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CALCULATIONS OF SYNCHROTRON EMISSION FROM THE TERRESTRIAL RADIATION BELTS

by

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ABSTRACT

In this paper a theoretical model is developed to allow for the calculation of the synchrotron emission arising from high energy electrons trapped in the Van Allen belts of a planet with a dipole magnetic field. The model is general enough to allow for the calculation of the intensity of radiation received by an observer at any distance from and any latitude about the planet.

The model is used to compute the emission from the Earth's Van Allen belts that one should expect at various latitudes at a distance of 1.92 Earth radii, the position of the Radio Astronomy Explorer (RAE) satellite that was launched in 1968 [12], for the frequencies 1.3 MHz and 2.2 MHz. The distribution of high energy electrons in the Earth's Van Allen belts used in the model was obtained from data published by Vette, Lucero and Wright [11].

The emission predicted by the theoretical model is compared with data on the distribution of radiation from the Earth's Van Allen belts gathered by the RAE satellite. These comparisons show that emissions from other regions of the magnetosphere interfere with the observance of the belts.

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I. INTRODUCTION

Since the discovery, by Van Allen [8] of the existance of the region above the Earth where there exist charged particles trapped in the magnetosphere, several attempts have been made to develope models that would allow one to compute the electromagnetic radiation emitted by these trapped charged particles (electrons).

These earlier models (by Vesecky [10] for example) would only allow one to compute the emission from the Van Allen belts in the geomagnetic equator. Also, when the earlier models were developed there was no available data on observations of the emission to enable one to make comparisons of the observations with the models' predicted values of the emission from the belts.

The model developed in this paper permits the computation of the emission from the belts one should observe at any position about the Earth. Comparisons of the model's predictions are also made with the data on the emissions from the belts gathered by the RAE (Radio Astronomy Explorer) satellite launched in July of 1968 [13].

II. THE EARTH'S MAGNETIC FIELD

The Earth's magnetic field can be approximated by a dipole field out to several Earth radii. Since we shall only be interested in the lower magnetosphere, (out to approximately two Earth radii) where there are high energy trapped electrons, the dipole approximation of the Earth's magnetic field shall be used in our model.

The strength of a dipole magnetic field is given in the equation below as

$$B(r,\lambda_e) = \frac{M}{r^3} [1. + 3. \sin^2 \lambda_e]^{1/2} \tag{1}$$

where r is the distance from the center of the dipole, λ_e is the magnetic latitude and M is the magnetic moment.

III. THE VAN ALLEN RADIATION BELTS

We now have a pretty complete picture of the radiation belts. We know now that the energetic particle distribution is made up of high energy electrons in the inner zone and energetic electrons in the outer zone. Lower energy particles have also been mapped. Particles in the outer zone especially, the electrons, show large time variations. The electron flux can vary more than an order of magnitude in less than a day, responding strongly to magnetic disturbances.

IV. THE DEVELOPMENT OF THE MODEL

a. The Coordinate System

The system of coordinates used in this paper are geomagnetic coordinates. The origin of this coordinate system is at the center of the magnetic dipole. For the coordinates of the observer, we shall use r_o, λ_o and O_o where r_o is the radial distance to the observer, λ_o is the latitude of the observer and O_o is the longitude of the observer. For the coördinates of the radiating electron we shall use r_e, λ_e and O_e where r_e is the radial distance to the radiating electron, λ_e is the latitude of the radiating electron and O_e is its longitude. Since the emitting electron is confined to move along a particular field line it is more convenient to express r_e in terms of that field line. For a dipole field the equation of a fiëld line is

$$r_e = re_o \cos^2 \lambda_e \tag{2}$$

where re_o is the distance from the center of the dipole to the point where the field line crosses the dipole equator. However, re_o can be expressed in terms of planetary radii, R_E where $re_o = LR_E$. Thus, L called the L-shell can be used to define the field line of the gyrating electron and in most places in this paper we shall use the coordinates L, λ_e, O_e instead of r_e, λ_e, O_e.

b. The Emission from a Gyrating Electron in a Magnetic Field

The differential rate at which energy is emitted spontaneously per unit solid angle per frequency interval $d\omega$, called the coefficient of spontaneous emission η_ω, by an electron gyrating about a magnetic field line is [1].

$$\eta_\omega(\omega, v, \theta) = \frac{e^2 \omega^2}{8\pi^2 \varepsilon_o C} \left[\sum_1^\infty \left(\frac{\cos\theta - \beta_{||}}{\sin\theta} \right)^2 J_m^2(x) + \beta_\perp^2 J_m'^{\,2}(x) \right] \delta(Y) \tag{3}$$

where e is the electronic charge, ε_o is the permittivity of free space, C is the speed of light, ω is the observing frequency, $I_m(x)$ is a Bessel function of order m and argument X, $J_m'(x) = dJ_m(x)/dx$ and θ is the angle between the direction of the emitted radiation and the magnetic field direction. The quantities $\beta_{||}$ and $\beta_\perp$ are

$$\beta_{||} = \beta \cos\alpha \text{ and } \beta_\perp = \beta \sin\alpha \tag{4}$$

where $\beta = V/C$ and α is the pitch angle of the gyrating electron. It is the angle between the particle's velocity vector $\vec{V}$ and the direction of the magnetic field. The quantity m is also the harmonic number for which radiation is being emitted, m = 1,2,3... The argument of the Bessel function, x is

$$X = \omega/\omega_o \, \beta_\perp \sin\theta \tag{5}$$

and the argument of the delta function, Y is

$$Y = m\omega_o - \omega \; (1 - \beta_{\parallel} \cos \theta) \tag{6}$$

the quantity ω_o is defined as

$$\omega_o = \omega_b \quad 1-\beta^2 \tag{7}$$

where ω_b, the gyrofrequency, is

$$\omega_b = eB/m \tag{8}$$

Assuming that the emission is not affected by the intevening medium between the observer and the radiating electron, the angle θ, in terms of geomagnetic coordinates can be found from the expression

$$\cos \theta = \sin \gamma \sin \lambda - \cos \gamma \cos \theta_e \cos \lambda \cos \theta - \cos \gamma \sin \theta_e \cos \lambda \sin \theta \tag{9}$$

where

$$\cos \theta = \frac{r_o \cos \lambda_o - r_e \cos \lambda_e \cos \theta_e}{(r_e^2 \cos^2 \lambda_e + r_o^2 \cos^2 \lambda_o - 2 r_o r_e \cos\lambda_o \cos\lambda_e \cos\theta_e)^{1/2}}$$

$$\sin \theta = \frac{r_e \cos \lambda_e \sin \theta_e}{(r_e^2 \cos^2 \lambda_e + r_o^2 \cos^2 \lambda_o - 2 r_o r_e \cos\lambda_o \cos\lambda_e \cos\theta_e)^{1/2}}$$

$$\sin \lambda = \frac{r_o \sin \lambda_o - r_e \sin \lambda_e}{(r_o^2 + r_e^2 - 2 r_o r_e (\cos\lambda_o \cos\lambda_e \cos\theta_e + \sin\lambda_o \sin\lambda_e))^{1/2}}$$

$$\cos \lambda = \frac{r_o \cos \lambda_o - r_e \cos \lambda_e}{(r_o^2 + r_e^2 - 2 r_o r_e (\cos\lambda_o \cos\lambda_e \cos\theta_e + \sin\lambda_o \sin\lambda_e))^{1/2}}$$

$$\sin \gamma = \frac{\cos^2 \lambda_e - 2. \sin^2 \lambda_2}{(1 + 3 \sin^2 \lambda_e)^{1/2}}$$

and

$$\cos \gamma = \frac{\cos \lambda_e \sin \lambda_2}{(1 + 3 \sin^2 \lambda_2)^{1/2}} .$$

c. The Particle Distribution Function

For one electron, if we assume that no radiation is absorbed by the medium, the power per unit area for a particular frequency ω or flux P_ω is

$$P_\omega(\lambda_e, L, r, \alpha, E, \theta) = \frac{\eta_\omega (\lambda_e, L, \alpha, E, \theta)}{|\vec{r}|^2} \quad (10)$$

The value of $|\vec{r}|$ is, in terms of geomagnetic coordinates;

$$|\vec{r}| = \left[r_o^2 + r_e^2 - 2r_o r_e (\cos\lambda_o \cos\lambda_e \cos\theta_e + \sin\lambda_o \sin\lambda_e \right]^{1/2} \quad (11)$$

For more than one electron, to obtain the total flux received by an observer, we must sum the individual contributions of each particle or electron. Let us assume that the radiating medium is continuous. This allows us to write

$$dP_\omega = P_\omega (\lambda_e, L, \vec{r}, \alpha, E, \theta) \; dN \quad (12)$$

where N is the total number of particles in the medium. But we can write dN as,

$$dN = \rho (\lambda_e, \theta_e, L, \alpha, E) \; dE \; d\alpha \; dV \quad (13)$$

where ρ is the electron density distribution function and dV is a differential volume element of the medium. In terms of geomagnetic coordinates

$$dV = L^2 R_E^3 \cos^7 \lambda_e \, d\lambda_e \, d\theta_e \, dL \tag{14}$$

where R_E is the radius of the Earth, L is the L-shell and λ_e and θ_e are the geomagnetic latitude and longitude respectively.

We now have for the differential power

$$dP_\omega = P_\omega(E,\lambda_e,\theta_e,L,\alpha,r_o) \, \rho \, (\lambda_e,\theta_e,L,\alpha,E) \; X$$
$$X L^2 R_E^3 \cos^7 \lambda_e \, d\lambda_e \, d\theta_e \, dL \, dE \, d\alpha \tag{15}$$

Considering the particle distribution function we shall assume azmutal symmetry by eleminating θ_e from the distribution function. This gives

$$\rho = \rho(\lambda_e,L,\alpha,E) \tag{16}$$

According to D. Chang [2], for a dipole magnetic field, considering Liouville's theorem

$$\rho(\lambda_e,L,\alpha,E) = \frac{(1 + 3 \sin^2 \lambda_e)^{1/4}}{\cos^3 \lambda_e} \rho(0^o,L,E,\alpha_E) \tag{17}$$

where $\rho(0^o,L,E,\alpha_E)$ is the electron distribution at the dipole equator. The variable α_E is the pitch angle of the electron at the equator.

Let us write $\rho(0^o,L,E,\alpha_E)$ as

$$\rho(0^o,L,E,\alpha_E) = \rho(L,E) \; f(\alpha_E) \tag{18}$$

and choose a value of $f(\alpha_E)$ in accordance with D. Chang [2] in his work on radiation from the planet Jupiter, that is

$$f(\alpha_E) = \sin^P \alpha_E \tag{19}$$

where P is any positive real number. Thus for our distribution function we have

$$\rho(\lambda_e, L, \alpha_E, E) = \frac{(1. + 3. \sin^2 \lambda_e)^{1/4}}{\cos^3 \lambda_e} \sin^P \alpha_E \, \rho(L,E) \tag{20}$$

Now, since $\sin \alpha_E = \sin\alpha \cos^3 \lambda_e / (1. + 3. \sin^2 \lambda_e)^{1/4}$, we have;

$$\rho(\lambda_e, L, \alpha, E) = \frac{(\cos \lambda_e)^{3(P-1)}}{(1 + 3 \sin^2 \lambda_e)^{1/4(P-1)}} \sin^P \alpha \, \rho(L,E) \tag{21}$$

For the limits on the pitch angle distribution we want α_1 and α_2, the limits on α , to be such that for a given λ_e and L, α_1 will be the minimum value of α that will allow the electron to mirror before crashing into the Earth and α_2 to be the maximum value of α that will allow the electron to mirror before crashing into the Earth.

For two point a and b on a dipole field line

$$\frac{\sin^2 \alpha_a}{\sin^2 \alpha_b} = \frac{(1 + 3 \sin^2 \lambda_{ea})^{1/2}}{(1 + 3 \sin^2 \lambda_{eb})^{1/2}} \; \frac{\cos^6 \lambda_{eb}}{\cos^6 \lambda_{ea}} \tag{22}$$

If we let the particle mirror at λ_{eb}, the latitude where the field line enters the surface of the Earth, then, we must have that

$$\sin \alpha_b = \sin {}^{\pi}/2 = 1 \text{ and } \cos \lambda_{eb} = \frac{1}{\sqrt{L}} \tag{23}$$

Letting $\alpha_m = \alpha_a$ and $\lambda_e = \lambda_{ea}$, we get that

$$\sin \alpha_m = \frac{[1. + 3. \sin^2 \lambda_e]^{1/4}}{L^{3/2}\,[4. - 3./L]} \; \frac{1}{\cos^3 \lambda_e} \tag{24}$$

It can easily be shown that $\alpha_2 = \pi - \alpha_m$, since the particle with pitch angle α_m on its return trip after mirroring, will have changed directions by 180 degrees.

Letting P = 2 in equation (21) will give the best description of the pitch angle distribution of electrons along a magnetic field line in the Earth's lower magnetosphere as experimental evidence has shown [4]. Thus we shall use $f(\alpha) = \sin^2\alpha$ throughout the paper.

We finally have for the distribution of electrons

$$\rho(\lambda_e, L, \alpha, E) = \frac{\cos^3 \lambda_e}{(1 + 3 \sin^2 \lambda_e)^{1/4}} \sin^2\alpha \; \rho(L, E) \tag{25}$$

where $\rho(L,E)$ will be determined by using experimental data.

d. The Limits of Observation of the Radiating Region of a Planet

Before proceeding to make computations of radiation from trapped electrons in a planet's dipole magnetic field, we must consider the fact that part of the radiating medium will be obscured by the planet. This being the case, we must allow for this fact when we make integrations over the entire observable radiating medium. The limits of integration over the latitude λ_e and longitude θ_e are a function of the position of the observer with respect to the planet.

In order for an observer to receive the emitted radiation from a particular section of the radiating medium about a planet, the condition

$$\cos\lambda_o \cos\lambda_e \cos(\theta_o - \theta_e) + \sin\lambda_o \sin\lambda_e$$

$$\geq \frac{r_e}{r_o} - \frac{1}{r_o r_e}\left\{(r_o^2 - R_E^2)\,(r_e^2 - R_E^2)\right\}^{1/2} \tag{26}$$

must be satisfied, where r_o, λ_o and θ_o are the position coordinates of the observer, r_e, λ_e and θ_e are the position coordinates of the emitting electron and R_E is the radius of the planet.

e. Radiation from Electrons in a Plasma Medium

The general formula for radiation in a plasma where there the observer is not in the medium is

$$I_\omega = I_\omega(\text{inc})\, e^{-\tau_o} + \int_o^\tau S_\omega(\tau)\, e^{-\tau}\, d\tau \tag{27}$$

where I_ω is the radiation intensity at the frequency ω, I_ω(inc) is the incident radiation upon the plasma medium, τ is the optical dept and τ_o is the total optical depth. The quantity $S_\omega(\tau)$ under the integral sign is called the source function.

For the quantity τ, the optical depth, we have

$$d\tau = -\alpha_\omega\, ds \tag{28}$$

where α_ω is the absorption coefficient and ds is the differential path length of the radiation.

For the source function S_ω, we have

$$S_\omega = \frac{1}{n_r^2} \frac{J_\omega}{\alpha_\omega} \tag{29}$$

where n_r is the ray refractive index and J_ω is the emission coefficient.

For J_ω, the emission coefficient, we have

$$J_\omega = \int \eta_\omega (P,\alpha,\lambda_o,L)\ f(P,\alpha,\lambda_o,L)\ 2\pi \sin\alpha\ P^2\ d\alpha\ dP \tag{30}$$

where P is the momentum of the electron and $f(P,\alpha,\lambda_o,L)$ is the distribution function of the electrons as a function of their momentum as well as α,λ_o,L. However for a fixed number of electrons N, we have

$$\begin{aligned} dN &= f(P,\alpha,\lambda_o,L)\ 2\pi \sin\alpha\ P^2\ d\alpha\ dP \\ &= \rho(E,\lambda_o,\alpha,L)\ dE\ d\alpha \end{aligned} \tag{31}$$

where E is the kinetic energy of the electron.

The absorption coefficient α_ω, is defined below as

$$\alpha_\omega = - \frac{8\pi^3 c^2}{n_r^2\ \omega_o^2} \int_\alpha \int_P \eta_\omega (\alpha,\lambda_o,L,E)\ \frac{\partial f(P)}{\partial E}\ 2\pi \sin\alpha\ P^2\ dP\ d\alpha \tag{32}$$

For the ray refractive index n_r, we have

$$n_r^2 = \left| n^2 \sin\theta \frac{\left[1. + \left(\frac{1}{n}\frac{\partial n}{\partial \theta}\right)_\omega^2\right]^{1/2}}{\frac{\partial}{\partial\theta}\left\{\frac{\cos\theta + \left(\frac{1}{n}\frac{\partial n}{\partial\theta}\right)_\omega \sin\theta}{\left[1. + \left(\frac{1}{n}\frac{\partial n}{\partial\theta}\right)_\omega^2\right]^{1/2}}\right\}} \right| \tag{33}$$

In equation (33) η is the refractive index and θ is the angle between the emission direction and the direction of the magnetic field.

For the value of the refractive index, η, we would solve for the real part of the dielectric constant, $\tilde{u}$ of the medium. For $\tilde{u}$ we have

$$\tilde{u} = (n + i\,\chi)^2 \tag{34}$$

where χ is the attenuation index. To solve for η, we would consider the Appelton equation for $\tilde{u}$. (see Heald and Wharton [13]. The Appelton equation is

$$\tilde{u} = \left[1 - \frac{(\omega_p^2/\omega^2)}{\left\{\left[1 - i\frac{\nu}{\omega} - \frac{(\omega_b^2/\omega^2)\sin^2\theta}{2(1 - \omega_p^2/\omega^2 - i\frac{\nu}{\omega}}\right] \pm \left[\frac{(\omega_b^4/\omega_4)\sin^4\theta}{4.(1.-\omega_p^2/\omega^2 - i\frac{\nu}{\omega})^2} \pm \frac{\omega_b^2}{\omega^2}\cos^2\theta\right]^{1/2}\right\}}\right]^{1/2} \tag{35}$$

where the + sign is for left circularly polarized waves and the - sign is for right circularly polarized waves. Also, ω is the observing frequency, ω_b is the gyrofrequency and ν is the collision frequency. The plasma frequency ω_p is defined as

$$\omega_p = \left(\frac{n_e e^2}{\epsilon_o m_e} \right)^{1/2} \tag{36}$$

where n_e is the local thermal electron density of the medium.

f. Intensity vs Frequency for Various Latitudes Using Experimental Data on High Energy Electron Distributions in the Lower Van Allen Belt.

We shall consider the intensity, I_ω as a function of frequency, ω for various observer latitudes at a height of 2 Earth radii.

We shall require that the observer look along a line connecting the geomagnetic dipole center and the observer.

For I_ω, we have

$$I_\omega \; (\lambda_e, \omega) = n^2(S_o) \int_{S_i}^{S_o} \frac{J_\omega \; (S, \omega, \lambda_e)}{n^2 \; (S)} \, dS \tag{37}$$

where $n(S_o)$ is the refractive index of the medium at the observer and $n(S)$ is the refractive index of the medium at the radiating source. If we neglect the effects of the magnetic field, we get for $n(S)$,

$$n^2(S) = 1. - \omega_p^2/\omega^2 \tag{38}$$

where ω_p is the plasma frequency of the medium at the positions. For ω_p^2 we have that

$$\omega_p^2 = 3.1762 \times 10^3 \times N_t \tag{39}$$

where N_t is the thermal electron density at the position S. The values of N_t used for various values of S for these calculations were taken from data published by Vesecky [10].

The positions along r_o, the observer's distance from the dipole center is determined by the formula

$$S = L\ R_E \cos^2 \lambda_e \tag{40}$$

where L is the L-shell containing the source and λ_e is the latitude of the source,

The value of the emission coefficient $J_\omega\ (S,\omega,\lambda_e)$ is determined using the formula

$$J_\omega(S,\omega,\lambda_e) = \int_{\alpha_1}^{\alpha_2} \int_{E_1}^{E_2} \eta_\omega(E,S,\alpha,\omega,\lambda_e)\ \rho(E,S,\alpha,\lambda_e)\ dE\ d\alpha \tag{41}$$

For the distribution function, ρ we will use the formula

$$\rho(E,S,\alpha,\lambda_e) = \frac{\cos^3 \lambda_e \sin^2 \alpha}{(1 + 3 \sin^2 \lambda_e)^{1/4}} N(S,E) \tag{42}$$

where the position, energy distribution function N(S,E) is determined by using data compiled and published by Vette, Lucero and Wright [11]. The limits on the energy of the electrons are E_1 = .5 Mev and E_2 = 7. MeV.

We will consider the Razin effect [5] in the formula for η_ω. The Razin effect is due to the fact that in a plasma medium, the phase velocity, V_ϕ of the transmitted radiation is,

$$V_\phi = \frac{c}{n} \tag{43}$$

and the dielectric constant ϵ is $\epsilon_o n^2$ where n is the refractive index of the medium. Thus we should replace c by c/n and ϵ_o by $n^2 \epsilon_o$ in the formula for the coefficient of spontaneous emission η_ω. This gives for η_ω

$$\eta_\omega = \frac{n_e^2 \omega^2}{8\pi^2 \epsilon_o c} \left[\sum_1^\infty \left(\frac{\cos\theta - \beta_{||}}{\sin\theta} \right)^2 J_M^2(x) + \beta_\perp^2 J_M'^2(x) \right] \delta(y) \tag{44}$$

In equation (37) we have neglected the term

$$e^{-\tau} = e^{-\alpha_\omega (S_o - S)}$$

because the self-absorption of the electrons, determined by α_ω, has been showed to be very small by Vesecky [10].

Considering figure (3), we see that the intensity decreases as we increase the observer's latitude, λ_o. This is to be expected from the nature of our electron distribution in terms of latitude λ_e. Also, as we move up in latitude the upper limit on our region of radiation, L = 2. moves deeper into the ambient plasma where the thermal electron density is greater. The mirroring of the electrons along field lines also contribute to this fall off in radiation for an increase in latitude. Notice that as we get up to the higher latitudes 35^{o} and above the radiation falls off more rapidly than for lower latitudes as we pass the peak frequency. This is due to the fact that near these latitudes, the radiating medium between L = 1.35 and L = 2. enters the atmosphere near the Earth's surface. Now, all electrons near these latitudes must have pitch angles around 90 degrees or become lost in the lower atmosphere. This restriction causes the delta function S(Y) in the formula for the coefficient of spontaneous emission equal to

$$\delta(r) = S\,(m\omega_o - \omega(1 - \beta \cos\alpha \cos\theta)) \tag{45}$$

to reduce to

$$\delta(Y) = S(m\omega_o - \omega), \tag{46}$$

because α = 90 degrees.

Considering equation (45) and (57), we see that as we increase the frequency ω for the low latitude case and the high latitude case, $m\omega_o$ in equation (46) must increase by a larger amount than for the low latitude case, equation (45). This larger increase in harmonic number causes a corresponding larger decrease in intensity as we look at higher frequencies.

V. OBSERVED RADIO EMISSION

We shall consider radiation received by the lower V antenna of the RAE-1 satellite. This is the antenna that is the primary receiver of the radiation coming from the direction of the lower Van Allen belt just below the satellite's orbit.

Since the Van Allen belt is just below the satellites orbit we will assume it to be an extended source. Thus we shall assume that the intensity of the radiation received by the lower V antenna does not vary over the main antenna lobe for a given observer frequency ω, and satellite latitude λ_o. This will allow us to replace the brightness temperature, T_b by the antenna temperature, T_A. This gives the intensity, $I_\omega (\lambda_o, \omega)$ as

$$I_\omega (\lambda_o, \omega) = \frac{\omega^2}{8\pi^3 C^2} K T_A \tag{47}$$

where K is the Boltzman constant and c is the speed of light.

Since the satellite is measuring the antenna temperature at different latitudes and observing frequencies, we have that

$$T_A = T_A (\omega, \lambda_o) \tag{48}$$

In making comparisons between the radiation received by the RAE-1 lower V antenna and the calculated value of the radiation due to synchrotron radiation from our model, we shall multiply $I_\omega(\lambda_o,\omega)$ determined from (47), by a factor of two. This is done because a linear type antenna only receives one degree of polarization of the radiation.

We shall only consider the observing frequencies of 1300 kHz and 2200 kHz for reasons stated below. Above 2200 kHz radiation from the Earth's surface is able to penetrate the plasma medium of the lower magnetosphere and distort the quiet continuum emission. Below 1300 kHz the radiation is unable to penetrate the medium above the region where the majority of the radiation is being produced, (at about L = 1.35). Thus, the majority of the radiation received below 1300 kHz is reflected radiation from the cosmic background. See figure (3).

Figures (4) and (5) show a comparison of the plot of intensity, I_ω vs latitude, λ_o (representing the average of twenty-five orbits of data) using the Ryle-Vonberg receiver data and a curve of intensity, I_ω vs latitude predicted by the theoretical model for frequencies 1300 kHz and 2200 kHz. Examination of these figures not only show an increase in intensity as we approach the higher latitudes for the RAE lower V data curve in contrast to the theoretical curve which shows a decrease in intensity as we approach the higher latitudes but also show that the intensity at any latitude is much higher than that predicted by the model. The fact that the lowest intensity of the cosmic background observed by the RAE is greater than the peak intensity predicted by the model by a factor of about five made it virtually impossible for the lower Van Allen radiation belt emission to be detected using the present available data. However, if the present population of energetic electrons was increased to about five times its present value, the synchrotron emission might be observed. The above statement was made because it has been observed that during intense solar activity the population of the lower belt of energetic electrons is increased substantially.

It has been showed experimentally that most of the electrons entering the polar regions of the magnetosphere have energies around 40 KeV [3]. We could not expect to obtain the quantity of synchrotron radiation observed by the RAE-1 in the auroral region (60 degrees latitude to

70 degrees latitude) from electrons with energy this low considering the cyclotron emission process. But if we assume that the velocity of the low energy electrons along the magnetic field lines is greater than the phase velocity of electromagnetic radiation in the plasma medium of thermal particles then we can consider the Cerenkov emission process. The energy transmitted by an electron to the plasma medium is

$$\frac{dE}{dS} = \frac{1}{4} \frac{\omega_p}{E_o} m_e e^2 \log \left[1 + \frac{E_o}{kT}\right] \tag{49}$$

where dE/dS is the energy per unit path length given up by the electron of energy E_o to the excitation of plasma oscillations, m_e and e are the mass and charge of the electron respectively, T is the temperature of the plasma medium and ω_p is the plasma frequency. Examination of the above equation shows that the amount of energy transmitted is increased as the energy of the electron is decreased. The intensity, I_ω for electromagnetic Cerenkov radiation of low energy electrons in a magnetic field is

$$I_\omega = \frac{1}{2\pi} 10^{-31} \omega J(\beta) \ell \left[1 - \frac{1}{\beta^2} \frac{Y - 1}{(Y + X - 1)}\right] \frac{\text{watts}}{m^2\text{-rad/sec-sr}} \tag{50}$$

where

$$X = \omega_p^2/\omega^2, \quad Y = \frac{\omega_b}{\omega},$$

$J(\beta)$ is the flux of the electrons and ℓ is the path length [9]. Here ω_p is the plasma frequency, ω_b is the gyrofrequency of the electron and ω is the observing frequency.

For a large beam of particles the equation above for I_ω no longer adequately describe the physical situation. Where as the first particle of the beam will be in a homogeneous plasma, the trailing particles will be affected by the plasma wave fields of the particles that precede them. These fields will move with the beam and so appear constant to the trailing particles within the Cerenkov cone. In this situation, the beam will become unstable and the particles will bunch. These bunched particles will radiate with partial coherence, greatly enhancing the total radiated energy.

The condition for Cerenkov emission is

$$\frac{1}{n\beta} = \cos\theta \leq 1 \tag{51}$$

where β = .374 for 40 KeV electrons. For the refractive index, n we have

$$n = \left(1. + \frac{f_p^{\,2}}{f(f_b \pm f)}\right)^{1/2} \tag{52}$$

where f_p, f_b and f are the plasma, gyro and observing frequencies respectively in Hertzs. We must use the minus sign to insure n to be greater than unity for frequencies

f not greater than the gyrofrequency, f_b.

Let us choose the observing frequency $f = 1.3$ MHz and also choose the gyrofrequency very close to f. This will insure a large enough value of n to satisfy the Cerenkov condition for emission.

For a latitude of 65^o, the position where f_b is equal to 1.3 MHz is at L-shell 1.2 or 2×10^3 Km above the Earth's surface. The plasma frequency at this position is 284 kHz well below the observing frequency.

Since f_p is much less than f, we have that

$$I_\omega \cong 1 \times 10^{-37} f_{MHz} J(\beta) \ell [1. - 1/\beta^2] \quad (53)$$

Now the observed electron flux density J(E > 40 Kev) is 4×10^{11} electron/m^2.sec.sr [10]. This gives for the intensity, I_ω

$$I_\omega \cong 10^{-20} \frac{\text{watts}}{\text{meter}^2\text{.sr.Hz}}$$

which is of the order of magnitude of the radiation observed by the RAE-1 satellite.

VI. CONCLUSION

The model for gyro-synchrotron emission that we have developed will allow the spectral and spatial distribution of gyro-synchrotron emission from a celestial body containing a dipole magnetic field to be computed given the distribution of the trapped particles and the strength of the dipole field.

We used an experimentally determined electron distribution to compute what the spectral and spatial distribution of gyro-synchrotron emission should be for an observer viewing the inner radiating region of the Earth's lower magnetosphere (the lower Van Allen belt) at various positions.

Comparisons were made with observations made with the RAE-1 satellite launched July 4, 1968. The satellite data showed that the cosmic background radiation is too large to allow for a direct observation of the Earth's lower Van Allen radiation belt.

It was also observed, upon examination of the RAE-1 data that the emission from the lower magnetosphere tended to increase as the observing latitude was increased which is opposite to what our model for gyro-synchrotron emission predicts. However, in Section V, a calculation was made to obtain an estimate of what the emission should be from the 40 KeV electrons found in the higher latitudes considering the Cerenkov emission process. This calculation showed that the emission obtained from this process produced an intensity of radiation of the same order of magnitude as was observed with the RAE-1 satellite. A more extensive investigation of the Cerenkov effect as a possible mechanism responsible for the observed intensity from the lower magnetosphere might be fruitful.

If one is allowed to speculate by saying that if, upon extensive investigation of the cosmic background emission received by the lower V antenna of the RAE-1 satellite, a region of the sky is found where the cosmic background emission is lower than or nearly equal to the synchrotron emission predicted by the theoretical model we have developed, then it might be possible to detect the lower Van Allen radiation belt.

Some specific examples of how the model developed in this paper might be useful are:

1) the investigation of the magnetospheres of other planets (such as Jupiter which has a strong magnetic field). Such an opportunity will present itself when NASA's Grand Tour Mission takes place and fly-bys of the outer planets of the solar system are made.

2) The investigation of type IV solar bursts which are suspected to be caused by electrons spiraling in localized magnetic loops eminating from sun

spot regions of the sun's surface [7].

3) The investigation of cosmic background which is due to gyro-synchrotron emission caused by charged particles spiraling about magnetic field lines in inner steller space.

4) The investigation of extra-galactic sources emitting synchrotron radiation, such as the strong radio-galaxies.

The model developed by Vesecky [10] for synchrotron emission from the Earth's magnetic field confined the observer to the Earth's dipole equatorial plane, restricted the pitch angles of the electrons to 90 degrees and only allows for the computation of emission from paths confined to the dipole equatorial plane.

The model developed by Chang [2] for synchrotron emission from the planet Jupiter does not allow for close up observation of the planet. It considers the planet to be a flat disk due to its great distance from the observer on the Earth.

The model developed in this paper is more general than the two models stated above. It does not contain their mentioned restrictions. However, it is restricted to the treatment of trapped particles in dipole magnetic fields only.

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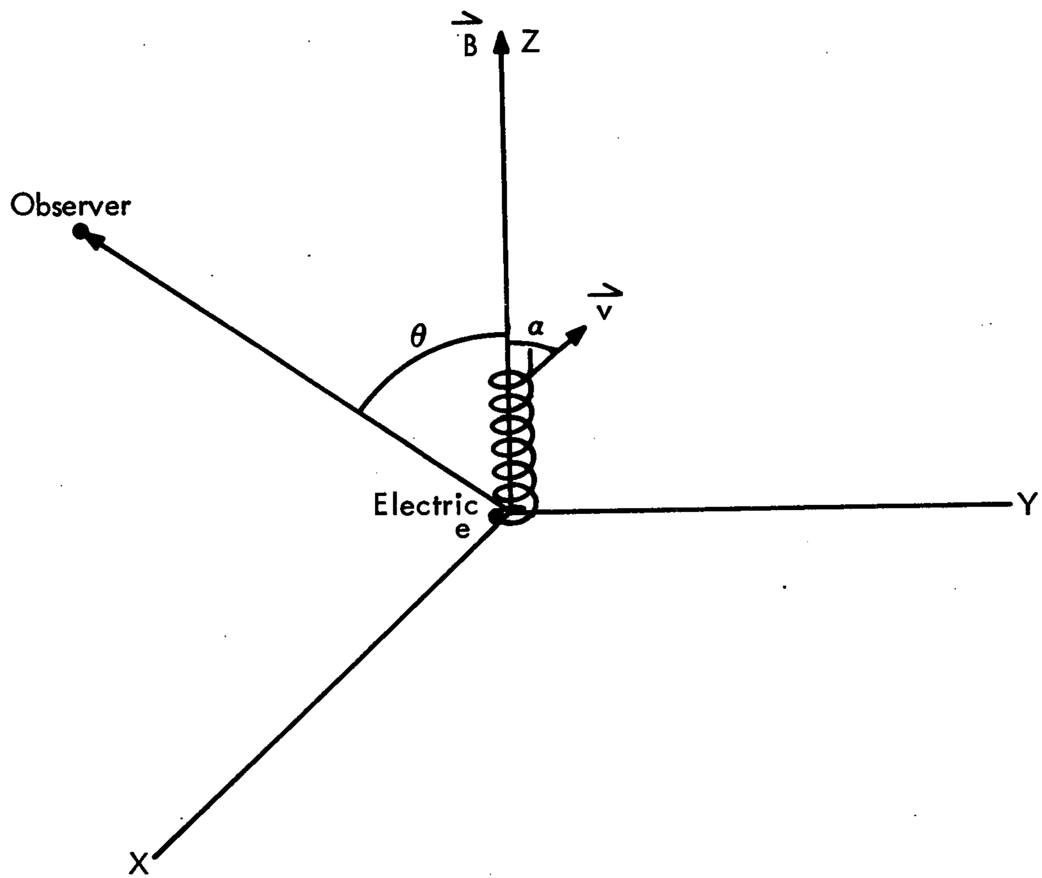

Figure 1 - The diagram shows the angle θ and the pitch angle α of the gyrating electron.

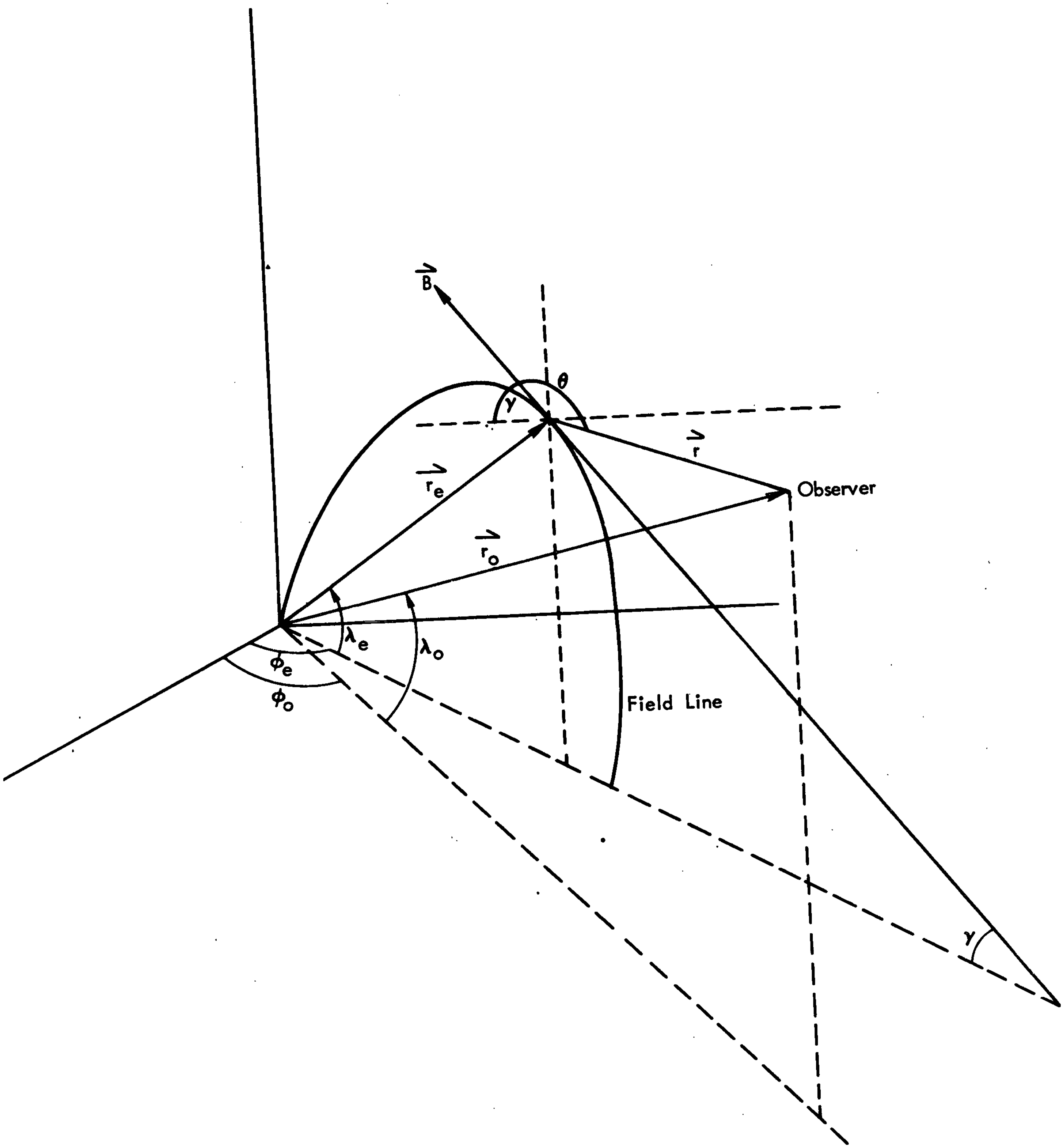

Figure 2 - Showing θ in terms of geomagnetic coordinates.

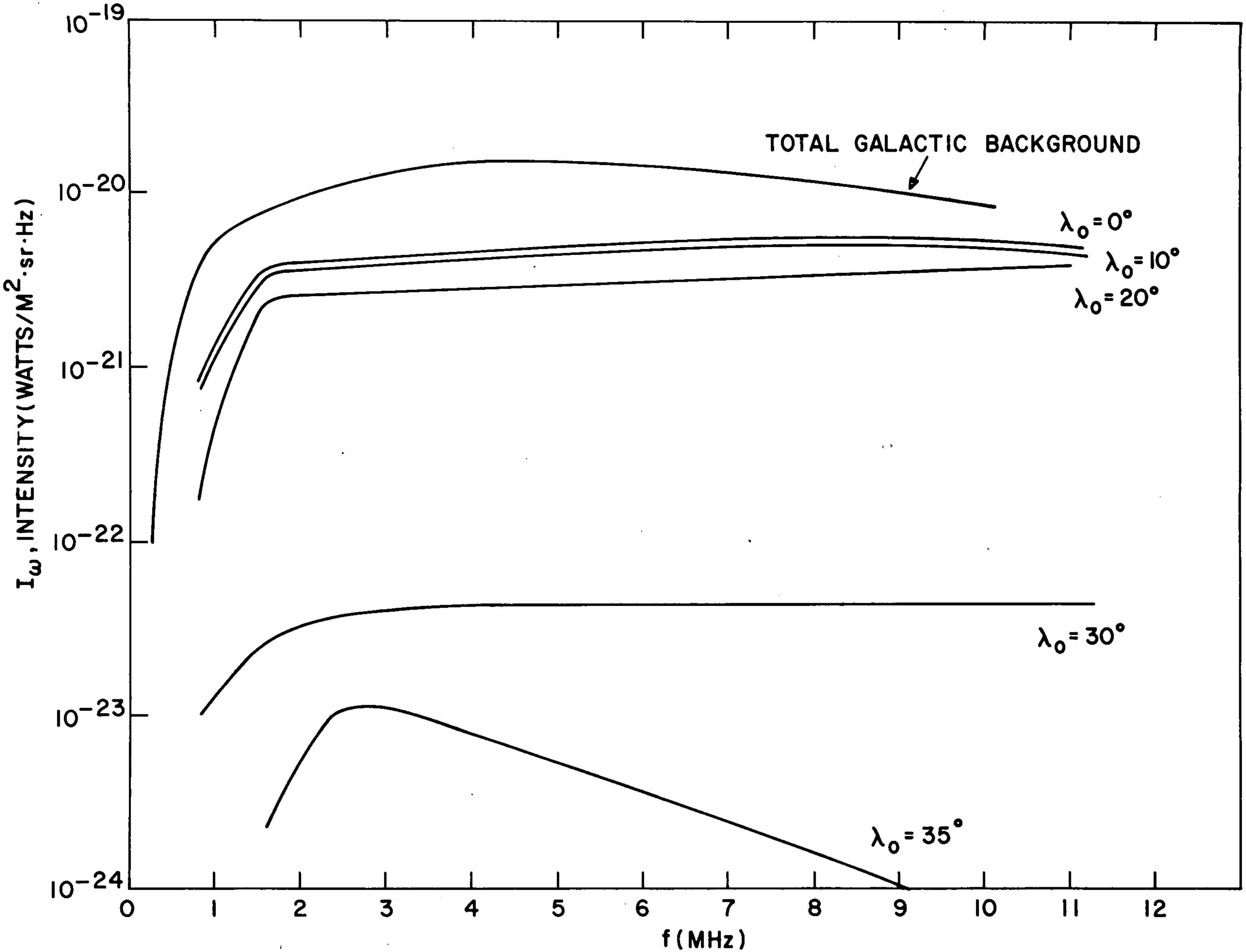

Figure 3 - Graphs showing the intensity I_ω vs. frequency f, at various latitudes λ_o, and a graph showing the total galactic background as a function of frequency [6].

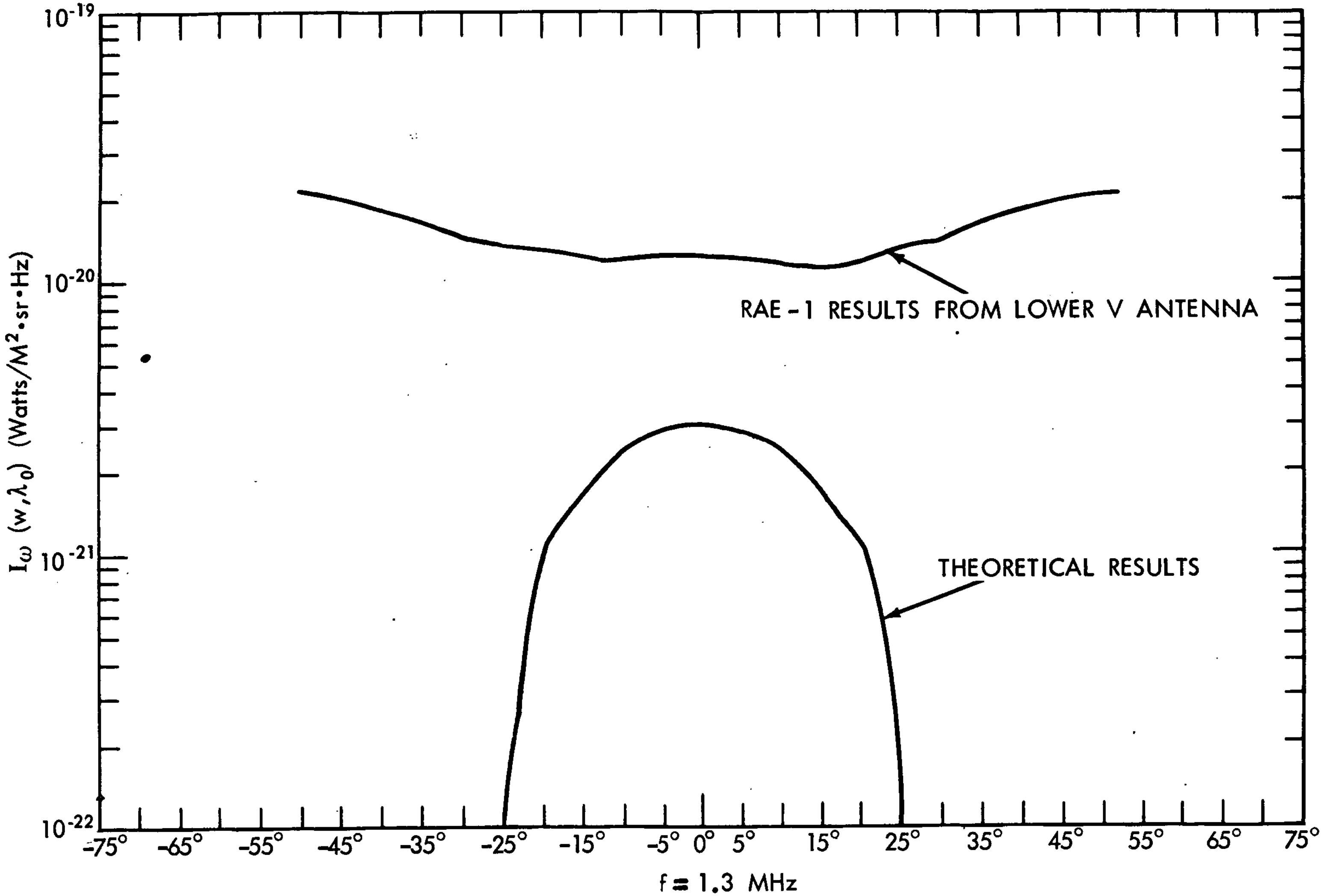

Figure 4 - A graph of $I_\omega(\omega,\lambda_o)$ vs λ_o for the case where experimental data is used to compute the expected radiation due to the synchrotron process compared with a graph of $I_\omega(\omega,\lambda_o)$ vs λ_o from the RAE-1 lower V results. Here f = 1.3 MHz.

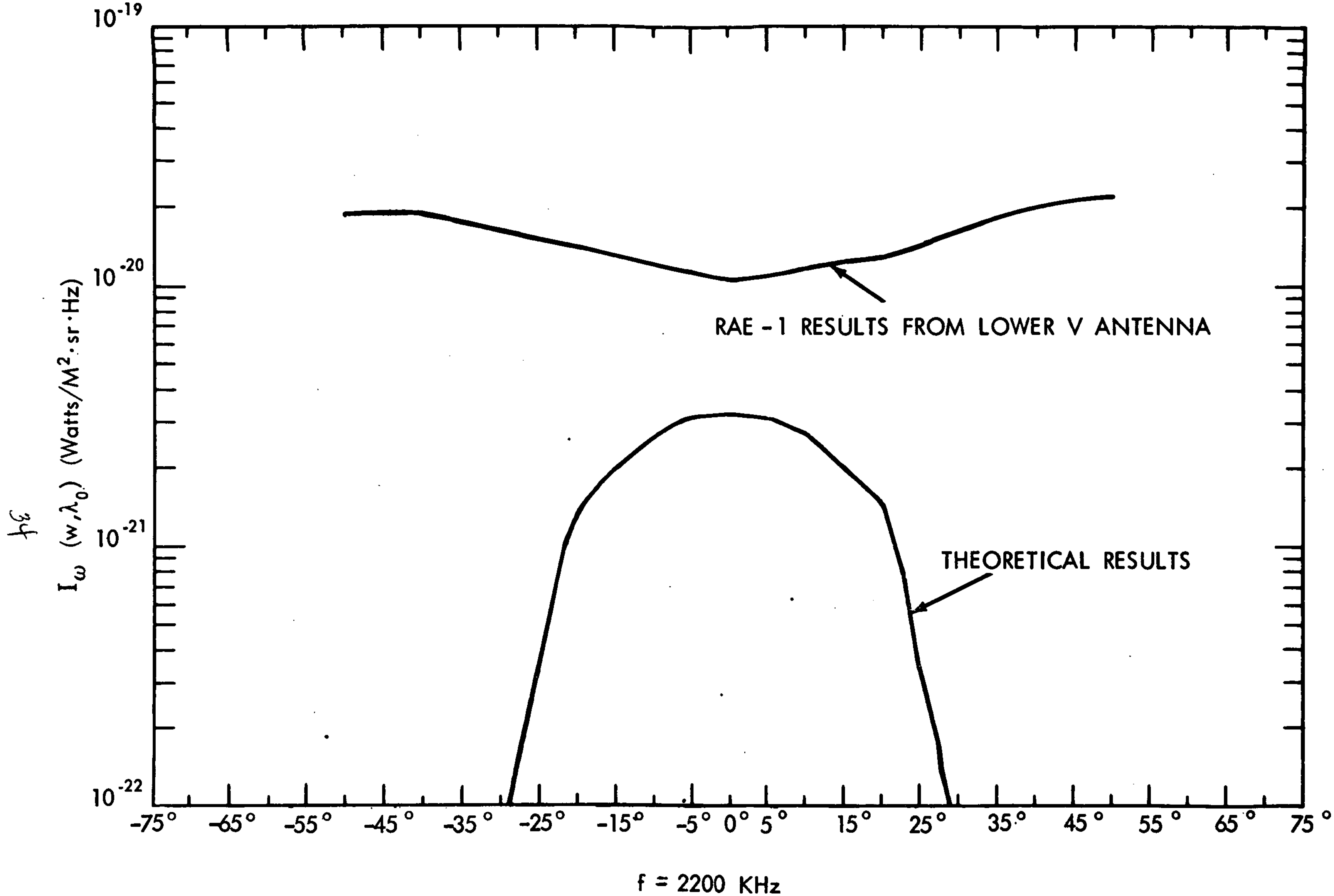

Figure 5 - A graph of $I_\omega(\omega,\lambda_o)$ vs λ_o for the case where experimental data is used to compute the expected radiation due to the synchrotron process compared with a graph of $I_\omega(\omega,\lambda_o)$ vs λ_o from the RAE-1 lower V results. Here f = 2.2 MHz.