NASA TECH BRIEF
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Minimum Switching Network for Generating the Weight of a Binary Vector

The problem:

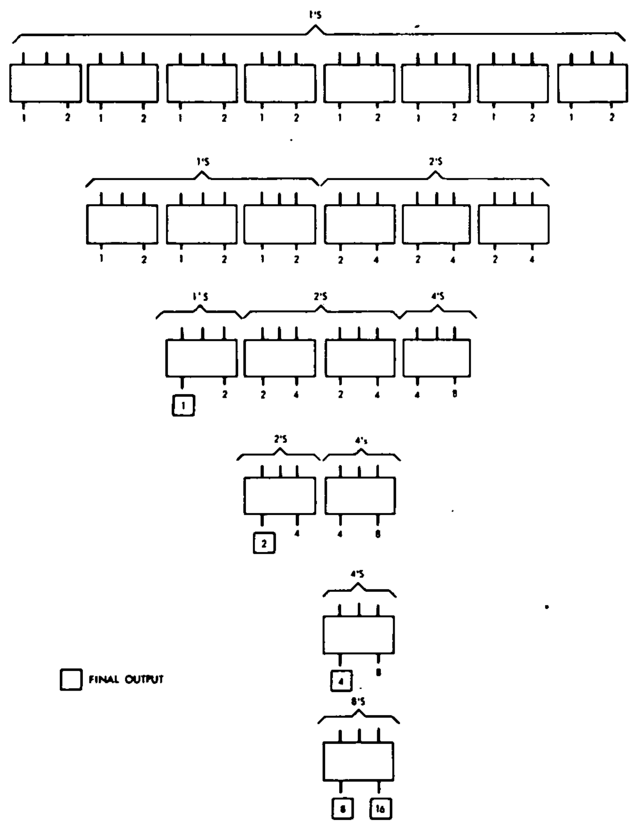
To devise a simple, iterative switching network for converting the weight of a binary vector to a binary number.

The solution:

The vector is divided into three variable sections, and each section is processed by a unary-to-binary decoder or adder. The outputs from the adders are then 2^0's and 2^1's. All the 2^0's are considered as a vector and treated in the same manner as the original vector; all the 2^1's are considered as another vector and treated in the same manner. The converter modules with 2^0 inputs have outputs which are 2^0 and 2^1, and the modules with 2^1 inputs have outputs that are 2^1 and 2^2. The resulting network then performs on an iterative collection process as opposed to a regular arithmetic addition process; all outputs of the same kind are collected in the same manner until there is only one 2^0, one 2^1, one 2^2, one 2^3, and one 2^4, etc., giving the binary weight of the input vector.

How it's done:

The weight of a binary vector is the number of ones it contains; the use of the weight function as a computing function is greatly enhanced when it is available as a binary vector, for example, as in algebraic decoding where a received word is correlated with all words in a dictionary and a search is made on the error vectors. If the weight of the error vector, i.e., the output of a bit-by-bit mod-2 comparison, can be made available in parallel in binary notation, the search for minimum or maximum weight can be made rapidly using a qualitative binary comparator. Other applications for the weight vector occur in searches for optimum comma-free vectors in block coding and in convolutional coding where searches are made to determine error correction capabilities.



The binary vector is first divided into three variable sections with each section converted in a unary-to-binary converter or decoder. This decoder is the

(continued overleaf)

module which is used in an iterative pattern for each level of the network. As shown in the diagram, the outputs from the first level modules are 2^0's and 2^1's, respectively. All 2^0 terms are collected and entered as inputs to the other set of second level modules in the same manner; only one kind of input terms can be entered to any one module. Collections of outputs of the same kind for entry to the modules in subsequent levels are continued until there is only one output of 2^0's, one 2^1's, one 2^2's, etc.; these outputs form the binary weight vector.

The most efficient unary-to-binary converter module is one wherein binary output terms are equally utilized. In terms of numbers of inputs, this then means 3, 7, 15, etc. Of these, the 3-input-2-output module, corresponding to a full adder of the commercial variety, is the most efficient.

It can be shown that the number of modules required is always less than the number of input variables, n. Each module with three inputs of significance 2^0 has one output of significance 2^0 and one of significance 2^1. Enough modules with 2^1's inputs are required to reduce the number of 2^1's to one. This clearly requires $n/2$ modules if n is even, and $(n-1)$ modules if n is odd or $[n/2]$ modules where [x] is the largest integer $\leq$ x. Accordingly, $[n/4]$ modules with 2^1's inputs are required, $[n/8]$ modules with 2^2's inputs are required, etc. From this argument, it follows that the minimum number of modules required for n input variables is

$$[n/2] + [n/4] + [n/8] + \cdots,$$

which series can be summed to $n-$m, where m is the "weight" of the binary representation of n. The number of modules required is then always $\leq n-1$.

Notes:

1. In combination with simple comparator gates, the weighting network can also be used as a majority network.
2. Requests for further information may be directed to:

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