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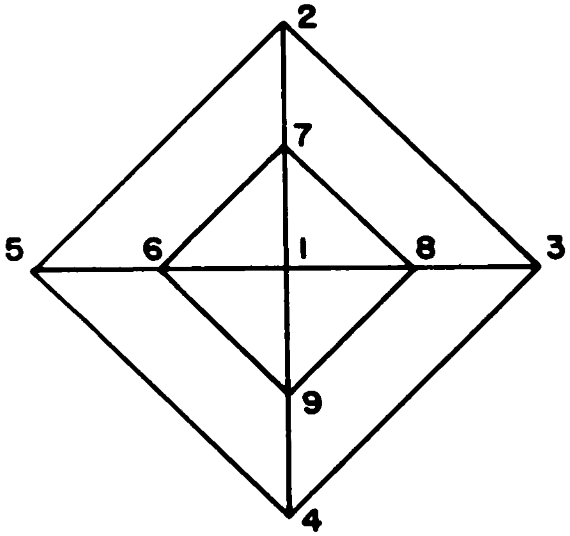
Node-Reordering Method for Stiffness Matrix Wavefront Reduction in Structural Analysis

It is anticipated that in the performance of structural analysis, the solution of structural matrices by means of the present bandwidth concepts will be replaced by the more efficient wavefront concepts. A method has been developed which fills an important void by providing a readily-implemented approach to automatic node relabeling that is consistent with requirements of the wavefront concept. Specific applications are in the analysis of aircraft, building structures, radar and surveillance structures, bridges, etc., or any other structure that is studied with the aid of a large and complex analytical model.



Sample Structure

The purpose of reordering for an analytical model of a particular structure is to minimize the number of degrees of freedom that determine the maximum of the bandwidth or the wavefront of the stiffness matrix.

	1	2	3	4	5	6	7	8	9	Wavefront
1										4
2										6
3		X								6
4			X							5
5		X		X						4
6	X				X					3
7	X	X				X				2
8	X		X				X			1
9	X			X		X		X		0

Nodal Connectivity Matrix

Frequently, the number of degrees of freedom per node is approximately the same for all nodes. Therefore, it is reasonable to perform reordering on the basis of the nodes and nodal connectivity, which is considerably simpler than on the basis of degrees of freedom and stiffness matrix connectivity. A new algorithm, described here, has been found effective in reducing the wavefront.

Wavefront counting for a sample structure is illustrated in the diagram. The wavefront at a row of the nodal connectivity matrix is the number of active

(continued overleaf)

columns that follow the diagonal element; a column becomes active at the row for which there is the first entry for that column and it remains active until that column is absorbed into the diagonal of the connectivity matrix. In the nodal connectivity matrix, heavy vertical lines are drawn for each column beginning at the row when the column becomes active and ending at the diagonal. To obtain the wavefront at each row, it is necessary only to count the number of these heavy lines crossing each row.

Alternatively, the wavefront at any row can be computed as equal to the wavefront at the preceding row, plus the number of new columns that become active. If the column corresponding to the row is currently in the wavefront, then the wavefront is reduced by one. The subtraction is made to account for columns that leave the wavefront by virtue of reaching the diagonal. The isolated column to the right of the connectivity matrix in the diagram gives the wavefront at the row. The maximum wavefront is equal to 6, which occurs at both the second and third rows; this can be reduced by application of the resequencing procedure.

The recommended resequencing approach can be called a "minimum growth" method; that is, assuming that resequencing has already been used to identify the nodes that will constitute the new first n nodes, then the node selected as the new $(n+1)$th node is the one for which the wavefront at the $(n+1)$th node will represent the smallest increase with respect to the wavefront at the nth node. In accordance with the alternative method described for wavefront counting, the increase can be a positive integer, zero, or negative unity. Very often, any one of several nodes will give the same minimum wavefront increase for the $(n+1)$th node; when this happens, the first node is selected, for this tends to make the new resequencing as close as possible to the original. Any one of several methods can be used to choose the node to be first in the new sequence.

The operations for resequencing can be conveniently organized in a tableau format that employs the original nodal connectivity matrix and a new table that is constructed to contain the change in wavefront for each possible selection of the next node to be placed in the new sequence. Tableau resequencing can be performed without difficulty for matrices with orders of from 20 to 30; when the order becomes much larger, the tableau becomes cumbersome and it is advisable to perform resequencing by computer.

A computer program has been written to automate the procedure by performing essentially the same operations as in executing the tableau. The largest model for which the program has been used to date has a connectivity matrix with over 500 nodes. In one test of the program for this model, the initial sequencing had been developed by inspection and study of the connectivity. The maximum wavefront was 43; resequencing by one computer cycle reduced the initial maximum wavefront to 30. In another test, the initial sequencing was scrambled at random, which produced a maximum wavefront of 92; one computer cycle reduced this to 40. Subsequently, as the result of six additional computer cycles, the maximum wavefront was reduced to 27. The computation time per cycle varies considerably, depending upon how quickly the connectivity matrix fills or upon the number of nodes that are processed before it can be determined that the current cycle is to be aborted because there will be no reduction of an existing maximum wavefront.

Experience with the computer program indicates that minimum growth sequencing is effective, rapid, and capable of producing economies, such as in the reduction or elimination of lengthy effort to study a particular structural model for the purpose of generating an acceptable sequencing pattern, and also in the reduction of computation time as the result of processing a relatively compact stiffness matrix.

Note:

Requests for further information may be directed to:

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