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DEFORMED BRUECKNER-HARTREE-FOCK CALCULATIONS

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ABSTRACT

The renormalized Brueckner-Hartree-Fock (RBHF) theory for many-body nuclear systems has been generalized to permit calculations for intrinsic states having permanent deformation. Both Hartree-Fock and Brueckner self-consistencies are satisfied, and details of the numerical techniques are discussed. The Hamada-Johnston interaction is used in a study of deformations, binding, size, and separation energies for several nuclei. Electromagnetic transition rates, moments, and electron scattering form factors are calculated using nuclear wave functions obtained by angular momentum projection. Comparison is made to experiment as well as to predictions of ordinary and density-dependent Hartree-Fock theory.

Deformed Brueckner-Hartree-Fock Calculations[+]

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INTRODUCTION

The connection between the nuclear many-body problem and the prediction of properties of finite nuclei has become a subject of great interest in the last few years. Advances in computational technology have made it possible to drop the "closed-shell-core" assumption which was common to most nuclear structure calculations.[1] Thus it has become feasible to attempt to bring the so-called "inert core" into active participation.

Our understanding of the properties of nuclei has been extended significantly by the Hartree-Fock method and variations thereof. The HF approach is of course a very natural starting point for microscopic studies of nuclear systems because of its simplicity and because it gives, by definition, the best independent-particle basis for the nucleus. HF calculations, in the

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past, have been restricted to the use of (1) phenomenological effective interactions[2] or (2) effective interactions based directly on nuclear matter calculations.[3] Such interactions have been used in studies of both spherical and deformed nuclei.[4] Some of the more recent calculations using interactions of the latter type have been quite successful in predicting a large number of nuclear properties.[5]

Although such studies have been invaluable in contributing to our knowledge of nuclear structure, one would nevertheless prefer to begin with a "realistic" nucleon-nucleon interaction and, with as few approximations as possible, calculate nuclear properties based on a many-body theory. The initial framework for such an approach was provided by the pioneering work of Brueckner and Goldstone[6], which was followed by a large number of papers clarifying their theory and extending it beyond applications to nuclear matter.[7] There still remain uncertainties in the many-body theory because of a lack of understanding of the two-body interaction and some questions regarding the importance of higher-order Brueckner-Goldstone diagrams and three-body clusters.[8] However, if one hopes to understand the structure of nuclei in terms of a true microscopic picture, a Brueckner-Hartree-Fock (BHF) calculation seems to be the most reasonable approach presently available.

Application of the BHF methods to finite nuclei was first carried out by the Oak Ridge groups.[9] They investigated

spherical, closed shell nuclei, and it was demonstrated by Davies et.al. that self consistent BHF calculations could be made easily and reliably, although they found the nuclei to be underbound and too small.[10] The inclusion of higher-order diagrams, representing occupation probabilities, in BHF calculations by Becker led to substantial improvement in the single-particle energies, and it was found that depletion factors for normally occupied single-particle orbits are of the order of 15 percent.[11] Subsequently Davies and McCarthy showed that the binding energies also improve when these higher-order diagrams are included.[12]

Until recently Brueckner-type calculations were restricted to infinite nuclear matter and spherical closed-shell nuclei. However, since most nuclei do not fall in this class, it is of interest to extend the BHF approach to include nonspherical and/or open-shell nuclei. The light deformed nuclei provide a good starting point for a study of this nature, since the number of particles involved is small enough to keep the problem tractable and the simple HF approximation is sufficiently well understood to provide guidelines for what is recognized to be a rather complicated problem.

The aim of this paper is to indicate how one goes about doing such a complicated calculation, to point out some of

the conceptual difficulties which arise for the case of deformed nuclei, and then to discuss some results for several light deformed nuclei. Some preliminary accounts of this work have been reported previously.[13]

BRUECKNER THEORY FOR DEFORMED SYSTEMS

The application of Brueckner theory to spherical closed-shell nuclei is relatively straight forward, since the uncorrelated ground states of such nuclei are simple and have good angular momentum. The single-particle Hamiltonians are therefore rotationally invariant, and standard techniques may be used to obtain a relative Bethe-Goldstone (BG) equation. Details of such Brueckner and BHF calculations have been presented by Becker, MacKellar, and Morris,[9] and by Davies and McCarthy.[10]

On the other hand, the treatment of open-shell nuclei is not at all simple. The formalisms developed thus far for nuclei with several particles outside a core involve either an energy-dependent effective Hamiltonian (Block-Horowitz[14] and Feshbach[15]), or more recently an energy-independent effective Hamiltonian arrived at by folded-diagram techniques (Baranger and Johnson,[16] and Kuo, Lee, and Ratcliff).[17] While these techniques are extremely valuable in providing a formal basis for the shell model, they are complicated in practice and it is at present desirable to look elsewhere for simpler means of calculating properties of open-shell nuclei.

The theory of rotational nuclei has been aided considerably by the concept of a deformed intrinsic state, and the present paper provides the first detailed discussion of Brueckner theory for nuclei for which such a model is reasonably valid. The formulation is restricted to the simplest case, that in which there is an isolated rotational band. We employ the approximation in which the states of good angular momentum in the band are obtained by projection from a single deformed, intrinsic state. This method appears promising because the simpler HF calculations with effective interactions have demonstrated that such projected wave functions yield reasonable dynamic as well as static properties of light nuclei.

Our approach is based on the non-degenerate Brueckner-Goldstone-Brandow renormalized[7] linked-cluster expansion (LCE), starting from a deformed, determinantal, intrinsic state, Φ_K, which is an eigenstate of an unperturbed Hamiltonian, $\mathcal{H}_0$, containing a deformed self-consistent field, U. The LCE generates from Φ_K a correlated, exact eigenstate, Ψ_K, of an intrinsic Hamiltonian, $\mathcal{H}_{in} = H_0 + H_1$. In order that Ψ_K may be a linear combination of the various states, Ψ_{JM}, of the rotational band, the Ψ_{JM} must be degenerate in intrinsic energy; so we define

$$\mathcal{H}_{in} \equiv \mathcal{H} - \vec{P}^2/2Am - \hbar^2\vec{J}^2/2\mathcal{J} \qquad (1)$$

where the proper choice of moment of inertia, $\mathcal{J}$, involves a self-consistency requirement.[18] Subtraction of the translational and rotational energy operators is important for convergence of the LCE. However, the effect of $-\vec{P}^2/2Am$ on Φ_K is not very great, and for light nuclei is almost cancelled by the Coulomb correction. We therefore treat these terms in first-order perturbation theory. The rotational energy, however, is expected to have an influence on the deformations, and consequently should be treated exactly.

The addition of the correction terms of course makes the BHF problem more complicated. In this case the Brueckner reaction matrix satisfies the equation

$$G'_{12}(\omega) = v'_{12} - v'_{12}\frac{Q}{h'_{12}-\omega}G'_{12}(\omega) \qquad (2)$$

where $$v'_{12} = v_{12} - \frac{\hbar^2}{\mathcal{J}}\vec{j}_1\cdot\vec{j}_2 - \frac{1}{Am}\vec{P}_1\cdot\vec{P}_2 \ , \qquad (3a)$$

$$h'_{12} = h'_1 + h'_2 \ ,$$

and $$h'_1 = \frac{1}{2m}\vec{P}_1^2 + U - \frac{1}{2Am}\vec{P}_1^2 - \frac{\hbar^2}{2\mathcal{J}}\vec{j}_1^2 \qquad (3b)$$

Here the Pauli operator, Q , prevents scattering into occupied states, v_{12} is a nucleon-nucleon interaction having short range repulsion, and $\vec{j}_1$ and $\vec{p}_1$ are the single particle total angular and linear momentum respectively. For each different matrix element of $G'_{12}(\omega)$ in the final HF representation, the starting

energy ω is a sum of single-particle energies which are calculated from G'_{12} in the course of achieving self-consistency. A Bethe-Goldstone (BG) equation appropriate to G'_{12} is obtained in the usual fashion by defining a correlated two-particle wave function ψ'_{12}. The equation is of the familiar form

$$\psi'_{12}(\omega) = \phi_{12} - \frac{Q}{h'_{12}-\omega} v'_{12} \psi'_{12}(\omega) \quad . \tag{4}$$

However the solution of this equation is substantially more difficult than the standard BG equation because the one-body Hamiltonians appearing in the Greens' function are no longer rotationally invariant. Furthermore, the Pauli operator depends on orbitals which are now deformed. Finally, v'_{12} depends on A and $\mathcal{I}$, and hence varies from nucleus to nucleus.

Since a direct solution of Equation (2) is impractical, we shall generate the matrix elements of $G'_{12}(\omega)$ by means of a two-step process. We begin by considering a reference equation which only involves rotationally invariant objects

$$G_{12}(\omega) = v'_{12} - v'_{12} \frac{Q^{(0)}}{h_{12}-\omega} G_{12}(\omega) , \tag{5}$$

$$h_{12} = h_1 + h_2 \quad .$$

Here $$h_1 \equiv \frac{1}{2m} \vec{p}_1^{\,2} + U_1^{(0)} \quad ,$$

and $Q^{(o)}$ are chosen to resemble h_1' and Q as much as possible and yet leave a soluble problem; i.e., harmonic oscillator functions are used and $U_1^{(o)}$ is a shifted oscillator well[9]. Having found $G_{12}(\omega)$, we may obtain $G_{12}'(\omega)$ from the relation

$$G_{12}'(\omega) = G_{12}(\omega) - G_{12}(\omega)\left[\frac{Q^{(o)}}{h_{12}-\omega} - \frac{Q}{h_{12}'-\omega}\right] G_{12}'(\omega) \tag{6}$$

either by iteration or inversion. (Note that the dimensions of the matrices to be inverted are very large, because of the deformation of the orbitals.)

We still have a problem, however, in that $G_{12}(\omega)$ changes from nucleus to nucleus because of the dependence of v_{12}' on A and $\mathcal{J}$. This difficulty can be circumvented by considering instead the quantity $G_{12}^{(o)}(\omega)$ which satisfies

$$G_{12}^{(o)}(\omega) = v_{12} - v_{12}\frac{Q^{(o)}}{h_{12}-\omega} G_{12}^{(o)}(\omega) . \tag{7}$$

It is clear that $G_{12}^{(o)}(\omega)$ is the usual G-matrix constructed for calculations on spherical nuclei, and is valid for any nucleus in the chosen model space. From it, by means of a transformation similar to the Gell-Mann-Goldberger two-potential theorem[19], we may obtain $G_{12}(\omega)$ as follows:

$$G_{12}(\omega) = G_{12}^{(o)}(\omega) + \left[1 + G_{12}^{(o)}(\omega)\frac{Q^{(o)}}{h_{12}-\omega}\right] G_{12}^{(1)}(\omega) \tag{8}$$

where
$$G_{12}^{(1)}(\omega) = \left(1 - V_1' \frac{Q^{(0)}}{h_{12}-\omega}\right)^{-1} V_1' \qquad (9)$$

and
$$V_1' = \Delta V \left(1 + \frac{Q^{(0)}}{h_{12}-\omega} G_{12}^{(0)}(\omega)\right), \qquad (10)$$

$$\Delta V = -\frac{\hbar^2}{\mathcal{J}} \vec{j}_1 \cdot \vec{j}_2 - \frac{1}{Am} \vec{P}_1 \cdot \vec{P}_2 \ .$$

These equations appear somewhat formidable, especially since a matrix inversion is required. However, we observe that all the quantities involved are rotationally invariant so the matrices are block-diagonal in J as well as parity and isospin. Consequently the manipulations may be carried out on the coupled matrix elements with little computational effort. Thus we have all that is required to obtain $G_{12}(\omega)$ and begin the self-consistency problem. (It should be noted that an exact evaluation of Eqs. 8-10 must include those matrix elements coupling to outside the model space and back again. Traditionally these are tacitly ignored, and all matrix multiplications and inversions are carried out within the model space. The same remarks apply to evaluation of G_{12}' and U.)

The set of self-consistent equations to be solved are

$$\sum_b \{\langle a|T|b\rangle + \langle a|U|b\rangle\} C_b^\lambda = e_\lambda C_a^\lambda \qquad (11)$$

where the coefficients C_k^λ express the deformed orbitals in terms of a spherical oscillator representation, that is,

$$|\lambda\rangle = \sum_k C_k^\lambda |k\rangle .$$

Note that the operator T in Eq. (11) differs from the one ordinarily ($\vec{p}^2/2m$) used in HF calculations [see Eq. (3b)]:

$$T = \left(\frac{A-1}{A}\right)\vec{p}^2/2m - \frac{\hbar^2}{2\mathcal{I}}\vec{j}^2 .$$

The method of Davies, Baranger, Tarbutton, and Kuo may be used to obtain the matrix elements of U in terms of $G'_{12}(\omega)$:

$$\langle a|U|b\rangle = \tfrac{1}{2}\left[\langle a|U'|b\rangle + \langle b|U'|a\rangle\right] \qquad (12)$$

with

$$\langle a|U'|b\rangle = \sum_\lambda^{occ.} P_\lambda \sum_{\gamma\delta} \sum_{\substack{k_1 k_2 \\ k_3 k_4}} C_a^\gamma C_{k_1}^\gamma C_{k_2}^\lambda \langle k_1 k_2|G'_{12}(e'_{\gamma\delta}+e_\lambda)|k_3 k_4\rangle \times C_{k_3}^\delta C_b^\delta C_{k_4}^\lambda . \qquad (13)$$

The RBHF single particle energy $e'_{\gamma\delta} = e_\gamma$ unless γ is a particle and δ a hole, in which case $e'_{\gamma\delta} = e_\delta$. P_λ is the occupation probability for orbit λ. The structure of U results in a double self-consistency requirement: the orbitals λ are eigenstates of $(T + U)$, and U itself depends on the <u>energies</u> of the filled orbitals. It therefore appears that, in order to do BHF correctly, we must recalculate the reaction matrix after each iteration in a self-consistency procedure. Fortunately this

can be avoided by the technique introduced by McCarthy and Davies , in which each matrix element of G is expressed as a power series:

$$\langle G(\omega) \rangle = \sum_{n=1}^{N} A_n \omega^n \qquad (14)$$

If the reaction matrix is calculated for several starting energies, the coefficients A_n may be obtained by a suitable fitting procedure. Once the reaction matrix is obtained as a function of starting energy, it would be possible after each iteration in the BHF problem to use equation (6) to make the Pauli corrections. As mentioned earlier, since J is not sharp the matrices involved are very large, so that a solution by matrix inversion is impractical. Happily, it has been found that the once-iterated form of (6) is quite accurate for light nuclei. Consequently an acceptable procedure would be to make use of $G_{12}(\omega)$ for $G_{12}(\omega')$ on the first iteration, and thereafter to precede each iteration by an "orbital correction" to the previous $G_{12}(\omega)$. This would continue until self-consistency is reached.

Once the RBHF problem has been solved for the intrinsic state, then states of good angular momentum are obtained by projection

$$\Psi_{JM} = P^{J}_{MK} \Phi_K \qquad (15)$$

using standard techniques. It should be noted that we project from the uncorrelated state. The problem of projection from the corre-

lated states has not yet been solved. However, for most of the properties which will be discussed here it is felt that this deficiency will not be too significant.

The RBHF method is known to provide very reliable predictions of nuclear separation energies. In the case of deformed nuclei one is generally resigned to the use of the "intrinsic separation energies" for comparison with experiment. It is well known that the rotational motion of the nucleus will affect these energies. A method for obtaining more physically meaningful separation energies has been suggested to us by D.J. Rowe.[20] If one assumes a purely rotational spectrum and that Koopmans' theorem is valid (which has been discussed by R.L. Becker and M.R. Patterson for RBHF[21]) then one can show that the corrected separation energy is given by

$$\mathcal{E}_i = e_i - \frac{\hbar^2}{2\mathcal{J}} J_{A-1}(J_{A-1}+1) \qquad (16)$$

where e_i is the intrinsic separation energy and J_{A-1} is the total angular momentum of the (A-1) particle system in its ground state.

APPROXIMATIONS

The program for obtaining the deformed G-matrix outlined in the previous section is quite ambitious, and will normally be carried out subject to certain simplifying approximations. The crudest of these is to ignore ΔV completely in obtaining

G'_{12}, i.e., set $G'_{12}=G^{(o)}_{12}$. Some measure of the effect of ΔV can be obtained by including it in the definition of U, so that

$$\langle a|U|b\rangle \longrightarrow \langle a|U|b\rangle - \sum_{\lambda}^{occ} \langle a\lambda|\frac{\hbar^2}{J}\vec{j}_1\cdot\vec{j}_2|b\lambda\rangle \qquad (17)$$

A more consistent approximation would be to treat ΔV perturbatively, using

$$G_{12}(\omega) \cong G^{(o)}_{12}(\omega) + \left[1 + G^{(o)}_{12}(\omega)\frac{Q^{(o)}}{\omega - h_{12}}\right]\Delta V \qquad (18)$$

Calculating the matrix elements of U by means of Eq. (13) may be greatly simplified if certain approximations are made concerning the energy dependence of the G-matrix. It is the exception, $e'_{\gamma\delta} = e_\delta$, in the G-matrix which prevents one from using closure with respect to the δ-sum. For the present calculations $e'_{\gamma\delta}$ is replaced by an energy independent of δ, thus enabling us to carry out the sums on k_3 and δ. This approximation has been investigated by Davies and Baranger and found to be quite reliable for light nuclei.[10]

It is well-known that the particle-particle matrix elements of U are off the energy shell; however, calculations are simplified if the "on-shell" prescription is used. It is felt this will not affect the hole states significantly,[10] and it is these in which we are most interested.

The matrix elements of $G_{12}^{(o)}(\omega)$ which are required in order to begin the self-consistency problem are obtained from single-oscillator-configuration (SOC) calculations of the type discussed by Dr. Becker in these proceedings. The SOC matrix elements are obtained by first solving the reference BG equation with a shifted oscillator spectrum and the Eden Emery approximation for the Pauli operator, and then making Pauli corrections to the reference G-matrix. The "off-shell" behavior of the particle-particle matrix elements was accounted for approximately as in method (1) of Dr. Becker's presentation, and the shift parameter, C, was chosen so as to make the low-lying "particle" states nearly self-consistent.

When the deformed nuclei of interest are of the semi-closed-shell variety (^{12}C, ^{28}Si, ^{32}S) one can carry out a SOC calculation and secure a set of matrix elements of $G_{12}^{(o)}(\omega)$ with which to begin the RBHF problem. In this case the shift parameter and "spherical-Pauli-corrections" probably give a reasonable approximation to the final values that would result from a self-consistent calculation.

Unfortunately, the situation is not quite simple for an open shell nucleus (e.g., ^{20}Ne, ^{24}Mg). Here the iterative procedure must begin with matrix elements of $G_{12}^{(o)}(\omega)$ for the nearest closed-shell or semi-closed-shell nucleus. In this case it is more important to make the Pauli corrections. If they are ignored one must rely on trends of shift parameters in other nuclei to

determine shift parameter for the nucleus of interest.

RESULTS

Reaction matrix elements have been calculated as functions of the starting energy for the Hamada-Johnston (HJ) and Reid soft-core (RSC) interactions assuming spherical configurations for ^{12}C and ^{16}O, and used to calculate $\bar{U}$ for the first iteration in the solution of Eq. (11). Intrinsic and projected properties of ^{12}C, ^{16}O, and ^{20}Ne have been calculated using the two interactions referred to above. The RBHF equations are solved subject to the conditions that the intrinsic states possess axial symmetry and that the single-particle orbitals be four-fold degenerate. Whenever possible the results are compared with the calculations of Zofka and Ripka (ZR) obtained with a density-dependent interaction,[4] and those of Lee and Cusson (LC) obtained with a velocity-dependent interaction.

Results for ^{12}C have been obtained using both $\mathcal{H}$ and $\mathcal{H} - \hbar^2\vec{J}^2/2\mathcal{I}$ as approximations to the intrinsic Hamiltonians, with $(\hbar^2/2\mathcal{I}) = 0.74$ MeV. from experiment. The lowest-energy self-consistent solution possesses an oblate shape. Intrinsic properties of ^{12}C are presented in Table 1. Expectation values are with respect to the uncorrelated intrinsic wave function. For both interactions the corrected Hamiltonians yield lower absolute values of binding energies and seperation energies, and larger radii and deformations (defined as $\delta_J = \langle Q_J^o \rangle / \langle r^J \rangle$). The binding energies are generally in good agreement with experiment and the

radii are slightly low compared to experiment. Both the ZR and LC predictions yield lower binding energies, and although LC obtain good agreement for the radius, ZR overestimates the radius by about 12%. Both the ZR and LC results yield larger radii and deformations than RBHF. The occupation probabilities are all of the order of 80%. The separation energies predicted by RBHF were obtained using Eq. (16), and while all of the results are in good agreement with experiment, the results obtained with the HJ interaction and $\mathcal{H} - \hbar^2\vec{J}^2/2\mathcal{I}$ are closest to the measured values. Results with density-dependent and velocity-dependent HF underestimate and overestimate, respectively, the energy of the most tightly bound positive parity state. Knowledge of the gap size enables us to determine the energy of the first unoccupied orbit in ^{12}C to be roughly 5MeV; this is in good agreement with the measured separation energy of the last neutron in ^{13}C. This is to be expected, as pointed out earlier by Dr. Becker, since an analogue of Koopmans' theorem for separation energies has been established for RBHF.[21]

There are, of course, uncertainties regarding the choice of shift parameter. The effect of the shift parameter on various intrinsic properties is demonstrated in Figure 2 where the calculations were made using the RSC interaction. It is noted that the energies increase linearly with choice of C-value (in this range), however the size and deformation is not affected appreciably.

Solutions other than the ground state were also obtained self-consistently. These are shown in Figure 1, and were obtained with the HJ interaction and with the intrinsic Hamiltonian, $\mathcal{H}$. The lowest prolate configuration is found to lie at about 7 MeV which is very near the measured 0^+ excited state at 7.65 MeV. A spherical 0^+ state is found at about 10 MeV, and another prolate solution is predicted at about 19 MeV.

Physical states of the nucleus are obtained by projecting states of good angular momentum from the uncorrelated intrinsic wave function. The projected energies are not presented since such a calculation would require the projection from a correlated state, or the construction of an effective interaction. Both of these methods are presently under investigation. The properties which may be obtained from consideration of single-particle operators are shown in Table 2. The results do not differ significantly for the two interactions used and it is noted that moments, E2, and E4 rates are larger when $\mathcal{H}_{in} = \mathcal{H} - \hbar^2\vec{J}^2/2\mathcal{I}$. The projected HF results of LC are all bigger than the largest of the RBHF results. However this is expected since their radius and deformations are larger. Another consequence of these differences is that the electron scattering form factors of Lee and Cusson are slightly better than the projected RBHF calculation. The elastic[23] and inelastic[24] electron scattering are shown in Figures 3 and 4. The Born approximation has been used to obtain the curves which are compared with experiment. None of the cal-

culations give particularly good results for the elastic scattering, but the inelastic form factors are fit rather well. The RBHF results with the HJ interaction and the largest deformation yield the best fit obtained with our wave functions; they are, however, too low by about 20%. In figure 5 the single particle densities are compared for the HJ and RSC interactions. The wave functions with the largest deformation and radius was used for each case.

The agreement between theory and experiment for separation energies, size, deformation and electron scattering for ^{12}C substantiates the rotational character of 0_1^+, 2_1^+, 4_1^+ as has been suggested recently by several authors.[24,26,27]

There has been a large number of BHF and RBHF studies which have included ^{16}O, so we feel that it is not necessary to include a detailed discussion of it here. However, for completeness a comparison of BHF and RBHF calculations is included in Table 3 for two oscillator lengths in order to demonstrate how various properties are affected. The RBHF results are better, as expected, and the radius increases with oscillator length. The RBHF single particle energies are in good agreement with experiment, while the ZR and LC results differ considerably from experiment for the most tightly bound state. Efforts to obtain a deformed excited 0^+ state in ^{16}O have thus far been unsuccessful. This state has been found, in standard HF studies, to be a 4p-4h state lying some 20-25 MeV above the ground state. The 4p-4h states which

we have investigated so far have been very unstable.

Of the even Z - even N nuclei in the s-d shell, ^{20}Ne has proven to be most amenable to description by means of standard HF theory. The various properties of this nucleus that are primarily long range in character may be obtained with good accuracy from a prolate intrinsic state with a rather large hexadecapole moment. Those properties primarily short range in character are usually not reproduced very well.

Our results for the prolate shape are compared with experiment[25] and with the results of Lee and Cusson[5] (SP energies are also compared with ZR) in Table 4. The binding energies agree for RBHF and velocity-dependent HF (VDHF), but are below the experimental value of 8.2 MeV. Of course the predicted results for binding energy will change considerably upon projection. The radius and deformation are lower for RBHF in spite of the fact that an oscillator length of 1.88 fm. has been used for the RBHF calculations while LC use 1.67 fm. The single particle energies differ greatly for RBHF and VDHF, but the density dependent calculation of Zofka and Ripka[4] yields results close to ours except for the most tightly bound state. Unfortunately there are no measured values for separation energies in ^{20}Ne.

The projected properties underestimate the measured moments and E2 rates, and are also smaller than the LC results. Some improvement in the ^{20}Ne results is expected when the Pauli and spectral corrections are made.

DISCUSSION

The aim of many-body theory is to provide a truly microscopic description of finite nuclei, and further to allow prediction of nuclear properties with as few approximations as possible. Of course a study of the type made here for deformed nuclei, and elsewhere for spherical nuclei[12], has elements of phenomenology introduced through the two-body interaction and the shift parameter. However, these are necessary steps required to gain an understanding of the connection between "realistic" interactions and nuclear phenomena.

Overall agreement with experiment, after accounting for the various approximations, is fairly good. As in the case of spherical nuclei, however, saturation is not achieved and the nuclei are underbound. This seems to be a defect of BHF theory as currently formulated or employed. One possibility is that three-body clusters are necessary to achieve saturation--- indeed, such consideration led Negele to adjust his potential phenomenologically to fit nuclear matter.[3] It has been suggested by Bethe[7] that such a technique be employed in BHF calculations. But until self-consistent Pauli corrections are included in BHF calculations, the question cannot be firmly answered. It is not unreasonable, for instance, to believe that the density-dependence employed by Negele[3], Meldner[28], and Zofka-Ripka[4] arises from the action of the operator Q in the BG equation, and that this is poorly approximated by current techniques.

One of the limitations of the present calculation is that uncorrelated states are used to calculate nuclear properties. Improvement in the agreement with experiment can be expected when this limitation is removed [29]. A precise theory for deformed nuclei will have to be developed for this purpose, however, inasmuch as our present methods, while based on successful HF studies, cannot be rigorously defended for BHF calculations. In spite of this it seems apparent that the deformations produced by HJ and RSC forces are inadequate to account for experimental observations. The successful results obtained by LC with the Saunier-Pearson force (which is adjusted to fit two-body data and nuclear matter) therefore tend to reinforce Bethe's suggestion that BHF calculations for finite nuclei should be made using an interaction which has been shown to be successful in BHF calculations for nuclear matter.

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TABLE CAPTIONS

Table 1. Intrinsic properties of ^{12}C. In the RBHF calculation b=1.57 fm. and $\hbar^2/2I$=0.74 MeV; ZR used b=1.67 fm. and LC used 1.54 fm .

Table 2. Projected properties of ^{12}C. In the RBHF calculation b=1.57 fm. and $\hbar^2/2I$=0.30 MeV; in the LC calculation b=1.54 fm .

Table 3. Intrinsic properties of ^{16}O. The Hamada-Johnston force was used in all BHF and RBHF calculations.

Table 4. Intrinsic and projected properties of ^{20}Ne. The RBHF calculation was made using the Hamada-Johnston force $\mathcal{H}_{in} = \mathcal{H} - \hbar^2\vec{J}^2/2\mathcal{J}$, $\hbar^2/2\mathcal{J}$ = 0.74 MeV, and b=1.88 fm ., and LC used b=1.67 fm .

FIGURE CAPTIONS

Figure 1. Energies of the self-consistent RBHF solutions using the HJ interaction.

Figure 2. The behavior of various intrinsic properties of ^{12}C as a function of the shift parameter. The RBHF calculation was performed with b=1.57 fm. and $\hbar^2/2\mathcal{I} = 0.74$ MeV; e_1 is the binding energy of the lowest orbit, R the uncorrected single-particle r.m.s. radius, and $\delta_L = \langle Q_L \rangle / \langle r^L \rangle$ the L-pole deformation.

Figure 3. Experimental and calculated elastic form factors for the scattering of electrons from ^{12}C. (————, RBHF with HJ interaction), (----, RBHF with RSC), and (——•——, Lee-Cusson).

Figure 4. Experimental and calculated inelastic form factors for electron scattering from ^{12}C. (————, RBHF with HJ interaction and $\mathcal{H} - \hbar^2\vec{J}^2/2\mathcal{I}$), (----, RBHF with RSC interaction and $\mathcal{H} - \hbar^2 J^2/2\mathcal{I}$), (—•—•—, RBHF with HJ interaction and H), and (——•——, Lee-Cusson).

Figure 5. Radial variation of the nuclear density of ^{12}C. The RBHF calculations are performed with b=1.57 fm. and

TABLE 1

INTRINSIC PROPERTIES

	HAMADA-JOHNSTON		REID SOFT CORE		ZOFKA-RIPKA	LEE-CUSSON	EXPT
	H	$H - \vec{h}^2\vec{J}^2/2I$	H	$H - \vec{h}^2\vec{J}^2/2I$			
-E/A, MeV	-7.28	-7.26	-7.53	-7.48	-6.4	-6.3	-7.7
Δ, MeV	10.59	11.78	11.24	12.52	9.8	13.9	
$\langle r^2 \rangle^{1/2}$, FM	2.39	2.41	2.36	2.38	2.68	2.47	2.40±0.03
$\langle r^4 \rangle^{1/4}$, FM	2.52	2.54	2.48	2.49			
δ_2	-5.43	-5.76	-5.38	-5.72	-6.0	-5.88	
δ_4	0.507	0.625	0.493	0.617			
$(m_\lambda^\pi, P_\lambda)$ e_λ, MeV	$(1/2^+, 0.782)$ -37.45	$(1/2^+, 0.782)$ -36.96	$(1/2^+, 0.830)$ -40.26	$(1/2^+, 0.829)$ -39.63	-30.8	-48.4	-38.3±1
	$(3/2^-, 0.796)$ -17.84	$(3/2^-, 0.795)$ -17.39)	$(3/2^-, 0.819)$ -19.43	$(3/2^-, 0.818)$ -18.91	-17.6	-20.8	-18.8±0.5
	$(1/2^-, 0.818)$ -16.44	$(1/2^-, 0.819)$ -16.83	$(1/2^-, 0.837)$ -17.90	$(1/2^-, 0.838)$ -18.27	-17.6	-20.1	-18.8±0.5
C, MeV	45.257	45.257	44.0	44.0			

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TABLE 2

PROJECTED PROPERTIES

		HAMADA-JOHNSTON		REID SOFT CORE		LEE-CUSSON	EXPT
		H	$H - \hbar^2\vec{J}^2/2I$	H	$H - \hbar^2\vec{J}^2/2I$		
R, FM		2.39	2.40	2.35	2.37	2.43	2.46±0.025
$\langle Q_2 \rangle$, FM^2	2^+	4.68	4.94	4.52	4.76	5.32	
	4^+	5.62	5.85	5.43	5.65	6.38	
$\langle Q_4 \rangle$, FM^4	2^+	0.89	0.98	0.71	0.90	1.02	
	4^+	2.16	2.68	1.98	2.48	2.75	
B(E2), $e^2 \cdot FM^4$	$0^+ \rightarrow 2^+$	27.3	30.7	25.32	28.52	35.4	41.8±4
	$2^+ \rightarrow 4^+$	13.9	16.14	12.77	14.87	18.2	
B(E4), $e^4 \cdot FM^8$	$0^+ \rightarrow 4^+$	226.5	348.6	188.4	296.5	393.0	
	$2^+ \rightarrow 4^+$	57.6	88.5	48.1	75.6	116.0	

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TABLE 3

^{16}O

b, FM	1.57		1.77		1.67	1.67	
CALCULATION	BHF	RBHF	BHF	RBHF	ZR	LC	EXPT
-E/A, MeV	6.2	7.3	6.2	7.0	7.5	7.9	7.98
Δ, MeV	18.0	16.9	17.2	15.8	17.0	20.6	
$\langle r^2 \rangle^{1/2}$	2.29	2.40	2.38	2.45	2.72	2.52	2.67±0.03
$(m_\lambda^\pi, P_\lambda)$ e_λ, MeV	$(1/2^+, 1.0)$ -56.7	$(1/2^+, 0.79)$ -43.9	$(1/2^+, 1.0)$ -54.9	$(1/2^+, 0.83)$ -43.6	(+) -37	$(1/2^+)$ -64.1	43±5
	$(3/2^-, 1.0)$ -30.0	$(3/2^-, 0.80)$ -21.3	$(3/2^-, 1.0)$ -28.3	$(3/2^-, 0.82)$ -20.9	(-) -21	$(3/2^-)$ -26.5	21.8
	$(1/2^-, 1.0)$ -30.0	$(1/2^-, 0.80)$ -21.3	$(1/2^-, 1.0)$ -28.3	$(1/2^-, 0.82)$ -20.9	(-) -21	$(1/2^-)$ -21.1	21.8
	$(1/2^-, 1.0$ -23.5	$(1/2^-, 0.80)$ -17.0	$(1/2^-, 1.0)$ -22.6	$(1/2^-, 0.82)$ -16.8	(-) -21	$(1/2^-)$ -21.1	15.7
C, MeV	50.46	48.64	42.62	40.64			

TABLE 4

	-E/A, MeV	Δ, MeV	R_2, FM	R_4, FM	δ_2	δ_4
RBHF	7.4	7.5	2.61	2.90	10.6	3.4
LC	7.4	9.9	2.69	----	11.7	---

	RBHF	LC	ZR
$(m_\lambda^\pi, P_\lambda)$ e_λ, MeV	$(1/2^+, 0)$ -48.02	$(1/2^+)$ -66.8	(+) -38
	$(1/2^-, 0)$ -28.65	$(1/2^-)$ -38.1	(-) -27
	$(3/2^-, 0)$ -22.93	$(3/2^-)$ -27.7	(-) -21
	$(1/2^-, 0)$ -20.41	$(1/2^-)$ -23.9	(-) -21
	$(1/2^+, 0)$ -15.23	$(1/2^+)$ -16.0	(+) -14

	R, FM	$\langle Q_2 \rangle_{2^+}$, FM^2	$\langle Q_2 \rangle_{4^+}$, FM^2	$B(E2; 0^+\text{-}2^+)$, $e^2 \cdot FM^4$	$B(E2; 2^+\text{-}4^+)$, $e^2 \cdot FM^4$	$B(E2; 4^+\text{-}6^+)$, $e^2 \cdot FM^4$
RBHF	2.72	-11.3	-14.5	152.7	77.3	66.9
LC	2.80	-13.1	-23.8	208.0	82.0	92.1
EXPT	2.80±0.05	-27±11	-----	285±40	128±13	95±11

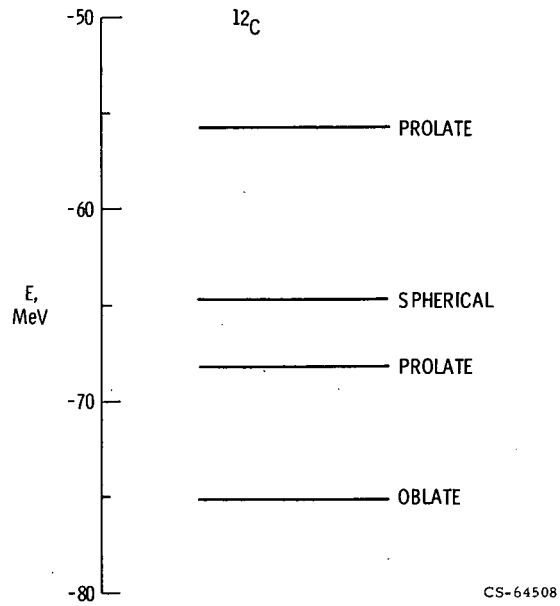

Figure 1.

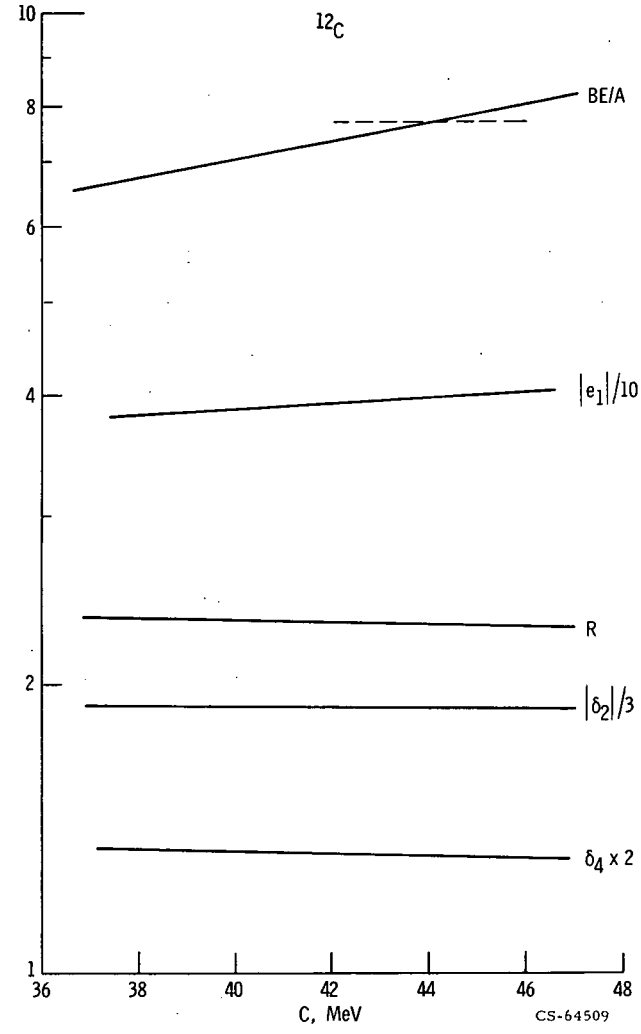

Figure 2.

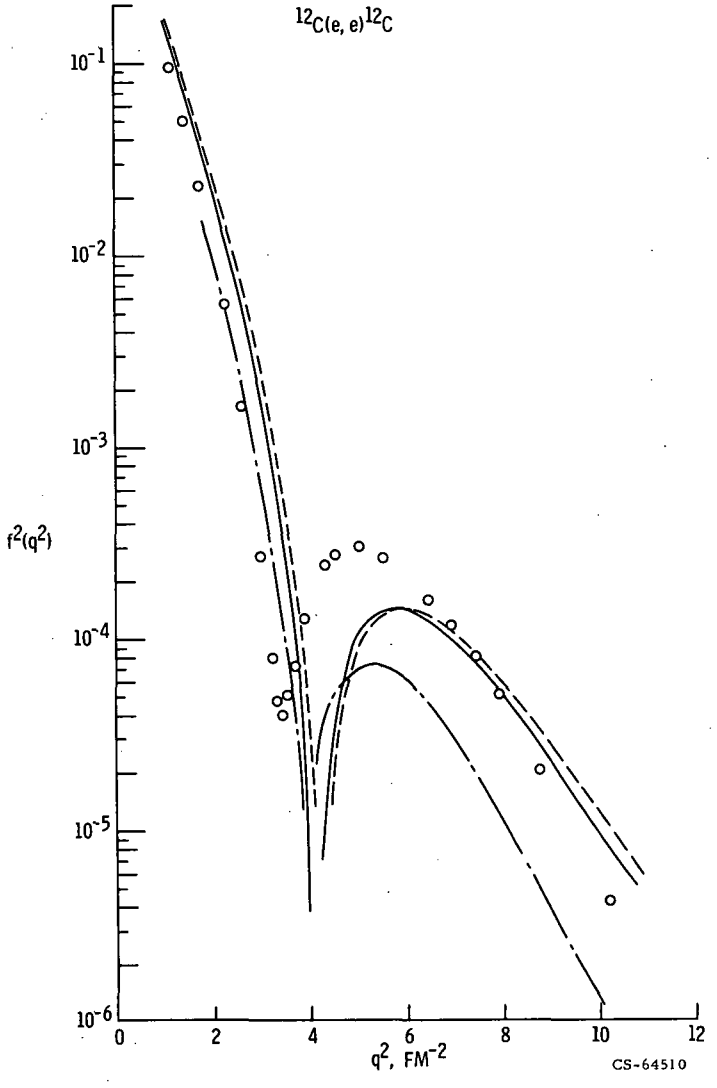

Figure 3.

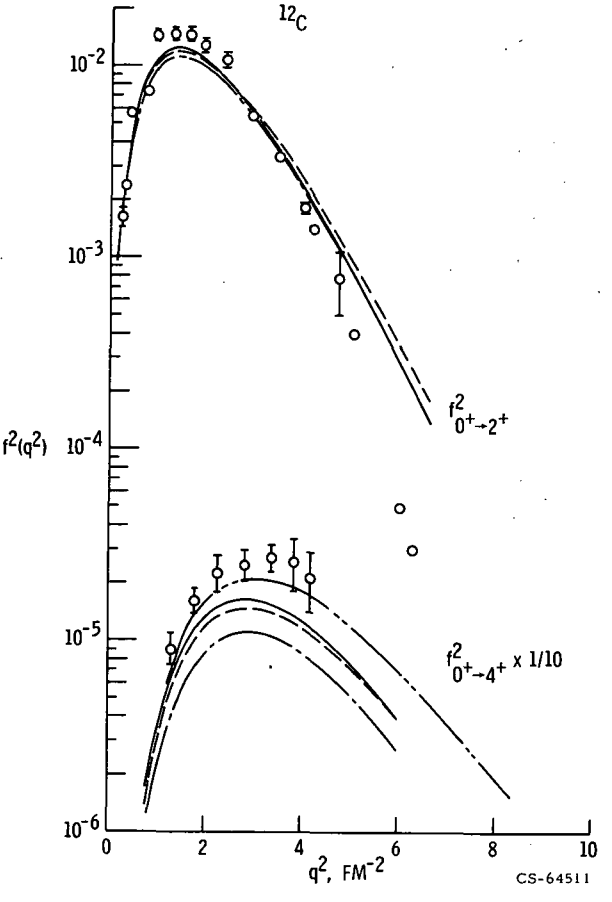

Figure 4.

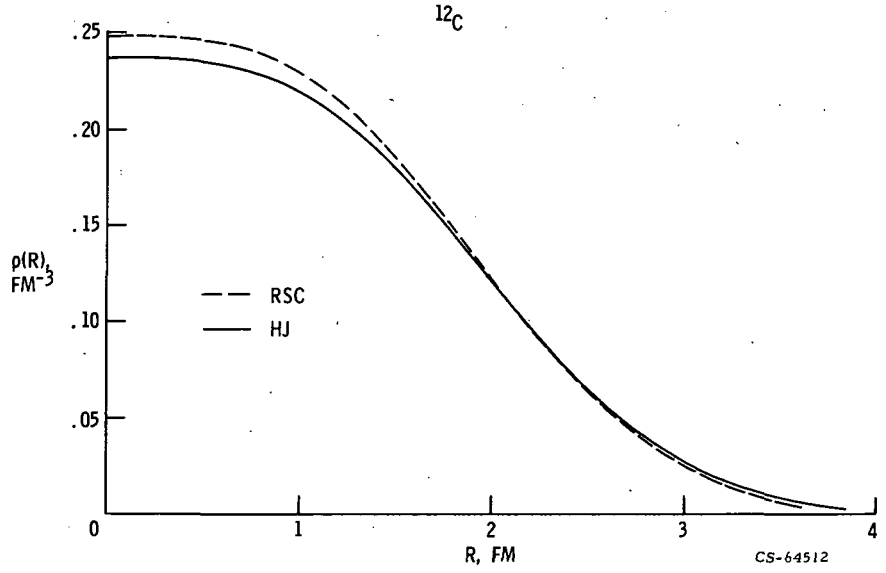

Figure 5.