANSION αf = 6.3 x 10⁻⁶ 1/°F (1.13 x 10⁻⁵ 1/°K)
TEMPERATURE τ (°F)
TEMPERATURE τ (°K)
FOIL TEMPERATURE τf
UPPER ESTIMATE — BY EQN. (26) — (A)
τg = τr = 70 °F (294.3 °K)
LOWER ESTIMATE — (B)
τf = τg = τr = 70 °F (294.3 °K)
— FOIL TEMPERATURE τf
— — SHIELD-DAMPER-GUIDE TEMPERATURE τg
RANGE OF τg - UPPER ESTIMATE - (D)
RANGE OF τg - LOWER ESTIMATE - (C)
RANGE OF τr
FOIL TEMPERATURE τf
UPPER ESTIMATE — (E)
τg = 410 °F (483.2 °K)
τr = 390 °F (472.0 °K)
LOWER ESTIMATE — (F)
τg = 350 °F (449.8 °K)
τr = 390 °F (472.0 °K)
INCREASING ROTATIONAL SPEED (RPS)
(ESTIMATES PERTINENT TO ACCELERATION)
(a)
INCREASING ROTOR TEMPERATURE τr (°F)
(ESTIMATES PERTINENT TO THERMAL TRANSIENT)
(b)
DECREASING ROTATIONAL SPEED (RPS)
(ESTIMATES PERTINENT TO COAST DOWN)
(c)

Geometrical Parameters

r_o = 0.8750 in (0.02222 m)
r_g = 0.1225 in (0.00311 m)
r_c = 1.0096 in (0.02564 m)
h_{go} = 0.0053 in (0.00013 m)

b = 0.1493 in (0.00379 m)
d_g = 0.1250 in (0.00317 m)
Θ = 88.8°
Θ_g = 133.5°

Elastic Characteristics

T_o = 2.0 lbf/in (350. N/m)
E = 30×10^6 psi (20.7×10^{10} N/m^2)

f = 0.15
ν = 0.3

Fluid and Thermal Characteristics

Working Fluid - Xe-He Mixture
p_a = 25.1 psi (17.3×10^4 N/m^2)
α_g = 5.6×10^{-6} $^{\circ}F^{-1}$ (1.01×10^{-5} $^{\circ}K^{-1}$)

M = 83.8
τ_{room} = $70^{\circ}F$ ($294.3^{\circ}K$)
α_r = 7.3×10^{-6} $^{\circ}F^{-1}$ (1.31×10^{-5} $^{\circ}K^{-1}$)

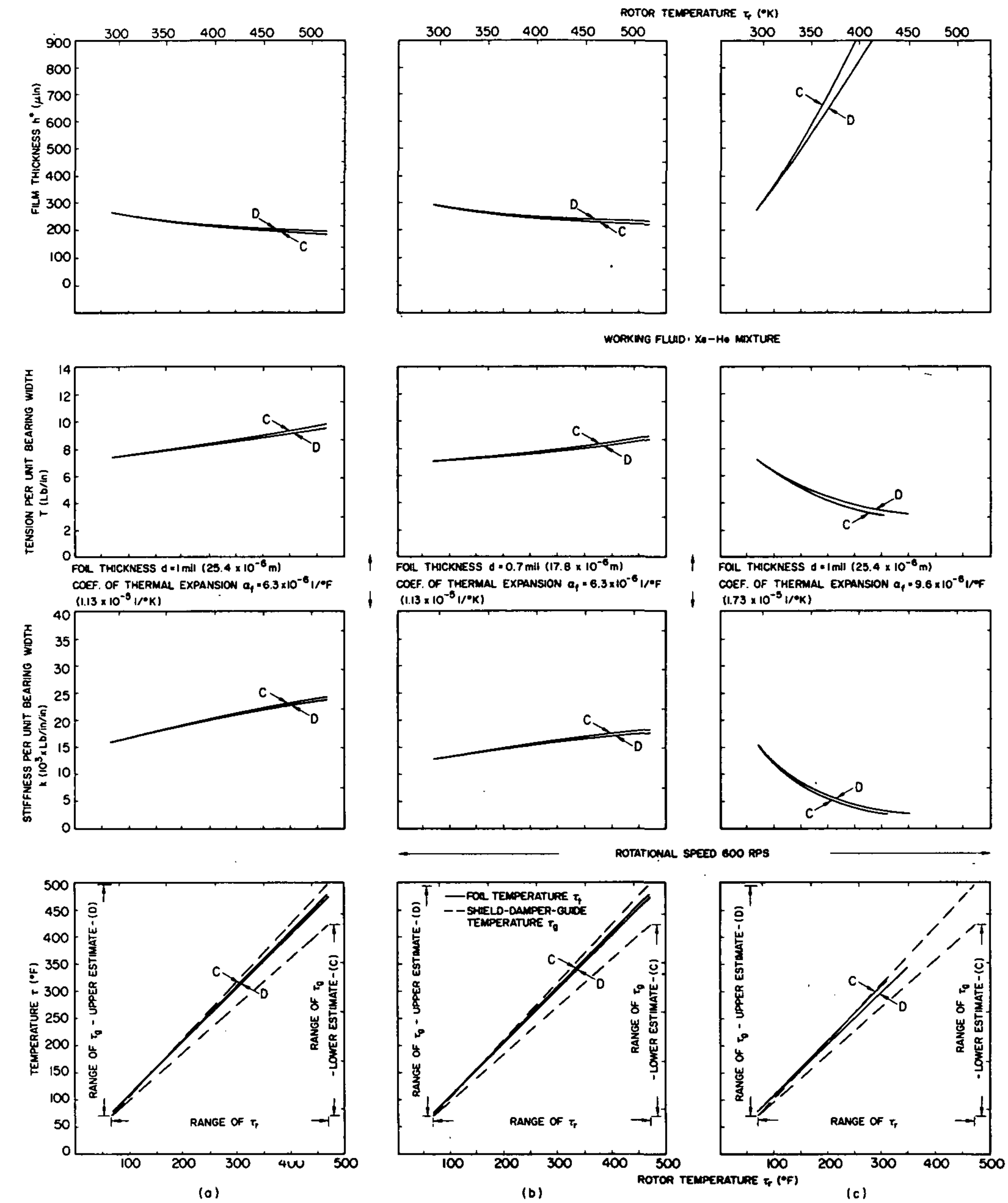
Viscosity Parameters (Eq. 29)

μ_b = 0.2475×10^{-8} lbf·sec/in^2 (1.706×10^{-5} N·s/m^2)
τ_b = $-100^{\circ}F$ ($200^{\circ}K$)
C_{μ} = $346.74^{\circ}F$ ($192.66^{\circ}K$)

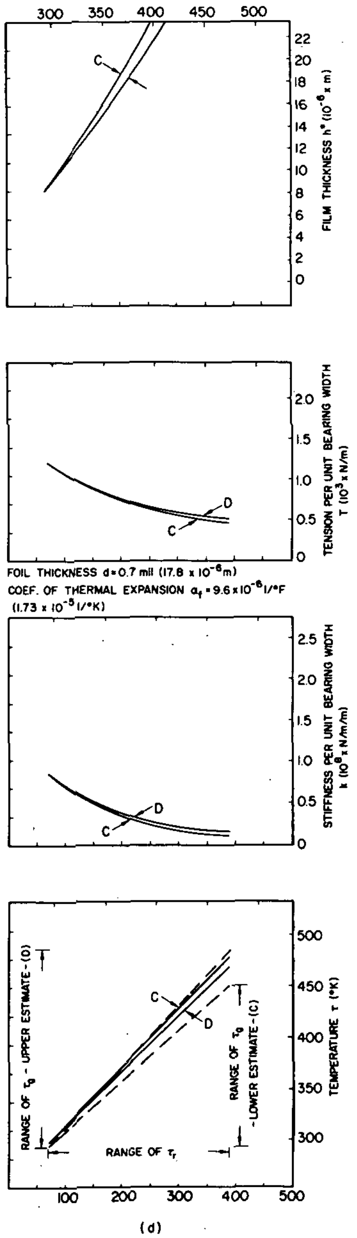
Thermal Conductivity Parameters (Eq. 30)

λ_b = 0.0110 BTU/(hr·ft $^{\circ}F$) (1.056 J/(m·s $^{\circ}K$))
τ_b = $-100^{\circ}F$ ($200^{\circ}K$)
C_{λ} = $168.16^{\circ}F$ ($93.42^{\circ}K$)

Fig. 7 Estimated Range of h^*, T, k, τ_f Pertinent to Operation with Xe-He Mixture (Acceleration, Thermal Transient at Nominal Speed, and Coast Down for One Combination of Foil Thickness d and Foil Thermal Expansion Coefficient α_f.)



ROTOR TEMPERATURE τ_r (°K)
FILM THICKNESS h^* (μin)
WORKING FLUID: Xe–He MIXTURE
TENSION PER UNIT BEARING WIDTH T (Lb/in)
FOIL THICKNESS d = 1 mil (25.4×10^{-6}m)
COEF. OF THERMAL EXPANSION $\alpha_f = 6.3 \times 10^{-6}$ 1/°F (1.13×10^{-5} 1/°K)
FOIL THICKNESS d = 0.7 mil (17.8×10^{-6}m)
COEF. OF THERMAL EXPANSION $\alpha_f = 6.3 \times 10^{-6}$ 1/°F (1.13×10^{-5} 1/°K)
FOIL THICKNESS d = 1 mil (25.4×10^{-6}m)
COEF. OF THERMAL EXPANSION $\alpha_f = 9.6 \times 10^{-6}$ 1/°F (1.73×10^{-5} 1/°K)
STIFFNESS PER UNIT BEARING WIDTH k (10^3 x Lb/in/in)
ROTATIONAL SPEED 600 RPS
TEMPERATURE τ (°F)
RANGE OF τ_g - UPPER ESTIMATE - (D)
RANGE OF τ_g - LOWER ESTIMATE - (C)
RANGE OF τ_r
FOIL TEMPERATURE τ_f
SHIELD-DAMPER-GUIDE TEMPERATURE τ_g
ROTOR TEMPERATURE τ_r (°F)
(a)
(b)
(c)



Geometrical Parameters

r_o = 0.8750 in (0.02222 m) b = 0.1493 in (0.00379 m)
r_g = 0.1225 in (0.00311 m) d_g = 0.1250 in (0.00317 m)
r_c = 1.0096 in (0.02564 m) Θ = 88.8°
h_{go} = 0.0053 in (0.00013 m) Θ_g = 133.5°

Elastic Characteristics

T_o = 2.0 lbf/in (350. N/m) f = 0.15
E = 30×10^6 psi (20.7×10^{10} N/m^2) ν = 0.3

Fluid and Thermal Characteristics

Working Fluid - Xe-He Mixture M = 83.8
p_a = 25.1 psi (17.3×10^4 N/m^2) τ_{room} = 70°F (294.3°K)
α_g = 5.6×10^{-6} °F^{-1} (1.01×10^{-5} °K^{-1}) α_r = 7.3×10^{-6} °F^{-1} (1.31×10^{-5} °K^{-1})

Viscosity Parameters (Eq. 29)

μ_b = 0.2475×10^{-8} lbf·sec/in^2 (1.706×10^{-5} N·s/m^2)
τ_b = -100°F (200°K)
C_μ = 346.74 °F (192.66°K)

Thermal Conductivity Parameters (Eq. 30)

λ_b = 0.0110 BTU/(hr·ft °F) (1.056 J/(m·s °K))
τ_b = -100°F (200°K)
C_λ = 168.16°F (93.42°K)

Fig. 8 Estimated Range of h^*, T, k, τ_f Pertinent to Operation with Xe-He Mixture (Thermal Transient at Nominal Speed for Four Combinations of Foil Thickness d and Foil Thermal Expansion Coefficient α_f.)

6.0 RESULTS

A computer program has been developed to give h^*, k, T, τ_f (referred to below as the "output parameters") for a given set of input parameters. Scanning capabilities for certain input parameters are built into the program. These capabilities are explained by comments in the program listing (Appendix B).

6.1 Method of Presentation

Values of the output parameters are shown for a range of conditions in Figs. 5 - 8. Estimates pertinent to operation with atmospheric air as the working fluid are presented in Figs. 5 and 6. In analogous fashion, estimates pertinent to operation with a mixture of xenon and helium (83.8 molecular weight) are given in Figs. 7 and 8. For the particular combination of foil thickness and thermal expansion coefficients used in the design, Figs. 5 and 7 show estimates pertinent to (a) start-up with rapid acceleration, (b) slow thermal transients which follow, with the machine at rated speed, and (c) coast-down. Figures 6 and 8 show a comparison of estimates pertinent to thermal transients for four combinations of foil thickness and coefficient of thermal expansion. (The first column in each figure pertains to the present design.)

(a) Results pertinent to rapid acceleration (a few seconds)

In order to study the bearing characteristics during start-up the output parameters are found as a function of operating speed with τ_r and τ_g assumed to remain substantially constant. These assumptions are justified by the rapidity of the process. The temperature of the low-thermal-inertia foil may range between its initial temperature (for infinitely fast acceleration) and the temperature predicted by Eq. (26) for sufficiently slow acceleration). The results are shown in Fig. 5a (air) and in Fig. 7a (Xe-He). At 600 rps, the stiffness predicted on the basis of the above assumptions is in the neighborhood of 20,000 lb/in/in (14×10^9 N/m^2) and the film thickness is approximately 250 microinches for air (6 microns) and 350 microinch for Xe-He (9 microns).

(b) Results pertinent to slow thermal transients at rated speed

Following start-up, while running at rated speed, the system warms up relatively slowly to its equilibrium temperatures. The shield-damper-guide temperature follows that of the rotor. A reasonable range of rotor and guide temperatures (based on available thermal maps [12] of the pivoted-shoe bearing version of the machine), is used to predict the range of the output parameters. It is assumed in this case that the foil temperature changes quasi-statically according to Eq. (26). All results are plotted versus the rotor temperature τ_r which may be regarded as a distorted time scale. By comparing Figs. 6a and 6b, 6c and 6d, 8a and 8b, 8c and 8d, one may observe that a reduction of foil thickness from of 1.0 to 0.7 mils (25 to 18 microns) causes little change in bearing stiffness and gap. On the other hand, comparison of Figs. 6a and 6c, 6b and 6d, 8a and 8c, 8b and 8d indicates that changes in foil material [thermal expansion coefficient 6.3×10^{-6} $^oF^{-1}$ (11.3×10^{-6} $^oK^{-1}$) to 9.6×10^{-6} $^oF^{-1}$ (17.3×10^{-6} $^oK^{-1}$)] may cause significant change in bearing behavior.* If the coefficient of thermal expansion of the foil is too low relative to that of the journal and of the foil support, the gap may be reduced excessively. Conversely, too high a value of the coefficient may produce a large clearance and an overly loose bearing. In the assumed ranges of temperatures, Figs. 6a and 8a confirm the correct selection of materials and other design parameters and predict favorable operation in terms of adequate stiffness and safe film thickness.**

* The curves in Figs. 8c and 8d were not carried to the upper temperature range due to convergence difficulties. It was not deemed worthwhile to make special program provisions for this range, since in this extremely relaxed foil state the accuracy of Eq. (29b) is marginal.

** Coefficients of thermal expansion of foil and foil supports were matched on the basis of a simplified calculation carried out by L. Licht prior to the present analysis. Metals of compatible thermal expansion and suitable elastic properties were then selected accordingly. The manufacture of foil bearings for the BRU has been completed at the time of drafting of this report.

(c) Results pertinent to coast-down

Coast-down is moderately rapid. Therefore, it is plausible to assume that during this process, the journal and the foil support do not experience appreciable changes in temperature. On the other hand, the thin foil is expected to follow quasi-statically the temperatures dictated by its more massive neighbors and by the heat generated in the film. Thus, the foil temperature is estimated on the basis of Eq. (26) for a range of rotor and foil support temperatures. The results are shown in Fig. 5c (Air) and Fig. 7c (Xe-He). The results indicate an increase in stiffness during coast-down, corresponding to a 10% increase in resonant frequency in comparison with start-up.

7.0 CONCLUSIONS

The parameter values selected in the design of the BRU foil journal-bearings have been tested by the computer program developed in this report. For the temperature ranges assumed for the journal, foil and foil supports, the design has been confirmed with regard to bearing stiffness and film thickness. Since foils can be easily replaced, the foil material is a sensitive variable for controlling the bearing characteristics, which remains at our disposal when other parameters cannot be altered.

APPENDIX A

EFFECT OF FOIL FLEXURE

In this appendix, a model is presented for the calculation of slack resulting from the nonvanishing foil stiffness.

Consider two parallel cylinders of radii r_o and r_g respectively (Fig. A1). The center to center distance of the two cylinders is r_c . The corresponding distance b is given by the equation

$$b = \sqrt{r_c^2 - (r_g + r_o)^2} \tag{A1}$$

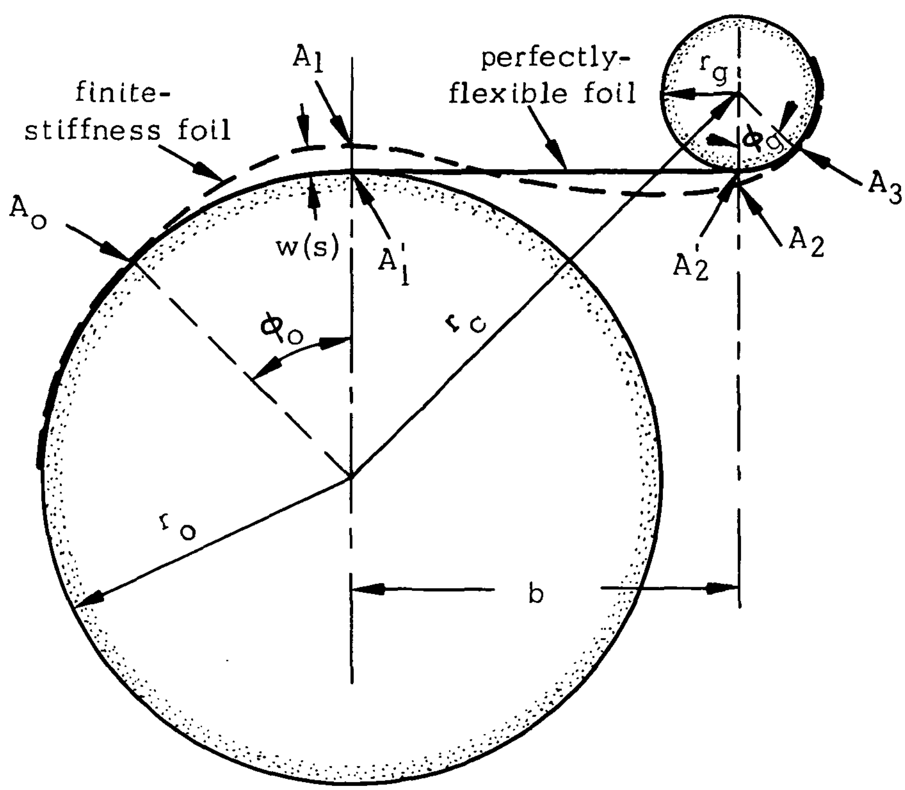


Fig. A1 Schematic View of a Foil Wrapped Around Two Cylinders

Let the foil be wrapped around the cylinders at a tension T The slack σ is defined as the excess of the length $A_0A_1A_2A_3$ of a finite-stiffness foil over the length $A_0A_1'A_2'A_3$ of a perfectly-flexible foil. In order to calculate σ, the contour of the foil must be found.

Let a coordinate s be measured along $A_0A_1A_2A_3$ with A_0 as the origin. At any point s, the distance w to the corresponding position of the perfectly-flexible foil is measured normal to the foil of finite stiffness. The local radius of curvature of the foil is R. R is taken positive when the curve is concave down (Fig. A1). Since there are no normal loads on the foil the differential equation for the curvature is

$$\frac{d^2\left(\frac{1}{R}\right)}{ds^2} - \frac{T}{D}\left(\frac{1}{R}\right) = 0 \tag{A2}$$

If the slope of the perfectly flexible foil does not differ much from that of the finite-stiffness foil, so that

$$\left(\frac{dw}{ds}\right)^2 \ll 1 \tag{A3}$$

it follows that

$$\frac{1}{R(s)} \cong \frac{1}{R_f(s)} - \frac{d^2w}{ds^2} \tag{A4}$$

where R_f is the local radius of curvature of the perfectly flexible foil at s.

It is convenient to use the following dimensionless notation.

$$\varepsilon = \sqrt{\frac{D}{Tr_o^2}} \tag{A5}$$

$$W = \frac{w}{r_o\,\varepsilon^2} \tag{A6}$$

$$\xi = \frac{s}{r_o \varepsilon} \tag{A7}$$

The problem is then formulated by the following differential equations:

$$\frac{r_o}{R} = \frac{r_o}{R_f} - \frac{d^2 W}{d \xi^2} \tag{A8}$$

where

$$\frac{r_o}{R_f} = \begin{cases} 1 & 0 < \xi < \xi_1 \\ 0 & \xi_1 < \xi < \xi_2 \\ -\frac{r_o}{r_g} & \xi_2 < \xi < \xi_3 \end{cases} \tag{A9}$$

and ξ_1, ξ_2, ξ_3 correspond to A_1, A_2, A_3 in Fig. A1, and

$$\frac{d^2\left(\frac{r_o}{R}\right)}{d \xi^2} - \frac{r_o}{R} = 0 \tag{A10}$$

The boundary conditions are:

at $\xi = 0$

$$\frac{r_o}{R} = 1 \tag{A11}$$

$$W = 0 \tag{A12}$$

$$\frac{dW}{d\xi} = 0 \tag{A13}$$

at $\xi = \xi_3$

$$\frac{r_o}{R} = -\frac{r_o}{r_g} \tag{A14}$$

$$W = 0 \tag{A15}$$

$$\frac{dW}{d\xi} = 0 \tag{A16}$$

An additional constraint is

$$\frac{b}{r_o \varepsilon} = \int_{\xi_1}^{\xi_2} \left[1 - \frac{\varepsilon^2}{2} \left(\frac{dW}{d\xi} \right)^2 \right] d\xi \approx \xi_2 - \xi_1 \tag{A17}$$

The above formulation contains four unknown integration constants and three unknown parameters ξ_1, ξ_2, ξ_3. The seven conditions (A11) to (A17) are used to determine these unknown constants. When the Eqs. (A8) through (A10) are integrated in the three regions, [Eq. (A9)], and the values of W and $dW/d\xi$ are matched at points A_1, A_2 the solutions are found to be:

$$W = \begin{cases} -A \sinh \xi - \cosh \xi + \dfrac{\xi^2}{2} + A\xi + 1 & 0 < \xi < \xi_1 \\ -A \sinh \xi - \cosh \xi + (A + \xi_1)\xi + 1 - \dfrac{\xi_1^2}{2} & \xi_1 < \xi < \xi_2 \\ -A \sinh \xi - \cosh \xi - \dfrac{r_o}{r_g} \dfrac{\xi^2}{2} + C\xi + E & \xi_2 < \xi < \xi_3 \end{cases} \tag{A18}$$

where

$$A = -\frac{\frac{r_o}{r_g} + \cosh \xi_3}{\sinh \xi_3} \tag{A19}$$

$$C = \frac{r_o}{r_g}\xi_3 + A\cosh\xi_3 + \sinh\xi_3 \tag{A20}$$

$$E = \frac{r_o}{r_g}\frac{\xi_3^2}{2} + A\sinh\xi_3 + \cosh\xi_3 - C\xi_3 \tag{A21}$$

The values of ξ_1, ξ_2, ξ_3 must be found from the simultaneous solution of the equations:

$$\xi_1 + \frac{r_o}{r_g}\xi_2 = \frac{r_o}{r_g}\xi_3 + A(\cosh\xi_3 - 1) + \sinh\xi_3 \tag{A22}$$

$$\xi_1^2 + \frac{r_o}{r_g}\xi_2^2 = 2 + \frac{r_o}{r_g}\xi_3^2 - 2A\sinh\xi_3 - 2\cosh\xi_3 + 2A\xi_3\cosh\xi_3 + 2\xi_3\sinh\xi_3 \tag{A23}$$

$$\xi_2 - \xi_1 \cong \frac{b}{r_o\varepsilon} \tag{A24}$$

The sought excess length σ may then be found from:

$$\sigma = \int_0^{s_3}\left[1 - \sqrt{1 - \left(\frac{dw}{ds}\right)^2}\right]ds \cong \frac{1}{2}\int_0^{s_3}\left(\frac{dw}{ds}\right)^2 ds \tag{A25}$$

In dimensionless form:

$$\frac{2\sigma}{r_o\varepsilon^3} \cong \int_0^{\xi_3}\left(\frac{dW}{d\xi}\right)^2 d\xi = \tag{A26}$$

$$= -\frac{A}{2} + (A+\xi_1)^2 \xi_2 + \frac{1}{3}\left(\frac{r_0}{r_g}\right)^2 \left(\xi_3^3 - \xi_2^3\right)$$

$$+ \frac{A^2-1}{2}\xi_3 + c^2(\xi_3 - \xi_2) - \frac{r_0}{r_g} c\left(\xi_3^2 - \xi_2^2\right)$$

$$- \frac{2r_0}{r_g}\left(\frac{\frac{r_0}{r_g}\cosh\xi_3 + \cosh(\xi_3 - \xi_2)}{\sinh \xi_3}\right)$$

$$\frac{A^2+1}{4}\sinh 2\xi_3 + \frac{A}{2}\cosh 2\xi_3 +$$

$$+ \begin{cases} -\frac{2}{\sinh\xi_2}\left[\frac{r_0}{r_g}\cosh\xi_1 + \cosh(\xi_3 - \xi_1)\right] - \frac{2}{3}\xi_1^3 - A\xi_1^2 & \xi_1 \geq 0 \\ -\frac{\xi_1^3}{3} + 2(A+\xi_1) & \xi_1 \leq 0 \end{cases}$$

APPENDIX B

```
C   PROGRAM TO EVALUATE FOIL BEARING OPERATING TENSION T, GAP HSTAR AND
C   STIFFNESS CAY FOR ARBITRARY PARAMETERS.  INITIAL INPUT   DATA IS
C   COMPILED IN HEONG SUBROUTINE.  DESIRED MODIFICATIONS IN PARAMETERS
C   SHOULD BE MADE THROUGH USE OF NAMELIST/DATA/. CALCULATION OF T, HSTAR
C   AND CAY IS MADE IN SUBROUTINE EQN1.AUXILIARY COMPUTATION OF SLACK
C   SIGMA AND ITS DERIVATIVES IS MADE IN SUBROUTINE SLACK.
C   COMPUTATION OF FOIL TEMPERATURE AND TEMP. - DEPENDENT FLUID-
C   VISCOSITY, CONDUCTIVITY AND DENSITY IS IN SUBROUTINE TEMPF.
C   PROGRAM OUTPUT IS IN (1) BRITISH UNITS AND (2) INTERNATIONAL(SI) UNITS.
C   BRITISH UNITS USED IN PROGRAM ARE INCH-LBF-SEC-DEGF OR AS
C   SPECIFIED.  BASIC SI UNITS ARE M-KG-S-K.  ALSO J,N.
C   E.G.  VISCOSITY IN NS/M2  (NEWTON*SECOND/METER**2)
C   VARIABLES USED FOR SI HAVE INT ENDING AND ARE PARENTHESISED  (BELOW)
C   INPUT OF PARAMETERS IS IN BRITISH UNIS. DEFINITIONS:
C   RPS = REVOLUTIONS PER SECOND OF ROTOR
C   RO = ROTOR RADIUS (PROGRAM ADDS TO THIS THIKNS/2.)              (ROINT)
C   RG = GUIDE RADIUS (PROGRAM ADDS TO THIS THIKNS/2.)              (RGINT)
C   RC = DISTANCE OF CENTER OF GUIDE FROM CENTER OF ROTOR           (RCINT)
C   SPACE = COLD SPACING BETWEEN ROTOR AND KIDNEY SHAPED GUIDE      (SPCINT)
C   TZ = INITIAL TENSION PER UNIT WIDTH (PRELOAD)                   (TZINT)
C   THIKNS=FOIL THICKNESS INCH                                      (TKINT)
C   E  = MODULUS OF ELASTICITY PSI                                  (EINT)
C   ENU = POISSON RATIO OF FOIL
C   D= BENDING RIGIDITY OF FOIL.  CALCULATED INTERNALLY FROM E AND
C   THIKNS.  IF D IS PRESCRIBED ZERO, FOIL IS ASSUMED PERFECTLY FLEXIBLE
C   FCOEF  = FRICTION COEF BETWEEN FOIL AND GUIDE
C   EMOL = MOLECULAR WEIGHT OF GAS
C   PA = AMBIENT PRESSURE PSI                                       (PAINT)
C   ALFAF = COEF.  OF EXPANSION OF FOIL 1/DEGF                      (AFINT)
C   ALFAG = COEF. OF EXPANSION OF GUIDE 1/DEGF                      (AGINT)
C   ALFAR = COEF. OF EXPANSION OF ROTOR                             (ARINT)
C   TROOM = ROOM TEMPERATURE DEG F                                  (TRMINT)
C   TGUIDE= FOIL GUIDE TEMPERATURE                                  (TGINT)
C   TROTOR= ROTOR TEMPERATURE                                       (TRINT)
C   TFOIL=FOIL TEMPERATURE.  IF(IFOIL.EQ.1) PROGRAM COMPUTES TFOIL(TFINT)
C   IFOIL IS ASSUMED TO BE 1 UNLESS OTHERWISE SPECIFIED
C   COND = INPUT THERMAL CONDUCTIVITY OF GAS BTU/(HR*FT*DEGF)       (CNDINT)
C   VIS= INPUT VISCOSITY OF GAS LBF*SEC/IN**2 (SEE CONSPR)          (VISINT)
C   CONSPR = LOGICAL VARIABLE.  ASSUMED TRUE UNLESS OTHERWISE SPECIFIED
C   IF TRUE, INPUT OF VIS AND COND IS ASSUMED CONSTANT
C   IF FALSE THESE INPUTS ARE ASSUMED TO BE AT REFERENCE
C   TEMPERATURES, TRVISC AND TRCOND, RESPECTIVELY.  GAS
C   TEMPERATURE ASSUMED AT AVERAGE OF TROTOR AND TFOIL.INTERNAL
C   VISCOSITY (EMU) AND AND CONDUCTIVITY(CONDCT)ARE ,THEN, CALCULATED
C   BY  THE SUTHERLAND FORMULA. CVISC AND CCOND ARE REQUIRED INPUTS OF
C   SUTHERLAND CONSTANTS
C   GOMTRY= LOGICAL VARIABLE. IF TRUE, OUTPUT FORMAT FOR GEOMETRY
C       VARIATIONS.
C   TERMO= LOGICAL VARIABLE.  IF TRUE, OUTPUT FORMAT FOR TEMPERATURE
C       VARIATIONS.
C   FLUID = LOGICAL VARIABLE.  IF TRUE, OUTPUT FORMAT FOR CHANGES
C       IN FLUID PROPERTIES
```

```
C   SOLID = LOGICAL VARIABLE.  IF TRUE, OUTPUT FORMAT FOR SOLID
C        PROPERTY CHANGES
C   IF ONE SPECIFIES GOMTRY =.TRUE. AND AT LEAST ONE CARD
C   LATER, ONE SPECIFIES GOMTRP= .FALSE. , THE COMPLETE
C   NAMELIST IS PRINTED.  SAME WITH FLUIDP =.FALSE.
C   FOLLOWING FLUID=.TRUE.  SAME WITH SOLIDP AND
C   TERMOP FOLLOWING SOLID AND TERMO RESPECTIVELY.
C   SKIP=.TRUE.  : LEAVES ONE  LINE SPACE BEFORE OUTPUT AND SWITCHES
C   SKIP BACK TO .FALSE.
C   READRC= .FALSE. : THETA1 (DEG) MUST BE PRESCRIBED, RC IS CALCULATED.
C   IF ISCAN=4 OR 5 READRC IS SWITCHED TO TRUE AFTER FIRST CASE
C   SCANNING OPTIONS:  INITIALLY, PROGRAM ASSUMES
C   ISCAN=1  I.E. EACH NAMELIST RECORD GIVES 1 OUTPUT RECORD
C   ISCAN=2  SCANS THIKNESS IN SOLID VARIATION FORMAT IN
C   (UNLESS OTHERWISE SPECIFIED) STEPS OF DTHIK=0.0002
C   TO THIKMX=0.002
C   ISCAN=3  SCANS PRELOAD TZ IN SOLID VARIATION FORMAT IN
C   (UNLESS OTHERWISE SPECIFIED)STEPS OF DTZ=0.5 TO TZMX=4.
C   ISCAN=4  SCANS RO,RC WITH CONST RADIAL SPACING IN GEOMETRY
C   VARIATION FORMAT IN STEPS (UNLESS OTHERWISE SPECIFIED) OF
C   DRO =0.25 TO ROMX=2.
C   ISCAN=5 SCANS RG,RC WITH CONST RADIAL SPACING IN GEOMETRY
C   VARIATION FORMAT IN STEPS (UNLESS OTHERWISE SPECIFIED) OF
C   DRG=0.05 TO RGMX=0.3
C   ISCAN =6 SCANS TROTOR AT RPS=CONST, AND UNLESS OTHERWISE
C   SPECIFIED AT STEPS DTROT=40. TO TROTMX=270., TGIDMX=200.
C   TGUIDE=TROTOR-(TROTMX - TGIDMX) * (TROTOR - TROOM)/(TROTMX- TROOM)
C   TERMO OUTPUT FORMAT
C   ISCAN= 7  SCANS RPS AT CONST TEMP.  DRPS=25. RPSMX=200.
C   UNLESS OTHERWISE SPECIFIED
      COMMON          THIKNS,RADIUS,DG,RGUIDE,RC,DD,SPACE,BB,RG,RO   ,
     1  T, TZ,  TT,    ET, TZP,TP,      EPS,EPS23,V,RPS,Y. CAY,
     2 HSTR,  HSTAR,  ETU,  PIR2,  THETA,  THETA1,  TH2,
     3  CTH2,   STH2 ,  THETAG,  THETG1, PI,DEG,FOA,FGA,
     4  PAIE,  COMP,  DENSTY,  PA,  EMU, CONDCT CF,EMOL, C,AIE,
     A  CONSPR,TRVISC,TRCOND,CVISC,CCOND, CONC,VIS,
     5  ELTOR,  RGBB  , ALF ,SIGBRO,SIGBR, DBDYK, DSGDB, DSBRDT,DSGDR ,
     B DBDR,
     6 THERML,  ALFAF, ALFAG, ALFAR, TROOM,  DTF, DTG, DTR,
     8 TROTOR ,DTROT,TROTMX,TGUIDE,  TGIDMX ,TFOIL,
     9 DTHIK, THIKMX,DTZ,TZMX,DRO,ROMX,DRG,RGMX,DRPS,RPSMX,
     7  PEXT,  E,  FCOEF, ENU, D ,SMAX
      LOGICAL TERMOP, FLUIDP,SOLIDP, GOMTRP ,CONSPR
      LOGICAL  TERMO, FLUID, SOLID, GOMTRY  ,SKIP ,READRC
      COMMON  TERMOP,FLUIDP,SOLIDP,GOMTRP                  ,ISCAN,IFOIL
      COMMON  TERMO,FLUID,SOLID,GOMTRY ,SKIP,READRC
      COMMON   AT(5), AX3(5), ASIGBR(5), ABSA(5)
      COMMON  AFP, AGP, ARP, AFINT, AGINT, ARINT, AFINTP,
     1 AGINTP, ARINTP, VISCP, TINT, HINTP, HINT,
     2 CYINTP, CYINT, SMINTP, SMINT, ROINT, RGINT, RCINT,
     3PAINT, PAINTP, TRMINT, DNSINT, VISINT, VSINTP, CNDINT,
     4TKINTP, TKINT, EINT, TZINT, TFINT, TGINT, TRINT,EP,EINTP,
     5 TCNV, CAYCNV, DNSCNV, VISCNV ,CONCNV
      CALL HEDNG (1)
    1 CALL EQN1
      CALL HEDNG(2)
      GO TO 1
      END
```

```
      SUBROUTINE HEDNG (J)
      NAMELIST/DATA/THIKNS , DTHIK, THIKMX,TT, ET,
     1RADIUS, RO,DRO,ROMX,RGUIDE,RG,DRG,RGMX, RC, Y,SPACE,
     2DG,BB,ALF,RGBB, ELTOR,
     3THETA,THETA1,TH2,CTH2,STH2,THETAG,THETG1,
     4T,TZ,DTZ,TZMX,TZP,TP,
     5PEXT,E,FCOEF,ENU,D,ALFAF,ALFAG,ALFAR,
     6PAIE,COMP,DENSTY,PA,VIS,EMOL,  COND,EMU, CONDCT, TRVISC,TRCOND,
     7RPS, DRPS, RPSMX, V, ETU, PIR2, CVISC,CCOND,
     8TFOIL,TGUIDE,TROTOR,DTROT,TROTMX,TGIDMX,TROOM,THERML,
     9TERMO,FLUID,SOLID,GOMTRY,TERMOP,FLUIDP,SOLIDP,GOMTRP,CONSPR,
     AISCAN,IFOIL,SKIP, READRC
      NAMELIST/SI/TKINT,ROINT,RGINT,RCINT,TINT,TZINT, SPCINT,
     1 EINT, AFINT, AGINT,ARINT, DNSINT, PAINT, CNDINT,
     2VISINT, TFINT, TGINT,TRINT, TRMINT
      COMMON              THIKNS,RADIUS,DG,RGUIDE,RC,DD,SPACE,BB,RG,RO   ,
     1  T, TZ,  TT,    ET, TZP,TP,       EPS,EPS23,V,RPS,Y, CAY,
     2 HSTR,  HSTAR,  ETU,  PIR2,  THETA,  THETA1,  TH2,
     3  CTH2,   STH2 ,  THETAG,  THETG1, PI,DEG,FOA,FGA,
     4  PAIE,  COMP,  DENSTY,  PA,  EMU, CONDCT,CF,EMOL, C,AIE,
     A  CONSPR,TRVISC,TRCOND,CVISC,CCOND, COND,VIS,
     5  ELTOR,  RGBB  , ALF ,SIGBRO,SIGBR, DBDYK, DSGDB, DSBRDT,DSGDR ,
     B DBDR,
     6 THERML,  ALFAF, ALFAG, ALFAR, TROOM,  DTF, DTG, DTR,
     8 TROTOR ,DTROT,TROTMX,TGUIDE,  TGIDMX ,TFOIL,
     9 DTHIK, THIKMX,DTZ,TZMX,DRO,ROMX,DRG,RGMX,DRPS,RPSMX,
     7  PEXT,  E,  FCOEF, ENU, D ,SMAX
      LOGICAL TERMOP, FLUIDP,SOLIDP, GOMTRP ,CONSPR
      LOGICAL  TERMO, FLUID, SOLID, GOMTRY  ,SKIP ,READRC
      COMMON  TERMOP,FLUIDP,SOLIDP,GOMTRP                 ,ISCAN,IFOIL
      COMMON  TERMO,FLUID,SOLID,GOMTRY ,SKIP,READRC
      COMMON   AT(5), AX3(5), ASIGBR(5), ABSA(5)
      COMMON  AFP, AGP, ARP, AFINT, AGINT, ARINT, AFINTP,
     1 AGINTP, ARINTP, VISCP, TINT, HINTP, HINT,
     2 CYINTP, CYINT, SMINTP, SMINT, ROINT, RGINT, RCINT,
     3PAINT, PAINTP, TRMINT, DNSINT, VISINT, VSINTP, CNDINT,
     4TKINTP, TKINT, EINT, TZINT, TFINT, TGINT, TRINT,EP,EINTP,
     5 TCNV, CAYCNV, DNSCNV, VISCNV ,CONCNV
      GO TO (1,2          ) , J
    1 CONTINUE
      TCNV= 4.4482/2.54E-2
      CAYCNV=(4.4482/2.54E-2**2)*1.E-9
      DNSCNV= 386.* 2.768E4
      VISCNV= 144.* 47.88*1.E4
      CF = 778.16/3600.
      CONCNV= 144.*5.1887E2/778.16
      C=100.
      AIE=0.2
      PI= 3.1415926536
      DEG= 180./PI
```

```
CCCCCCCCCCCCCCCCCCC INITIALIZATION OF DATA CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     ASSUMED INITIAL VALUES. MAY BE MODIFIED BY STATEMENT 110
      THIKNS   =  0.001
      DTHIK=0.0002
      THIKMX=0.002
      EMOL   =   83.8
      RPS = 600.
      DRPS=25.
      RPSMX=200.
      PA = 25.1
      ALFAF= 6.3 E-6
      ALFAG = 5.6E-6
      ALFAR = 7.3E-6
      TROOM = 70.
      TFOIL=470.
      TGUIDE=330.
      TGIDMX=200.
      TROTOR=400.
      DTROT=40.
      TROTMX=270.
      E = 30.E6
      ENU = 0.3
      FCOEF=0.15
      VIS =5.1E-9
      CVISC= 346.792
      CCOND = 168.176
      TRCOND= -100.
      TRVISC=-100.
      CONSPR= .TRUE.
      COND = 0.0123
      RO = 0.875
      DRO=0.25
      ROMX=2.
      RG= 0.1225
      DRG=0.05
      RGMX=0.3
      RC     = 1.0096
      SPACE=0.00525
      TZ= 2.0
      DTZ=0.5
      TZMX=4.
      IFOIL = 1
      ISCAN=1
      GOMTRY=.FALSE.
      GOMTRP=.FALSE.
      TERMO=.FALSE.
      TERMOP=.FALSE.
      FLUID=.FALSE.
      FLUIDP=.FALSE.
      SOLID=.FALSE.
      SOLIDP=.FALSE.
      SKIP=.FALSE.
      READRC=.TRUE.
CCCCCCCCCCCCCCCCCCC END OF DATA INITIALIZATION CCCCCCCCCCCCCCCCCCCCCCCCC
```

```
      T= 2.*TZ
      D=0.003
      FOA= 2./DEG
      FGA= 2./DEG
      HSTR= 0.0005
      DTF= TFOIL-TROOM
    2 GO TO (  110,120,130,140,150,160,170                              ),ISCAN
  120 THIKNS=THIKNS + DTHIK
      IF(THIKNS.GT.THIKMX) GO TO 110
      GO TO 118
  130 TZ= TZ+DTZ
      IF(TZ.GT.TZMX) GO TO 110
      GO TO 118
  140 RO=RO+DRO
      RC=RC+DRO
      IF (RO.GT.ROMX) GO TO 110
      GO TO 118
  150 RG=RG+DRG
      RC=RC+DRG
      READRC=.TRUE.
      IF (RG.GT.RGMX) GO TO 110
      GO TO 118
  160 TROTOR= TROTOR + DTROT
      TGUIDE=TROTOR-(TROTMX-TGIDMX)*(TROTOR-TROOM)/(TROTMX-TROOM)
      IF(TROTOR.GT. TROTMX) GO TO 110
      GO TO 118
  170 RPS=RPS+ DRPS
      IF (RPS.GT.RPSMX) GO TO 110
      GO TO 118
  110 READ (5,DATA,END=401)
      GO TO (118,128,128,148,148,168,168,168                        ),ISCAN
  128 SOLID=.TRUE.
      GO TO 118
  148 GOMTRY=.TRUE.
      GO TO 118
  168 TERMO=.TRUE.
  118 DTG=TGUIDE-TROOM
      RADIUS  = RO      + THIKNS/2.
      RGUIDE = RG     + THIKNS/2.
      DG= RG+ 0.0025
      RGPRO=RG+RO
      IF(READRC) GO TO 8
      THETA=THETA1/DEG
      RC=SQRT( DG**2 +(RGPRO)**2 -2.*SIN(PI/3.-THETA/2.)*(RGPRO)*DG)/
     1  COS(PI/3. - THETA/2.)
      GO TO 9
    8 THETA =2.*PI/3. - 2. * ( ARCOS((   RGPRO            ) /RC) +
     1 ARSIN ( DG/RC))
      THETA1= THETA  *DEG
    9 TH2 =THETA /2.0
      STH2 = SIN(TH2)
      CTH2 = COS(TH2)
```

```
      AFP=ALFAF*1.E6
      AGP=ALFAG*1.E6
      ARP=ALFAR*1.E6
      AFINTP= ALFAF*1.8E6
      AGINTP= ALFAG*1.8E6
      ARINTP=ALFAR*1.8E6
      AFINT=AFINTP*1.E-6
      AGINT= AGINTP*1.E-6
      ARINT= ARINTP*1.E-6
      ROINT=RO*2.54E-2
      RGINT=RG*2.54E-2
      RCINT=RC*2.54E-2
      SPCINT = SPACE *2.54E-2
      PAINTP=PA*6.8947E-1
C   PAINT=PA*6.8947E3* (1.E-4 FOR PRINTING)
      PAINT=PAINTP* 1.E4
      TRMINT=(TROOM + 459.67)/1.8
      TKINTP= THIKNS*2.54E4
      TKINT = TKINTP*1.E-6
      EINTP= E* 6.8947E-7
      EINT=EINTP*1.E10
      EP=E*1.E-6
      TZINT= TZ*TCNV
      TGINT=(TGUIDE+459.67)/1.8
      TRINT=(TROTOR+459.67)/1.8
      DTR=TROTOR- TROOM
      CONDCT= COND*CF
      TT = THIKNS*1000.
      ET= E*THIKNS
      TINT = TCNV*T
      TP= T/ET
      IF  (RC.GT.(RADIUS+RGUIDE+0.005)) GO TO 4
      PRINT 10, RC,RO,RG
   10 FORMAT (26H GUIDE TOO CLOSE TO ROTOR    /10X,2HRC,18X,2HRO,18X,
     1    2HRG/ 3E20.5)
      GO TO 2
    4 BB =   SQRT ( RC**2- ( RADIUS+ RGUIDE)**2)
      DBDYK= - STH2  + CTH2* (RADIUS +  RGUIDE )/BB
      DBDR=-(RADIUS+RGUIDE)/BB
      RGBB =  RGUIDE/BB
      ALF   = 2.*BB +  RADIUS *THETA
C     ALF = FREE-FOIL LENGTH
      ELTOR=ALF/RADIUS
      PIR2=PI*RADIUS*2.0
      V= RPS*PIR2
      THETAG=ARCOS(DG/( 2.*RGUIDE)) +ARCOS(DG/RC)-ARCOS((RG+RO)/RC)
      THETG1  = THETAG*DEG
C     Y= (ACTUAL Y) /RADIUS    SGBR= SIGMA/RADIUS
      Y  = 0.
      IF(D.EQ.0.) GO TO 5
      D= E*THIKNS   **3/(12.*(1.-ENU**2))
      II=1
```

```
      CALL SLACK (TZ,SIGBRO,RADIUS,RGUIDE,RC,II )
      II=1
      CALL SLACK (TZ,SIGBRO,RADIUS,RGUIDE,RC,II )
    5 DD=  SPACE - DTR*ALFAR*RO + DTG*ALFAG* (RC-RG) - THIKNS
      CALL TEMPF
      COMP = PA *RADIUS/ET
      PEXT =2.*(RGUIDE/RADIUS)*(1.-EXP (-FCOEF*THETAG))/FCOEF
      TZP= TZ/ET
      IF( GOMTRY. AND.. NOT. GOMTRP) GO TO 200
      IF ( FLUID. AND.. NOT. FLUIDP) GO TO 210
      IF ( SOLID.  AND.. NOT. SOLIDP) GO TO 220
      IF ( TERMO.  AND.. NOT. TERMOP) GO TO 230
      GO TO 300
  200 GOMTRP=.TRUE.
      WRITE (6,400)
      WRITE (6,DATA)
      WRITE (6, SI)
      FLUID = .FALSE.
      FLUIDP=.FALSE.
      SOLID = .FALSE.
      SOLIDP=.FALSE.
      TERMOP=.FALSE.
      TERMO = .FALSE.
      PRINT 201
  201 FORMAT (50X,35HVARIATION OF GEOMETRY                        //
     1 60H RPS          T          H*       GAP            K           SMAX  ,
     2 60H          RO                 RG                RC       THETA THETAG/
     3 60H          LB/IN  N/M        MINCH E-6*M LB/IN2 E9*N/M2 PSI  E8*,
     4 60HN/M2  IN        M        IN       M       IN       M    DEG     DEG  )
      GO TO 300
  210 GOMTRP=.FALSE.
      WRITE (6,400)
      WRITE (6,DATA)
      WRITE (6, SI)
      GOMTRY= .FALSE.
      FLUIDP=.TRUE.
      SOLIDP=.FALSE.
      SOLID =  .FALSE.
      TERMOP=.FALSE.
      TERMO = .FALSE.
      PRINT 211
  211 FORMAT (50X,35HVARIATION OF FLUID PROPERTIES               //
     1 60H RPS          T          H*       GAP            K             PA  ,
     2 60H       TROOM       M          DENSITY       VISCOSITY          CON,
     3 10HDUCTIVITY          /
     4 60H          LB/IN  N/M        MINCH E-6*M LB/IN2 E9*N/M2 PSI   E4*,
     5 60HN/M2 F     K       LBF*S2/IN4  KG/M3 E-8*REYN E-4*NS/M2 B/H*F,
     6 10HT*F J/MSK      )
      GO TO 300
  220 GOMTRP=.FALSE.
      WRITE (6,400)
      WRITE (6,DATA)
```

```
      WRITE (6, SI)
      GOMTRY= .FALSE.
      FLUIDP=.FALSE.
      FLUID = .FALSE.
      SOLIDP=.TRUE.
      TERMOP=.FALSE.
      TERMO = .FALSE.
      PRINT 221
  221 FORMAT (50X,35HVARIATION OF  SOLID  PROPERTIES          //
     1 60H RPS         T          H*       GAP           K          SMAX  ,
     2 60H    THIKNESS            E              TO       ALFAF      ALFAG    ,
     3 10H  ALFAR          /
     4 60H          LB/IN  N/M        MINCH E-6*M LB/IN2 E9*N/M2 PSI E8*N/,
     5 60HM2  MIL E-6*M E6*PSI E10*N/M2 LB/IN N/M E-6/F  /K   E-6/F  /,
     6 11HK  E-6/F /K     )
      GO TO 300
  230 GOMTRP=.FALSE.
      WRITE (6,400)
      WRITE (6,DATA)
      WRITE (6, SI)
      GOMTRY= .FALSE.
      FLUIDP=.FALSE.
      FLUID = .FALSE.
      SOLIDP=.FALSE.
      SOLID = .FALSE.
      TERMOP=.TRUE.
      PRINT 231
  231 FORMAT( 47X,35HVARIATION OF TEMPERATURE/SPEED           //
     1 60H   RPS         T          H*       GAP           K         TFOIL ,
     2 60H     TGUIDE     TROTOR       VISCOSITY     CONDUCTIVITY        ,
     3 10H  DENSITY           /
     4 60H          LB/IN  N/M        MINCH E-6*M LB/IN2 E9*N/M2  F      K ,
     5 60H        F     K     F     K     E-8*REYN NS/M2 B/HR*FT*F   J/MSK  ,
     6 10HLBF*S2/IN4          )
 300  RETURN
  401 STOP
 400  FORMAT (1H1)
      END
```

```
      SUBROUTINE EQN1
      COMMON             THIKNS,RADIUS,DG,RGUIDE,RC,DD,SPACE,BB,RG,RO   ,
     1  T, TZ,   TT,    ET, TZP,TP,       EPS,EPS23,V,RPS,Y, CAY,
     2 HSTR,   HSTAR,  ETU,  PIR2,  THETA,  THETA1,  TH2,
     3  CTH2,   STH2 ,  THETAG,  THETG1, PI,DEG,FOA,FGA,
     4  PAIE,  COMP,  DENSTY,  PA,  EMU, CONDCT,CF,EMOL, C,AIE,
     A  CONSPR,TRVISC,TRCOND,CVISC,CCOND, COND,VIS,
     5  ELTOR,  RGBB  , ALF ,SIGBRO,SIGBR, DBDYK, DSGDB, DSBRDT,DSGDR ,
     B DBDR ,
     6 THERML,  ALFAF, ALFAG, ALFAR, TROOM,  DTF, DTG, DTR,
     8 TROTOR ,DTROT,TROTMX,TGUIDE,   TGIDMX ,TFOIL,
     9 DTHIK, THIKMX,DTZ,TZMX,DRO,ROMX,DRG,RGMX,DRPS,RPSMX.
     7  PEXT,   E,   FCOEF, ENU,  D ,SMAX
      LOGICAL TERMOP, FLUIDP,SOLIDP, GOMTRP ,CONSPR
      LOGICAL  TERMO, FLUID, SOLID, GOMTRY   ,SKIP ,READRC
      COMMON  TERMOP,FLUIDP,SOLIDP,GOMTRP                  ,ISCAN,IFOIL
      COMMON  TERMO,FLUID,SOLID,GOMTRY ,SKIP,READRC
      COMMON   AT(5), AX3(5), ASIGBR(5), ABSA(5)
      COMMON  AFP, AGP, ARP, AFINT, AGINT, ARINT, AFINTP,
     1 AGINTP, ARINTP, VISCP, TINT, HINTP, HINT,
     2 CYINTP, CYINT, SMINTP, SMINT, ROINT, RGINT, RCINT,
     3PAINT, PAINTP, TRMINT, DNSINT, VISINT, VSINTP, CNDINT,
     4TKINTP, TKINT, EINT, TZINT, TFINT, TGINT, TRINT,EP,EINTP,
     5 TCNV, CAYCNV, DNSCNV, VISCNV ,CONCNV
      IF(V.NE.0.) GO TO 999
      T=TZ
      TP=T/ET
      TZP=TP
  999 AVG=4.0
    1 DO 10 I=1,25
      IF (V.NE.0.)  GO TO 5
      EPS23 =0.
      HS= 0.643
      GO TO 6
    5 EPS=V/ETU
      AIE= PAIE*EPS**2/TP
      C=COMP/TP
      EPS23=(EPS/TP)**0.6666666667
      HS= 0.643+0.286*AIE+1.905*AIE*AIE-0.183 /C
    6 IF(D.NE.0.) GO TO 7
      DSGBR=0.
      GO TO 8
    7 II=1
      CALL SLACK (T,SIGBR,RADIUS, RGUIDE, RC,II)
      DSGBR= SIGBR-SIGBRO
    8 TPN= TZP + (   -2.*Y*STH2
     1  +THETA * HS*EPS23+ DSGBR*2. -THERML ) /(ELTOR+ PEXT)
      HSTR=RO* HS* EPS23
      IF(TPN.GT.0.) GO TO 9
      TEMP=TP
      TP = TP/2.
      GO TO 91
```

```
    9  IF (ABS (( TPN-TP)/TPN).LT.0.0005 ) GO TO 11
       TEMP=TP
       TP=(TPN+AVG*TP)/(1.+AVG)
   91  IF(IFOIL.EQ.1) CALL TEMPF
       T= TP*ET
   10  CONTINUE
       IF ( AVG. GT. 4. ) GO TO 101
       AVG=8.
       GO TO 1
  101  PRINT 80, TEMP,TP
   80  FORMAT( 35H NO CONVERGENCE AFTER 50 STEPS,             6HTPOLD= ,
     1  E16.8, 6HTPNEW=  E16.8)
   11  CONTINUE
       HSTAR = HSTR*1.E 6
       IF( D.NE.0.) GO TO 27
       DSBRDT=0.
       DSGDB=0.
       DSGDR=0.
       GO TO 28
   27  XXX=((ASIGBR(1) - ASIGBR(3))/(AT(1) -AT(3))    -
     1      ( ASIGBR(2)-ASIGBR(3))/(AT(2)- AT(3))) / (AT(1) -AT(2))
       YYY = ((ASIGBR(1) -ASIGBR(3))/ (AT(1)**2 - AT(3)**2) -
     1      (ASIGBR(2)- ASIGBR(3)) / (AT(2)**2  - AT(3)**2))
     2     *  (  AT(1)+  AT(3)) * (AT(2) + AT(3)) / (AT(2) - AT(1))
       DSBRDT  = 2.   *  XXX * T   + YYY
       T=TP*ET
       II =4
       CALL SLACK ( T, SIGBR, RADIUS, RGUIDE, RC,II)
   28  CAY = 3.*STH2*((2.*STH2 - DSGDB * DBDYK) / RADIUS ) /
     1((ELTOR+PEXT)/ET    - EPS23 *((-.286*AIE -3.81*AIE**2 - 0.183/C
     2 -0.666667*HS) / T ) * (THETA+2.*(DSGDR+DSGDB*DBDR))-2.*DSBRDT)
       SMAX= T/THIKNS+ ET/(2.*RGUIDE)
       SMINTP= SMAX*6.8947E-5
C   SMINTP=SMAX*6.8947E3 *(1.E-8  FOR PRINTING)
       TINT=TCNV*T
       HINTP=HSTAR*2.54E-2
       HINT=HINTP*1.E-6
       CYINTP=CAY*CAYCNV
       CYINT = CYINTP*1.E9
       ROUND=100.
       IF(CAY.GT.10000.) GO TO 13
       ROUND=10.
   13  CAY= ROUND*IFIX((CAY+ROUND/2.)/ROUND)
C   IF(SKIP) LEAVE A BLANK OUTPUT LINE
       IF(.NOT.SKIP) GO TO 15
       SKIP=.FALSE.
       WRITE (6,14)
   14  FORMAT(1H )
   15  CONTINUE
       IF(GOMTRY)           GO TO 200
       IF(FLUID)           GO TO 210
       IF(SOLID           )GO TO 220
```

```
      IF(TERMO)           GO TO 230
  200 PRINT 201; RPS, T, TINT,HS,HSTAR, HINTP, CAY, CYINTP, SMAX,
     1 SMINTP, RO,ROINT, RG, RGINT, RC, RCINT, THETA1, THETG1
  201 FORMAT(1X,F6.0, F6.2,F6.0 ,F6.3,F6.0,F5.1,F7.0,F6.3,1X,
     1 F8.0,F5.1, F8.4, F7.4, F8.4, F7.4,F8.4,F7.4,F8.2, F7.2 )
      GO TO 300
 210  COND1= CONDCT/CF
      PRINT 211,RPS,T , TINT, HS,HSTAR, HINTP, CAY, CYINTP,
     1 PA, PAINTP, TROOM, TRMINT, EMOL, DENSTY, DNSINT, VISCP,
     2 VSINTP, COND1, CNDINT
  211 FORMAT (1X,F6.0,F6.2,F6.0,F6.3,F6.0,F5.1, F7.0,F6.3, 1X ,
     1F6.1, F6.1, F5.1, F6.1, F5.1, E10.3, F7.2, F7.3, F7.3, E10.3,
     2 E10.3)
      GO TO 300
  220 PRINT 221, RPS, T, TINT, HS, HSTAR, HINTP, CAY, CYINTP,
     1SMAX, SMINTP, TT     , TKINTP,EP, EINTP, TZ,TZINT,
     2AFP , AFINTP, AGP , AGINTP, ARP, ARINTP
  221 FORMAT(1X,F6.0,F6.2,F6.0,F6.3, F6.0, F5.1, F7.0, F6.3, 1X ,
     1 F8.0, F5.1, F5.2,F5.1, F7.1, F7.2, F5.1, F6.1, F5.1, F5.1,
     2F6.1, F5.1, F6.1, F5.1  )
      GO TO 300
 230  COND1= CONDCT/CF
      PRINT 231, RPS, T, TINT,HS, HSTAR, HINTP, CAY, CYINTP,
     1 TFOIL, TFINT, TGUIDE, TGINT, TROTOR, TRINT,
     2 VISCP,  VSINTP, COND1, CNDINT, DENSTY
  231 FORMAT(1X,F6.0,F6.2, F6.0, F6.3, F6.0,F5.1, F7.0, F6.3, 1X,
     1 F6.1, F6.1, F6.1, F6.1, F6.1, F6.1, F7.3, F7.3, E10.3,E10.3,
     2 E10.3)
 300  RETURN
      END
```

```
      SUBROUTINE   SLACK ( TX, SIGBRX,ROX, RGX,RCX,I)
      COMMON          THIKNS,RADIUS,DG,RGUIDE,RC,DD,SPACE,BB,RG,RO   ,
     1  T, TZ,  TT,    ET, TZP,TP,      EPS,EPS23,V,RPS,Y, CAY,
     2 HSTR,  HSTAR,  ETU,  PIR2,  THETA,  THETA1,  TH2,
     3  CTH2,   STH2 ,  THETAG,  THETG1, PI,DEG,FOA,FGA,
     4  PAIE,  COMP,  DENSTY,  PA,  EMU, CONDCT,CF,EMOL,CZ,AIE,
     A  CONSPR,TRVISC,TRCOND,CVISC,CCOND, COND,VIS,
     5  ELTOR,  RGBB  , ALF ,SIGBRO,SIGBR, DBDYK, DSGDB, DSBRDT,DSGDR ,
     B DBDR,
     6 THERML,  ALFAF, ALFAG, ALFAR, TROOM,  DTF, DTG, DTR,
     8 TROTOR ,DTROT,TROTMX,TGUIDE,  TGIDMX ,TFOIL,
     9 DTHIK, THIKMX,DTZ,TZMX,DRO,ROMX,DRG,RGMX,DRPS,RPSMX,
     7  PEXT,  E,  FCOEF, ENU, D ,SMAX
      LOGICAL TERMOP, FLUIDP,SOLIDP, GOMTRP ,CONSPR
      LOGICAL  TERMO, FLUID, SOLID, GOMTRY  ,SKIP ,READRC
      COMMON  TERMOP,FLUIDP,SOLIDP,GOMTRP                 ,ISCAN,IFOIL
      COMMON  TERMO,FLUID,SOLID,GOMTRY ,SKIP,READRC
      COMMON   AT(5), AX3(5), ASIGBR(5), ABSA(5)
      COMMON  AFP, AGP, ARP, AFINT, AGINT, ARINT, AFINTP,
     1 AGINTP, ARINTP, VISCP, TINT, HINTP, HINT,
     2 CYINTP, CYINT, SMINTP, SMINT, ROINT, RGINT, RCINT,
     3PAINT, PAINTP, TRMINT, DNSINT, VISINT, VSINTP, CNDINT,
     4TKINTP, TKINT, EINT, TZINT, TFINT, TGINT, TRINT,EP,EINTP,
     5 TCNV, CAYCNV, DNSCNV, VISCNV ,CONCNV
      DOUBLE PRECISION A, A2, C, SH3, CH3, TH3, PART1, PART2, P,Q,
     1 X1,X1A,X2,X2A, X3, BSA,EPSA,TA,SH1,CH1,SH2,CH2,
     2        TERM2,        TERM4,TERM5,      TERM7,SIBRE3,SIGM
      EQUIVALENCE (X1,X1A), (X2,X2A)  ,(SIBRE3,SIGM)
C     WRITE (6,9)
    9 FORMAT (1H , 3X,7HTENSION,5X,3HEPS, 6X, 2HFO,    6X, 2HFG, 4X,
     1 4HSO/B, 4X, 4HSG/B,1X,7HB/(R*E)
     2      2H J , 1X,7HTENSION,5X,3HEPS, 6X, 2HFO,    6X, 2HFG, 4X,
     3 4HSO/B, 4X, 4HSG/B,1X,7HB/(R*E)                )
      BBX =  SQRT (  RCX**2 - (ROX + RGX)** 2  )
      IF(I.NE. 4) GO TO 1
      ROBB = 1.10* ROX/BBX
      ROBRG = 1.10* ROX/RGX
      BBYRO = BBX / (ROX*1.10)
      ROBRG2 = ROBRG**2
      RGBRO = 1./ ROBRG
      X3 = AX3(3)
      GO TO 3
    1 ROBB = ROX /BBX
      ROBRG= ROX/RGX
      BBYRO= BBX/ROX
      ROBRG2 = ROBRG**2
      RGBRO = RGX/ROX
      TA = TX  +0.35
C  ESTIMATED FIRST GUESS
    2 X3 = ( RGX*FGA+BBX+ROX* FOA)*DSQRT(TA/D)
    3 J=0
      IF(X3.LT.85.)GO TO 110
```

```
      SH3=1.E43
      CH3=1.E43
      TH3=1.
      A=-1.
      A2=1.
      C=ROBRG*(X3-1.)
      GO TO 120
 110  CH3=DCOSH(X3)
      SH3 =DSINH(X3)
      TH3 =DTANH(X3)
      A= - ROBRG/SH3-1./TH3
      A2= A**2
      C= ROBRG*X3-ROBRG/TH3 -1./SH3
  120 P= ROBRG*X3-(ROBRG-1.)*DTANH(X3/2.)
      Q = 2.+ ROBRG*(X3**2+2.) -2.*X3 *(1./SH3+ ROBRG/ DTANH(X3)
      PART1  =  P*ROBRG
      PART2=          Q*(ROBRG2+ROBRG)  -P**2*ROBRG
      IF  (PART2.GE.0)  GO TO 5
      PART2=0.
      J=1
    5 PART2  =DSQRT(PART2)
      X2 A= (PART1  +  PART2)/(ROBRG2  + ROBRG)
      X1A= P-ROBRG*X2A
      BSA=  X2A - X1A
      EPSA=  BBYRO/BSA
      FGA=  ROBRG*EPSA*(X3 -X2A)
      FOA=X1A*EPSA
      FO=FOA*DEG
      FG= FGA*DEG
C     SOBBA = SO/B , SOLUTION A ETC
      SOBBA = X1A* EPSA *ROBB
      SGBBA=(X3 -X2A)*EPSA*ROBB
      BBROEA=  BBYRO/EPSA
      SH1 =DSINH(X1)
      CH1 =DCOSH(X1)
      SH2  =DSINH(X2)
      CH2 =DCOSH(X2)
      IF(X3.LT.85.)GO TO 210
 219  TERM2= - ROBRG*DEXP(X2-X3)+ DEXP(-X2)
      TERM4= 2.*ROBRG * (-ROBRG*DEXP(X2-X3)-DEXP(-X2))
      IF(X1.LT.0.)GO TO 220
      TERM7=-2.*(ROBRG*DEXP(X1-X3) + DEXP(-X1))
     1 -(0.666667*X1 + A)*X1**2
      GO TO 225
  220 TERM7=-X1**3/3. +2.*(A+X1)
      GO TO 225
  210 TERM2 = (-ROBRG*SH2 + DSINH(X3-X2))/SH3
      TERM4= 2.*ROBRG*(-ROBRG*CH2 - DCOSH(X3-X2))/SH3
      IF(X1.LT.0.) GO TO 240
      TERM7=-2.*(ROBRG*CH1 +DCOSH(X3-X1))/SH3-
     1( 0.666667*X1     +A)*X1**2
      GO TO 225
```

```
 240 TERM7= -X1**3/3.+2.*(A+X1)
 225 IF(X3.GT.16.)GO TO 221
     TERM5 = (A2+1.)*DSINH(2.*X3)/4.+A*DCOSH(2.*X3)/2.
     GO TO 228
221  TERM5 = 0.5*ROBRG2
228  SIGM  =(-A/2.+(A+X1)**2*X2+ ROBRG2*(X3**3-X2**3)/3.
    1 +(A2-1.)*X3/2.+C**2*(X3-X2)-ROBRG*C*(X3**2-X2**2)
    2                            +TERM4+TERM5       +TERM7)/2.
     SIR2B3  =  SIBRE3 /BSA**3
     SIGBRX= SIR2B3/ROBB**3
     TA= (BSA/BBX)**2*D
     AT(I)= TA
     AX3(I)= X3
     ASIGBR(I)= SIGBRX
     ABSA(I)=BSA
     GO TO (24,25,26,34,35                               ), I
 24  I=2
     TA= TA-0.70
     GO TO 2
 25  X3 = AX3(1)+ (AX3(2)- AX3(1))*(TX-AT(1)) /(AT(2)-AT(1))
     I=3
     GO TO 3
  34 X3 = AX3(3)*1.10
     DSGDB= (ASIGBR(2)-ASIGBR(1))/(( ABSA(2)-ABSA(1))*EPSA)
     I=5
     GO TO 3
  35 BSIGBR = ASIGBR(4)+(ASIGBR(5)-  ASIGBR(4))* (ABSA(3) -ABSA(4)) /
    1  ( ABSA(5)- ABSA(4))
     DSGDR=(BSIGBR*1.10   - ASIGBR(3)) / 0.1
 26  RETURN
     END
```

```
      SUBROUTINE TEMPF
      COMMON             THIKNS,RADIUS,DG,RGUIDE,RC,DD,SPACE,BB,RG,RO  ,
     1  T, TZ,  TT,    ET, TZP,TP,       EPS,EPS23,V,RPS,Y, CAY,
     2 HSTR,  HSTAR,  ETU,  PIR2,  THETA,  THETA1,  TH2,
     3  CTH2,   STH2 ,  THETAG,  THETG1, PI,DEG,FOA,FGA,
     4  PAIE,  COMP,  DENSTY,  PA,  EMU, CUNDCT,CF,EMOL, C,AIE,
     A  CONSPR,TRVISC,TRCOND,CVISC,CCOND, COND,VIS,
     5  ELTOR,  RGBB  , ALF ,SIGBRO,SIGBR, DBDYK, DSGDB, DSBRDT,DSGDR ,
     B DBDR,
     6 THERML,  ALFAF, ALFAG, ALFAR, TROOM,  DTF, DTG, DTR,
     8 TROTOR ,DTROT,TROTMX,TGUIDE,  TGIDMX ,TFOIL,
     9 DTHIK, THIKMX,DTZ,TZMX,DRO,ROMX,DRG,RGMX,DRPS,RPSMX,
     7  PEXT,  E,  FCOEF, ENU, D ,SMAX
      LOGICAL TERMOP, FLUIDP,SOLIDP, GOMTRP ,CONSPR
      LOGICAL  TERMO, FLUID, SOLID, GOMTRY  ,SKIP ,READRC
      COMMON  TERMOP,FLUIDP,SOLIDP,GOMTRP                 ,ISCAN,IFOIL
      COMMON  TERMO,FLUID,SOLID,GOMTRY ,SKIP,READRC
      COMMON   AT(5), AX3(5), ASIGBR(5), ABSA(5)
      COMMON  AFP, AGP, ARP, AFINT, AGINT, ARINT, AFINTP,
     1 AGINTP, ARINTP, VISCP, TINT, HINTP, HINT,
     2 CYINTP, CYINT, SMINTP, SMINT, RCINT, RGINT, RCINT,
     3PAINT, PAINTP, TRMINT, DNSINT, VISINT, VSINTP, CNDINT,
     4TKINTP, TKINT, EINT, TZINT, TFINT, TGINT, TRINT,EP,EINTP,
     5 TCNV, CAYCNV, DNSCNV, VISCNV ,CONCNV
      IF(CONSPR) GO TO 121
      EMU= VIS*(TRVISC+460.+CVISC)*((  TROOM+(DTR+DTF)/2.+460.)/(
     2TRVISC+460. ))**1.5/  (TROOM  +(DTF +DTR)/2. +460.+ CVISC)
      CONDCT= COND* CF* (TRCOND+460.+CCOND)*((TROOM+(DTF+DTR)/2. +460.)/
     1  (TRCOND+460.))**1.5/(TROOM+(DTF+DTR)/2.+460.+CCOND)
      GO TO 122
  121 EMU=VIS
      CONDCT=COND*CF
  122 CONTINUE
      IF(IFOIL.NE.1) GO TO 10
      TFOIL= (EMU*V**2/(2.*CONDCT   )+TROTOR)*(DD-HSTR)/DD
     1  +  TGUIDE* HSTR / DD
   10 DTF=TFOIL-TROOM
C     DENSTY = LBF*SEC**2/IN**4
      ETU=ET/(6.0*EMU)
      DENSTY = (PA/(1544.*(460.+  TROOM+ (DTF + DTR)/2.)/EMOL))
     1 /(32.2 *144.)
      PAIE=DENSTY  *ET *RADIUS/(72.   *EMU**2)
    5 THERML =(ALFAF* ALF*DTF
     1      + 2.*ALFAF  * (RGUIDE *THETAG) *DTG
     2 -ALFAG*2.*BB*DTG -ALFAR*THETA*RADIUS*DTR -2.*ALFAG*THETAG
     4  *RGUIDE*DTG) /RADIUS
      TFINT=(TFOIL+459.67)/1.8
      DNSINT= DENSTY* DNSCNV
      CNDINT=CONDCT*CONCNV
      VISCP=EMU*1.E8
      VSINTP=EMU*VISCNV
      VISINT= VSINTP* 1.E-4
      RETURN
      END
```

SAMPLE OUTPUT

Namelist, British Units

```
&DATA
THIKNS= 0.99999993E-03,DTHIK= 0.19999999E-03,THIKMX= 0.19999999E-02,TT= 0.99999988     ,ET=  29999.996     ,RADIUS= 0.87549996
RO= 0.87500000     ,DRO= 0.25000000     ,ROMX=   2.0000000     ,RGUIDE= 0.12299991     ,RG= 0.12249994     ,DRG= 0.49999997E-01,RGMX=
 0.29999995     ,RC=  1.0095997     ,Y= 0.0                  ,SPACE= 0.52499995E-02,DG= 0.12499994     ,BB= 0.14929491     ,ALF=
  1.6435175     ,RGBB= 0.82387203     ,ELTOR=  1.8772326     ,THETA=  1.5361834     ,THETAI=  88.016815     ,TH2= 0.76809168     ,CTH2=
 0.71923780     ,STH2= 0.69476396     ,THETAG=  2.3294735     ,THETG1=  133.46898     ,T=  4.0000000     ,TZ=  2.0000000     ,DTZ=
 0.50000000     ,TZMX=   4.0000000     ,TZP= 0.66666675E-04,TP= 0.13333335E-03,PEXT= 0.55241752     ,E=  30000000.     ,FCOEF=
 0.14999998     ,ENU= 0.29999995     ,D= 0.27472500E-02,ALFAF= 0.62999998E-05,ALFAG= 0.55999999E-05,ALFAR= 0.72999992E-05,PAIE=
 0.91413283E 13,COMP= 0.73250127E-03,DENSTY= 0.55143539E-06,PA=  25.099991     ,VIS= 0.24750000E-08,EMOL=  83.799988     ,COND=
 0.11000000E-01,EMU= 0.46909960E-08,CONDCT= 0.40374398E-02,TRVISC= -100.00000     ,TRCOND= -100.00000     ,RPS=  600.00000     ,DRPS=
  25.000000     ,RPSMX=  200.00000     ,V=  3300.5562     ,ETU= 0.10658715E 13,PIR2=  5.5009270     ,CVISC=  346.79199     ,CCOND=
  168.17599     ,TFOIL=  75.583969     ,TGUIDE=  70.000000     ,TROTOR=  70.000000     ,DTROT=  40.000000     ,TROTMX=  470.00000     ,
TGIDMX=  495.00000     ,TROOM=  70.000000     ,THERML= 0.66039182E-04,TERMO=T,FLUID=F,SOLID=F,GOMTRY=F,TERMOP=F,FLUIDP=F,SOLIDP=F,
GOMTRP=F,CONSPR=F,ISCAN=          6,IFOIL=          1,SKIP=F,READRC=T
&END
```

Namelist, International System

```
&SI
TKINT= 0.25399975E-04,ROINT= 0.22224996E-01,RGINT= 0.31114982E-02,RCINT= 0.25643829E-01,TINT=  700.50366     ,TZINT=  350.25171
SPCINT= 0.13334997E-03,EINT= 0.20684092E 12,AFINT= 0.11339992E-04,AGINT= 0.10079993E-04,ARINT= 0.13139990E-04,DNSINT=
  5.8917999     ,PAINT=  173056.75     ,CNDINT= 0.38766629     ,VISINT= 0.32343058E-04,TFINT=  297.36304     ,TGINT=  294.26099
TRINT=  294.26099     ,TRMINT=  294.26099
&END
```

VARIATION OF TEMPERATURE/SPEED

RPS	T		H*	GAP		K		TFOIL		TGUIDE		TROTOR		VISCOSITY		CONDUCTIVITY		DENSITY
	LB/IN	N/M		MINCH	E-6*M	LB/IN2	E9*N/M2	F	K	F	K	F	K	E-8*REYN	NS/M2	B/HR*FT*F	J/MSK	LBF*S2/IN4
600.	7.37	1291.	0.926	367.	9.3	15900.	0.110	75.5	297.3	70.0	294.3	70.0	294.3	0.358	0.247	0.149E-01	0.310E 00	0.551E-06
600.	7.53	1319.	0.874	356.	9.0	16900.	0.116	115.8	319.7	112.5	317.9	110.0	316.5	0.382	0.263	0.157E-01	0.327E 00	0.513E-06
600.	7.72	1352.	0.830	346.	8.8	17800.	0.123	156.1	342.1	155.0	341.5	150.0	338.7	0.405	0.279	0.165E-01	0.343E 00	0.479E-06
600.	7.93	1389.	0.794	337.	8.6	18700.	0.129	196.3	364.4	197.5	365.1	190.0	360.9	0.428	0.295	0.173E-01	0.359E 00	0.450E-06
600.	8.16	1429.	0.763	328.	8.3	19600.	0.135	236.6	386.8	240.0	388.7	230.0	383.1	0.449	0.310	0.180E-01	0.374E 00	0.424E-06
600.	8.40	1471.	0.737	321.	8.1	20500.	0.141	276.8	409.2	282.5	412.3	270.0	405.4	0.471	0.325	0.187E-01	0.389E 00	0.401E-06
600.	8.66	1516.	0.714	314.	8.0	21300.	0.147	317.1	431.5	325.0	435.9	310.0	427.6	0.492	0.339	0.194E-01	0.403E 00	0.380E-06
600.	8.93	1563.	0.695	307.	7.8	22100.	0.152	357.3	453.9	367.5	459.5	350.0	449.8	0.512	0.353	0.201E-01	0.417E 00	0.361E-06
600.	9.21	1612.	0.678	301.	7.7	22800.	0.158	397.5	476.2	410.0	483.2	390.0	472.0	0.532	0.367	0.208E-01	0.431E 00	0.344E-06
600.	9.50	1663.	0.663	296.	7.5	23500.	0.162	437.7	498.6	452.5	506.8	430.0	494.3	0.552	0.380	0.214E-01	0.444E 00	0.329E-06
600.	9.80	1715.	0.650	290.	7.4	24200.	0.167	477.9	520.9	495.0	530.4	470.0	516.5	0.571	0.394	0.220E-01	0.457E 00	0.315E-06

SYMBOLS

a_c, a_{ck}	distance between origin "O" and line connecting the centers of the guides, Fig. 3 (k refers to kth foil sector, subscript k is omitted for axisymmetric case), in. (m)
A	function defined in Eq. (20b) lb/in^2 (N/m^2); also, integration constant Eq. (A19)
A_o, A_1, etc.	points defined in Fig. A1
b_c, b_{ck}	distance between centers of guides, Fig. 3 (k refers to kth foil sector; subscript k is omitted in axisymmetric case), in. (m)
b, b_{ik}, b_{ek}	partial foil length defined in Fig. 3 and Fig. A1 (k refers to the kth foil sector, i and e refer to inlet and exit respectively), in. (m)
C	integration constant, Eq. (A20)
C_λ	Sutherland constant for thermal conductivity, Eq. (30), ^oF (^oK)
C_μ	Sutherland constant for viscosity, Eq. (29), ^oF (^oK)
d	foil thickness, in. (m)
d_g	distance defined in Fig. 2 for reference state, in. (m)
D	flexural rigidity of foil = $Ed^3/[12(1-\nu^2)]$, lb/in (N/m)
E	Young's modulus, psi (N/m^2); also integration constant Eq. (A21)
e	subscript referring to the exit region
F_x, F_y	components of resultant force of foil support system on the rotor, lb/in (N/m)
g	subscript referring to guide or shield-damper, Fig. 2
h_g, h_{gk}	distance of rotor to kth shield-damper, Fig. 2 (subscript k is omitted in axisymmetric case), in. (m)
h_{go}	distance of rotor to shield-damper in reference state, in. (m)

h^*, h_k^*	clearance in uniformity region of kth foil sector (subscript k is omitted for axisymmetric case), in. (m)
H^*, H_k^*	dimensionless clearance in uniformity region of kth foil sector (subscript k omitted for axisymmetric case)
i	subscript referring to the inlet region
k	bearing stiffness per unit width lb/in^2 (N/m^2); also subscript referring to kth foil sector
ℓ_k	total foil length between clamping points on foil guides, referred to kth foil sector, in. (m)
ℓ_o	magnitude of foil length l_k in the reference state, in. (m)
ℓ_{eff}	effective foil length defined by Eq. (11), in. (m)
M	molecular weight of lubricating fluid
O	position of rotor center in reference state, Fig. 2
p_a	pressure of fluid entering the bearing, absolute, psi (N/m^2)
R	local radius of curvature of foil, in. (m)
R_f	local radius of curvature of perfectly flexible foil, in. (m)
r_c	distance of center of guide from origin O, Fig. 3, in. (m)
r_g	radius of foil guide, Fig. 3, in. (m)
r_o	radius of rotor in reference state, in. (m)
$\overline{R}$	universal gas constant, in lb/lb-mole ^{o}F (m N/kg-mole ^{o}K)
s	length along foil, in. (m)
s_1, s_2, s_3	values of s at points A_1, A_2, A_3, Fig. A1, in. (m)
T_o	initial tension per unit width of foil (preload), lb/in (N/m)
T, T_k	tension per unit width of kth foil segment (subscript k may be omitted for axisymmetric case), lb/in (N/m)
w	deviation of finite stiffness foil contour from flexible foil contour, in. (m)
W	$w/(r_o \varepsilon^2)$
U	surface velocity of rotor, in/sec (m/s)

x, y — components of displacement of rotor center, Fig. 2, in (m)

x_k, y_k — component of displacement of rotor center in the kth auxiliary coordinate system, Fig. 2, in. (m)

z — coordinate normal to foil, Fig. 4, in. (m)

α_f, α_{fk} — coefficient of thermal expansion of kth foil, (subscript k is omitted in axisymmetric case), $1/^{o}F$ $(1/^{o}K)$

α_g, α_{gk} — coefficient of thermal expansion of kth shield-damper-guide, (subscript k is omitted in axisymmetric case), $1/^{o}F$ $(1/^{o}K)$

α_r — coefficient of thermal expansion of rotor, $1/^{o}F$ $(1/^{o}K)$

α_{ik}, α_{ek} — inlet and exit angles defined in Fig. 3, referred to kth foil sector, rad (rad)

β_{ik}, β_{ek} — inlet and exit angles defined in Fig. 3, referred to the kth foil sector, rad (rad)

δa_{ck} — change in a_{ck} relative to reference state, in. (m)

$\delta \ell_k$ — elongation of kth foil $= \ell_k - \ell_o$, in. (m)

γ_k — angular position of kth foil sector referred to the foil sector (k = 1), Fig. 2, rad (rad)

ε — $\sqrt{D/(T r_o^2)}$

$\delta b, \delta b_{ck}, \delta b_{ik}, \delta b_{ek}$ — change in b, b_{ck}, b_{ik}, b_{ek} with respect to reference state, in. (m)

$\delta \ell_{\tau k}$ — equivalent length change of kth foil sector defined by Eq. (10), in. (m)

δr_o — perturbation in rotor radius (thermal and centrifugal), in. (m)

δr_{gk} — perturbation in guide radius, referred to the kth foil sector, (subscript k is omitted for axisymmetric case), in. (m)

$\delta\sigma_{ik}, \delta\sigma_{ek}, \delta\sigma$ — difference between value of σ_{ik}, σ_{ek} or σ, and the value σ_o (reference state), in (m)

$\delta\tau$ — excess temperature over reference, $\tau - \tau_o$, ^{o}F (^{o}K)

$\delta\tau_{fk}$ — $\tau_{fk} - \tau_o$ (subscript k is omitted for axisymmetric case), $^{\circ}F$ $(^{\circ}K)$

$\delta\tau_{gk}$	$\tau_{gk} - \tau_o$ (subscript k is omitted for axisymmetric case), °F (°K)
$\delta\tau_{rk}$	$\tau_{rk} - \tau_o$ (subscript k is omitted for axisymmetric case), °F (°K)
ϕ_o, ϕ_g	angles defined in Fig. A1, rad (rad)
Θ	nominal angle of wrap in reference state, rad (rad)
Θ_g	nominal angle of wrap around guide in reference state, rad (rad)
λ_a	thermal conductivity of lubricating fluid at temperature τ_a, BTU/(ft hr °F) (J/(m·s °K))
λ_b	thermal conductivity of lubricating fluid at base temperature τ_b, BTU/(ft·hr °F) (J/(m·s °K))
μ_a	viscosity of lubricating fluid at temperature τ_a, lb sec/in^2 (N.S/m^2)
μ_b	viscosity of lubricating fluid at base temperature τ_b, lb sec/in^2 (N.S/m^2)
ν	Poisson's ratio of foil material
ρ_a	density of lubricating fluid at pressure p_a and temperature τ_a, lb·sec^2/in^4 (kg/m^3)
σ	excess of length of finite stiffness foil over a perfectly flexible foil defined in Eq. (A25), in. (m)
σ_{ik}, σ_{ek}	value of σ referred to inlet and exit branch of kth foil sector respectively, in. (m)
τ_a, τ_{ak}	temperature of working fluid; assumed: $\tau_a = (\tau_r + \tau_{fk})/2$, (subscript k is omitted for axisymmetric case. The subscript (abs) is added when absolute temperature is used). °F (°K)
ξ	$s/(r_o \varepsilon)$
ξ_1, ξ_2, ξ_3	values of ξ at points A_1, A_2, A_3, Fig. A1

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