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Analysis of Nonlinear Vibrations of Cylinders

Geometric nonlinear vibrations of structural systems are different from other types of vibrations for they occur when vibration amplitudes are large. Because of the growing appreciation of the importance of nonlinear effects in determining the stability and response of structural systems, the geometric nonlinear vibrations of structural systems have received renewed attention; moreover, there now exists the capability to handle more complex descriptions of structural systems by means of high-speed digital computers.

It is often considered that mathematical treatment of nonlinear vibration is simply an extension of the treatment of linear vibration because the nonlinear terms frequently are the smaller terms of an equation for linear vibration; it is of interest to note that the equations of motion for a thin, circular cylinder become nonlinear when finite displacements are considered. Ordinarily, solutions of equations for linear vibration use either a finite difference method or an iterative method to take care of the nonlinear terms, but a solution is obtained somewhat blindly. In a study of geometric nonlinear vibrations, an infinite, long, thin-walled cylinder was analyzed under periodic, dynamic loading to demonstrate that some nonlinear phenomena cannot be obtained by the straightforward numerical solution methods used heretofore. Although the geometric nonlinearity is small because of the thinness of the cylinder wall, some of the special phenomena exhibited by nonlinear vibrations can be obtained only by careful mathematical considerations.

Accordingly, in order to modify the cylinder equations so that they can apply to an infinite long cylinder, it was assumed that (1) the displacements as well as the radial load are functions of only the circumferential coordinate and time, (2) the cylinder shell is thin and of constant thickness, (3) the forces are assumed to be zero throughout the shell. With these assumptions, the nonlinear equations of a vibrating, infinitely long cylinder were derived and reduced to a sequence of linear equations by the perturbation method. The linearized, governing, differential equations were then solved, and the stability of the solutions was investigated by using the results of the Mathieu equation. Although only one mode was directly driven by the forcing function used in the solution, it was necessary to include two vibration modes in the calculations because, under certain conditions, nonlinear coupling caused the companion mode to respond and participate in the motion.

The results of this analysis demonstrate that nonlinear phenomenon in a large amplitude vibration (such as a jump phenomenon) traveling-wave condition can be predicted by the analysis; the results are in good agreement with available experimental results. Furthermore, this analysis provides a systematic approach to a nonlinear analysis, which is of importance for more complicated problems for which the automation of analysis by digital computers becomes necessary.

(continued overleaf)

Reference:

Chen, J. C.: Nonlinear Vibration of an Infinite Long Circular Cylinder. Jet Propulsion Laboratory SPS 37-62, Vol. III, pp. 182-7; SPS 37-64, Vol. III, pp. 96-100.

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