

N73-10111

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## 7. Digital Modeling of Human Operator Dynamics Via a Class of Liapunov Functions\*

RALPH MEKEL AND PATRICK PERUO, JR.

*The City College of The City University of New York*

This paper describes a technique which is utilized for modeling human operator dynamics. The technique is based upon a model-reference system configuration and a class of Liapunov functions formulated for this purpose. Since human operator characteristics change with changes in environment as well as with variable processes within the operator himself, it is therefore desirable that the class of Liapunov functions with their time derivatives also possess variable characteristics. It is shown in this paper how such a class of Liapunov functions and their time derivatives is formulated. The crux of the formulation lies in three variable positive definite matrices used for the construction of the class of Liapunov functions. The form and order of these matrices depend upon the form and order of the error differential equation of the human operator model-reference system. These matrices can be modified to include nonlinear functions of the human operator model.

The purpose of this study is to synthesize a controller dedicated to identify parameter variations and provide corrective dynamics to the mathematical model of the human operator. In order to describe briefly the modeling technique, consider the human operator model-reference system depicted in figure 1. The mathematical model system is described by the vector differential equation

$$\dot{x} = Bx + Cr \quad (1)$$

where  $x$  is the model's output state vector,  $\dot{x} = dx/dt$  and  $r$  is the input vector. The square matrices  $B$  and  $C$  consist of zeros, ones and the

postulated parameters of the model system including parameter variations. It is assumed that the reference system is a human operator whose response due to an input  $r$  can be obtained experimentally. The output vector of the human operator is denoted by  $z$ . The difference between the mathematical model output and the human operator output is the model-reference system error and is denoted by vector  $e$ . The error differential equation is given by

$$\dot{e} = Ae + bu^T(e+z) + dw^Tr \quad (2)$$

where  $\dot{e} = de/dt$  and  $A$  is a square matrix consisting of zeros, ones and the postulated nominal values of the model parameters. It is the  $B$  matrix without the parameter variations. The controller outputs are denoted by vectors  $u$  and  $w$ . These vectors when modified by the vectors  $b$  and  $d$  provide the corrective dynamics for the postulated parameters in the mathematical model. Note that the superscript  $T$  denotes the transpose.

The Liapunov function and its time derivative, denoted by  $V$  and  $\dot{V}$  respectively, have the forms

$$V = e^T M e + u^T N u + w^T Q w \quad (3)$$

and

$$\dot{V} = -e^T D e + 2[\dot{u}^T N + (e+z)^T (b^T M e)]u + 2[\dot{w}^T Q + r^T (d^T M e)]w \quad (4)$$

where

$$D = (A^T M + M A + \dot{M}). \quad (5)$$

One way to comply with Liapunov's criterion for stability is to constrain the elements of the  $D$  matrix, denoted by  $d_{ii}$  and  $d_{ij}$ , to satisfy the conditions

$$d_{ii} \geq 0 \quad (6)$$

\* This research is supported by NASA-Langley Research Center grant NGR-33-013-053.

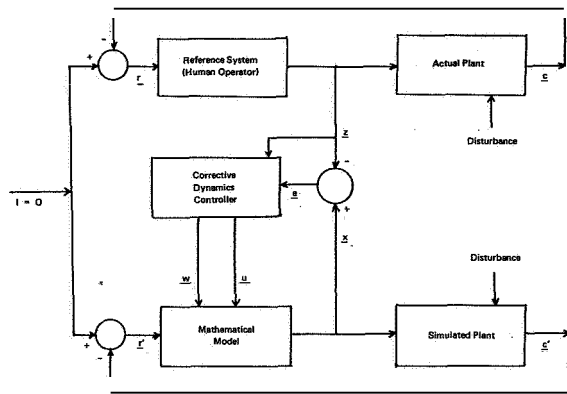


FIGURE 1.—Block diagram of the model-reference system.

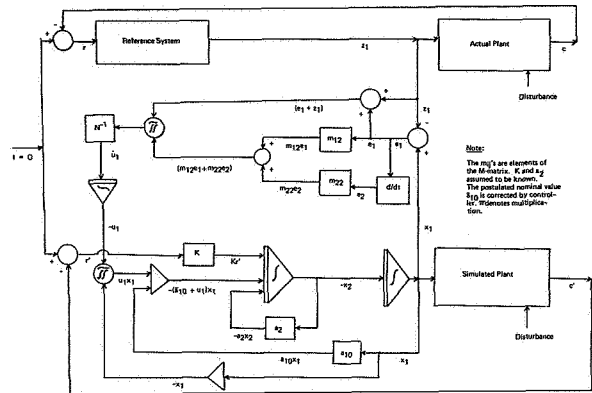


FIGURE 2.—Controller structure which provides corrective dynamics for a second-order mathematical model.

and

$$d_{ij} + d_{ji} = 0 \quad (7)$$

where  $i$  and  $j$  denote row and column respectively and let

$$\dot{\underline{u}} = -N^{-1}(\underline{e} + \underline{z})(\underline{b}^T M \underline{e}) \quad (8)$$

and

$$\dot{\underline{v}} = -Q^{-1} \underline{r}(\underline{d}^T M \underline{e}). \quad (9)$$

The conditions given by equations (6) and (7) permit the evaluation of the elements in the  $D$  and  $M$  matrices in terms of the mathematical model's nominal parameters. At this point a test is performed to insure the positive definiteness of these matrices. Matrices  $N$  and  $Q$  must satisfy the condition  $N > 0, Q > 0$  and are design parameter matrices at the discretion of the investigator. Equations (8) and (9), when integrated and modified by vectors  $\underline{b}$  and  $\underline{d}$  respectively, provide the corrective dynamics for the postulated mathematical model of the human operator.

A very useful feature of this study is the development of a digital computer program which is easily implemented and modified concurrent with experimentation. In this way the modeling process interacts with the experimentation process in a mutually beneficial way. It is the systematic and logical use of the digital computer that permits one to effectively apply the class of Liapunov functions to the modeling of the human operator.

An illustrative example shown in figure 2 describes a controller structure which provides corrective dynamics to a second order mathematical model.

#### ACKNOWLEDGMENT

The first author wishes to thank James J. Adams, NASA-Langley Research Center, for experimental data and useful discussions.