N72-32834

NASA TECHNICAL NOTE

NASA TN D-6949

NASA TN D-6949

MARS GRAVITATIONAL FIELD ESTIMATION ERROR

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1972

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-6949		

4. Title and Subtitle	5. Report Date
MARS GRAVITATIONAL FIELD ESTIMATION ERROR	October 1972
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Harold R. Compton and Edward F. Daniels	L-8143
9. Performing Organization Name and Address	10. Work Unit No.
NASA Langley Research Center Hampton, Va. 23365	790-91-42-01
	11. Contract or Grant No.
12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered
National Aeronautics and Space Administration Washington, D.C. 20546	Technical Note
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Mars gravitational field Aeroanalysis	Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	41	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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SUMMARY

The error covariance matrices associated with a weighted least-squares differential correction process have been analyzed for accuracy in determining the gravitational coefficients through degree and order five in the Mars gravitational potential function. The results are presented in terms of standard deviations for the assumed estimated parameters. The covariance matrices were calculated by assuming Doppler tracking data from a Mars orbiter, a priori statistics for the estimated parameters, and model error uncertainties for tracking-station locations, the Mars ephemeris, the astronomical unit, the Mars gravitational constant (G_M), and the gravitational coefficients of degrees six and seven. Model errors were treated by using the concept of consider parameters.

In general, there appears to be sufficient signature in the Doppler tracking data for improving current uncertainties in some of the gravitational coefficients through degree and order five. Indications are that current estimates of the mass cannot be significantly improved. Model error parameter uncertainties of the order assumed have been shown to cause significant degradation in solution accuracy in some cases. For the range of orbital elements investigated, the estimation accuracy of the gravitational coefficients was found to be slightly dependent on the semimajor axis and inclination and essentially independent of the eccentricity and nodal position. Combining Mariner 1971 Doppler tracking data with Viking Doppler tracking data appears to be an effective way of improving the coefficient estimates. However, in order to obtain this improvement, it may be necessary to extend the gravitational-coefficient solution set to a higher degree and order.

INTRODUCTION

Current plans for planetary research include two Mars orbiter-lander exploratory missions. These two missions are scheduled in the Mars 1975 Viking Project. Whereas the Mariner 1971 spacecraft was the first United States spacecraft to orbit a planet other than the earth or moon, the Viking lander will be the first United States spacecraft to soft-land on a planet other than the earth or moon. The purpose of the Mars exploratory missions is to increase the scientific knowledge of Mars by returning data from experiments carried onboard the orbiters and landers. For successful completion of these missions, it will be necessary to calculate accurately the ephemerides for the orbiters. In order to do this, an accurate knowledge of the Mars gravitational field is required. This knowledge would also be useful for calculating the positions of the landed spacecraft and would be useful to those concerned with the figure and internal structure of Mars.

Present knowledge of the Mars gravitational field is limited mainly to estimates of the mass of Mars and the second-degree spherical harmonic coefficients in the Mars gravitational potential function (refs. 1 and 2). From past experience in earth and lunar gravitational field estimation, it is expected that these estimates are insufficient, in terms of the number of coefficients estimated, for accurate mission control, and certainly so in defining the fine structure of the Mars gravitational field. It is therefore desirable to extend these estimates to a gravitational field of high degree and order. The gravitational fields of the earth and moon have been estimated through analysis of the tracking data from close earth and lunar satellites. However, the planned Mars missions do not include close satellites such as those for the earth and moon; therefore, the tracking data from these planned Mars missions may not yield as much information about the Mars gravitational field. The purpose of this paper is to present the results of an error analysis to define the accuracy with which the gravitational coefficients of Mars can be determined by using the Doppler tracking data from a Mars orbiter in the Viking project series.

SYMBOLS

A matrix containing partial derivatives of a given data type with respect to estimated parameters

a semimajor axis of Mars satellite orbit

C matrix containing partial derivatives of a given data type with respect to model error parameters

$C_{n,m}$, $S_{n,m}$ coefficients of spherical harmonic in gravitational potential function (n is degree and m is order)

e eccentricity of Mars satellite orbit

G_M product of universal gravitational constant and mass of Mars

i areographic inclination of the Mars satellite orbital plane

R mean radius of Mars

r distance from center of Mars to Mars satellite

$P_{n,m}$ associated Legendre polynomials (n is degree and m is order)

t_o time of periapsis passage

γ ratio of solution parameter standard deviation to a priori standard deviation for that parameter

$\Lambda_{\hat{x}}$ covariance matrix for solution parameters

Λ_α covariance matrix for model error parameters

Λ_ϵ covariance matrix for tracking data noise

Λ_o covariance matrix of a priori statistics for solution parameters

λ areographic longitude of Mars satellite

ϕ areographic latitude of Mars satellite

Ω areographic longitude of ascending node of Mars satellite orbital plane

ω argument of periapsis, angle measured in orbital plane from ascending node to point of periapsis

Superscripts:

-1 matrix inverse

T transpose of matrix

ANALYSIS

Physical Model and Satellite Geometry

An approximation of the Mars gravitational field can be made by determining a sufficient number of coefficients $C_{n,m}$ and $S_{n,m}$ in the expansion, in spherical harmonics, of the Mars gravitational potential function,

$$U = \frac{G_M}{r}\left[1 + C_{0,0} + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n P_{n,m}(\sin \phi)\left(C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda\right)\right]$$

The coefficient $C_{0,0}$ is used to account for a deviation of the mass of Mars from a nominal value assumed in G_M.

The Kepler elements of the simulated Viking-type orbit used as a reference or nominal orbit in this report are

$a = 21\ 767.8$ km

$e = 0.7752$

$i = 33^{\circ}$

$\Omega = 102.7^{\circ}$

$\omega = 38^{\circ}$

$t_o = 0$ hr GMT

The angles and time are referred to the areographic coordinate system at 0 hr GMT, July 23, 1976. The period of this orbit (27.085 hr) is such that the spacecraft ground track advances through 360° of longitude in approximately 10 orbits. The nominal periapsis altitude is 1500 km.

Methods and Statistical Considerations

A parametric error analysis was made to investigate how the accuracies of estimating the gravitational coefficients are affected by the semimajor axis, eccentricity, inclination, and nodal position of the spacecraft orbit, the solution set, the tracking data arc length, and the model errors (parameters considered but not estimated). It should be noted that the coefficients were never actually determined; only the covariance matrix for the assumed estimated parameters was calculated.

For the purpose of this report, it was assumed that the Doppler tracking data were used in a weighted least-squares differential correction process to estimate the parameters of interest. It was also assumed that a priori estimates of the gravitational coefficients, with known standard deviations, were available for use in the differential correction process. The error covariance matrices associated with this process have been analyzed, and the results are presented in this report in terms of the standard deviations associated with each estimate.

In order to account for an incomplete solution set, a statistical model characteristic of those in current use in orbit determination processes was used. This model made use of the consider or model error concept. This concept allows the uncertainty in some nonestimated parameters to propagate into the statistics on the estimated parameters. The nonestimated parameters are usually referred to as model error parameters. The error covariance matrix for the statistical model was calculated from the following equation:

$$\Lambda_{\hat{x}} = \left(A^T \Lambda_\epsilon^{-1} A + \Lambda_o^{-1}\right)^{-1} + \left(A^T \Lambda_\epsilon^{-1} A + \Lambda_o^{-1}\right)^{-1} A^T \Lambda_\epsilon^{-1} C \Lambda_\alpha C^T \Lambda_\epsilon^{-1} A \left(A^T \Lambda_\epsilon^{-1} A + \Lambda_o^{-1}\right)^{-1} \tag{1}$$

The matrices A and C contain the partial derivatives of the data type (Doppler) with respect to the estimated and model error parameters, respectively. The covariance matrices for the solution set, the a priori estimates of the solution set, the tracking data measurement errors, and the model error parameters are $\Lambda_{\hat{x}}$, Λ_o, Λ_ϵ, and Λ_α, respectively. Details of the development of the above equation are given in reference 3. The error covariance matrix $\Lambda_{\hat{x}}$ without a priori statistics on the solution parameters can be found by setting Λ_o^{-1} equal to zero, and model errors can be neglected by excluding the second term on the right in equation (1).

For calculations of the error covariance results presented in this report, Doppler tracking data having a random noise with standard deviation of 0.001 m/sec were assumed to have been taken at 10-minute intervals during the acquisition periods by the Deep Space Network tracking stations. The tracking data errors were assumed to be uncorrelated in time, unbiased, and of equal weight, and the weighting matrix in the estimation process was assumed to be the inverse of the measurement covariance matrix.

The model errors assumed for the nongravity parameters are the same as those used in reference 4 and are presented in table I along with the a priori standard deviations used for the six state variables. The assumed a priori standard deviations for the Mars gravitational coefficients are given in table II. These values are based on information presented in reference 5 and are similar to those in reference 6.

Solution Parameter Selection

The gravitational coefficients represent an infinite set of parameters which must be specified to uniquely define the gravitational field. However, the number of parameters which can be estimated is limited by such factors as the mathematical approach, computer storage, computer accuracy, program software, and computer time. It is therefore necessary to choose a finite set of estimation parameters from the available set. To aid in this selection, some preliminary analyses of simulated Doppler tracking data were made. In these analyses, the simulated data were treated as being representative of real Doppler tracking data having a known random noise level. It was assumed that if a particular solution vector could be used to fit the simulated Doppler tracking data to the extent that the Doppler residual was below the assumed random noise level, then all the detectable signal due to the physical system had been removed and that particular set of solution parameters was an adequate model for the physical system.

Doppler tracking data were simulated for 10 orbits by a computer program using the nominal spacecraft trajectory and a gravitational potential function which included the gravitational coefficients through degree and order seven as given in table II. This particular potential function was assumed to represent the real gravitational field of Mars and was the only source of a disturbing force on the orbiting spacecraft. The choice of a 10-orbit tracking interval was made to allow the spacecraft ground track to advance through 360° in areographic longitude. Four different sets of parameters were then used as a solution vector for fitting the simulated tracking data. The four different sets, each of which included the six Cartesian state variables, were the gravitational coefficients through degree and order 3, 4, 5, and 6, excluding $C_{0,0}$. The root-mean-square (rms) residual for each of these solution sets was 15, 6, 0.5, and 0.3 mm/sec, respectively. The solution set containing the six state variables and the gravitational coefficients through degree and order five, except for $C_{0,0}$, was found to be an adequate representation of the real generating model in that it could be used to reduce the average tracking data residual to approximately the assumed noise level of 1.0 mm/sec. Therefore, this set of parameters was chosen as the solution set for this report. Note that the rms residual for the solution set containing the coefficients of sixth degree and order is also less than the assumed tracking data noise level. However, in the absence of any knowledge of the gravitational field, there would probably be no rationale for including more parameters.

For most of the results presented in this report, the $C_{0,0}$ coefficient was not included in the solution set because of its correlation with the other parameters. In some cases the normal matrix with $C_{0,0}$ included was noninvertible. Therefore, it was decided to present results for a set of gravitational coefficients whose normal matrix was invertible under all conditions investigated. However, it is important to know whether or not the tracking data can be reduced to estimate $C_{0,0}$, and so a brief discussion on the estimation of this coefficient is given in a subsequent section of this report.

In the rest of this report the phrase "a priori information" applies only to those parameters which were assumed to have been estimated, and the phrase "model errors" applies only to nonestimated parameters.

RESULTS AND DISCUSSION

The results presented in the subsequent sections of this report were obtained from one of three different calculations of the covariance matrix associated with the estimation process. These three methods are

(a) Covariance assuming no a priori information and no model errors – This matrix is based entirely on the Doppler data and is therefore a measure of the information content of the data.

(b) Covariance assuming a priori information and no model errors – The standard deviations obtained from this matrix represent the minimum uncertainty that can be expected.

(c) Covariance assuming both a priori information and model errors – The standard deviations obtained from this matrix are expected to be the most representative of the estimation process.

For convenience, the results in all figures are presented in terms of a logarithmic ratio. Plotted along the ordinate axis is the common logarithm of the ratio of the solution parameter standard deviation to the a priori standard deviation for that particular parameter ($\log \gamma$). A value of zero indicates that the solution parameter standard deviation is exactly equivalent to the a priori standard deviation, and a positive or negative value of 1, 2, 3, . . . indicates that the solution parameter standard deviation is, respectively, 1, 2, or 3 orders of magnitude larger or smaller than the a priori.

Effects of Tracking Data Arc Length

The accuracy of estimating the solution parameters is strongly dependent upon the time span over which the data are taken, and therefore its variation with data arc length is of interest. In order to analyze this variation, normal matrices were formed and processed in equation (1) for up to 27 complete revolutions of tracking data. This represents a maximum of 30 days of tracking since the period is approximately 27 hours. The results of these calculations are presented in figure 1 for the three different solution techniques mentioned in the previous section of this report.

Figure 1 is presented to show the accuracy variations that might be expected for tracking data taken from a typical Mars orbiter in the Viking project. As expected, curve 1 decreases monotonically with time which is in the direction of increased estimation accuracy. The standard deviations for estimating the gravitational coefficients vary by as much as three orders of magnitude, and with few exceptions (for example, $C_{5,0}$), no less than two orders of magnitude during the 30-day tracking interval. This is an indication of the maximum accuracy improvement which might be expected. Curve 1 also indicates how the information content of the tracking data varies with time. It can be seen that the steepest gradient or largest information rate is contained in the first 10 orbits of tracking, and that with the exception of the fifth-degree sectorials, a much smaller gradient occurs thereafter for up to 27 orbits or 30 days of tracking. The fifth-degree-sectorial coefficients appear to be very sensitive to tracking data arc length up to about 18 orbits. With a few exceptions, the standard deviations for the coefficients very closely approximate the $\sqrt{N}$ law after about 20 orbits of tracking where N is the total number of observations.

A comparison of curves 1 and 2 gives an indication of the effect of a priori information on the solution set estimation accuracy. As might be expected, the effects of a priori information appear to be the most significant during the first orbits of tracking. The two curves are nearly identical after nine orbits of tracking. Therefore, in the ideal situation of no model errors and long tracking data arcs, one might expect the a priori to be of little consequence. However, for short tracking data arcs, the a priori significantly increases the estimation accuracy for the first few orbits and should be used as an aid to the parameter estimation process.

The difference between curves 2 and 3 in each plot of figure 1 is a direct consequence of assuming the model errors given in tables I and II. In general, this difference is much less than an order of magnitude for about the first five to 10 orbits of tracking, but it increases to a separation of about one order of magnitude after 20 orbits for most of the coefficients except for the two fifth-degree sectorials which have a maximum separation of about two orders of magnitude after 20 orbits of tracking. This is an indication that the solution set estimation accuracy will be significantly degraded by model errors, especially for the longer tracking data arcs. However, it appears that current uncertainties in the second- and third-degree coefficients can be improved by an order of magnitude and that current uncertainties in the fourth-degree coefficients can be improved slightly. The results indicate virtually no improvement for the fifth-degree coefficients. The results also indicate that virtually no improvement occurs after 10 orbits for any of the coefficients. This appears to be consistent with the method used to select the solution set for the fifth degree and order.

Effects of Orbit Parameters

Semimajor axis.- The variation of the standard deviation in the estimates of the gravitational coefficients with the semimajor axis after 10 orbits of tracking is presented in figure 2. In order to obtain the standard deviations, covariance matrices were calculated after 10 orbits of tracking for each value of the semimajor axis investigated with i, Ω, ω, e, and t_0 held constant at the nominal values. The semimajor axis was varied from approximately 17 300 km to approximately 22 700 km, corresponding to a periapsis altitude range from 500 km to 1700 km. Even though the period of the orbits changed as the semimajor axis changed, the Doppler measurement rate was held at one measurement per 10 minutes.

With few exceptions, the standard deviations, assuming a priori and no model errors, increased with increasing values of the semimajor axis. The primary reason for the increase in the standard deviation is the inverse proportionality between the partial derivatives which comprise the A matrix and the nth power of the semimajor axis, where n is the degree of the coefficient. As the semimajor axis increases, the numerical values of the derivatives decrease, resulting in less sensitive derivatives and hence in less information about the gravitational coefficients. The important point to note in curve 1 of figure 2 is that with only two exceptions, the standard deviations for the gravitational coefficients varied at most one order of magnitude over the range of semimajor axes investigated. Thus, the semimajor axis does not have a major effect on the basic data content for this range.

In general, the difference between curves 1 and 2 is expected to decrease as the semimajor axis increases because at the higher altitudes the data are less sensitive to the nonestimated parameters and hence are expected to contribute less to the standard deviations of the estimated parameters. In figure 2 this trend is evident for all the gravitational coefficients up to a semimajor axis of about 21 700 km. The increase in the standard deviations between 21 700 km and 22 700 km is due to significant changes in the contributions of certain of the model error parameters (especially those due to the sixth-order coefficients) to the standard deviations of the estimated gravitational coefficient.

Eccentricity.- The eccentricity interval studied was $0.7614 \leqq e \leqq 0.8211$, corresponding to the Viking operation range of periapsis altitude from 500 to 1800 km. The results of the eccentricity investigation are presented in figure 3. With the exception of the $C_{5,5}$ coefficient, the standard deviations obtained with the assumption of no model errors behave as expected in that they show a monotonic decrease (periapsis altitude decreases and hence data sensitivity increases) with increasing values of the eccentricity. However, it can be seen that this variation is small (much less than an order of magnitude) over the entire range of eccentricities. Therefore, for this range of eccentricities, the accuracy of estimating the gravitational coefficients appears to be nearly independent of the eccentricity.

A comparison of curves 1 and 2 in each plot in figure 3 shows that for most of the gravitational coefficients, the difference between these two curves increases monotonically with increasing values of the eccentricity. This monotonic increase is as it should be, and although the variation of the difference between the two curves is small, it appears that model error effects are somewhat dependent on eccentricity.

Inclination.- Five values of inclination (15°, 33°, 45°, 60°, and 85°) were investigated and the solution covariance matrices were calculated for each value. The results are presented in figure 4. It can be seen that there is no unique inclination at which all the gravity coefficients are best determined. The maximum variation of the standard deviations, without consideration of model errors, is slightly larger than one order of magnitude. For the second-degree coefficients, the information content of the tracking data (curve 1, fig. 4) increases with inclination up to about 60°. The low-degree coefficients appear to be determined best at inclinations between 45° and 60°.

The effects of model errors as a function of inclination can also be observed in figure 4. With few exceptions, the variation of the difference between curves 1 and 2 is much less than an order of magnitude. For most of the coefficients, the estimation accuracy decreases with increasing inclination because of model errors. However, even with model errors, it appears that the optimal inclination for estimating the lower degree coefficients is between 45° and 60°. Model error effects for the third-, fourth-, and fifth-degree sectorials appear to be the most dependent upon inclination. The four sectorial coefficients $C_{3,3}$, $S_{3,3}$, $C_{4,4}$, and $S_{4,4}$ have near-monotonic increases in standard deviations with increasing inclination because of model errors. It appears that the largest model error effects associated with changes in the inclination occur with the higher degree coefficients.

Nodal position.- In general, it might be expected that the accuracy of estimating the gravitational coefficients would be affected by the spacecraft orbit nodal position since this parameter positions the orbit relative to the earth-based tracking stations. For example, the Doppler shift is a maximum when the spacecraft and tracking-station relative position vector lies in the spacecraft orbital plane and a minimum when this vector is normal to the orbital plane. The effects of nodal position on standard deviations for the estimated coefficients are presented in figure 5. The nodal positions shown on the abscissa of figure 5 are equivalent to the initial nodal position at the beginning of the tracking interval. It can be seen that the standard deviations were not significantly dependent upon nodal position.

There are no major variations in the difference between curves 1 and 2 in each plot in figure 5 for any of the coefficients. This indicates that model error effects are also independent of nodal position. Curve 2 for several of the coefficients contains an abrupt change in slope at about 315°. These abrupt changes are due to the strong coupling of certain of the estimated coefficients with certain of the nonestimated coefficients at a particular nodal position.

Inclusion of Mariner '71 Tracking Data

The Mariner '71 tracking data are currently being analyzed for determining the Mars gravitational field (ref. 2), and it is of interest to know if these tracking data can be combined with those from the Viking orbiter to significantly improve the overall estimates of the gravitational coefficients. To answer this question, it was assumed that 60 orbits (approximately 30 days) of Mariner '71 tracking data would be combined with the tracking data taken from the Viking orbiter for several different tracking data arc lengths. The results are presented in figure 6, and it should be noted that because of software limitations, the solution set did not include the six Cartesian state variables as in previous cases. There are four curves presented in each plot of figure 6. Curves 1 and 3 represent the standard deviations obtained assuming a priori information but no model errors, whereas curves 2 and 4 assume both a priori information and model errors. Another distinction between the curves is that curves 1 and 2 represent the standard deviations assuming a combination of Viking and Mariner data, whereas curves 3 and 4 assume Viking data only. The abcissa represents the number of Viking orbits assumed to have been tracked, and for the two curves containing Mariner data, the assumption is that at each point 60 orbits of Mariner data have been combined (by adding normal matrices) with the given number of Viking orbits.

A direct comparison of curves 1 and 3 reveals a difference between the two curves of approximately one to two orders of magnitude, and in several cases during the first few orbits of tracking, nearly three orders of magnitude. This difference is due entirely to the inclusion of Mariner data and represents a significant increase in accuracy. With the exception of the zonal coefficients, the difference between the two curves decreased as the number of orbits tracked increased. However, it appears that including Mariner '71 tracking data in the parameter estimation process may be a very effective way of increasing the solution estimation accuracy, especially when only short arcs of Viking tracking data are available.

A comparison of curves 2 and 4 shows that except for a few isolated cases such as $C_{2,0}$, $C_{2,2}$, $S_{2,2}$, and $C_{4,3}$, the difference between these two curves is much less than an order of magnitude. This difference is due entirely to the inclusion of Mariner '71 data and in general is not as large as those differences between curves 1 and 3. This is a clear indication that even though one might expect significant accuracy increases when Mariner data are combined with Viking data, this accuracy increase may not be forthcoming in the presence of model errors of the order assumed. Therefore, in order to utilize the Mariner data to the best advantage, it may be necessary to extend the gravitational-coefficient solution set to a higher degree and order.

Estimation of the Mass of Mars

One of the more important physical parameters of any planet is its mass, which is usually denoted in terms of the parameter G_M. In terms of the gravitational spherical harmonic coefficients, any deviation of this parameter from some nominal value can be accounted for through the coefficient $C_{0,0}$. When attempts were made to include $C_{0,0}$

in the solution set, the normal matrix was noninvertible for most of the cases investigated and presented in this report. Hence, all the results presented so far have been obtained by excluding this coefficient from the solution set. However, the normal matrix was not always noninvertible, and it is important to know whether or not the tracking data can be reduced to obtain accurate estimates of $C_{0,0}$.

The results presented in figure 7 were obtained in exactly the same manner as those presented in figure 1, except that $C_{0,0}$ was retained in the solution set. The covariance matrix at the end of nine orbits was not positively definite; therefore the matrix was noninvertible. However, the important thing to note from the results presented in figure 7 is that without the consideration of a priori or model errors, the standard deviation for $C_{0,0}$ has been reduced to the a priori level after approximately seven orbits of tracking and to about one order of magnitude less than the a priori after 27 orbits. These results indicate that the data information content is sufficient to allow for a slight improvement in the current uncertainty in the mass of Mars. However, model error effects on the estimation of $C_{0,0}$ over the time interval considered are such that the expected improvement is significantly reduced. In fact, after approximately 20 orbits of tracking, there is no improvement over current uncertainties, and after 27 orbits of tracking, the improvement is less than a factor of three. Thus, it appears that very little improvement in the mass estimation can be expected from the Doppler data.

CONCLUSIONS

The error covariance matrices associated with a weighted least-squares differential correction process have been analyzed for the accuracy of determining the coefficients of the spherical harmonics in the expansion of the Mars gravitational potential function. The use of a priori information has been included and unmodeled error sources have been treated by using the concept of consider parameters.

The parametric analysis presented in this report was made to determine the effects of tracking data arc length, model errors, a priori information, semimajor axis, eccentricity, inclination, nodal position, and the inclusion of Mariner '71 data on the accuracy of estimating the Mars gravitational field. The following results were noted:

1. With no model errors, there appears to be sufficient signature in the Doppler tracking data for improving current uncertainties in most of the Mars gravitational coefficients through degree and order five. However, model error effects are such that this improvement is limited to about an order of magnitude for the second- and third-degree coefficients. Estimates of the fourth-degree coefficients can be improved slightly, whereas those for the fifth-degree coefficients show virtually no improvement.

2. The gravity information content of the tracking data occurs mainly in the first 10 orbits. However, the standard deviations for most of the coefficients decrease faster than the $\sqrt{N}$ law (where N is the total number of observations) up to about 20 orbits.

3. Over the range of semimajor axes investigated (17 300 km to 22 700 km) the accuracy of estimating the gravitational coefficients varied by at most one order of magnitude except for the two fifth-degree sectorial coefficients, which had slightly larger variations. Thus, over this range, the semimajor axis does not have a very large effect on the accuracy. Model error effects on several of the gravitational coefficients were also found to be dependent upon the semimajor axis.

4. The accuracy of estimating the gravitational coefficients was independent of eccentricity over the small range of eccentricities investigated. However, model error effects were found to be slightly dependent on eccentricity.

5. The accuracy of estimating the gravitational coefficients showed a slight dependence on inclination. The variations of the standard deviations were usually less than an order of magnitude. Model error effects were somewhat dependent on inclination, especially for the third-, fourth-, and fifth-degree sectorials, but usually these effects were less than one order of magnitude.

6. The estimation accuracy of the gravitational coefficients and model error effects were essentially independent of the nodal position.

7. Combining Mariner '71 Doppler tracking data with the Viking tracking data appears to be an effective way of increasing the gravitational-coefficient estimation accuracy, especially for short arcs of Viking tracking data; however, model error effects are such that these expected increases may not be realized unless the gravitational-coefficient solution set is extended to a higher degree and order.

8. It appears that current estimates of the mass of Mars cannot be significantly improved by using the information contained in the Dopper tracking data from a Viking-type Mars orbiter.

It should be noted that the results presented herein were restricted to a very particular type of orbital geometry. To be specific, the semimajor axis is very large and the eccentricity is very high. Any major dispersions from this orbit will result in very different answers. The choice of orbital geometries investigated was made on the basis of what was considered feasible in the Viking series of spacecraft.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., August 30, 1972.

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TABLE I.- PARAMETER STANDARD DEVIATIONS

	Standard deviation
Solution parameters:	
Orbiter position (each component)	1000 km
Orbiter velocity (each component)	1 km/sec
Model error parameters:	
Tracking-station distance off spin axis	0.0015 km
Tracking-station Z-component	0.025 km
Tracking-station longitude	4.7×10^{-7} rad
Mars ephemeris position (each component)	5 km
Mars ephemeris velocity (each component)	5×10^{-7} km/sec
Astronomical unit	2 km
Mars gravitational constant, G_M	1.43 km^3/sec^2
Doppler tracking data noise	0.001 m/sec

TABLE II.- STANDARD DEVIATIONS OF MARS GRAVITATIONAL FIELD COEFFICIENTS

Solution coefficients, C and S		Standard deviation
n	m	
2	0	3.88×10^{-5}
2	1	2.24×10^{-5}
2	2	1.12×10^{-5}
3	0	2.04×10^{-5}
3	1	8.33×10^{-6}
3	2	2.64×10^{-6}
3	3	1.08×10^{-6}
4	0	1.30×10^{-5}
4	1	4.12×10^{-6}
4	2	9.71×10^{-7}
4	3	2.59×10^{-7}
4	4	9.17×10^{-8}
5	0	9.21×10^{-6}
5	1	2.38×10^{-6}
5	2	4.50×10^{-7}
5	3	9.18×10^{-8}
5	4	2.16×10^{-8}
5	5	6.84×10^{-9}
Model error coefficients, C and S		**Standard deviation**
n	m	
6	0	6.96×10^{-6}
6	1	1.52×10^{-6}
6	2	2.40×10^{-7}
6	3	4.00×10^{-8}
6	4	7.30×10^{-9}
6	5	1.56×10^{-9}
6	6	4.49×10^{-10}
7	0	5.49×10^{-6}
7	1	1.04×10^{-6}
7	2	1.41×10^{-7}
7	3	2.00×10^{-8}
7	4	3.01×10^{-9}
7	5	5.02×10^{-10}
7	6	9.84×10^{-11}
7	7	2.63×10^{-11}
0	0	3.33×10^{-5}

1 No a priori and no model errors
2 A priori and no model errors
3 A priori and model errors

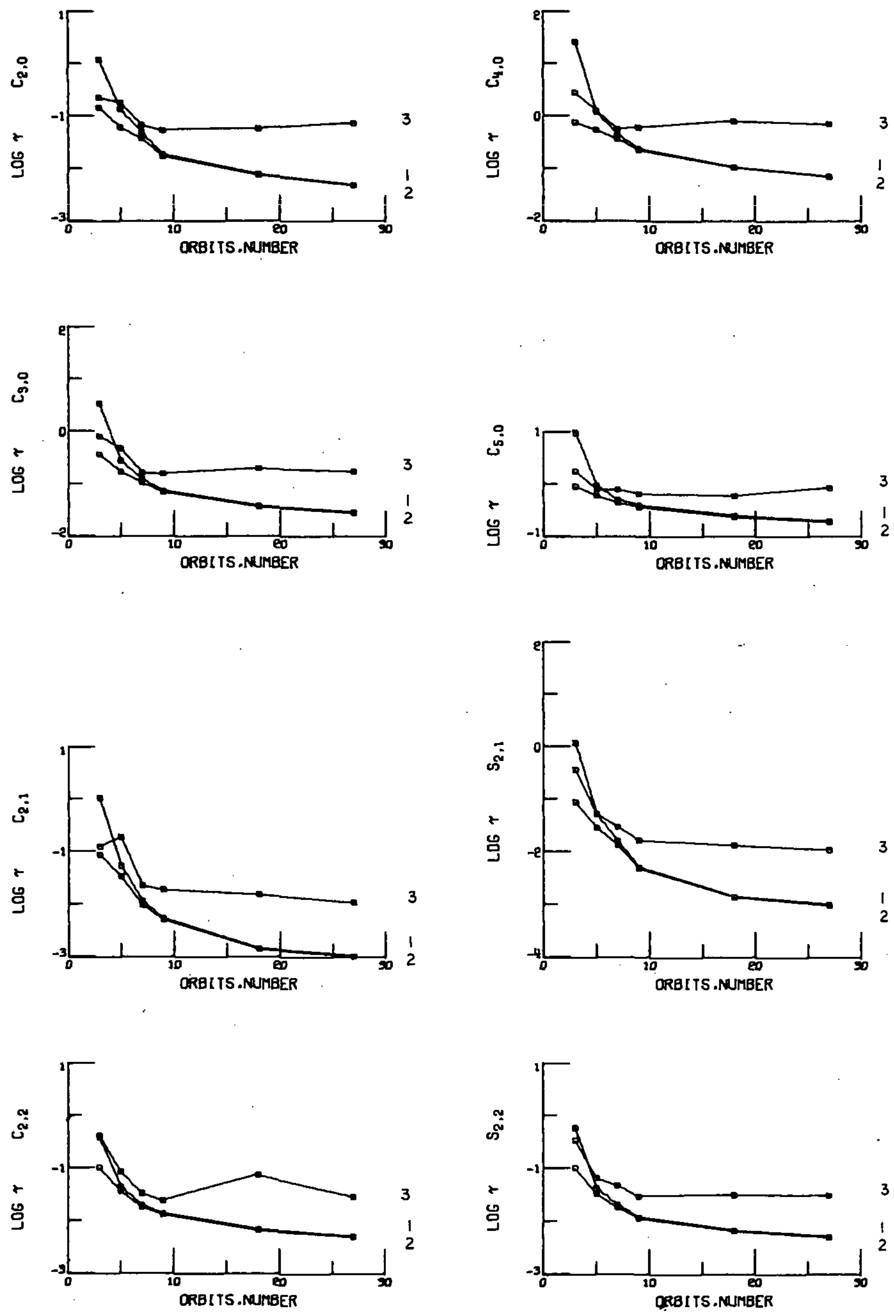


Figure 1.- Variation of standard deviation of Mars gravitational coefficients with number of orbits assumed to have been tracked.

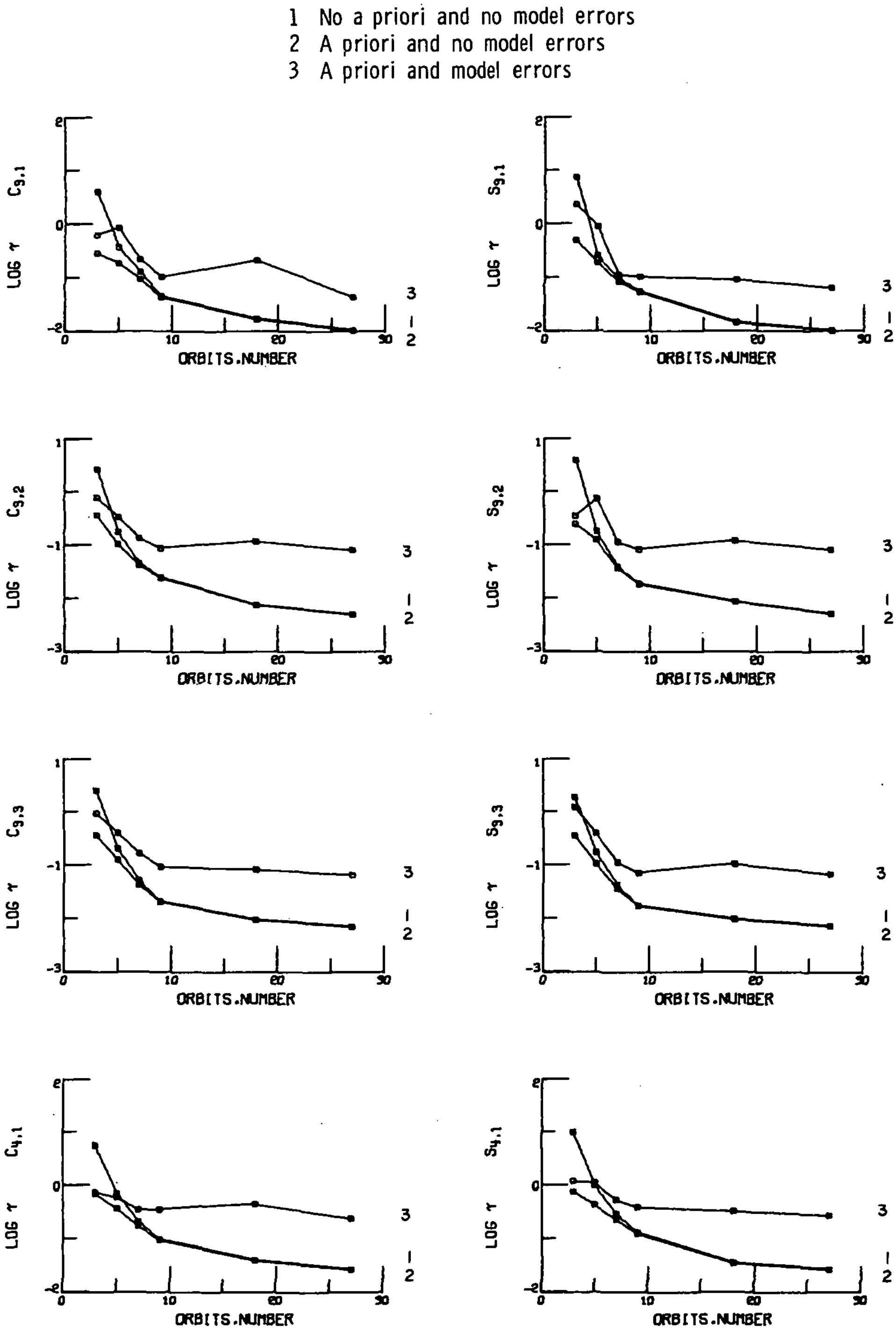


Figure 1.- Continued.

1 No a priori and no model errors
2 A priori and no model errors
3 A priori and model errors

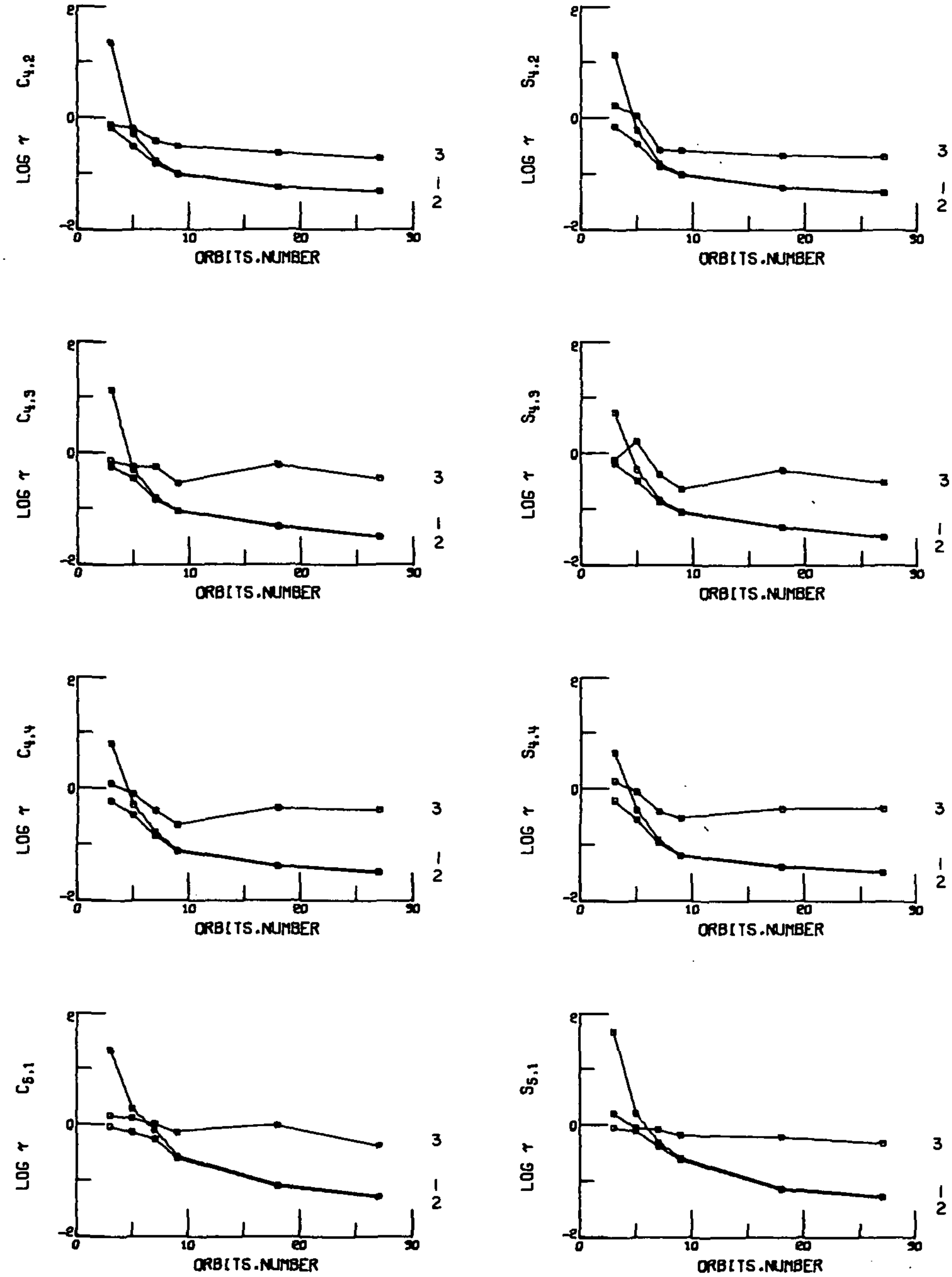


Figure 1.- Continued.

1 No a priori and no model errors
2 A priori and no model errors
3 A priori and model errors

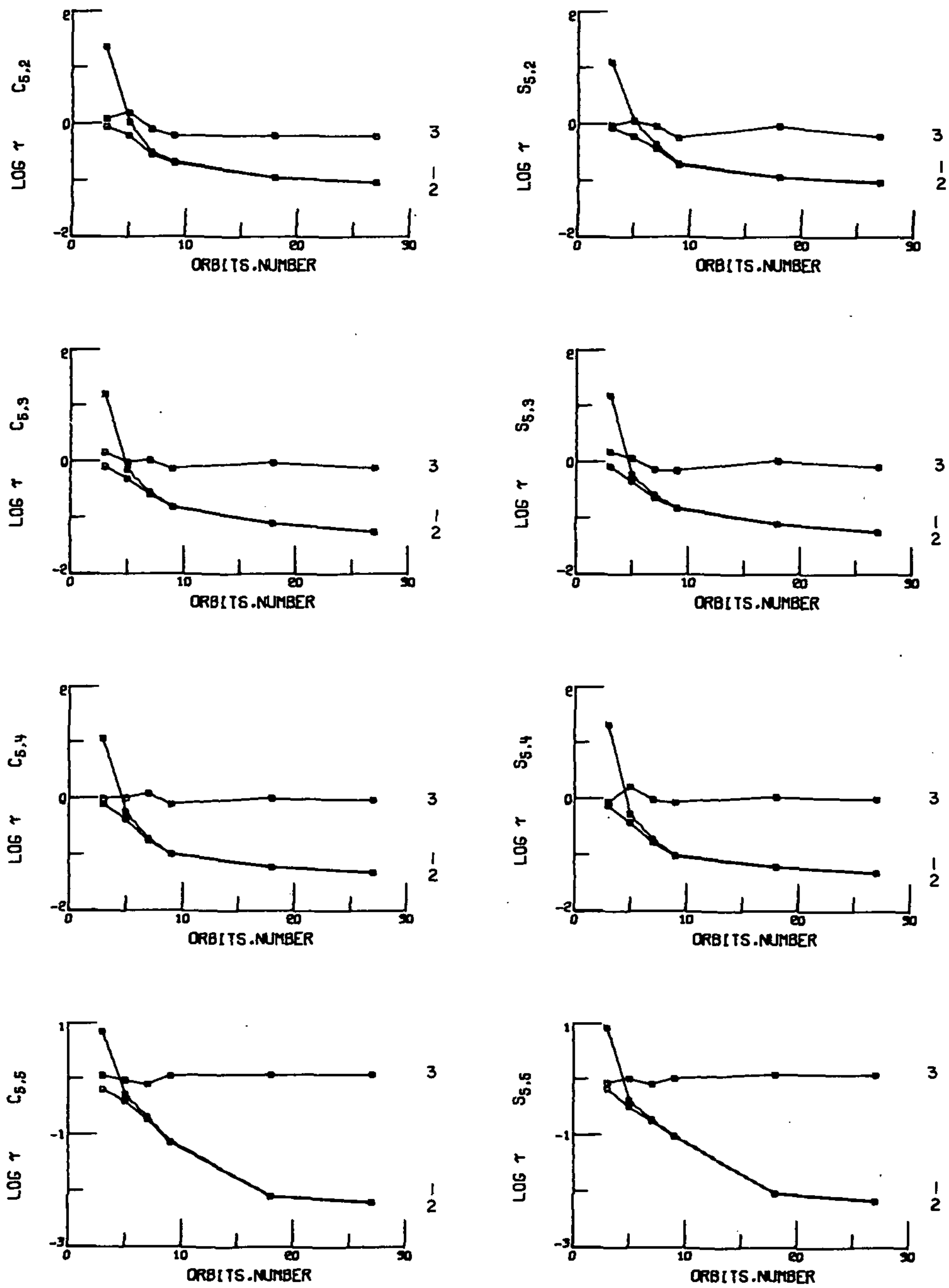


Figure 1.- Concluded.

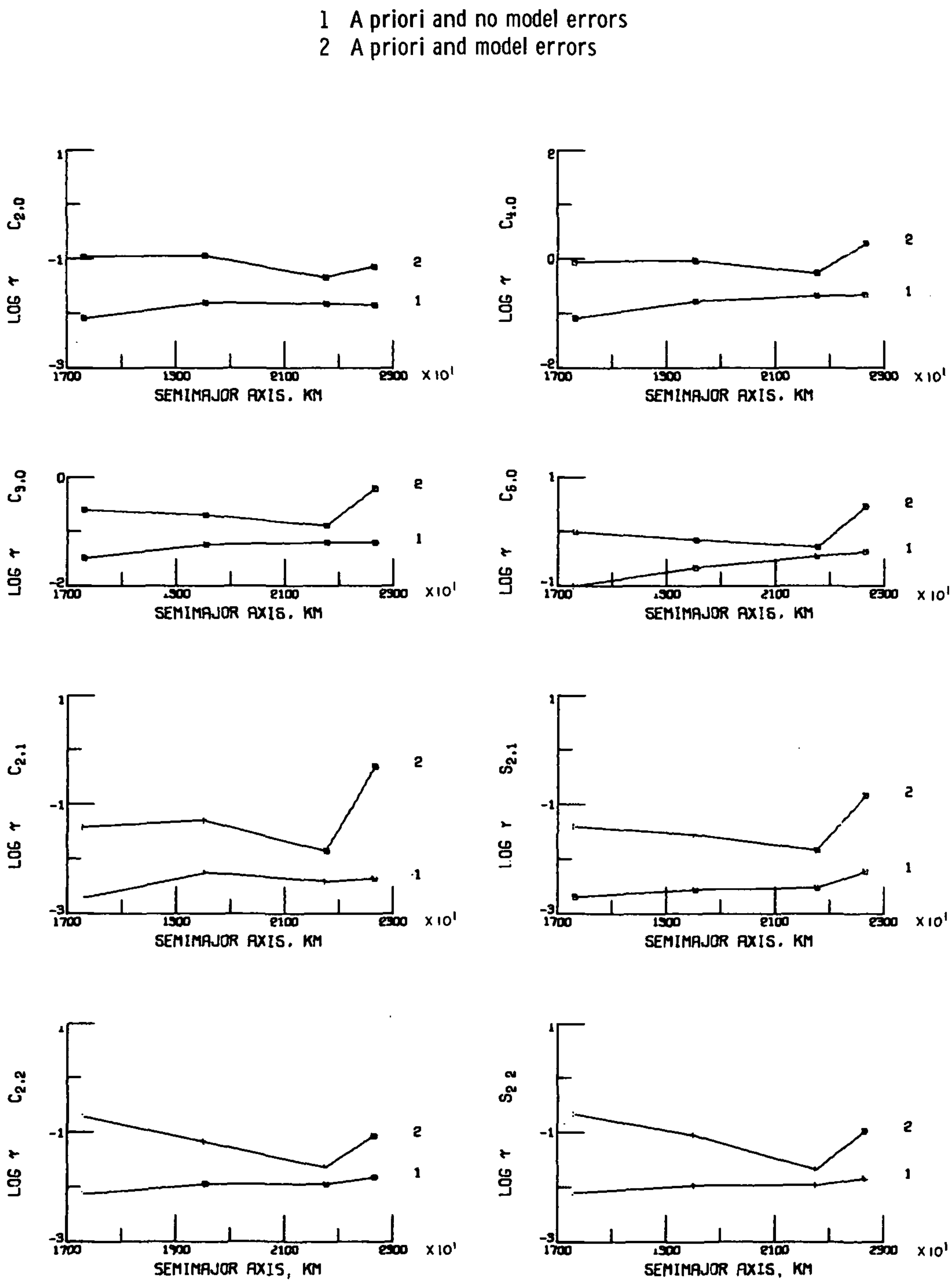


Figure 2.- Variation of standard deviation of Mars gravitational coefficients with semimajor axis.

1 A priori and no model errors
2 A priori and model errors

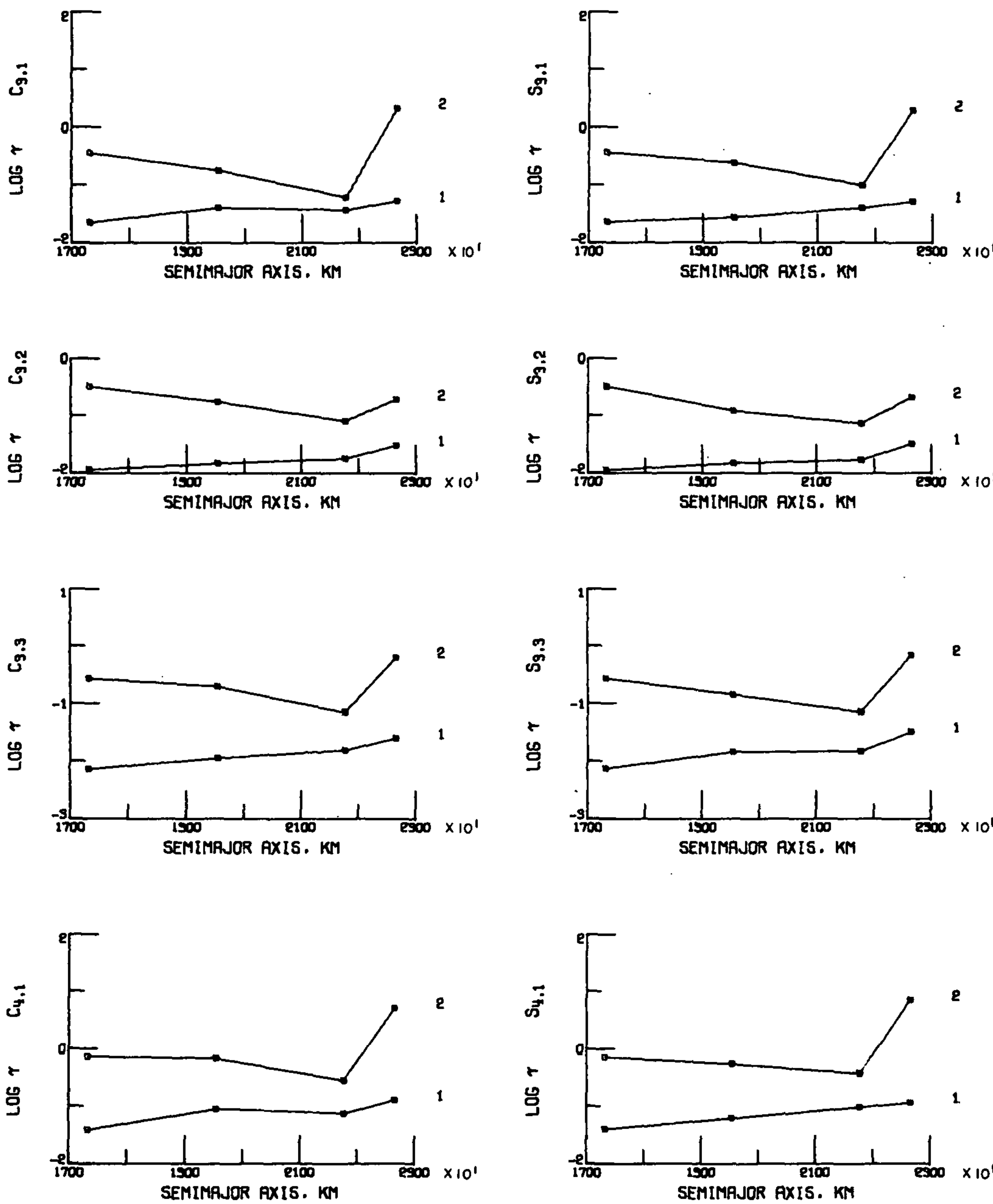


Figure 2.- Continued.

1 A priori and no model errors
2 A priori and model errors

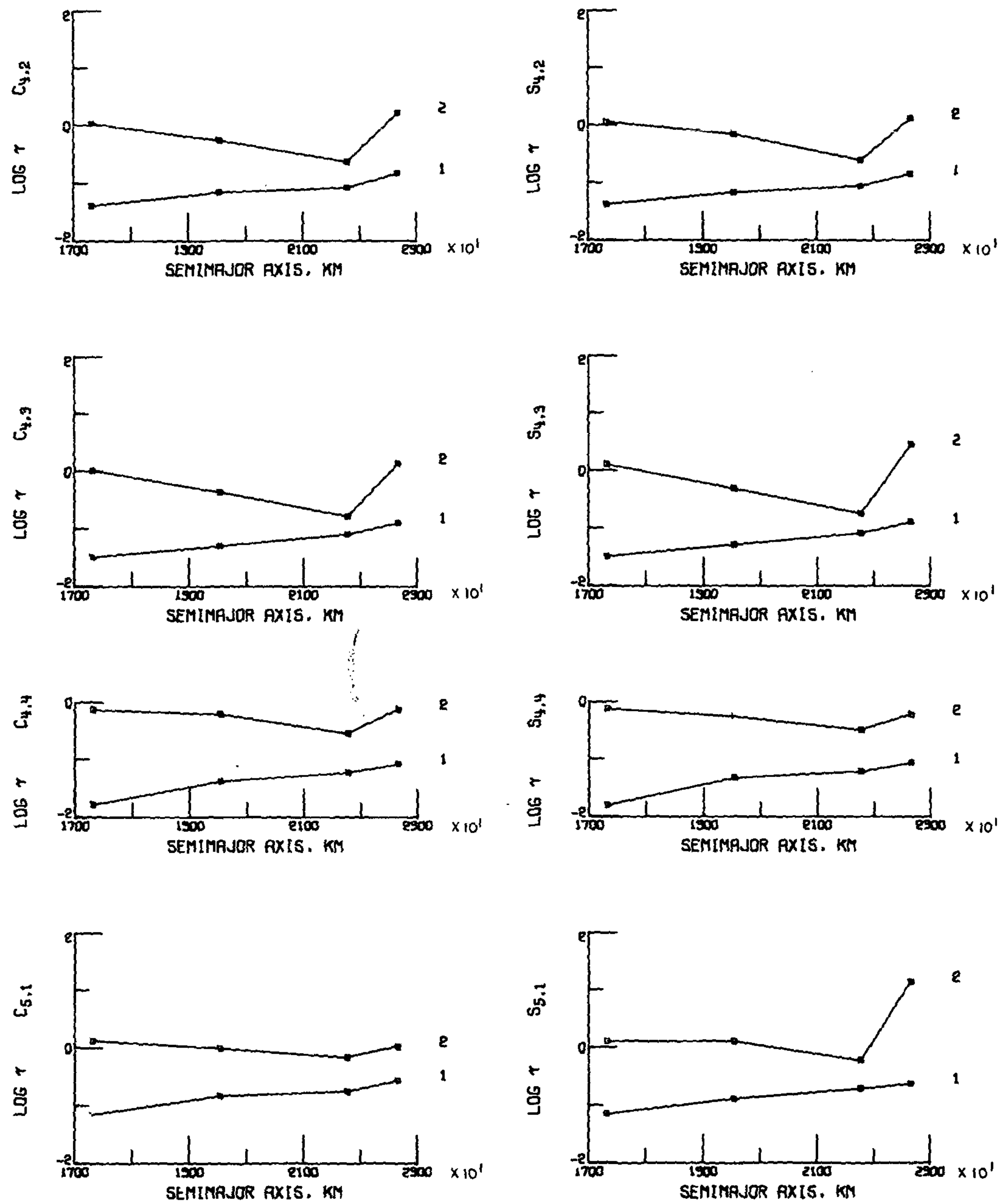
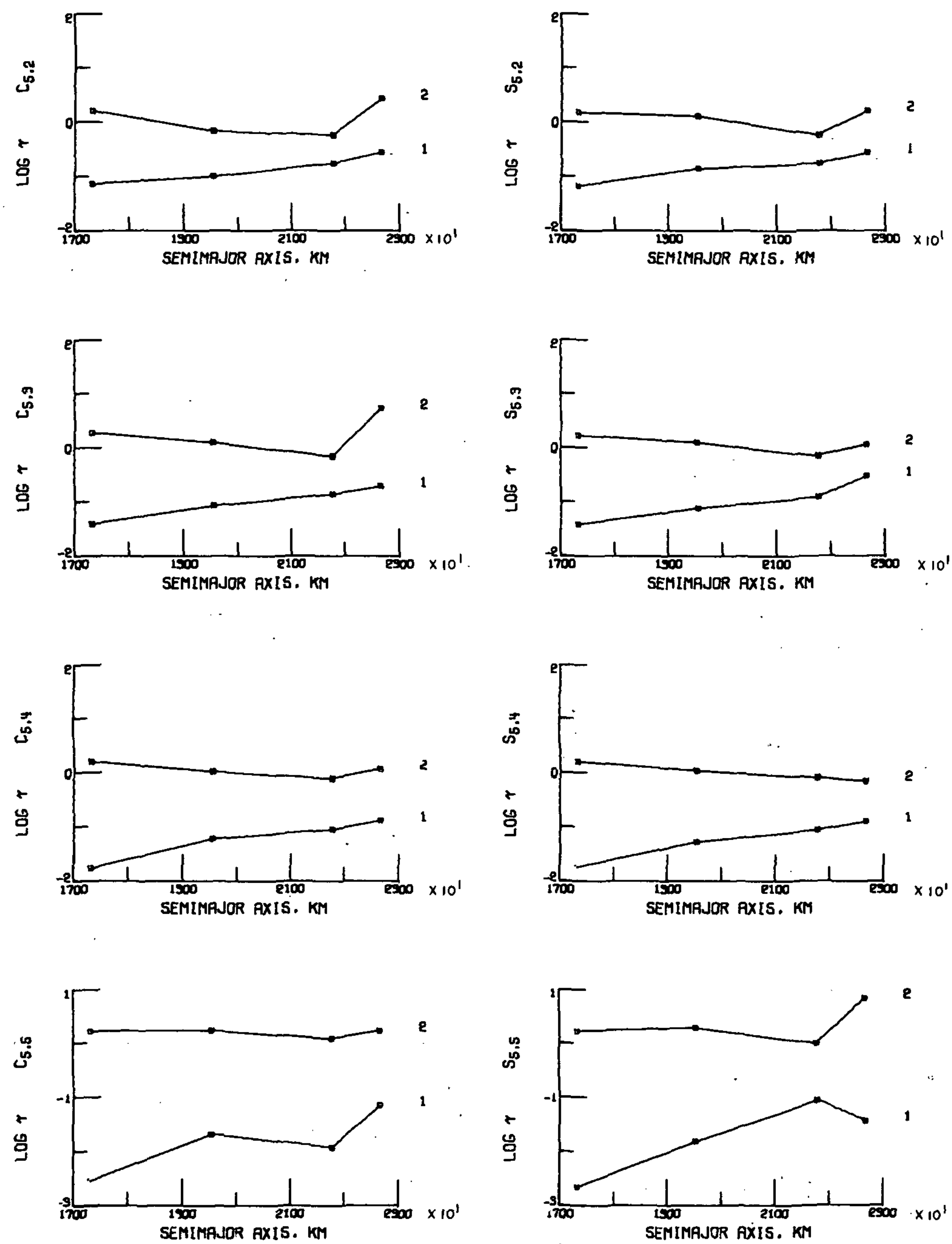


Figure 2.- Continued.

1 A priori and no model errors
2 A priori and model errors



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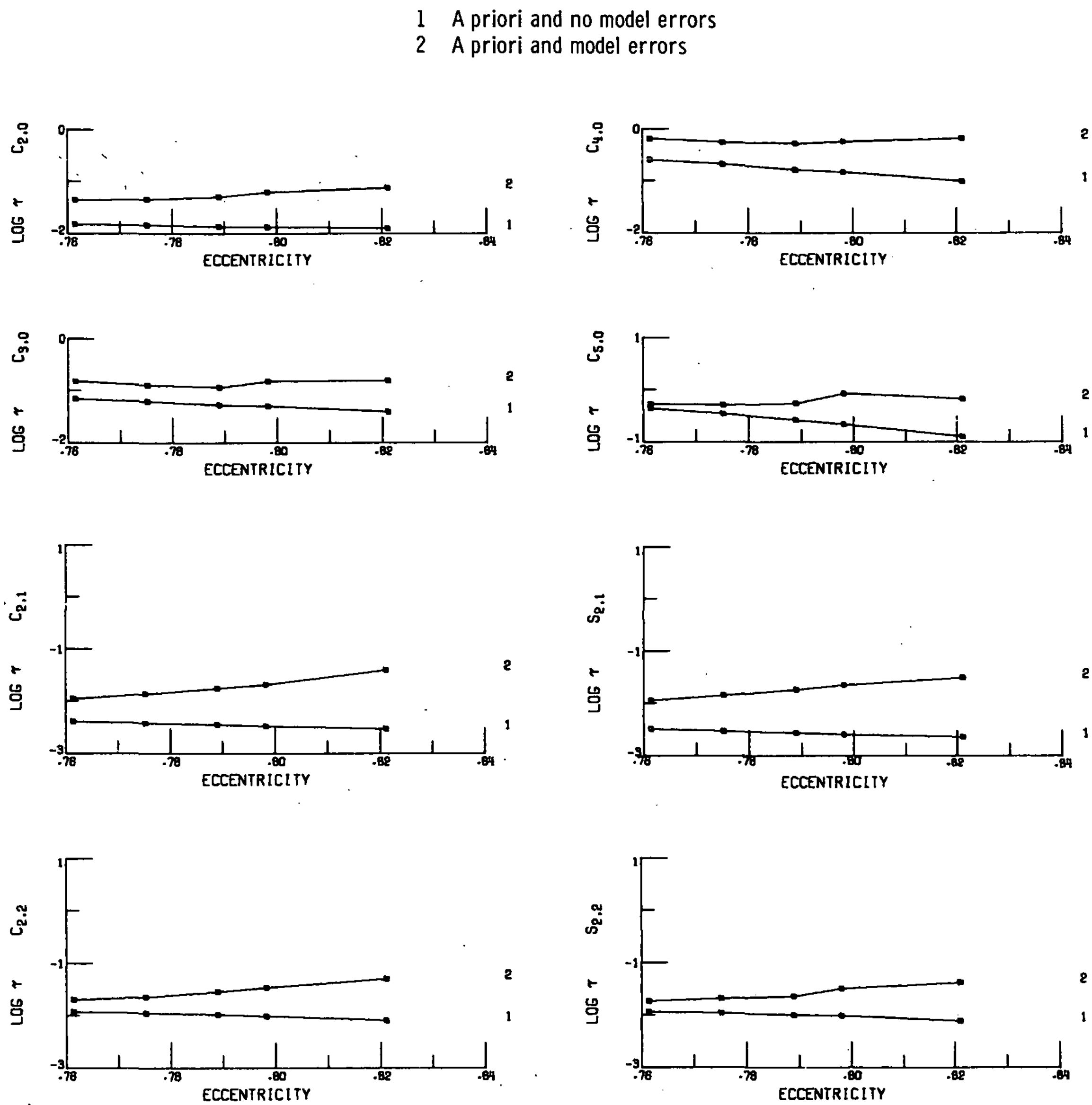


Figure 3.- Variation of standard deviation of Mars gravitational coefficients with eccentricity.

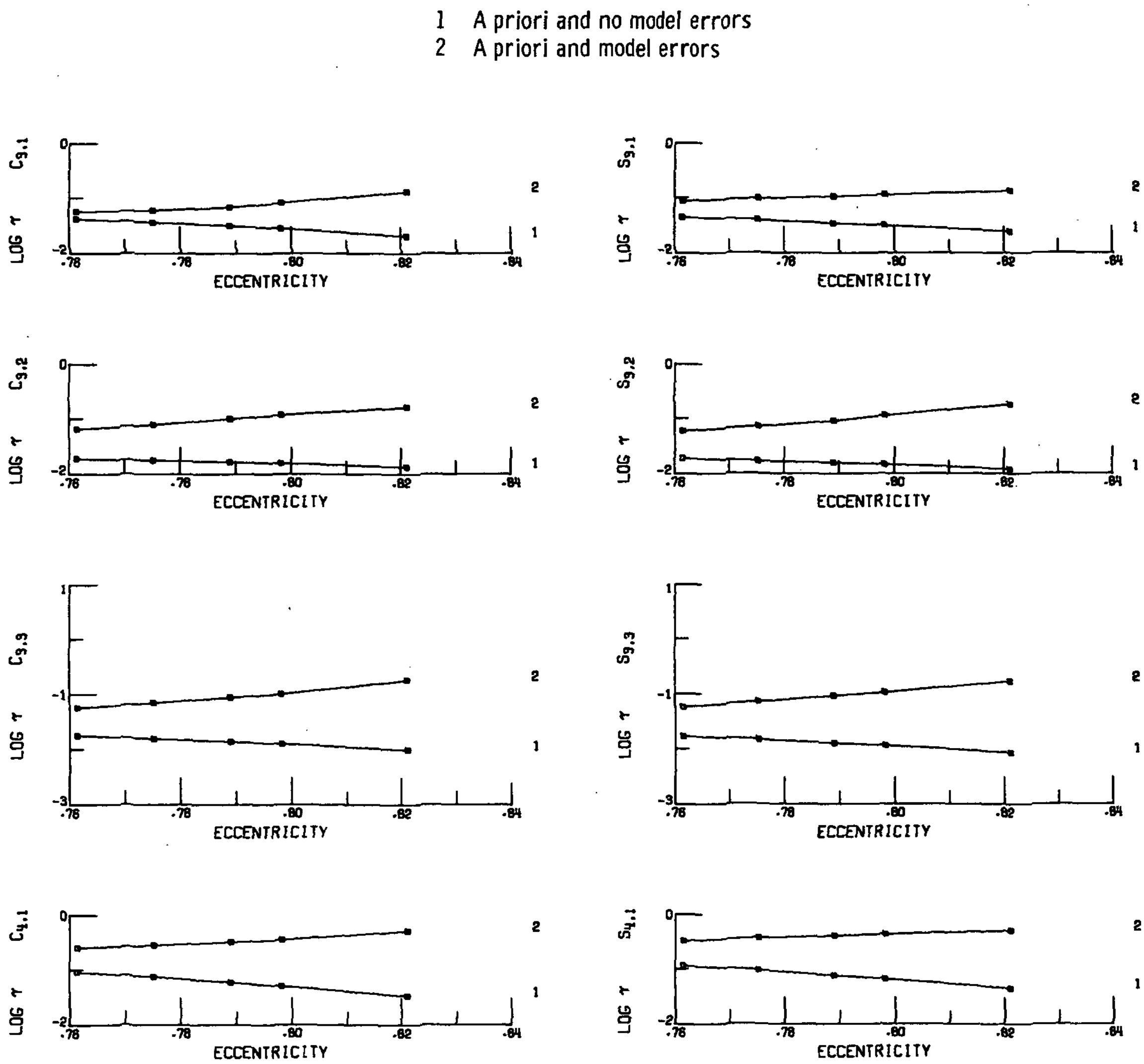


Figure 3.- Continued.

1 A priori and no model errors
2 A priori and model errors

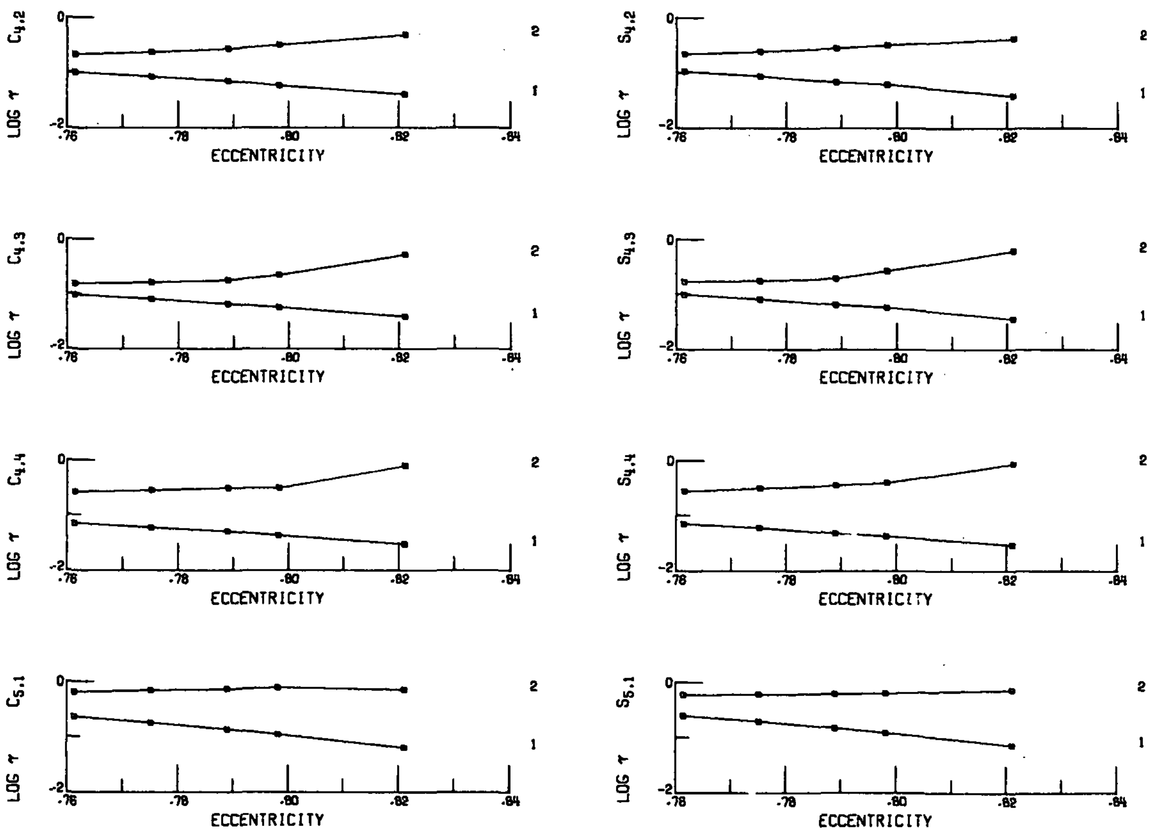


Figure 3.- Continued.

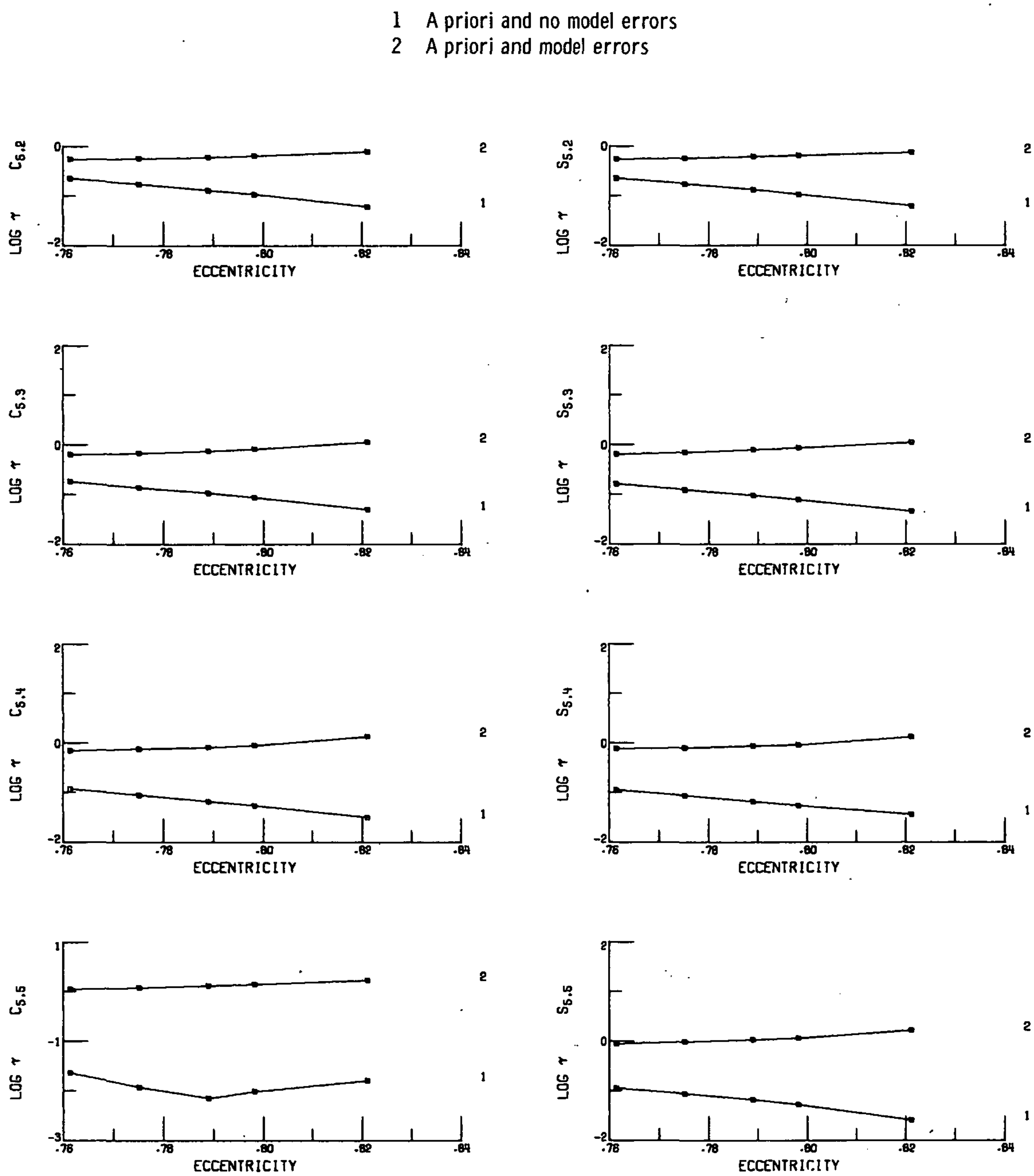


Figure 3.- Concluded.

1 A priori and no model errors
2 A priori and model errors

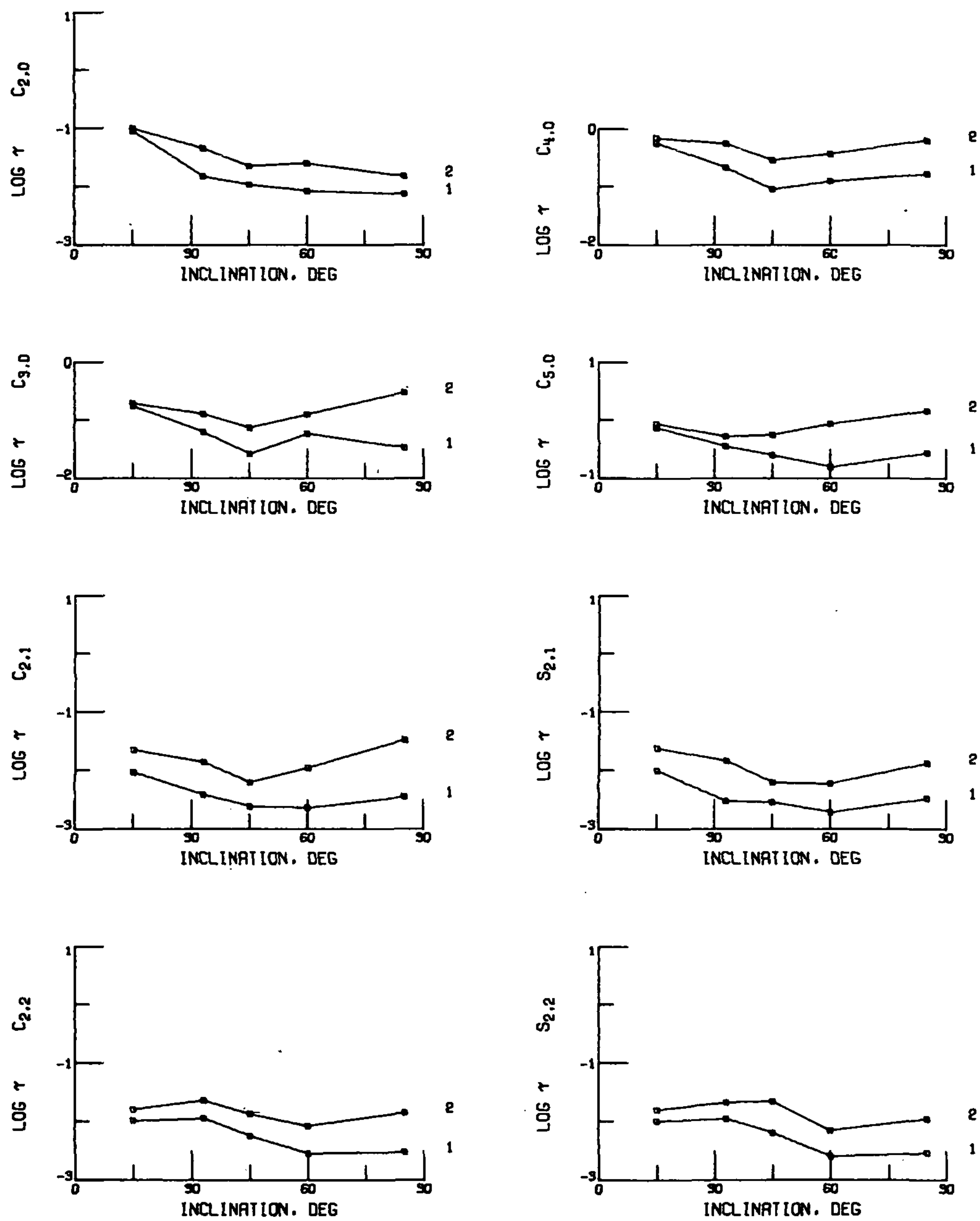


Figure 4.- Variation of standard deviation of Mars gravitational coefficients with inclination.

1 A priori and no model errors
2 A priori and model errors

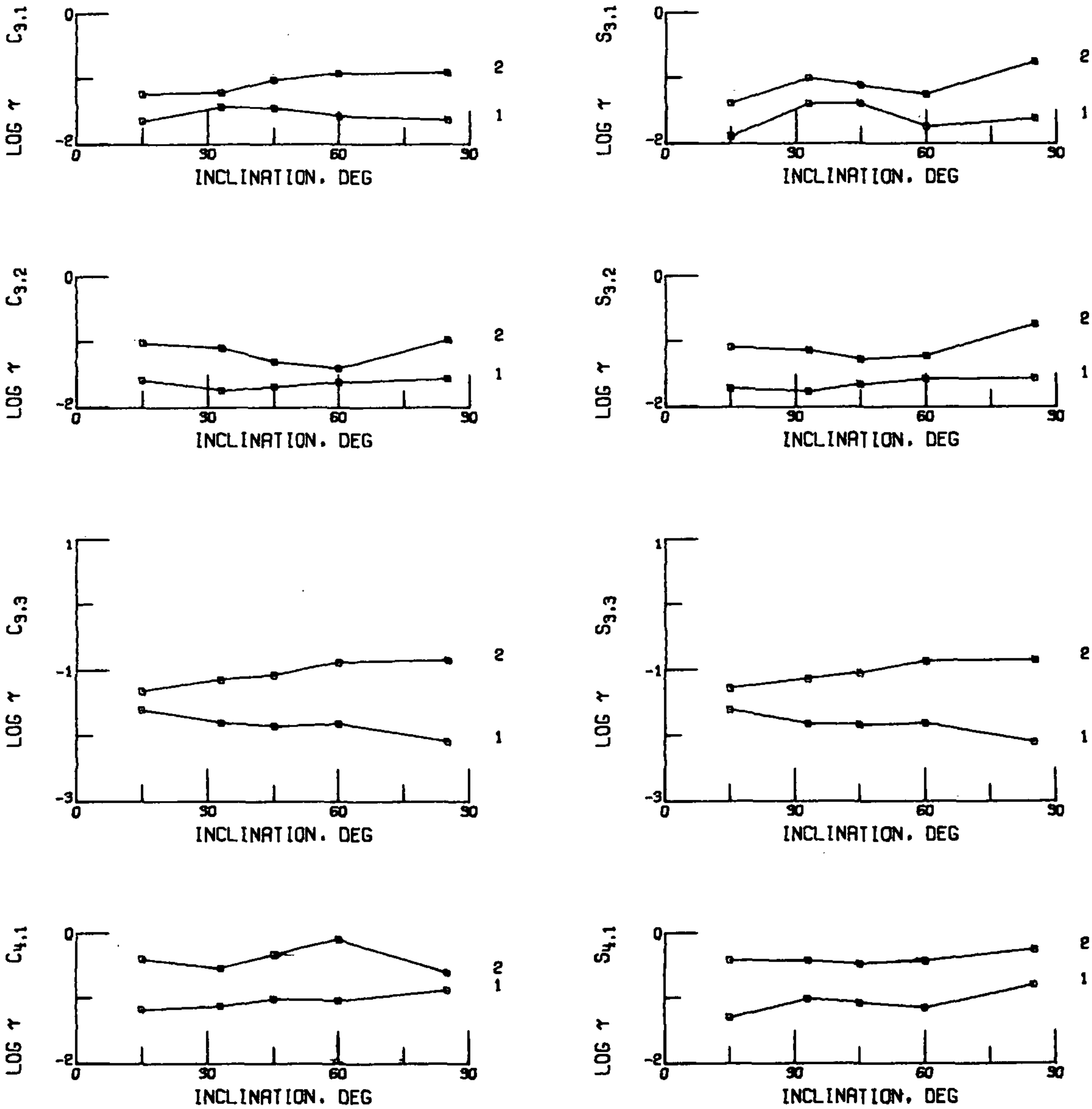


Figure 4.- Continued.

1 A priori and no model errors
2 A priori and model errors

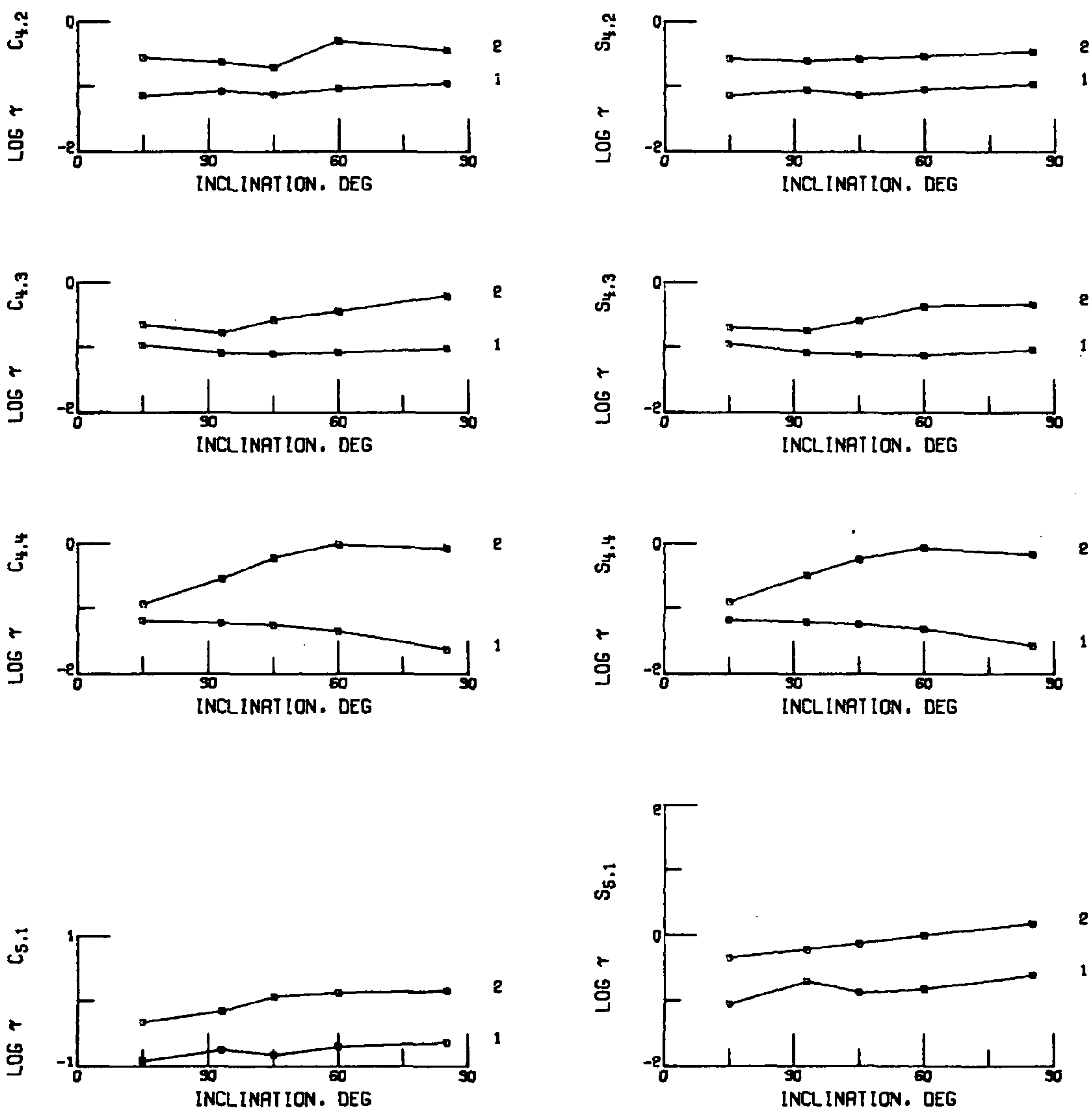
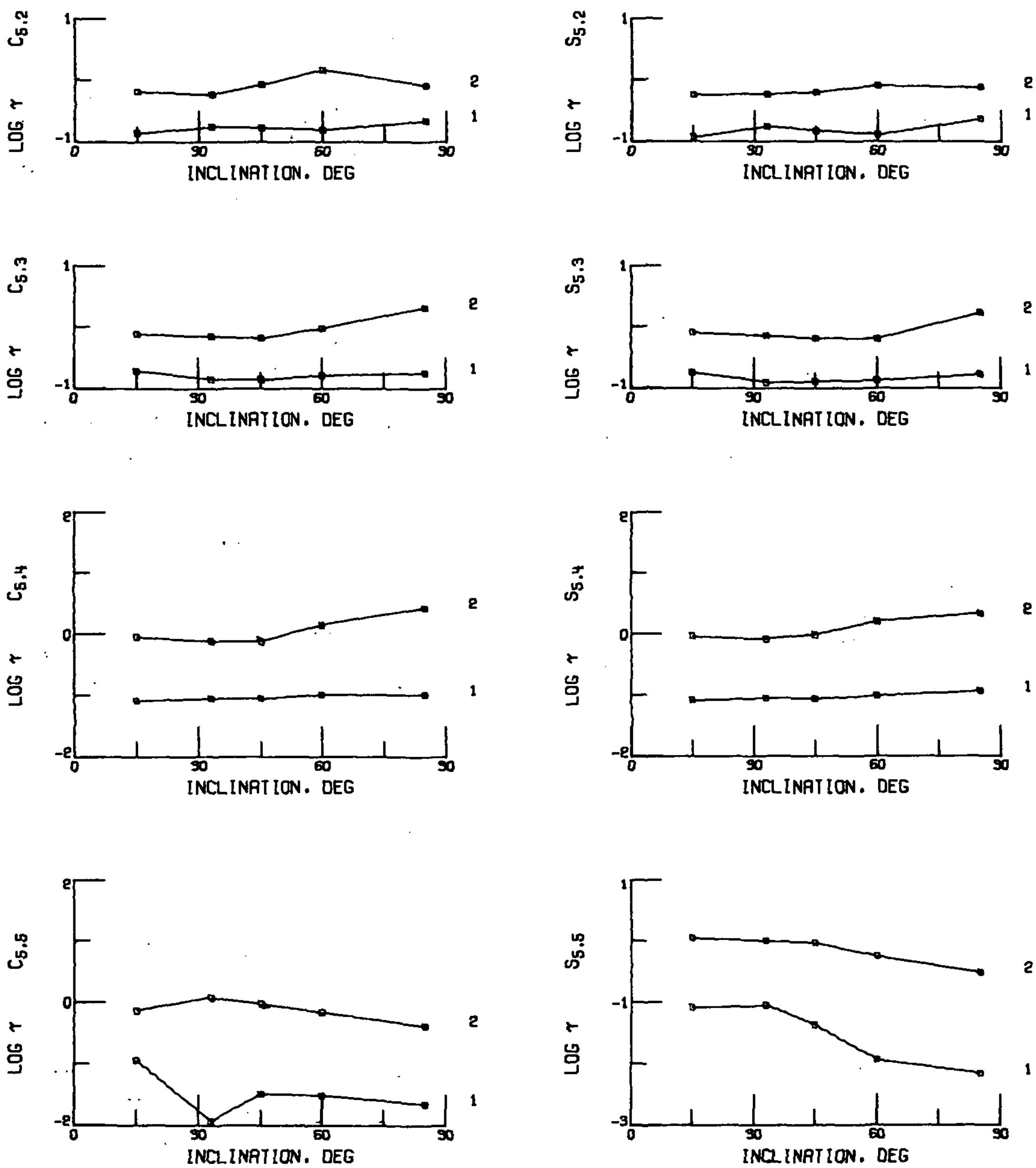


Figure 4.- Continued.

1 A priori and no model errors
2 A priori and model errors



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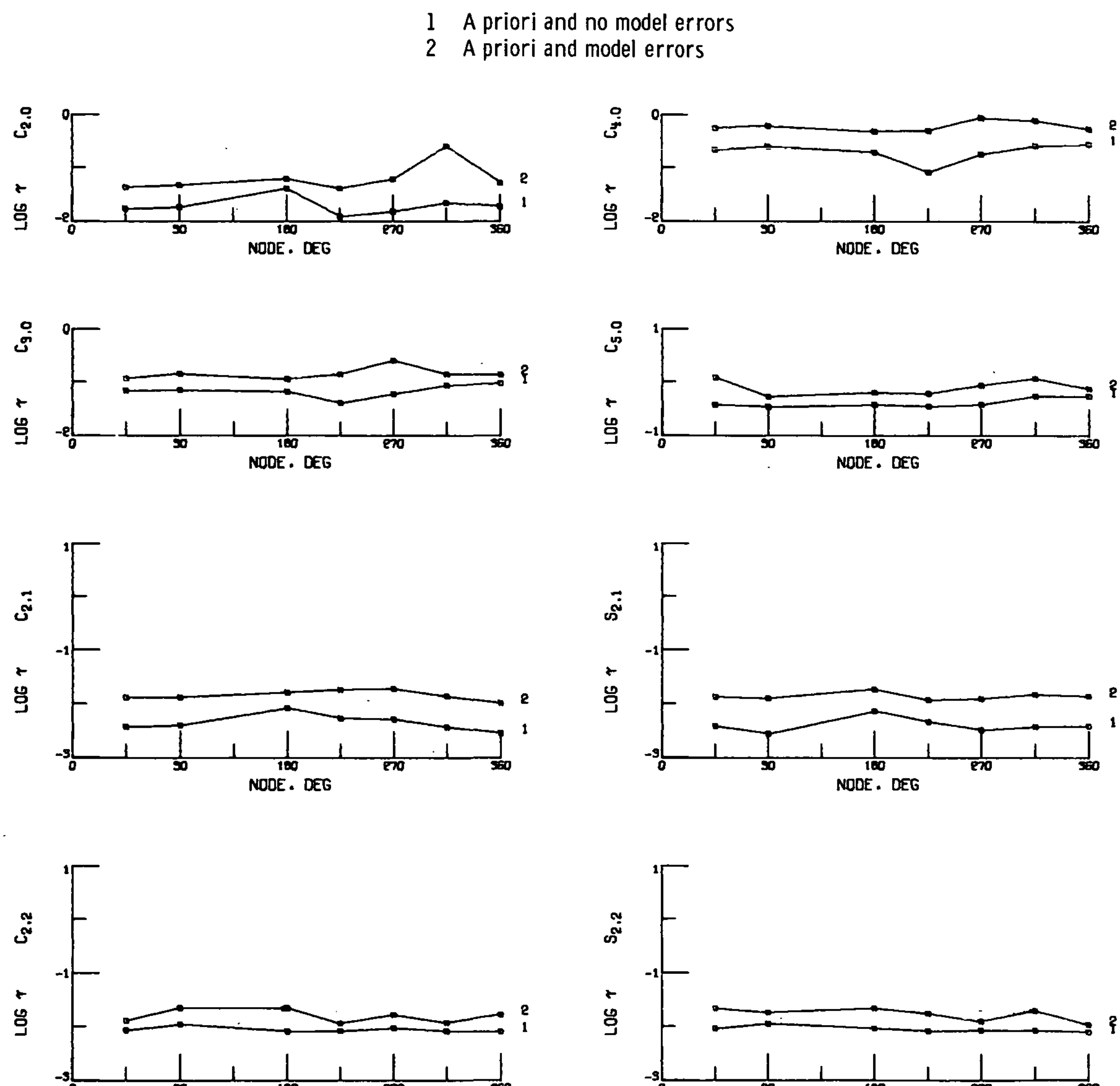


Figure 5.- Variation of standard deviation of Mars gravitational coefficients with node.

1 A priori and no model errors
2 A priori and model errors

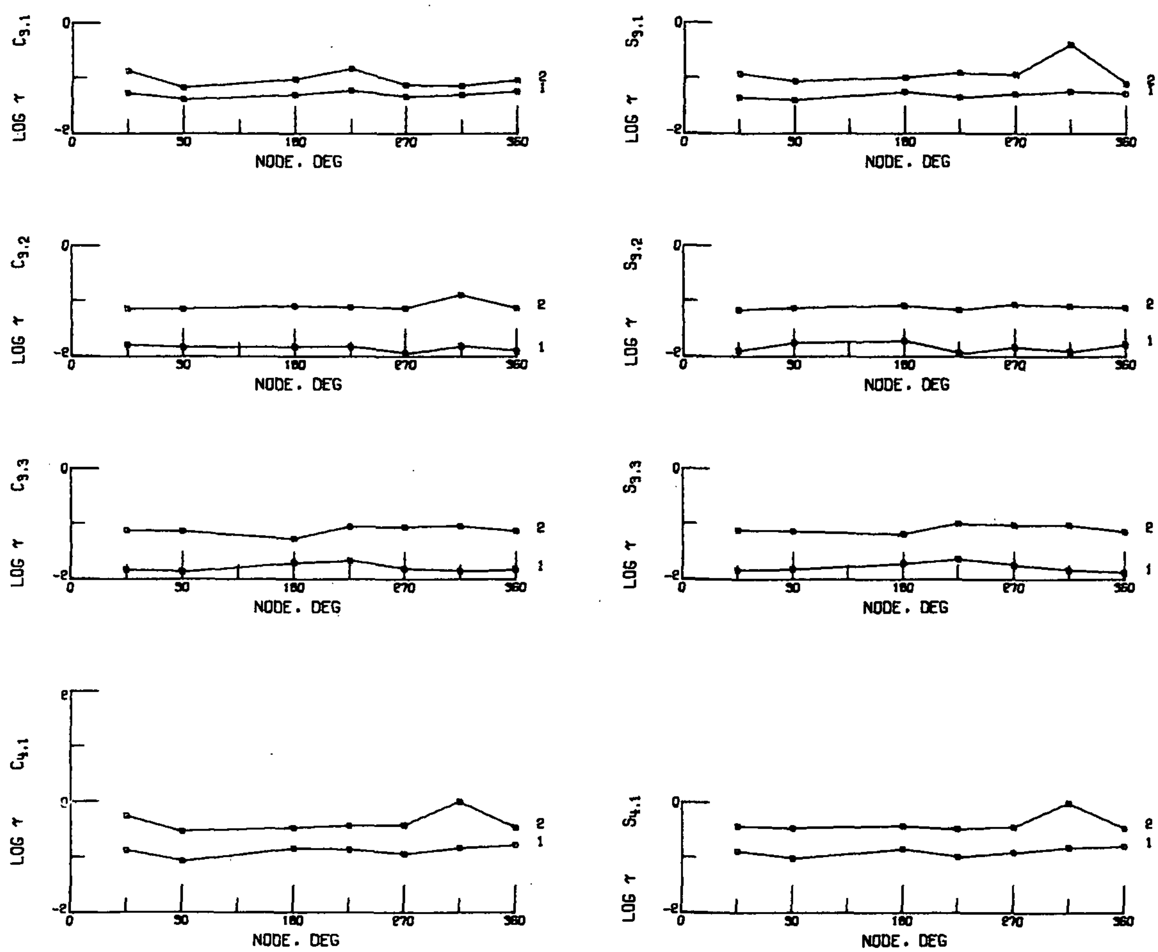
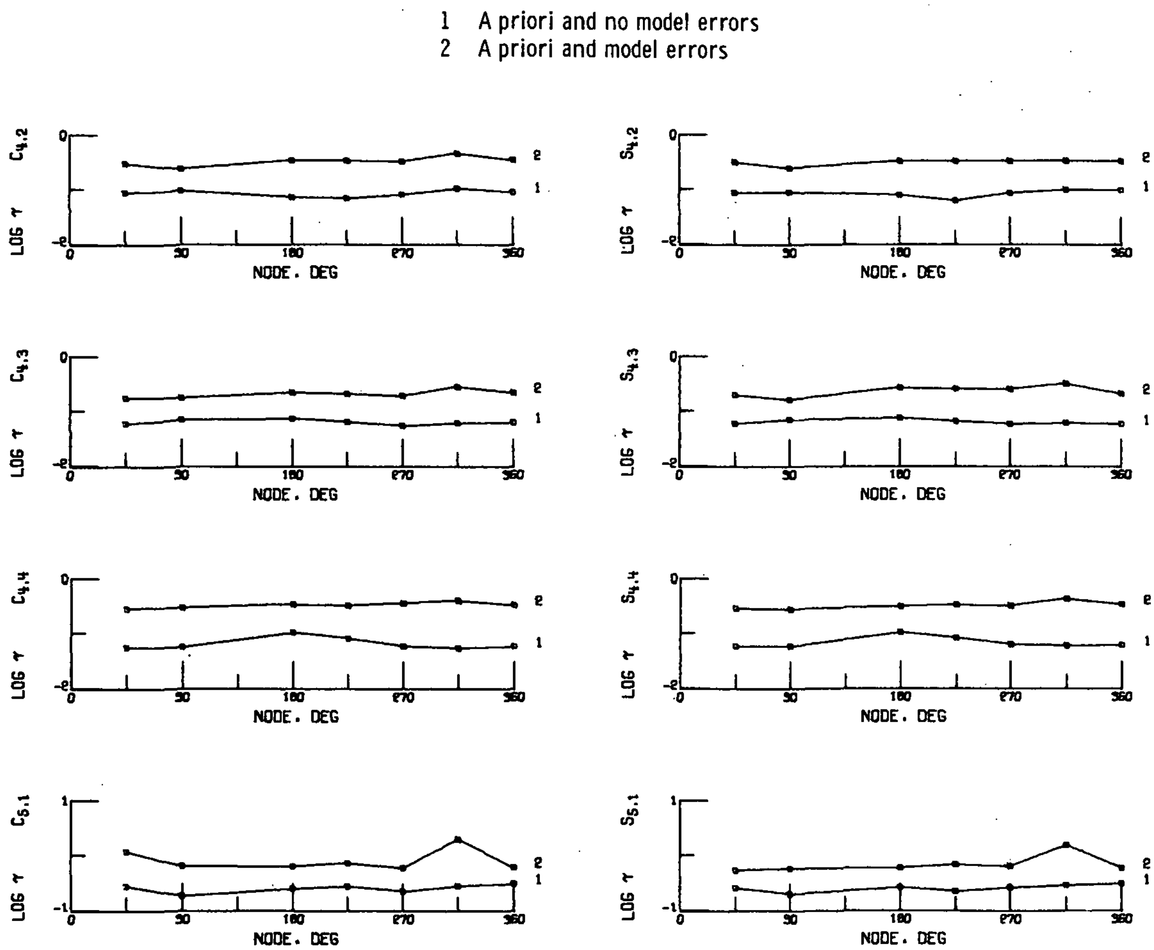


Figure 5.- Continued.



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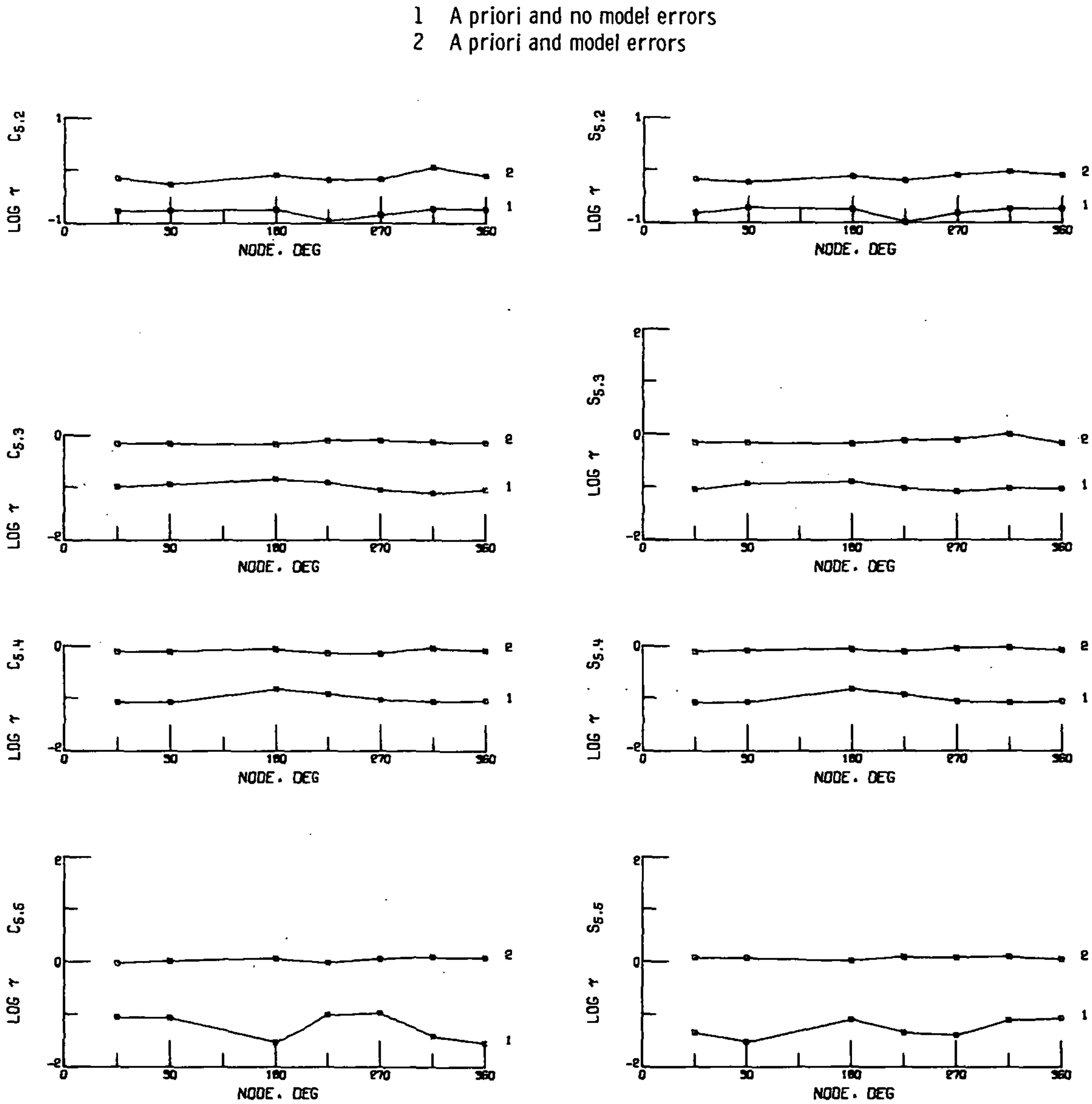

Figure 5.- Concluded.

1 A priori and no model errors (Viking and Mariner data)
2 A priori and model errors (Viking and Mariner data)
3 A priori and no model errors (Viking data)
4 A priori and model errors (Viking data)

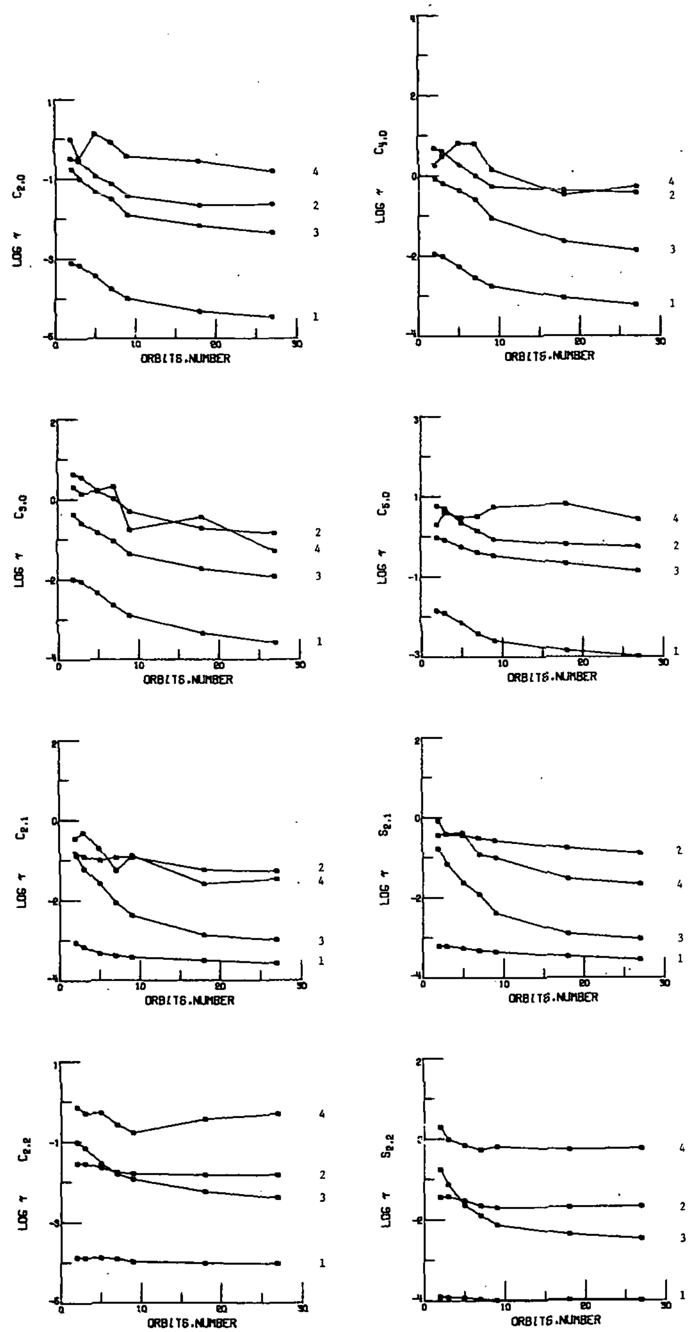

Figure 6.- Variation of standard deviation of Mars gravitational coefficients with and without Mariner data, with number of orbits assumed to have been tracked.

1 A priori and no model errors (Viking and Mariner data)
2 A priori and model errors (Viking and Mariner data)
3 A priori and no model errors (Viking data)
4 A priori and model errors (Viking data)

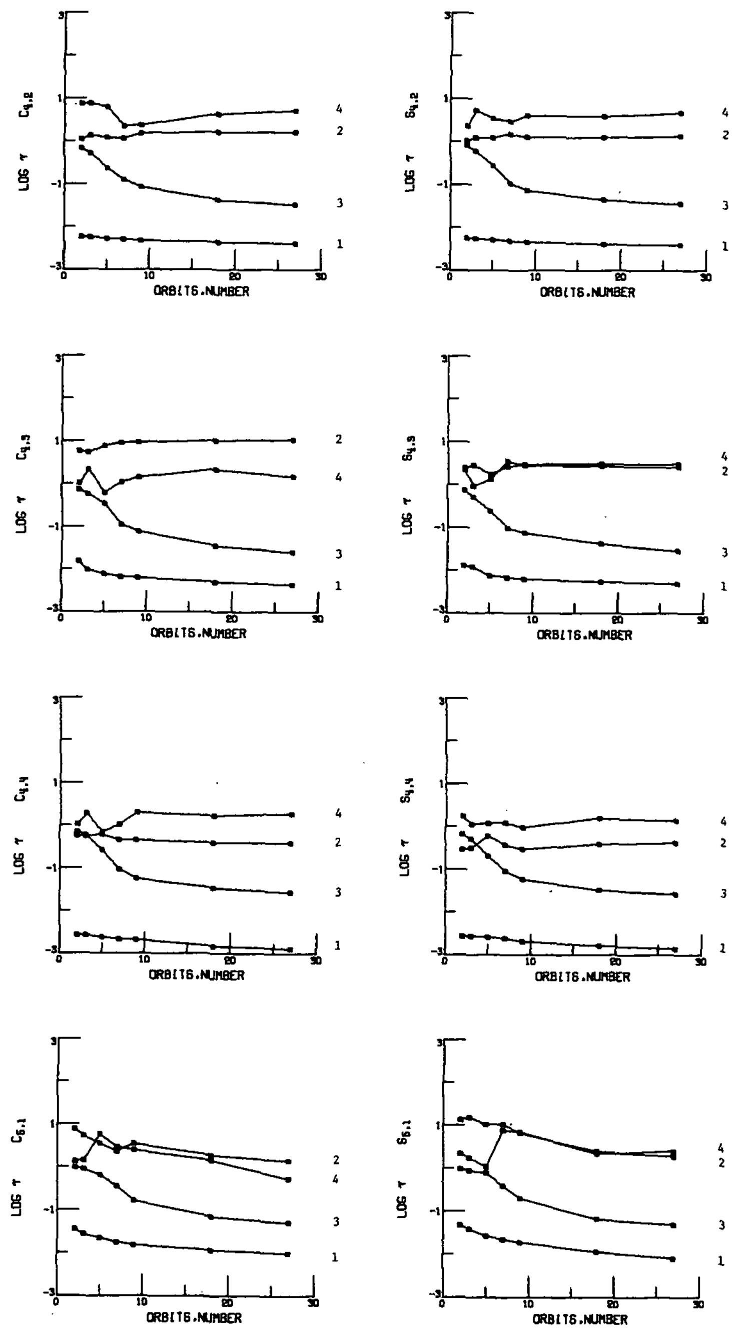

Figure 6.- Continued.

1 A priori and no model errors (Viking and Mariner data)
2 A priori and model errors (Viking and Mariner data)
3 A priori and no model errors (Viking data)
4 A priori and model errors (Viking data)

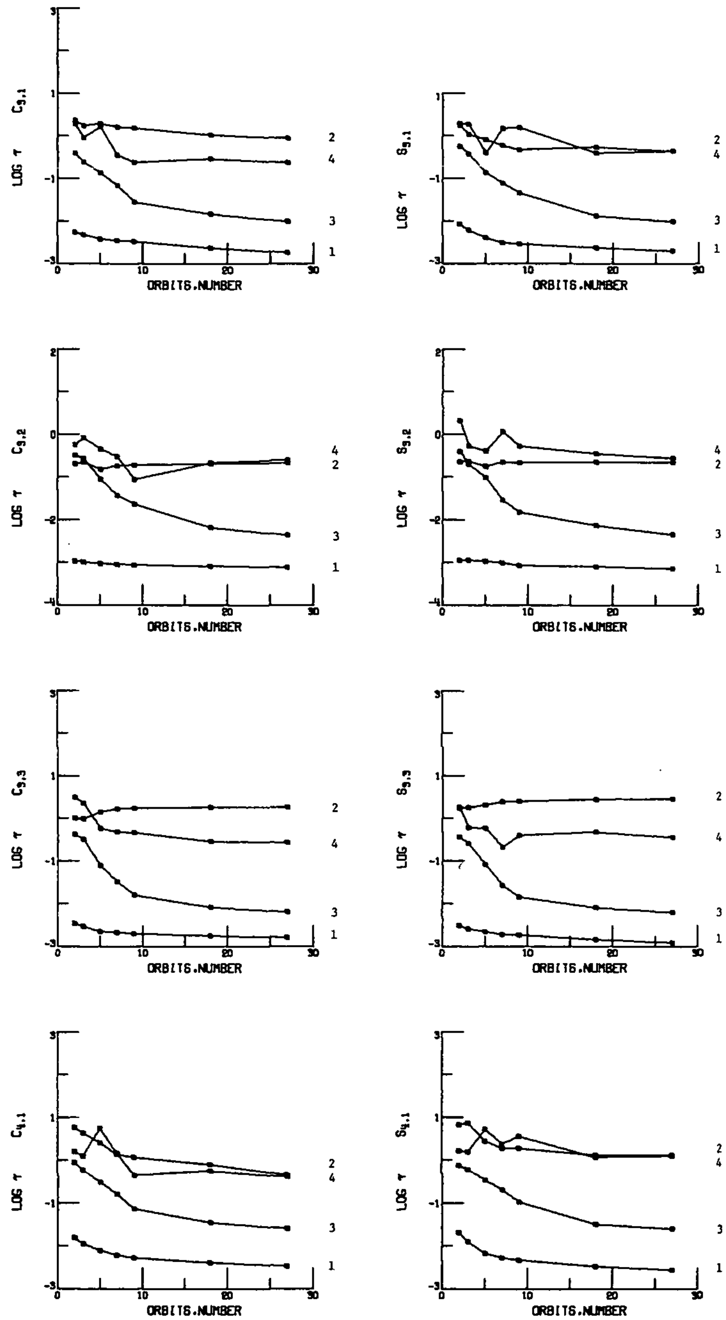

Figure 6.- Continued.

1 A priori and no model errors (Viking and Mariner data)
2 A priori and model errors (Viking and Mariner data)
3 A priori and no model errors (Viking data)
4 A priori and model errors (Viking data)

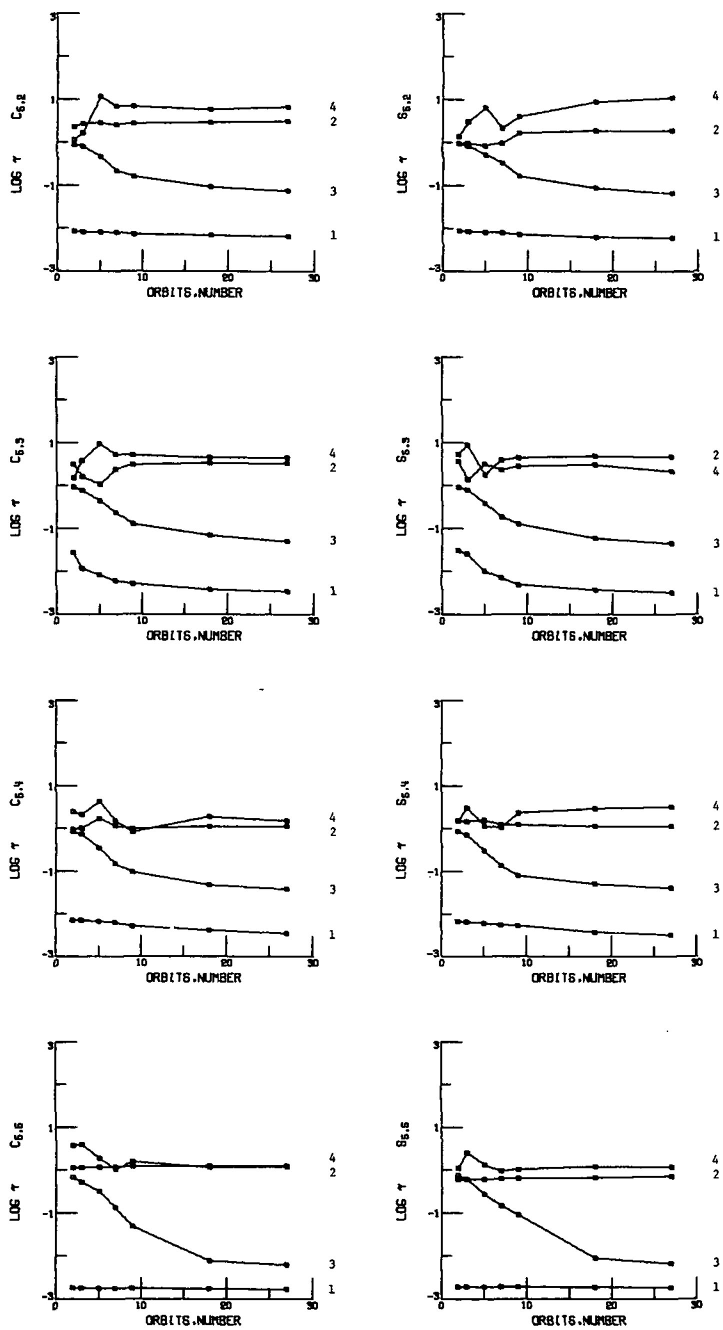

Figure 6.- Concluded.

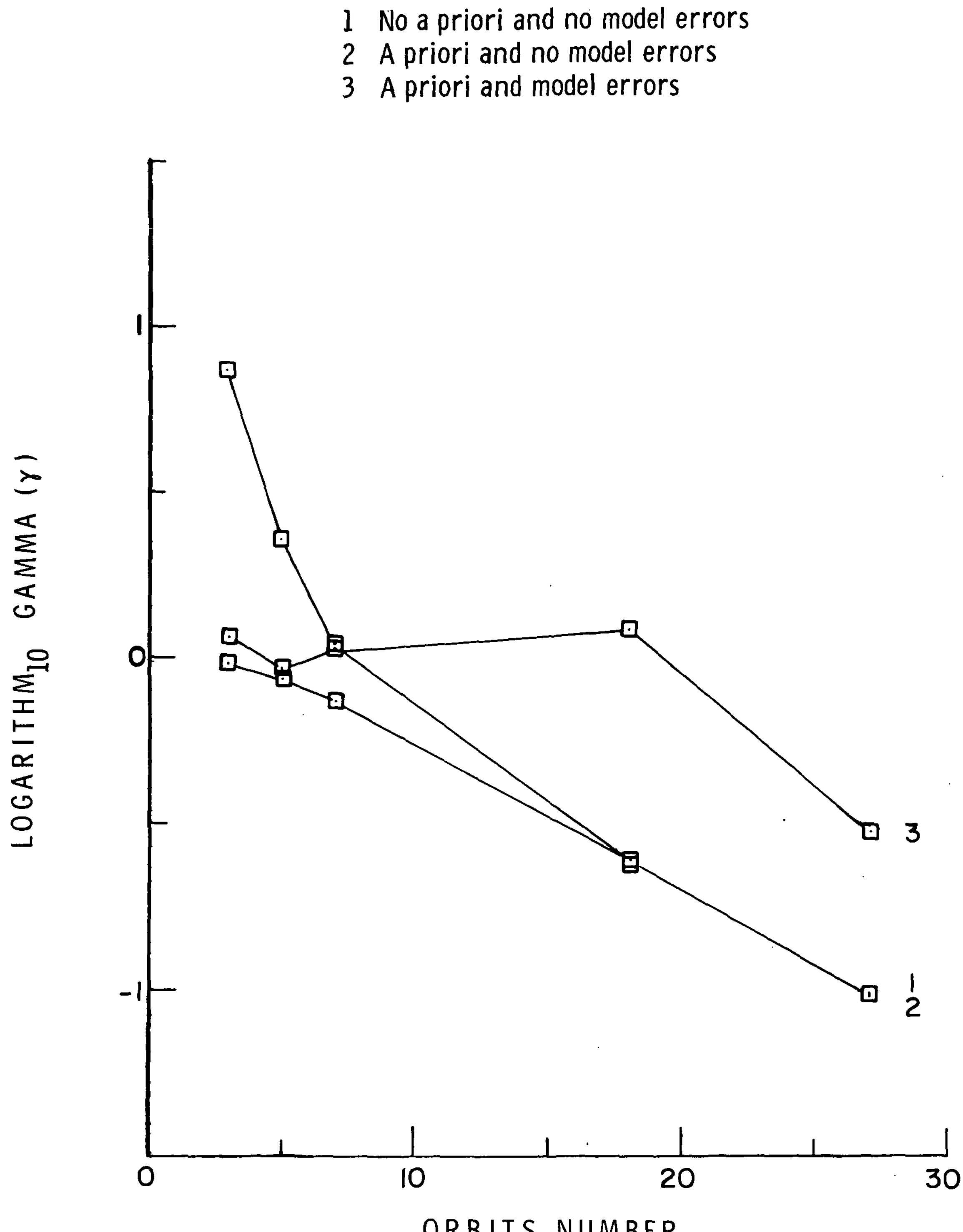

Figure 7.- Variation of standard deviation of mass $(C_{0,0})$ of Mars with number of orbits assumed to have been tracked.

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