APPLICATION OF THE LEADING-EDGE-SUCTION ANALOGY TO PREDICTION OF LONGITUDINAL LOAD DISTRIBUTION AND PITCHING MOMENTS FOR SHARP-EDGED DELTA WINGS

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SUMMARY

The leading-edge-suction analogy of Polhamus has been used to develop the longi-
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be similar in shape to that of the potential-flow longitudinal loading for delta wings having
aspect ratios of 2 or less. The totals of the two theoretical distributions for delta wings
with an aspect ratio near 1 are in good agreement with the experimentally determined
loadings over the angle-of-attack range from 0° to 30°. The corresponding predicted
pitching moments show slightly more stability than those measured, because of loss of
lift near the wing tips.

INTRODUCTION

The nonlinear behavior of lift, drag, and pitching moment with angle of attack is of
considerable interest because of its occurrence for the class of sharp-edged highly swept
wings desirable for use in supersonic cruise vehicles and, possibly, in hypersonic
vehicles. Many theoretical methods have been developed to predict the nonpotential, or
vortex-lift, behavior (refs. 1 to 6). Most of these methods have limited applicability
because of inherent assumptions, such as conical flow and slender wings (refs. 2 to 4),
or because additional information is required to implement them (ref. 5). However, the
leading-edge-suction analogy, originally postulated by Polhamus in reference 6 for the
vortex lift on delta wings, has been shown to be a versatile technique which is useful over
a broad range of planforms (refs. 6 to 8) and speeds (ref. 8). This analogy proposes that
the low pressures associated with the leading-edge vortices produce a normal force
which has the same magnitude as the potential-flow leading-edge-suction force that is lost
because of the separation at the sharp leading edge. The suction analogy has been applied
by Polhamus to predict the drag due to lift of delta wings (ref. 9) as well as arrow and
diamond wings (ref. 7).

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Recently, Nangia and Hancock (ref. 10) concluded, from wind-tunnel tests at angles of attack at which the vortex lift was a significant portion of the total lift, that the longitudinal distribution of the vortex lift should be the same as for the potential lift on a slender delta wing. In order for this conclusion to be consistent with the leading-edge-suction analogy, the longitudinal distribution of the potential-flow leading-edge suction must be the same as the longitudinal distribution of the potential-flow lift. To evaluate their conclusion, the present study utilizes the vortex-lattice method of reference 11 to predict the distribution of the potential-flow leading-edge suction and then, by use of the Polhamus analogy, predicts the longitudinal distribution of vortex lift. The longitudinal distributions of the theoretical vortex lift and potential lift are compared and the total lift distribution is compared with experimental results. Ranges of aspect ratios and angles of attack were studied to determine limits of the validity of the Nangia and Hancock (ref. 10) conclusion.

SYMBOLS

\[ A \quad \text{aspect ratio,} \quad \frac{b^2}{S_{\text{ref}}} \]

\[ b \quad \text{wing span} \]

\[ b_L \quad \text{local wing span, that is, the spanwise distance from one leading edge to the other at any chordwise position} \]

\[ c \quad \text{local chord} \]

\[ \bar{c} \quad \text{reference chord,} \quad \frac{2}{3}c_r \]

\[ c_r \quad \text{root chord} \]

\[ c_s \quad \text{section suction-force coefficient,} \quad \frac{1}{q_c} \frac{dS}{dy} \]

\[ c_{s}^{i} \quad \text{station suction-force coefficient,} \quad \frac{2}{q_{b_L}} \frac{dS}{dx} \]

\[ C_{L} \quad \text{lift coefficient,} \quad \frac{\text{Lift}}{qS_{\text{ref}}} \]

\[ C_{LL} = C_{NL} \cos \alpha \]

\[ C_m \quad \text{pitching-moment coefficient about} \quad \frac{c_r}{2}, \quad \frac{\text{Pitching moment}}{q \bar{c} S_{\text{ref}}} \]
\(C_N\) normal-force coefficient, \(\frac{N}{qS_{\text{ref}}}\) (see eq. (7))

\(C_{NL}\) longitudinal normal-loading coefficient, \(C_{NL} = 2 \int_0^1 \left(\frac{x}{c_r}\right) \Delta C_p \, d\left(\frac{2y}{b_l}\right) = \frac{3N}{q} \left(\frac{b}{b_l}\right)

\(C_S\) suction-force coefficient, \(\frac{S}{qS_{\text{ref}}}\) (see eq. (12))

\(C_{SL}\) longitudinal suction-force coefficient, \(c_S(c/c_r)\)

\(\Delta C_p\) pressure-difference coefficient, \(\frac{p_{\text{lower}} - p_{\text{upper}}}{q}\)

\(K_p, K_v\) coefficients used in Polhamus' suction analogy (see eqs. (2) and (3))

\(N\) normal force

\(p\) static pressure

\(q\) free-stream dynamic pressure

\(S\) suction force

\(S_{\text{ref}}\) reference area, \(\frac{bc_r}{2}\)

\(V\) free-stream velocity

\(x\) chordwise coordinate in wing plane, origin at apex and positive aft (see fig. 1(b))

\(y\) spanwise coordinate, origin at center line and positive toward right wing tip (see fig. 1(b))

\(\alpha\) angle of attack, deg

Subscripts:

\(p\) potential flow

\(v\) vortex flow
DISCUSSION AND RESULTS

Vortex-Lift Theory

From flight and wind-tunnel studies, the flow around wings having sharp leading edges has been observed to separate at all but the very lowest angles of attack. For wings having leading-edge sweepback, this separated flow rolls up into basically two helical vortices, the cores of which are inboard of the leading edges and above the wing-chord plane. As shown in figure 1(a), the flow reattaches inboard of the vortices. Data indicate that when leading-edge separation occurs, the wing lift generated exceeds that predicted by potential-flow solutions. Some authors have postulated that the total lift could be thought of as being composed of the potential-flow lift and a lift term associated with the vortex flow, that is, \( CL = C_{L,p} + C_{L,v} \).

The vortex-flow lift expression developed in reference 6 employs this postulate and is based on the concept that the leading-edge suction force, which exists in attached flow, is rotated into a normal force when the leading-edge flow separates. This concept led to

\[
C_L = \frac{C_{L,p}}{K_p \sin \alpha \cos^2 \alpha} + \frac{C_{L,v}}{K_v \sin^2 \alpha \cos \alpha}
\]

(1)

where

\[
K_p = \frac{\partial C_N}{\partial (\sin \alpha \cos \alpha)}
\]

(2)

\[
K_v = \frac{\partial C_S}{\partial (\sin^2 \alpha)}
\]

(3)

and \( K_p \) and \( K_v \) depend only on planform and Mach number. Comparisons of Polhamus' expression with experiment presented in references 6, 8, 12, and 13 show good agreement over the range of \( \alpha \), including the cases in which a large proportion of the total lift comes from the vortex flow. Extensive experimental data were available from reference 14 for the \( A = 1.147 \) flat delta wing which was used in the present study for comparison with the calculated lift curves. Figure 2 shows the excellent agreement of these data with the predicted values from equation (1).

Longitudinal Lift Distribution

For slender wings, it is useful to consider the load distribution in the longitudinal direction. This force distribution has been termed variously: chordwise (refs. 10, 15,
and 16); stream (ref. 10), longitudinal (ref. 10), lengthwise (ref. 10), and cross-load (ref. 17) distribution. In this paper, this loading is termed longitudinal loading in contrast to chordwise loading, which will be reserved for a given spanwise position.

The longitudinal loadings used in this paper are determined by the procedures outlined by Nangia and Hancock (ref. 10) and detailed here. Normally, $N$ is determined from wing pressure data by the expression

$$N = q \frac{b}{2} \int_{-1}^{1} c \int_{le}^{te} \Delta C_p \, dx \, d\left(\frac{2y}{b}\right)$$

(4)

where $le$ and $te$ refer to the leading and trailing edge, respectively. However, it can alternately be expressed as

$$N = q c r \left(\frac{b t}{2}\right) \int_{0}^{1} \Delta C_p \, d\left(\frac{2y}{b t}\right) \, d\left(\frac{x}{c r}\right)$$

(5)

From the geometric relationships which exist on a delta wing, it is known that

$$\frac{b t}{b} = \frac{x}{c r}$$

(6)

Hence, by combining the preceding equations, the results can be written in coefficient form as

$$C_N = \int_{0}^{1} C_{NL} \, d\left(\frac{x}{c r}\right)$$

(7)

where

$$C_{NL} = 2 \int_{0}^{1} \left(\frac{x}{c r}\right) \Delta C_p \, d\left(\frac{2y}{b t}\right) = \frac{\delta N}{q \frac{b}{2}} \frac{\delta x}{\delta x}$$

(8)

The vortex-lift longitudinal loading, however, is obtained by a different procedure. This difference is due to the unique character of the vortex lift in that, rather than being the result of distributed pressure over an area, it is better regarded as a force per length distributed along each of the leading edges. The magnitude of this force is the same as that of the leading-edge suction force – the distribution of which can be determined by many computer programs based on lifting-surface theory. The vortex-lattice computer program of reference 11 was employed to provide the suction distributions used herein. The lattice arrangement selected for representing the delta wings in this study consisted of 10 chordwise positions at each of 12 spanwise stations on a half-wing, as shown in
This choice of vortex lattice was the result of a systematic study of the effects of varying the number of chordwise and spanwise stations (with the maximum number of vortices limited to 120). Two criteria were used, the first being comparison of the calculated value of $K_V$ with that cited in reference 6. The second criterion was that the normalized $C_{SL}$ curve must not have negative values and should go to zero, with the slope approaching infinity at the apex, as indicated by reference 18.

It is conventional to express the leading-edge suction force in terms of a section coefficient $c_S$ defined by

$$\frac{S}{2} = \int_0^{b/2} c_S c q \, dy = \int_0^1 c_S \frac{b}{2} c q \, d\left(\frac{y}{b/2}\right)$$

(9)

This section suction-force coefficient is provided by the vortex-lattice program as a function of $\frac{y}{b/2}$. The suction force may also be expressed in terms of a station suction coefficient $c'_s$:

$$\frac{S}{2} = \int_0^{c_T} c'_s q \frac{b_T}{2} \, dx = \int_0^1 c'_s c_T \frac{b_T}{2} q \, d\left(\frac{x}{c_T}\right)$$

(10)

Because of the delta planform, $\frac{x}{c_T} = \frac{y}{b/2}$. Equations (9) and (10) may be combined to relate the two coefficients:

$$c'_s c_T b_T = c_S c_T$$

(11)

In order to compare the vortex-lift longitudinal loading with the potential-lift longitudinal loading, it is desirable to use the same form as equation (7):

$$C_S = 2 \int_0^1 C_{SL} d\left(\frac{x}{c_T}\right)$$

(12)

and

$$S = C_S q S_{ref}$$

(13)

Equations (9) to (13) lead to the equation used to obtain $C_{SL}$ from the computer program:

$$C_{SL} = c_S \left(1 - \frac{x}{c_T}\right) = c_S \left(\frac{c}{c_T}\right)$$

(14)
The results are presented in terms of \( \frac{2C_{SL}}{C_S} \) which by use of the suction analogy is equivalent to \( \left( \frac{C_{NL}}{C_N} \right)_v \) and, consequently, to \( \left( \frac{C_{LL}}{C_L} \right)_v \). The longitudinal vortex-loading coefficient as defined by equations (12) and (14) is consistent with the previously defined potential loading.

In reference 10, Nangia and Hancock reported the results of a series of wind-tunnel tests conducted on flat and cambered sharp-edged delta-wing models of \( A = 1 \). With the use of experimental pressure data \( \frac{C_{LL}}{C_L} \), which is equivalent to \( \frac{C_{NL}}{C_N} \), was obtained by evaluation of equation (8). The longitudinal loading was compared with a similar distribution predicted by the subsonic lifting-surface theory of Taylor (ref. 16). Since the normalized curve of potential-lift longitudinal loading was very similar to the normalized curve of total lift, Nangia and Hancock concluded that the variation of vortex lift must also have the same shape.

The present report is concerned with this conclusion. To determine its validity limits, a systematic study of the longitudinal-loading characteristics of flat delta wings was performed by using the vortex-lattice program of reference 11. The normalized potential-lift and vortex-lift curves obtained with this program for an \( A = 1 \) delta wing are compared with data from Nangia and Hancock (ref. 10) in figure 3. Figure 4 shows the potential-lift and vortex-lift longitudinal loadings for an \( A = 1.147 \) wing compared with data reported in reference 14. Both figures 3 and 4 indicate general agreement in the forms of the vortex-lift and potential-lift curves and close agreement with the experimental data for total lift. The experimental data show some slight variation in the shape of the longitudinal loadings with angle of attack, whereas the theoretical loadings are independent of \( \alpha \).

**Effects of Angle of Attack and Aspect Ratio**

In addition to the changes in the forms of the normalized longitudinal-loading variation with angle of attack, a second angle-of-attack effect is shown in figure 5. An examination of this figure shows that the potential- and vortex-lift longitudinal loadings change proportions with \( \alpha \) in such a way that a theoretical total is produced which, in general, agrees in shape with the experimental distribution even though the two curves have slightly different centroids. Note, in particular, the more rapid growth of the vortex-lift longitudinal loading with \( \alpha \) than that for the potential lift — a characteristic of the lift development in this separated flow regime.

The variations of the potential- and vortex-lift longitudinal loadings with aspect ratio are presented in figure 6. The most interesting feature noted in a comparison of the two sets of curves ((a) and (b) parts) is that as the aspect ratio decreases, the two kinds of
loadings approach one another. In fact, it is easy to show from slender-wing theory that
the loadings are identical in the limit as \( A \to 0 \). However, as the aspect ratio is
increased above say 2, the similarity in shape of the two loadings no longer exists. Thus,
the conclusion of reference 10 holds only over a limited range of aspect ratio.

Figure 6 also shows that the variation of longitudinal loading with aspect ratio is
greater for the vortex lift than for the potential lift. This occurs because the vortex-lift
distribution is obtained from the leading-edge-suction distribution, which is really asso-
ciated with a point on the leading edge rather than along either the span or the root chord.
Hence, it may be graphed for a delta wing in either normalized coordinate direction with-
out changing the shape. The reason for this behavior is that the leading-edge suction
exists at the leading edge and is not distributed along the chord or span, even though it is
dependent upon the negative pressure rise along a normal to the edge. With delta wings
of small aspect ratio, normal to the leading edge is almost the same as normal to the root
chord. Hence, graphing the vortex-lift loading along the root chord is appropriate. How-
ever, as the aspect ratio increases to some value (\( A = 16 \) illustrated in fig. 6) for which
the sweepback angle becomes low, then the normal direction to the leading edge is nearly
normal to the span (rather than to the chord), and hence, graphing the loading along the
root chord is not appropriate. In fact, for the \( A = 16 \) delta wing, the vortex-lift loading
resembles a span load distribution more than it does the potential-lift longitudinal loading.

Pitching-Moment Prediction

The pitching-moment coefficient for the potential and vortex lift can be predicted
by using the appropriate longitudinal loadings in the following equation:

\[
(C_m)_{p,v} = \frac{3}{2} \frac{C_N}{2} - \int_0^1 \frac{x}{c_r} C_{NL} \left[ \frac{X}{c_r} \right]_{p,v}
\]

The results of computations made with this equation are shown in figure 7 as a function
of \( \alpha \). From the figure, it is seen that the vortex lift produces a stabilizing moment and
becomes the larger contributor at the high angles of attack. The sum of these curves is
compared in figure 8 with the experimental data of reference 17 (\( A = 1.0 \)) and in figure 9
with the experimental data of references 13 and 14 (\( A = 1.147 \)). In each case, the theory
predicts a nose-down moment of larger magnitude than is obtained from experiment. The
difference is in the direction expected to be caused by the actual loading centroid being
forward of the theoretical — caused by loss of lift near the model wing tips. The theo-
retical curve predicted in reference 13 by use of the computer program described in
reference 19 coincides with the present prediction in figure 9.
CONCLUDING REMARKS

The leading-edge-suction analogy of Polhamus is used to develop the vortex-lift longitudinal load distribution for delta wings. This type of loading has been combined with that obtained from potential-flow concepts and the combination has been compared with that determined experimentally. Good agreement has been determined for delta wings with aspect ratios of 1.0 and 1.147 at several different angles of attack.

The Nangia and Hancock conclusion (in C.P. No. 1129, Brit. A.R.C.) that similar longitudinal load distributions result for vortex-flow and potential-flow lifts has been verified by the suction analogy for delta wings with aspect ratios of 2 or less.

The theoretical longitudinal load distributions have also been used to predict the nonlinear pitching-moment behavior of delta wings. A comparison with experiment shows that the theory predicts slightly greater stability than measured.

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REFERENCES


Figure 1. - Separated and attached flow-field representations for the delta wing.

(a) Leading-edge separation vortices.

(b) Vortex-lattice representation.
Figure 2. Variation of $C_L$ as measured and predicted for an $A = 1.147$ delta wing.
Figure 3.- Effect of angle of attack on the theoretical and experimental normalized longitudinal loadings for an $A = 1$ delta wing.
Figure 4.- Effect of angle of attack on the theoretical and experimental normalized longitudinal loadings for an $A = 1.147$ delta wing.
Figure 5.- Combinations of the vortex- and potential-lift longitudinal loadings along with the experimental loadings at several angles of attack for an $A = 1.147$ delta wing.
Figure 6. - Effect of aspect ratio on the normalized longitudinal loadings.

(a) Potential-lift loading.

(b) Vortex-lift loading.
Figure 7.- Combinations of pitching-moment contributions from the potential- and vortex-lift longitudinal loadings at various angles of attack for an $A = 1$ delta wing.
Figure 8.- Variation of $C_m$ as measured and predicted for an $A = 1$ delta wing.
Figure 9. - Variation of $C_m$ as measured and predicted for an $A = 1.147$ delta wing.
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