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TROPOSPHERIC RANGE-RATE TRACKING-DATA CORRECTION

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GREENBELT, MARYLAND

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ABSTRACT

A formula for correcting the tropospheric error in range-rate satellite tracking data is given. The formula is based on the method, already published, which was used to obtain corrections for elevation-angle and range data.

In addition, an improved method is given for calculating some of the parameters required in the correction formulas.

CONTENTS

ILLUSTRATIONS

TROPOSPHERIC RANGE-RATE TRACKING-DATA CORRECTION

INTRODUCTION

The corrections given here are based on a method already published [1, 2]. The method employs a ratio of polynomials that is adjusted to agree simultaneously with two different series expansions, one of which holds at small values of the elevation angle, while the other, an asymptotic expansion, holds at large values of the angle.

Most satellite tracking is done at elevation angles above about 10 degrees. At these higher angles, the magnitude of the tropospheric correction depends mainly on the coefficients of the terms in the asymptotic expansion, and these may be calculated or estimated provided only that the surface refractivity and the zenith integral of the refractivity from the ground station up is known. The surface refractivity can be calculated and the zenith integral can be estimated from measurements of pressure temperature, and relative humidity, taken at ground level at the tracking station.

CALCULATION OF SURFACE REFRACTIVITY

The surface refractivity N_0 can be calculated from the surface pressure P_0 (millibars), the temperature T (degrees Celsius), and the relative humidity R_h (percent). The formula for refractivity is [3]

$$N = \frac{77.6\,P}{T + 273.15} + \frac{3.73 \times 10^5\, e}{(T + 273.15)^2} \tag{1}$$

where e (millibars) is the partial pressure of the water vapor, which can be calculated from the relative humidity and temperature using [4]

$$e = \frac{R_h}{100} \times 6.11 \times 10^{\frac{7.5\,T}{237.3 + T}} \tag{2}$$

Alternatively if the wet bulb temperature T_w (°C) of a psychrometer is given instead of the relative humidity, e may be calculated from [3, 5]

$$e = 6.11 \times 10^{\frac{7.5\,T_w}{237.3 + T_w}} - 0.00067\, P\,(T - T_w) \qquad (3)$$

Finally, if the dew-point temperature T_d (°C) is given

$$e = 6.11 \times 10^{\frac{7.5\,T_d}{237.3 + T_d}} \qquad (4)$$

ESTIMATION OF THE ZENITH INTEGRAL

The zenith integral is defined as

$$I_z = \int_{h_0}^{\infty} N(h)\, dh \qquad (5)$$

where N (h) is the refractivity at the height h (km) above sea level, and h_0 is the height of the surface where the tracking station is located.

This integral is evaluated by separating the refractivity N into two parts

$$N = N_d + N_w \qquad (6)$$

where N_d is the contribution to N from the polarizability (nominally the dry part) of the air and N_w is the contribution from the (dipole moment of the) water vapor that is present. The zenith integral becomes

$$I_z = I_{zd} + I_{zw} = \int_{h_0}^{\infty} N_d\, dh + \int_{h_0}^{\infty} N_w\, dh \qquad (7)$$

The dry part may be evaluated from a knowledge of surface pressure P_0 (mb) [6, 7]

$$I_{zd} = k\, P_0 \qquad (\mathrm{km}) \qquad (8)$$

where k equals 2.2757 at 45° latitude and varies with latitude as shown in Figure 1.

Reproduced from Reference 7

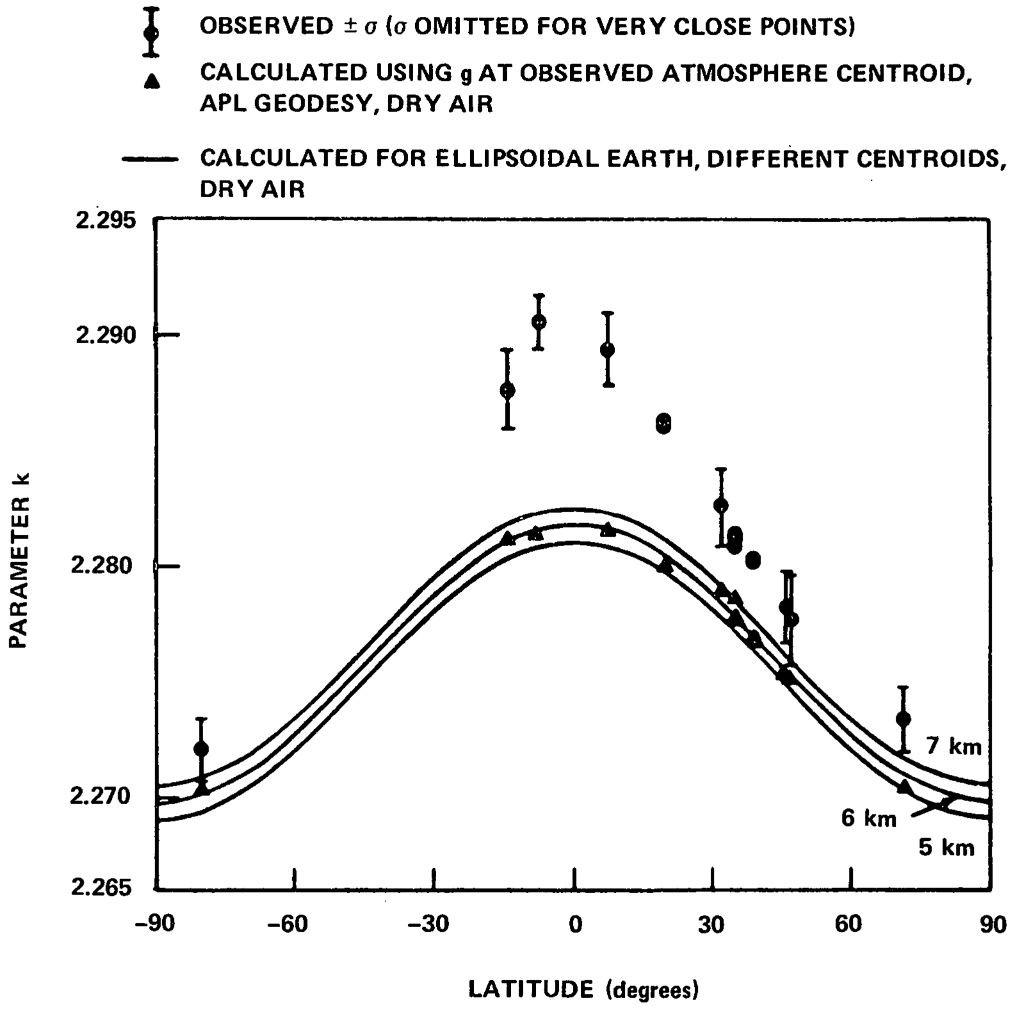

Figure 1. Latitude Variation of Tropospheric Range Error Parameter k (Where $\int N_d \, dh = k \, P_s$)

The wet part is not so easily evaluated, since there is no simple theoretical relationship between the integral of N_w and the surface pressure of the water vapor in the air. Fortunately the contribution of the wet part is somewhat smaller than that of the dry, so that the relative error in N caused by inaccuracies in N_w is not as serious as it might otherwise be. As an interim measure, the wet part of the zenith integral may be estimated from

$$I_{zw} = N_{0w} \int_{h_0}^{\infty} (N_w / N_{0w})\, dh \equiv N_{0w} H_w \doteq 2 N_{0w} \quad \text{(km)} \qquad (9)$$

where N_{0w} is the surface value of the wet part of the refractivity (the second term in Eq. 1), and the factor 2 arises from assuming an equivalent height $(h_w - h_0)$ [6] of 10 km, or an exponential scale height H_w of 2 km for the wet term. Work is in progress to provide a better estimate of I_{hw} at each tracking station through statistical analysis of appropriate radiosonde data.

CALCULATION OF THE SCALE HEIGHT

The correction equations do not employ the zenith integral I_z directly, but are formulated instead using the scale height H defined as

$$H = I_z / N_0 \qquad (10)$$

with the surface refractivity calculated from (1) using (2), (3) or (4), and the scale height calculated from (7), (8), (9) and (10), the calculation of the corrections is a straight-forward application of the equations already published [2]* together with the range-rate correction derived in the next section.

The programs in APL language [8] are given in Appendix 1.

DERIVATION OF RANGE-RATE CORRECTION

The range-rate correction $\Delta\dot{R}$ can be obtained by differentiation of the range-correction. If (43) of reference [2] is differentiated with respect to time, and if the dependence of the correction on $\dot{R}$ is neglected, there results

*The method for calculating one of the polynomial ratios has been modified. This change is described in Appendix 2.

$$\frac{\Delta \dot{R}}{r_0} = \frac{1}{2} 10^{-6} N_0 \cos\theta_0 \left[M'(\alpha) - \rho\, Q\, L(\alpha)\, L'(\alpha)\right] \dot{\theta}_0 \tag{11}$$

Here $M(\alpha)$ has the expansions

$$M(\alpha) \sim (1/\alpha) - M_1(1/\alpha)^3 + M_2(1/\alpha)^5 - \dots \tag{12}$$

$$M(\alpha) = m_0 - m_1\alpha + m_2\alpha^2 - \dots \tag{13}$$

where the values of M_1, M_2, m_0, and m_1 are given in [2] and m_2 has the value

$$m_2 = i_0\left(1 + \frac{1}{2}\, q\, i_1\right)^2 + q\left(1 + \frac{1}{4}\, q\, i_0^2\right) i_2 \tag{14}$$

and where

$$i_2 \equiv \frac{1}{2} I''(0)$$

has, for an exponential profile, the approximation

$$i_2 = \sqrt{\pi}\,(1 - 1.023\, q)^{-1.8} \pm 0.15\% \quad 0 < q < 0.7 \tag{15}$$

Consequently $M'(\alpha)$ in (11) has the expansions, obtained by differentiating (12) and (13),

$$M'(\alpha) \sim -(1/\alpha)^2 + 3M_1(1/\alpha)^4 - 5M_2(1/\alpha)^5 + \dots \tag{16}$$

$$M'(\alpha) = -m_1 + 2m_2\alpha - \dots \tag{17}$$

which can be modelled as shown in Appendix 6 of [2]. Greater accuracy, however, is obtained by using the procedure given in Appendix 2 of this report.

The faction $L'(\alpha)$ in (11) equals

$$L'(\alpha) = -\alpha\, I'(\alpha) - I(\alpha) + \frac{1}{2}\, q\, I(\alpha)\, I'(\alpha) \tag{18}$$

in which $I'(\alpha)$ can be modelled by the same procedure used for $M'(\alpha)$.

Similarly, (67) of [2] gives

$$\frac{\Delta\dot{R}}{r_0} = \frac{1}{2}\, 10^{-6}\, N_0 \cos E \left[W' - Q\rho \left(1 - \beta U - \frac{1}{4}\, q U^2\right) \cdot \left(U + \beta U' + \frac{1}{2}\, q U U'\right)\right] \dot{E} \tag{19}$$

and W' can be modelled in the same way using the result that

$$w_2 = \frac{1}{2}\, W''(0) = \left(u_0 \quad 1 - \frac{1}{2}\, q u_1\right) \tag{20}$$

Similarly, $U'(\beta)$ is modelled using Equation (65) of [2] which should read

$$u_2 = \frac{1}{2}\, U''(0) = \frac{1}{2}\, I''\left(\frac{1}{2}\, q u_0\right) \Big/ \left[1 - \frac{1}{2}\, q I'\left(\frac{1}{2}\, q u_0\right)\right]^3 \tag{21}$$

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APPENDIX 1

APL PROGRAMS FOR CORRECTIONS

Inputs to PREPASS 1

Surface refractivity NN0

Scale height HH (km)

Distance R0 (km) of tracking station from the center of the earth

Outputs of PREPASS 1

The 16 constants I1 - I4, M1 - M4, S1 - S4, and IP1 - IP4.

Inputs to CORR 1

The 16 constants I1 - I4, M1 - M4, S1 - S4, and IP1 - IP4 from PREPASS 1

Surface refractivity NN0

Scale height HH (km)

Arrival angle THETA0 (mrad) of the wave from the satellite.

Arrival angle rate THETA0 DOT (mrad/sec)

Range RR (km) of the satellite from the tracking station.

Outputs of CORR 1

Elevation angle correction DEE (mrad)

$$E = \theta_0 - DEE$$

Range correction DRR (km)

$$R = R_{measured} - DRR$$

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Range-rate correction DRRDOT (m/s)

$$\dot{R} = \dot{R}_{measured} - DRRDOT$$

Inputs to PREPASS 2

Surface refractivity NN0

Scale height HH (km)

Distance R0 (km) of tracking station from the center of the earth.

Outputs of PREPASS 2

The 16 constants $\underline{U}1$ - $\underline{U}4$, $\underline{U}P1$ - $\underline{U}P4$, $\underline{W}1$ - $\underline{W}4$, and $\underline{I}1$ - $\underline{I}4$.

Inputs to CORR 2

The 16 constants $\underline{U}1$ - $\underline{U}4$, $\underline{U}P1$ - $\underline{U}P4$, $\underline{W}1$ - $\underline{W}4$, and $\underline{T}1$ - $\underline{T}4$ from PREPASS 2.

Surface refractivity NN0

Scale height HH (km)

Elevation angle E (milliradians) of the satellite above the tracking station

Elevation angle rate EDOT (mrad/sec)

Range RR (km) to satellite from the tracking station

Outputs of CORR 2

Elevation angle correction DEE (mrad)

$$E = \theta_0 - DEE$$

Range correction DRR (km)

$$R = R_{measured} - DRR$$

Range-Rate correction DRRDOT (m/s)

$$\mathring{R} = \mathring{R}_{measured} - DRRDOT$$

```
      ∇PREPASS1[⎕]∇
    ∇ PREPASS1
[1]   PSQUARED←2×HH÷R0
[2]   P←PSQUARED*0.5
[3]   Q←1E¯6×NN0×R0÷HH
[4]   F0←I0←((○1)*0.5)×(1-0.9206×Q)*¯0.4468
[5]   F1←I1←2÷1-Q
[6]   FF1←II1←0.5×1-0.5×Q
[7]   FF2←II2←0.75×1+Q×¯0.75+Q÷6
[8]   CALCULATEF̲
[9]   I̲1←PSQUARED×F̲1
[10]  I̲2←PSQUARED×F̲2
[11]  I̲3←PSQUARED×F̲3
[12]  I̲4←P×F̲4
[13]  K0←((○2)*0.5)×(1-0.9408×Q)*¯0.4759
[14]  F0←M0←(I0×1+Q×1+Q×I0×I0÷12)-Q×K0÷2
[15]  F1←M1←2×(1+Q×I0×I0÷4)÷1-Q
[16]  FF1←MM1←0.5×1-0.75×Q
[17]  FF2←MM2←0.75×1+Q×(¯25÷24)+11×Q÷36
[18]  CALCULATEF̲
[19]  M̲1←PSQUARED×F̲1
[20]  M̲2←PSQUARED×F̲2
[21]  M̲3←PSQUARED×F̲3
[22]  M̲4←P×F̲4
[23]  I2←((○1)*0.5)×(1-1.023×Q)*¯1.8
[24]  M2←(I0÷(1-Q)*2)+Q×I2×1+0.25×Q×I0*2
[25]  G0←M1
[26]  G1←2×M2
[27]  GG1←3×MM1
[28]  GG2←5×MM2
[29]  PREPAREF
[30]  CALCULATEF̲
[31]  S̲1←PSQUARED×F̲1
[32]  S̲2←PSQUARED×F̲2
[33]  S̲3←PSQUARED×F̲3
[34]  S̲4←P×F̲4
[35]  G0←I1
[36]  G1←2×I2
[37]  GG1←3×II1
[38]  GG2←5×II2
[39]  PREPAREF
[40]  CALCULATEF̲
[41]  I̲P1←PSQUARED×F̲1
[42]  I̲P2←PSQUARED×F̲2
[43]  I̲P3←PSQUARED×F̲3
[44]  I̲P4←P×F̲4
    ∇
```

```
      ∇CORR1[⎕]∇
    ∇ CORR1;SIN;COS;I;LL;M;S
[1]   SIN←1○THETA0×0.001
[2]   COS←2○THETA0×0.001
[3]   I←÷SIN+I̲1÷SIN+I̲2÷SIN+I̲3÷SIN+I̲4
[4]   LL←1-I×SIN-0.5×1E¯6×NN0×I
[5]   DEE←0.001×NN0×COS×I-R0×LL÷RR
[6]   M←÷SIN+M̲1÷SIN+M̲2÷SIN+M̲3÷SIN+M̲4
[7]   DRR←1E¯6×NN0×HH×M-0.5×1E¯6×NN0×((R0×COS×LL)*2)÷RR×HH
[8]   S←÷SIN+S̲1÷SIN+S̲2÷SIN+S̲3÷SIN+S̲4
[9]   IP←(÷SIN+I̲P1÷SIN+I̲P2÷SIN+I̲P3÷SIN+I̲P4)*2
[10]  TERM←Q×(R0÷RR)×(COS*2)×LL×I+(1E¯6×NN0×I×IP)-SIN×IP
[11]  DRRDOT←¯1E¯9×THETA0DOT×NN0×HH×COS×(S*2)-TERM
    ∇


      ∇PREPAREF[⎕]∇
    ∇ PREPAREF
[1]   F0←G0*0.5
[2]   F1←G1÷2×F0
[3]   FF1←GG1÷2
[4]   FF2←0.5×GG2-FF1*2
    ∇


      ∇CALCULATEF̲[⎕]∇
    ∇ CALCULATEF̲
[1]   F̲1←FF1
[2]   F̲2←(FF2÷F̲1)-F̲1
[3]   F̲3←F̲2÷(F0×F0×F̲1×1+F̲1÷F̲2)-1+F1×F̲1
[4]   F̲4←F0×F̲1×F̲3÷F̲2
    ∇
```

```
        ∇PREPASS2[⎕]∇
      ∇ PREPASS2
[1]     PSQUARED←2×HH÷R0
[2]     P←PSQUARED*0.5
[3]     Q←1E¯6×NN0×R0÷HH
[4]     IPU←¯2×(1+1.482×Q)*¯0.3826
[5]     DEN←1-0.5×Q×IPU
[6]     IPPU←2×((○1)*0.5)×(1+1.71×Q)*0.1
[7]     F0←U0←((○1)*0.5)×(1+1.4844×Q)*¯0.39144
[8]     F1←U1←-IPU÷DEN
[9]     FF1←UU1←0.5×1+0.5×Q
[10]    FF2←UU2←0.75×1+Q×(7÷12)+Q÷6
[11]    CALCULATEF̲
[12]    U̲1←PSQUARED×F̲1
[13]    U̲2←PSQUARED×F̲2
[14]    U̲3←PSQUARED×F̲3
[15]    U̲4←P×F̲4
[16]    G0←U1
[17]    G1←2×U2←0.5×IPPU÷DEN*3
[18]    GG1←3×UU1
[19]    GG2←5×UU2
[20]    PREPAREF
[21]    CALCULATEF̲
[22]    U̲P1←PSQUARED×F̲1
[23]    U̲P2←PSQUARED×F̲2
[24]    U̲P3←PSQUARED×F̲3
[25]    U̲P4←P×F̲4
[26]    KU←((○2)*0.5)×(1+1.6454×Q)*¯0.583
[27]    F0←W0←(U0×1+Q×1-Q×U0×U0÷6)-0.5×Q×KU
[28]    F1←W1←2×1-0.25×Q×U0×U0
[29]    FF1←WW1←0.5×1+0.25×Q
[30]    FF2←WW2←0.75×1+Q×(7÷24)+Q÷18
[31]    CALCULATEF̲
[32]    W̲1←PSQUARED×F̲1
[33]    W̲2←PSQUARED×F̲2
[34]    W̲3←PSQUARED×F̲3
[35]    W̲4←P×F̲4
[36]    W2←U0×1-0.5×Q×U1
[37]    G0←W1
[38]    G1←2×W2
[39]    GG1←3×WW1
[40]    GG2←5×WW2
[41]    PREPAREF
[42]    CALCULATEF̲
[43]    T̲1←PSQUARED×F̲1
[44]    T̲2←PSQUARED×F̲2
[45]    T̲3←PSQUARED×F̲3
[46]    T̲4←P×F̲4
      ∇
```

```
      ∇CORR2[⎕]∇
    ∇ CORR2;SINE;COSE;U;SRUP;PAREN;W;T
[1]   SINE←1○EE×0.001
[2]   COSE←2○EE×0.001
[3]   U←÷SINE+U̲1÷SINE+U̲2÷SINE+U̲3÷SINE+U̲4
[4]   UP←(÷SINE+U̲P1÷SINE+U̲P2÷SINE+U̲P3÷SINE+U̲P4)*2
[5]   PAREN←1-U×SINE+0.5×1E¯6×NN0×U
[6]   DEE←0.001×NN0×COSE×U-PAREN×(1-1E¯6×NN0×UP)×R0÷RR
[7]   W←÷SINE+W̲1÷SINE+W̲2÷SINE+W̲3÷SINE+W̲4
[8]   DRR←1E¯6×NN0×HH×W+0.5×1E¯6×NN0×(÷HH×RR)×(R0×COSE×PAREN)*2
[9]   T←÷SINE+T̲1÷SINE+T̲2÷SINE+T̲3÷SINE+T̲4
[10]  TERM←Q×(R0÷RR)×(COSE*2)×PAREN×U-(1E¯6×NN0×U×UP)+SINE×UP
[11]  DRRDOT←¯1E¯9×EDOT×NN0×HH×COSE×(T*2)+TERM
    ∇
```

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APPENDIX 2

APPROXIMATION TO $G(\alpha)$

Let the function $G(\alpha)$ have the expansions

$$G(\alpha) \sim (1/\alpha)^2 - G_1(1/\alpha)^4 + G_2(1/\alpha)^6 \cdots \tag{2-1}$$

$$G(\alpha) = g_0 - g_1\alpha + \ldots \tag{2-2}$$

for α large and small respectively. The modelling of $G(\alpha)$ is accomplished by approximating its square root as a ratio of polynomials. The square root of $G(\alpha)$ has the expansions

$$\sqrt{G(\alpha)} \sim (1/\alpha) - \frac{1}{2} G_1(1/\alpha)^3 + \frac{1}{2}\left(G_2 - \frac{1}{4} G_1^2\right)(1/\alpha)^5 - \ldots \tag{2-3}$$

$$\sqrt{G(\alpha)} = \sqrt{g_0} - \frac{1}{2}\frac{g_1}{\sqrt{g_0}}\alpha + \ldots \tag{2-4}$$

as may be verified by squaring (2-3) and (2-4) to obtain (2-1) and (2-2). Expansions (2-3) and (2-4), however have the form of (38) and (39) of reference [2], and so may be modelled by (33) of [2].

Hence

$$G(\alpha) \doteq F^2\left(\alpha;\ \frac{1}{2} G_1,\ \frac{1}{2}\left(G_2 - \frac{1}{4} G_1^2\right);\ \sqrt{g_0},\ g_1/2\sqrt{g_0}\right)$$

where F is defined by (33)-(37) of [2].

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