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A PASSIVE BALANCER FOR A CLASS OF ROTATING SPACECRAFT

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16. Abstract

Equations of motion are derived for a flexibly connected dual-spin spacecraft equipped with four pendulumlike "passive controllers" for mass balance and spin axis control. These equations are constrained and used to determine the conditions required for successful steady-state operation of the controllers with a spinning, rigid-body spacecraft. Generalized controller sizing criteria are derived and related to limiting values of system time constant for an example spacecraft unbalance condition.

A generalized real-time computer simulation of a large, slowly spinning rigid-body spacecraft equipped with passive controllers is also presented. Numerical results of this simulation show that passive controllers can successfully balance a class of rotating rigid bodies undergoing large internal mass and inertial disturbances. The ratio of total controller mass to spacecraft mass need not be greater than 1 to 2 percent.

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A PASSIVE BALANCER FOR A CLASS OF ROTATING SPACECRAFT

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SUMMARY

Equations of motion have been derived for a flexibly connected dual-spin spacecraft equipped with four pendulumlike "passive controllers" for mass balance and spin axis control. The derived equations, simplified by eliminating hub and flexibility terms, were analyzed to determine the conditions required for successful steady-state operation of the controllers with a spinning, rigid-body spacecraft. Results indicated that spacecraft inertia about the intended spin axis must be less than spacecraft inertia about the transverse axes. Also, positive damping of controller motion relative to the body is required.

A generalized real-time computer simulation of a large, slowly spinning rigid-body spacecraft equipped with passive controllers has also been presented. Numerical results of this simulation show that passive controllers can successfully balance a class of rotating rigid bodies undergoing large internal mass and inertial disturbances. Results also indicate a reduction in spacecraft attitude error due to the action of the controllers. The ratio of total controller mass to spacecraft mass need not be greater than 1 to 2 percent.

INTRODUCTION

Man in space may be unable to function over extended time periods without artificial gravity. A practical method for providing an artificial gravity environment, as well as a means of stabilization, is that of rotating the entire spacecraft or an appreciable part of the spacecraft, as is done in a dual-spin application. (See ref. 1.) It is anticipated that rotating space stations will require a means of preserving both the location of their mass center and the orientation of their axis of rotation. This requirement would insure that docking ports remain centered about the rotation axis and that steady observations could be made from any nonrotating part of the station.

The stabilization problem arises because of a necessity for crew members to move about the station and for supplies and equipment to be distributed and relocated during operation. Also, resupply vehicles occasionally will be coupled to the station. All these activities alter the mass center of the station and thereby the location of the rotational axis. Also, the mass redistribution introduces products of inertia that cause dynamic unbalance. The resultant wobbling and circling motion of the station may interfere with docking activities and pointing requirements.

Existing technology for unmanned satellites is not directly applicable for controlling the axis of rotation and mass center of a manned space station. Wobbling of the station can be prevented by an active momentum storage system, but the associated weight increase may be prohibitive and such a system would be unable to prevent static unbalance and unwanted circling of the nonrotating part of the station.

The proposed technique for spin axis and mass center control (that is, control of static and dynamic balance) of manned rotating space stations uses two sets of "passive controllers." Each set consists of two pendulumlike masses free to rotate concentrically about the desired spin axis in planes perpendicular to the spin axis. See figure 1. If the actual spin axis initially is not coincident with the desired spin axis, the centrifugal forces generated by the spinning motion will automatically deflect the controllers in such a way as to drive the actual spin axis toward the desired location. The passive controllers should incorporate sufficient damping to minimize their settling time after introduction of an unbalance. Once in operation, the controllers rotate with the spinning part of the station and need only gradual relative movements to perform their function automatically.

As part of an overall study, this paper develops equations of motion for a flexibly connected dual-spin spacecraft equipped with passive controllers. However, the intent of this paper is to investigate controller and spacecraft dynamics for a rigid-body spacecraft. Thus, the derived equations of motion first were simplified by eliminating hub and flexibility terms. A steady-state analysis of the resulting equations was performed to define design conditions required for successful operation of the controllers with a rigid-body spacecraft. Also, the simplified equations were used in a digital computer simulation to obtain the dynamic response of the spacecraft and controller system to large crew motion disturbances.

SYMBOLS

A bar over a symbol indicates a vector quantity. A dot over a symbol indicates a derivative with respect to time. A prime with a symbol denotes a derivative with respect to T. A symbol within braces $\{\ \}$ also indicates a vector. A symbol within brackets $[\ ]$ indicates a square matrix. If this symbol is a vector quantity, however, its use in brackets indicates a particular type of skew symmetric matrix as illustrated by the following example:

Let

$$\bar{r} = \{r\} = \begin{Bmatrix} r_x \\ r_y \\ r_z \end{Bmatrix}$$

Then

$$[r] = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$

$\{A\}$ disk Euler rate vector, $\begin{bmatrix}\dot{\phi} & \dot{\theta} & \dot{\psi}\end{bmatrix}^T$

$-\{A_1\}$ location of total mass center in disk coordinates, $\{A_1\} = [D_1]^{-1}\{R\}$

$A_{1,xy}$ total mass center offset in x,y-plane, $\sqrt{A_{1,x}^2 + A_{1,y}^2}$

a_1, b_1, a_2, b_2 defined in equations (43) to (46), respectively

$\{B\}$ hub Euler rate vector relative to disk, $\begin{bmatrix}\dot{\phi}_h & \dot{\theta}_h & \dot{\psi}_h\end{bmatrix}^T$

$[C]$ translational damping constant matrix between disk and hub

$[C_R]$ rotational damping constant matrix between disk and hub

C_j jth controller damping coefficient where $j = 1, 2, 3, 4$

$[D]$ transformation matrix, disk Euler rates to disk body rates

$[D_1]$ orthogonal transformation matrix, disk components to inertial components

$[D_2]$ orthogonal transformation matrix, hub components to disk components

$[D_3]$ orthogonal transformation matrix, controller components to disk components

$[D_h]$ transformation matrix, disk-relative hub Euler rates to disk-relative hub body rates

$\{F\}$ components of total external force parallel to disk coordinate axes, $\{F_d\} + [D_2]\{F_h\}$

$\{F_d\}$ external force components applied to disk along x-, y-, and z-axes

$\{F_h\}$ external force components applied to hub along x_h-, y_h-, and z_h-axes

F_d dissipation function

G dimensionless quantity (see eqs. (17) and (33))

h distance along z-axis from x,y,z origin to controller pivot point

h_j z coordinate of jth controller mass center, $(-1)^j h$ where j is an exponent

$$[I] = [I_d] + [I_c] + \sum_{j=1}^{4}\left([D_3][I_j][D_3]^{-1}\right)$$

$[I_c]$ crew inertia matrix about x-, y-, and z-axes at the crew mass center

$[I_d]$ disk inertia matrix about x-, y-, and z-axes at disk mass center

$[I_h]$ hub inertia matrix about hub mass center referred to x_h-, y_h-, and z_h-axes

$[I_j]$ jth controller inertia matrix about controller axes at jth controller mass center

$I_{r,z}$ total dynamic unbalance of spacecraft without controllers,

$$\sqrt{\left(I_{xz} - \sum_{j=1}^{4} I_{xz,j}\right)^2 + \left(I_{yz} - \sum_{j=1}^{4} I_{yz,j}\right)^2}$$

$[K]$ translational spring constant matrix between disk and hub

$[K_R]$ rotational spring constant matrix between disk and hub

ℓ distance from controller pivot point to controller mass center

m controller mass (see eqs. (10))

m_c crew mass

m_d disk mass

m_h	hub mass
m_j	jth controller mass ($j = 1, 2, 3, 4$)
m_k	mass of additional spacecraft crew members
m_T	total spacecraft mass, $m_d + m_c + m_h + \sum_{j=1}^{4} m_j$
Q_i	generalized "force" associated with ith generalized coordinate
q_i	ith generalized coordinate
$\overline{R}$	inertial coordinates of disk coordinate axes system relative to overall mass center
$\overline{R}_c$	inertial coordinates of crew mass center relative to overall mass center
$\overline{R}_d$	inertial coordinates of disk mass center relative to overall mass center
$\overline{R}_g$	inertial coordinates of spacecraft mass center, $\left\{x' \quad y' \quad z'\right\}^T$
$\overline{R}_h$	inertial coordinates of hub mass center relative to overall mass center
$\overline{R}_j$	inertial coordinates of jth controller relative to overall mass center
$\bar{r}$	disk coordinates of hub coordinate axis system
$\bar{r}_c$	disk coordinates of crew mass center
$\bar{r}_d$	disk coordinates of disk mass center
$\bar{r}_f$	hub coordinates of hub mass center
$\bar{r}_h$	disk coordinates of hub mass center
$\bar{r}_j$	disk coordinates of jth controller mass center
r_{kx}, r_{ky}, r_{kz}	spacecraft coordinates of crew mass m_k

$s(\ ), c(\ )$ sin( ) and cos( ), respectively

$\{T\}$ components of total external torque along x-, y-, and z-axes, $\{T_d\} + [D_2]\{T_h\}$

$\{T_d\}$ external torque applied to disk about x-, y-, and z-axes

$\{T_h\}$ external torque applied to hub about x_h-, y_h-, and z_h-axes

T kinetic energy; also nondimensional angle ξt in equation (28)

t time, sec

V potential energy

x,y,z disk coordinate axes, with origin at center of figure of disk

x',y',z' inertial coordinate axes

x_h,y_h,z_h hub coordinate axes, with origin at center of figure of hub

x_j,y_j,z_j coordinate axes of jth controller

$\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$

α_j angular location of jth controller in x,y-plane (see fig. (12(d))

$\dot{\alpha}_j$ jth controller rotation rate about z-axis with respect to disk coordinate system

$\Delta\alpha$ see equations (22)

$\Delta\alpha_j$ incremental rotation of jth controller about its steady-state value

$\epsilon = \tan^{-1} \frac{I_{yz}}{I_{xz}}$

$\eta = \sqrt{{\eta_x}^2 + {\eta_y}^2}$

η_x principal axis misalinement in x,z-plane

η_y principal axis misalinement in y,z-plane

λ orientation of ω_{xy} in x,y-plane, $\tan^{-1} \frac{\omega_{y,o}}{\omega_{x,o}}$

ξ coning rate less spin rate, $\omega_z\left(\frac{I_z}{I} - 1\right)$

$\bar{\rho}_j$ vector from spacecraft mass center to jth controller expressed in spacecraft coordinates

ϕ,θ,ψ disk Euler angles (see fig. 12(a))

ϕ_h,θ_h,ψ_h hub Euler angle rotations with respect to disk axes x, y, and z (see fig. 12(b))

ϕ_I,θ_I,ψ_I hub Euler angle rotations with respect to inertial axes x', y', and z' (see fig. 12(c))

ϕ_1 orientation of $I_{r,z}$ in x,y-plane

$\{\omega\}$ disk inertially referenced body rates $\{\omega_x \quad \omega_y \quad \omega_z\}^T$

$\{\omega_h\}$ hub inertially referenced body rates $\{\omega_{h,x} \quad \omega_{h,y} \quad \omega_{h,z}\}^T$

$\{\omega_j\}$ inertial attitude rate of jth passive controller about controller axes

$\omega_{x,o}$ initial attitude rate about x-axis

$\omega_{y,o}$ initial attitude rate about y-axis

$\omega_{xy} = \sqrt{\omega_{x,o}^2 + \omega_{y,o}^2}$

$\frac{d}{dt}(\ ) = (\dot{\ }) + \Omega(\ )$, where Ω is angular inertial rate vector of coordinate system used to express ( )

$[\ ]^T$ transpose of bracketed matrix

$[\ ]^{-1}$ inverse of bracketed matrix

$\left\{ \right\}^T$ transpose of braced vector

$\times$ indicates a vector cross product operation

Subscripts:

c crew

d rotor or disk

h hub

j jth controller (j = 1, 2, 3, 4)

o initial conditions

s steady-state value of subscripted quantity

x,y,z components of subscripted quantity along x-, y-, and z-axes

ANALYSIS

This section describes a mathematical model of a flexibly connected dual-spin spacecraft equipped with four pendulus masses designed to provide passive balance and spin axis control. Equations of motion, derived by the method of Lagrange, are presented. The derived equations of motion, simplified by eliminating hub and flexibility terms, are then analyzed to define design conditions required for successful steady-state operation of the controllers with a spinning rigid-body space station and crew. Finally, controller sizing criteria are determined as a function of total static and dynamic unbalance.

Mathematical Model

The schematic model of the dual-spin spacecraft used in this study is shown in figure 2. The model consists of a nonrotating (zero gravity) "hub," a slowly spinning rotor or "disk," and four pendulum-like arms (with end masses) free to rotate concentrically about the desired spin axis (z-axis). The rotating arms or passive controllers are deployed in two pairs on either side of the overall mass center along the z-axis. In normal operation the controllers rotate with the disk and exhibit gradual relative movements only to counteract mass and/or inertial disturbances. Viscous dampers are incorporated between the controllers and the disk to minimize settling time of the controllers after the introduction of a disturbance.

The hub mass is connected flexibly to the disk mass through an arrangement of springs and dampers attached to the hub side of a bearing as shown in figure 3. Thus, spring and damping restraint exists for relative translations of the hub and disk along the x-, y-, and z-axes and for relative rotation of the hub and disk about the x- and y-axes. Relative rotations about the z-axis are unrestrained because of the bearing; frictional effects about the z-axis are assumed to be effectively compensated by application of an internal torque between the hub and disk. Disk, hub, and controllers are assumed to be rigid bodies. Flexibility exists only in the hub-disk connection previously described. Gravity gradient effects are assumed to be negligible for this analysis.

Equations of Motion

The equations of motion for a dual-spin spacecraft equipped with four passive controllers are derived in appendix A. The final form of these equations and the degrees of freedom represented are summarized below.

x', y', and z' translational degrees of freedom of entire spacecraft:

$$m_T \begin{Bmatrix} \ddot{x}' \\ \ddot{y}' \\ \ddot{z}' \end{Bmatrix} = [D_1]\{F\} \tag{1}$$

Equation (1) corresponds to equation (A24) and is written in the inertial system.

ϕ, θ, and ψ rotational degrees of freedom of entire spacecraft:

$$\begin{aligned} &[I]\{\dot{\omega}\} + [\dot{I}]\{\omega\} + [\omega][I]\{\omega\} - m_h[D_2][r_f][D_2]^T[D_1]^T\{\ddot{R}_h + \ddot{R}_g\} - \left([D_2]\left[[D_h]^T\right]^{-1}\right)[C_R]\{B\} \\ &+ [K_R]\left(\begin{Bmatrix} \phi_h \\ \theta_h \\ \psi_h \end{Bmatrix}\right) + m_d[r_d][D_1]^T\{\ddot{R}_d\} + \sum_{j=1}^{4}\left(m_j[r_j][D_1]^{-1}\{\ddot{R}_j\}\right) + m_c[r_c][D_1]^{-1}\{\ddot{R}_c\} \\ &+ m_h[r_h][D_1]^{-1}\{\ddot{R}_h\} + \sum_{j=1}^{4}\left(I_{j,z}\begin{Bmatrix} \dot{\alpha}_j\omega_y \\ -\dot{\alpha}_j\omega_x \\ \ddot{\alpha}_j \end{Bmatrix}\right) = \{T_d\} + [A_1]\{F\} + [r][D_2]\{F_h\} \end{aligned} \tag{2}$$

Equation (2) corresponds to equation (A30) and is written in the disk coordinate system.

r_x, r_y, and r_z translational degrees of freedom of hub with respect to disk: These equations are written in the disk coordinate system and correspond to equation (A25).

$$m_h\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\} + \left[C\right]\left\{\dot{r}\right\} + \left[K\right]\left\{r\right\} = \left[D_2\right]\left\{F_h\right\} \tag{3}$$

ϕ_h, θ_h, and ψ_h rotational degrees of freedom of hub with respect to disk: These equations are written in the hub coordinate system and correspond to equation (A31).

$$\left[I_h\right]\left\{\dot{\omega}_h\right\} + m_h\left[r_f\right]\left[D_2\right]^T\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\} + \left[\omega_h\right]\left[I_h\right]\left\{\omega_h\right\} + \left[\dot{I}_h\right]\left\{\omega_h\right\}$$

$$+ \left[\left[D_h\right]^T\right]^{-1}\left\{\left[C_R\right]\left\{\dot{B}\right\} + \left[K_R\right]\begin{Bmatrix}\phi_h\\ \theta_h\\ \psi_h\end{Bmatrix}\right\} = \left\{T_h\right\} \tag{4}$$

$\alpha_j (j = 1, 2, 3, 4)$ rotational degree of freedom of jth passive controller relative to disk coordinate system: This equation is written in the disk coordinate system and corresponds to equation (A19).

$$I_{j,z}\left(\dot{\omega}_z + \ddot{\alpha}_j\right) - \left(I_{j,x} - I_{j,y}\right)\left[\left(\omega_y^2 - \omega_x^2\right) s\alpha_j\, c\alpha_j - \omega_x\omega_y\left(s^2\alpha_j - c^2\alpha_j\right)\right]$$

$$+ m_j\begin{Bmatrix}-\ell\, s\alpha_j\\ \ell\, c\alpha_j\\ 0\end{Bmatrix}^T\left[D_1\right]^T\left(\left\{\ddot{R}_j + \ddot{R}_g\right\}\right) + C_j\dot{\alpha}_j = 0 \qquad (j = 1, 2, 3, 4) \tag{5}$$

Stability Analysis of Passive Controller Operation With Rigid-Body Spacecraft

This analysis considers a rigid-body space station equipped with four passive controllers. The steady-state controller response to static and dynamic unbalance of the spacecraft is derived along with conditions required for stable controller operation. Also, the effects of spacecraft coning on controller response are determined and controller sizing criteria are developed for spinning rigid-body spacecraft.

A schematic of the space station is shown in figure 4 illustrating the vector location of the jth passive controller and the overall mass center in disk body coordinates. The three vectors have the relationship:

$$\bar{\rho}_j = \overline{A}_1 + \bar{r}_j$$

The inertial acceleration of m_j is

$$\frac{d^2\bar{\rho}_j}{dt^2} = \ddot{\bar{\rho}}_j + \dot{\bar{\omega}} \times \bar{\rho}_j + 2\bar{\omega} \times \dot{\bar{\rho}}_j + \bar{\omega} \times \left(\bar{\omega} \times \bar{\rho}_j\right)$$

where $\bar{\omega}$ is the inertial angular velocity of the disk coordinate system written in disk coordinates. Substituting $\bar{A}_1$ and $\bar{r}_j$ and their derivatives for $\bar{\rho}_j$, $\dot{\bar{\rho}}_j$, and $\ddot{\bar{\rho}}_j$ and writing the acceleration in matrix form yields

$$\left\{\frac{d^2\rho_j}{dt^2}\right\} = \left\{\ddot{A}_1\right\} + \left\{\ddot{r}_j\right\} + \left[\dot{\omega}\right]\left(\left\{A_1\right\} + \left\{r_j\right\}\right) + 2\left[\omega\right]\left(\left\{\dot{A}_1\right\} + \left\{\dot{r}_j\right\}\right) + \left[\omega\right]\left[\omega\right]\left(\left\{A_1\right\} + \left\{r_j\right\}\right)$$

The moment equation for the passive controller about the z-axis can be written

$$I_{j,z}\left(\dot{\omega}_z + \ddot{\alpha}_j\right) + \left(m_j\left[r_j\right]\left\{\frac{\partial^2\rho_j}{\partial t^2}\right\}\right)_{z \text{ component}} + C_j\dot{\alpha}_j = 0$$

or

$$I_{j,z}\left(\dot{\omega}_z + \ddot{\alpha}_j\right) + m_j\begin{Bmatrix}-\ell\, s\alpha \\ \ell\, c\alpha \\ 0\end{Bmatrix}^T\left(\left\{\ddot{A}_1\right\} + \left\{\ddot{r}_j\right\} + \left[\dot{\omega}\right]\left(\left\{A_1\right\} + \left\{r_j\right\}\right) + 2\left[\omega\right]\left(\left\{\dot{A}_1\right\} + \left\{\dot{r}_j\right\}\right)\right.$$

$$\left. + \left[\omega\right]\left[\omega\right]\left(\left\{A_1\right\} + \left\{r_j\right\}\right)\right) + C_j\dot{\alpha}_j = 0$$

Substituting the components of $\left\{A_1\right\}$, $\left\{\dot{A}_1\right\}$, $\left\{\ddot{A}_1\right\}$, $\left\{r_j\right\}$, $\left\{\dot{r}_j\right\}$, $\left\{\ddot{r}_j\right\}$, $\left[\omega\right]$, and $\left[\dot{\omega}\right]$ in this equation and collecting terms yields the basic equation governing controller motion

$$\begin{aligned}
&\left(I_{j,z} + m_j\ell^2\right)\ddot{\alpha}_j + C_j\dot{\alpha}_j + m_j\left(-\ddot{A}_{1,x}\ell\, s\alpha_j + \ddot{A}_{1,y}\ell\, c\alpha_j\right) + m_j\left(A_{1,z} + (-1)^j h\right)\left(-\dot{\omega}_x\ell\, c\alpha_j\right. \\
&\left. - \dot{\omega}_y\ell\, s\alpha_j\right) + \dot{\omega}_z I_{j,z} + m_j\dot{\omega}_z\left(\ell^2 + A_{1,y}\ell\, s\alpha_j + A_{1,x}\ell\, c\alpha_j\right) + 2m_j\omega_z\left(\dot{A}_{1,y}\ell\, s\alpha_j + \dot{A}_{1,x}\ell\, c\alpha_j\right) \\
&- 2m_j\dot{A}_{1,z}\left(\omega_y\ell\, s\alpha_j + \omega_x\ell\, c\alpha_j\right) + m_j\omega_z^2\left(A_{1,x}\ell\, s\alpha_j - A_{1,y}\ell\, c\alpha_j\right) + m_j\omega_y^2\ell\, s\alpha_j A_{1,x} \\
&- m_j\omega_x^2\ell\, c\alpha_j A_{1,y} + m_j\ell^2\, s\alpha_j\, c\alpha_j\left(\omega_y^2 - \omega_x^2\right) + m_j\omega_x\omega_y\left[\ell^2\left(c^2\alpha_j - s^2\alpha_j\right) - \ell\, s\alpha_j A_{1,y}\right. \\
&\left. + \ell\, c\alpha_j A_{1,x}\right] + m_j\left(A_{1,z} + (-1)^j h\right)\left(-\omega_x\omega_z\ell\, s\alpha_j + \omega_y\omega_z\ell\, c\alpha_j\right) = 0 \qquad (6)
\end{aligned}$$

Equation (6) will be used first to examine the stability of controller steady-state response to spacecraft dynamic unbalance. Stability of controller response to static unbalance or mass center offset will follow. Finally, the effects of coning on controller response will be discussed.

Response to dynamic unbalance.- Equation (6) is simplified by limiting inputs to pure inertia product disturbances $\left(\text{that is, } \bar{A}_1 = \bar{\dot{A}}_1 = \bar{\ddot{A}}_1 = 0\right)$. Also, in the solution area of interest, $\dot{\omega}_z \approx 0$ and, since $(\omega_x,\omega_y) << \omega_z$, second-order terms of ω_x and ω_y are negligible. These conditions reduce equation (6) to

$$\left(I_{j,z} + m_j \ell^2\right)\ddot{\alpha}_j + C_j\dot{\alpha}_j + I_{xz,j}\left(-\dot{\omega}_x + \omega_y\omega_z\right) + I_{yz,j}\left(-\dot{\omega}_y - \omega_x\omega_z\right) = 0 \tag{7}$$

where the inertia products are defined as

$$\left.\begin{aligned} I_{xz,j} &= m_j \ell \; c\alpha_j (-1)^j h \\ I_{yz,j} &= m_j \ell \; s\alpha_j (-1)^j h \end{aligned}\right\} \tag{7a}$$

Equation (7) governs the motion of the jth controller as a function of the spacecraft angular velocities and accelerations. The angular motion of the spacecraft, in turn, is governed by other equations which depend upon controller motion. The simultaneous solution of these equations is easily accomplished (for any particular set of conditions) by numerical methods, but a general solution is very difficult to obtain analytically. For this reason, a steady-state solution was sought analytically to determine the conditions required for successful operation of the controllers.

The approximate spacecraft motion for fixed location of the controllers is given in terms of the following body rates and accelerations (ref. 2):

$$\left.\begin{aligned} \omega_x &= \omega_{x,o} \cos \xi t - \omega_{y,o} \sin \xi t - \frac{\omega_z}{I - I_z}\left[I_{xz}(\cos \xi t - 1) - I_{yz} \sin \xi t\right] \\ \omega_y &= \omega_{x,o} \sin \xi t + \omega_{y,o} \cos \xi t - \frac{\omega_z}{I - I_z}\left[I_{xz} \sin \xi t + I_{yz}(\cos \xi t - 1)\right] \\ \omega_z &= \omega_{z,o} \\ \dot{\omega}_x &= -\xi\left(\omega_{x,o} \sin \xi t + \omega_{y,o} \cos \xi t\right) + \frac{\xi\omega_z}{I - I_z}\left(I_{xz} \sin \xi t + I_{yz} \cos \xi t\right) \\ \dot{\omega}_y &= \xi\left(\omega_{x,o} \cos \xi t - \omega_{y,o} \sin \xi t\right) - \frac{\xi\omega_z}{I - I_z}\left(I_{xz} \cos \xi t - I_{yz} \sin \xi t\right) \end{aligned}\right\} \tag{8}$$

where

$$I = I_x = I_y$$

$$\xi = \omega_z\left(\frac{I_z}{I} - 1\right)$$

The quantities I_{xz} and I_{yz} are inertia products of the entire configuration. Equation (7) can be written for each of the four controllers, and the equations summed thusly:

$$\sum_{j=1}^{4}\left[\left(I_{j,z} + m_j\ell^2\right)\ddot{\alpha}_j + C_j\dot{\alpha}_j + I_{xz,j}\left(-\dot{\omega}_x + \omega_y\omega_z\right) + I_{yz,j}\left(-\dot{\omega}_y - \omega_x\omega_z\right)\right] = 0 \tag{9}$$

By defining $\alpha = \sum_{j=1}^{4} \alpha_j$ and stipulating that

$$\left.\begin{aligned} &I_{1,z} = I_{2,z} = I_{3,z} = I_{4,z} = I_{j,z} \\ &C_1 = C_2 = C_3 = C_4 = C \\ &m_1 = m_2 = m_3 = m_4 = m \end{aligned}\right\} \tag{10}$$

equation (9) combined with equations (7a) becomes

$$\begin{aligned} &\left(I_{j,z} + m\ell^2\right)\ddot{\alpha} + C\dot{\alpha} + m\ell h\left(c\alpha_1 - c\alpha_2 + c\alpha_3 - c\alpha_4\right)\left(\dot{\omega}_x - \omega_y\omega_z\right) \\ &+ m\ell h\left(s\alpha_1 - s\alpha_2 + s\alpha_3 - s\alpha_4\right)\left(\dot{\omega}_y + \omega_x\omega_z\right) = 0 \end{aligned} \tag{11}$$

For the solution resulting from pure inertia product inputs, the initial attitude rates $\left(\omega_{x,o}, \omega_{y,o}\right)$ of equations (8) are set equal to zero and the following conditions apply:

$$\left.\begin{aligned} &\alpha_4 = \alpha_1 \pm 180^\circ \\ &\alpha_2 = \alpha_3 \pm 180^\circ \end{aligned}\right\} \tag{12}$$

Equation (11) becomes

$$\left(I_{j,z} + m\ell^2\right)\ddot{\alpha} + C\dot{\alpha} + 2m\ell h\left(c\alpha_1 + c\alpha_3\right)\left(\dot{\omega}_x - \omega_y\omega_z\right) + 2m\ell h\left(s\alpha_1 + s\alpha_3\right)\left(\dot{\omega}_y + \omega_x\omega_z\right) = 0$$

The steady-state solution to this equation will occur when the forcing terms are zero; that is,

$$2m\ell h\left(c\alpha_1 + c\alpha_3\right)\left(\dot{\omega}_x - \omega_y\omega_z\right) + 2m\ell h\left(s\alpha_1 + s\alpha_3\right)\left(\dot{\omega}_y + \omega_x\omega_z\right) = 0 \tag{13}$$

By combining equations (8) and (13) with $\omega_{x,o} = \omega_{y,o} = 0$, the steady-state condition can be written as

$$2m\ell h \frac{\omega_z^2}{I - I_z}\frac{I_z}{I}\left\{\left[\sqrt{I_{xz}^2 + I_{yz}^2}\,\sin(\xi t + \epsilon) - I_{yz}\frac{I}{I_z}\right]\left(c\alpha_1 + c\alpha_3\right)\right.$$

$$\left. - \left[\sqrt{I_{xz}^2 + I_{yz}^2}\,\cos(\xi t + \epsilon) - I_{xz}\frac{I}{I_z}\right]\left(s\alpha_1 + s\alpha_3\right)\right\} = 0 \tag{14}$$

where

$$\epsilon = \tan^{-1}\left(\frac{I_{yz}}{I_{xz}}\right)$$

The inertia product terms include products of inertia of the spacecraft without controllers and products of inertia of the controllers; thus,

$$\left.\begin{aligned} I_{xz} &= I_{r,z}\, c\phi_1 + \sum_{j=1}^{4} I_{xz,j} = I_{r,z}\, c\phi_1 - 2m\ell h\left(c\alpha_1 + c\alpha_3\right) \\ I_{yz} &= I_{r,z}\, s\phi_1 + \sum_{j=1}^{4} I_{yz,j} = I_{r,z}\, s\phi_1 - 2m\ell h\left(s\alpha_1 + s\alpha_3\right) \end{aligned}\right\} \tag{15}$$

Combining equations (14) and (15) yields

$$-\frac{\omega_z^2}{I - I_z}\frac{I_z}{I}\sqrt{I_{xz}^2 + I_{yz}^2}\left[\sin\left(\alpha_1 - \xi t - \epsilon_1\right) + \sin\left(\alpha_3 - \xi t - \epsilon_1\right)\right]$$

$$-\frac{\omega_z^2}{I - I_z} I_{r,z}\left[\sin\left(\phi_1 - \alpha_1\right) + \sin\left(\phi_1 - \alpha_3\right)\right] = 0 \tag{16}$$

The first bracketed term of this equation cannot remain zero for steady-state α values, except for the trivial case of $\alpha_1 = -\alpha_3$ and $\alpha_2 = -\alpha_4$ which applies only when inertia products of the spacecraft (the controllers being neglected) are zero. However, the radical can equal zero for $I_{xz} = 0$ and $I_{yz} = 0$. These conditions lead to the steady-state requirement

$$\alpha_1 - \alpha_3 = \cos^{-1}G \tag{17}$$

where

$$G = \frac{1}{2}\left(\frac{I_{r,z}}{2m\ell h}\right)^2 - 1$$

An obvious constraint is

$$-1 \leqq G \leqq 1 \tag{18}$$

or

$$0 \leqq \frac{I_{r,z}}{2m\ell h} \leqq 2$$

This constraint can be written as

$$\sum_{j=1}^{4} m_j \ell h > \text{Total dynamic unbalance} \tag{19}$$

The other steady-state requirement stems from setting the second term of equation (16) to zero which leads to

$$\sin\left(\phi_1 - \alpha_1\right) + \sin\left(\phi_1 - \alpha_3\right) = 2\sin\left[\frac{1}{2}\left(2\phi_1 - \alpha_1 - \alpha_3\right)\right]\cos\left[\frac{1}{2}\left(-\alpha_1 + \alpha_3\right)\right] = 0$$

The cosine term cannot be zero without violating equation (17). Setting the sine term equal to zero results in the following additional requirement for steady state:

$$\alpha_1 + \alpha_3 = 2\phi_1$$

Combining the two requirements leads to

$$\left.\begin{aligned} \alpha_1 &= \phi_1 + \frac{1}{2}\cos^{-1}G \\ \alpha_3 &= \phi_1 - \frac{1}{2}\cos^{-1}G \end{aligned}\right\} \tag{20}$$

and because of the angular relationships of the controllers for a pure inertia product input,

$$\alpha_2 = \phi_1 - \frac{1}{2}\cos^{-1}G \pm 180^{\circ}$$

$$\alpha_4 = \phi_1 + \frac{1}{2}\cos^{-1}G \pm 180^{\circ}$$

The quantities α_1, α_2, α_3, and α_4 are the steady-state angular locations of the four passive controllers in response to a pure inertia product disturbance to the rigid-body spacecraft.

To determine stability conditions for the previous steady-state solution, each of the controllers is given an incremental perturbation in angle, angular rate, and angular acceleration from the steady-state condition and the system response is examined. By using an additional subscript s to indicate the previous steady-state solution, controller angles can be defined as

$$\left.\begin{aligned} \alpha_j &= \alpha_{js} + \Delta\alpha_j \\ \dot{\alpha}_j &= \Delta\dot{\alpha}_j \\ \ddot{\alpha}_j &= \Delta\ddot{\alpha}_j \end{aligned}\right\} \qquad (j = 1, 2, 3, 4) \tag{21}$$

The pure dynamic unbalance condition is maintained with the relationships

$$\left.\begin{aligned} \Delta\alpha_1 &= -\Delta\alpha_2 = -\Delta\alpha_3 = \Delta\alpha_4 = \frac{\Delta\alpha}{4} \\ \Delta\dot{\alpha}_1 &= -\Delta\dot{\alpha}_2 = -\Delta\dot{\alpha}_3 = \Delta\dot{\alpha}_4 = \frac{\Delta\dot{\alpha}}{4} \\ \Delta\ddot{\alpha}_1 &= -\Delta\ddot{\alpha}_2 = -\Delta\ddot{\alpha}_3 = \Delta\ddot{\alpha}_4 = \frac{\Delta\ddot{\alpha}}{4} \end{aligned}\right\} \tag{22}$$

Also, by considering equations (12) and (21),

$$\left.\begin{aligned} s\alpha_{1s} + s\alpha_{4s} &= 0 \\ s\alpha_{2s} + s\alpha_{3s} &= 0 \\ c\alpha_{1s} + c\alpha_{4s} &= 0 \\ c\alpha_{2s} + c\alpha_{3s} &= 0 \end{aligned}\right\} \tag{23}$$

Equations (21) are substituted into equation (7) to yield four moment equations – one for each controller. The equations for controllers 2 and 3 are summed and subtracted from the sum of the equations for controllers 1 and 4. This result is combined with equations (10), (22), and (23) with the result

$$\left(I_{j,z} + m\ell^2\right)\Delta\ddot{\alpha} + C\,\Delta\dot{\alpha} + 2m\ell h\left(\dot{\omega}_x - \omega_y\omega_z\right)\left[\left(c\alpha_{1s} - c\alpha_{3s}\right) - \frac{\Delta\alpha}{4}\left(s\alpha_{1s} + s\alpha_{3s}\right)\right]$$

$$+ 2m\ell h\left(\dot{\omega}_y + \omega_x\omega_z\right)\left[\left(s\alpha_{1s} - s\alpha_{3s}\right) + \frac{\Delta\alpha}{4}\left(c\alpha_{1s} + c\alpha_{3s}\right)\right] = 0 \tag{24}$$

where

$$\left.\begin{aligned}\dot{\omega}_x - \omega_y\omega_z &= \frac{\omega_z{}^2}{I - I_z}\frac{I_z}{I}\left[I_{xz}\sin\xi t + I_{yz}\cos\xi t - \frac{I}{I_z}I_{yz}\right]\\ \dot{\omega}_y + \omega_x\omega_z &= -\frac{\omega_z{}^2}{I - I_z}\frac{I_z}{I}\left[I_{xz}\cos\xi t - I_{yz}\sin\xi t - \frac{I}{I_z}I_{xz}\right]\end{aligned}\right\} \tag{25}$$

and during the perturbation about steady state,

$$\left.\begin{aligned}I_{xz} &= \frac{m\ell h}{2}\,\Delta\alpha\left(s\alpha_{1s} - s\alpha_{3s}\right)\\ I_{yz} &= \frac{m\ell h}{2}\,\Delta\alpha\left(c\alpha_{1s} - c\alpha_{3s}\right)\end{aligned}\right\} \tag{26}$$

Substituting equations (25) and (26) into equation (24), ignoring the higher order $(\Delta\alpha)^2$ term, and simplifying leads to

$$\left(I_{j,z} + m\ell^2\right)\Delta\ddot{\alpha} + C\,\Delta\dot{\alpha} + \Delta\alpha\left(c_1 + c_2\cos\xi t\right) = 0 \tag{27}$$

where

$$c_1 = \frac{2\omega_z{}^2}{I - I_z}(m\ell h)^2(1 - G)$$

$$c_2 = -\frac{2\omega_z{}^2}{I - I_z}\frac{I_z}{I}(m\ell h)^2(3 - G)$$

This nonlinear equation can be related to a Mathieu equation with known solutions by defining a new independent variable

$$T = \xi t$$

and a new dependent variable

$$P = \Delta\alpha \exp\left[-\frac{1}{2}\int \frac{-C\ dT}{\left(I_{j,z} + m\ell^2\right)\xi}\right] \tag{28}$$

with the result

$$P'' + K_1 P' + \left(K_2 \cos T\right)P = 0$$

where

$$K_1 = \frac{c_1}{\left(I_{j,z} + m\ell^2\right)\xi^2} - \frac{1}{4}\left[\frac{C}{\left(I_{j,z} + m\ell^2\right)\xi}\right]^2$$

$$K_2 = \frac{c_2}{\left(I_{j,z} + m\ell^2\right)\xi^2}$$

$$\frac{d(\ )}{dT} = (\ ')$$

The solution to this equation is discussed in reference 3. In general, the regions for a stable solution are defined by the condition $K_1 \geqq 0$ for small values of K_2. This definition leads to

$$C^2 \leqq \frac{8\omega_z^{\ 2}}{I - I_z} m^2\ell^4h^2(1 - G) \tag{29}$$

Consideration of equations (18) and (29) indicates that $G < 1$ for finite damping. Thus, the vital condition for a stable solution is $I > I_z$.

These conditions are for the P solution. Of more importance is the $\Delta\alpha$ solution which is modified by a stabilizing exponential (eq. (28)). However, for normal values of damping coefficient C, the exponential coefficient is much smaller than unity and the two solutions will have about the same degree of stability and the same stability conditions.

Controller response to static unbalance.- Equation (6) can be simplified to determine stability of controller response to static unbalance by setting $A_{1,z} = 0$ and eliminating other small terms involving ω_x, ω_y, $\dot{\omega}_z$, $\dot{A}_1$, and $\ddot{A}_1$ with the result

$$\left(I_{j,z} + m_j \ell^2\right)\ddot{\alpha}_j + C_j\dot{\alpha}_j + m_j \omega_z^2 \left(A_{1,x} \ell \; s\alpha_j - A_{1,y} \ell \; c\alpha_j\right) = 0$$

Conditions for a stable steady-state response of the controllers to static unbalance were determined from this equation by the method of the previous section to be

$$\left.\begin{array}{l} C_j > 0 \\ \sum\limits_{j=1}^{4} m_j \ell \geqq \text{Total static unbalance} \end{array}\right\} \tag{30}$$

Effects of attitude rate on controller response.- The response of the controllers to a coning motion can be examined by determining the steady-state α_j response of equation (6) for the conditions of no unbalance and some initial attitude rate. For these conditions with $\left(\omega_x, \omega_y\right) << \omega_z$ and $\dot{\omega}_z \approx 0$, equation (6) reduces to

$$\left(I_{j,z} + m_j \ell^2\right)\ddot{\alpha}_j + C_j\dot{\alpha}_j + m\ell h_j\left[\left(-\dot{\omega}_x + \omega_y\omega_z\right)c\alpha_j - \left(\dot{\omega}_y + \omega_x\omega_z\right)s\alpha_j\right]$$
$$+ m_j \ell^2 \; s\alpha_j \; c\alpha_j\left(\omega_y^2 - \omega_x^2\right) + m_j\omega_x\omega_y \ell^2\left(c^2\alpha_j - s^2\alpha_j\right) = 0$$

Substituting $\omega_x = \omega_{x,o} \; c(\xi t) - \omega_{y,o} \; s(\xi t)$ and $\omega_y = \omega_{x,o} \; s(\xi t) + \omega_{y,o} \; c(\xi t)$ into this equation and simplifying yields

$$\left(I_{j,z} + m_j \ell^2\right)\ddot{\alpha}_j + C_j\dot{\alpha}_j + m\ell h_j\omega_{xy}\omega_z \frac{I_z}{I} \sin\left(\xi t + \lambda - \alpha_j\right)\omega_{xy}^2 \frac{m\ell^2}{2} \sin\left[2\left(\xi t + \lambda - \alpha_j\right)\right] = 0 \tag{31}$$

where

$$\omega_{xy} = \sqrt{\omega_{x,o}^2 + \omega_{y,o}^2}$$

$$\lambda = \tan^{-1}\left(\frac{\omega_{y,o}}{\omega_{x,o}}\right)$$

The two forcing terms disappear for

$$\alpha_j = \xi t + \tan^{-1}\left(\frac{\omega_{y,o}}{\omega_{x,o}}\right)$$

That is, the coning motion generates controller forces and torques tending to stabilize the location of the controllers along the cross spin rate vector ω_{xy}. This vector precesses in inertial space at the coning rate $\omega_z \frac{I_z}{I}$. The controllers cannot follow this motion because the damping term $C_j \dot{\alpha}_j$ in equation (31) produces sufficiently large moments to keep the controllers essentially rotating along with the spinning body. As the body and controllers spin around the spin axis, the coning-generated forces in effect move around the body at the rate ξ (the coning rate less the spin rate). Thus, the controllers experience a cyclic torque from these forces and respond with a small-amplitude oscillation of frequency ξ. An approximation of this amplitude was determined from equation (31) to be

$$\text{Controller oscillation amplitude due to coning effect} = \pm \frac{m \ell h \omega_{xy} \omega_z \left(I_z/I\right)}{\xi \sqrt{m^2 \ell^4 \xi^2 + C_j^2}} \approx \pm \frac{h}{\ell} \omega_z \frac{I_z}{I} \frac{\omega_{xy}}{\xi^2} \tag{32}$$

This result is an important one as it represents the lower limit of controller activity during spacecraft coning motions. This effect will be illustrated in the "Computer Simulation Results" section. The trim value of this oscillation is determined by balance requirements as previously explained. Coning-induced controller oscillations cause only very small variations in spacecraft balance (less than two parts per thousand of the initial unbalance).

In the section entitled "Response to Dynamic Unbalance," conditions required to counteract inertia product inputs were determined for $\omega_{x,o} = \omega_{y,o} = 0$, since the coning effect on equations (12) was unknown. When the effect was determined to be small, the analysis was repeated to include the coning effects. Results were basically the same except that a new G incorporating attitude rates was defined

$$G = \frac{\left(\frac{I - I_z}{\omega_z}\right)^2 \omega_{xy}^2 + I_{r,z}^2 - 2I_{r,z} \frac{I - I_z}{\omega_z} \omega_{xy} \cos\left(\phi_1 - \lambda\right)}{2(2m\ell h)^2} - 1 \tag{33}$$

where $I_{r,z} \neq 0$ because equations (12) are not valid for $I_{r,z} = 0$. This value of G must satisfy the inequality $-1 \leqq G \leqq 1$ and equation (29).

Steady-State Response of the Controllers to Combined Static and Dynamic Crew Unbalance

The steady-state response of the controllers to combined static and dynamic crew unbalance can be determined for steady spin about the desired spin axis as follows. The addition of coning motion has little effect on these results because of the small coning forces inherent in this application.

The condition of static balance about the x- and y-axes is given by

$$m_c r_{c,x} + m_k r_{k,x} + \sum_{j=1}^{4} m_j \ell \, c\alpha_{js} = 0 \tag{34}$$

$$m_c r_{c,y} + m_k r_{k,y} + \sum_{j=1}^{4} m_j \ell \, s\alpha_{js} = 0 \tag{35}$$

The dynamic balance conditions about the x- and y-axes are given by

$$m_c r_{c,y}\left(r_{c,z} + A_{1,z}\right) + m_k r_{k,y}\left(r_{k,z} + A_{1,z}\right) + \sum_{j=1}^{4} \left[m_j \ell \, s\alpha_{js}\left(h_j + A_{1,z}\right)\right] = 0 \tag{36}$$

$$m_c r_{c,x}\left(r_{c,z} + A_{1,z}\right) + m_k r_{k,x}\left(r_{k,z} + A_{1,z}\right) + \sum_{j=1}^{4} \left[m_j \ell \, c\alpha_{js}\left(h_j + A_{1,z}\right)\right] = 0 \tag{37}$$

Let

$$\left.\begin{aligned} m &= m_1 = m_2 = m_3 = m_4 \\ h &= -h_1 = h_2 = -h_3 = h_4 \end{aligned}\right\} \tag{38}$$

From equations (34), (37), and (38),

$$c\alpha_{1s} - c\alpha_{2s} + c\alpha_{3s} - c\alpha_{4s} = \frac{1}{m\ell h}\left(m_c r_{c,x} r_{c,z} + m_k r_{k,x} r_{k,z}\right) \tag{39}$$

$$c\alpha_{1s} + c\alpha_{2s} + c\alpha_{3s} + c\alpha_{4s} = -\frac{1}{m\ell}\left(m_c r_{c,x} + m_k r_{k,x}\right) \tag{40}$$

From equations (35), (36), and (38),

$$s\alpha_{1s} - s\alpha_{2s} + s\alpha_{3s} - s\alpha_{4s} = \frac{1}{m\ell h}\left(m_c r_{c,y} r_{c,z} + m_k r_{k,y} r_{k,z}\right) \tag{41}$$

$$s\alpha_{1s} + s\alpha_{2s} + s\alpha_{3s} + s\alpha_{4s} = -\frac{1}{m\ell}\left(m_c r_{c,y} + m_k r_{k,y}\right) \tag{42}$$

Combining equations (39) and (40) yields

$$c\alpha_{2s} + c\alpha_{4s} = a_1 = -\frac{1}{2m\ell h}\left[m_c r_{c,x}\left(h + r_{c,z}\right) + m_k r_{k,x}\left(h + r_{k,z}\right)\right] \tag{43}$$

$$c\alpha_{1s} + c\alpha_{3s} = b_1 = -\frac{1}{2m\ell h}\left[m_c r_{c,x}\left(h - r_{c,z}\right) + m_k r_{k,x}\left(h - r_{k,z}\right)\right] \tag{44}$$

Combining equations (41) and (42) yields

$$s\alpha_{2s} + s\alpha_{4s} = a_2 = -\frac{1}{2m\ell h}\left[m_c r_{c,y}\left(h + r_{c,z}\right) + m_k r_{k,y}\left(h + r_{k,z}\right)\right] \tag{45}$$

$$s\alpha_{1s} + s\alpha_{3s} = b_2 = -\frac{1}{2m\ell h}\left[m_c r_{c,y}\left(h - r_{c,z}\right) + m_k r_{k,y}\left(h - r_{k,z}\right)\right] \tag{46}$$

Finally, from equations (43) and (45), the steady-state responses of controllers 2 and 4 result

$$\left.\begin{aligned} \alpha_{2s} &= \tan^{-1}\left(\frac{a_2}{a_1}\right) \mp \cos^{-1}\left(\frac{+\sqrt{a_1^2 + a_2^2}}{2}\right) \\ \alpha_{4s} &= \tan^{-1}\left(\frac{a_2}{a_1}\right) \pm \cos^{-1}\left(\frac{+\sqrt{a_1^2 + a_2^2}}{2}\right) \end{aligned}\right\} \tag{47a}$$

Similarly, from equations (44) and (46), the responses for controllers 1 and 3 result

$$\left.\begin{aligned} \alpha_{1s} &= \tan^{-1}\left(\frac{b_2}{b_1}\right) \mp \cos^{-1}\left(\frac{+\sqrt{b_1^2 + b_2^2}}{2}\right) \\ \alpha_{3s} &= \tan^{-1}\left(\frac{b_2}{b_1}\right) \pm \cos^{-1}\left(\frac{+\sqrt{b_1^2 + b_2^2}}{2}\right) \end{aligned}\right\} \tag{47b}$$

These results compare closely with the computer simulation of a spinning and coning spacecraft reported in the "Computer Simulation Results" section.

Controller Sizing Criteria for Combined Static and Dynamic Unbalance

Coning motion is not considered in this analysis. However, the effects on control sizing are negligible.

A diagram of mass centers and connective geometry showing the steady-state response of the controllers to a combination static and dynamic unbalance imposed by crew mass offsets, $r_{c,y}$ and $r_{c,z}$, on a spinning spacecraft, is shown in figure 5. The static balance equation (moments about the z-axis) is

$$m_c r_{c,xy} = \left(m_1 + m_3\right)\Delta x_1 + \left(m_2 + m_4\right)\Delta x_2$$

The dynamic balance equation (moments of centrifugal forces about an axis perpendicular to the plane of fig. 5 through m_{T2}) is

$$m_c \omega_z^2 r_{c,xy}\left(r_{c,z} - \Delta h\right) + \left(m_1 + m_3\right)\omega_z^2\left(\Delta x_1\right)(h + \Delta h) = \left(m_2 + m_4\right)\omega_z^2\left(\Delta x_2\right)(h - \Delta h)$$

Combining these equations with $m_1 = m_2 = m_3 = m_4 = m_j$ results in

$$\Delta x_1 = \frac{m_c r_{c,xy}\left(h_j - r_{c,z}\right)}{4m_j h}$$

$$\Delta x_2 = \frac{m_c r_{c,xy}\left(h_j + r_{c,z}\right)}{4m_j h}$$

For $r_{c,z} > 0$, Δx_2 is larger than Δx_1 and should be used to size the controllers.[1] Since, in the extreme case, Δx_2 cannot exceed the controller length ℓ, an inequality can be written

$$\ell \geqq \frac{m_c r_{c,xy}\left(h + r_{c,z}\right)}{4m_j h}$$

and

$$m_j \ell \geqq \frac{1}{4}\left(m_c r_{c,xy}\right) + \frac{1}{4h}\left(m_c r_{c,xy} r_{c,z}\right)$$

---

[1]For $r_{c,z} \leqq 0$, the Δx_1 equation leads to the same result.

or

$$\sum_{j=1}^{4} m_j \ell \geqq \text{(Total static unbalance)} + \frac{1}{h}\text{(Total dynamic unbalance)} \tag{48}$$

This relation is the sum of the static criteria of inequality equations (30) and the dynamic criteria of inequality (19) and shows that h controls the relative sensitivity of the controllers to dynamic unbalance and static unbalance. For example, increasing h increases the effectiveness of the controllers to reduce or eliminate dynamic unbalance without directly affecting their ability to control static unbalance. It should be pointed out that a violation of inequality (48) means only that the controllers are unable to counteract the excess of unbalance.

In summary, conditions required for successful operation of the controllers as static and dynamic balancers of large rigid-body spacecraft include

(1) $C_j > 0$

(2) $I_Z < I$

(3) $\sum_{j=1}^{4} m_j \ell \geqq \text{(Total static unbalance)} + \frac{1}{h}\text{(Total dynamic unbalance)}$

COMPUTER SIMULATION RESULTS

The computer simulation (appendix B) considered a large, rigid-body space vehicle equipped with four controllers. Mass and inertial properties are presented in table I. Initially, the vehicle is assumed to be spinning slowly about its axis of symmetry in a balanced condition. At a given time ($t = 10$), 20 crew members (1500 kg) start moving radially outward from the mass center in a direction midway between the x- and y-axes at a speed of about 0.85 m/s. Twenty seconds later they arrive at point $x, y, z = 12, 12, 0$. They immediately change their motion to 0.6 m/s in the z-direction and continue for 20 more seconds at which time ($t = 50$) they stop at the spacecraft location $x, y, z = 12, 12, 12$. These motions of the crew introduce static and dynamic unbalance to the spacecraft.

Typical simulation results are shown in figures 6 to 10. Figure 6 presents the angular motion history for each controller. To illustrate the frequency content of these curves, the second derivative of α_1 is also given. A basic period of some 60 seconds is evident throughout the simulation. This period represents the mass center translation mode. More noticeable over the last 300 seconds is the precessional motion mode characterized by a period of about 25 seconds. This effect was described in the "Analysis" section. The controller is being driven by the coning motion of the spacecraft. The effect does not show up in the early part of the simulation because the restoring moments on the controllers due to the balancing action are overpowering. Controller oscillation amplitude associated with this response was measured to be about $\pm 0.037^{\circ}$. The computed value from equation (32) is $\pm 0.03^{\circ}$. As previously mentioned, the coning motion effect on the controllers is important in that it controls the lower limit of controller activity. The controllers cannot come completely to rest with respect to the spacecraft until the coning motion ceases.

Figure 7 presents mass center offset in the x- and y-directions from the desired z-axis location. Also, the vector sum of these curves is shown to illustrate total offset of the mass center. Mass center offset levels for the same simulation without controllers are also indicated for comparison on these plots. Comparisons show that the passive controllers effectively reduce the static unbalance throughout the simulation.

A similar result is evident from figure 8 which presents histories of principal-axis misalinement about the x- and y-axes and total principal-axis misalinement. These quantities are a measure of dynamic balance. Again, results of the same simulation with controllers eliminated are shown for comparison. The ability of the controllers to simultaneously reduce or eliminate the static and dynamic unbalance is reflected in figures 7 and 8.

The inertial attitude response of the spacecraft to the crew motion disturbances is presented in figure 9. Part (a) of figure 9 shows the trace of the z-axis in the $\phi\theta$-plane for the spacecraft with controllers and for the spacecraft without controllers, both in the interval $650 \leqq t \leqq 680$. The presence of the controllers clearly has eliminated much of the unwanted heading angle.

The $\phi\theta$-response for "no controllers" in figure 9(a) is periodic and repeats every precession cycle (about 25.5-second period). The response with controllers also is cyclic at the precession frequency but changes from cycle to cycle due to movement of the controllers and the resultant change in mass and inertial properties of the overall spacecraft. This condition is evident from figure 9(b) which presents the history of the resultant heading angle. By $t = 700$, this heading response has reached its steady-state character (a small-amplitude coning motion) since controller motion has essentially ceased.

Attitude Instability

Energy dissipation results in attitude instability for a torque-free gyroscopically stabilized body if the spin axis is not the axis of maximum moment of inertia. (See ref. 4, for example.) As pointed out in the analysis section, the use of passive controllers with a rigid body is limited to the case where the moment of inertia about the spin axis is smaller than the moment of inertia about the transverse axes. Thus, in the rigid-body application, the use of passive balancers implies a certain amount of attitude instability. Although there was no indication of attitude instability (cone-angle growth) in the crew-motion disturbance simulations, some of which extended up to 1200 seconds duration, the expected instability is very slow acting and would likely require a small corrective control torque over a long term history of disturbances.

This type of attitude instability can be passively controlled for the case of the dual-spin vehicle previously described. Reference 1 shows that it is only necessary to provide a wobble damper in the hub or hub side of the bearing which will have an energy dissipation rate sufficient to dominate the energy dissipation of the disk (structural plus controller damping). The spacecraft motion will then be stable and cone angle will gradually decrease.

System Time Constants

The computer simulation results presented in figures 6 to 9 represent a spacecraft, crew, and controllers with properties listed in table I. Responses to a given disturbance were plotted for some 700 seconds. The system time constant for this simulation was about 200 seconds. The ratio of total controller mass to total spacecraft mass was about 3.5 percent. This ratio is unnecessarily large and can be reduced considerably. Figure 10 shows the effect of controller mass and length on the system time constant. The upper plot illustrates a linear increase in the system time constant with controller length and the lower plot a linear increase in time constant with controller mass. The relationship in equation form is

$$\text{System time constant} = 86 + 10\ell + 0.0425m_j \tag{49}$$

Simulation results presented in previous figures are represented by a shaded symbol in each of the plots in figure 10. Note that controller mass could have been halved (to 1600 kg) with an improved response time. Also, controller length can be decreased to improve response time. The only disadvantage to reducing controller mass and/or length is in violating the stability limits of equation (48). These limits are indicated in both plots of figure 10 by dashed curves developed from equations (48) and (49). These curves indicate combinations of ℓ and m_j for which two or more of the controllers are exercising all their balancing capacity. Operations beyond this limit are not desirable because the excess of unbalance will cause small unwanted coning and nutational motions of the spacecraft. However, these motions will cease when the excess unbalance is removed.

Ideally, controllers should be designed to operate near the dashed lines of figure 10 and at a minimum time constant. However, each unbalance input will have different limiting curves and the design may have to be based on the largest unbalance anticipated. This condition could result in large system response times for applications having widely varying balance requirements.

A means of avoiding large response times and controller ineffectiveness is to design the system for the relatively low levels of unbalance experienced in normal operations with provision for temporarily increasing controller length (during operation) in preparation for occasional activities requiring a relatively high level of balancing capability such as docking and resupply operations. This provision could be accomplished with a controller design incorporating telescoping arm sections. Variable controller mass would also be a solution to this problem – possibly through fluid transfer.

The effect of controller damping on system time constant is presented in figure 11. As would be expected, increased damping in the range of practical interest tends to reduce the system time constant. For impractically large damping coefficients, however, the effect reverses, especially at near steady-state controller angles where the corrective centrifugal torques are relatively weak and unable to move the controllers against the damping at an adequate rate.

CONCLUDING REMARKS

Equations of motion have been derived for a flexibly connected dual-spin spacecraft equipped with four pendulumlike "passive controllers" for mass balance and spin axis control. The derived equations, simplified by eliminating hub and flexibility terms, were analyzed to determine the conditions required for successful steady-state operation of the controllers with a spinning, rigid-body spacecraft. Results indicated that spacecraft inertia about the desired spin axis must be less than spacecraft inertia about the transverse axes. Positive damping of controller motion relative to the spacecraft is also required. The analysis also indicated that spacecraft coning motion induces very small controller oscillations which prevent the controllers from eliminating about two parts per thousand of the initial unbalance. Controller sizing criteria were determined as a function of balance requirements and related to limiting values of system time constant for a given unbalance condition.

A generalized real-time computer simulation of a large, slowly spinning rigid-body spacecraft incorporating passive controllers has also been presented. Numerical results of this simulation show that passive controllers can successfully balance a class (spin inertia less than transverse inertia) of rotating rigid bodies undergoing large internal mass and inertial disturbances. These results also indicate a large reduction in spacecraft attitude error due to the action of the controllers. The ratio of total controller mass to spacecraft mass need not be more than 1 or 2 percent.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., August 7, 1972.

APPENDIX A

EQUATIONS OF MOTION

Mathematical Model

The mathematical model of a generalized dual-spin space station with passive controllers is shown in figure 2. The model consists of a nonrotating hub, a slowly spinning disk or rotor, and four pendulumlike arms (with end masses) constrained to rotational freedom about the desired spin axis (z-axis). The rotating arms or passive controllers are deployed in two pairs along the z-axis and their motions relative to the disk are damped.

The hub mass is connected flexibly to the disk mass through an arrangement of springs and viscous dampers attached to the inner race of a bearing as shown in figure 3. Thus, spring and damping restraint exists for relative translations of the hub and disk along the x-, y-, and z-axes and for relative rotation of the hub and disk about the x- and y-axes. The presence of the bearing permits relative rotations about the z-axis to be unrestrained; frictional effects about the z-axis are assumed to be effectively compensated by application of an internal torque between the hub and disk. (See fig. 3.) Matrix representation of the spring and damping constants is as follows:

Translational spring constant, newtons/meter:

$$[K] = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{bmatrix}$$

Translational damping constant, newton-sec/meter:

$$[C] = \begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{bmatrix}$$

Rotational spring constant, newton-meters/radian:

$$[K_R] = \begin{bmatrix} K_{R,x} & K_{R,xy} & K_{R,xz} \\ K_{R,xy} & K_{R,y} & K_{R,yz} \\ K_{R,xz} & K_{R,yz} & 0 \end{bmatrix}$$

Rotational damping constant, newton-meter-sec/radian:

$$[C_R] = \begin{bmatrix} C_{R,x} & C_{R,xy} & C_{R,xz} \\ C_{R,xy} & C_{R,y} & C_{R,yz} \\ C_{R,xz} & C_{R,yz} & 0 \end{bmatrix}$$

Although certain off-diagonal elements are listed as zeros, this is not a limitation of the mathematical model. Other coefficients could easily be used in these locations.

Reference coordinates.- Four coordinate axis systems are used (fig. 12): Inertially fixed reference axes x', y', and z'; disk body fixed axes x, y, and z (origin at disk center of figure); hub body fixed axes x_h, y_h, and z_h; and controller fixed axes x_j, y_j, and z_j. Origin of the hub axis system is fixed coincident with the disk axis system when the spring-damper suspension system is undeflected. Disk angular motion is defined relative to the inertial axes by successive Euler rotations ϕ, θ, and ψ, as shown in figure 12(a). Similarly, hub angular motion is defined relative to the disk system by successive Euler rotations ϕ_h, θ_h, and ψ_h as shown in figure 12(b). Hub angular motion relative to the inertial axes is also computed as discussed in the section entitled "Hub Inertial Angles."

Transfer matrices.- Quantities expressed relative to disk body coordinates can be referenced to the inertial coordinate system by premultiplication with the transfer matrix $[D_1]$; that is,

$$\{r_{inertial}\} = [D_1]\{r_{disk}\}$$

where

$$[D_1] = \begin{bmatrix} c\psi\, c\theta & -s\psi\, c\theta & s\theta \\ c\psi\, s\theta\, s\phi + s\psi\, c\phi & c\psi\, c\phi - s\psi\, s\theta\, s\phi & -c\theta\, s\phi \\ s\psi\, s\phi - c\psi\, s\theta\, c\phi & c\psi\, s\phi + s\psi\, s\theta\, c\phi & c\theta\, c\phi \end{bmatrix}$$

Similarly, quantities expressed relative to the hub coordinate system are referred to disk coordinates by the transfer matrix $[D_2]$; that is,

$$\{r_{disk}\} = [D_2]\{r_{hub}\}$$

where

$$
\left[D_2\right] = \begin{bmatrix} c\psi_h\, c\theta_h & -s\psi_h\, c\theta_h & s\theta_h \\ c\psi_h\, s\theta_h\, s\phi_h + s\psi_h\, c\phi_h & c\psi_h\, c\phi_h - s\psi_h\, s\theta_h\, s\phi_h & -c\theta_h\, s\phi_h \\ s\psi_h\, s\phi_h - c\psi_h\, s\theta_h\, c\phi_h & c\psi_h\, s\phi_h + s\psi_h\, s\theta_h\, c\phi_h & c\theta_h\, c\phi_h \end{bmatrix}
$$

Note that $\left[D_2\right]$ is $\left[D_1\right]$ with h-subscripted Euler angles.

Quantities expressed relative to controller coordinate systems are referred to disk coordinates by the transfer matrix $\left[D_3\right]$; that is,

$$
\left\{r_{disk}\right\} = \left[D_3\right]\left\{r_{controller}\right\}
$$

where

$$
\left[D_3\right] = \begin{bmatrix} c\alpha_j & -s\alpha_j & 0 \\ s\alpha_j & c\alpha_j & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

the matrices $\left[D_1\right]$, $\left[D_2\right]$, and $\left[D_3\right]$ are all orthogonal transformations and the matrix inverse is equal to the matrix transpose.

The relationship between Euler rates and inertial body rates for the disk is

$$
\left\{\omega\right\} = \left[D\right]\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}
$$

where

$$
\left[D\right] = \begin{bmatrix} c\psi\, c\theta & s\psi & 0 \\ -s\psi\, c\theta & c\psi & 0 \\ s\theta & 0 & 1 \end{bmatrix}
$$

The matrix transferring disk-relative hub Euler rates to disk-relative hub body rates is

$$\left[D_h\right] = \begin{bmatrix} c\psi_h\, c\theta_h & s\psi_h & 0 \\ -s\psi_h\, c\theta_h & c\psi_h & 0 \\ s\theta_h & 0 & 1 \end{bmatrix}$$

Note that $\left[D_h\right]$ is $\left[D\right]$ with h-subscripted angles. Neither $\left[D\right]$ nor $\left[D_h\right]$ is orthogonal.

Hub inertial body rates can be expressed in terms of Euler rates of the disk and hub as follows:

$$\left\{\omega_h\right\} = \left[D_h\right]\begin{Bmatrix} \dot{\phi}_h \\ \dot{\theta}_h \\ \dot{\psi}_h \end{Bmatrix} + \left[D_2\right]^T\left[D\right]\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}$$

Finally, controller inertial body rates are expressed in terms of disk rates and controller rate relative to the disk as

$$\left\{\omega_j\right\} = \left[D_3\right]^T\left\{\omega\right\} + \begin{Bmatrix} 0 \\ 0 \\ \dot{\alpha}_j \end{Bmatrix} = \begin{Bmatrix} \omega_x\, c\alpha_j + \omega_y\, s\alpha_j \\ -\omega_x\, s\alpha_j + \omega_y\, c\alpha_j \\ \omega_z + \dot{\alpha}_j \end{Bmatrix}$$

All transfer matrices, some of their derivatives, and their inverses are listed for convenience in appendix C.

Position coordinates of mass centers.- The mass centers of the disk, hub, crew, and individual balance masses are all located in disk coordinates as shown in figure 2 by the subscripted r vectors. The same mass centers are located in inertial coordinates relative to the total (or overall) mass center by the subscripted R vectors. The hub mass center is also located in hub coordinates $\left\{r_f\right\}$, as shown in figure 2 where the origin of the hub axis system is shown displaced from the disk axis system. The following relationships can be determined from figure 2 and knowledge of the transformation matrices

$$\left\{R_d\right\} = \left\{R\right\} + \left[D_1\right]\left\{r_d\right\}$$

$$\left\{R_j\right\} = \left\{R\right\} + \left[D_1\right]\left\{r_j\right\}$$

$$\left\{R_c\right\} = \left\{R\right\} + \left[D_1\right]\left\{r_c\right\}$$

$$\left\{R_h\right\} = \left\{R\right\} + \left[D_1\right]\left\{r_h\right\} = \left\{R\right\} + \left[D_1\right]\left(\left\{r\right\} + \left[D_2\right]\left\{r_f\right\}\right)$$

Taking derivatives $\left(\left\{\dot{r}_d\right\} = \left\{\dot{r}_f\right\} = 0\right)$ yields

$$\left\{\dot{R}_d\right\} = \left\{\dot{R}\right\} + \left[\dot{D}_1\right]\left\{r_d\right\}$$

$$\left\{\dot{R}_j\right\} = \left\{\dot{R}\right\} + \left[\dot{D}_1\right]\left\{r_j\right\} + \left[D_1\right]\left\{\dot{r}_j\right\}$$

$$\left\{\dot{R}_c\right\} = \left\{\dot{R}\right\} + \left[\dot{D}_1\right]\left\{r_c\right\} + \left[D_1\right]\left\{\dot{r}_c\right\}$$

$$\left\{\dot{R}_h\right\} = \left\{\dot{R}\right\} + \left[\dot{D}_1\right]\left(\left\{r\right\} + \left[D_2\right]\left\{r_f\right\}\right) + \left[D_1\right]\left(\left\{\dot{r}\right\} + \left[\dot{D}_2\right]\left\{r_f\right\}\right)$$

and

$$\left\{\ddot{R}_d\right\} = \left\{\ddot{R}\right\} + \left[\ddot{D}_1\right]\left\{r_d\right\}$$

$$\left\{\ddot{R}_j\right\} = \left\{\ddot{R}\right\} + \left[\ddot{D}_1\right]\left\{r_j\right\} + 2\left[\dot{D}_1\right]\left\{\dot{r}_j\right\} + \left[D_1\right]\left\{\ddot{r}_j\right\}$$

$$\left\{\ddot{R}_c\right\} = \left\{\ddot{R}\right\} + \left[\ddot{D}_1\right]\left\{r_c\right\} + 2\left[\dot{D}_1\right]\left\{\dot{r}_c\right\} + \left[D_1\right]\left\{\ddot{r}_c\right\}$$

$$\left\{\ddot{R}_h\right\} = \left\{\ddot{R}\right\} + \left[\ddot{D}_1\right]\left(\left\{r\right\} + \left[D_2\right]\left\{r_f\right\}\right) + 2\left[\dot{D}_1\right]\left(\left\{\dot{r}\right\} + \left[\dot{D}_2\right]\left\{r_f\right\}\right) + \left[D_1\right]\left(\left\{\ddot{r}\right\} + \left[\ddot{D}_2\right]\left\{r_f\right\}\right)$$

The mass balance equations for the entire spacecraft can be determined from figure 2. Taking mass moments about the disk origin and expressing quantities in inertial coordinates results in

$$m_T\left\{R\right\} + \left[D_1\right]\left(m_d\left\{r_d\right\} + m_c\left\{r_c\right\} + \sum_{j=1}^{4}\left(m_j\left\{r_j\right\}\right) + m_h\left\{r_h\right\}\right) = 0$$

or

$$\left\{R\right\} = \left[D_1\right]\left\{A_1\right\}$$

and

$$\left\{\dot{R}\right\} = \left[\dot{D}_1\right]\left\{A_1\right\} + \left[D_1\right]\left\{\dot{A}_1\right\}$$

$$\left\{\ddot{R}\right\} = \left[\ddot{D}_1\right]\left\{A_1\right\} + 2\left[\dot{D}_1\right]\left\{\dot{A}_1\right\} + \left[D_1\right]\left\{\ddot{A}_1\right\}$$

where

$$\left\{A_1\right\} = -\frac{1}{m_T}\left(m_d\left\{r_d\right\} + m_c\left\{r_c\right\} + \sum_{j=1}^{4}\left(m_j\left\{r_j\right\}\right) + m_h\left\{r_h\right\}\right)$$

the location of total mass center in disk coordinates. Also $m_T = m_d + m_c + m_h + \sum_{j=1}^{4} m_j$, $\left\{r_d\right\}$ and $\left\{r_f\right\}$ are given constants, and $\left\{r_c\right\}$ is input as a time function. For the controllers,

$$\left\{r_j\right\} = \begin{Bmatrix} \ell\, c\alpha_j \\ \ell\, s\alpha_j \\ h_j \end{Bmatrix}$$

$$\left\{\dot{r}_j\right\} = \begin{Bmatrix} -\ell\dot{\alpha}_j\, s\alpha_j \\ \ell\dot{\alpha}_j\, c\alpha_j \\ 0 \end{Bmatrix}$$

$$\left\{\ddot{r}_j\right\} = \begin{Bmatrix} -\ell\, s\alpha_j\ddot{\alpha}_j \\ \ell\, c\alpha_j\ddot{\alpha}_j \\ 0 \end{Bmatrix} + \begin{Bmatrix} -\ell\, c\alpha_j\dot{\alpha}_j^{\,2} \\ -\ell\, s\alpha_j\dot{\alpha}_j^{\,2} \\ 0 \end{Bmatrix}$$

External forces and moments.- External forces are assumed to be zero during normal operation of the space station. However, there are occasional periods during which orbit corrections, docking impacts, etc., will require application of external forces and moments to the station. Therefore, terms have been included in the equations of motion to supply external forces and moments both to the disk and to the hub.

Inertia properties.- The hub, disk, crew, and passive controllers are assumed to have constant inertial properties about their own axes as follows:

Hub:

$$\left[I_h\right] = \begin{bmatrix} I_{h,x} & -I_{h,xy} & -I_{h,xz} \\ -I_{h,xy} & I_{h,y} & -I_{h,yz} \\ -I_{h,xz} & -I_{h,yz} & I_{h,z} \end{bmatrix}$$

Rotor or disk:

$$\left[I_d\right] = \begin{bmatrix} I_{d,x} & -I_{d,xy} & -I_{d,xz} \\ -I_{d,xy} & I_{d,y} & -I_{d,yz} \\ -I_{d,xz} & -I_{d,yz} & I_{d,z} \end{bmatrix}$$

Controllers:

$$\left[I_j\right] = \begin{bmatrix} I_{j,x} & 0 & 0 \\ 0 & I_{j,y} & 0 \\ 0 & 0 & I_{j,z} \end{bmatrix}$$

Crew:

$$\left[I_c\right] = \begin{bmatrix} I_{c,x} & -I_{c,xy} & -I_{c,xz} \\ -I_{c,xy} & I_{c,y} & -I_{c,yz} \\ -I_{c,xz} & -I_{c,yz} & I_{c,z} \end{bmatrix}$$

Langrange's Equations of Motion

The space station with passive controllers has 16 degrees of freedom; one rotational degree for each of the controllers and three translational and three rotational degrees for both the disk and the hub. The analysis of this system has been simplified by choosing 16 independent generalized coordinates to represent it. These independent coordinates are the controller rotation angles α_1, α_2, α_3, and α_4, disk Euler angles ϕ, θ, and ψ, inertial coordinates of disk x', y', and z', disk coordinates of hub r_x, r_y, and r_z, and disk-relative hub Euler angles ϕ_h, θ_h, and ψ_h.

The equations of motion are derived by substitutions of the appropriate terms in Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial F_d}{\partial \dot{q}_i} = Q_i \qquad (i = 1, 2, \ldots, 16) \qquad \text{(A1)}$$

where the q_i represent the 16 generalized independent coordinates. Before illustrating the method with an arbitrarily selected coordinate, it will be necessary to define T, V, F_d, and Q in terms of the basic physical quantities.

Kinetic energy.- The kinetic energy of the space station includes the translatory and rotational kinetic energies of the disk, hub, crew, and passive controllers. It can be written in the form

$$\begin{aligned} T = {} & \frac{1}{2} m_d \left\{\dot{R}_d\right\}^T \left\{\dot{R}_d\right\} + \frac{1}{2}\left\{\omega\right\}^T \left[I_d\right]\left\{\omega\right\} + \frac{1}{2} m_h \left\{\dot{R}_h\right\}^T \left\{\dot{R}_h\right\} + \frac{1}{2}\left\{\omega_h\right\}^T \left[I_h\right]\left\{\omega_h\right\} \\ & + \frac{1}{2} m_c \left\{\dot{R}_c\right\}^T \left\{\dot{R}_c\right\} + \frac{1}{2}\left\{\omega\right\}^T \left[I_c\right]\left\{\omega\right\} + \frac{1}{2}\sum_{j=1}^{4}\left(m_j\left\{\dot{R}_j\right\}^T\left\{\dot{R}_j\right\}\right) + \frac{1}{2}\sum_{j=1}^{4}\left(\left\{\omega_j\right\}^T\left[I_j\right]\left\{\omega_j\right\}\right) \\ & + \frac{1}{2} m_T \left\{\dot{R}_g\right\}^T \left\{\dot{R}_g\right\} \end{aligned} \qquad \text{(A2)}$$

For the rotational degrees of freedom, a more workable form of the energy equation will be used; namely,

$$\begin{aligned} T = {} & \frac{1}{2} m_d \left\{\dot{R}_d\right\}^T \left\{\dot{R}_d\right\} + \frac{1}{2} m_h \left\{\dot{R}_h\right\}^T \left\{\dot{R}_h\right\} + \frac{1}{2} m_c \left\{\dot{R}_c\right\}^T \left\{\dot{R}_c\right\} + \frac{1}{2}\sum_{j=1}^{4}\left(m_j\left\{\dot{R}_j\right\}^T\left\{\dot{R}_j\right\}\right) \\ & + \frac{1}{2} m_T \left\{\dot{R}_g\right\}^T \left\{\dot{R}_g\right\} + \frac{1}{2}\left\{\omega\right\}^T \left[I\right]\left\{\omega\right\} + \frac{1}{2}\left\{\omega_h\right\}^T \left[I_h\right]\left\{\omega_h\right\} + \frac{1}{2}\sum_{j=1}^{4}\left(2I_{j,z}\omega_z\dot{\alpha}_j + I_{j,z}\dot{\alpha}_j^{2}\right) \end{aligned} \qquad \text{(A3)}$$

where

$$\left[I\right] = \left[I_d\right] + \left[I_c\right] + \sum_{j=1}^{4}\left(\left[D_3\right]\left[I_j\right]\left[D_3\right]^{-1}\right)$$

Potential energy.- The potential energy of the space station consists only of the strain energy of the hub support springs due to relative displacement of the disk and hub. The potential energy can be written as

$$V = \frac{1}{2}\{r\}^T[K]\{r\} + \frac{1}{2}\begin{Bmatrix}\phi_h \\ \theta_h \\ \psi_h\end{Bmatrix}^T [K_R] \begin{Bmatrix}\phi_h \\ \theta_h \\ \psi_h\end{Bmatrix}$$

$$= \frac{1}{2}\left[K_x r_x{}^2 + K_y r_y{}^2 + K_z r_z{}^2 + K_{R,x}\phi_h{}^2 + K_{R,y}\theta_h{}^2\right]$$

$$+ K_{R,xy}\phi_h\theta_h + K_{R,xz}\phi_h\psi_h + K_{R,yz}\theta_h\psi_h \tag{A4}$$

Dissipation function.- The dissipation function for the space base system involves translational and rotational terms generated by the hub support dampers and a rotational term for each of the passive controllers as follows:

$$F_d = \frac{1}{2}\{\dot{r}\}^T[C]\{\dot{r}\} + \frac{1}{2}\begin{Bmatrix}\dot{\phi}_h \\ \dot{\theta}_h \\ \dot{\psi}_h\end{Bmatrix}^T [C_R] \begin{Bmatrix}\dot{\phi}_h \\ \dot{\theta}_h \\ \dot{\psi}_h\end{Bmatrix} + \frac{1}{2}\sum_{j=1}^{4} C_j\dot{\alpha}_j{}^2 \tag{A5}$$

Generalized forces.- The generalized force Q_i associated with a generalized coordinate q_i is given by

$$Q_i = \sum_{j=1}^{n}\left(F_j \frac{\partial x_j}{\partial q_i}\right) \qquad (i = 1, 2, \ldots, 16)$$

where the forces F_j are applied at and along the coordinates x_j. For the dual-spin spacecraft application, the 16 independent generalized coordinates are α_1, α_2, α_3, α_4, ϕ, θ, ψ, x', y', z', r_x, r_y, r_z, ϕ_h, θ_h, and ψ_h. The forces are components of the external forces and torques applied to the disk and hub. The twelve x_j coordinates are the inertial locations of the external force applications.

The generalized forces were determined to be

$$Q_{\alpha,j} = -\frac{m_j}{m_T}\begin{Bmatrix}-\ell\, s\alpha_j \\ \ell\, c\alpha_j \\ 0\end{Bmatrix}^T \{F\} \qquad (j = 1, 2, 3, 4) \tag{A6}$$

$$\begin{Bmatrix} Q_\phi \\ Q_\theta \\ Q_\psi \end{Bmatrix} = \left[D\right]^T \left(\left\{T\right\} + \left[A_1\right]\left\{F\right\} + \left[r\right]\left[D_2\right]\left\{F_h\right\}\right) \tag{A7}$$

$$\begin{Bmatrix} Q_{x'} \\ Q_{y'} \\ Q_{z'} \end{Bmatrix} = \left[D_1\right]\left\{F\right\} \tag{A8}$$

$$\begin{Bmatrix} Q_{r,x} \\ Q_{r,y} \\ Q_{r,z} \end{Bmatrix} = -\frac{m_h}{m_T}\left\{F\right\} + \left[D_2\right]\left\{F_h\right\} \tag{A9}$$

$$\begin{Bmatrix} Q_{\phi,h} \\ Q_{\theta,h} \\ Q_{\psi,h} \end{Bmatrix} = -\frac{m_h}{m_T}\left[D_h\right]^T\left[r_f\right]\left[D_2\right]^{-1}\left\{F\right\} + \left[D_h\right]^T\left\{T_h\right\} \tag{A10}$$

Selecting α_j as the generalized coordinate to illustrate the Lagrange method of derivation yields

$$\frac{\partial T}{\partial \dot{\alpha}_j} = m_d\left\{\dot{R}_d\right\}^T\left\{\frac{\partial \dot{R}_d}{\partial \dot{\alpha}_j}\right\} + m_h\left\{\dot{R}_h\right\}^T\left\{\frac{\partial \dot{R}_h}{\partial \dot{\alpha}_j}\right\} + m_c\left\{\dot{R}_c\right\}^T\left\{\frac{\partial \dot{R}_c}{\partial \dot{\alpha}_j}\right\} + \sum_{j=1}^{4}\left(m_j\left\{\dot{R}_j\right\}^T\right)\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}$$
$$+ m_j\left\{\dot{R}_j\right\}^T\left[D_1\right]\left\{\frac{\partial \dot{r}_j}{\partial \dot{\alpha}_j}\right\} + \left\{\omega_j\right\}^T\left[I_j\right]\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \tag{A11}$$

By using the relationships

$$\left[D_1\right]\left\{\frac{\partial \dot{r}_j}{\partial \dot{\alpha}_j}\right\} = -\frac{m_T}{m_j}\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}$$

$$\left\{\frac{\partial \dot{R}_d}{\partial \dot{\alpha}_j}\right\} = \left\{\frac{\partial \dot{R}_c}{\partial \dot{\alpha}_j}\right\} = \left\{\frac{\partial \dot{R}_h}{\partial \dot{\alpha}_j}\right\} = \left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}$$

and

$$m_d\left\{\dot{R}_d\right\} + \sum_{j=1}^{4}\left(m_j\left\{\dot{R}_j\right\}\right) + m_c\left\{\dot{R}_c\right\} + m_h\left\{\dot{R}_h\right\} = 0$$

equation (A11) reduces to

$$\left\{\frac{\partial T}{\partial \dot{\alpha}_j}\right\} = \left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}^T\left(-m_T\right)\left\{\dot{R}_j\right\} + \left\{\omega_j\right\}^T\left[I_j\right]\begin{Bmatrix}0\\0\\1\end{Bmatrix}$$

and the derivative is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}_j}\right) = -m_T\frac{d}{dt}\left(\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}^T\right)\left\{\dot{R}_j\right\} - m_T\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}^T\left\{\ddot{R}_j\right\} + \left\{\dot{\omega}_j\right\}^T\left[I_j\right]\begin{Bmatrix}0\\0\\1\end{Bmatrix} \tag{A12}$$

Evaluation of the next term in equation (A1) yields

$$\left\{\frac{\partial T}{\partial \alpha_j}\right\} = m_d\left\{\dot{R}_d\right\}^T\left\{\frac{\partial \dot{R}_d}{\partial \alpha_j}\right\} + m_h\left\{\dot{R}_h\right\}^T\left\{\frac{\partial \dot{R}_h}{\partial \alpha_j}\right\} + m_c\left\{\dot{R}_c\right\}^T\left\{\frac{\partial \dot{R}_c}{\partial \alpha_j}\right\} + \sum_{j=1}^{4}\left(m_j\left\{\dot{R}_j\right\}^T\right)\left\{\frac{\partial \dot{R}}{\partial \alpha_j}\right\}$$

$$+ m_j\left\{\dot{R}_j\right\}\left\{\left[\dot{D}_1\right]\left\{\frac{\partial r_j}{\partial \alpha_j}\right\} + \left[D_1\right]\left\{\frac{\partial \dot{r}_j}{\partial \alpha_j}\right\}\right\} + \left\{\omega_j\right\}^T\left[I_j\right]\left[\frac{\partial D_3}{\partial \alpha_j}\right]^T\left\{\omega\right\} \tag{A13}$$

By using the relations

$$\left[\dot{D}_1\right]\left\{\frac{\partial r_j}{\partial \alpha_j}\right\} + \left[D_1\right]\left\{\frac{\partial \dot{r}_j}{\partial \alpha_j}\right\} = -\frac{m_T}{m_j}\left\{\frac{\partial \dot{R}}{\partial \alpha_j}\right\}$$

$$\left\{\frac{\partial \dot{R}_d}{\partial \alpha_j}\right\} = \left\{\frac{\partial \dot{R}_c}{\partial \alpha_j}\right\} = \left\{\frac{\partial \dot{R}_h}{\partial \alpha_j}\right\} = \left\{\frac{\partial \dot{R}}{\partial \alpha_j}\right\}$$

and

$$m_d\left\{\dot{R}_d\right\} + m_c\left\{\dot{R}_c\right\} + m_h\left\{\dot{R}_h\right\} + \sum_{j=1}^{4}\left(m_j\left\{\dot{R}_j\right\}\right) = 0$$

equation (A13) reduces to

$$\left\{\frac{\partial T}{\partial \alpha_j}\right\} = -m_T\left\{\frac{\partial \dot{R}}{\partial \alpha_j}\right\}^T\left\{\dot{R}_j\right\} + \left\{\omega_j\right\}^T\left[I_j\right]\left[\frac{\partial D_3}{\partial \alpha_j}\right]^T\left\{\omega\right\} \tag{A14}$$

For the potential functions,

$$\frac{\partial V}{\partial \alpha_j} = 0 \tag{A15}$$

and from the dissipation functions,

$$\frac{\partial F_d}{\partial \dot{\alpha}_j} = C_j\dot{\alpha}_j \tag{A16}$$

The generalized force for this degree of freedom is

$$Q_{\alpha,j} = -\frac{m_j}{m_T}\begin{Bmatrix} -\ell\ s\alpha_j \\ \ell\ c\alpha_j \\ 0 \end{Bmatrix}^T\left\{F\right\} \tag{A17}$$

Substituting equations (A12), (A14), (A15), (A16), and (A17) into equation (A1) and simplifying yields

$$-m_T\frac{d}{dt}\left(\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}^T\right)\left\{\dot{R}_j\right\} - m_T\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\}^T\left\{\ddot{R}_j\right\} + \left\{\dot{\omega}_j\right\}^T\left[I_j\right]\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} + m_T\left\{\frac{\partial \dot{R}}{\partial \alpha_j}\right\}^T\left\{\dot{R}_j\right\}$$

$$-\left\{\omega_j\right\}^T\left[I_j\right]\left[\frac{\partial D_3}{\partial \alpha_j}\right]^T\left\{\omega\right\} + C_j\dot{\alpha}_j = -\frac{m_j}{m_T}\begin{Bmatrix} -\ell\ s\alpha_j \\ \ell\ c\alpha_j \\ 0 \end{Bmatrix}^T\left\{F\right\} \tag{A18}$$

By noting that

$$\frac{d}{dt}\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\} = \left\{\frac{\partial \dot{R}}{\partial \alpha_j}\right\}$$

$$\left\{\frac{\partial \dot{R}}{\partial \dot{\alpha}_j}\right\} = -\frac{m_j}{m_T}\left[D_1\right]\left\{\frac{\partial \dot{r}_j}{\partial \dot{\alpha}_j}\right\} = -\frac{m_j}{m_T}\left[D_1\right]\begin{Bmatrix} -\ell\ s\alpha_j \\ \ell\ c\alpha_j \\ 0 \end{Bmatrix}$$

and from equation (A24)

$$\left\{F\right\} = m_T\left[D_1\right]^T\left\{\ddot{R}_g\right\}$$

equation (A18) can be written as

$$I_{j,z}\left(\dot{\omega}_z + \ddot{\alpha}_j\right) + m_j\begin{Bmatrix} -\ell\ s\alpha_j \\ \ell\ c\alpha_j \\ 0 \end{Bmatrix}^T\left[D_1\right]^T\left(\left\{\ddot{R}_j\right\} + \left\{\ddot{R}_g\right\}\right) + C_j\dot{\alpha}_j$$

$$= +\left(I_{j,x} - I_{j,y}\right)\left[\left(\omega_y^2 - \omega_x^2\right)s\alpha_j\ c\alpha_j - \omega_x\omega_y\left(s^2\alpha_j - c^2\alpha_j\right)\right] \qquad (j = 1, 2, 3, 4) \qquad \text{(A19)}$$

This equation is the equation of motion for each of the four passive controllers.

Equations of motion for the other 12 degrees of freedom, determined by the same method, are as follows: For the disk rotational degree of freedom ϕ, the Lagrange equation is

$$m_d\left\{\ddot{R}_d\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{A_1 + r_d\right\} + m_d\left\{\dot{R}_d\right\}^T\left(\left[\frac{d}{dt}\left(\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right) - \frac{\partial \dot{D}_1}{\partial \phi}\right]\left\{A_1 + r_d\right\} + \left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}} - \frac{\partial D_1}{\partial \phi}\right]\left\{\dot{A}_1 + \dot{r}_d\right\}\right) + \sum_{j=1}^{4}\left(m_j\left\{\ddot{R}_j\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{A_1 + r_j\right\}\right)$$

$$+ \sum_{j=1}^{4}\left[m_j\left\{\dot{R}_j\right\}^T\left(\left[\frac{d}{dt}\left(\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right) - \frac{\partial \dot{D}_1}{\partial \phi}\right]\left\{A_1 + r_j\right\} + \left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}} - \frac{\partial D_1}{\partial \phi}\right]\left\{\dot{A}_1 + \dot{r}_j\right\}\right)\right] + m_c\left\{\ddot{R}_c\right\}^T\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\left\{A_1 + r_c\right\} + m_c\left\{\dot{R}_c\right\}^T\left(\left[\frac{d}{dt}\left(\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right)\right.\right.$$

$$\left.- \frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{A_1 + r_c\right\} + \left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}} - \frac{\partial D_1}{\partial \phi}\right]\left.\left\{\dot{A}_1 + \dot{r}_c\right\}\right) + m_h\left\{\ddot{R}_h\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{A_1 + r_h\right\} + m_h\left\{\dot{R}_h\right\}^T\left(\left[\frac{d}{dt}\left(\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right) - \frac{\partial \dot{D}_1}{\partial \phi}\right]\left\{A_1 + r_h\right\} + \left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}} - \frac{\partial D_1}{\partial \phi}\right]\left\{\dot{A}_1 + \dot{r}_h\right\}\right)$$

$$+ \left\{\omega\right\}^T\left[I\right]\left\{\frac{d}{dt}\left(\frac{\partial \omega}{\partial \dot{\phi}}\right) - \frac{\partial \omega}{\partial \phi}\right\} + \left\{\dot{\omega}\right\}^T\left[I\right]\left\{\frac{\partial \omega}{\partial \dot{\phi}}\right\} + \left\{\omega\right\}^T\left[\dot{I}\right]\left\{\frac{\partial \omega}{\partial \dot{\phi}}\right\} + \left\{\omega_h\right\}^T\left[I_h\right]\left\{\frac{d}{dt}\left(\frac{\partial \omega_h}{\partial \dot{\phi}}\right) - \frac{\partial \omega_h}{\partial \phi}\right\}$$

$$+ \left\{\dot{\omega}_h\right\}^T\left[I_h\right]\left\{\frac{\partial \omega_h}{\partial \dot{\phi}}\right\} + \left\{\omega_h\right\}^T\left[\dot{I}_h\right]\left\{\frac{\partial \omega_h}{\partial \dot{\phi}}\right\} + \sum_{j=1}^{4}\left(I_{j,z}\left\{\dot{\alpha}_j\frac{d}{dt}\left(\frac{\partial \omega_z}{\partial \dot{\phi}}\right) + \ddot{\alpha}_j\frac{\partial \omega_z}{\partial \dot{\phi}} - \dot{\alpha}_j\frac{\partial \omega_z}{\partial \phi}\right\}\right) = Q_\phi$$

This equation can be shortened considerably by the use of the following equivalences:

$$\frac{d}{dt}\left(\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right) = \frac{\partial \dot{D}_1}{\partial \phi}$$

$$\frac{\partial \dot{D}_1}{\partial \dot{\phi}} = \frac{\partial D_1}{\partial \phi}$$

$$\frac{d^2}{dt^2}\left[m_d R_d + \sum_{j=1}^{4} m_j R_j + m_c R_c + m_h R_h\right] = 0$$

The shortened equation with partial derivatives of the ω terms expressed as functions of the transformation matrices becomes

$$m_d\left\{\ddot{R}_d\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{r_d\right\} + \sum_{j=1}^{4} m_j\left\{\ddot{R}_j\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{r_j\right\} + m_c\left\{\ddot{R}_c\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{r_c\right\} + m_h\left\{\ddot{R}_h\right\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\left\{r_h\right\}$$

$$+ \left\{\dot{\omega}\right\}^T\left[I\right]\left[D\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix} + \left\{\omega\right\}^T\left(\left[I\right]\left[\dot{D}\right] + \left[\dot{I}\right]\left[D\right]\right)\begin{Bmatrix}1\\0\\0\end{Bmatrix} - \left\{\omega\right\}^T\left[I\right]\left[\frac{\partial D}{\partial \phi}\right]\left\{A\right\} + \left\{\dot{\omega}_h\right\}^T\left[I_h\right]\left[D_2\right]^T\left[D\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix}$$

$$+ \left\{\omega_h\right\}^T\left[I_h\right]\left(\left[\dot{D}_2\right]^T\left[D\right] + \left[D_2\right]^T\left[\dot{D}\right]\right)\begin{Bmatrix}1\\0\\0\end{Bmatrix} + \left\{\omega_h\right\}^T\left[\dot{I}_h\right]\left[D_2\right]^T\left[D\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix}$$

$$- \left\{\omega_h\right\}^T\left[I_h\right]\left[D_2\right]^T\left[\frac{\partial D}{\partial \phi}\right]\left\{A\right\} + \sum_{j=1}^{4}\left[I_{j,z}\left(\dot{\alpha}_j\dot{\theta}\ c\theta + \ddot{\alpha}\ s\theta\right)\right] = Q_\phi$$

Combining this equation with similar equations obtained for the θ and ψ degrees of freedom results in the following matrix equation governing rotational motion about the overall mass center:

$$m_d\left[M_d\right]\left\{\ddot{R}_d\right\} + \sum_{j=1}^{4}\left(m_j\left[M_j\right]\left\{\ddot{R}_j\right\}\right) + m_c\left[M_c\right]\left\{\ddot{R}_c\right\} + m_h\left[M_h\right]\left\{\ddot{R}_h\right\} + \left[D\right]^T\left[I\right]\left\{\dot{\omega}\right\} + \left(\left[D\right]^T\left[\dot{I}\right]\right.$$

$$\left. + \left[\dot{D}\right]^T\left[I\right]\right)\left\{\omega\right\} - \left[M_1\right]\left[I\right]\left\{\omega\right\} + \left[D\right]^T\left[D_2\right]\left[I_h\right]\left\{\dot{\omega}_h\right\} + \left(\left[\dot{D}\right]^T\left[D_2\right] + \left[D\right]^T\left[\dot{D}_2\right]\right)\left[I_h\right]\left\{\omega_h\right\}$$

$$+ \left[D\right]^T\left[D_2\right]\left[\dot{I}_h\right]\left\{\omega_h\right\} - \left[M_1\right]\left[D_2\right]\left[I_h\right]\left\{\omega_h\right\} + \sum_{j=1}^{4}\left(I_{j,z}\begin{Bmatrix} \dot{\alpha}_j\dot{\theta}\ c\theta + \ddot{\alpha}\ s\theta \\ -\dot{\phi}\dot{\alpha}_j\ c\theta \\ \ddot{\alpha}_j \end{Bmatrix}\right) = \begin{Bmatrix} Q_\phi \\ Q_\theta \\ Q_\psi \end{Bmatrix} \tag{A20}$$

where

$$\left[M_d\right] = \begin{bmatrix} \left\{\left[\dfrac{\partial D_1}{\partial \phi}\right]\left\{r_d\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D_1}{\partial \theta}\right]\left\{r_d\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D_1}{\partial \psi}\right]\left\{r_d\right\}\right\}^T \end{bmatrix} = \left[D\right]^T\left[D_1\right]^T\left[D_1 r_d\right] = \left[D\right]^T\left[r_d\right]\left[D_1\right]^{-1} \tag{A21}$$

with similar relationships for $\left[M_j\right]$, $\left[M_c\right]$, and $\left[M_h\right]$. Also

$$\left[M_1\right] = \begin{bmatrix} \left\{\left[\dfrac{\partial D}{\partial \phi}\right]\left\{A\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D}{\partial \theta}\right]\left\{A\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D}{\partial \psi}\right]\left\{A\right\}\right\}^T \end{bmatrix} = \left[\dot{D}\right]^T - \left[D\right]^T\left[\omega\right] \tag{A22}$$

Equation (A22) is derived from equation (A20) and the first of the derivative equations:

$$\left.\begin{aligned}
\frac{d}{dt}\left(\left[D_1\right]\langle r\rangle\right) &= \left[D_1\right]\left\{\frac{d}{dt}\left(\langle r\rangle\right) + \left[\omega\right]\langle r\rangle\right\} \\
\frac{d}{dt}\left(\left[D_1\right]\left[D_2\right]\left\{r_f\right\}\right) &= \left[D_1\right]\left[D_2\right]\left\{\frac{d}{dt}\left(\left\{r_f\right\}\right) + \left[\omega_h\right]\left\{r_f\right\}\right\}
\end{aligned}\right\} \tag{A23}$$

Expressions (A23) are pertinent applications of the general rule that the transformation to inertial coordinates of the total derivative of a vector which is expressed in a rotating coordinate system is equal to the derivative of the transformed (from rotating to inertial coordinates) vector. The Lagrange equation for the x' coordinate is

$$m_T\ddot{x}' = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^T \left[D_1\right]\langle F\rangle$$

This equation plus similar equations obtained for the y' and z' degrees of freedom are combined to yield the following matrix equation governing translational motions of the overall mass center along the inertial axes:

$$m_T\left\{\ddot{R}_g\right\} = \left[D_1\right]\langle F\rangle \tag{A24}$$

where

$$\left\{\ddot{R}_g\right\} = \begin{Bmatrix} \ddot{x}' \\ \ddot{y}' \\ \ddot{z}' \end{Bmatrix}$$

For the r_x degree of freedom, the Lagrange equation is

$$m_h\left(\left[D_1\right]\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}\right)^T\left\{\ddot{R}_h\right\} + K_xr_x + C_x\dot{r}_x = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}^T\left(-\frac{m_h}{m_T}\langle F\rangle + \left[D_2\right]\left\{F_h\right\}\right)$$

This equation and similar equations for r_y and r_z are combined to form the matrix equation governing relative translation of the disk and hub

$$m_h\left[D_1\right]^T\left\{\ddot{R}_h\right\} + \left[C\right]\left\{\dot{r}\right\} + \left[K\right]\left\{r\right\} = -\frac{m_h}{m_T}\left\{F\right\} + \left[D_2\right]\left\{F_h\right\}$$

which because of equation (A24) can be written as

$$m_h\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\} + \left[C\right]\left\{\dot{r}\right\} + \left[K\right]\left\{r\right\} = \left[D_2\right]\left\{F_h\right\} \tag{A25}$$

The Lagrange equation derived for the hub-disk rotational degree of freedom ϕ_h is

$$m_h\left\{\ddot{R}_h\right\}^T\left[D_1\right]\left[\frac{\partial\dot{D}_2}{\partial\dot{\phi}_h}\right]\left\{r_f\right\} + \left\{\dot{\omega}_h\right\}^T\left[I_h\right]\left[D_h\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix} + \left\{\omega_h\right\}^T\left[I_h\right]\left[\dot{D}_h\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix} + \left\{\omega_h\right\}^T\left[\dot{I}_h\right]\left[D_h\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix}$$

$$- \left\{\omega_h\right\}^T\left[I_h\right]\left[\frac{\partial D_h}{\partial\phi_h}\right]\begin{Bmatrix}\dot{\phi}_h\\\dot{\theta}_h\\\dot{\psi}_h\end{Bmatrix} - \left\{\omega_h\right\}^T\left[I_h\right]\left[\frac{\partial D_2}{\partial\phi_h}\right]^T\left\{\omega\right\} + C_{R,x}\dot{\phi}_h + C_{R,xy}\dot{\theta}_h + C_{R,xz}\dot{\psi}_h$$

$$+ K_{R,x}\phi_h + K_{R,xy}\theta_h + K_{R,xz}\psi_h = -\frac{m_h}{m_T}\left\{F\right\}^T\left[\frac{\partial D_2}{\partial\phi_h}\right]\left\{r_f\right\} + \begin{Bmatrix}1\\0\\0\end{Bmatrix}^T\left[D_h\right]^T\left\{T_h\right\} \tag{A26}$$

By using the identity $\left[\frac{\partial\dot{D}_2}{\partial\dot{\phi}_h}\right] = \left[\frac{\partial D_2}{\partial\phi_h}\right]$ and equation (A24), the first terms on both sides of the equation can be combined.

The resulting equation along with similar equations derived for the θ_h and ψ_h degrees of freedom can be written in the combined form

$$m_h\left[M_r\right]\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\} + \left[D_h\right]^T\left[I_h\right]\left\{\dot{\omega}_h\right\} + \left(\left[\dot{D}_h\right]^T\left[I_h\right] + \left[D_h\right]^T\left[\dot{I}_h\right]\right)\left\{\omega_h\right\} - \left[M_2\right]\left[I_h\right]\left[\omega_h\right]$$

$$+ \left[C_R\right]\left\{B\right\} + \left[K_R\right]\begin{Bmatrix}\phi_h\\\theta_h\\\psi_h\end{Bmatrix} = \left[D_h\right]^T\left\{T_h\right\} \tag{A27}$$

where

$$\left[M_r\right] = \begin{bmatrix} \left\{\left[\dfrac{\partial D_2}{\partial \phi_h}\right]\left\{r_f\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D_2}{\partial \theta_h}\right]\left\{r_f\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D_2}{\partial \psi_h}\right]\left\{r_f\right\}\right\}^T \end{bmatrix} = \left[D_h\right]^T\left[D_2\right]^T\left[D_2 r_f\right] = \left[D_h\right]^T\left[r_f\right]\left[D_2\right]^T \tag{A28}$$

$$\left[M_2\right] = \begin{bmatrix} \left\{\left[\dfrac{\partial D_h}{\partial \phi_h}\right]\left\{B\right\} + \left[\dfrac{\partial D_2}{\partial \phi_h}\right]^T\left\{\omega\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D_h}{\partial \theta_h}\right]\left\{B\right\} + \left[\dfrac{\partial D_2}{\partial \theta_h}\right]^T\left\{\omega\right\}\right\}^T \\ \left\{\left[\dfrac{\partial D_h}{\partial \psi_h}\right]\left\{B\right\} + \left[\dfrac{\partial D_2}{\partial \psi_h}\right]^T\left\{\omega\right\}\right\}^T \end{bmatrix} = \left[\dot{D}_h\right]^T - \left[D_h\right]^T\left[\omega_h\right] \tag{A29}$$

Equation (A29) is derived from equation (A27) and the second of equations (A23).

Equation conditioning.- The ϕ, θ, and ψ equations involve both disk and hub angular acceleration terms. These terms must be separated for purposes of solution. The hub acceleration term is eliminated by substitution of the ϕ_h, θ_h, and ψ_h equations as follows:

Premultiplying equation (A27) by $\left[D\right]^T\left[D_2\right]\left[\left[D_h\right]^T\right]^{-1}$, combining with equations (A28) and (A29), and solving for the $\left\{\dot{\omega}_h\right\}$ term yield

$$\left[D\right]^T\left[D_2\right]\left[I_h\right]\left\{\dot{\omega}_h\right\} = -\left[D\right]^T\left[D_2\right]\left[\omega_h\right]\left[I_h\right]\left\{\omega_h\right\} - \left[D\right]^T\left[D_2\right]\left[\left[D_h\right]^T\right]^{-1}\left(\left[C_R\right]\left\{B\right\} + \left[K_R\right]\begin{Bmatrix}\phi_h \\ \theta_h \\ \psi_h\end{Bmatrix}\right)$$

$$- \left[D\right]^T\left[D_2\right]\left(\left[\dot{I}_h\right]\left\{\omega_h\right\} + m_h\left[r_f\right]\left[D_2\right]^T\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\}\right) + \left[D\right]^T\left[D_2\right]\left\{T_h\right\}$$

This relationship is substituted for the $\left\{\dot{\omega}_h\right\}$ term in equation (A20). The resulting equation is simplified by means of equations (A7), (A21), (A22), and the identities $\left[\omega\right] = \left[D_1\right]^T\left[\dot{D}_1\right]$ and $\left[\omega_h\right] = \left[D_2\right]^T\left[D_1\right]^T\left[\dot{D}_1 D_2 + D_1\dot{D}_2\right]$ developed from equations (A23) with the result

$$\left[I\right]\left\{\dot{\omega}\right\} + \left[\dot{I}\right]\left\{\omega\right\} + \left[\omega\right]\left[I\right]\left\{\omega\right\} - m_h\left[D_2\right]\left[r_f\right]\left[D_2\right]^T\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\} - \left[D_2\right]\left[\left[D_h\right]^T\right]^{-1}\left\{\left[C_R\right]\left\{B\right\}\right.$$

$$\left. + \left[K_R\right]\begin{Bmatrix}\phi_h \\ \theta_h \\ \psi_h\end{Bmatrix}\right\} + m_d\left[r_d\right]\left[D_1\right]^T\left\{\ddot{R}_d\right\} + \sum_{j=1}^{4}\left(m_j\left[r_j\right]\left[D_1\right]^T\left\{\ddot{R}_j\right\}\right) + m_c\left[r_c\right]\left[D_1\right]^T\left\{\ddot{R}_c\right\}$$

$$+ m_h\left[r_h\right]\left[D_1\right]^T\left\{\ddot{R}_h\right\} + \sum_{j=1}^{4}\left(I_{j,z}\begin{Bmatrix}\dot{\alpha}_j\omega_y \\ -\dot{\alpha}_j\omega_x \\ \ddot{\alpha}_j\end{Bmatrix}\right) = \left\{T_d\right\} + \left\{A_1\right\}\left\{F\right\} + \left[r\right]\left[D_2\right]\left\{F_h\right\} \tag{A30}$$

Similarly, a combination of equations (A27), (A28), and (A29) allows the hub angular degrees of freedom to be expressed simply by the matrix equation

$$\left[I_h\right]\left\{\dot{\omega}_h\right\} + \left[\dot{I}_h\right]\left\{\omega_h\right\} + \left[\omega_h\right]\left[I_h\right]\left\{\omega_h\right\} + m_h\left[r_f\right]\left[D_2\right]^T\left[D_1\right]^T\left\{\ddot{R}_h + \ddot{R}_g\right\}$$

$$+ \left[\left[D_h\right]^T\right]^{-1}\left\{\left[C_R\right]\left\{B\right\} + \left[K_R\right]\begin{Bmatrix}\phi_h \\ \theta_h \\ \psi_h\end{Bmatrix}\right\} = \left\{T_h\right\} \tag{A31}$$

Hub inertial angles.- Hub angles with respect to an inertial frame of reference can be determined by two methods. After establishing an ordered set of Euler rotations ϕ_I, θ_I, and ψ_I (see fig. 12(c)), the first method is to integrate the Euler inertial rates to obtain inertial angles from the expression

$$\begin{Bmatrix}\dot{\phi}_I \\ \dot{\theta}_I \\ \dot{\psi}_I\end{Bmatrix} = \begin{bmatrix}\dfrac{c\psi_I}{c\theta_I} & \dfrac{-s\psi_I}{c\theta_I} & 0 \\ s\psi_I & c\psi_I & 0 \\ \dfrac{-c\psi_I\ s\theta_I}{c\theta_I} & \dfrac{s\psi_I\ s\theta_I}{c\theta_I} & 1\end{bmatrix}\begin{Bmatrix}\omega_{h,x} \\ \omega_{h,y} \\ \omega_{h,z}\end{Bmatrix}$$

The second method, derived in reference 5, is the method used in the present investigation. A vector quantity expressed in hub coordinates is transformed to inertial coordinates in terms of the ϕ, θ, and ψ and ϕ_h, θ_h, and ψ_h systems as indicated by the equation

$$\left\{r_{inertial}\right\} = \left[D_1\right]\left[D_2\right]\left\{r_{hub}\right\}$$

This vector transformation can also be expressed as functions of the hub inertial Euler angles ϕ_I, θ_I, and ψ_I. Equating the transformations yields

$$\begin{bmatrix}c\psi_I\ c\theta_I & -s\psi_I\ c\theta_I & s\theta_I \\ c\psi_I\ s\theta_I\ s\phi_I + s\psi_I\ c\phi_I & c\psi_I\ c\phi_I - s\psi_I\ s\theta_I\ s\phi_I & -c\theta_I\ s\phi_I \\ s\psi_I\ s\phi_I - c\psi_I\ s\theta_I\ c\phi_I & c\psi_I\ s\phi_I + s\psi_I\ s\theta_I\ c\phi_I & c\theta_I\ c\phi_I\end{bmatrix} = \left[D_1\right]\left[D_2\right]$$

Equating comparable elements on the right and left sides of the equal sign provides a means of determining ϕ_I, θ_I, and ψ_I in terms of the angles ϕ, θ, ψ, ϕ_h, θ_h, and ψ_h. These relationships are given in reference 5.

APPENDIX B

DESCRIPTION AND LISTING OF SIMULATION PROGRAM

Computer Simulation

The simulation was programed on a CDC 6600 digital computer which operates in a real-time mode and can be linked to actual control system or sensor hardware. The simulation was controlled from a program control station shown in figure 13 which includes a data entry keyboard, an on-line typewriter and time-history recorder, and a cathode ray tube (CRT) display console.

The present program includes the spacecraft rotating element (disk) dynamics and the dynamics of four passive controller masses. Equations pertaining to the zero-gravity hub and isolation spring system are not included. These elements are being incorporated into a more extensive program for further use in control studies. The simulation required a storage of approximately 45 000 octal words and operated at 16 iterations (computer cycles) per second. A fourth-order Runge-Kutta integration scheme was used for the spacecraft and passive controller dynamics. A basic computing (integration) interval of 0.03125 second was used. A flow diagram of the simulation follows.

INITIALIZATION

Setup output parameter table
Initialize input-output signals
Initialize scale factors and simulation parameters

RESET MODE

Resets $t = 0.0$
Resets derivative equations
Resets CRT plot
Unload CRT option
Insert new data from keyboard
Calculate initial r_j for use in disk equations

HOLD MODE

Derivative equations held at present values

OPERATE MODE

Mass and/or inertial disturbance
Option to erase and reset CRT display
Rotating element (disk) equations of motion

- Compute disk Euler rates
- Compute $\ddot{x}'$, $\ddot{y}'$, and $\ddot{z}'$
- Compute $\{A_1\}$
- Compute total torques on disk
- Compute $\ddot{R}_d$, $\ddot{R}_c$, and $\ddot{R}_j$
- Compute total mass acceleration
- Compute total inertia matrix
- Solution of disk derivative equations or Euler solution for disk (checkout option)
- Runge-Kutta disk integration

4 pass integration loop

Passive controller equations of motion

- Compute $\bar{r}_j$, $\dot{\bar{r}}_j$, and $\ddot{\bar{r}}_j$
- Compute $\ddot{\alpha}_j$
- Runge-Kutta integration of $\ddot{\alpha}_j$

4 pass integration loop

Auxiliary calculation
Time-history recorder output
CRT plot
Real-time mode controls

PRINTER OUTPUT MODE

Input

Input for the simulation was supplied by the operator from the program control station through a data entry keyboard. The data entry keyboard provided capability to change parameters in central memory without removing the program from the computer and simultaneously displayed the value of the parameters on a digital display located on the program control station. These input variables were defined in a specific array VAR described in table II.

Output

Data output facilities included Brush time-history recorders, CRT display, and a high-speed line printer. A parameter listing and description, output formats, and explanations of output options are presented in tables III and IV.

Recorder output.- Time-history recordings of spacecraft parameters (figs. 6 to 9) were generated by time-history recorders located adjacent to the program control station. Each time-history recorder had eight analog and nine discrete (event) channels. The analog channels were used to record desired data parameters. Time-history recorder channel assignments are shown in table III.

Printer output.- A block of output data was stored on a disk file at specified time intervals denoted by the integer variable NT in terms of iteration cycles. Upon completion of the run, all output was routed to the high-speed printer by depressing the "PRINT" control button located on the program control station. Output variables are identified by an asterisk in table IV which presents and defines all significant program variables.

CRT output.- Another form of output was provided by a CRT display which generated x,y-plots of spacecraft angles ϕ as functions of θ as shown in figure 9. CRT plotting was done while the simulation was in a real-time status with a plotting frequency of FREQ in terms of iteration cycles. Since the amount of data required for a typical run ($\approx$700 sec) exceeded the limit on the CRT controller instructions, an option was included to erase the plot at any time and reinitialize the CRT so that only the desired part of the run was displayed. A hard copy of the CRT plot could be obtained if desired.

Program Listing

The program listing is as follows:

```
-
      PROGRAM SPBASE(INPUT,OUTPUT)
C
C
C    *     ***LDISI***                    OPTION
C              33     -    ALPHA D DOTS = 0.0 TO LOCK BALANCE ARMS
C              34     -    CREW RATE * SRCDX
C              35     -    CREW RATE * SRCDY
C              36     -    CREW RATE * SRCDZ
C              40     -    EULER DISK EQUATIONS FOR CHECKOUT
C              41     -    OPTION TO PLOT ON CRT (PHDK VS THDK IN DEGS)
C              45     -    SETUP CRT IN OPERATE
C              46     -    UNLODE CRT DISPLAY
C
C
      COMMON/REALTIM/ANALGIN(32),DIGOUT(64),LDISI(108),LDISO(196),
     1 NOPER,NHOLD,NRESET,NTERM,NPRINT,NREAD
      LOGICAL LDISI,LDISO,LOGIC,VARCHNG
      DIMENSION VAR(40),INTEG(1),LOGIC(4),IVARBUF(5)
      DIMENSION RR(6,24),TXX(7),TYY(7),TZZ(7)
      DIMENSION ARDDX(7),ARDDY(7),ARDDZ(7),  BUFF(1),TIM(3)
      DIMENSION IMAT(3,3),EIGV(3),EVEC(3,3)
C
      REAL IMAT,IXYCG
      REAL         MASSDI,MCI,M1I,M2I,M3I,M4I,MTI,I1,I2,I3,I4
      REAL        MASSD,MASSC,M1,M2,M3,M4,MT
      REAL IDXX,IDXY,IDXZ,IDYY,IDYZ,IDZZ,IDXXO,IDYYO,IDZZO,IDXYO
      REAL IDDXX,IDDYY,IDDZZ,IDDXY,IDDYZ,IDDXZ
      REAL M1O,M2O,M3O,M4O,I1O,I2O,I3O,I4O
      REAL  IZTOT,IYTOT,IXTOT,IDIFX,IDIFY,IXYTOT,IXZTOT,IYZTOT
      REAL IDXZO,IDYZO,  IXCG,IYCG,IZCG,IXZCG,IYZCG,MDUM
      REAL I1X,I2X,I3X,I4X,I1Y,I2Y,I3Y,I4Y
      LOGICAL DUMCG
C
      EQUIVALENCE  (VAR( 1),PHDKO  ),(VAR( 2),THDKO  ),(VAR( 3),PSDKO  )
      EQUIVALENCE  (VAR( 4),WXDKO  ),(VAR( 5),WYDKO  ),(VAR( 6),WZDKO  )
      EQUIVALENCE  (VAR( 7),XPRO   ),(VAR( 8),YPRO   ),(VAR( 9),ZPRO   )
      EQUIVALENCE  (VAR(10),XPRDO  ),(VAR(11),YPRDO  ),(VAR(12),ZPRDO  )
      EQUIVALENCE  (VAR(13),IDXXO  ),(VAR(14),IDYYO  ),(VAR(15),IDZZO  )
      EQUIVALENCE(VAR(16),A10),(VAR(17),A20),(VAR(18),A30),(VAR(19),A40)
      EQUIVALENCE(VAR(20),I10),(VAR(21),I20),(VAR(22),I30),(VAR(23),I40)
      EQUIVALENCE(VAR(24),M10),(VAR(25),M20),(VAR(26),M30),(VAR(27),M40)
      EQUIVALENCE  (VAR(28),EL     ),(VAR(29),DISTZ  ),(VAR(30),CJO    )
      EQUIVALENCE  (VAR(31),SRCDXO ),(VAR(32),SRCDYO ),(VAR(33),SRCDZO )
      EQUIVALENCE  (VAR(34),FREQ   ),(VAR(35),PLGAIN )
      EQUIVALENCE  (VAR(37),MASSDO ),(VAR(38),MASSCO )
      EQUIVALENCE  (RR(6,1 ), A1 ),(RR(6,2 ), A2 ),(RR(6,3 ), A3 )
      EQUIVALENCE  (RR(6,4 ), A4 ),(RR(6,5 ),ADOT1),(RR(6,6 ),ADOT2)
      EQUIVALENCE  (RR(6,7 ),ADOT3),(RR(6,8 ),ADOT4)
      EQUIVALENCE  (RR(6,13),XPR ),(RR(6,14),YPR ),(RR(6,15),ZPR )
      EQUIVALENCE  (RR(6,16),XPRD),(RR(6,17),YPRD),(RR(6,18),ZPRD)
      EQUIVALENCE  (RR(6,19),PHDK),(RR(6,20),THDK),(RR(6,21),PSDK)
      EQUIVALENCE  (RR(6,22),WXDK),(RR(6,23),WYDK),(RR(6,24),WZDK)
      EQUIVALENCE  (INTEG(1),ISCAN  )
      EQUIVALENCE  (LDISI(48),INTABLS)
```

```
C
C**** SECTION C.     INITIALIZATION OF REAL TIME SYSTEM
      CALL CYCLE (90006S)
      NT=32
      CALL READOUT(4,NT,T,              SRCX,SRCY,SRCZ)
      CALL READOUT(6,NT,WXDK,WYDK,WZDK,PHDK,THDK,PSDK)
      CALL READOUT(6,NT,WXDDK,WYDDK,WZDDK,A1X,A1Y,A1Z)
      CALL READOUT(6,NT,A1A,A2A,A3A,A4A,ETAXZ,ETAYZ)
      CALL READOUT(6,NT,ADOT1,ADOT2,ADOT3,ADOT4,DELE,ETAXYZ)
      CALL READOUT(6,NT,ADDOT1,ADDOT2,ADDOT3,ADDOT4,CMO,CON)
      CALL READOUT(6,NT,THETH,DELH,THETZ,DELZ,THETI,DELI)
      CALL READOUT(3,NT,EIGV(1),EIGV(2),EIGV(3))
      CALL RTROUTE(MF,90034S)
      CALL INOUT(ANALGIN,32,DIGOUT,42,LDISI,88,LDISO,196)
      CALL XDSPLAY(LDISI,LDISO,VARCHNG,ITYPE,IVARBUF,INTABLS)
      CALL DATABLX(VAR,40,INTEG,1,LOGIC,4 ,ANALGIN,32,DIGOUT,42,
     1             LDISI(1),88,LDISO(1),196)
C
C**** CLEAR INDICATOR LITES
      DO 85 IND=1,196
   85 LDISO(IND) = .F.
C**** CLEAR DISCRETE INPUTS
      DO 86 IND=1,108
   86 LDISI(IND) = .F.
C**** CLEAR DA CONVERTERS
      DO 87 IND=1,42
   87 DIGOUT(IND) = 0.
      CALL NAMECRT(6LCRTTPE,ERR)
      ASSIGN 90001 TO NOPER
      ASSIGN 90002 TO NHOLD
      ASSIGN 90003 TO NRESET
      ASSIGN 90004 TO NTERM
      ASSIGN 90014 TO NPRINT
      ASSIGN 90015 TO NREAD
C
C**** SECTION D. CONSTANTS AND INITIAL PARAMETERS
C     PRINT 16
   16 FORMAT(6X* SPACE BASE SIMULATION*5X*JOB,43,77777,75000.   A2718,
     1 13043,1,C.W.MARTZ,B1232 R125*)
      TIM(1)=4RXTIM  $  TIM(2)=4RXE=   $  TIM(3)=4RX.
      NUMBER=INTEG=KOUNT=0
      ISCAN = 32
C      ******        RECORDER SCALE FACTORS
      SFA1Y=10.    $   SFA1Z=10.    $   SFA1X=10.
      SFTH=5.   $   SFTI=5.   $   SFDELI=1./180.
      SFCON=5.
      SFCMO=5.
      SFETA=5.
      SFETAX=5.
      SFETAY=5.
      SFCONE=5.
      SFTHETZ=2.5
      SFMBA=1./180.
      SFCREW=.01
      SFANG=1.
      SFACC=1000.
      SFRATE=1.
```

```
C
C     *****************  INITIALIZATION
       TINC=HH=.03125
       SX=1./6.
       BILL=50.     $    TMARTZ=0.0
       TIMER=10.
       DUMCG=.F.
       FREQ=16.
       PLGAIN=.2
       CTIMF=100.
       SRXO=SRYO=SRZO=0.0
       SRDXO=SRDYO=SRDZO=0.0
C     *****************  INITIALIZATION DISK **********
       MASSDO=350000.
       PHDKO=THDKO=PSDKO=0.0
       WXDKO=WYDKO=0.0
       WZDKO=.5
       IDXXO=IDYYO=380000000.
       IDZZO=190000000.
       IDXYO=IDXZO=IDYZO=0.0
       IDXYO=0.
       IDXY=IDYZ=IDXZ=0.0
       SRSX=SRSDX=SRSDDX=0.0
       SRSY=SRSDY=SRSDDY=0.0
       SRSZ=SRSDZ=SRSDDZ=0.0
       RDDX=RDDY=RDDZ=0.0
       FXDK=FYDK=FZDK=0.0
       TXDK=TYDK=TZDK=0.0
       XPRO=YPRO=ZPRO=XPRDO=YPRDO=ZPRDO=0.0
       XPRDD=YPRDD=ZPRDD=0.0
C      **********************INITIALIZATION FOR  MASS BALANCE SYSTEM  ***
       A10=A20=1.570796
       A30=A40=-1.570796
       M10=M20=M30=M40=3200.
       CJO=4000.
       EL=15.     $     DISTZ=7.5
       I1X=I2X=I3X=I4X=710.
       I1Y=I2Y=I3Y=I4Y=7800.
       I10=I20=I30=I40=7800.
C      **********************INITIALIZATION FOR  CREW
       MASSCO=1500.
       SRCDXO=SRCDYO=SRCDZO=.6
C
90003 CONTINUE
      CALL READY
C**** SECTION E.  INITIALIZATION OF INTEGRALS
C    *****************  RESET LOOP
      T=0.0
      TINC=HH
      NFREQ=FREQ
      N2=10*NFREQ
      TCOUNT=0.
C
C      ******       SETUP CRT PLOT  (PH  VS  TH)
      IF(DUMCG) GO TO 17
      CALL HALT
      CALL ENDPLOT
      CALL UNLODE
      CALL CLRPLOT
```

```
      CALL CRTPLOT(1,1,NFREQ,0,0,THDEG,PLGAIN,0,10LTHDK       ,PHDEG,PLGA
     1IN,0,10LPHDK       )
      CALL CRTPLOT(1,1,N2,308,1, THDEG,PLGAIN,0,10LTHDK       ,PHDEG,PLGA
     1IN,0,10LPHDK       )
      CALL READY
   17 CONTINUE
      DUMCG=.T.
C
C      ******       UNLODE CRT SCREEN
      IF(.NOT. LDISI(46)) GO TO 50003
      CALL HALT
      CALL UNLODE
      CALL READY
50003 CONTINUE
C
C     ******       SET IN INITIAL CONDITIONS
      TSAVE=T
      TMARTZ=T
      XPR=XPRO    $   YPR=YPRO    $   ZPR=ZPRO
      XPRD=XPRDO    $   YPRD=YPRDO    $   ZPRD=ZPRDO
C     ************************     DISK    ***
      PHDK=PHDKO  $  THDK=THDKO  $  PSDK=PSDKO
      WXDK=WXDKO  $  WYDK=WYDKO  $  WZDK=WZDKO
      SRX=SRXO  $  SRY=SRYO  $  SRZ=SRZO
      SRDX=SRDXO  $  SRDY=SRDYO  $  SRDZ=SRDZO
      SRCX=SRCDX=SRCDDX=0.0
      SRCY=SRCDY=SRCDDY=0.0
      SRCZ=SRCDZ=SRCDDZ=0.0
      TXX(1)=TXX(2)=TXX(3)=TXX(4)=TXX(5)=TXX(6)=TXX(7)=0.0
      TYY(1)=TYY(2)=TYY(3)=TYY(4)=TYY(5)=TYY(6)=TYY(7)=0.0
      TZZ(1)=TZZ(2)=TZZ(3)=TZZ(4)=TZZ(5)=TZZ(6)=TZZ(7)=0.0
C     ************************     MASSBAL.    ***
      WXDDK=WYDDK=WZDDK=0.0
      ADOT1=ADOT2=ADOT3=ADOT4=0.0
      ADDOT1=ADDOT2=ADDOT3=ADDOT4=0.0
      WXDHOLD=WYDHOLD=WZDHOLD=0.0
      MASSD=MASSDO  $  MASSC=MASSCO
      A1=A10  $  A2=A20  $  A3=A30  $  A4=A40
      I1=I10  $  I2=I20  $  I3=I30  $  I4=I40
      M1=M10  $  M2=M20  $  M3=M30  $  M4=M40
      CJ1=CJ2=CJ3=CJ4=CJO
      I1X=I2X=I3X=I4X=710.
      I1Y=I2Y=I3Y=I4Y=7800.
      SR1X=SR1DX=SR1DDX=0.0
      SR1Y=SR1DY=SR1DDY=0.0
      SR1Z=SR1DZ=SR1DDZ=0.0
      SR2X=SR2DX=SR2DDX=0.0
      SR2Y=SR2DY=SR2DDY=0.0
      SR2Z=SR2DZ=SR2DDZ=0.0
      SR3X=SR3DX=SR3DDX=0.0
      SR3Y=SR3DY=SR3DDY=0.0
      SR3Z=SR3DZ=SR3DDZ=0.0
      SR4X=SR4DX=SR4DDX=0.0
      SR4Y=SR4DY=SR4DDY=0.0
      SR4Z=SR4DZ=SR4DDZ=0.0
C     ******       CALCULATE INITIAL CONDITIONS
      CA1=COS(A1)  $ CA2=COS(A2) $ CA3=COS(A3) $ CA4=COS(A4)
      SA1=SIN(A1)  $ SA2=SIN(A2) $ SA3=SIN(A3) $ SA4=SIN(A4)
      SR1X=EL*CA1
```

```
      SR1Y=EL*SA1
      SR1Z=-DISTZ
      SR2X=EL*CA2
      SR2Y=EL*SA2
      SR2Z=    DISTZ
      SR3X=EL*CA3
      SR3Y=EL*SA3
      SR3Z=   -DISTZ
      SR3Z=-DISTZ-5.
      SR3Z=-DISTZ*1.2
      SR4X=EL*CA4
      SR4Y=EL*SA4
      SR4Z=    DISTZ
      SR4Z=DISTZ+5.
      SR4Z=DISTZ*1.2
      MT=MASSD         + MASSC + M1 + M2 + M3 + M4
      IF(MT .NE. 0.0) MTI=1./MT
      CPSDK=COS(PSDK)
      CTHDK=COS(THDK)
      CPHDK=COS(PHDK)
      SPHDK=SIN(PHDK)
      STHDK=SIN(THDK)
      SPSDK=SIN(PSDK)
      SECTHDK=1./CTHDK
C     *********   THE  D  MATRIX
      D11=CPSDK*CTHDK
      D12=SPSDK
      D21=-SPSDK*CTHDK
      D22=CPSDK
      D31=STHDK
      D33=1.0
C    **********  THE  D - 1  MATRIX
      DO11=D11
      DO12=D21
      DO13=STHDK
      DO21=CPSDK*STHDK*SPHDK + SPSDK*CPHDK
      DO22=CPSDK*CPHDK - SPSDK*STHDK*SPHDK
      DO23=-CTHDK*SPHDK
      DO31=SPSDK*SPHDK - CPSDK*STHDK*CPHDK
      DO32=CPSDK*SPHDK + SPSDK*STHDK*CPHDK
      DO33=CTHDK*CPHDK
C
C    ********************************** DISK EULER RATES ******
      DPHDK=CPSDK*SECTHDK*WXDK - SPSDK*SECTHDK*WYDK
      DTHDK=SPSDK*WXDK + CPSDK*WYDK
      DPSDK=-CPSDK*STHDK*SECTHDK*WXDK + SPSDK*STHDK*SECTHDK*WYDK+WZDK
C    **********  THE D - 1 -DOT  MATRIX
      DOD11=-DTHDK*CPSDK*STHDK - DPSDK*SPSDK*CTHDK
      DOD12=+DTHDK*SPSDK*STHDK - DPSDK*CPSDK*CTHDK
      DOD13=+DTHDK*CTHDK
      DOD21=+DPHDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK) + DTHDK*CPSDK*CTHDK
     1*SPHDK - DPSDK*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK)
      DOD22=-DPHDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK) - DTHDK*SPSDK*CTHDK
     2*SPHDK - DPSDK*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK)
      DOD23=-DPHDK*CTHDK*CPHDK + DTHDK*STHDK*SPHDK
      DOD31=+DPHDK*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK) - DTHDK*CPSDK*CTHDK
     3*CPHDK + DPSDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK)
      DOD32=-DPHDK*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK) + DTHDK*SPSDK*CTHDK
     4*CPHDK + DPSDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK)
      DOD33=-DPHDK*CTHDK*SPHDK - DTHDK*STHDK*CPHDK
```

```
C
C
90002 CONTINUE
C**** SECTION F.  HOLD CONTROL
      WXDDK=WXDHOLD
      WYDDK=WYDHOLD
      WZDDK=WZDHOLD
C
C
90006 CONTINUE
C**** SECTION G.  OPERATE LOOP
      IF(LDISI(17)) DUMCG=.F.
      SRCDX=SRCDY=SRCDZ=0.0

C      ******       CREW MOTION DISTURBANCE
      IF(LDISI(34))  SRCDX=SRCDX0
      IF(LDISI(35))  SRCDY=SRCDY0
      IF(LDISI(36))  SRCDZ=SRCDZ0
      IF(T.GE.10..AND.T.LT.30.)SRCDX=.6
      IF(T.GE.10..AND.T.LT.30.)SRCDY=.6
      IF(T.GE.30..AND.T.LT.50.)SRCDZ=.6
      IF (.NOT. LDISI(17)) GO TO 40
      SRCX=SRCX + HH*SRCDX
      SRCY=SRCY + HH*SRCDY
      SRCZ=SRCZ + HH*SRCDZ
   40 CONTINUE
C
C      ******       ERASE AND SETUP CRT DISPLAY
C
      IF(.NOT. LDISI(45)) GO TO 42
      CALL HALT
      CALL ENDPLOT
      CALL UNLODE
      CALL CLRPLOT
      ITIM=T
      JTIM=(T+HH - ITIM)*100.
   41 CONTINUE
      CALL ENABLE(41S)
      CALL CRTCODE(2,TIM(1),100.,990.)
      CALL ENCODEI(ITIM,4,150.,990.)
      CALL CRTCODE(1,TIM(3),190.,990.)
      CALL ENCODEI(JTIM,2,196.,990.)
      CALL MARK250
      CALL CRTPLOT(1,1,NFREQ,0,1,THDEG,PLGAIN,0,10LTHDK      ,PHDEG,PLGA
     1IN,0,10LPHDK      )
      CALL CRTPLOT(1,1,N2,30B,1, THDEG,PLGAIN,0,10LTHDK      ,PHDEG,PLGA
     1IN,0,10LPHDK      )
      CALL READY
   42 CONTINUE
C
C      ******       BEGIN DISK CALCULATIONS
      INT=1
   27 CONTINUE
      MASSDI=1./MASSD
      CPSDK=COS(PSDK)
      CTHDK=COS(THDK)
      CPHDK=COS(PHDK)
      SPHDK=SIN(PHDK)
      STHDK=SIN(THDK)
```

```
      SPSDK=SIN(PSDK)
      SECTHDK=1./CTHDK
C     *********   THE  D  MATRIX
      D11=CPSDK*CTHDK
      D12=SPSDK
      D21=-SPSDK*CTHDK
      D22=CPSDK
      D31=STHDK
      D33=1.0
C    **********   THE  D- DOT  MATRIX
      DD11=-CPSDK*DTHDK*STHDK - CTHDK*DPSDK*SPSDK
      DD12=DPSDK*CPSDK
      DD21=SPSDK*DTHDK*STHDK - CTHDK*DPSDK*CPSDK
      DD22=-DPSDK*SPSDK
      DD31=DTHDK*CTHDK
C    **********   THE  D -  INVERSE  MATRIX
      DI11=CPSDK*SECTHDK
      DI12=-SPSDK*SECTHDK
      DI21=SPSDK
      DI22=CPSDK
      DI31=-CPSDK*STHDK*SECTHDK
      DI32=SPSDK*STHDK*SECTHDK
      DI33=1.0
C     ****************************************  DISK  EULER  RATES
      DPHDK= DI11*WXDK+  DI12*WYDK
      DTHDK= DI21*WXDK +  DI22*WYDK
      DPSDK= DI31*WXDK +  DI32*WYDK + WZDK
      DTHDK2=DTHDK*DTHDK
      DPHDK2=DPHDK*DPHDK
      DPSDK2=DPSDK*DPSDK
      PDA1=DD11*DPHDK + DD12*DTHDK
      PDA2=DD21*DPHDK + DD22*DTHDK
      PDA3=DD31*DPHDK
      DDPHDK= DI11*(WXDDK-PDA1)+DI12*(WYDDK-PDA2)
      DDTHDK= DI21*(WXDDK-PDA1)+DI22*(WYDDK-PDA2)
      DDPSDK= DI31*(WXDDK-PDA1)+DI32*(WYDDK-PDA2)+WZDDK-PDA3
C    **********   THE  D - 1  MATRIX
      DO11=D11
      DO12=D21
      DO13=STHDK
      DO21=CPSDK*STHDK*SPHDK + SPSDK*CPHDK
      DO22=CPSDK*CPHDK - SPSDK*STHDK*SPHDK
      DO23=-CTHDK*SPHDK
      DO31=SPSDK*SPHDK - CPSDK*STHDK*CPHDK
      DO32=CPSDK*SPHDK + SPSDK*STHDK*CPHDK
      DO33=CTHDK*CPHDK
C    **********   THE D - 1 -DOT  MATRIX
      DOD11=-DTHDK*CPSDK*STHDK - DPSDK*SPSDK*CTHDK
      DOD12=+DTHDK*SPSDK*STHDK - DPSDK*CPSDK*CTHDK
      DOD13=+DTHDK*CTHDK
      DOD21=+DPHDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK) + DTHDK*CPSDK*CTHDK
     1*SPHDK - DPSDK*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK)
      DOD22=-DPHDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK) - DTHDK*SPSDK*CTHDK
     2*SPHDK - DPSDK*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK)
      DOD23=-DPHDK*CTHDK*CPHDK + DTHDK*STHDK*SPHDK
      DOD31=+DPHDK*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK) - DTHDK*CPSDK*CTHDK
     3*CPHDK + DPSDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK)
      DOD32=-DPHDK*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK) + DTHDK*SPSDK*CTHDK
```

```
      4*CPHDK + DPSDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK)
       DOD33=-DPHDK*CTHDK*SPHDK - DTHDK*STHDK*CPHDK
C     ********** THE D - 1 - DOUBLE DOT MATRIX
       DODD11=-DDTHDK*CPSDK*STHDK - DDPSDK*SPSDK*CTHDK + 2.*DTHDK*DPSDK*
      1SPSDK*STHDK - (DTHDK2 + DPSDK2)*CPSDK*CTHDK
       DODD12=+DDTHDK*SPSDK*STHDK - DDPSDK*CPSDK*CTHDK + 2.*DPSDK*DTHDK*
      2CPSDK*STHDK + (DTHDK2 + DPSDK2)*SPSDK*CTHDK
       DODD13=+DDTHDK*CTHDK - DTHDK2*STHDK
       DODD21=+DDPHDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK) + DDTHDK*CPSDK*
      3CTHDK*SPHDK - DDPSDK*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK) -(DPHDK2 +
      4DPSDK2)*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK) - DTHDK2*CPSDK*STHDK*
      5SPHDK + 2.*DPHDK*DTHDK*CPSDK*CTHDK*CPHDK - 2.*DTHDK*DPSDK*SPSDK
      6*CTHDK*SPHDK - 2.*DPSDK*DPHDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK)
       DODD22=-DDPHDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK) - DDTHDK*SPSDK*
      7CTHDK*SPHDK - DDPSDK*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK) + (DPHDK2 +
      8DPSDK2)*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK) + DTHDK2*SPSDK*STHDK*
      9SPHDK - 2.*DPHDK*DTHDK*SPSDK*CTHDK*CPHDK - 2.*DTHDK*DPSDK*CPSDK*
      ACTHDK*SPHDK - 2.*DPHDK*DPSDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK)
       DODD23=-DDPHDK*CTHDK*CPHDK + DDTHDK*STHDK*SPHDK + (DPHDK2+ DTHDK2)
      B*CTHDK*SPHDK + 2.*DPHDK*DTHDK*STHDK*CPHDK
       DODD31=+DDPHDK*(CPHDK*STHDK*SPHDK + SPSDK*CPHDK) - DDTHDK*CPSDK*
      CCTHDK*CPHDK + DDPSDK*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK) + (DPHDK2 +
      DDPSDK2)*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK) + DTHDK2*CPSDK*STHDK*
      ECPHDK + 2.*DPHDK*DTHDK*CPSDK*CTHDK*SPHDK - 2.*DPHDK*DPSDK*(SPSDK*
      FSTHDK*SPHDK - CPSDK*CPHDK) + 2.*DTHDK*DPSDK*SPSDK*CTHDK*CPHDK
       DODD32=-DDPHDK*(SPSDK*STHDK*SPHDK - CPSDK*CPHDK) + DDTHDK*SPSDK*
      GCTHDK*CPHDK + DDPSDK*(CPSDK*STHDK*CPHDK - SPSDK*SPHDK) - (DPHDK2 +
      HDPSDK2)*(SPSDK*STHDK*CPHDK + CPSDK*SPHDK) - DTHDK2*SPSDK*STHDK*
      ICPHDK - 2.*DPHDK*DTHDK*SPSDK*CTHDK*SPHDK + 2.*DTHDK*DPSDK*CPSDK*
      JCTHDK*CPHDK - 2.*DPHDK*DPSDK*(CPSDK*STHDK*SPHDK + SPSDK*CPHDK)
       DODD33=-DDPHDK*CTHDK*SPHDK - DDTHDK*STHDK*CPHDK - (DPHDK2 + DTHDK2
      K)*CTHDK*CPHDK + 2.*DPHDK*DTHDK*STHDK*SPHDK
C     ********** THE PARTIAL OF D - 1 - DOT WRT PHI DOT
       DODPD21=+CPSDK*STHDK*CPHDK - SPSDK*SPHDK
       DODPD22=-SPSDK*STHDK*CPHDK - CPSDK*SPHDK
       DODPD23=-CTHDK*CPHDK
       DODPD31=+CPSDK*STHDK*SPHDK + SPSDK*CPHDK
       DODPD32=-SPSDK*STHDK*SPHDK + CPSDK*CPHDK
       DODPD33=-CTHDK*SPHDK
C     ********** THE PARTIAL OF D - 1 - DOT WRT THETA DOT
       DODTD11=-CPSDK*STHDK
       DODTD12=+SPSDK*STHDK
       DODTD13=+CTHDK
       DODTD21=+CPSDK*CTHDK*SPHDK
       DODTD22=-SPSDK*CTHDK*SPHDK
       DODTD23=+STHDK*SPHDK
       DODTD31=-CPSDK*CTHDK*CPHDK
       DODTD32=+SPSDK*CTHDK*CPHDK
       DODTD33=-STHDK*CPHDK
C     ********** THE PARTIAL OF D - 1 - DOT WRT PSI DOT
       DODSD11=-SPSDK*CTHDK
       DODSD12=-CPSDK*CTHDK
       DODSD21=-SPSDK*STHDK*SPHDK + CPSDK*CPHDK
       DODSD22=-CPSDK*STHDK*SPHDK - SPSDK*CPHDK
       DODSD31=+SPSDK*STHDK*CPHDK + CPSDK*SPHDK
       DODSD32=+CPSDK*STHDK*CPHDK - SPSDK*SPHDK
C     *********************
       XPRDD=(DO11*FXDK+DO12*FYDK+DO13*FZDK)*MTI
       YPRDD=(DO21*FXDK+DO22*FYDK+DO23*FZDK)*MTI
       ZPRDD=(DO31*FXDK+DO32*FYDK+DO33*FZDK)*MTI
```

```
C     *************  A 1
       A1X=-MTI*(MASSD*SRSX +                  MASSC*SRCX + M1*SR1X +
     1          M2*SR2X + M3*SR3X + M4*SR4X)
       A1Y=-MTI*(MASSD*SRSY +                  MASSC*SRCY + M1*SR1Y +
     2          M2*SR2Y + M3*SR3Y + M4*SR4Y)
       A1Z=-MTI*(MASSD*SRSZ +                  MASSC*SRCZ + M1*SR1Z +
     3          M2*SR2Z + M3*SR3Z + M4*SR4Z)
C     *************  A 1 DOT
       A1DX=-MTI*(            MASSC*SRCDX + M1*SR1DX + M2*SR2DX +
     1          M3*SR3DX + M4*SR4DX)
       A1DY=-MTI*(            MASSC*SRCDY + M1*SR1DY + M2*SR2DY +
     2          M3*SR3DY + M4*SR4DY)
       A1DZ=-MTI*(            MASSC*SRCDZ + M1*SR1DZ + M2*SR2DZ +
     3          M3*SR3DZ + M4*SR4DZ)
C     *************  A 1 DOUBLE DOT
       A1DDX=-MTI*(             MASSC*SRCDDX + M1*SR1DDX + M2*SR2DDX +
     1    M3*SR3DDX + M4*SR4DDX)
       A1DDY=-MTI*(             MASSC*SRCDDY + M1*SR1DDY + M2*SR2DDY +
     2    M3*SR3DDY + M4*SR4DDY)
       A1DDZ=-MTI*(             MASSC*SRCDDZ + M1*SR1DDZ + M2*SR2DDZ +
     3    M3*SR3DDZ + M4*SR4DDZ)
C
C
C     **********  TORQUE  TRANSFORMATION
C     *********  TOTAL TORQUES ON DISK IN EULER COORDINATES
       CSTORX=(TXDK - FYDK*A1Z + FZDK*A1Y)*CPSDK*CTHDK -
     1       (TYDK + FXDK*A1Z - FZDK*A1X)*SPSDK*CTHDK +
     2       (TZDK - FXDK*A1Y + FYDK*A1X)*STHDK
       CSTORY=CPSDK*(TYDK + FXDK*A1Z - FZDK*A1X) + SPSDK*(TXDK - FYDK*A1Z
     1 + FZDK*A1Y)
       CSTORZ=FYDK*A1X -FXDK*A1Y + TZDK
C      ****************************************  MASS ACCELERATIONS CALCULATIONS
  200 IF(MASSD .EQ. 0.0) GO TO 201
      RDUMX=SRSX  $  RDUMY=SRSY  $  RDUMZ=SRSZ
      RDUMDX=SRSDX  $  RDUMDY=SRSDY  $  RDUMDZ=SRSDZ
      RDUMDDX=SRSDDX  $  RDUMDDY=SRSDDY  $  RDUMDDZ=SRSDDZ
      MDUM=MASSD
      ICK=2
      GO TO 207
  201 CONTINUE
  202 IF(MASSC .EQ. 0.0) GO TO 203
      RDUMX=SRCX  $  RDUMY=SRCY  $  RDUMZ=SRCZ
      RDUMDX=SRCDX  $  RDUMDY=SRCDY  $  RDUMDZ=SRCDZ
      RDUMDDX=SRCDDX  $  RDUMDDY=SRCDDY  $  RDUMDDZ=SRCDDZ
      MDUM=MASSC
      ICK=3
      GO TO 207
  203 IF(M1 .EQ. 0.0) GO TO 204
      RDUMX=SR1X  $  RDUMY=SR1Y  $  RDUMZ=SR1Z
      RDUMDX=SR1DX  $  RDUMDY=SR1DY  $  RDUMDZ=SR1DZ
      RDUMDDX=SR1DDX  $  RDUMDDY=SR1DDY  $  RDUMDDZ=SR1DDZ
      MDUM=M1
      ICK=4
      ADOT1SE=ADOT1*ADOT1*EL
      TERX=-ADOT1SE*CA1
      TERY=-ADOT1SE*SA1
      GO TO 207
  204 IF(M2 .EQ. 0.0) GO TO 205
      RDUMX=SR2X  $  RDUMY=SR2Y  $  RDUMZ=SR2Z
      RDUMDX=SR2DX  $  RDUMDY=SR2DY  $  RDUMDZ=SR2DZ
      RDUMDDX=SR2DDX  $  RDUMDDY=SR2DDY  $  RDUMDDZ=SR2DDZ
```

```
      MDUM=M2
      ICK=5
      ADOT2SE=ADOT2*ADOT2*EL
      TERX=-ADOT2SE*CA2
      TERY=-ADOT2SE*SA2
      GO TO 207
  205 IF(M3 .EQ. 0.0) GO TO 206
      RDUMX=SR3X  $  RDUMY=SR3Y  $  RDUMZ=SR3Z
      RDUMDX=SR3DX  $  RDUMDY=SR3DY  $  RDUMDZ=SR3DZ
      RDUMDDX=SR3DDX  $  RDUMDDY=SR3DDY  $  RDUMDDZ=SR3DDZ
      MDUM=M3
      ICK=6
      ADOT3SE=ADOT3*ADOT3*EL
      TERX=-ADOT3SE*CA3
      TERY=-ADOT3SE*SA3
      GO TO 207
  206 IF(M4 .EQ. 0.0) GO TO 208
      RDUMX=SR4X  $  RDUMY=SR4Y  $  RDUMZ=SR4Z
      RDUMDX=SR4DX  $  RDUMDY=SR4DY  $  RDUMDZ=SR4DZ
      RDUMDDX=SR4DDX  $  RDUMDDY=SR4DDY  $  RDUMDDZ=SR4DDZ
      MDUM=M4
      ICK=7
      ADOT4SE=ADOT4*ADOT4*EL
      TERX=-ADOT4SE*CA4
      TERY=-ADOT4SE*SA4
  207 CONTINUE
C
      ANX=A1X + RDUMX
      ANY=A1Y + RDUMY
      ANZ=A1Z + RDUMZ
      PART1X=DODD11*ANX+ DODD12*ANY + DODD13*ANZ
      PART1Y=DODD21*ANX+ DODD22*ANY + DODD23*ANZ
      PART1Z=DODD31*ANX+ DODD32*ANY + DODD33*ANZ
C
      ANDX=A1DX + RDUMDX
      ANDY=A1DY + RDUMDY
      ANDZ=A1DZ + RDUMDZ
      PART2X=2.*(DOD11*ANDX + DOD12*ANDY + DOD13*ANDZ)
      PART2Y=2.*(DOD21*ANDX + DOD22*ANDY + DOD23*ANDZ)
      PART2Z=2.*(DOD31*ANDX + DOD32*ANDY + DOD33*ANDZ)
C
      ANDDX=A1DDX + RDUMDDX
      ANDDY=A1DDY + RDUMDDY
      ANDDZ=A1DDZ + RDUMDDZ
      PART3X=DO11*ANDDX + DO12*ANDDY + DO13*ANDDZ
      PART3Y=DO21*ANDDX + DO22*ANDDY + DO23*ANDDZ
      PART3Z=DO31*ANDDX + DO32*ANDDY + DO33*ANDDZ
C
      IF(ICK .LE. 3) GO TO 210
      ANEWX=A1DDX + TERX
      ANEWY=A1DDY + TERY
      ANEW3X=DO11*ANEWX + DO12*ANEWY + DO13*A1DDZ
      ANEW3Y=DO21*ANEWX + DO22*ANEWY + DO23*A1DDZ
      ANEW3Z=DO31*ANEWX + DO32*ANEWY + DO33*A1DDZ
      ARDDX(ICK)=PART1X + PART2X + ANEW3X
      ARDDY(ICK)=PART1Y + PART2Y + ANEW3Y
      ARDDZ(ICK)=PART1Z + PART2Z + ANEW3Z
  210 CONTINUE
```

```
C     ******* PARTIAL OF D1 DOT WRT PHI DOT * A1 VECTOR
      RDDXA= PART1X + PART2X + PART3X
      RDDYA= PART1Y + PART2Y + PART3Y
      RDDZA= PART1Z + PART2Z + PART3Z
      TOTX=MDUM*RDDXA
      TOTY=MDUM*RDDYA
      TOTZ=MDUM*RDDZA
      PART4Y=DODPD21*RDUMX + DODPD22*RDUMY + DODPD23*RDUMZ
      PART4Z=DODPD31*RDUMX + DODPD32*RDUMY + DODPD33*RDUMZ
      PART5X=DODTD11*RDUMX + DODTD12*RDUMY + DODTD13*RDUMZ
      PART5Y=DODTD21*RDUMX + DODTD22*RDUMY + DODTD23*RDUMZ
      PART5Z=DODTD31*RDUMX + DODTD32*RDUMY + DODTD33*RDUMZ
      PART6X=DODSD11*RDUMX + DODSD12*RDUMY
      PART6Y=DODSD21*RDUMX + DODSD22*RDUMY
      PART6Z=DODSD31*RDUMX + DODSD32*RDUMY
C
      TXX(ICK)=TOTY*PART4Y + TOTZ*PART4Z
      TYY(ICK)=TOTX*PART5X + TOTY*PART5Y + TOTZ*PART5Z
      TZZ(ICK)=TOTX*PART6X + TOTY*PART6Y + TOTZ*PART6Z
C
      GO TO(201,202,203,204,205,206,208),ICK
  208 CONTINUE
C     **************************************** TOTAL MASS ACCEL. TERMS
      TX= TXX(1)+TXX(2) +TXX(3)+TXX(4)+TXX(5)+TXX(6)+TXX(7)
      TY=TYY(1)+TYY(2)+TYY(3)+TYY(4)+TYY(5)+TYY(6)+TYY(7)
      TZ=TZZ(1)+TZZ(2)+TZZ(3)+TZZ(4)+TZZ(5)+TZZ(6)+TZZ(7)
C
C     ******      TOTAL INERTIA MATRIX
      IDXX=IDXXO+I1X*CA1*CA1+I1Y*SA1*SA1+I2X*CA2*CA2+I2Y*SA2*SA2
     1+I3X*CA3*CA3+I3Y*SA3*SA3+I4X*CA4*CA4+I4Y*SA4*SA4
      IDYY=IDYYO+I1X*SA1*SA1+I1Y*CA1*CA1+I2X*SA2*SA2+I2Y*CA2*CA2
     1+I3X*SA3*SA3+I3Y*CA3*CA3+I4X*SA4*SA4+I4Y*CA4*CA4
      IDZZ=IDZZO+I1+I2+I3+I4
      IDXY=IDXYO+(I1Y-I1X)*SA1*CA1+(I2Y-I2X)*SA2*CA2
     1+(I3Y-I3X)*SA3*CA3+(I4Y-I4X)*SA4*CA4
      IDXZ=IDXZO  $  IDYZ=IDYZO
      IDDXX=2.*(ADOT1*CA1*SA1*(I1Y-I1X)+ADOT2*CA2*SA2*(I2Y-I2X)
     1+ADOT3*CA3*SA3*(I3Y-I3X)+ADOT4*CA4*SA4*(I4Y-I4X))
      IDDYY=-IDDXX
      IDDXY=ADOT1*(CA1*CA1-SA1*SA1)*(I1Y-I1X)+ADOT2*(CA2*CA2-SA2*SA2)*
     1(I2Y-I2X)+ADOT3*(CA3*CA3-SA3*SA3)*(I3Y-I3X)+ADOT4*
     1(CA4*CA4-SA4*SA4)*(I4Y-I4X)
C
C     ******      SOLUTION OF DISK DERIVATIVE EQUATIONS
      YDX=WXDK*(+IDDXX*D11 - IDDXY*D21 - IDDXZ*D31) +
     1    WYDK*(-IDDXY*D11 + IDDYY*D21 - IDDYZ*D31)+
     2    WZDK*(-IDDXZ*D11 - IDDYZ*D21 + IDDZZ*D31)
      YDY=WXDK*(+IDDXX*D12 - IDDXY*D22) + WYDK*(-IDDXY*D12 + IDDYY*D22)+
     1    WZDK*(-IDDXZ*D12 - IDDYZ*D22)
      YDZ=WXDK*(-IDDXZ) + WYDK*(-IDDYZ) + WZDK*IDDZZ
C
C
      COMPX=+WZDK*(IDYY*WYDK-IDYZ*WZDK-IDXY*WXDK) -WYDK*(IDZZ*WZDK-IDXZ*
     1 WXDK-IDYZ*WYDK)
      COMPY=+WXDK*(IDZZ*WZDK-IDXZ*WXDK-IDYZ*WYDK) -WZDK*(IDXX*WXDK-IDXY*
     2 WYDK-IDXZ*WZDK)
      COMPZ=+WYDK*(IDXX*WXDK-IDXY*WYDK-IDXZ*WZDK) -WXDK*(IDYY*WYDK-IDYZ*
     3 WZDK-IDXY*WXDK)
```

```
C
      COA1=+IDXX*D11  - IDXY*D21  - IDXZ*D31
      COB1=-IDXY*D11  + IDYY*D21  - IDYZ*D31
      COC1=-IDXZ*D11  - IDYZ*D21  + IDZZ*D31
      COA2=+IDXX*D12  - IDXY*D22
      COB2=-IDXY*D12  + IDYY*D22
      COC2=-IDXZ*D12  - IDYZ*D22
      COA3=-IDXZ
      COB3=-IDYZ
      COC3=+IDZZ
C     ******      EULER SOLUTION CHECKOUT OPTION
      IF(.NOT. LDISI(40)) GO TO 2
      COA1=IDXX
      COB1=-IDXY
      COC1=-IDXZ
      COA2=-IDXY
      COB2=IDYY
      COC2=-IDYZ
      GO TO 3
    2 CONTINUE
      FUL1=D12*COMPX
      FUL2=D22*COMPY
      COMPX=CSTORX        - TX -YDX + D11*COMPX + D21*COMPY + D31*COMPZ
     1-(D11*WYDK+D21*WXDK)*(I1*ADOT1+I2*ADOT2+I3*ADOT3+I4MADOT4)-D31*
     2(ADDOT1+ADDOT2+ADDOT3+ADDOT4)
      COMPY=CSTORY        - TY -YDY + FUL1      + D22*COMPY
     1-(D12*WYDK+D22*WXDK)*(I1*ADOT1+I2*ADOT2+I3*ADOT3+I4*ADOT4)
      COMPZ=CSTORZ        - TZ - YDZ + COMPZ
     1-D33*(ADDOT1+ADDOT2+ADDOT3+ADDOT4)
    3 CONTINUE
      DET=COA1*(COB2*COC3-COC2*COB3) -COB1*(COA2*COC3-COA3*COC2) +
     1  COC1*(COA2*COB3-COA3*COB2)
      DETI=1./DET
C    **********************    CRAMER$S RULE    ********
      WXDDK=(COMPX*(COB2*COC3-COC2*COB3) -COB1*(COMPY*COC3-COMPZ*COC2) +
     1 COC1*(COMPY*COB3-COMPZ*COB2))*DETI
      WYDDK=(COA1*(COMPY*COC3-COC2*COMPZ)-COMPX*(COA2*COC3-COA3*COC2) +
     1 COC1*(COA2*COMPZ-COA3*COMPY))*DETI
      WODDK=(COA1*(COB2*COMPZ-COMPY*COB3) -COB1*(COA2*COMP0-COA3*COMPY)+
     1  COMPX*(COA2*COB3-COA3*COB2))*DETI
      WXDHOLD=WXDDK
      WYDHOLD=WYDDK
      WZDHOLD=WZDDK
C
C     ************         DISK  EQUATIONS  *******
C    ************** RUNGE KUTTA INTEGRATION SCHEME    ***OM
      IF( .NOT. LDISI(17)) GO TO 301
      GO TO(161,150,151,152),INT
  161 RR(1,13) = XPR
      RR(1,14) = YPR
      RR(1,15) = ZPR
      RR(1,16) = XPRD
      RR(1,17) = YPRD
      RR(1,18) = ZPRD
      RR(1,19) = PHDK
      RR(1,20) = THDK
      RR(1,21) = PSDK
      RR(1,22) = WXDK
      RR(1,23) = WYDK
      RR(1,24) = WZDK
```

```
      XY = 0.5
      L = 2
      INT = 2
  160 RR(L,13) = XPRD*HH
      RR(L,14) = YPRD*HH
      RR(L,15) = ZPRD*HH
      RR(L,16) = XPRDD*HH
      RR(L,17) = YPRDD*HH
      RR(L,18) = ZPRDD*HH
      RR(L,19) = DPHDK*HH
      RR(L,20) = DTHDK*HH
      RR(L,21) = DPSDK*HH
      RR(L,22) = WXDDK*HH
      RR(L,23) = WYDDK*HH
      RR(L,24) = WZDDK*HH
      IF(L.EQ.5)  GO TO 153
  154 RR(6,13) = RR(1,13)+XY*RR(L,13)
      RR(6,14) = RR(1,14)+XY*RR(L,14)
      RR(6,15) = RR(1,15)+XY*RR(L,15)
      RR(6,16) = RR(1,16)+XY*RR(L,16)
      RR(6,17) = RR(1,17)+XY*RR(L,17)
      RR(6,18) = RR(1,18)+XY*RR(L,18)
      RR(6,19) = RR(1,19)+XY*RR(L,19)
      RR(6,20) = RR(1,20)+XY*RR(L,20)
      RR(6,21) = RR(1,21)+XY*RR(L,21)
      RR(6,22) = RR(1,22)+XY*RR(L,22)
      RR(6,23) = RR(1,23)+XY*RR(L,23)
      RR(6,24) = RR(1,24)+XY*RR(L,24)
      IA = L
      GO TO 27
  150 L = 3
      INT = 3
      GO TO 160
  151 L = 4
      INT = 4
      XY = 1.0
      GO TO 160
  152 L = 5
      GO TO 160
  153 RR(6,13)=RR(1,13)+(RR(2,13)+2.*RR(3,13)+2.*RR(4,13)+RR(5,13))*SX
      RR(6,14)=RR(1,14)+(RR(2,14)+2.*RR(3,14)+2.*RR(4,14)+RR(5,14))*SX
      RR(6,15)=RR(1,15)+(RR(2,15)+2.*RR(3,15)+2.*RR(4,15)+RR(5,15))*SX
      RR(6,16)=RR(1,16)+(RR(2,16)+2.*RR(3,16)+2.*RR(4,16)+RR(5,16))*SX
      RR(6,17)=RR(1,17)+(RR(2,17)+2.*RR(3,17)+2.*RR(4,17)+RR(5,17))*SX
      RR(6,18)=RR(1,18)+(RR(2,18)+2.*RR(3,18)+2.*RR(4,18)+RR(5,18))*SX
      RR(6,19)=RR(1,19)+(RR(2,19)+2.*RR(3,19)+2.*RR(4,19)+RR(5,19))*SX
      RR(6,20)=RR(1,20)+(RR(2,20)+2.*RR(3,20)+2.*RR(4,20)+RR(5,20))*SX
      RR(6,21)=RR(1,21)+(RR(2,21)+2.*RR(3,21)+2.*RR(4,21)+RR(5,21))*SX
      RR(6,22)=RR(1,22)+(RR(2,22)+2.*RR(3,22)+2.*RR(4,22)+RR(5,22))*SX
      RR(6,23)=RR(1,23)+(RR(2,23)+2.*RR(3,23)+2.*RR(4,23)+RR(5,23))*SX
      RR(6,24)=RR(1,24)+(RR(2,24)+2.*RR(3,24)+2.*RR(4,24)+RR(5,24))*SX
      IA = L
      T=T+HH
  301 CONTINUE
      SUBW=WYDK*WYDK-WXDK*WXDK
      WXWY=WXDK*WYDK
C  *  **********  CAP R , R-DOT , R-DDOT     ****** (D1*A1)
      RDD1X=ARDDX(4)   $   RDD1Y=ARDDY(4)   $   RDD1Z=ARDDZ(4)
      RDD2X=ARDDX(5)   $   RDD2Y=ARDDY(5)   $   RDD2Z=ARDDZ(5)
```

```
      RDD3X=ARDDX(6)    $    RDD3Y=ARDDY(6)    $    RDD3Z=ARDDZ(6)
      RDD4X=ARDDX(7)    $    RDD4Y=ARDDY(7)    $    RDD4Z=ARDDZ(7)
C
C
C     **************************  MASS BALANCE SYSTEM EQUATIONS  **
      INT=1
   28 CONTINUE
      IF(LDISI(19)) ADDOT1=ADDOT2=ADDOT3=ADDOT4=0.0
      IF(.NOT. LDISI(18)) GO TO 77
      ADDOT1=ADDHLD1
      ADDOT2=ADDHLD2
      ADDOT3=ADDHLD3
      ADDOT4=ADDHLD4
   77 CONTINUE
      CA1=COS(A1)  $ CA2=COS(A2) $ CA3=COS(A3) $ CA4=COS(A4)
      SA1=SIN(A1)  $ SA2=SIN(A2) $ SA3=SIN(A3) $ SA4=SIN(A4)
      IF(EL .EQ. 0.0) GO TO 227
      IF(M1 .EQ. 0.0 ) GO TO 29
C     *************************  MBS - MASS 1   **
      ADOT1S=ADOT1*ADOT1
C     ************ SMALL R
      SR1X=EL*CA1
      SR1Y=EL*SA1
      SR1Z=-DISTZ
C     ************ SMALL R-DOT
      SR1DX=-EL*ADOT1*SA1
      SR1DY=+EL*ADOT1*CA1
C     ************ SMALL R-DOUBLE DOT
      SR1DDX=-EL*ADOT1S*CA1-EL*ADDOT1*SA1
      SR1DDY=-EL*ADOT1S*SA1+EL*ADDOT1*CA1
C     ************ TERM 2 OF EQ. 4
      AR1DDX=RDD1X + XPRDD
      AR1DDY=RDD1Y + YPRDD
      AR1DDZ=RDD1Z + ZPRDD
      FINA1=M1*(-EL*SA1*(DO11*AR1DDX+DO21*AR1DDY+DO31*AR1DDZ) +
     1           EL*CA1*(DO12*AR1DDX+DO22*AR1DDY+DO32*AR1DDZ))
C     ************ COEFF. OF ALPHA DOUB. DOTS
      FINNA1=M1*EL*EL
      IF(FINNA1 .EQ. 0.0) GO TO 29
      RIGHT1=(I1X-I1Y)*(SURW*SA1*CA1-WXWY*(SA1*SA1-CA1*CA1))
      ADDOT1=(-WZDDK*I1 - FINA1 - CJ1*ADOT1 + RIGHT1)/( I1 + FINNA1)
   29 CONTINUE
C
C     *************************  MBS - MASS 2   **
      IF(M2 .EQ. 0.0 ) GO TO 30
      ADOT2S=ADOT2*ADOT2
      SR2X=EL*CA2
      SR2Y=EL*SA2
      SR2Z=    DISTZ
      SR2DX=-EL*ADOT2*SA2
      SR2DY=+EL*ADOT2*CA2
      SR2DDX=-EL*ADOT2S*CA2-EL*ADDOT2*SA2
      SR2DDY=-EL*ADOT2S*SA2+EL*ADDOT2*CA2
      AR2DDX=RDD2X + XPRDD
      AR2DDY=RDD2Y + YPRDD
      AR2DDZ=RDD2Z + ZPRDD
      FINA2=M2*(-EL*SA2*(DO11*AR2DDX+DO21*AR2DDY+DO31*AR2DDZ) +
     1           EL*CA2*(DO12*AR2DDX+DO22*AR2DDY+DO32*AR2DDZ))
```

```
      FINNA2=M2*EL*EL
      IF(FINNA2 .EQ. 0.0) GO TO 30
      RIGHT2=(I2X-I2Y)*(SUBW*SA2*CA2-WXWY*(SA2*SA2-CA2*CA2))
      ADDOT2=(-WZDDK*I2 - FINA2 - CJ2*ADOT2 + RIGHT2)/( I2 + FINNA2)
   30 CONTINUE
C
C     ************************** MBS - MASS 3  **
      IF(M3 .EQ. 0.0 ) GO TO 31
      ADOT3S=ADOT3*ADOT3
      SR3X=EL*CA3
      SR3Y=EL*SA3
      SR3Z=    -DISTZ
      SR3Z=-DISTZ-5.
      SR3Z=-DISTZ*1.2
      SR3DX=-EL*ADOT3*SA3
      SR3DY=+EL*ADOT3*CA3
      SR3DDX=-EL*ADOT3S*CA3-EL*ADDOT3*SA3
      SR3DDY=-EL*ADOT3S*SA3+EL*ADDOT3*CA3
      AR3DDX=RDD3X+ XPRDD
      AR3DDY=RDD3Y+ YPRDD
      AR3DDZ=RDD3Z+ ZPRDD
      FINA3=M3*(-EL*SA3*(D011*AR3DDX+D021*AR3DDY+D031*AR3DDZ) +
     1            EL*CA3*(D012*AR3DDX+D022*AR3DDY+D032*AR3DDZ))
      FINNA3=M3*EL*EL
      IF(FINNA3 .EQ. 0.0) GO TO 31
      RIGHT3=(I3X-I3Y)*(SUBW*SA3*CA3-WXWY*(SA3*SA3-CA3*CA3))
      ADDOT3=(-WZDDK*I3 - FINA3 - CJ3*ADOT3 + RIGHT3)/( I3 + FINNA3)
   31 CONTINUE
C
C     ************************** MBS - MASS 4  **
      IF(M4 .EQ. 0.0 ) GO TO 32
      ADOT4S=ADOT4*ADOT4
      SR4X=EL*CA4
      SR4Y=EL*SA4
      SR4Z=     DISTZ
      SR4Z=DISTZ+5.
      SR4Z=DISTZ*1.2
      SR4DX=-EL*ADOT4*SA4
      SR4DY=+EL*ADOT4*CA4
      SR4DDX=-EL*ADOT4S*CA4-EL*ADDOT4*SA4
      SR4DDY=-EL*ADOT4S*SA4+EL*ADDOT4*CA4
      AR4DDX=RDD4X + XPRDD
      AR4DDY=RDD4Y + YPRDD
      AR4DDZ=RDD4Z + ZPRDD
      FINA4=M4*(-EL*SA4*(D011*AR4DDX+D021*AR4DDY+D031*AR4DDZ) +
     1            EL*CA4*(D012*AR4DDX+D022*AR4DDY+D032*AR4DDZ))
      FINNA4=M4*EL*EL
      IF(FINNA4 .EQ. 0.0) GO TO 32
      RIGHT4=(I4X-I4Y)*(SUBW*SA4*CA4-WXWY*(SA4*SA4-CA4*CA4))
      ADDOT4=(-WZDDK*I4 - FINA4 - CJ4*ADOT4 + RIGHT4)/( I4 + FINNA4)
   32 CONTINUE
C
      IF(LDISI(33))ADDOT1=ADDOT2=ADDOT3=ADDOT4=ADOT1=ADOT2=ADOT3=ADOT4=0
      ADDHLD1=ADDOT1
      ADDHLD2=ADDOT2
      ADDHLD3=ADDOT3
      ADDHLD4=ADDOT4
C
C     *********************INTEGRATION SCHEME FOR MASS BALANCING SYSTEM
```

```
C
      IF(M1.EQ.0.0.AND.M2.EQ.0.0.AND.M3.EQ.0.0.AND.M4.EQ.0.0) GO TO 227
      IF(.NOT. LDISI(17)) GO TO 227
      GO TO (121,110,111,112),INT
  121 RR(1, 1)=A1
      RR(1, 2)=A2
      RR(1, 3)=A3
      RR(1, 4)=A4
      RR(1, 5)=ADOT1
      RR(1, 6)=ADOT2
      RR(1, 7)=ADOT3
      RR(1, 8)=ADOT4
      XY = 0.5
      L = 2
      INT = 2
  120 RR(L, 1)=ADOT1*HH
      RR(L, 2)=ADOT2*HH
      RR(L, 3)=ADOT3*HH
      RR(L, 4)=ADOT4*HH
      RR(L, 5)=ADDOT1*HH
      RR(L, 6)=ADDOT2*HH
      RR(L, 7)=ADDOT3*HH
      RR(L, 8)=ADDOT4*HH
      IF(L.EQ.5)  GO TO 113
  114 RR(6, 1) = RR(1, 1)+XY*RR(L, 1)
      RR(6, 2) = RR(1, 2)+XY*RR(L, 2)
      RR(6, 3) = RR(1, 3)+XY*RR(L, 3)
      RR(6, 4) = RR(1, 4)+XY*RR(L, 4)
      RR(6, 5) = RR(1, 5)+XY*RR(L, 5)
      RR(6, 6) = RR(1, 6)+XY*RR(L, 6)
      RR(6, 7) = RR(1, 7)+XY*RR(L, 7)
      RR(6, 8) = RR(1, 8)+XY*RR(L, 8)
      IA = L
      GO TO 28
  110 L = 3
      INT = 3
      GO TO 120
  111 L = 4
      INT = 4
      XY = 1.0
      GO TO 120
  112 L = 5
      GO TO 120
  113 RR(6, 1)=RR(1, 1)+(RR(2, 1)+2.*RR(3, 1)+2.*RR(4, 1)+RR(5, 1))*SX
      RR(6, 2)=RR(1, 2)+(RR(2, 2)+2.*RR(3, 2)+2.*RR(4, 2)+RR(5, 2))*SX
      RR(6, 3)=RR(1, 3)+(RR(2, 3)+2.*RR(3, 3)+2.*RR(4, 3)+RR(5, 3))*SX
      RR(6, 4)=RR(1, 4)+(RR(2, 4)+2.*RR(3, 4)+2.*RR(4, 4)+RR(5, 4))*SX
      RR(6, 5)=RR(1, 5)+(RR(2, 5)+2.*RR(3, 5)+2.*RR(4, 5)+RR(5, 5))*SX
      RR(6, 6)=RR(1, 6)+(RR(2, 6)+2.*RR(3, 6)+2.*RR(4, 6)+RR(5, 6))*SX
      RR(6, 7)=RR(1, 7)+(RR(2, 7)+2.*RR(3, 7)+2.*RR(4, 7)+RR(5, 7))*SX
      RR(6, 8)=RR(1, 8)+(RR(2, 8)+2.*RR(3, 8)+2.*RR(4, 8)+RR(5, 8))*SX
      IA = L
  227 CONTINUE
C
C      ******       TIC MARKS FOR ACTUAL PROGRAM TIME
      LDISO(31)=LDISO(103)=.F.
      IF(T .EQ. 0.0) LDISO(31)=LDISO(103)=.T.
      IF((T-TSAVE) .LT. TIMER) GO TO 90
      LDISO(31)=LDISO(103)=.T.
      TSAVE=T
```

```
   90 CONTINUE
C
C
C      ******       AUXILLIARY CALCULATIONS
       CMO=SQRT(A1X*A1X+A1Y*A1Y)
       IXCG=IDXX+MASSD*((A1Y+SRSY)*(A1Y+SRSY)+(A1Z+SRSZ)*(A1Z+SRSZ))
      1+MASSC*((A1Y+SRCY)*(A1Y+SRCY)+(A1Z+SRCZ)*(A1Z+SRCZ))
      2+M1*((A1Y+SR1Y)*(A1Y+SR1Y)+(A1Z+SR1Z)*(A1Z+SR1Z))
      3+M2*((A1Y+SR2Y)*(A1Y+SR2Y)+(A1Z+SR2Z)*(A1Z+SR2Z))
      4+M3*((A1Y+SR3Y)*(A1Y+SR3Y)+(A1Z+SR3Z)*(A1Z+SR3Z))
      5+M4*((A1Y+SR4Y)*(A1Y+SR4Y)+(A1Z+SR4Z)*(A1Z+SR4Z))
       IYCG=IDYY+MASSD*((A1X+SRSX)*(A1X+SRSX)+(A1Z+SRSZ)*(A1Z+SRSZ))
      1+MASSC*((A1X+SRCX)*(A1X+SRCX)+(A1Z+SRCZ)*(A1Z+SRCZ))
      2+M1*((A1X+SR1X)*(A1X+SR1X)+(A1Z+SR1Z)*(A1Z+SR1Z))
      3+M2*((A1X+SR2X)*(A1X+SR2X)+(A1Z+SR2Z)*(A1Z+SR2Z))
      4+M3*((A1X+SR3X)*(A1X+SR3X)+(A1Z+SR3Z)*(A1Z+SR3Z))
      5+M4*((A1X+SR4X)*(A1X+SR4X)+(A1Z+SR4Z)*(A1Z+SR4Z))
       IZCG=IDZZ+MASSD*((A1X+SRSX)*(A1X+SRSX)+(A1Y+SRSY)*(A1Y+SRSY))
      1+MASSC*((A1X+SRCX)*(A1X+SRCX)+(A1Y+SRCY)*(A1Y+SRCY))
      2+M1*((A1X+SR1X)*(A1X+SR1X)+(A1Y+SR1Y)*(A1Y+SR1Y))
      3+M2*((A1X+SR2X)*(A1X+SR2X)+(A1Y+SR2Y)*(A1Y+SR2Y))
      4+M3*((A1X+SR3X)*(A1X+SR3X)+(A1Y+SR3Y)*(A1Y+SR3Y))
      5+M4*((A1X+SR4X)*(A1X+SR4X)+(A1Y+SR4Y)*(A1Y+SR4Y))
       IXYCG=IDXY+MASSD*((A1X+SRSX)*(A1Y+SRSY))
      1          +MASSC*((A1X+SRCX)*(A1Y+SRCY))
      2+M1*((A1X+SR1X)*(A1Y+SR1Y))+M2*((A1X+SR2X)*(A1Y+SR2Y))
      3+M3*((A1X+SR3X)*(A1Y+SR3Y))+M4*((A1X+SR4X)*(A1Y+SR4Y))
       IXZCG=IDXZ+MASSD*((A1X+SRSX)*(A1Z+SRSZ))
      1          +MASSC*((A1X+SRCX)*(A1Z+SRCZ))
      2+M1*((A1X+SR1X)*(A1Z+SR1Z))+M2*((A1X+SR2X)*(A1Z+SR2Z))
      3+M3*((A1X+SR3X)*(A1Z+SR3Z))+M4*((A1X+SR4X)*(A1Z+SR4Z))
       IYZCG=IDYZ+MASSD*((A1Y+SRSY)*(A1Z+SRSZ))
      1          +MASSC*((A1Y+SRCY)*(A1Z+SRCZ))
      2+M1*((A1Y+SR1Y)*(A1Z+SR1Z))+M2*((A1Y+SR2Y)*(A1Z+SR2Z))
      3+M3*((A1Y+SR3Y)*(A1Z+SR3Z))+M4*((A1Y+SR4Y)*(A1Z+SR4Z))
       IMAT(1,1)=IXCG  $  IMAT(1,2)=-IXYCG  $  IMAT(1,3)=-IXZCG
       IMAT(2,1)=-IXYCG  $  IMAT(2,2)= IYCG  $  IMAT(2,3)=-IYZCG
       IMAT(3,1)=-IXZCG  $  IMAT(3,2)= -IYZCG $  IMAT(3,3)=  IZCG
       CALL JACTV(3,3,1,IMAT,EIGV,EVEC,B,C,W1,W2,NERR)
       IF(NERR .EQ. 1) PRINT 100
  100 FORMAT(10X*NON CONVERGENCE AFTER 100 ITERATIONS*)
       ETAXZ=.15514022    $    ETAYZ=.15514022
       IF(EVEC(3,3) .NE. 0.0) ETAXZ=ATAN2(EVEC(1,3),EVEC(3,3))
       IF(EVEC(3,3) .NE. 0.0) ETAYZ=ATAN2(EVEC(2,3),EVEC(3,3))
       ETAXYZ=SQRT(ETAXZ*ETAXZ+ ETAYZ*ETAYZ)
       DELE=.15514022
       IF(ETAXZ.NE. 0.0) DELE=ATAN2(ETAYZ,ETAXZ)
       TIAD=I1*ADOT1+I2*ADOT2+I3*ADOT3+I4*ADOT4
       AIWX=IXCG*WXDK-IXYCG*WYDK-IXZCG*WZDK
      1-(A1X+SR1X)*(A1Z+SR1Z)*ADOT1*M1-(A1X+SR2X)*(A1Z+SR2Z)*ADOT2*M2
      1-(A1X+SR3X)*(A1Z+SR3Z)*ADOT3*M3-(A1X+SR4X)*(A1Z+SR4Z)*ADOT4*M4
       AIWY=-IXYCG*WXDK+IYCG*WYDK-IYZCG*WZDK
      1-(A1Y+SR1Y)*(A1Z+SR1Z)*ADOT1*M1-(A1Y+SR2Y)*(A1Z+SR2Z)*ADOT2*M2
      1-(A1Y+SR3Y)*(A1Z+SR3Z)*ADOT3*M3-(A1Y+SR4Y)*(A1Z+SR4Z)*ADOT4*M4
       AIWZ=-IXZCG*WXDK-IYZCG*WYDK+IZCG*WZDK
      1+((A1X+SR1X)*(A1X+SR1X)+(A1Y+SR1Y)*(A1Y+SR1Y))*ADOT1*M1
      1+((A1X+SR2X)*(A1X+SR2X)+(A1Y+SR2Y)*(A1Y+SR2Y))*ADOT2*M2
      1+((A1X+SR3X)*(A1X+SR3X)+(A1Y+SR3Y)*(A1Y+SR3Y))*ADOT3*M3
      1+((A1X+SR4X)*(A1X+SR4X)+(A1Y+SR4Y)*(A1Y+SR4Y))*ADOT4*M4+ TIAD
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```
      AIWX2=AIWX*AIWX
      AIWY2=AIWY*AIWY
      ASQR=SQRT(AIWX2+AIWY2)
      THETH=.15514022
      IF(AIWZ .NE. 0.)THETH=ATAN2(ASQR,AIWZ)
      DELH=.15514022
      IF(WXDK .NE. 0.0)DELH=ATAN2(AIWY,AIWX)
      THETZ=SQRT(PHDK*PHDK + THDK*THDK)
      DELZ=.15514022
      IF(PHDK.NE.0.0)DELZ=ATAN2(THDK,PHDK)+1.570795
      BIWX=WXDK*WXDK
      BIWY=WYDK*WYDK
      BSQR=SQRT(BIWX+BIWY)
      THETI=.15514022
      IF(WZDK .NE. 0)THETI=ATAN2(BSQR,WZDK)
      DELI=.15514022
      IF(WXDK.NE.0.0)DELI=ATAN2(WYDK,WXDK)
      CONSQ=THETH*THETH + ETAXYZ*ETAXYZ - 2.*THETH*ETAXYZ*COS(DELH-DELE)
      CON=57.295780*SQRT(CONSQ)
      DELE=DELE   *57.295780
      ETAXZ=ETAXZ *57.295780
      ETAYZ=ETAYZ *57.295780
      ETAXYZ=ETAXYZ*57.295780
      THETH=57.29578*THETH
      DELH=57.29578*DELH
      THETZ=57.29578*THETZ
      DELZ=57.29578*DELZ
      THETI=57.29578*THETI
      DELI=57.29578* DELI
      A1A=57.29578*A1
      A2A=57.29578*A2
      A3A=57.29578*A3
      A4A=57.29578*A4
C
C     ******       RECORDER CHANNEL  OUTPUTS
      DIGOUT( 1)=CMO*SFCMO
      DIGOUT(2)=A1X*SFA1X
      DIGOUT( 3)=A1Y*SFA1Y
      DIGOUT( 4)=ETAXYZ*SFETA
      DIGOUT( 5)=ETAXZ *SFETAX
      DIGOUT( 6)=ETAYZ *SFETAY
      DIGOUT( 7)=THETZ*SFTHETZ
      DIGOUT( 8)=CON*SFCON
      DIGOUT(9)=THETH*SFTH
      DIGOUT(10)=THETI*SFTI
      DIGOUT(11)=ADDOT1*SFACC
      DIGOUT(12)=0.0
      DIGOUT(13)=A1A*SFMBA
      DIGOUT(14)=A2A*SFMBA
      DIGOUT(15)=A3A*SFMBA
      DIGOUT(16)=A4A*SFMBA
C  ***      SCANNER FUNCTION**********
90047 LDISO(124)=LDISI(22)
C**** COMMUNICATION WITH REAL TIME DISPLAY
      IF(LDISI(22))  CALL SCANNER(ISCAN)
      CALL DSPLAY
      IF(LDISI(17))  GO TO 90050
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C**** RETURN TO MODE CONTROL SUBROUTINE
      LDISO(59)=.F.
      LDISO(60)=.F.
      LDISO(102)=.F.
      LDISO(110)=.F.
C
90050 CONTINUE
C
C       ******        REAL TIME CRT PLOT    (PH  VS TH  -  IN DEGREES)
      THDEG=THDK*57.295780
      PHDEG=PHDK*57.295780
      IF(THDEG .EQ. 0. .AND.PHDEG .EQ. 0.) GO TO 50002
      IF( .NOT. LDISI(41)) GO TO 50002
      CALL RITECRT(LDISI(17),.T.,10)
50002 CONTINUE
      CALL RTMODE
C**** RETURN FROM MODE CONTROL INTO OPERATE LOOP
90001 CONTINUE
      LDISO(59)=.T.
      LDISO(60)=.T.
      LDISO(102)=.T.
      LDISO(110)=.T.
      IZZ = .F.
      CALL RECORD
      CALL RECYCLE
      GO TO 90006
C**** SECTION H.   PRINT CONTROL
90014 CONTINUE
      NUMBER=(NUMBER+1)
      WRITE(MF,89)NUMBER
   89 FORMAT(1H0,12HRUN NUMBER =,2X,I5)
      WRITE(MF,130)  HH,(VAR(I),I=1,38)
  130 FORMAT(5X*HH=*E12.4,10X*VAR BLOCK (1 THRU 38)*/(10E13.5))
      WRITE(MF,132)RFX,TXDK,MASSD,IDXYO,SRSX,SRCX,RFY,TYDK,MASSD,
     1IDXZO,SRSY,SRCY,RFZ,TZDK,MASSC,IDYZO,SRSZ,SRCZ,FXDK,FYDK,FZDK
  132 FORMAT(///2X*RFX=*E12.5,2X*TXDK=*E12.5,2X*MASSD=*E12.5,
     A         9X*IDXYO=*E12.4,2X*RSX=*E12.4,2X*RCX=*E12.4/
     1          2X*RFY=*E12.5,2X*TYDK=*E12.5,2X*MASSD=*E12.5,
     B         9X*IDXZO=*E12.4,2X*RSY=*E12.4,2X*RCY=*E12.4/
     2          2X*RFZ=*E12.5,2X*TZDK=*E12.5,2X*MASSC=*E12.5,
     C         9X*IDYZO=*E12.4,2X*RSZ=*E12.4,2X*RCZ=*E12.4//
     D         2X*FXDK=*E12.4,2X*FYDK=*E12.4,2X*FZDK=*E12.4)
      WRITE(MF,133) I1X,I2X,I3X,I4X,I1Y,I2Y,I3Y,I4Y
  133 FORMAT(2X*I1X=*E12.5,2X*I2X=*E12.5,3X*I3X=*E12.5,4X*I4X=*E12.5/
     1 2X*I1Y=*E12.5,2X*I2Y=*E12.5,3X*I3Y=*E12.5,4X*I4Y=*E12.5///)
      WRITE(MF,131)
  131 FORMAT(26X*TIME*44X*SRCX*12X*SRCY*12X*SRCZ*/
     1         26X*WXDK*12X*WYDK*12X*WZDK*12X*PHDK*12X*THDK*12X*PSDK*/
     1         26X*WXDDK*11X*WYDDK*11X*WZDDK*11X*A1X*13X*A1Y*13X*A1Z*/
     1         26X*A1*14X*A2*14X*A3*14X*A4*14X*ETAY*12X*ETAX*/
     1       26X*ADOT1*11X*ADOT2*11X*ADOT3*11X*ADOT4*11X*CMO*/
     1       26X*ADDOT1*10X*ADDOT2*10X*ADDOT3*10X*ADDOT4*10X*ETA*13X*DEL
     2TA*/26X*THETH*11X*DELH*12X*THETZ*11X*DELZ*12X*THETI*11X*DELI*///)
90030 CALL PLAYBAK(90032S,NFILE)
      WRITE(MF,1800) T,              SRCX,SRCY,SRCZ
      WRITE(MF,1801) WXDK,WYDK,WZDK,PHDK,THDK,PSDK
      WRITE(MF,1801) WXDDK,WYDDK,WZDDK,A1X,A1Y,A1Z
      WRITE(MF,1801)    A1A,A2A,A3A,A4A,ETAXZ,ETAYZ
```

```
      WRITE(MF,1801)     ADOT1,ADOT2,ADOT3,ADOT4,DELE,ETAXYZ
      WRITE(MF,1801)     ADDOT1,ADDOT2,ADDOT3,ADDOT4,CMO,CON
      WRITE(MF,1801)THETH,DELH,THETZ,DELZ,THETI,DELI
      WRITE(MF,1801)     EIGV(1),EIGV(2),EIGV(3)
 1800 FORMAT(/23XE12.5,32X,3(4XE12.5))
 1801 FO-MAT(19X,6(4XE12.5))
      GO TO 90030
90032 CALL APRINT
C**** SECTION I.  READ CONTROL
90015 CONTINUE
C**** ANY READ STATEMENTS CAN BE PLACED HERE TO INITIALIZE FOR A NEW RUN
C***  READ **,A,B,C
C   ** FORMAT (8E16.8)
      CALL AREAD
C**** SECTION J. TERMINATE
90004 CONTINUE
C**** ANY POST PROCESSING
      CALL ATERM
90034 FORMAT(6X* SPACE BASE SIMULATION*5X*JOB,43,77777,75000,   A2718,
     1 13043,1,C.W.MARTZ,B1232 R125*)
      END
-
-
```

APPENDIX C

TRANSFORMATION MATRICES AND DERIVATIVES

The following transfer matrices and derivatives, collected for convenience, are used in the simulation. For identification or explanations, see section of appendix A entitled "Transfer Matrices."

$$\left[D_1\right] = \begin{bmatrix} c\psi\, c\theta & -s\psi\, c\theta & s\theta \\ c\psi\, s\theta\, s\phi + s\psi\, c\phi & c\psi\, c\phi - s\psi\, s\theta\, s\phi & -c\theta\, s\phi \\ s\psi\, s\phi - c\psi\, s\theta\, c\phi & c\psi\, s\phi + s\psi\, s\theta\, c\phi & c\theta\, c\phi \end{bmatrix}$$

$\left[D_2\right]$ is the same as $\left[D_1\right]$ with subscript h on angles.

$$\left[D\right] = \begin{bmatrix} c\psi\, c\theta & s\psi & 0 \\ -s\psi\, c\theta & c\psi & 0 \\ s\theta & 0 & 1 \end{bmatrix}$$

$\left[D_h\right]$ is the same as $\left[D\right]$ with subscript h on angles.

$$\left[D_3\right] = \begin{bmatrix} c\alpha_j & -s\alpha_j & 0 \\ s\alpha_j & c\alpha_j & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\dot{D}\right] = \begin{bmatrix} -\dot{\psi}\, s\psi\, c\theta - \dot{\theta}\, c\psi\, s\theta & \dot{\psi}\, c\psi & 0 \\ -\dot{\psi}\, c\psi\, c\theta + \dot{\theta}\, s\psi\, s\theta & -\dot{\psi}\, s\psi & 0 \\ \dot{\theta}\, c\theta & 0 & 0 \end{bmatrix}$$

$\left[\dot{D}_h\right]$ is the same as $\left[\dot{D}\right]$ with subscript h on angles.

$$\left[\dot{D}_1\right] = \begin{bmatrix} d'_{11} & d'_{12} & d'_{13} \\ d'_{21} & d'_{22} & d'_{23} \\ d'_{31} & d'_{32} & d'_{33} \end{bmatrix}$$

where

$$d'_{11} = -\dot{\theta}\ c\psi\ s\theta - \dot{\psi}\ s\psi\ c\theta$$

$$d'_{12} = \dot{\theta}\ s\psi\ s\theta - \dot{\psi}\ c\psi\ c\theta$$

$$d'_{13} = \dot{\theta}\ c\theta$$

$$d'_{21} = \dot{\phi}(c\psi\ s\theta\ c\phi - s\psi\ s\phi) + \dot{\theta}\ c\psi\ c\theta\ s\phi - \dot{\psi}(s\psi\ s\theta\ s\phi - c\psi\ c\phi)$$

$$d'_{22} = -\dot{\phi}(s\psi\ s\theta\ c\phi + c\psi\ s\phi) - \dot{\theta}\ s\psi\ c\theta\ s\phi - \dot{\psi}(c\psi\ s\theta\ s\phi + s\psi\ c\phi)$$

$$d'_{23} = -\dot{\phi}\ c\theta\ c\phi + \dot{\theta}\ s\theta\ s\phi$$

$$d'_{31} = \dot{\phi}(c\psi\ s\theta\ s\phi + s\psi\ c\phi) - \dot{\theta}\ c\psi\ c\theta\ c\phi + \dot{\psi}(s\psi\ s\theta\ c\phi + c\psi\ s\phi)$$

$$d'_{32} = -\dot{\phi}(s\psi\ s\theta\ s\phi - c\psi\ c\phi) + \dot{\theta}\ s\psi\ c\theta\ c\phi + \dot{\psi}(c\psi\ s\theta\ c\phi - s\psi\ s\phi)$$

$$d'_{33} = -\dot{\phi}\ c\theta\ s\phi - \dot{\theta}\ s\theta\ c\phi$$

$\left[\dot{D}_2\right]$ is the same as $\left[\dot{D}_1\right]$ with h subscripted angles and angular rates.

$$\left[\ddot{D}_1\right] = \begin{bmatrix} d''_{11} & d''_{12} & d''_{13} \\ d''_{21} & d''_{22} & d''_{23} \\ d''_{31} & d''_{32} & d''_{33} \end{bmatrix}$$

where

$$d''_{11} = -\theta''\ c\psi\ s\theta - \psi''\ s\psi\ c\theta + 2\dot{\theta}\dot{\psi}\ s\psi\ s\theta - (\dot{\theta}^2 + \dot{\psi}^2)c\psi\ c\theta$$

$$d''_{12} = \theta''\ s\psi\ s\theta - \psi''\ c\psi\ c\theta + (\dot{\theta}^2 + \dot{\psi}^2)s\psi\ c\theta + 2\dot{\psi}\dot{\theta}\ c\psi\ s\theta$$

$$d''_{13} = \theta''\ c\theta - \dot{\theta}^2\ s\theta$$

$$d''_{21} = \phi''(c\psi\ s\theta\ c\phi - s\psi\ s\phi) + \theta''\ c\psi\ c\theta\ s\phi - \psi''(s\psi\ s\theta\ s\phi - c\psi\ c\phi) - (\dot{\phi}^2 + \dot{\psi}^2)(c\psi\ s\theta\ s\phi$$
$$+ s\psi\ c\phi) - \dot{\theta}^2\ c\psi\ s\theta\ s\phi + 2\dot{\phi}\dot{\theta}\ c\psi\ c\theta\ c\phi - 2\dot{\theta}\dot{\psi}\ s\psi\ c\theta\ s\phi - 2\dot{\psi}\dot{\phi}(s\psi\ s\theta\ c\phi + c\psi\ s\phi)$$

$$d''_{22} = -\phi''(s\psi\, s\theta\, c\phi + c\psi\, s\phi) - \theta''\, s\psi\, c\theta\, s\phi - \psi''(c\psi\, s\theta\, s\phi + s\psi\, c\phi) + (\dot{\phi}^2 + \dot{\psi}^2)(s\psi\, s\theta\, s\phi - c\psi\, c\phi) + \dot{\theta}^2\, s\psi\, s\theta\, s\phi - 2\dot{\phi}\dot{\theta}\, s\psi\, c\theta\, c\phi - 2\dot{\theta}\dot{\psi}\, c\psi\, c\theta\, s\phi - 2\dot{\phi}\dot{\psi}(c\psi\, s\theta\, c\phi - s\psi\, s\phi)$$

$$d''_{23} = -\phi''\, c\theta\, c\phi + \theta''\, s\theta\, s\phi + (\dot{\phi}^2 + \dot{\theta}^2)c\theta\, s\phi + 2\dot{\phi}\dot{\theta}\, s\theta\, c\phi$$

$$d''_{31} = \phi''(c\psi\, s\theta\, s\phi + s\psi\, c\phi) - \theta''\, c\psi\, c\theta\, c\phi + \psi''(s\psi\, s\theta\, c\phi + c\psi\, s\phi) + (\dot{\phi}^2 + \dot{\psi}^2)(c\psi\, s\theta\, c\phi - s\psi\, s\phi) + \dot{\theta}^2\, c\psi\, s\theta\, c\phi + 2\dot{\phi}\dot{\theta}\, c\psi\, c\theta\, s\phi - 2\dot{\phi}\dot{\psi}(s\psi\, s\theta\, s\phi - c\psi\, c\phi) + 2\dot{\theta}\dot{\psi}\, s\psi\, c\theta\, c\phi$$

$$d''_{32} = -\phi''(s\psi\, s\theta\, s\phi - c\psi\, c\phi) + \theta''\, s\psi\, c\theta\, c\phi + \psi''(c\psi\, s\theta\, c\phi - s\psi\, s\phi) - (\dot{\phi}^2 + \dot{\psi}^2)(s\psi\, s\theta\, c\phi + c\psi\, s\phi) - \dot{\theta}^2\, s\psi\, s\theta\, c\phi - 2\dot{\phi}\dot{\theta}\, s\psi\, c\theta\, s\phi + 2\dot{\theta}\dot{\psi}\, c\psi\, c\theta\, c\phi - 2\dot{\phi}\dot{\psi}(c\psi\, s\theta\, s\phi + s\psi\, c\phi)$$

$$d''_{33} = -\phi''\, c\theta\, s\phi - \theta''\, s\theta\, c\phi - (\dot{\phi}^2 + \dot{\theta}^2)c\theta\, c\phi + 2\dot{\phi}\dot{\theta}\, s\theta\, s\phi$$

$$\left[D\right]^{-1} = \begin{bmatrix} \frac{c\psi}{c\theta} & \frac{-s\psi}{c\theta} & 0 \\ s\psi & c\psi & 0 \\ \frac{-c\psi\, s\theta}{c\theta} & \frac{s\psi\, s\theta}{c\theta} & 1 \end{bmatrix}$$

$\left[D_h\right]^{-1}$ is the same as $\left[D\right]^{-1}$ with h subscripted angles.

$$\left[D_1\right]^{-1} = \left[D_1\right]^T$$

$$\left[D_2\right]^{-1} = \left[D_2\right]^T$$

$$\left[D_3\right]^{-1} = \left[D_3\right]^T$$

$$\left[\frac{\partial D}{\partial \theta}\right] = \begin{bmatrix} -c\psi\, s\theta & 0 & 0 \\ s\psi\, s\theta & 0 & 0 \\ c\theta & 0 & 0 \end{bmatrix}$$

$\left[\frac{\partial D_h}{\partial \theta_h}\right]$ is the same as $\left[\frac{\partial D}{\partial \theta}\right]$ with h subscripted angles.

$$\left[\frac{\partial D}{\partial \psi}\right] = \begin{bmatrix} -s\psi\ c\theta & c\psi & 0 \\ -c\psi\ c\theta & -s\psi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\left[\frac{\partial D_h}{\partial \psi_h}\right]$ is the same as $\left[\frac{\partial D}{\partial \psi}\right]$ with h subscripted angles.

$$\left[\frac{\partial D}{\partial \phi}\right] = \left[\frac{\partial D_h}{\partial \phi_h}\right] = 0$$

$$\left[\frac{\partial D_1}{\partial \theta}\right] = \left[\frac{\partial \dot{D}_1}{\partial \dot{\theta}}\right] = \begin{bmatrix} -c\psi\ s\theta & s\psi\ s\theta & c\theta \\ c\psi\ c\theta\ s\phi & -s\psi\ c\theta\ s\phi & s\theta\ s\phi \\ -c\psi\ c\theta\ c\phi & s\psi\ c\theta\ c\phi & -s\theta\ c\phi \end{bmatrix}$$

$\left[\frac{\partial D_2}{\partial \theta_h}\right]$ is the same as $\left[\frac{\partial D_1}{\partial \theta}\right]$ with h subscripted angles.

$$\left[\frac{\partial D_1}{\partial \psi}\right] = \left[\frac{\partial \dot{D}_1}{\partial \dot{\psi}}\right] = \begin{bmatrix} -s\psi\ c\theta & -c\psi\ c\theta & 0 \\ -s\psi\ s\theta\ s\phi + c\psi\ c\phi & -c\psi\ s\theta\ s\phi - s\psi\ c\phi & 0 \\ s\psi\ s\theta\ c\phi + c\psi\ s\phi & c\psi\ s\theta\ c\phi - s\psi\ s\phi & 0 \end{bmatrix}$$

$\left[\frac{\partial D_2}{\partial \psi_h}\right]$ is the same as $\left[\frac{\partial D_1}{\partial \psi}\right]$ with h subscripted angles.

$$\left[\frac{\partial D_1}{\partial \phi}\right] = \left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right] = \begin{bmatrix} 0 & 0 & 0 \\ c\psi\ s\theta\ c\phi - s\psi\ s\phi & -s\psi\ s\theta\ c\phi - c\psi\ s\phi & -c\theta\ c\phi \\ c\psi\ s\theta\ s\phi + s\psi\ c\phi & -s\psi\ s\theta\ s\phi + c\psi\ c\phi & -c\theta\ s\phi \end{bmatrix}$$

$\left[\frac{\partial D_2}{\partial \phi_h}\right]$ is the same as $\left[\frac{\partial D_1}{\partial \phi}\right]$ with h subscripted angles.

$$\left[\frac{\partial \dot{D}_1}{\partial \theta}\right] = \frac{d}{dt}\left(\left[\frac{\partial \dot{D}_1}{\partial \dot{\theta}}\right]\right) = \begin{bmatrix} \dot{\psi}\, s\psi\, s\theta - \dot{\theta}\, c\psi\, c\theta & \dot{\psi}\, c\psi\, s\theta + \dot{\theta}\, s\psi\, c\theta & -\dot{\theta}\, s\theta \\ -\dot{\psi}\, s\psi\, c\theta\, s\phi - \dot{\theta}\, c\psi\, s\theta\, s\phi + \dot{\phi}\, c\psi\, c\theta\, c\phi & -\dot{\psi}\, c\psi\, c\theta\, s\phi + \dot{\theta}\, s\psi\, s\theta\, s\phi - \dot{\phi}\, s\psi\, c\theta\, c\phi & \dot{\theta}\, c\theta\, s\phi + \dot{\phi}\, s\theta\, c\phi \\ \dot{\psi}\, s\psi\, c\theta\, c\phi + \dot{\theta}\, c\psi\, s\theta\, c\phi + \dot{\phi}\, c\psi\, c\theta\, s\phi & \dot{\psi}\, c\psi\, c\theta\, c\phi - \dot{\theta}\, s\psi\, s\theta\, c\phi - \dot{\phi}\, s\psi\, c\theta\, s\phi & -\dot{\theta}\, c\phi\, c\theta + \dot{\phi}\, s\theta\, s\phi \end{bmatrix}$$

$$\left[\frac{\partial \dot{D}_1}{\partial \psi}\right] = \frac{d}{dt}\left(\left[\frac{\partial \dot{D}_1}{\partial \dot{\psi}}\right]\right) = \begin{bmatrix} -\dot{\psi}\, c\psi\, c\theta + \dot{\theta}\, s\psi\, s\theta & \dot{\psi}\, s\psi\, c\theta + \dot{\theta}\, c\psi\, s\theta & 0 \\ -\dot{\psi}(c\psi\, s\theta\, s\phi + s\psi\, c\phi) - \dot{\theta}\, s\psi\, c\theta\, s\phi - \dot{\phi}(s\psi\, s\theta\, c\phi + c\psi\, s\phi) & \dot{\psi}(s\psi\, s\theta\, s\phi - c\psi\, c\phi) - \dot{\theta}\, c\psi\, c\theta\, s\phi - \dot{\phi}(c\psi\, s\theta\, c\phi - s\psi\, s\phi) & 0 \\ \dot{\psi}(c\psi\, s\theta\, c\phi - s\psi\, s\phi) + \dot{\theta}\, s\psi\, c\theta\, c\phi - \dot{\phi}(s\psi\, s\theta\, s\phi - c\psi\, c\phi) & -\dot{\psi}(s\psi\, s\theta\, c\phi + c\psi\, s\phi) + \dot{\theta}\, c\psi\, c\theta\, c\phi - \dot{\phi}(c\psi\, s\theta\, s\phi + s\psi\, c\phi) & 0 \end{bmatrix}$$

$$\left[\frac{\partial \dot{D}_1}{\partial \phi}\right] = \frac{d}{dt}\left(\left[\frac{\partial \dot{D}_1}{\partial \dot{\phi}}\right]\right) = \begin{bmatrix} 0 & 0 & 0 \\ -\dot{\psi}(s\psi\, s\theta\, c\phi + c\psi\, s\phi) + \dot{\theta}\, c\psi\, c\theta\, c\phi - \dot{\phi}(c\psi\, s\theta\, s\phi + s\psi\, c\phi) & -\dot{\psi}(c\psi\, s\theta\, c\phi - s\psi\, s\phi) - \dot{\theta}\, s\psi\, c\theta\, c\phi + \dot{\phi}(s\psi\, s\theta\, s\phi - c\psi\, c\phi) & \dot{\theta}\, s\theta\, c\phi + \dot{\phi}\, c\theta\, s\phi \\ -\dot{\psi}(s\psi\, s\theta\, s\phi - c\psi\, c\phi) + \dot{\theta}\, c\psi\, c\theta\, s\phi + \dot{\phi}(c\psi\, s\theta\, c\phi - s\psi\, s\phi) & -\dot{\psi}(c\psi\, s\theta\, s\phi + s\psi\, c\phi) - \dot{\theta}\, s\psi\, c\theta\, s\phi - \dot{\phi}(s\psi\, s\theta\, c\phi + c\psi\, s\phi) & \dot{\theta}\, s\theta\, s\phi - \dot{\phi}\, c\theta\, c\phi \end{bmatrix}$$

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TABLE I.- PHYSICAL CONSTANTS USED IN BASIC COMPUTER SIMULATION

Constants:

$m_d = 350\ 000$ kg

$m_c = 1500$ kg

$m_1 = m_2 = m_3 = m_4 = 3200$ kg

$I_{d,x} = I_{d,y} = 3.8 \times 10^8$ kg-m^2

$I_{d,z} = 1.9 \times 10^8$ kg-m^2

$I_{d,xy} = I_{d,xz} = I_{d,yz} = 0$

$I_{c,x} = I_{c,y} = I_{c,z} = I_{c,xy} = I_{c,xz} = I_{c,yz} = 0$

$I_{j,x} = 710$ km^2 (j = 1, 2, 3, 4)

$I_{j,y} = I_{j,z} = 7800$ kg-m^2 (j = 1, 2, 3, 4)

$\ell = 15$ m

$$\left.\begin{aligned} h_1 &= -7.5 \text{ m} \\ h_2 &= 7.5 \text{ m} \\ h_3 &= -9 \text{ m} \\ h_4 &= 9 \text{ m} \end{aligned}\right\} \text{(Note the separation of controllers along the z-axis for collision avoidance purposes)}$$

$C_1 = C_2 = C_3 = C_4 = 4000$ N-m-s

Initial conditions:

$\omega_x = \omega_y = 0$

$\omega_z = 0.5$ rad/s

$\alpha_1 = \alpha_2 = 1.57080$ rad

$\alpha_3 = \alpha_4 = -1.57080$ rad

$\dot{\alpha}_1 = \dot{\alpha}_2 = \dot{\alpha}_3 = \dot{\alpha}_4 = 0$

TABLE II.- KEYBOARD INPUT VARIABLES

Variable (VAR)	Variable name	Symbol or Description
1	PHDKØ	ϕ_o, θ_o, and ψ_o
2	THDKØ	
3	PSDKØ	
4	WXDKØ	Initial value of ω_d
5	WYDKØ	
6	WZDKØ	
7	XPRØ	x'_o, y'_o, and z'_o
8	YPRØ	
9	ZPRØ	
10	XPRDØ	$\dot{x}'_o$, $\dot{y}'_o$, and $\dot{z}'_o$
11	YPRDØ	
12	ZPRDØ	
13	IDXXØ	Initial disk inertias about x-, y-, and z-axes
14	IDYYØ	
15	IDZZØ	
16	A1Ø	Initial values of α_j
17	A2Ø	
18	A3Ø	
19	A4Ø	
20	I1Ø	Initial controller inertias about z-axis
21	I2Ø	
22	I3Ø	
23	I4Ø	
24	M1Ø	Initial values of m_j
25	M2Ø	
26	M3Ø	
27	M4Ø	
28	EL	ℓ
29	DISTZ	h
30	CJØ	Initial damping coefficient for jth controller

TABLE II.- KEYBOARD INPUT VARIABLES – Concluded

Variable (VAR)	Variable name	Symbol or Description
31	SRCDXØ	Initial value of $\{\dot{r}_c\}$
32	SRCDYØ	
33	SRCDZØ	
34	FREQ	CRT real-time plotting frequency (number of iterations per plot point)
35	PLGAIN	CRT plot x- and y-axis gain (units of x and y, full scale)
36	--------	
37	MASSDØ	Disk mass
38	MASSCØ	Crew mass

TABLE III.- TIME-HISTORY RECORDER OUTPUT

Recorder channel	Symbol	Parameter	Scale factor	Range
1	$\sqrt{A_{1,x}^2 + A_{1,y}^2}$	CMO	SFCMO	0 to 0.1 m
2	$A_{1,x}$	A1X	SFA1X	±0.1 m
3	$A_{1,y}$	A1Y	SFA1Y	±0.1 m
4	η	ETA	SFETA	0 to 0.1°
5	η_x	ETAX	SFETAX	±0.1°
6	η_y	ETAY	SFETAY	±0.1°
7	θ_z	THETZ	SFTHETZ	0 to 0.2°
8	------------	--------	---------	--------------
9	θ_h	THETH	SFTH	0 to 0.2°
10	θ_I	THETI	SFTI	0 to 0.2°
11	$\ddot{\alpha}_1$	ADDOT1	SFACC	±0.001 rad/sec^2
12	------------	--------	---------	--------------
13	α_1	A1A	SFMBA	±180°
14	α_2	A2A	SFMBA	±180°
15	α_3	A3A	SFMBA	±180°
16	α_4	A4A	SFMBA	±180°

TABLE IV.- PROGRAM SYMBOL LISTING

[An asterisk denotes printed output]

FORTRAN notation	Symbol definition
*T	Time (sec)
*PHDK, THDK, PSDK	ϕ, θ, and ψ
*WXDK, WYDK, WZDK	ω_d
*WXDDK, WYDDK, WZDDK	$\dot{\omega}_d$
SRX, SRY, SRZ	$\{r\}$
SRSX, SRSY, SRSZ	$\{r_d\}$
*SRCX, SRCY, SRCZ	$\{r_c\}$
SR1X, SR1Y, SR1Z SR2X, SR2Y, SR2Z SR3X, SR3Y, SR3Z SR4X, SR4Y, SR4Z	$\{r_j\}$
SRDX, SRDY, SRDZ	$\{\dot{r}\}$
SRSDX, SRSDY, SRSDZ	$\{\dot{r}_d\}$
SRCDX, SRCDY, SRCDZ	$\{\dot{r}_c\}$
SR1DX, SR1DY, SR1DZ SR2DX, SR2DY, SR2DZ SR3DX, SR3DY, SR3DZ SR4DX, SR4DY, SR4DZ	$\{\dot{r}_j\}$
SRSDDX, SRSDDY, SRSDDZ	$\{\ddot{r}_d\}$
SRCDDX, SRCDDY, SRCDDZ	$\{\ddot{r}_c\}$
SR1DDX, SR1DDY, SR1DDZ SR2DDX, SR2DDY, SR2DDZ SR3DDX, SR3DDY, SR3DDZ SR4DDX, SR4DDY, SR4DDZ	$\{\ddot{r}_j\}$
*A1, A2, A3, A4	α_j
*ADOT1, ADOT2, ADOT3, ADOT4	$\dot{\alpha}_j$
*ADDOT1, ADDOT2, ADDOT3, ADDOT4	$\ddot{\alpha}_j$
M1, M2, M3, M4	m_j
CJ1, CJ2, CJ3, CJ4	c_j
MASSD	m_d
MASSC	m_c
MT	m_T

TABLE IV.- PROGRAM SYMBOL LISTING – Continued

[An asterisk denotes printed output]

FORTRAN notation	Symbol definition
D11, D12, etc.	$[D]$
DØ11, DØ12, etc.	$[D_1]$
DD11, DD12, etc.	$[\dot{D}]$
DI11, DI12, etc.	$[D]^{-1}$
DØD11, DØD12, etc.	$[\dot{D}_1]$
DØDD11, DØDD12, etc.	$[\ddot{D}_1]$
DØDPD11, DØDPD12, etc.	$\frac{\partial[\dot{D}_1]}{\partial\dot{\phi}}$
DØDTD11, DØDTD12, etc.	$\frac{\partial[\dot{D}_1]}{\partial\dot{\theta}}$
DØDSD11, DØDSD12, etc.	$\frac{\partial[\dot{D}_1]}{\partial\dot{\psi}}$
XPR, YPR, ZPR	x', y', and z'
XPRD, YPRD, ZPRD	$\dot{x}'$, $\dot{y}'$, and $\dot{z}'$
XPRDD, YPRDD, ZPRDD	$\ddot{x}'$, $\ddot{y}'$, $\ddot{z}'$, and $\{\ddot{R}_g\}$
*A1X, A1Y, A1Z	$\{A_1\}$
A1DX, A1DY, A1DZ	$\{\dot{A}_1\}$
A1DDX, A1DDY, A1DDZ	$\{\ddot{A}_1\}$
TXDK, TYDK, TZDK	$\{T_d\}$
FXDK, FYDK, FZDK	$\{F_d\}$
RDD1X, RDD1Y, RDD1Z RDD2X, RDD2Y, RDD2Z RDD3X, RDD3Y, RDD3Z RDD4X, RDD4Y, RDD4Z	$\{\ddot{R}_j\}$
ARDDX(1), ARDDY(1), ARDDZ(1)	$\{\ddot{R}_d\}$
ARDDX(3), ARDDY(3), ARDDZ(3)	$\{\ddot{R}_c\}$

TABLE IV.- PROGRAM SYMBOL LISTING – Concluded

[An asterisk denotes printed output]

FORTRAN notation	Symbol definition
TXX(1), TYY(1), TZZ(1)	$m_d\{\ddot{R}_d\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\theta}}\right]\{r_d\}$ (See eq. (A21))
TXX(3), TYY(3), TZZ(3)	$m_c\{\ddot{R}_c\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\theta}}\right]\{r_c\}$ (See eq. (A21))
TXX(4), TYY(4), TZZ(4) TXX(5), TYY(5), TZZ(5) TXX(6), TYY(6), TZZ(6) TXX(7), TYY(7), TZZ(7)	$m_j\{\ddot{R}_j\}^T\left[\frac{\partial \dot{D}_1}{\partial \dot{\theta}}\right]\{r_j\}$ (See eq. (A21))
IDXX, IDYY, etc.	$[I]$
IDDXX, IDDYY, etc.	$[\dot{I}]$
PHDEG, THDEG	ϕ and θ in degrees for CRT plot
EL	ℓ
DISTZ	h_j
KX, KY, KZ	$[K]$
KRX, KRY, KRZ	$[K_R]$
CX, CY, CZ	$[C]$
CRX, CRY, CRZ	$[C_R]$
I1, I2, I3, I4	$I_{j,z}$
I1X, I2X, I3X, I4X	$I_{j,x}$
I1Y, I2Y, I3Y, I4Y	$I_{j,y}$
*ETA	η
*ETAX	η_x
*ETAY	η_y
*CMØ	$\sqrt{A_{1,x}^2 + A_{1,y}^2}$

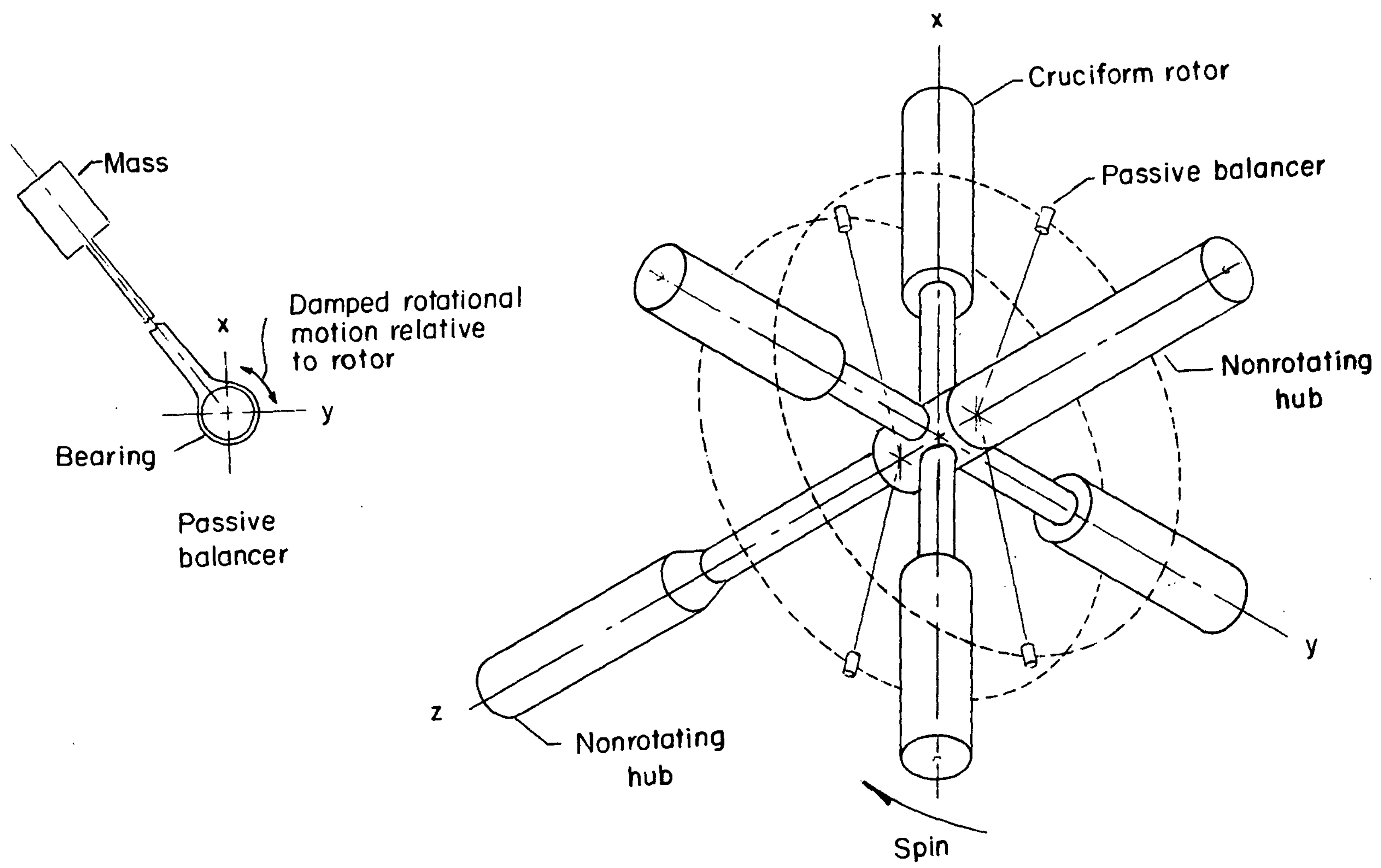


Figure 1.- Dual-spin spacecraft equipped with passive balancing.

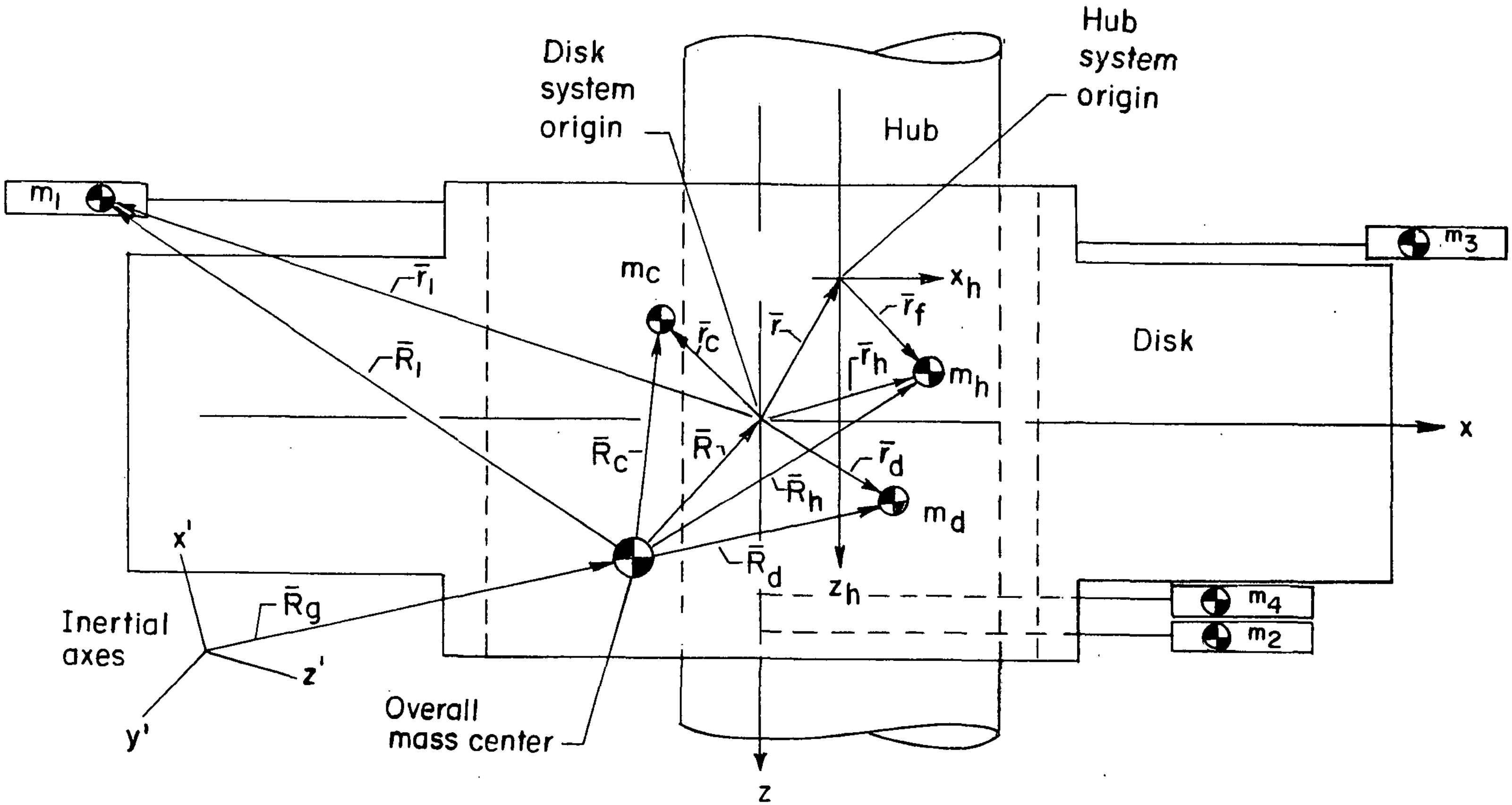


Figure 2.- Analytical representation of spacecraft illustrating vector relationships of mass centers and origins of axis systems.

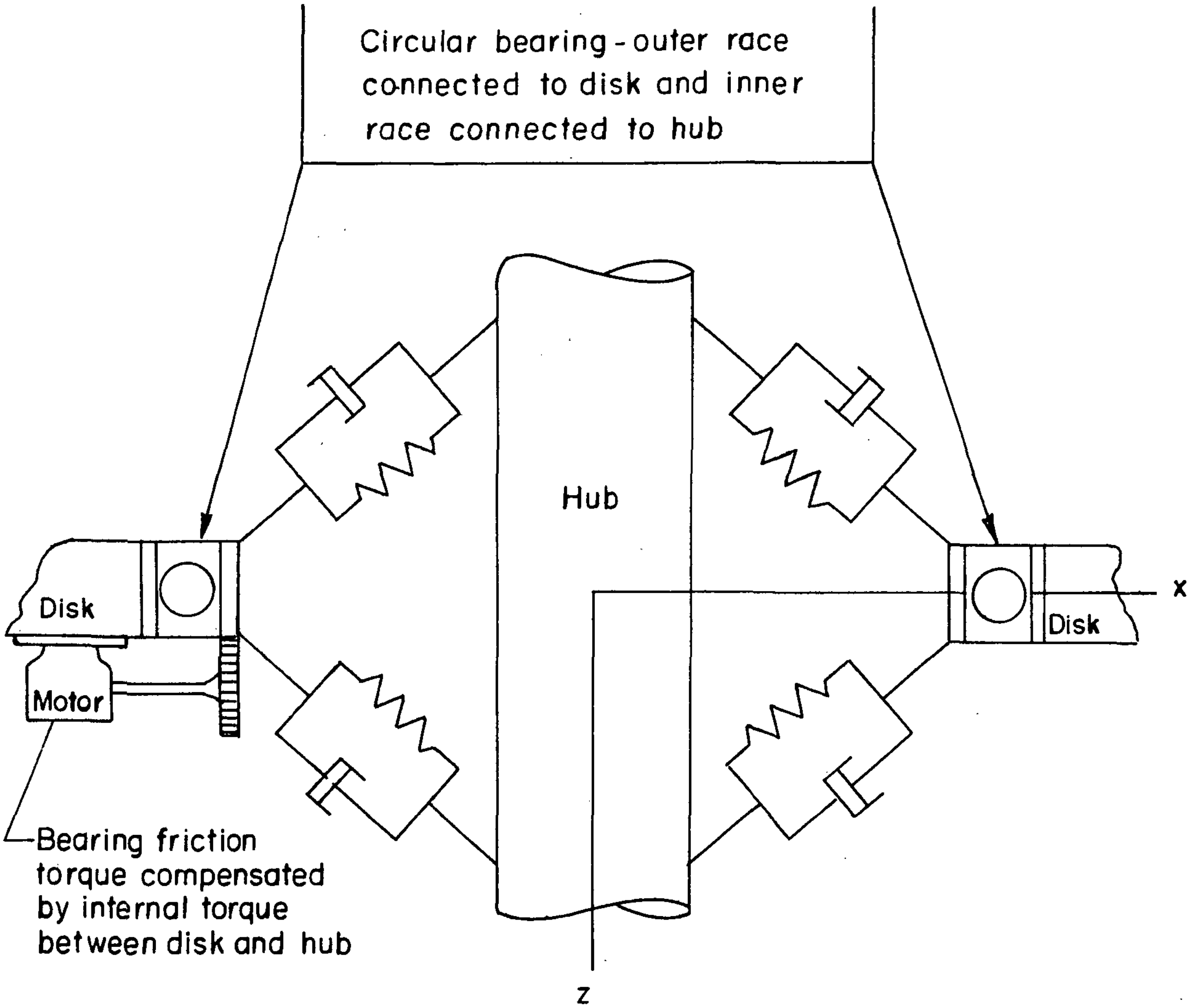


Figure 3.- Schematic showing hub and disk connected through springs, dampers, and a bearing. Note that y,z-plane is similar to x,z-plane shown.

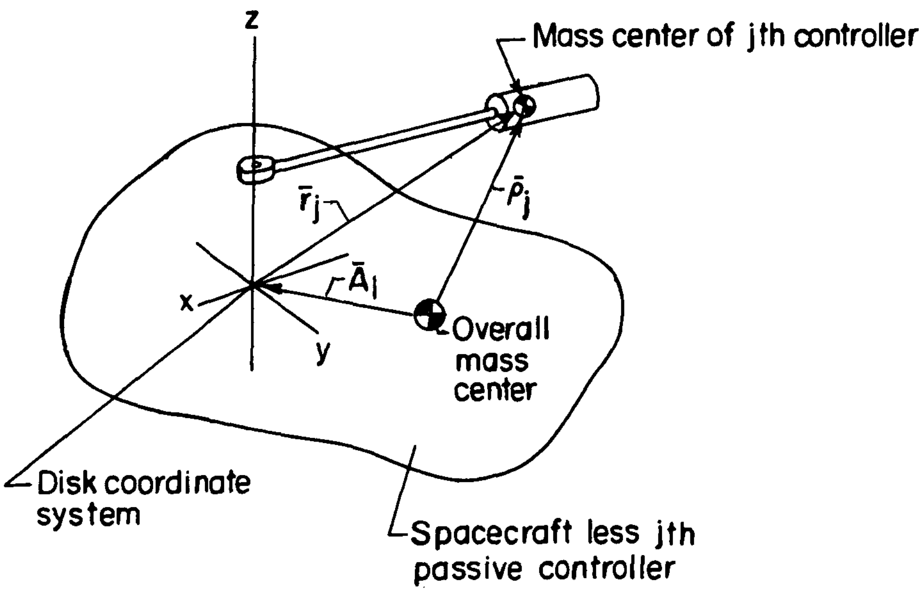
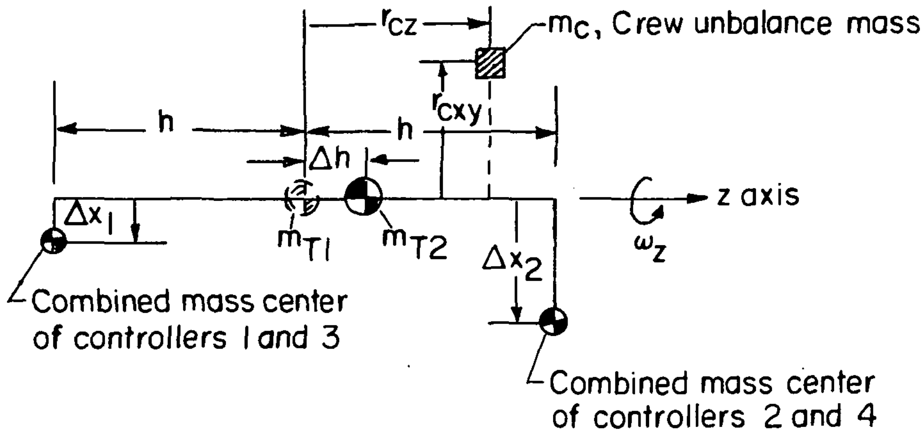


Figure 4.- Vector relationship of jth controller, overall mass center, and disk coordinate axes.



m_{T1} - Spacecraft mass center for controllers balanced about z axis and crew undeployed ($r_{cz} = r_{cxy} = 0$)

m_{T2} - Spacecraft mass center with crew unbalance counteracted by controllers

$$r_{cxy} = \sqrt{r_{cx}^2 + r_{cy}^2}$$

$$\Delta h = \frac{m_c r_{cz}}{m_T}$$

Figure 5.- Mass center geometry of controller steady-state response to combined static and dynamic crew unbalance.

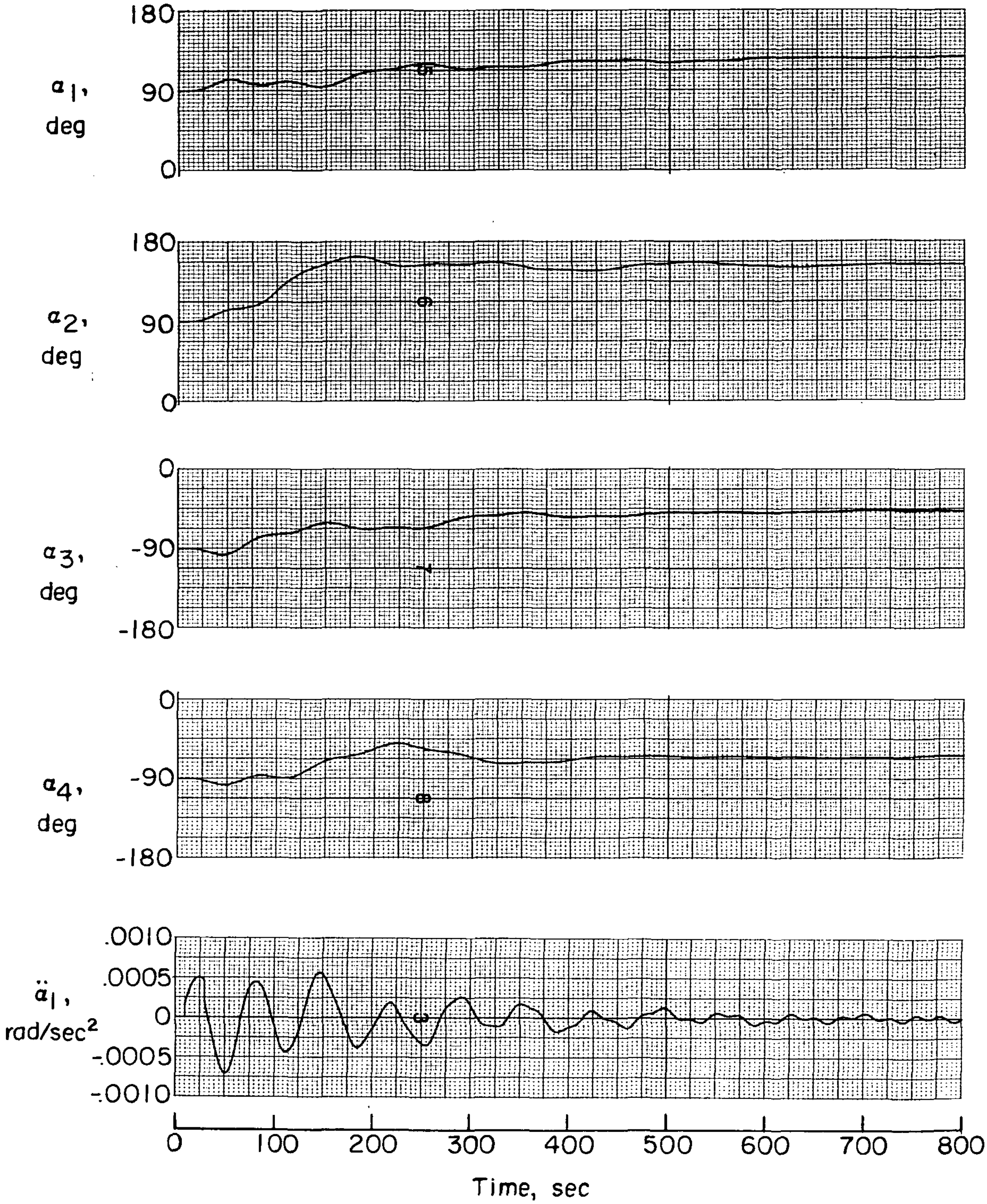


Figure 6.- Angular response of controllers during simulation.

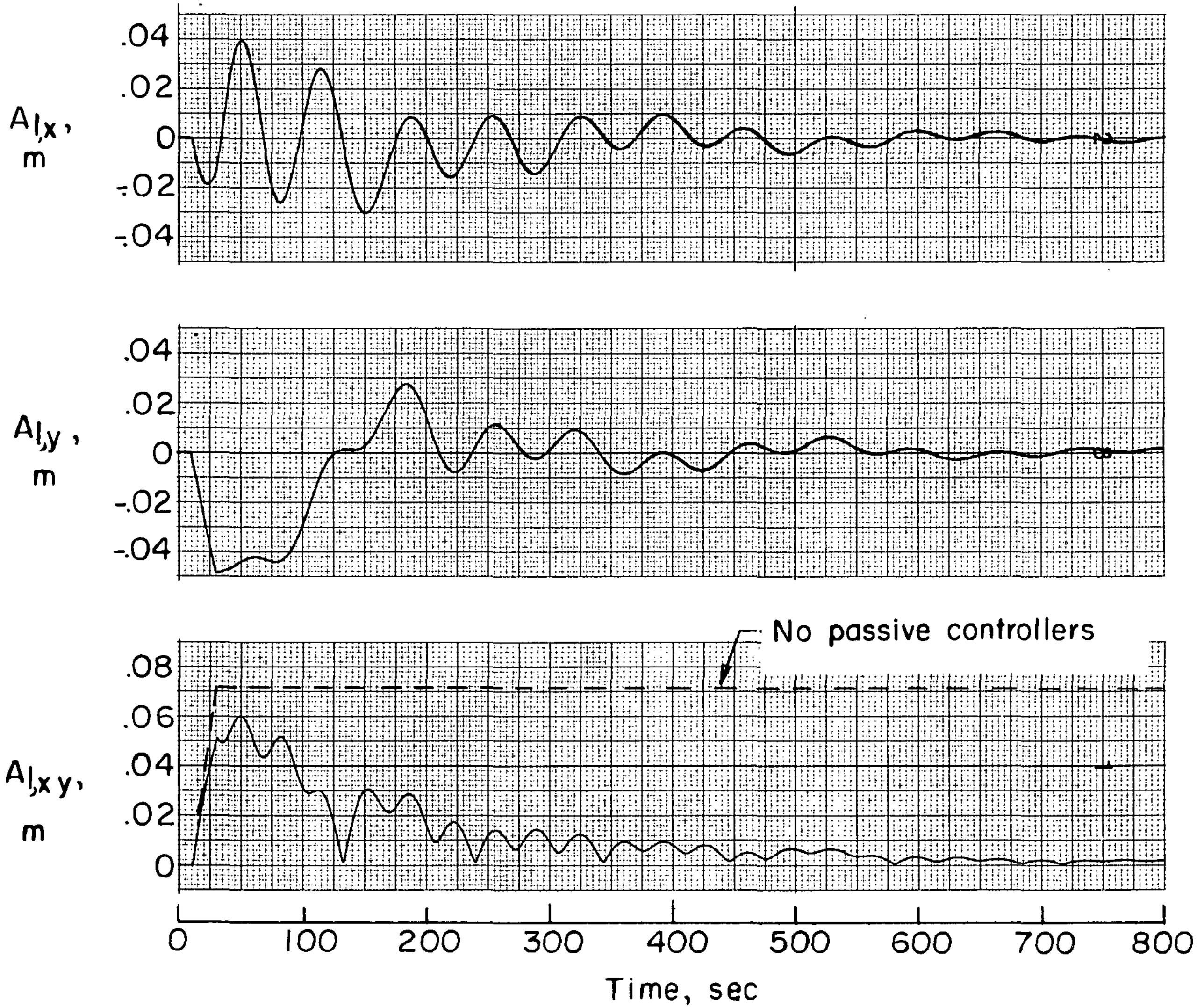


Figure 7.- History of overall mass center offset for crew disturbance simulations with and without passive controllers.

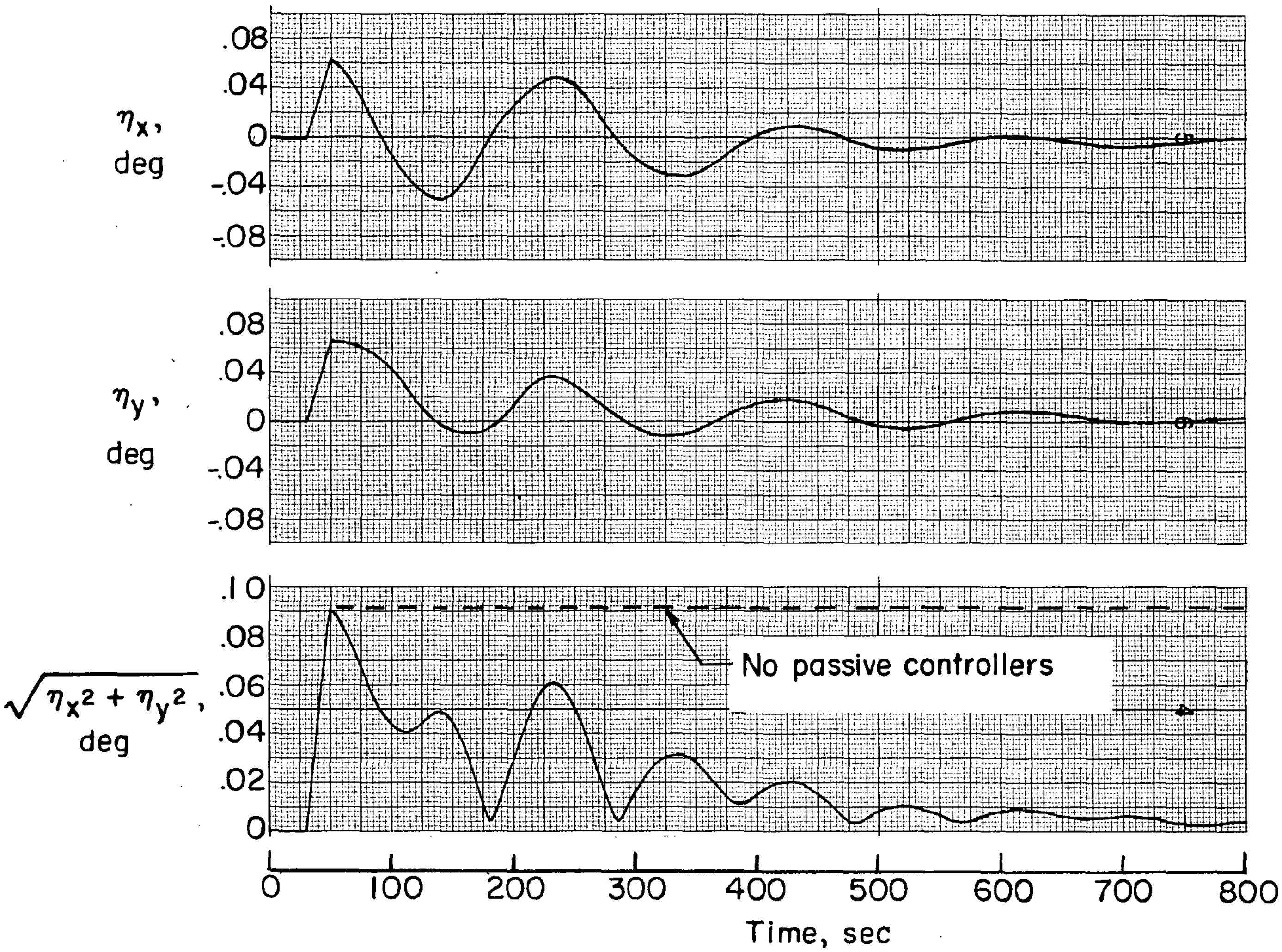
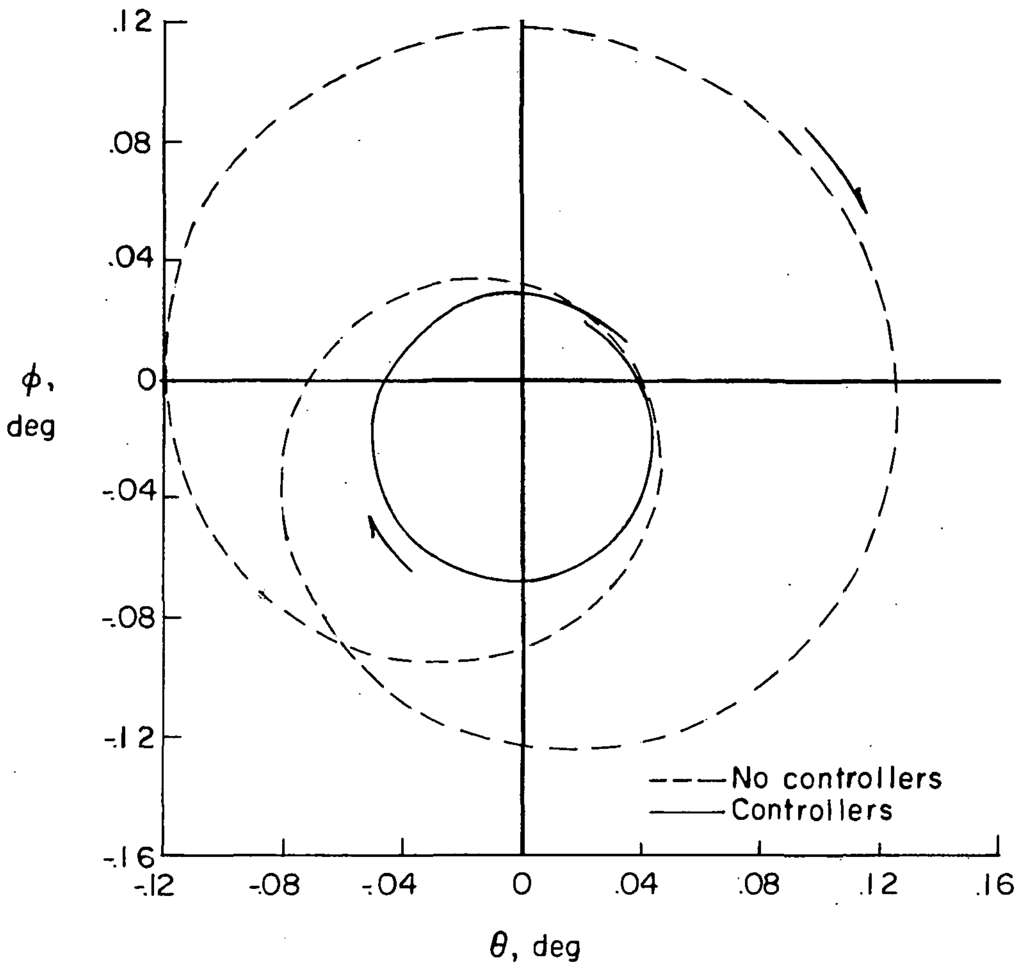
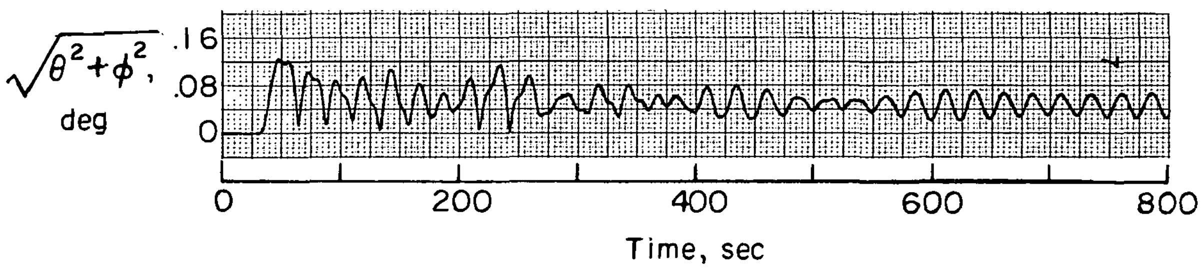


Figure 8.- Simulation history of principal-axis misalinement with and without passive controllers.



(a) ϕ as a function of θ for $650 \leqq T \leqq 680$.



(b) Resultant heading angle (with controllers).

Figure 9.- Spacecraft inertial pointing response to crew motion disturbances.

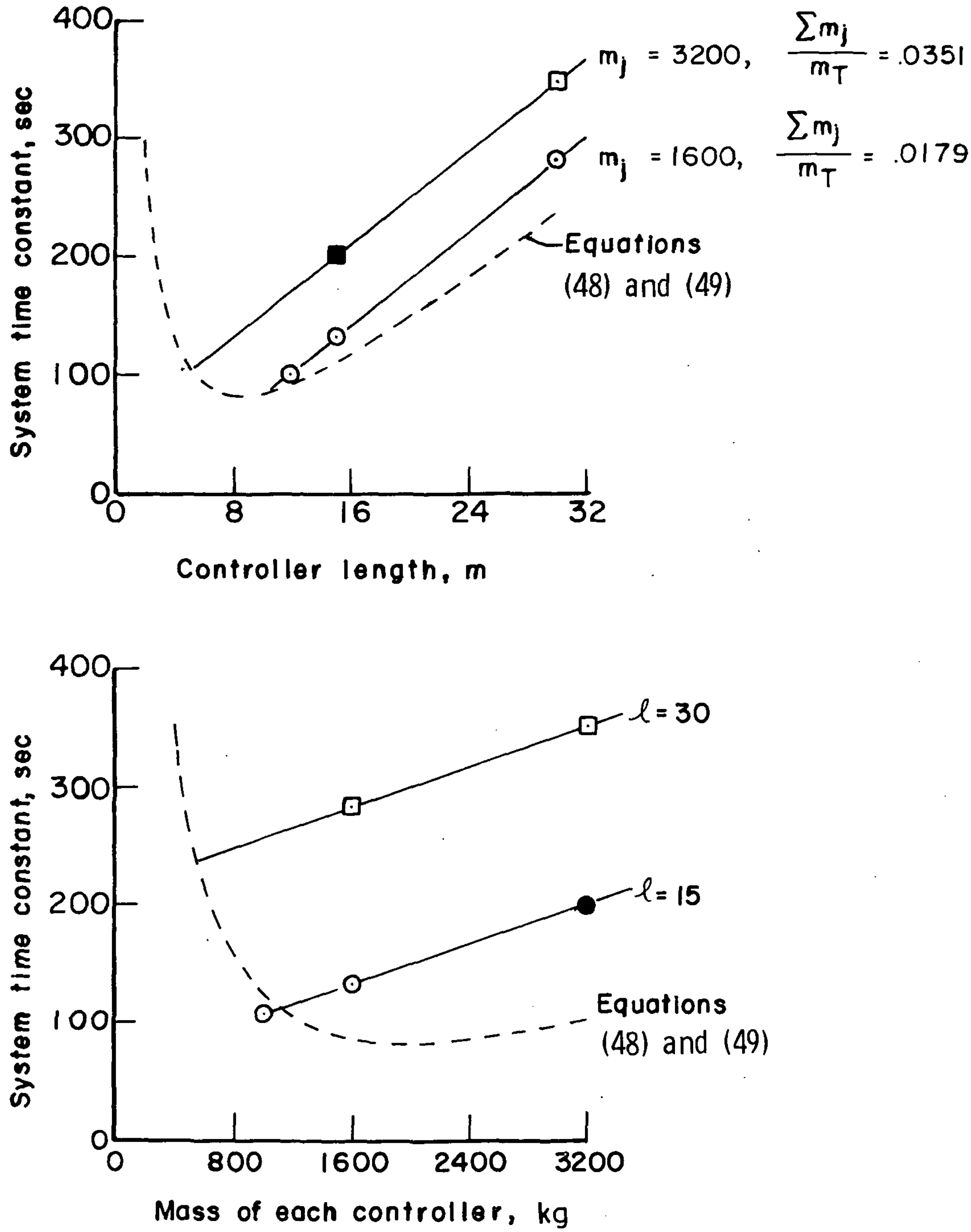


Figure 10.- Effect of controller mass and length on system time constant. See table I for system constants.

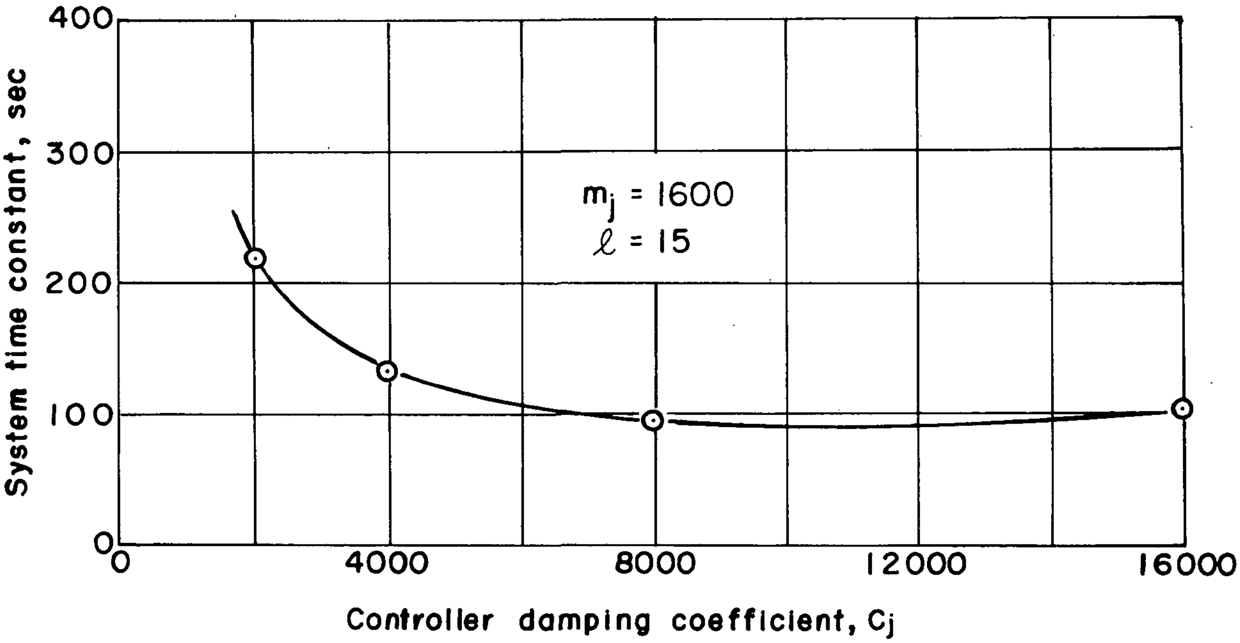
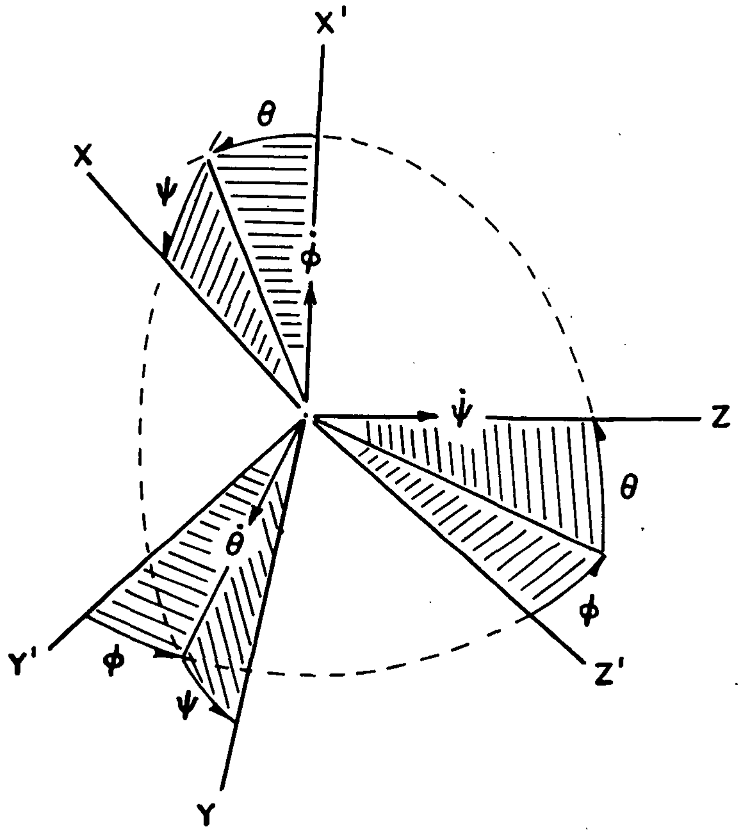
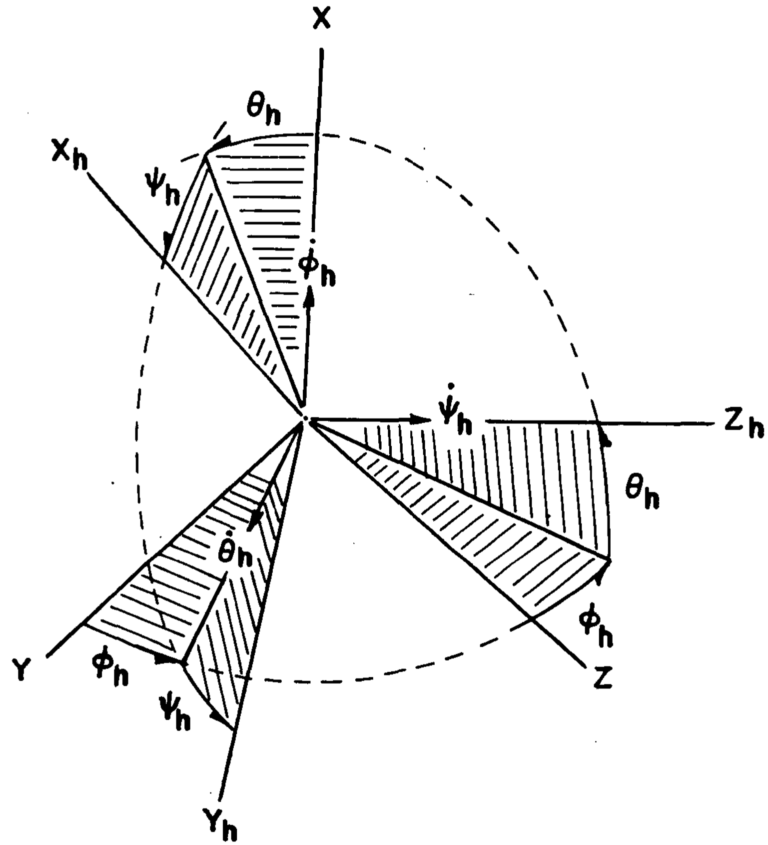


Figure 11.- Effect of controller damping on system time constant.

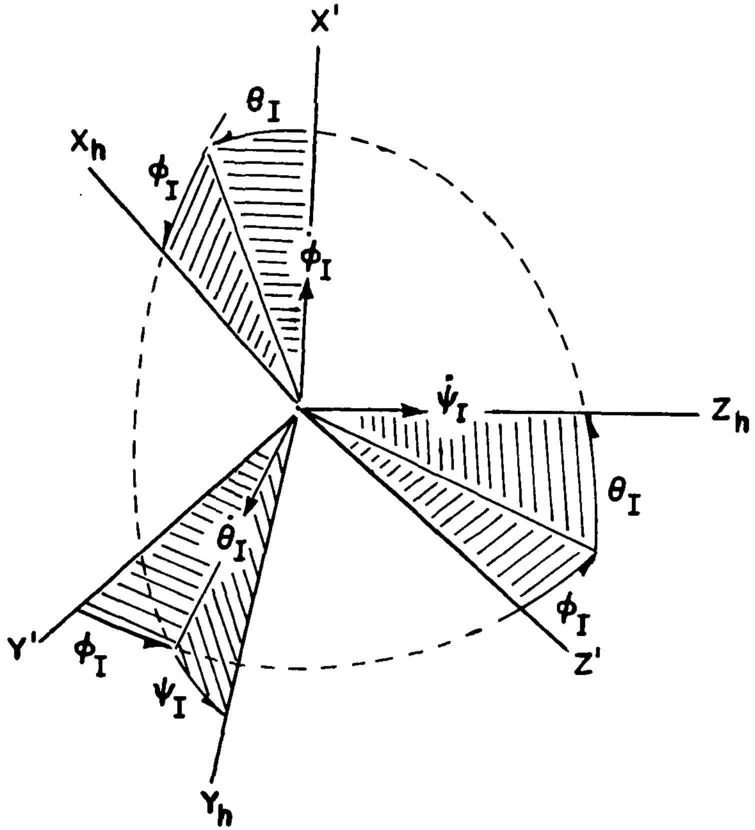


(a) Inertial and rotor axes systems showing Euler rotations ϕ, θ, and ψ.

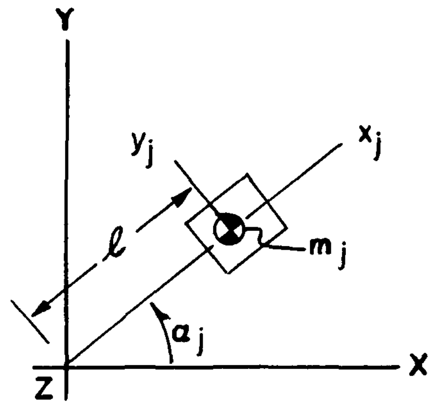


(b) Rotor and hub axes systems showing Euler relative rotations ϕ_h, θ_h, and ψ_h.

Figure 12.- Orientation of coordinate axes.



(c) Inertial and hub axes systems showing Euler rotations ϕ_I, θ_I, and ψ_I.



(d) Rotor and controller axes systems showing controller degree of freedom α_j.

Figure 12.- Concluded.

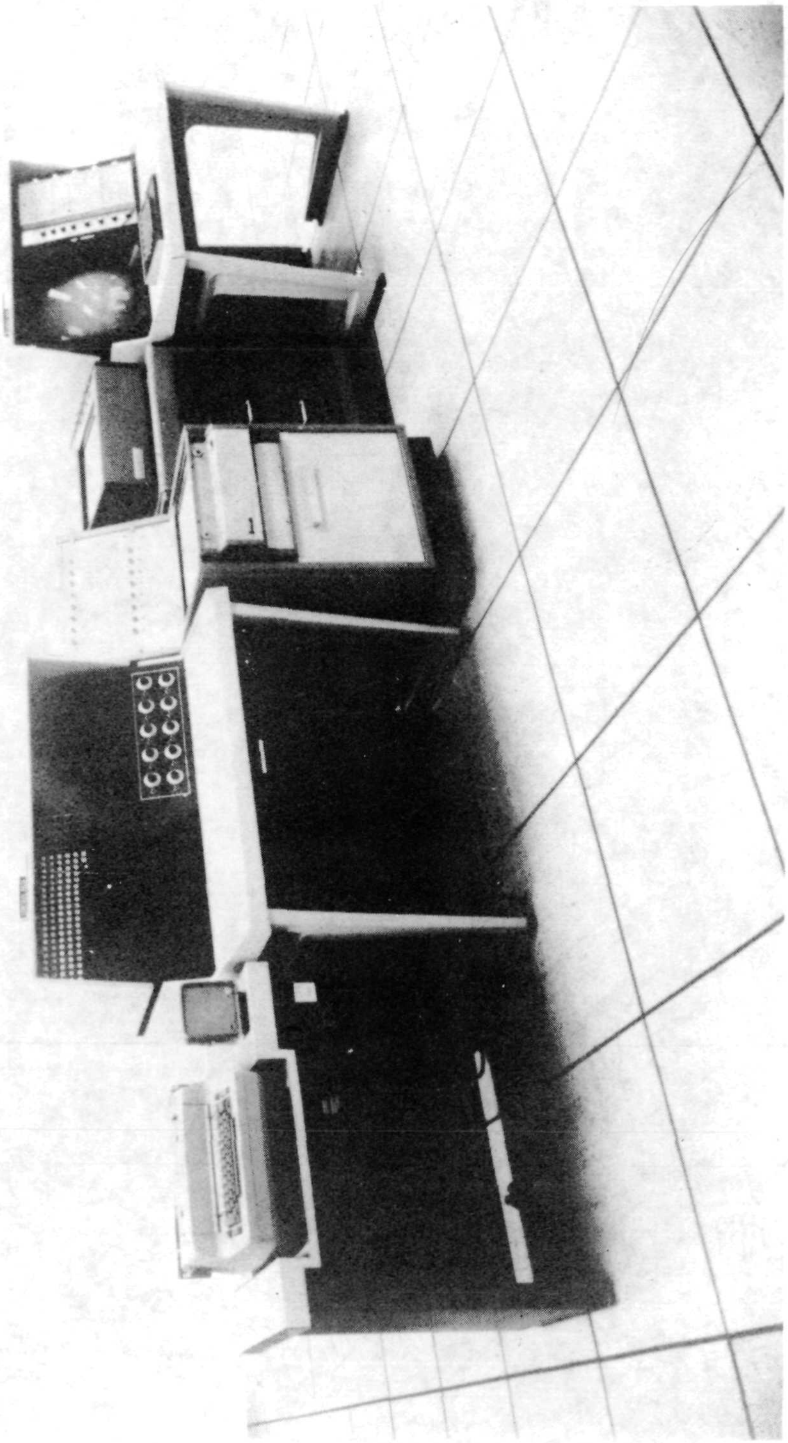


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Figure 13.- Simulation control station.

NASA-Langley, 1972 —— 31 L-8282

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