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Theory and Calculus of Cubical Complexes

Combination switching networks with multiple outputs may be represented by Boolean functions. A report has been prepared which describes the derivation and use of an extraction algorithm that may be adapted to the simplification of such simultaneous Boolean functions, a problem that often arises in logic minimization and fault diagnosis.

The algorithms are derived through cubical complexes, an n-variable, algebraic and topological system that may be used to represent a Boolean function of n-variables.

The particular problem that motivates the formulation of the extraction algorithm is a "covering" problem, i.e.: given a Boolean function, find an equivalent one in disjunctive form (the logical sum of logical products) that requires a minimum number of Boolean variables.

Each term of a Boolean function may be represented as:

$$X_1^{a_1} X_2^{a_2} \ldots X_n^{a_n}$$

where $X_i^{a_i} = X_1$ if $a_i = 1$, and $X_i^{a_i} = X_i'$ if $a_i = 0$

Each vertex of an n-cube (an n-dimensional hypercube) can be represented by the coordinates $(a_1, a_2 \ldots, a_n)$ where $a_i = 0, 1$. Thus a one-to-one correspondence exists between the 2^n possible terms of the Boolean function and the 2^n vertices of an n-cube.

In developing the extraction algorithm, several concepts are defined. A 0-cube is a single vertex, a 1-cube consists of two zero cubes that differ in exactly one place (e.g., 1101 and 1100 which may be written as 110X where X is a free or "don't care" variable). A 2-cube would be of the form 11XX, and by induction the collection of r-cubes K^r is defined for all r where $0 \leq r \leq n$.

A cubical complex is defined as $K^0, K^1, \ldots K^n$ and two operators, the face and co-face operators. Several other operations (a star product, a sharp product, and a "less than" operation) are defined for us in the extraction algorithm.

Several types of subcomplexes are defined and, along with the various operations, are used in an extraction algorithm to find a K-cover of minimum cost for a subset of K. This procedure, then, is analogous to finding the minimum number of literals and (logical) product terms necessary to represent a Boolean function in disjunctive form.

Note:

Requests for further information may be directed to:

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