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NASA TECH BRIEF

Lewis Research Center

Computer Program to Determine Roots of Polynomials by Ratio of Successive Derivatives

The Problem:

Development of a fast accurate method of determining the roots of a polynomial by successive ratios.

The Solution:

A computer program and equations have been developed which find the roots of polynomials with real number coefficients.

How It's Done:

High-speed computing has made some formerly laborious mathematical procedures, such as solving for the roots of rather high degree polynomials, somewhat more practical for broad engineering application. Before the advent of computers, solving high degree polynomials for roots had an element of art to complement the science. The solution of general polynomials on computers, however, requires completely logical steps. Many methods have been developed and programmed for general use. Almost all of these programs use iterative procedures and require the evaluation of the polynomial at each trial root. Most of the programs work well for the vast majority of cases; however, they usually either compute an inaccurate solution or fail to converge to a solution for some root combinations. These difficulties are usually caused by multiple roots at a point or by two or more very close roots.

Roots of polynomials are the arguments for which the polynomial is zero. When roots of polynomials are very close together or when multiple roots occur at a point, the polynomial approaches these roots at very nearly zero slope. With these low slopes, very poor resolution capability is possible with the commonly used polynomial-equal-zero tolerance criterion. Polynomial derivatives give clues as to the nature of the root or roots approached.

The ratios of successive polynomial derivatives give the following very useful information: (1) the multiplicity of a root, that is, the number of roots at the root point approached, (2) the closeness of a trial point to the root approached, and (3) a good approximation as to where the next nearest root is when at a root. Thus, an approach using the ratios of successive polynomial derivatives provides accurate roots-of-polynomial computer programs with very high reliability.

The input is the number, N, of polynomial coefficients A(I), which are read in order, beginning with the high degree end of the polynomial. The output is the real and imaginary parts of each root and the polynomial remainder left when a root is divided out. Since the remainder should be zero, the residual value is a check on the validity of the root.

The root resolution capability of the program is a function of the number of significant figures in the polynomial coefficients as well as the computational methods. For example, the resolution of a cluster of nearby roots can be approximated by an evenly spaced group of roots, m, on a circle in the complex plane. On the circle, the resolution distance is roughly the number of meaningful significant figures of a computed derivative divided by m.

The advantage of the derivative ratio method over other methods is that root analysis can still be done even though the polynomial and its lower order derivatives cannot be evaluated with sufficient accuracy. Upon near range approach to a cluster, individual roots can usually be resolved; but in the cases where resolution cannot be made, the remaining group is treated as a multiroot located near the centroid of the group.

Notes:

1. The computer program is written in FORTRAN IV language for use on an IBM-7044-7094 or 360 system.
2. Inquiries concerning this program should be directed to:

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   Information Services
   112 Barrow Hall
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