N73-10315

NASA TECHNICAL NOTE

NASA TN D-6945

NASA TN D-6945

CHARTS FOR PREDICTING TURBULENT SKIN FRICTION FROM THE VAN DRIEST METHOD (II)

by Edward J. Hopkins

Ames Research Center
Moffett Field, Calif. 94035

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1972

1. Report No. NASA TN D-6945	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle CHARTS FOR PREDICTING TURBULENT SKIN FRICTION FROM THE VAN DRIEST METHOD (II)		5. Report Date October 1972
		6. Performing Organization Code
7. Author(s) Edward J. Hopkins		8. Performing Organization Report No. A-4488
9. Performing Organization Name and Address NASA Ames Research Center Moffett Field, Calif. 94035		10. Work Unit No. 136-13-01-03-00-21
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Charts are presented for rapidly estimating local or average turbulent skin friction on flat plates from Reynolds numbers based on either length or momentum thickness. These charts facilitate, therefore, the conversion from one Reynolds number to the other. Ranges of variables covered are: Mach number, 0 to 10; length Reynolds number, 10^5 to 10^9; and wall-to-adiabatic-wall temperature ratio, 0.2 to 1.0.

17. Key Words (Suggested by Author(s)) Turbulent skin friction on nonadiabatic flat plates Mach number effects Reynolds number effects	18. Distribution Statement Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	47	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

NOTATION

A	constant defined by equation (14) and used in equations (12) and (13)
B	constant defined by equation (15) and used in equations (12) and (13)
C_f	local skin-friction coefficient, $\dfrac{\tau_w}{1/2\, \rho_e U_e^2}$
C_F	average skin-friction coefficient, $\int_0^1 C_f \, d\dfrac{x}{L}$
F	temperature ratio, $\dfrac{T_w}{T_e}$
F_c	transformation function for skin-friction coefficient, $\dfrac{\overline{C}_f}{C_f}$ or $\dfrac{\overline{C}_F}{C_F}$
F_x	transformation function for length Reynolds number, $\dfrac{\overline{R}e_x}{Re_x}$
F_θ	transformation function for momentum-thickness Reynolds number, $\dfrac{\overline{R}e_\theta}{Re_\theta}$
h	static enthalpy
H	total enthalpy
L	length of turbulent boundary-layer flow
m	Mach number factor used in equations (8), (14), and (15), $0.2M_e^2$
M	Mach number
r	temperature recovery factor for a turbulent boundary layer, $r = 0.88$
Re_x	Reynolds number based on length of turbulent boundary-layer flow, $\dfrac{\rho_e U_e L}{\mu_e}$
Re_θ	Reynolds number based on momentum thickness, $\dfrac{\rho_e U_e \theta}{\mu_e}$
T	absolute temperature

U	velocity
x	longitudinal distance
y	distance measured normal to surface
α	constant defined by equation (12) and used in equation (8)
β	constant defined by equation (13) and used in equation (8)
δ	boundary–layer thickness
μ	viscosity of air as given by Keyes' formula, reference 6
ρ	mass density
τ	local shear stress
θ	boundary–layer momentum thickness, $\int_0^{\delta_e} \frac{\rho U}{\rho_e U_e}\left[1 - \left(\frac{U}{U_e}\right)\right] dy$
$(\bar{\ })$	incompressible or variable transformed to equivalent constant property case

Subscripts

aw	adiabatic wall
e	boundary–layer edge
w	wall

CHARTS FOR PREDICTING TURBULENT SKIN FRICTION

FROM THE VAN DRIEST METHOD (II)

Edward J. Hopkins

Ames Research Center

SUMMARY

Charts are presented for rapidly estimating local or average turbulent skin friction on flat plates from Reynolds numbers based on either length or momentum thickness. These charts facilitate, therefore, the conversion from one Reynolds number to the other. Ranges of variables covered are: Mach number, 0 to 10; length Reynolds number, 10^5 to 10^9; and wall-to-adiabatic-wall temperature ratio, 0.2 to 1.0.

INTRODUCTION

An evaluation in reference 1 of the methods for predicting turbulent skin friction on flat surfaces, with and without heat transfer throughout a Mach number range from 1.5 to 9, indicated that the Van Driest method (II)[1] (ref. 2) gave the most satisfactory agreement with the experimental results. Other methods examined in reference 1 were those of Sommer and Short (ref. 3), Spalding and Chi (ref. 4), and Coles (ref. 5). This evaluation was based, wherever possible, on direct measurements of skin friction by balances and the use of the momentum-thickness Reynolds number.

Van Driest presented his method in the form of a nomogram for estimating local and average skin friction from the length Reynolds number only. Skin friction was not presented by Van Driest as a function of momentum-thickness Reynolds number, a form of presentation that has become widely used in recent years for the analysis of skin-friction data. The present charts, therefore, present skin friction as a function of both the length Reynolds number and the momentum-thickness Reynolds number to facilitate conversion from one Reynolds number to the other. Effects are indicated for employing the more accurate viscosity formula of Keyes (ref. 6), instead of a power viscosity formula, and a temperature recovery factor of 0.88, instead of 1.0 assumed by Van Driest.

VAN DRIEST METHOD (II)

The Van Driest method (II) can be thought of as a transformation method, for which it is necessary to choose an incompressible formula relating the skin-friction coefficient to the

[1]The (II) refers to the second theory of Van Driest in which the von Kármán mixing length is assumed.

Reynolds number. In reference 7, it is shown that the Schoenherr formula (ref. 8) for average skin friction gives a good representation of incompressible data over a length Reynolds number range from about 3 × 10^5 to 4.5 × 10^8. Outside this range of Reynolds numbers, insufficient data exist to corroborate the Schoenherr formula. This formula, also known as the Kármán-Schoenherr formula, is

$$\frac{0.242}{\sqrt{\overline{C}_F}} = \log_{10}(\overline{R}e_x \overline{C}_F) \tag{1}$$ [2]

By differentiating equation 1 with respect to length, Schoenherr found the relation between the local and average skin friction to be

$$\overline{C}_f = \frac{0.242\,\overline{C}_F}{0.242 + 0.8686\sqrt{\overline{C}_F}} \tag{2}$$

Also, from the simplified boundary-layer continuity and momentum equations, the average skin-friction coefficient on surfaces without longitudinal pressure gradients is

$$\overline{C}_F = \frac{2\overline{R}e_\theta}{\overline{R}e_x} \tag{3}$$

Skin friction for compressible boundary-layer flow was calculated from equations (1), (2), and (3), as well as the Van Driest (II) transformation functions given in reference 1, as defined by the following equations:

$$\overline{C}_f = F_c C_f \tag{4}$$

$$\overline{C}_F = F_c C_F \tag{5}$$

$$\overline{R}e_\theta = F_\theta Re_\theta \tag{6}$$

$$\overline{R}e_x = F_x Re_x \tag{7}$$

where

$$F_c = rm/(\sin^{-1}\alpha + \sin^{-1}\beta)^2 \qquad (\text{for } M_e \neq 0) \tag{8}$$

$$F_c = [(1 + \sqrt{T_w/T_e})/2]^2 \qquad (\text{for } M_e \to 0) \tag{9}$$

$$F_\theta = \frac{\mu_e}{\mu_w} = \sqrt{\frac{T_e}{T_w}}\left(\frac{1 + \frac{122}{T_w} \times 10^{-5/T_w}}{1 + \frac{122}{T_e} \times 10^{-5/T_e}}\right); \quad T, \text{°K} \tag{10}$$

[2] In reference 9 (pp. 127 and 128), it is shown how the form of equation (1) was derived by von Kármán from the law-of-the-wall and velocity defect formulas; however, the constant of integration, which appears in both the original von Kármán and Schoenherr formulas, was found to be zero by Schoenherr and is not included.

$$F_x = \frac{F_\theta}{F_c} \tag{11}$$

$$\alpha = \frac{2A^2 - B}{(4A^2 + B^2)^{1/2}} \tag{12}$$

$$\beta = \frac{B}{(4A^2 + B^2)^{1/2}} \tag{13}$$

$$A = \left(\frac{rm}{F}\right)^{1/2} \tag{14}$$

$$B = \frac{1 + rm - F}{F} \tag{15}$$

$$F = \frac{T_w}{T_e} \tag{16}$$

$$m = 0.2M_e^2 \tag{17}$$

Equation (10) is from the Keyes formula for viscosity (ref. 6), which should give more accurate values than the Sutherland formula at low temperatures, $T_e \cong 55.6°$ K (100° R). At temperatures above about 111° K (200° R), these two formulas give values of viscosity that agree within 3 percent. Equations (14) and (15) contain a temperature recovery factor (r), which was assumed to be 1.0 for the nomogram of reference 2. Analysis of skin-friction data in reference 1 indicates that assuming $r = 0.88$ results in improved predictions of such data.

Also, the adiabatic wall temperature used in the wall-to-adiabatic-wall temperature ratios given on each figure corresponds to the adiabatic-wall enthalpy given by

$$H_{aw} = h_e + \frac{rU_e^2}{2} \tag{18}$$

PRESENTATION OF RESULTS

Local and average skin-friction coefficients are presented both as a function of the length Reynolds numbers (Re_x) and the momentum-thickness Reynolds number (Re_θ) for two different temperatures: $T_e = 55.6°$ K (100° R) in figures 1 and 2; $T_e = 222°$ K (400° R) in figures 3 and 4. Figure 5 can be used to estimate the effect on skin friction of temperatures other than 222° K (400° R). Although correction factors in figure 5 were calculated for $M_e = 5$, it was found that they also apply to $0 < M_e < 10$ within 1 percent. Figure 5 is used by multiplying the C_f (or C_F's) from figures 3 and 4, respectively, times the factor from figure 5 for a particular T_e. For convenience, Re_x is presented as a function of Re_θ for $T_e = 55.6°$ K (100° R) and $10^6 < Re_x < 10^9$, in figure 6. For other temperatures considered herein, the Reynolds number conversion obtained from figure 6 is within 5 percent of the correct value. For $Re_x < 10^6$, or for more accurate conversions, figures 1 through 4 should be used.

The figures in which the results appear are summarized in table 1.

TABLE 1.– DATA SUMMARY

Figure	T_e	Quantities plotted
1	55.6° K (100° R)	C_f versus Re_θ and Re_x
2	55.6° K (100° R)	C_F versus Re_θ and Re_x
3	222° K (400° R)	C_f versus Re_θ and Re_x
4	222° K (400° R)	C_F versus Re_θ and Re_x
5	27.8° K–222° K (50° R–400° R)	$\dfrac{C_f}{C_{f,T_e = 222° K(400° R)}}$ and $\dfrac{C_F}{C_{F,T_e = 222° K(400° R)}}$ versus T_e
6	55.6° K (100° R)	Re_x versus Re_θ

VISCOSITY AND TEMPERATURE RECOVERY-FACTOR EFFECTS

Effects related to the choice of temperature recovery factor and viscosity formula are indicated for the extreme flow conditions of the charts in table 2.

In general, the calculated effects on skin friction resulting from changes in the recovery factor or viscosity formula were less than 6 percent; however, for some isolated conditions, for example, $M_e = 10$ and $T_w/T_{aw} = 0.2$, the effects were greater than 6 percent.

TABLE 2.– EFFECTS OF CHANGES IN VISCOSITY AND TEMPERATURE RECOVERY-FACTOR ON C_f

M_e	$\dfrac{T_w}{T_{aw}}$	T_e, °K (° R)	$\dfrac{C_f,\ r = 0.88;\ \text{Keyes}\ \mu}{C_f,\ r = 1.0;\ \text{Keyes}\ \mu}$		$\dfrac{C_f,\ r = 0.88;\ \text{Keyes}\ \mu}{C_f,\ r = 1.0;\ \mu \sim T^{0.76}}$	
			$Re_x = 10^5$	$Re_x = 10^9$	$Re_x = 10^5$	$Re_x = 10^9$
5	0.2	55.6 (100)	1.039	1.046	1.044	1.049
5	1.0	55.6 (100)	1.020	1.019	1.089	1.054
5	.2	222 (400)	1.040	1.046	1.040	1.046
5	1.0	222 (400)	1.021	1.027	0.994	1.013
10	0.2	55.6 (100)	1.051	1.066	1.117	1.100
10	1.0	55.6 (100)	1.026	1.032	1.078	1.057
10	.2	222 (400)	1.053	1.067	1.044	1.061
10	1.0	222 (400)	1.028	1.033	0.934	0.984

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, California 94035, July 20, 1972

REFERENCES

1. Hopkins, Edward J.; and Inouye, Mamoru: An Evaluation of Theories for Predicting Turbulent Skin Friction and Heat Transfer on Flat Plates at Supersonic and Hypersonic Mach Numbers. AIAA J., vol. 9, no. 6, June 1971, pp. 993-1003.

2. Van Driest, E. R.: Problem of Aerodynamic Heating. Aeron. Eng. Rev., vol. 15, no. 10, Oct. 1956, pp. 26-41.

3. Sommer, Simon C.; and Short, Barbara J.: Free-Flight Measurements of Turbulent Boundary-Layer Skin Friction in the Presence of Severe Aerodynamic Heating at Mach Numbers from 2.8 to 7.0. NACA TN 3391, 1955. (Also J. Aeron. Sci., vol. 23, no. 6, June 1956, pp. 536-542.)

4. Spalding, D. B.; and Chi, S. W.: The Drag of a Compressible Turbulent Boundary Layer on a Smooth Flat Plate With and Without Heat Transfer. J. Fluid Mech., vol. 28, pt. 1, Jan. 1964, pp. 117-143.

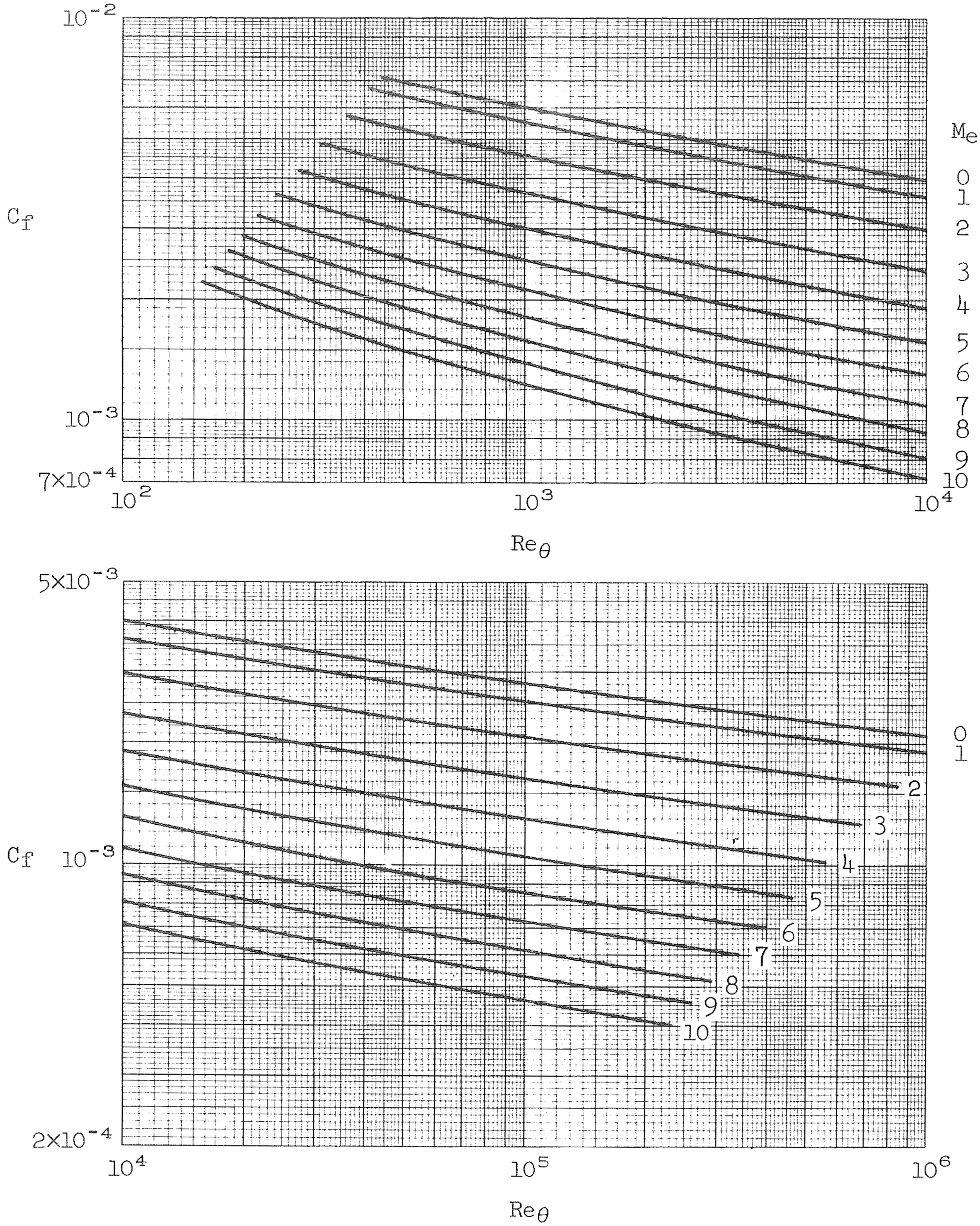
5. Coles, D. E.: The Turbulent Boundary Layer in a Compressible Fluid. The Physics of Fluids, vol. 7, no. 9, Sept. 1964, pp. 1403-1423. (Also Rep. R-403-PR, 1962, Rand Corp.)

6. Keyes, F. G.: A Summary of Viscosity and Heat Conduction Data for H_e, A, H_2, O_2, N_2, CO_2, H_2O and Air. Trans. Amer. Soc. Mech. Engrs., vol. 73, 1951, pp. 589-596. (Also, Keyes, F. G.: The Heat Conductivity, Viscosity and Viscosity Specific Heat and Prandtl Number for Thirteen Gases. Project Squid, Mass. Insti. Tech., TR 37 (1952).)

7. Nestler, D. E.; and Goetz, R.: Survey of Theoretical and Experimental Determinations of Skin Friction in Compressible Boundary Layers, Part II. The Turbulent Boundary on a Flat Plate. Gen. Electric, Missile and Ordnance Systems Dept., Tech. Info. Series No. R58SD270, Jan. 29, 1959.

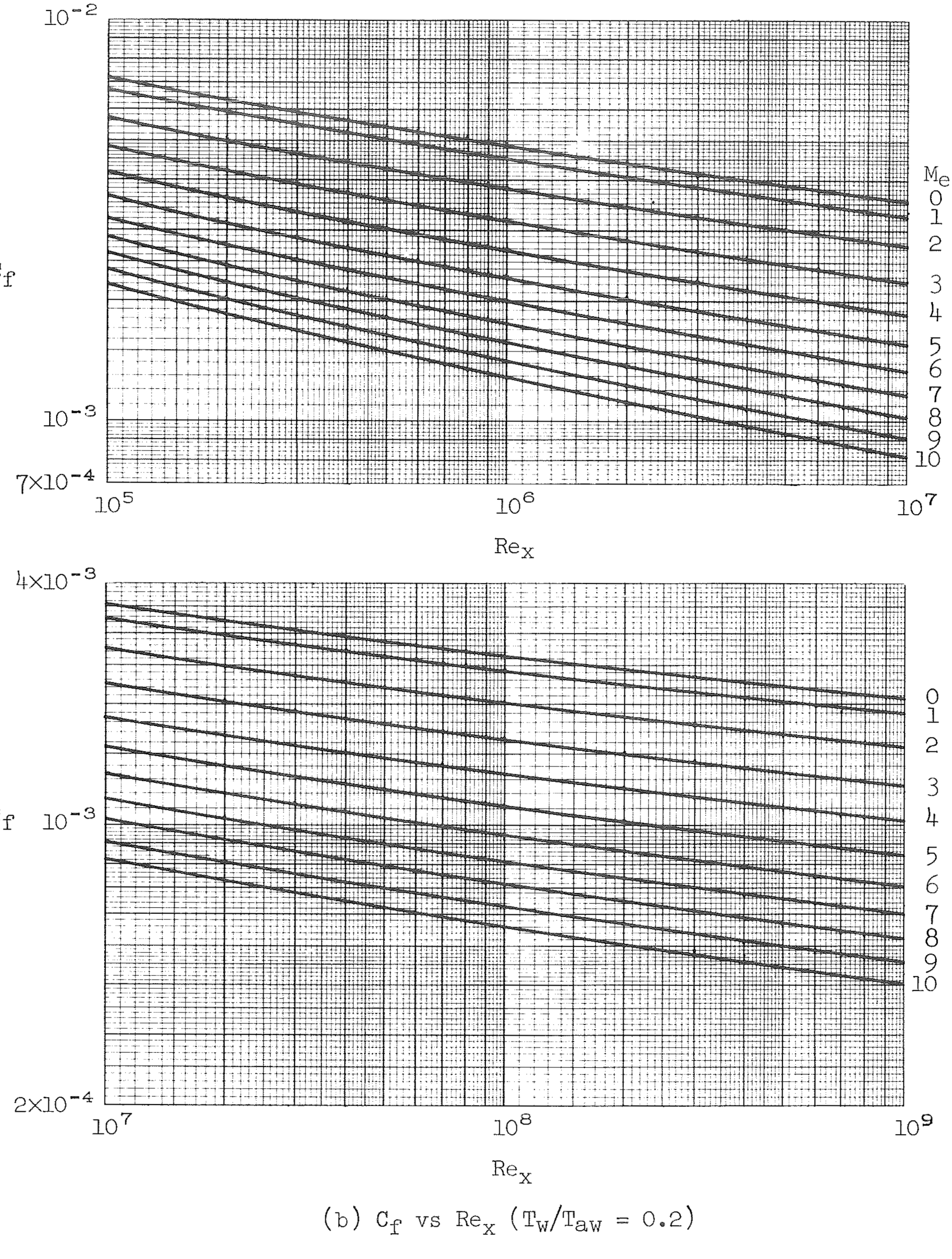
8. Schoenherr, Karl E.: Resistance of Flat Surfaces Moving Through a Fluid. Soc. Naval Architects and Marine Engineers, vol. 40, 1932, pp. 279-313.

9. Schubauer, G. B.; and Tchen, G. M.: Turbulent Flow. Section B, Chap. 1 of vol. V, entitled "High Speed Aerodynamics and Jet Propulsion – Turbulent Flows and Heat Transfer," edited by C. C. Lin, Princeton, New Jersey, Princeton University Press, 1959.



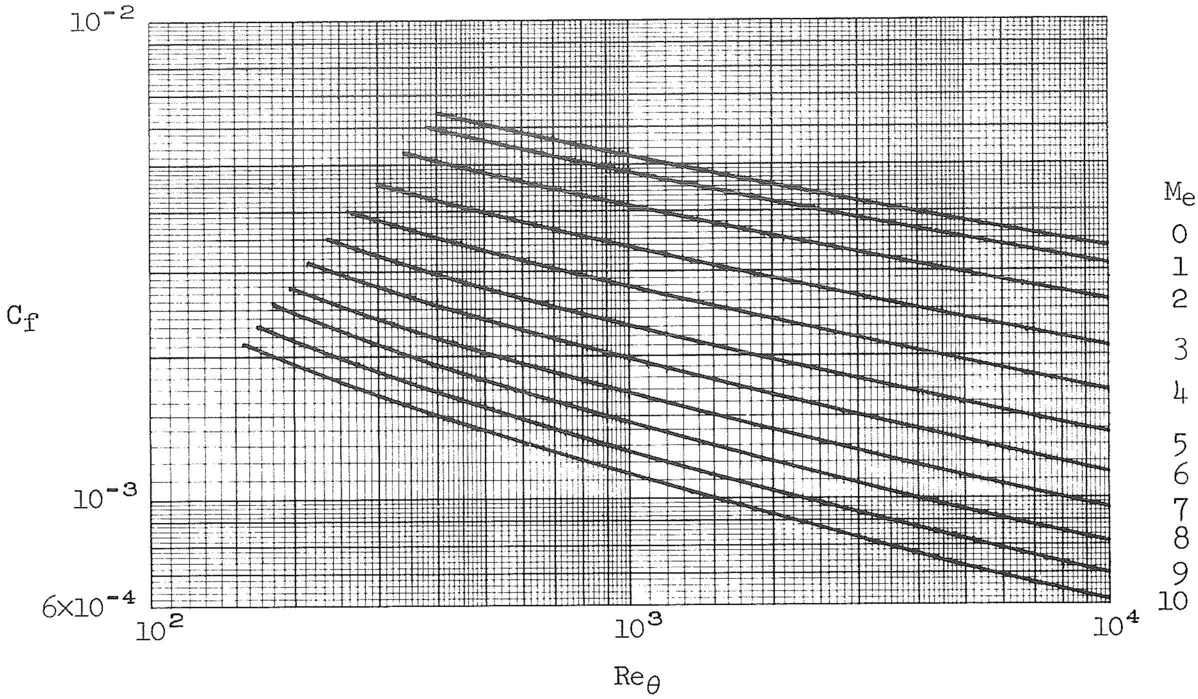
(a) C_f vs Re_θ (T_w/T_{aw} = 0.2)

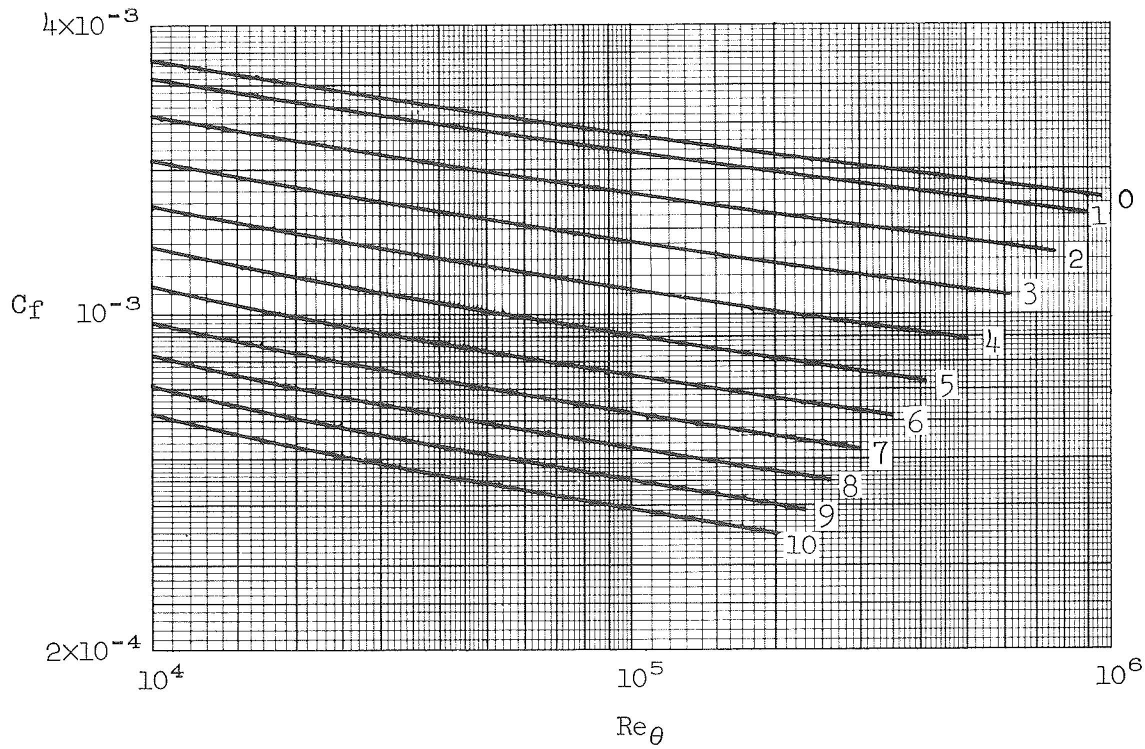
Figure 1.- Local skin friction predicted by the Van Driest method (II); T_e = 55.6°K (100°R).



(b) C_f vs Re_x (T_w/T_{aw} = 0.2)

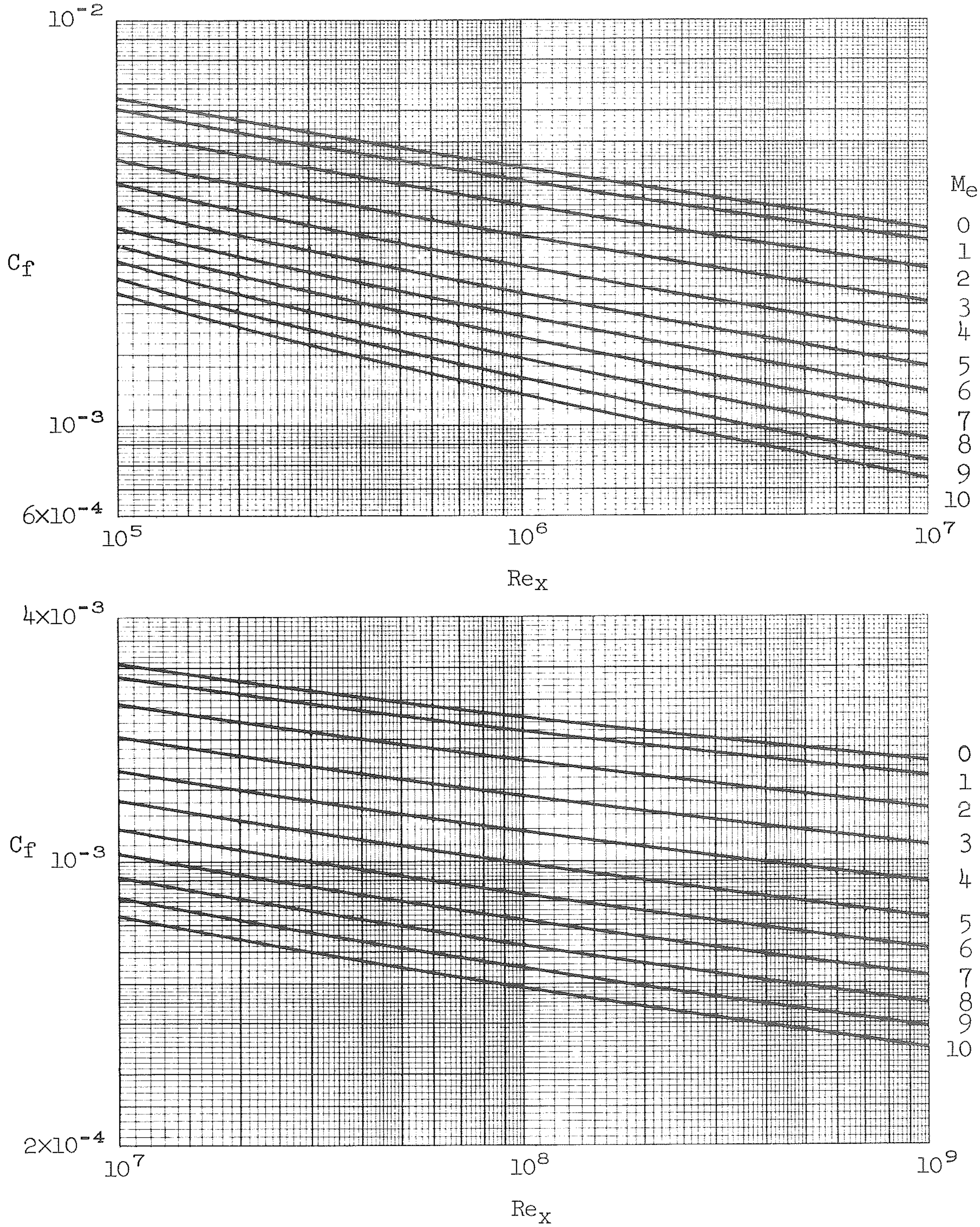
Figure 1.- Continued.





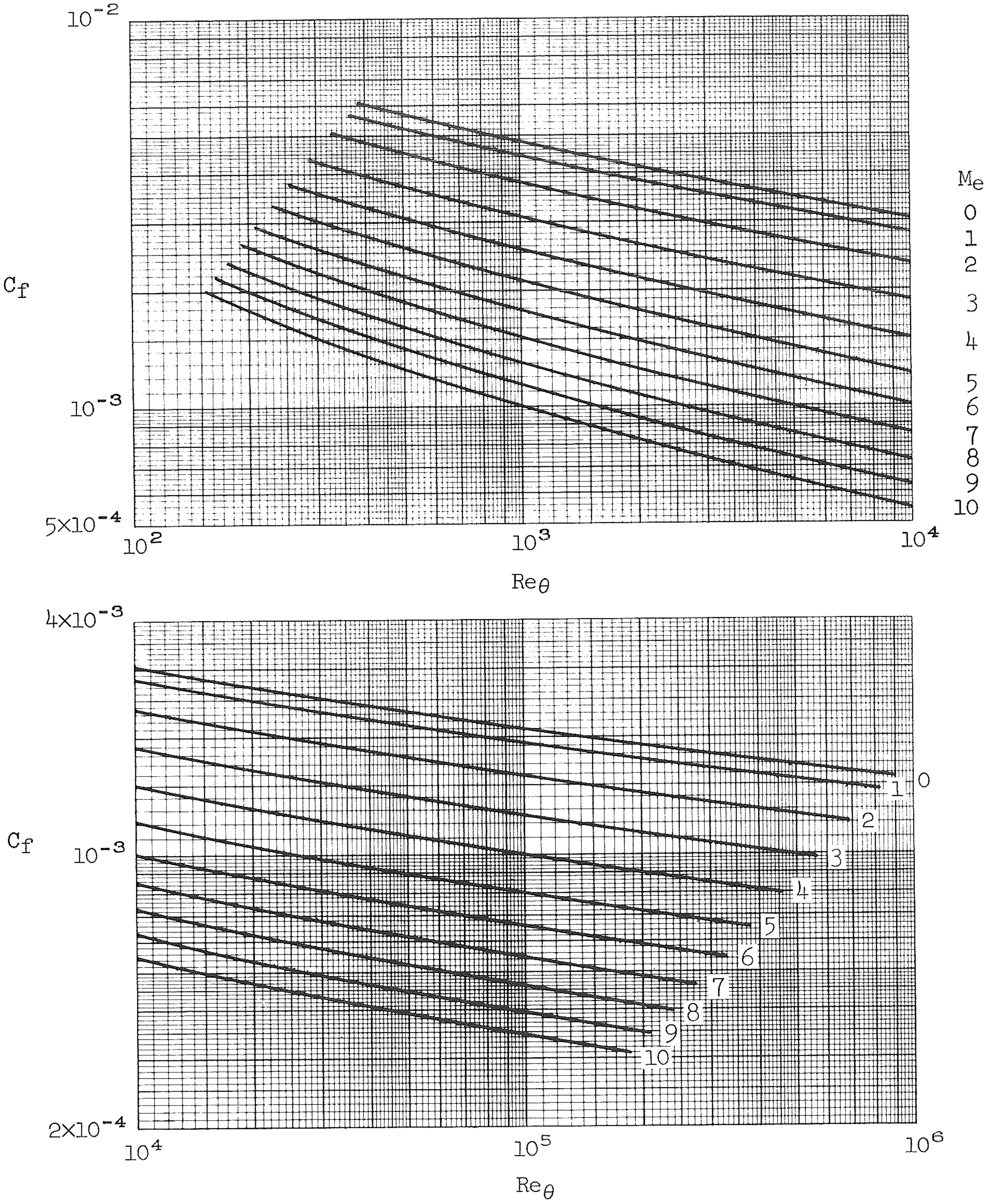
(c) C_f vs Re_θ ($T_w/T_{aw} = 0.4$)

Figure 1.- Continued.



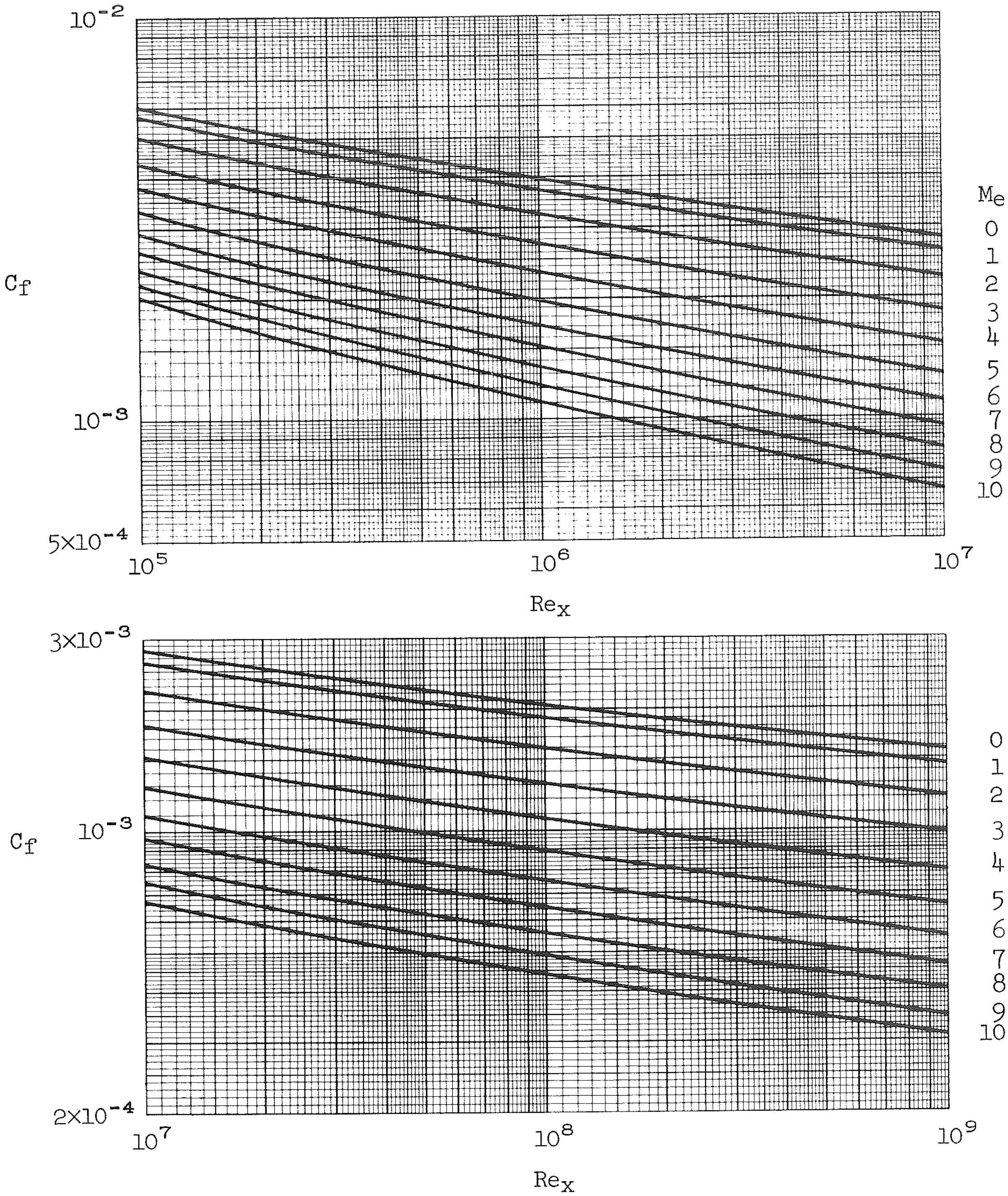
(d) C_f vs Re_x $(T_w/T_{aw} = 0.4)$

Figure 1.- Continued.



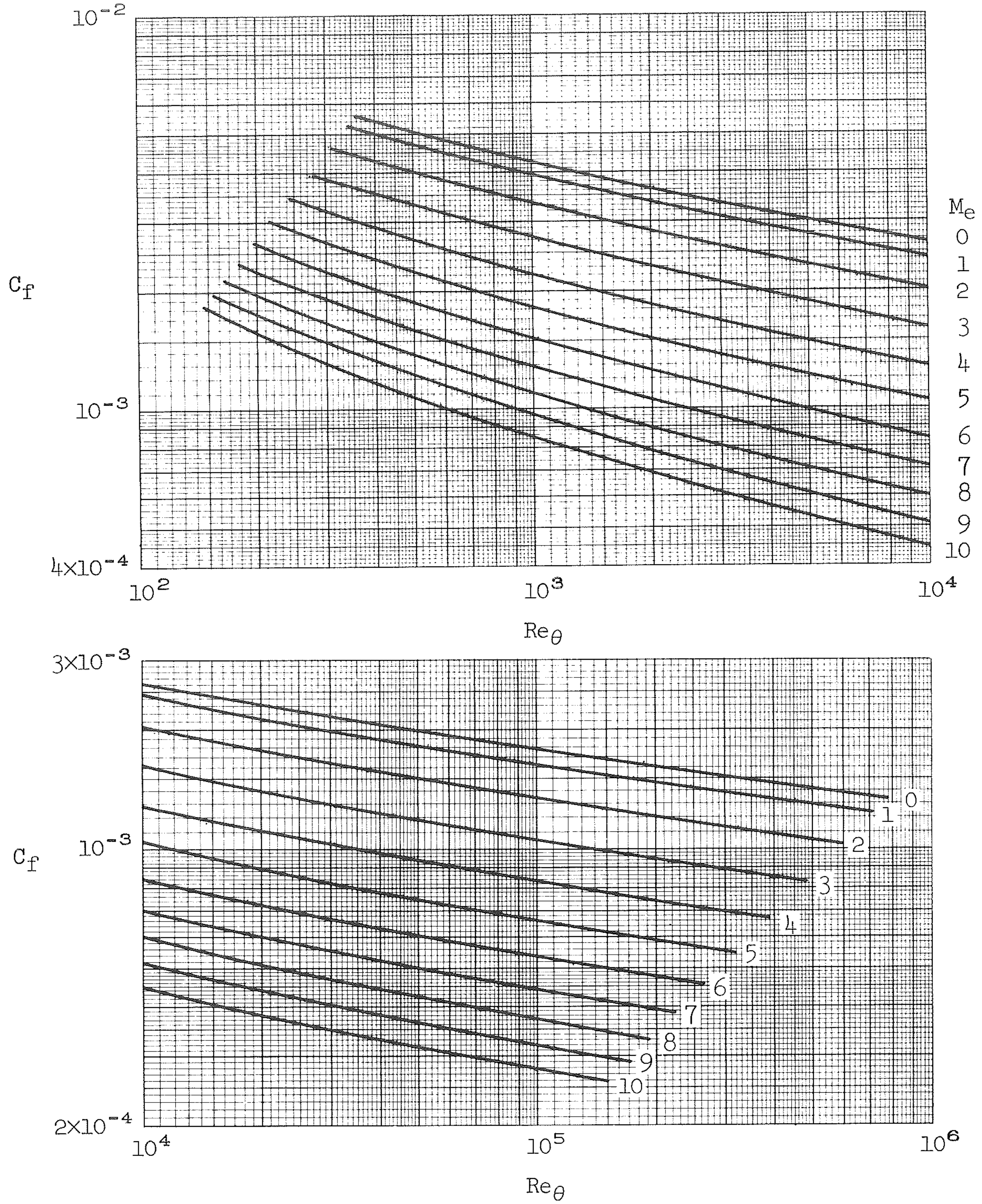
(e) C_f vs Re_θ ($T_w/T_{aw} = 0.6$)

Figure 1.- Continued.



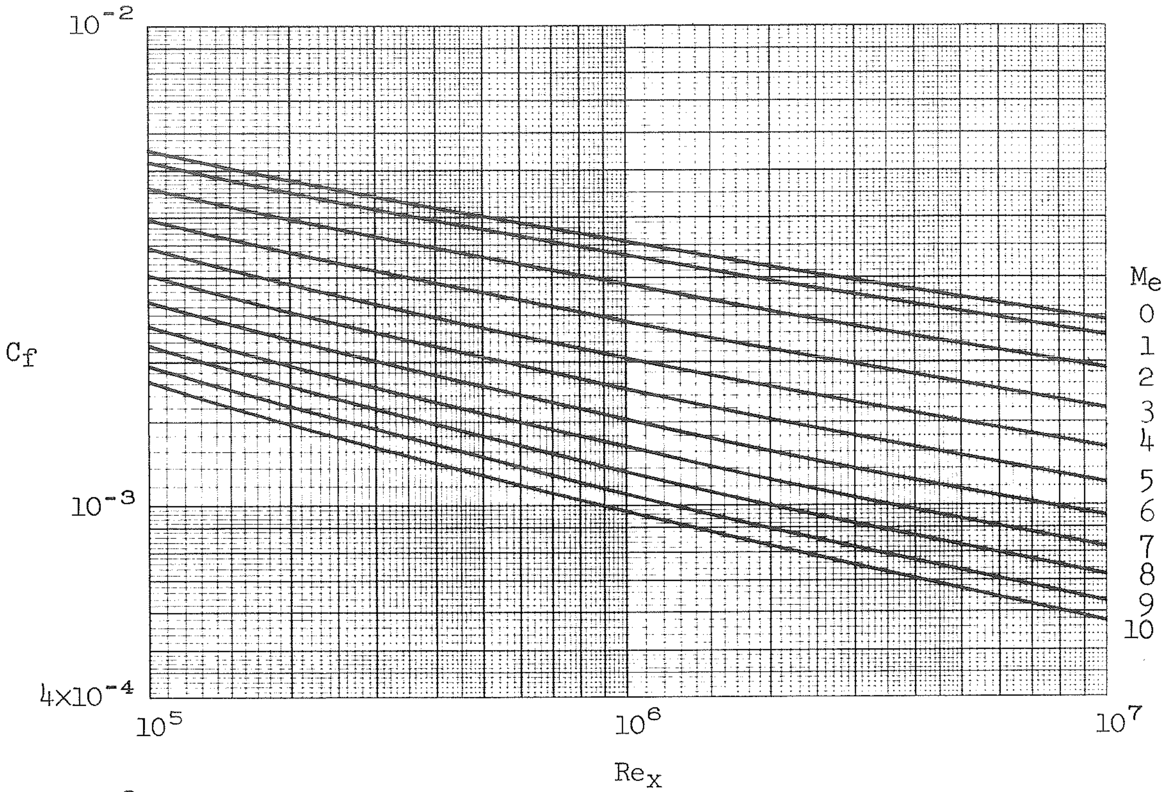
(f) C_f vs Re_x (T_w/T_{aw} = 0.6)

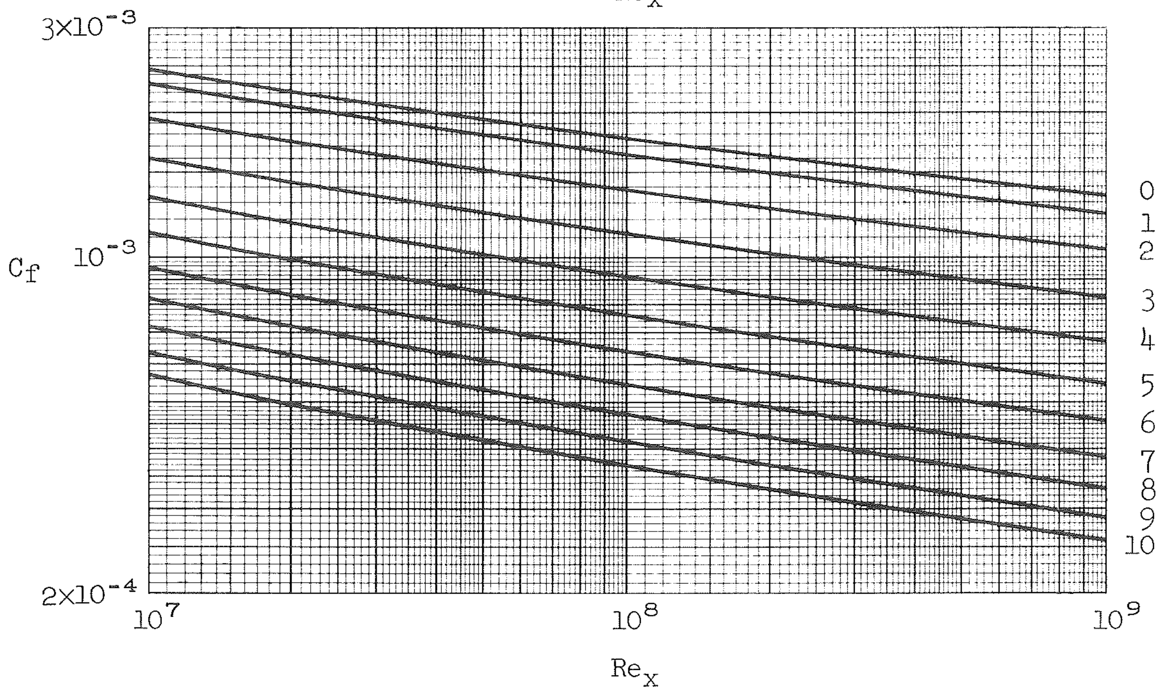
Figure 1.- Continued.



(g) C_f vs Re_θ (T_w/T_{aw} = 1.0)

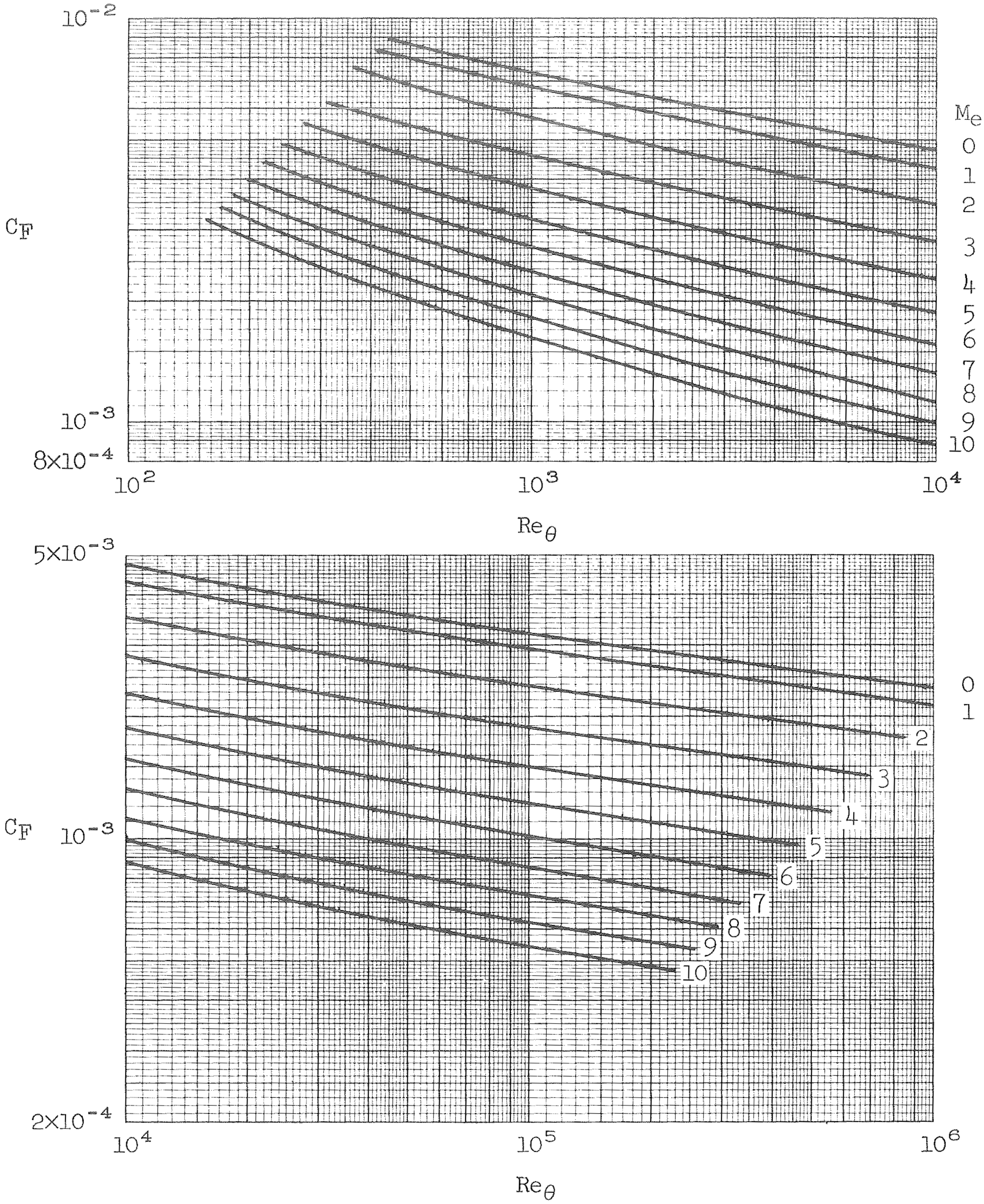
Figure 1.- Continued.





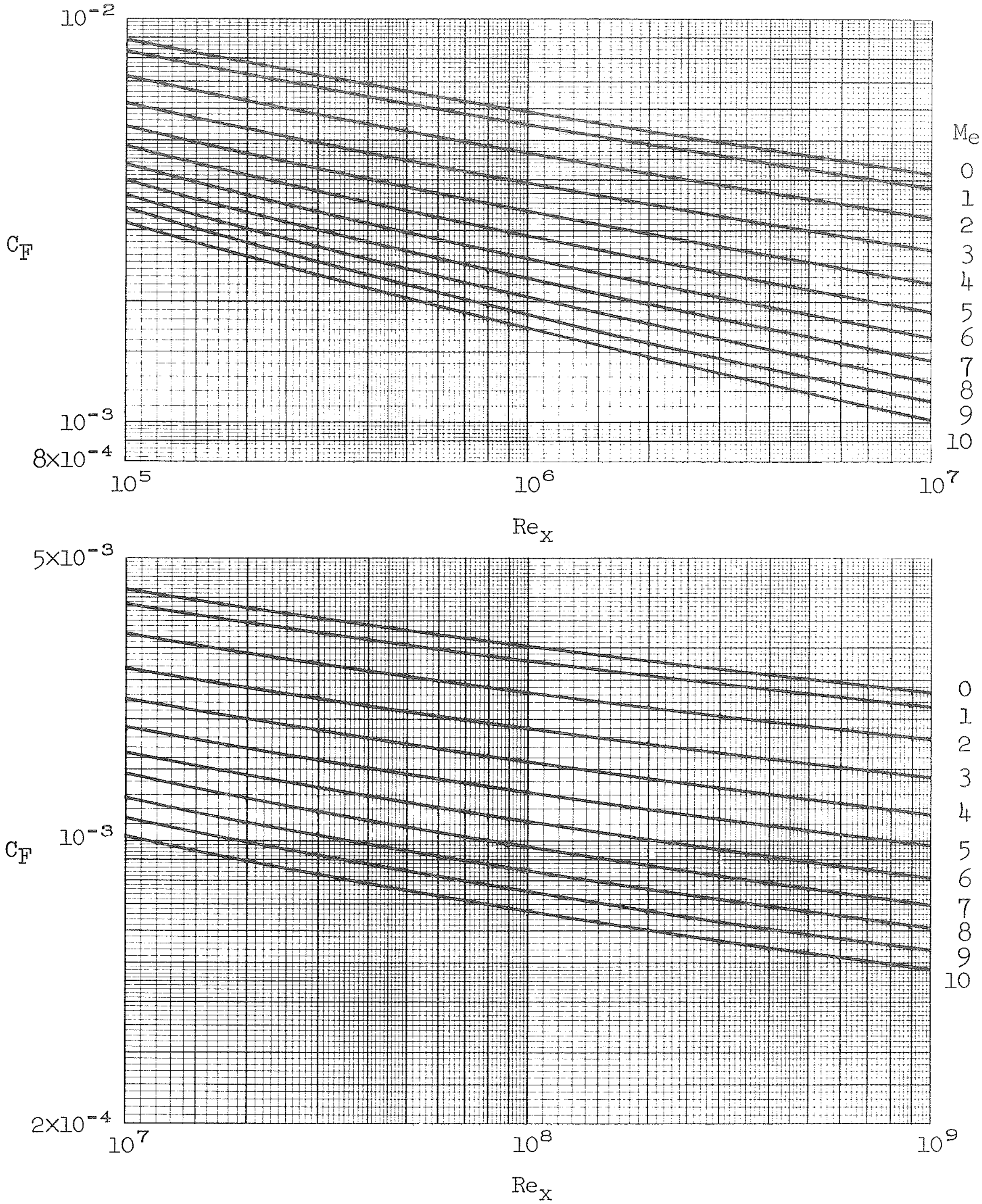
(h) C_f vs Re_x ($T_w/T_{aw} = 1.0$)

Figure 1.- Concluded.



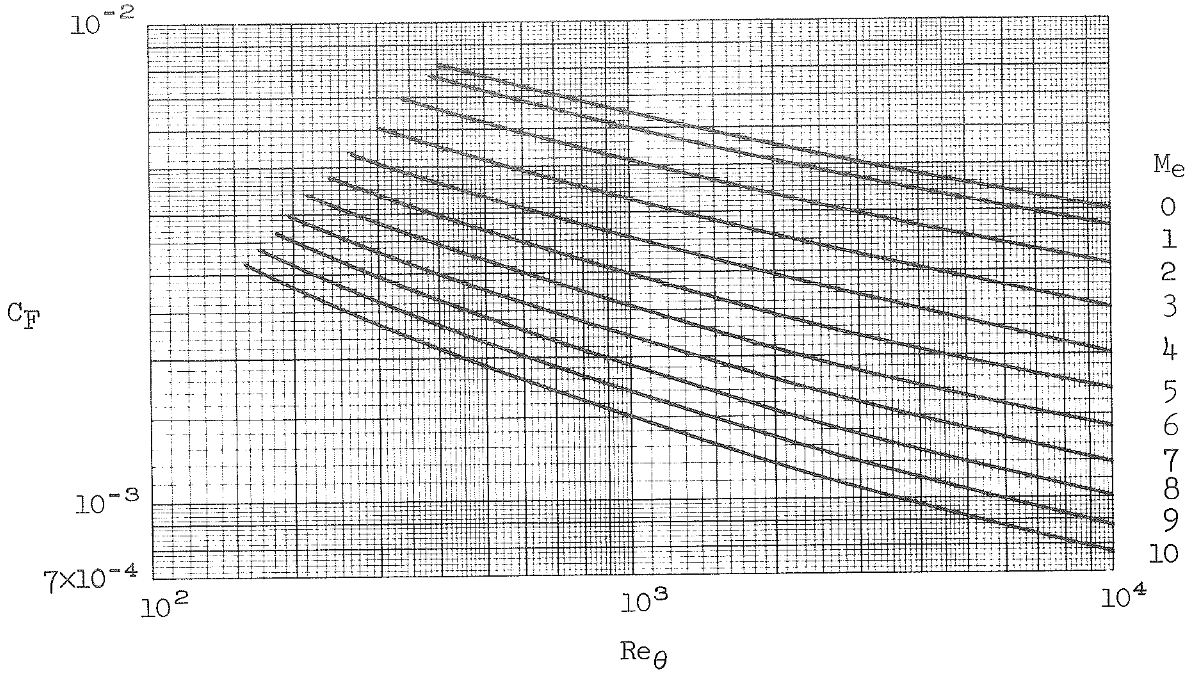
(a) C_F vs Re_θ (T_w/T_{aw} = 0.2)

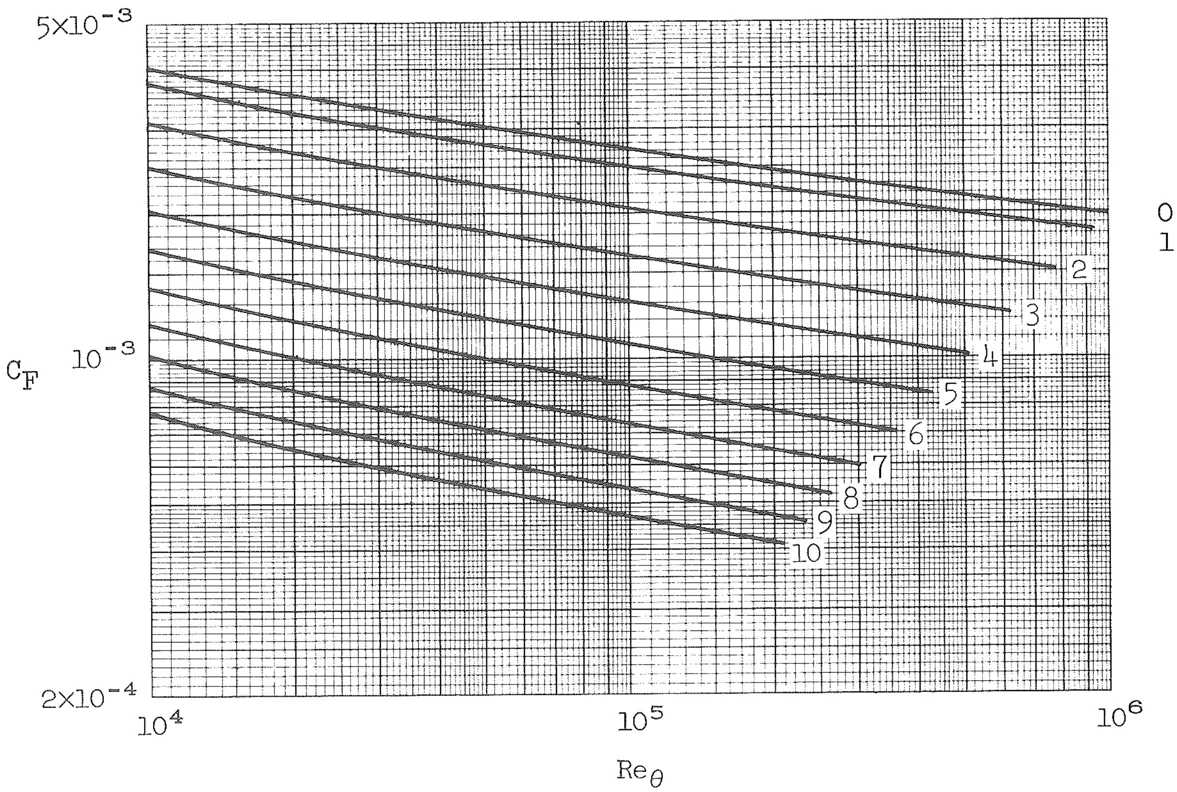
Figure 2.- Average skin friction predicted by the Van Driest method (II); T_e = 55.6°K (100°R).



(b) C_F vs Re_x (T_w/T_{aw} = 0.2)

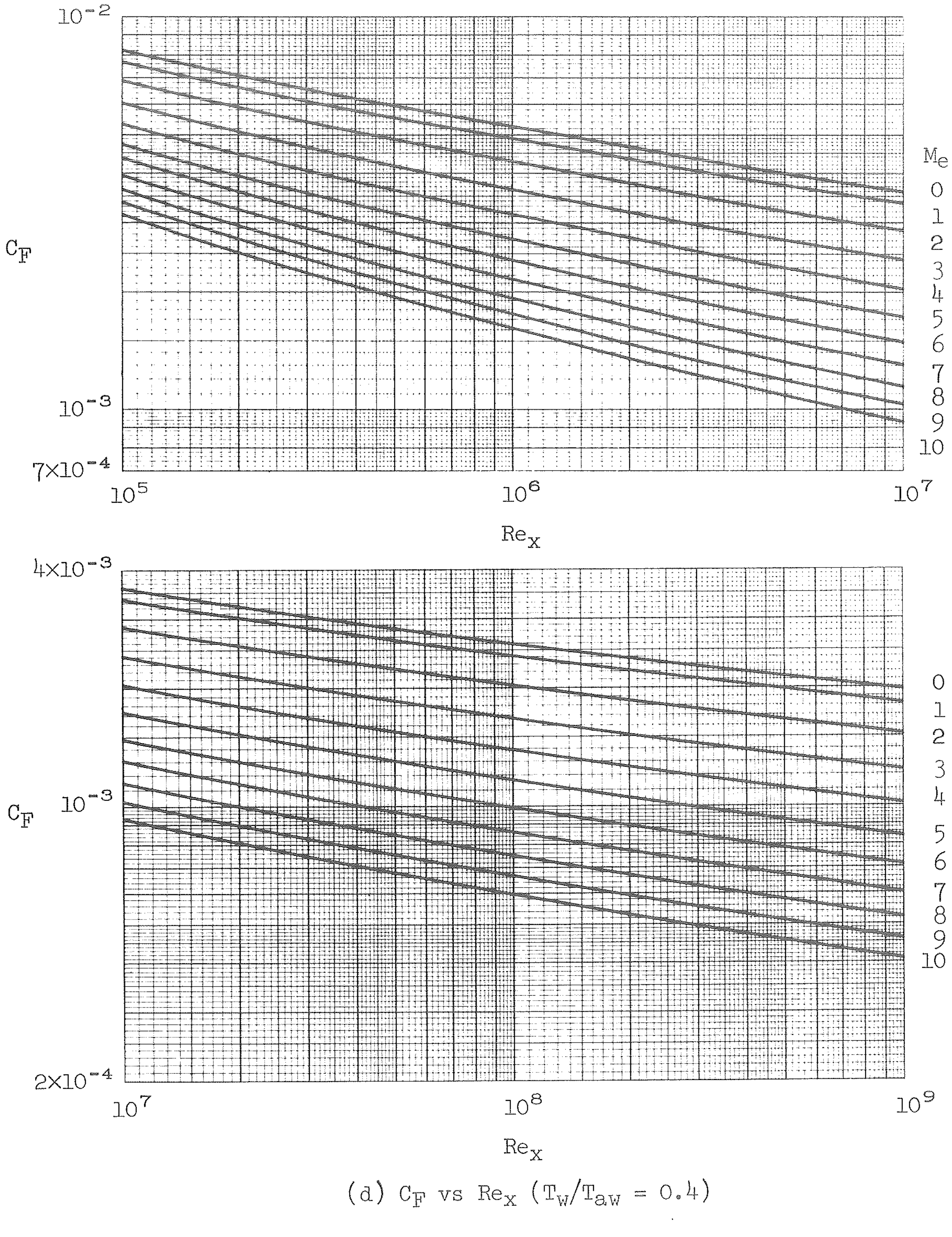
Figure 2.- Continued.





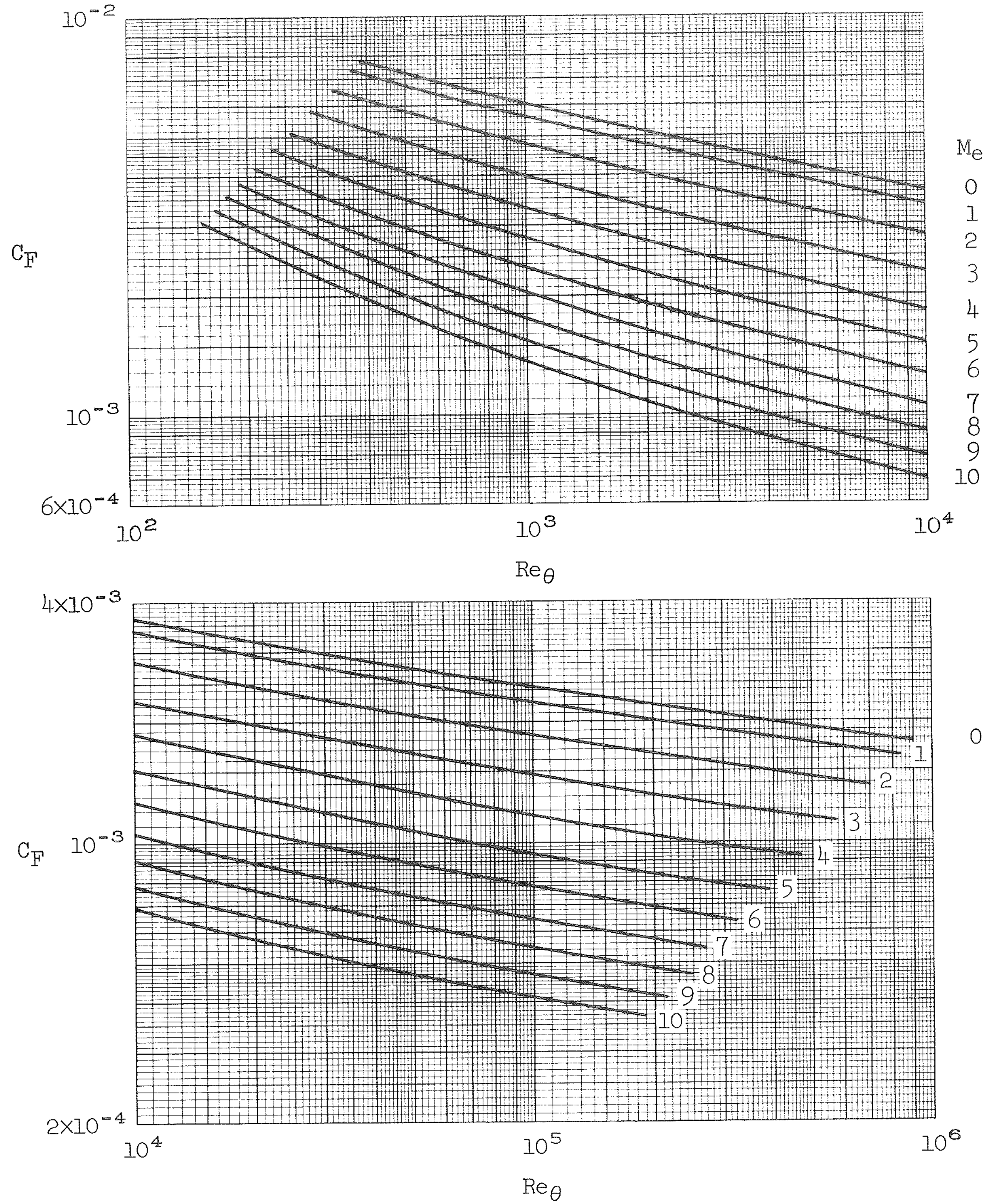
(c) C_F vs Re_θ (T_w/T_{aw} = 0.4)

Figure 2.- Continued.



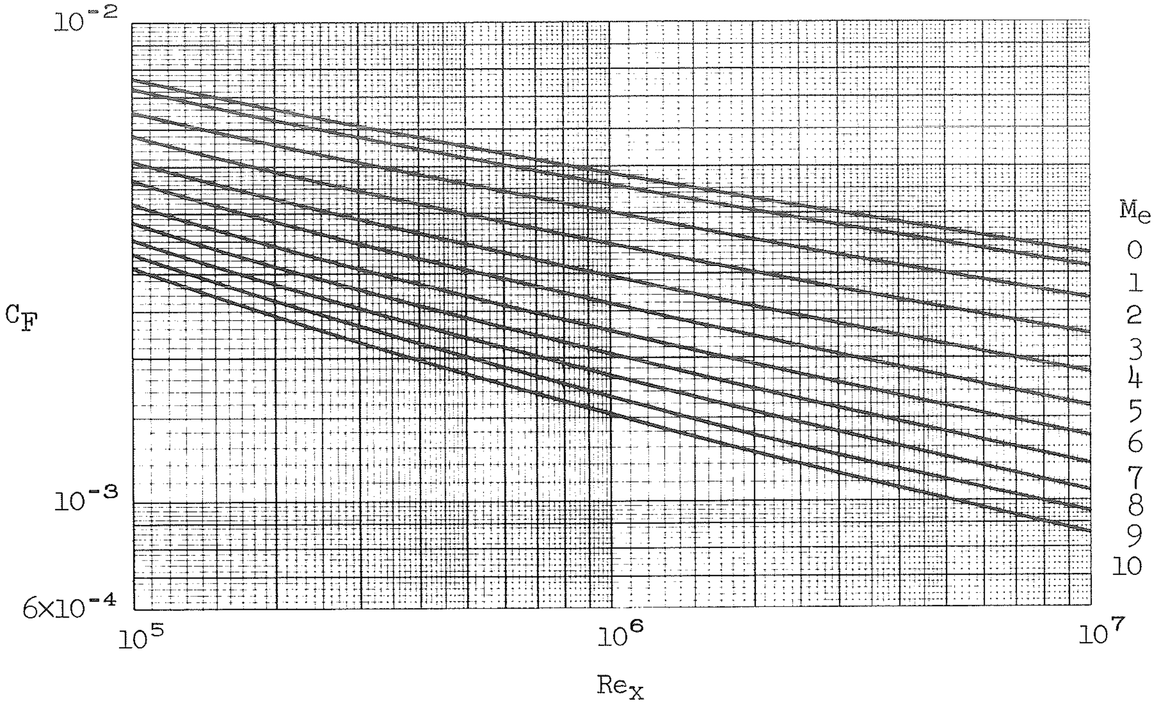
(d) C_F vs Re_x $(T_w/T_{aw} = 0.4)$

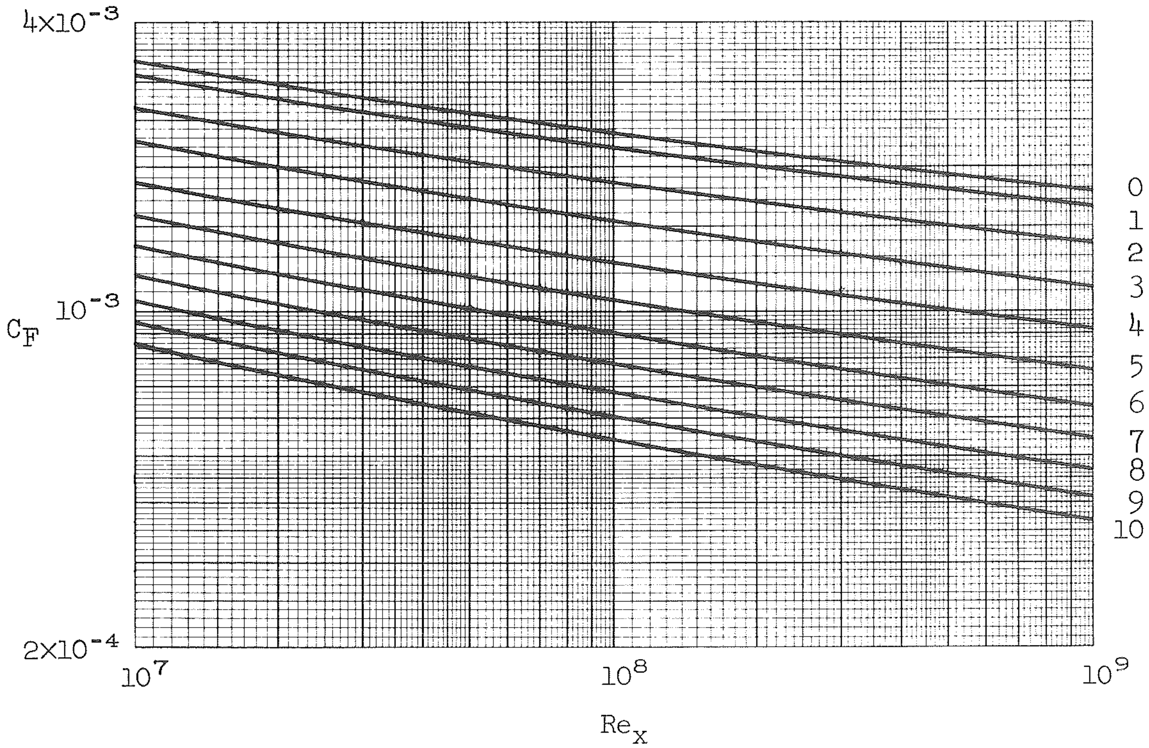
Figure 2.- Continued.



(e) C_F vs Re_θ (T_w/T_{aw} = 0.6)

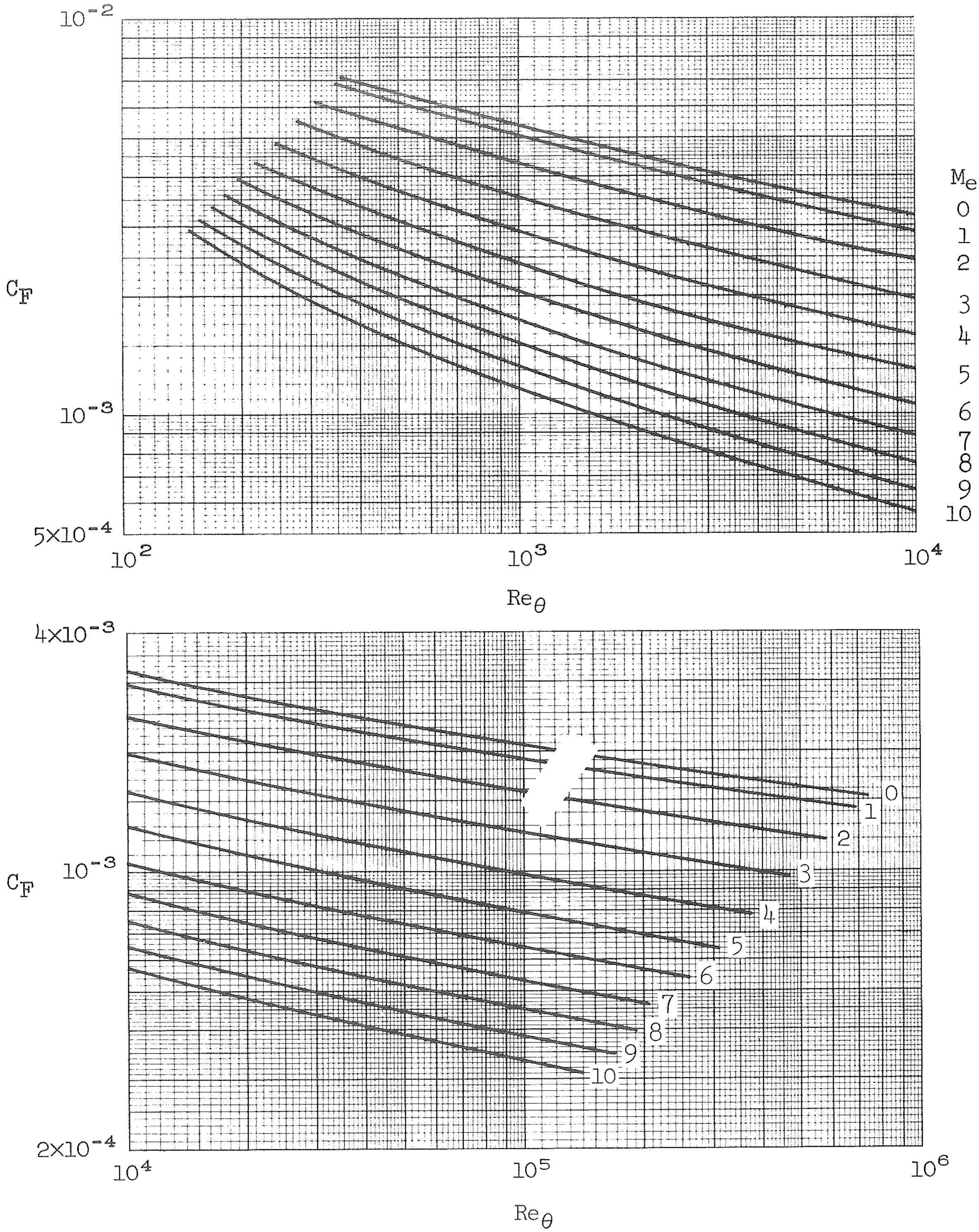
Figure 2.- Continued.





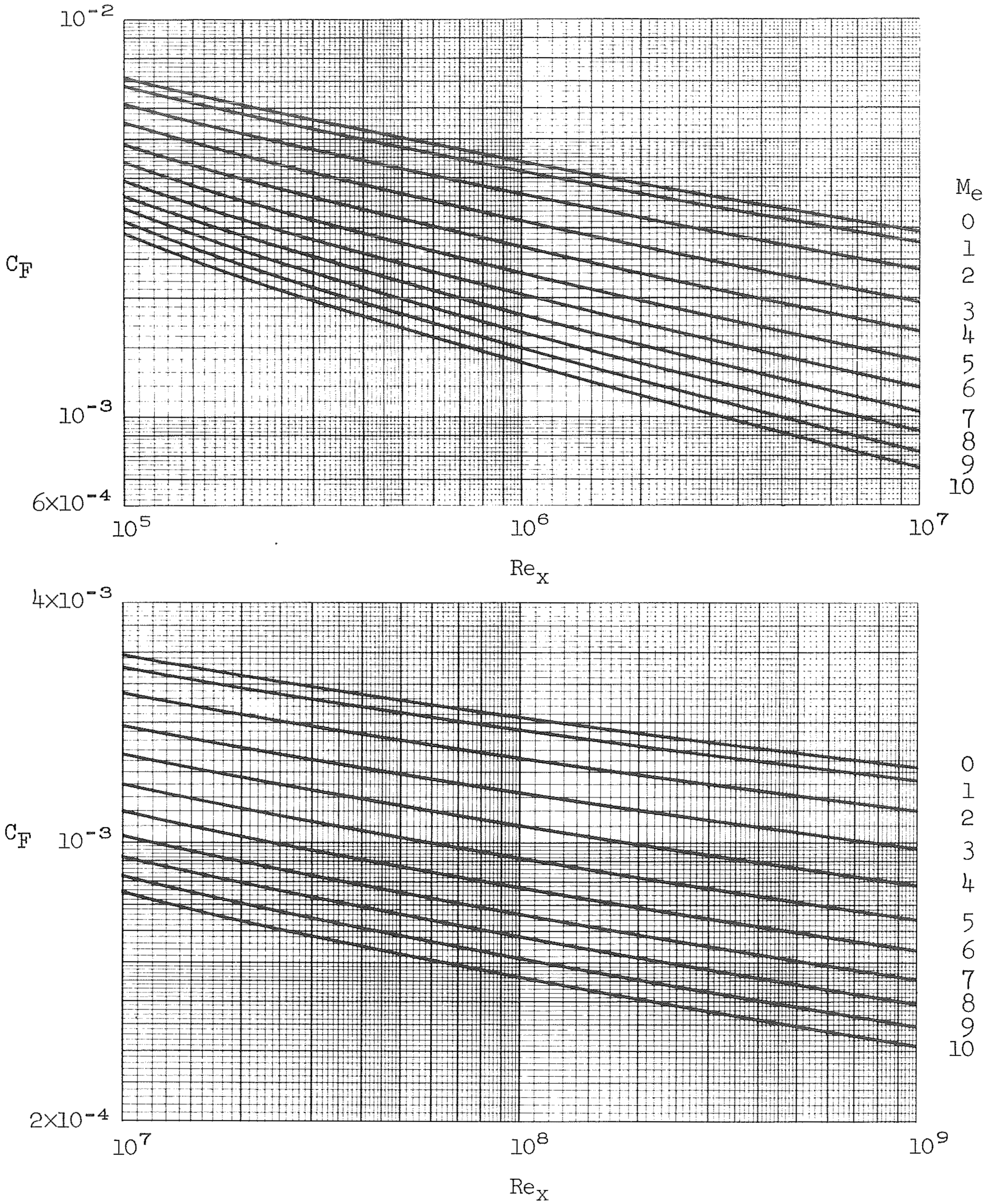
(f) C_F vs Re_x ($T_w/T_{aw} = 0.6$)

Figure 2.- Continued.



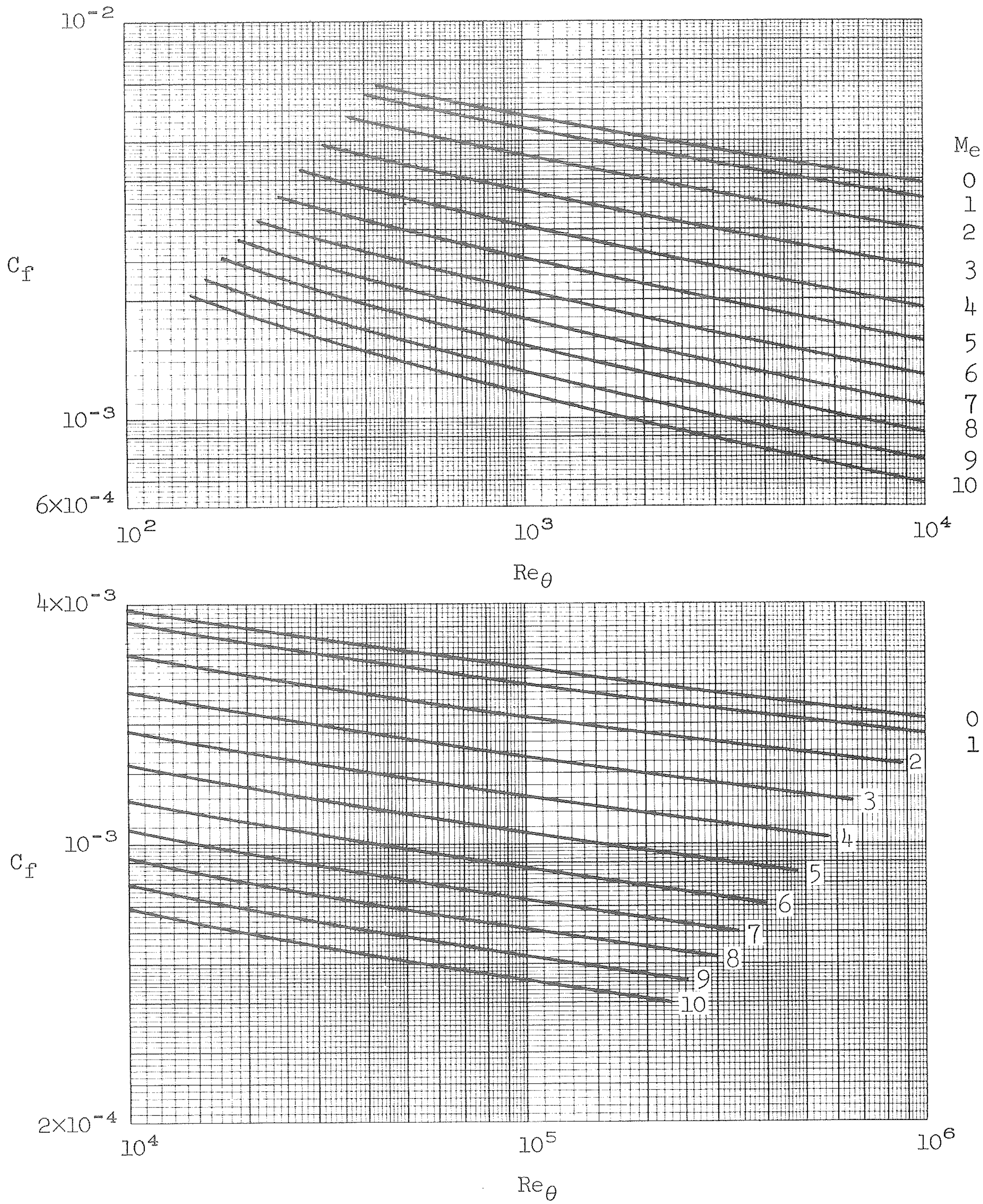
(g) C_F vs Re_θ $(T_w/T_{aw} = 1.0)$

Figure 2.- Continued.



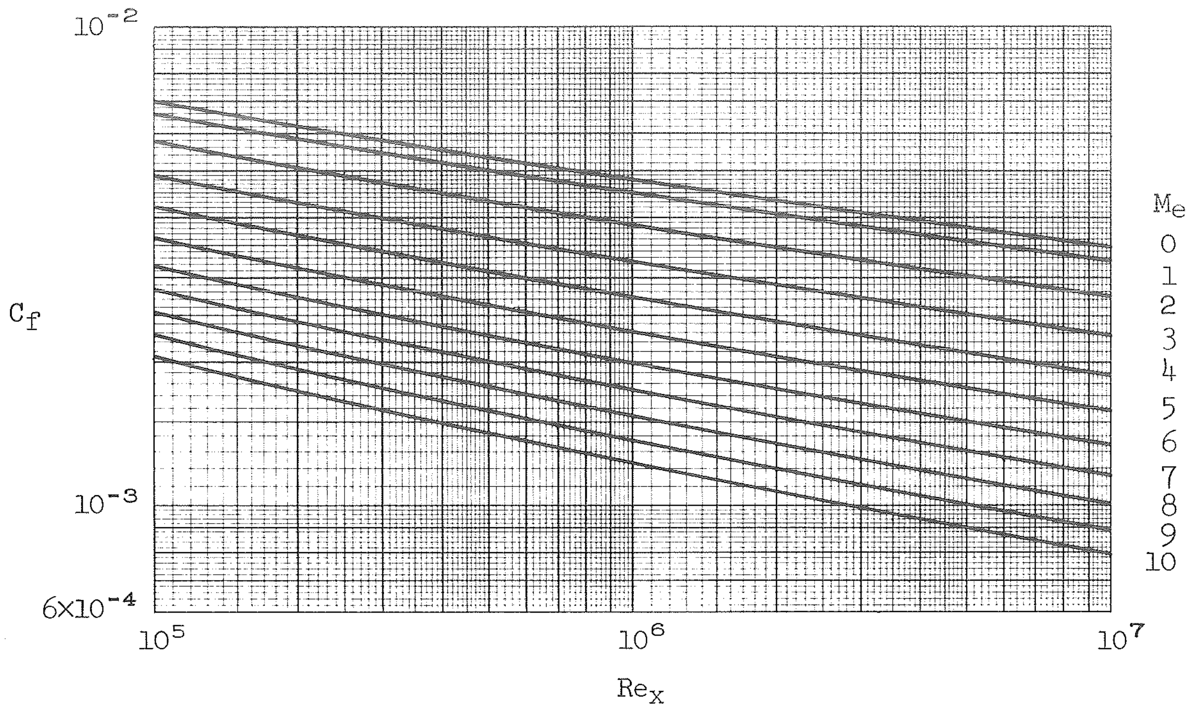
(h) C_F vs Re_x ($T_w/T_{aw} = 1.0$)

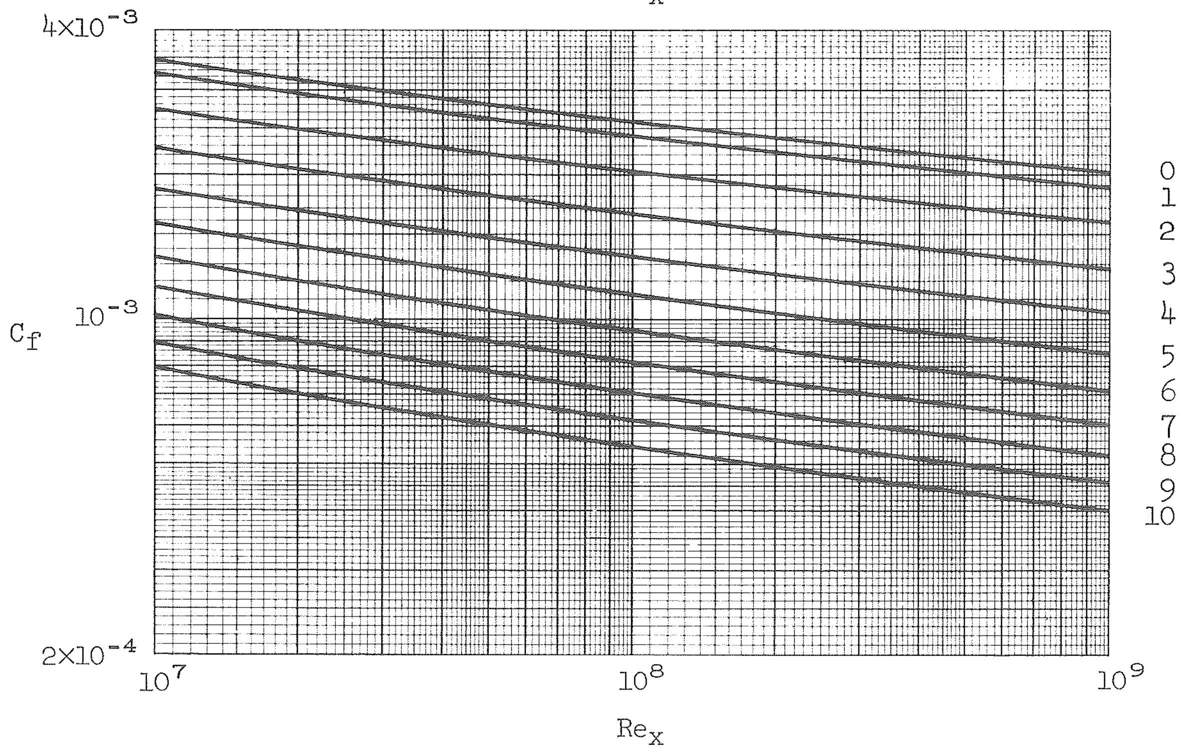
Figure 2.- Concluded.



(a) C_f vs Re_θ (T_w/T_{aw} = 0.2)

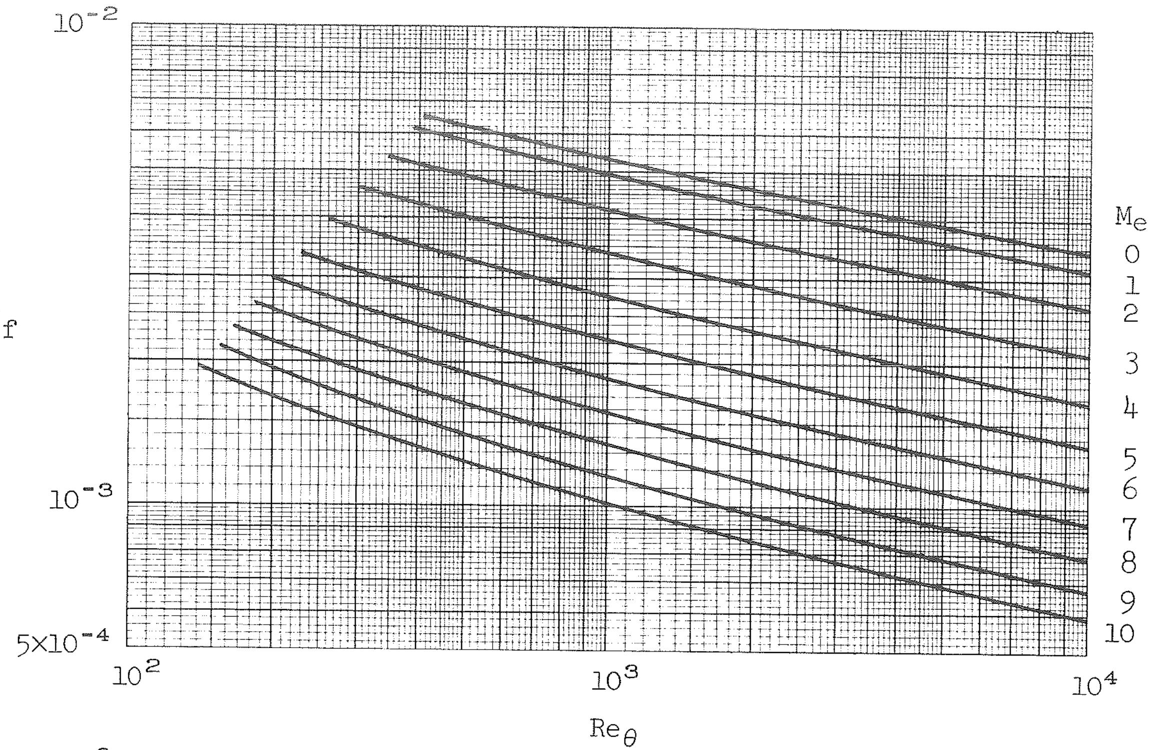
Figure 3.- Local skin friction predicted by the Van Driest method (II); T_e = 222°K (400°R).

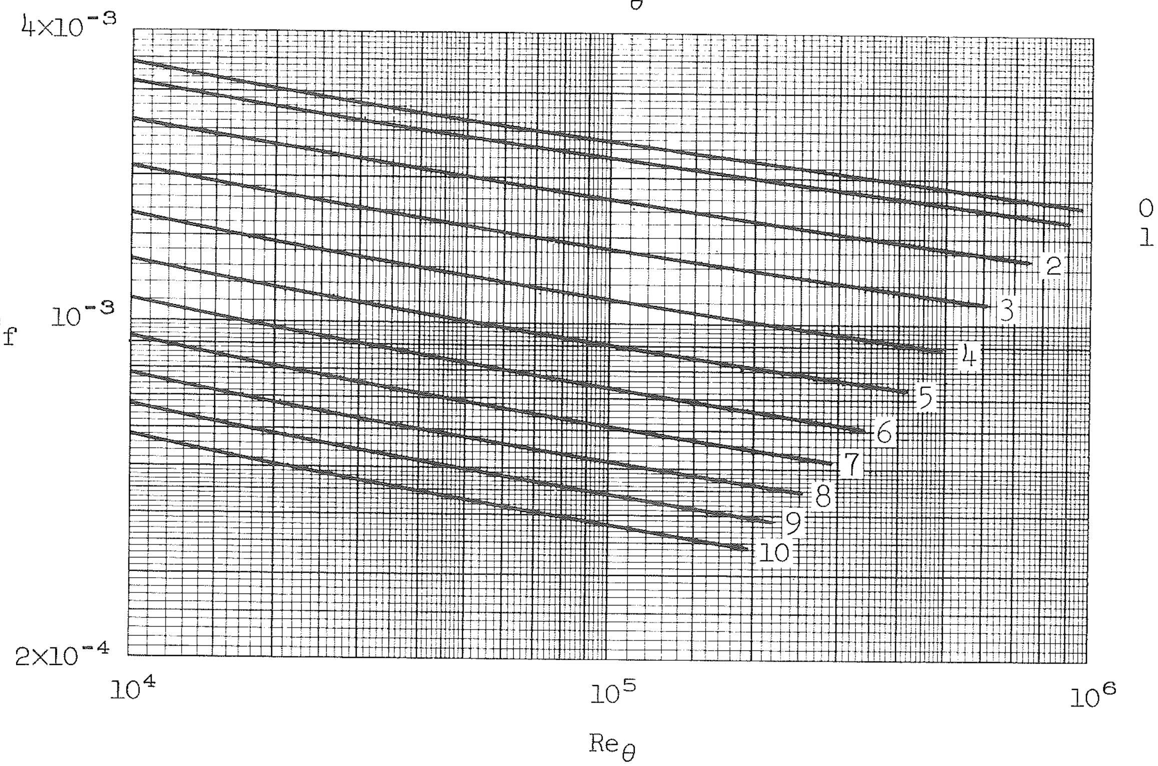




(b) C_f vs Re_x ($T_w/T_{aw} = 0.2$)

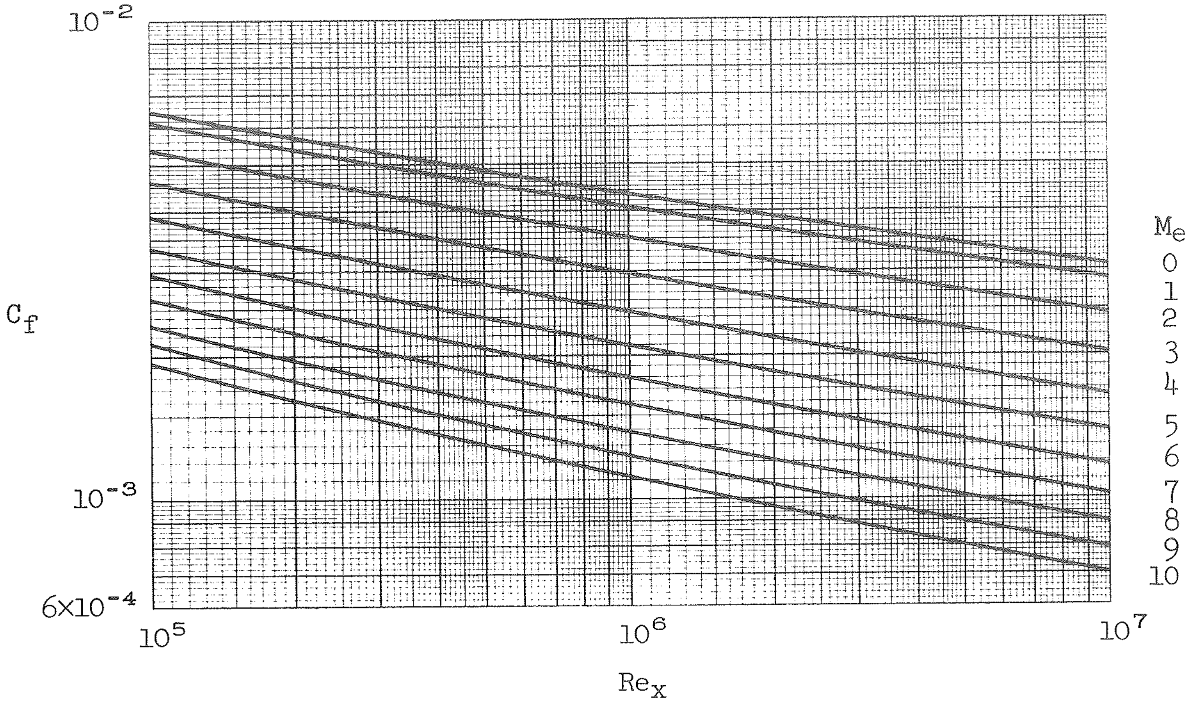
Figure 3.- Continued.

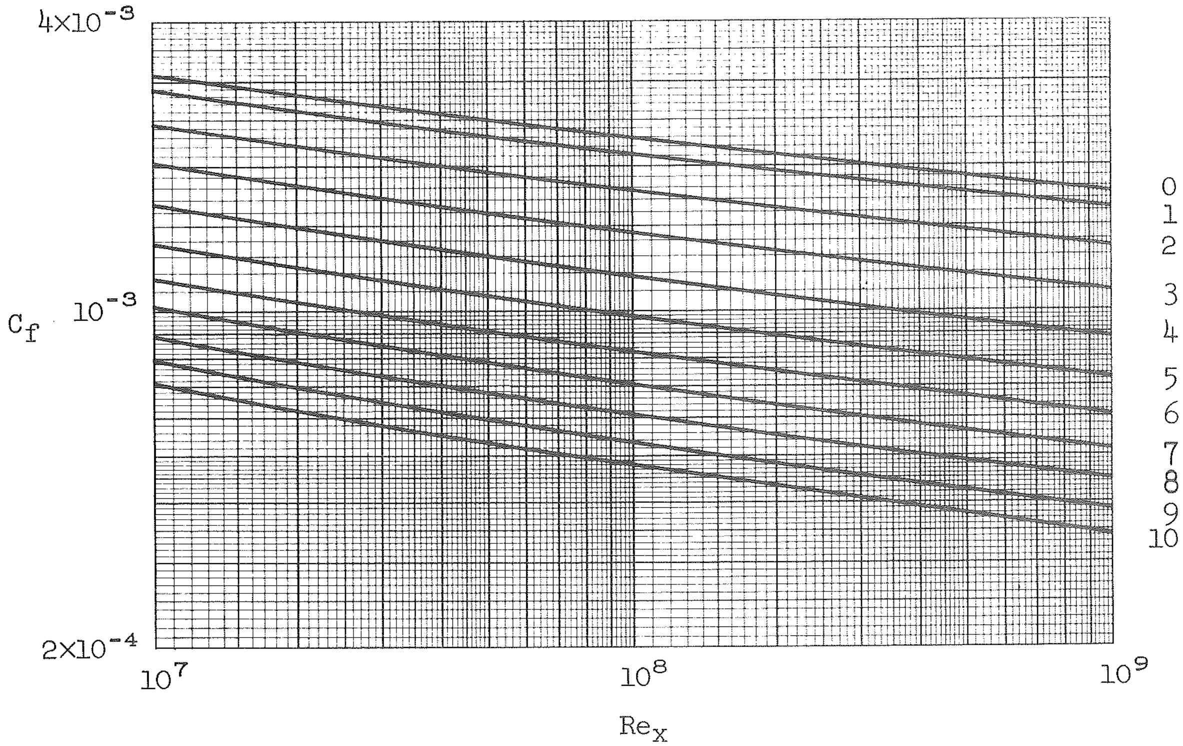




(c) C_f vs Re_θ (T_w/T_{aw} = 0.4)

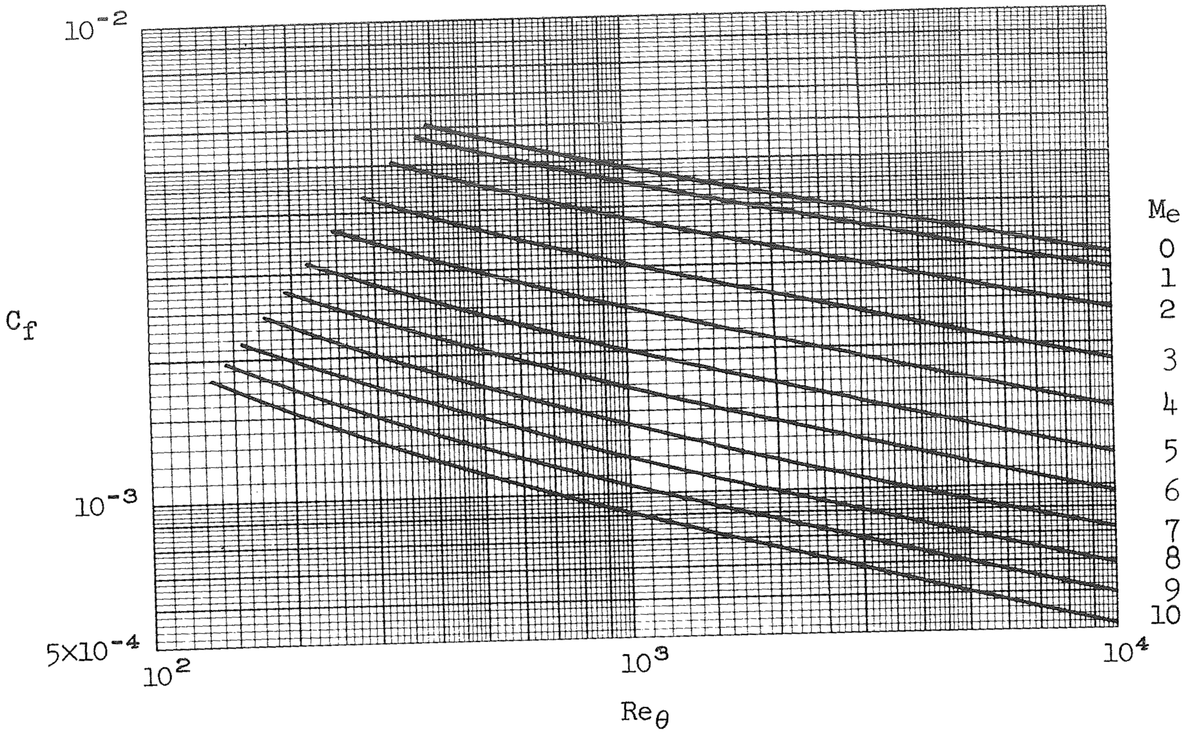
Figure 3.- Continued.

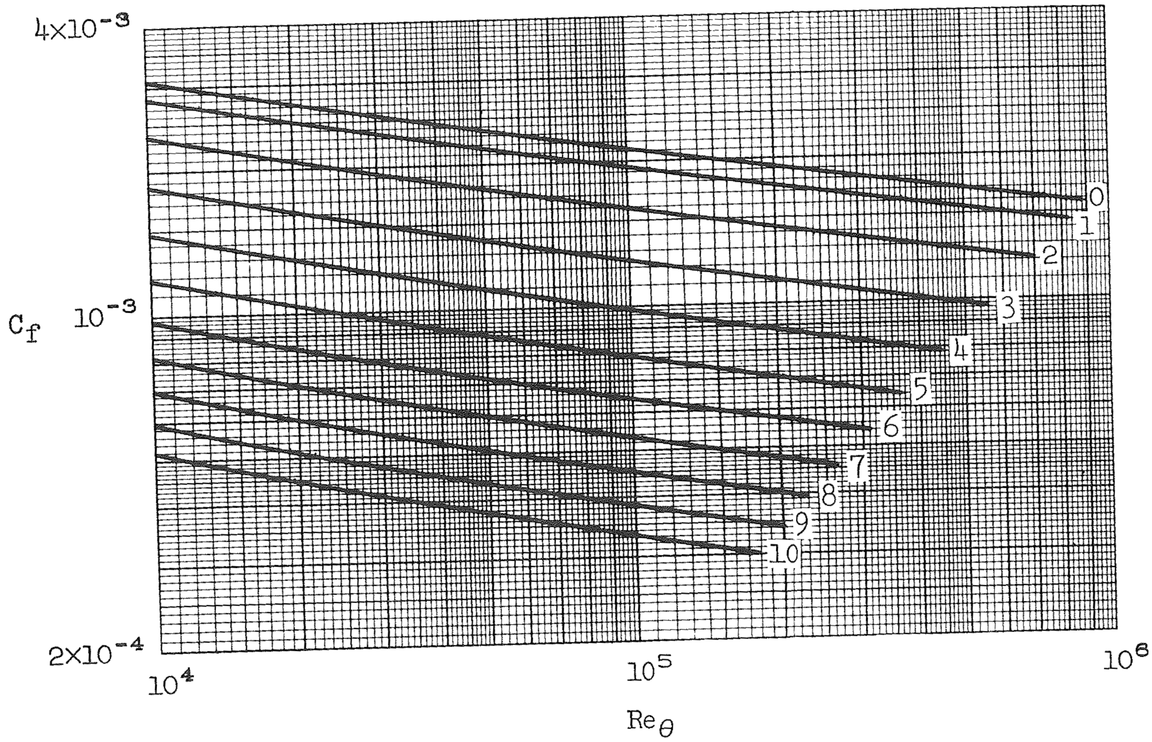




(d) C_f vs Re_x (T_w/T_{aw} = 0.4)

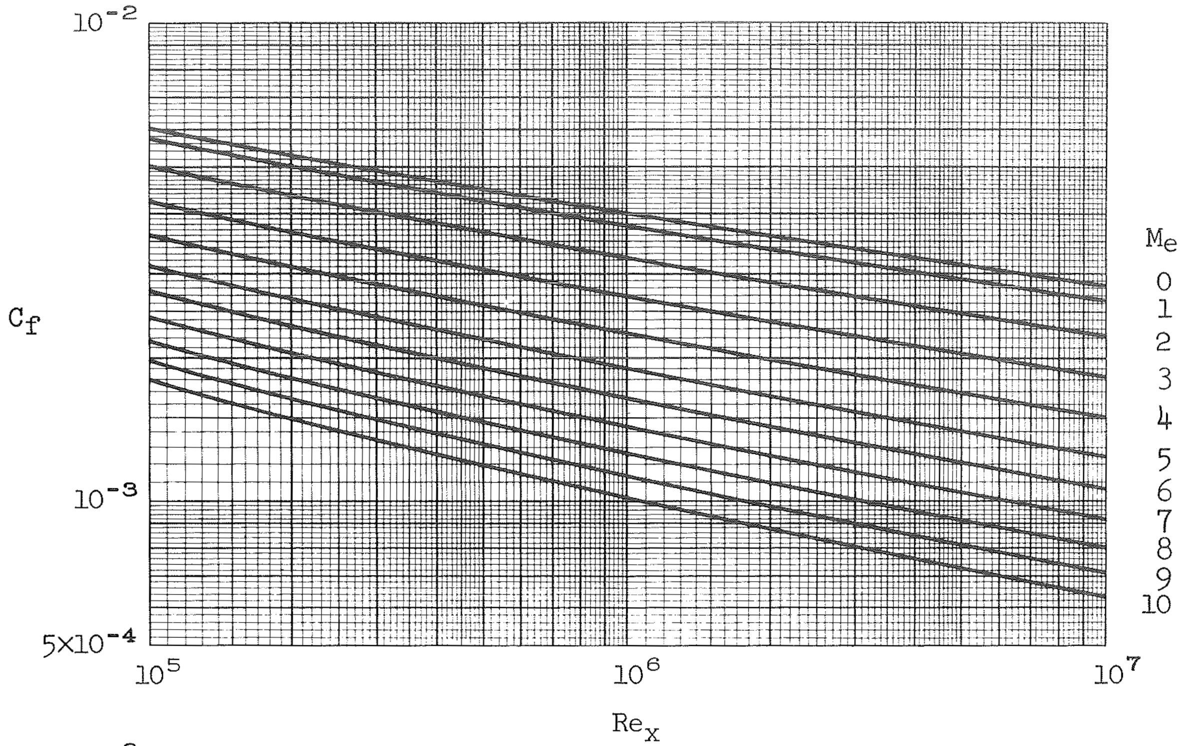
Figure 3.- Continued.

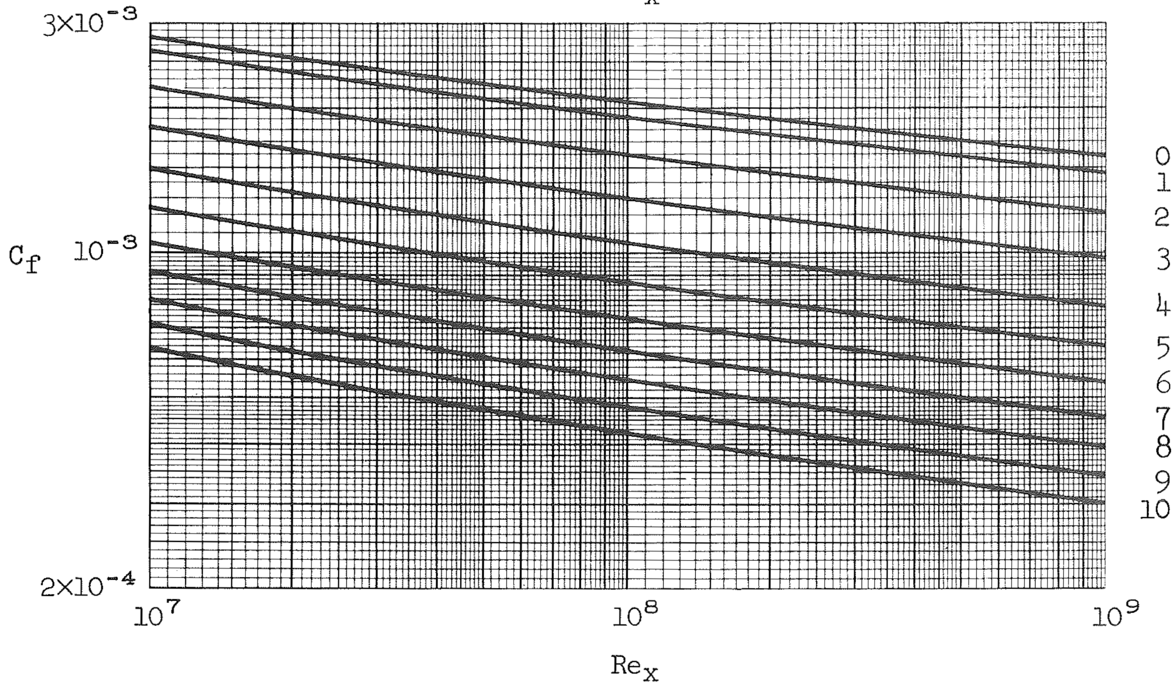




(e) C_f vs Re_θ (T_w/T_{aw} = 0.6)

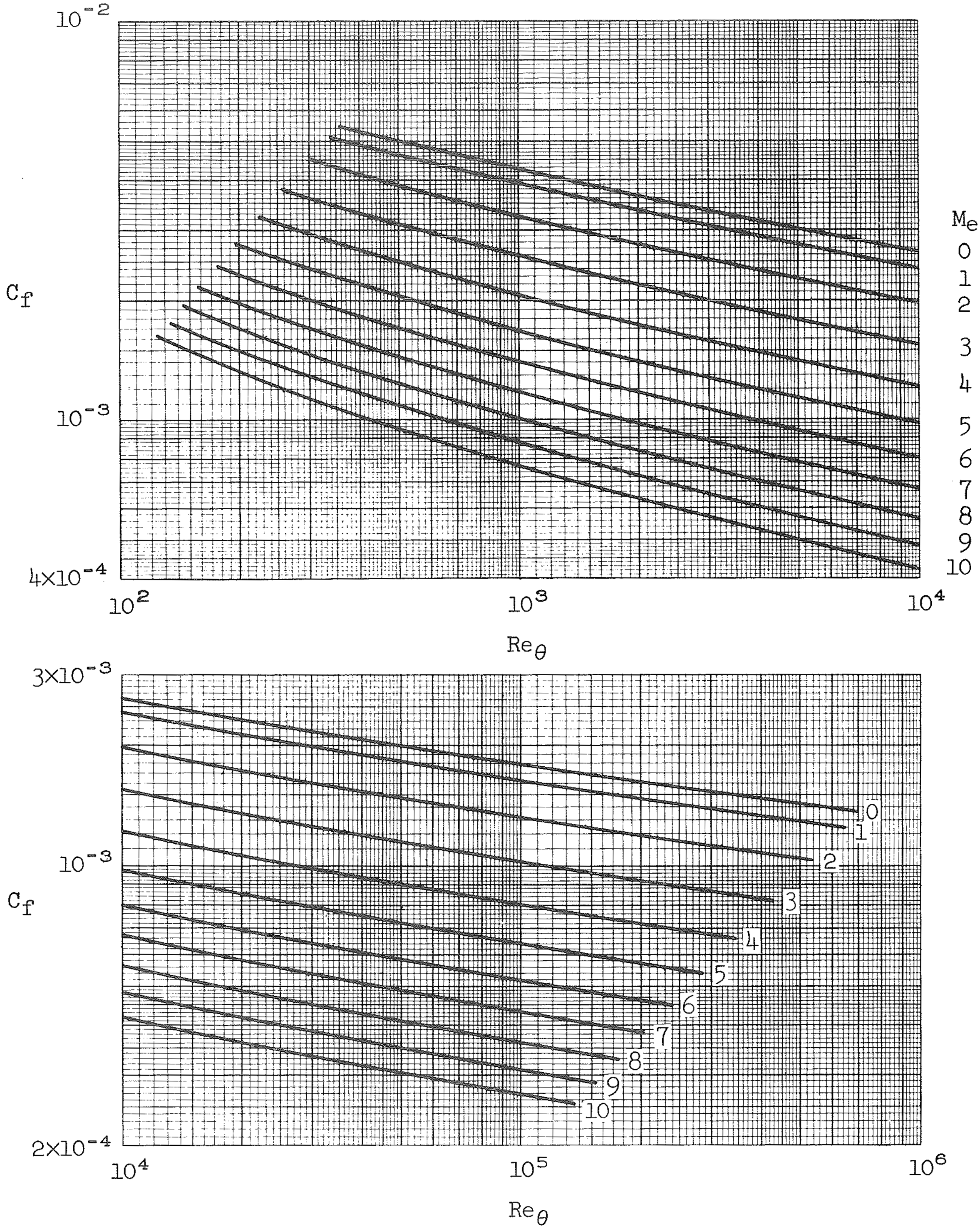
Figure 3.- Continued.





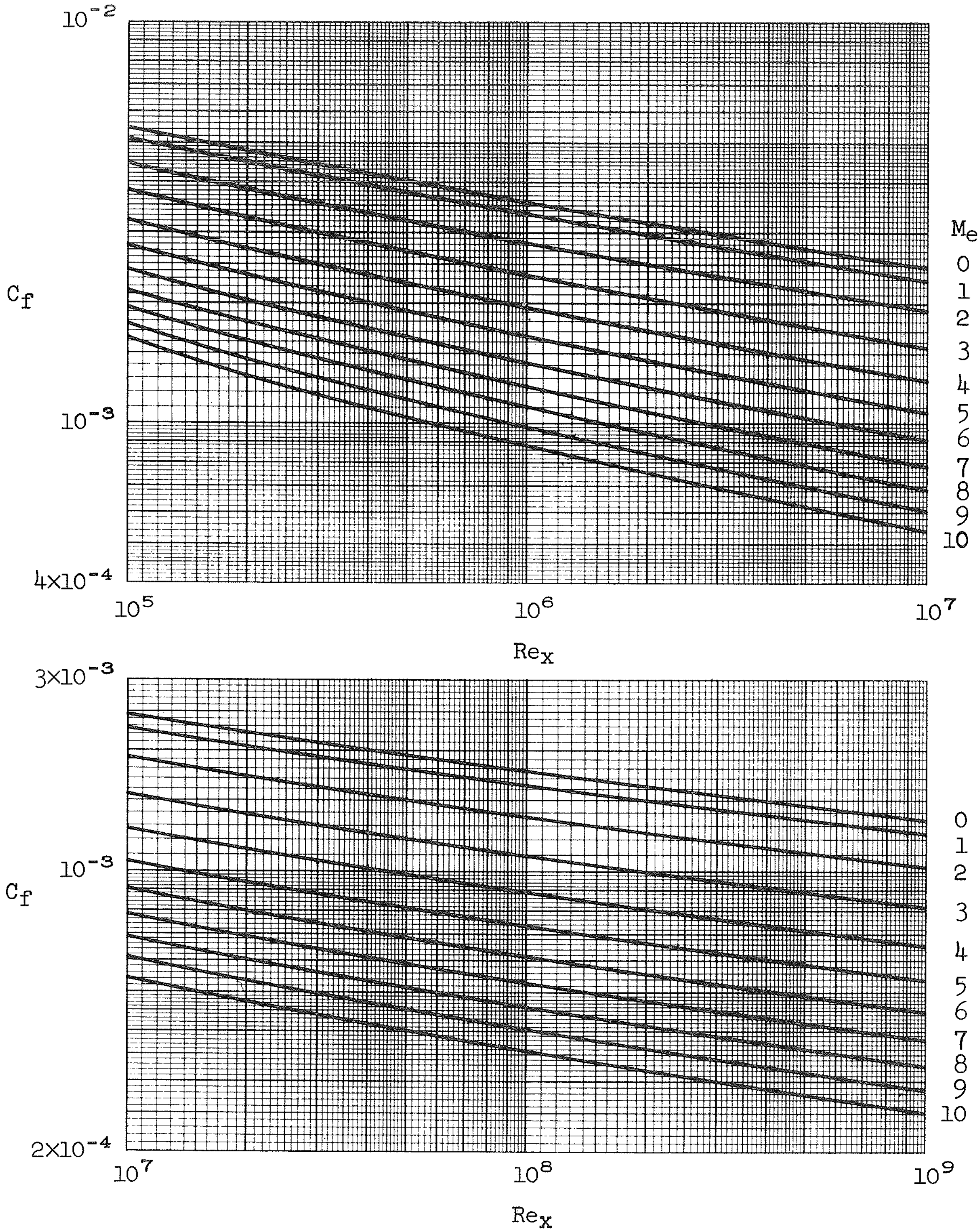
(f) C_f vs Re_x ($T_w/T_{aw} = 0.6$)

Figure 3.- Continued.



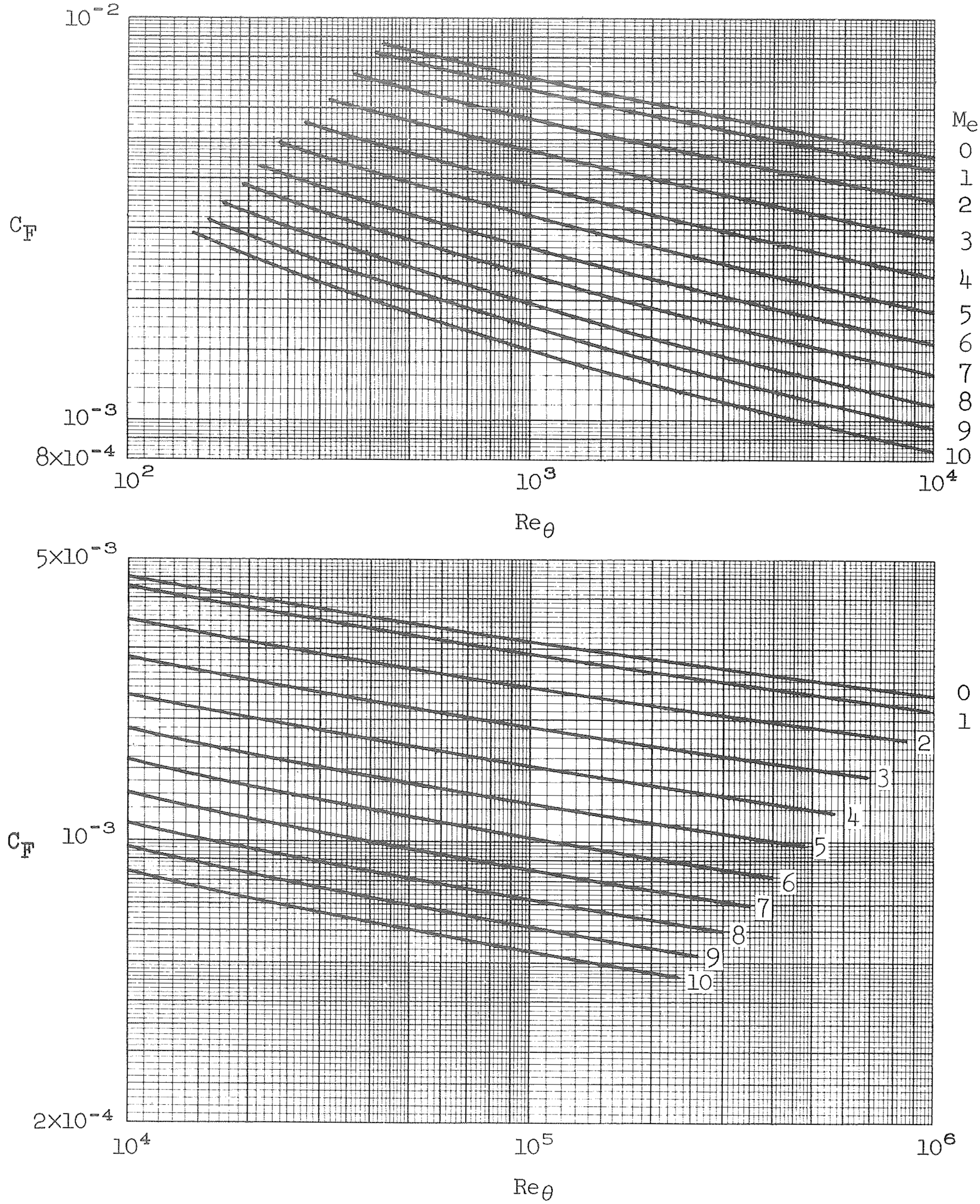
(g) C_f vs Re_θ (T_w/T_{aw} = 1.0)

Figure 3.- Continued.



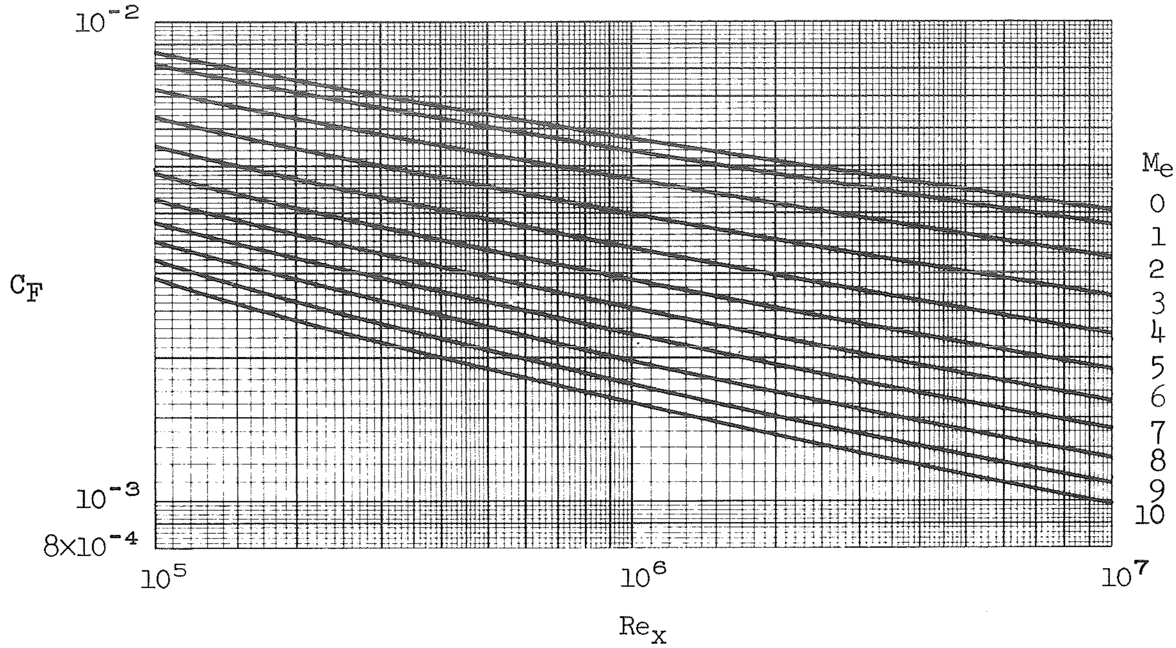
(h) C_f vs Re_x $(T_w/T_{aw} = 1.0)$

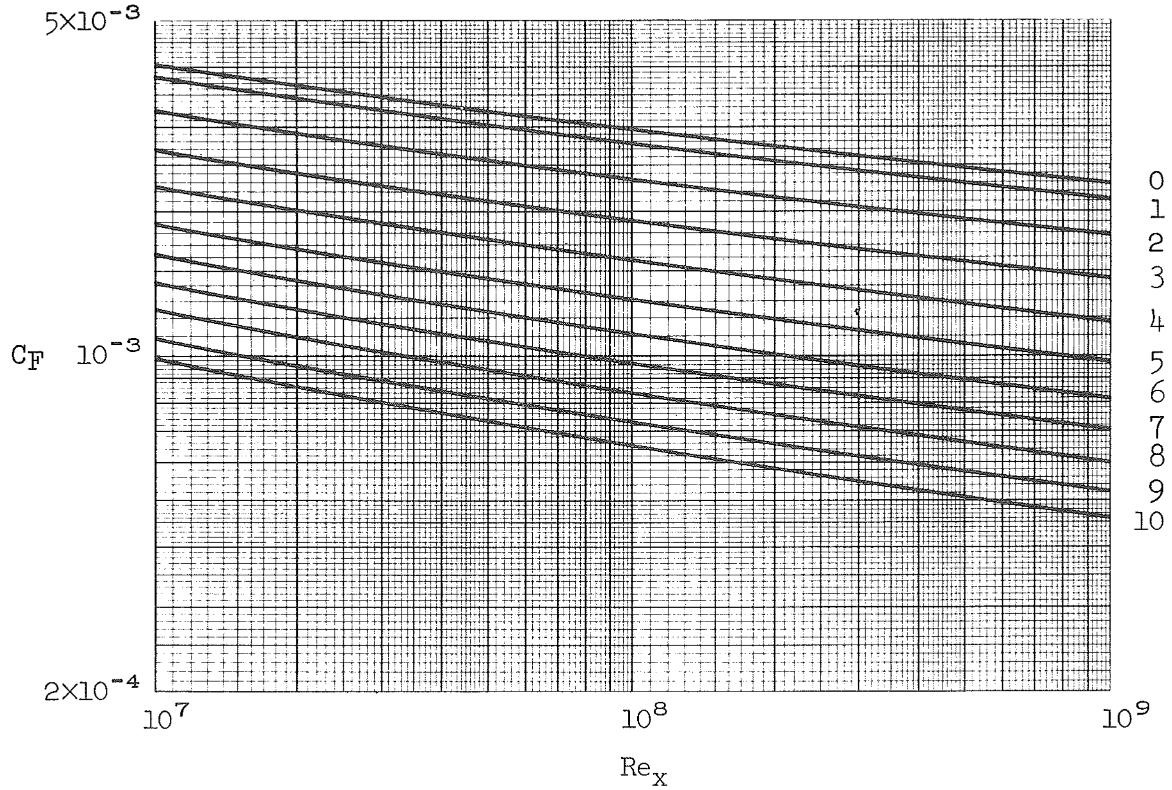
Figure 3.- Concluded.



(a) C_F vs Re_θ ($T_w/T_{aw} = 0.2$)

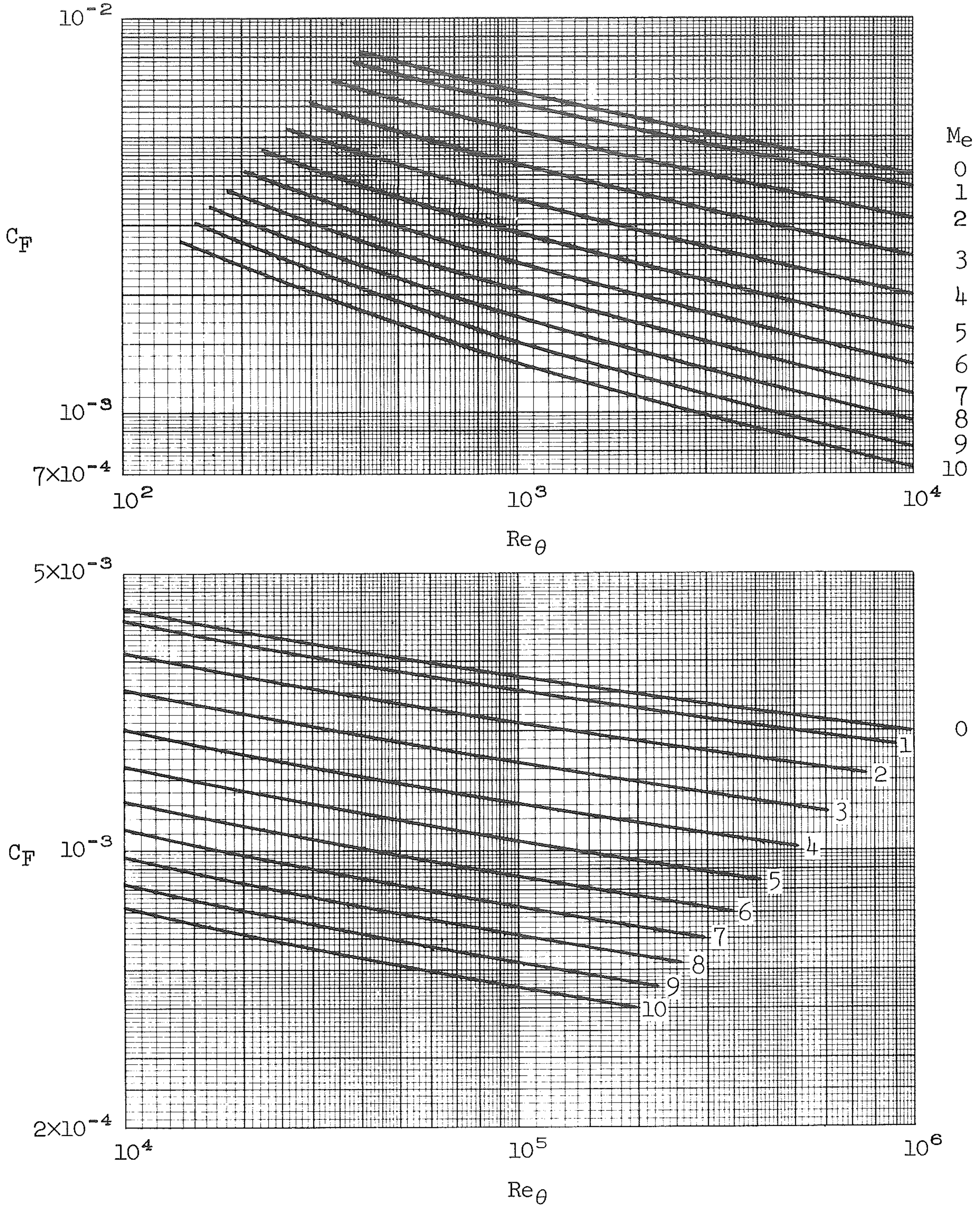
Figure 4.- Average skin friction predicted by the Van Driest method (II); $T_e = 222°K$ (400°R).





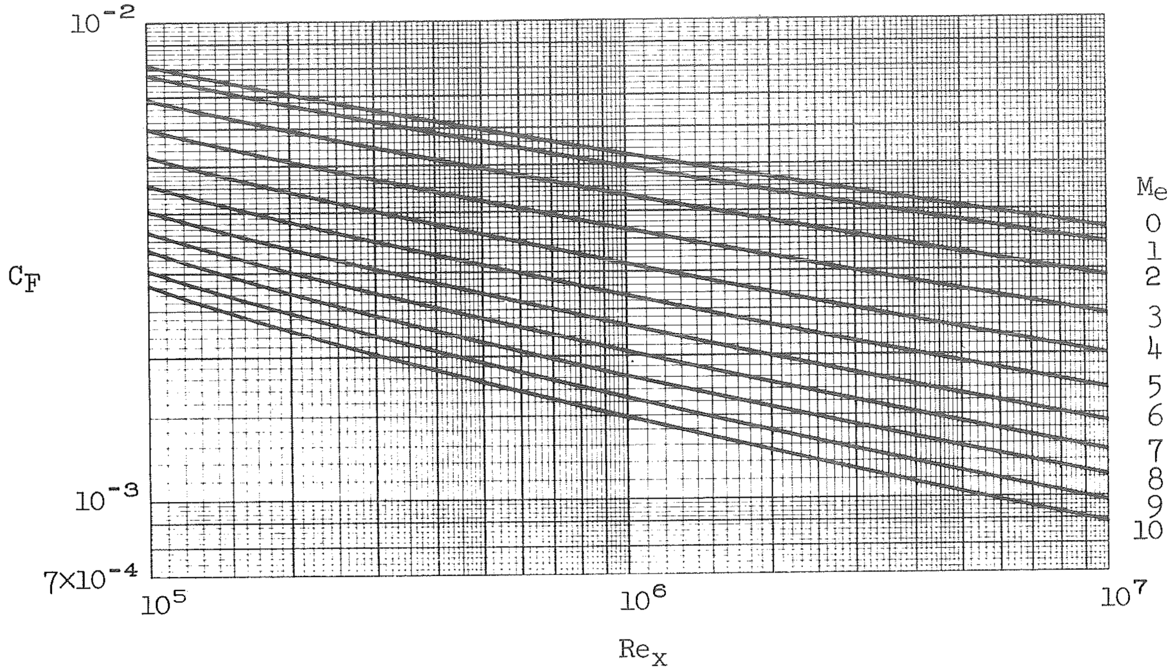
(b) C_F vs Re_x ($T_w/T_{aw} = 0.2$)

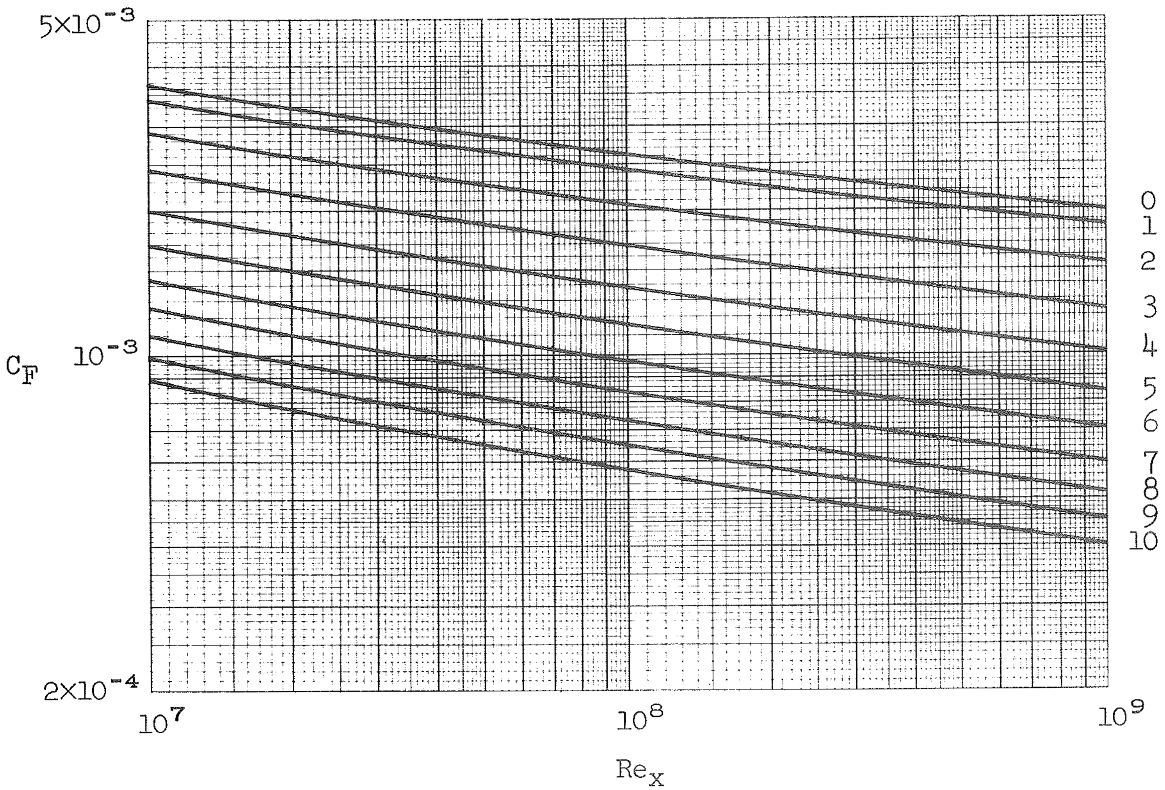
Figure 4.- Continued.



(c) C_F vs Re_θ (T_w/T_{aw} = 0.4)

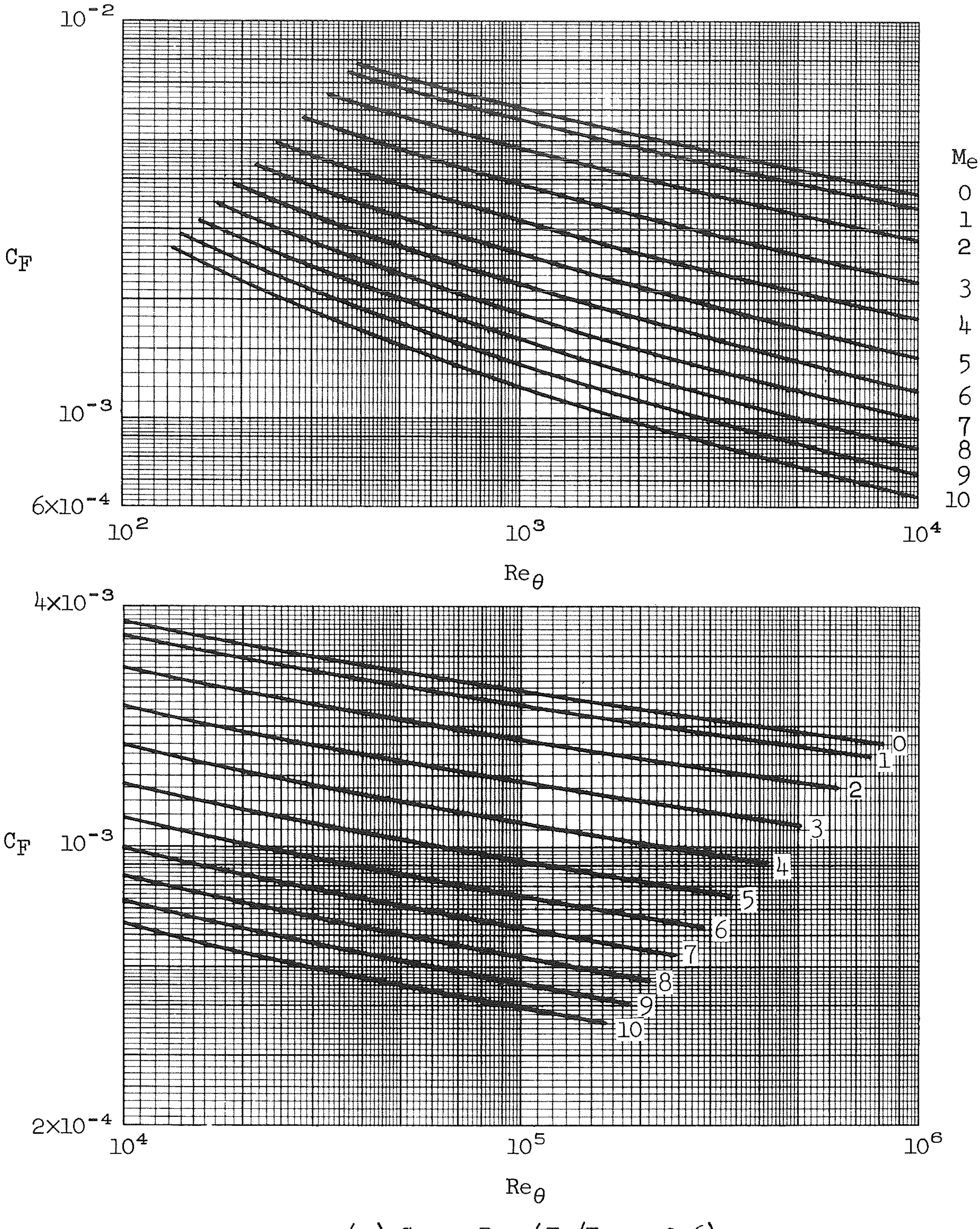
Figure 4.- Continued.





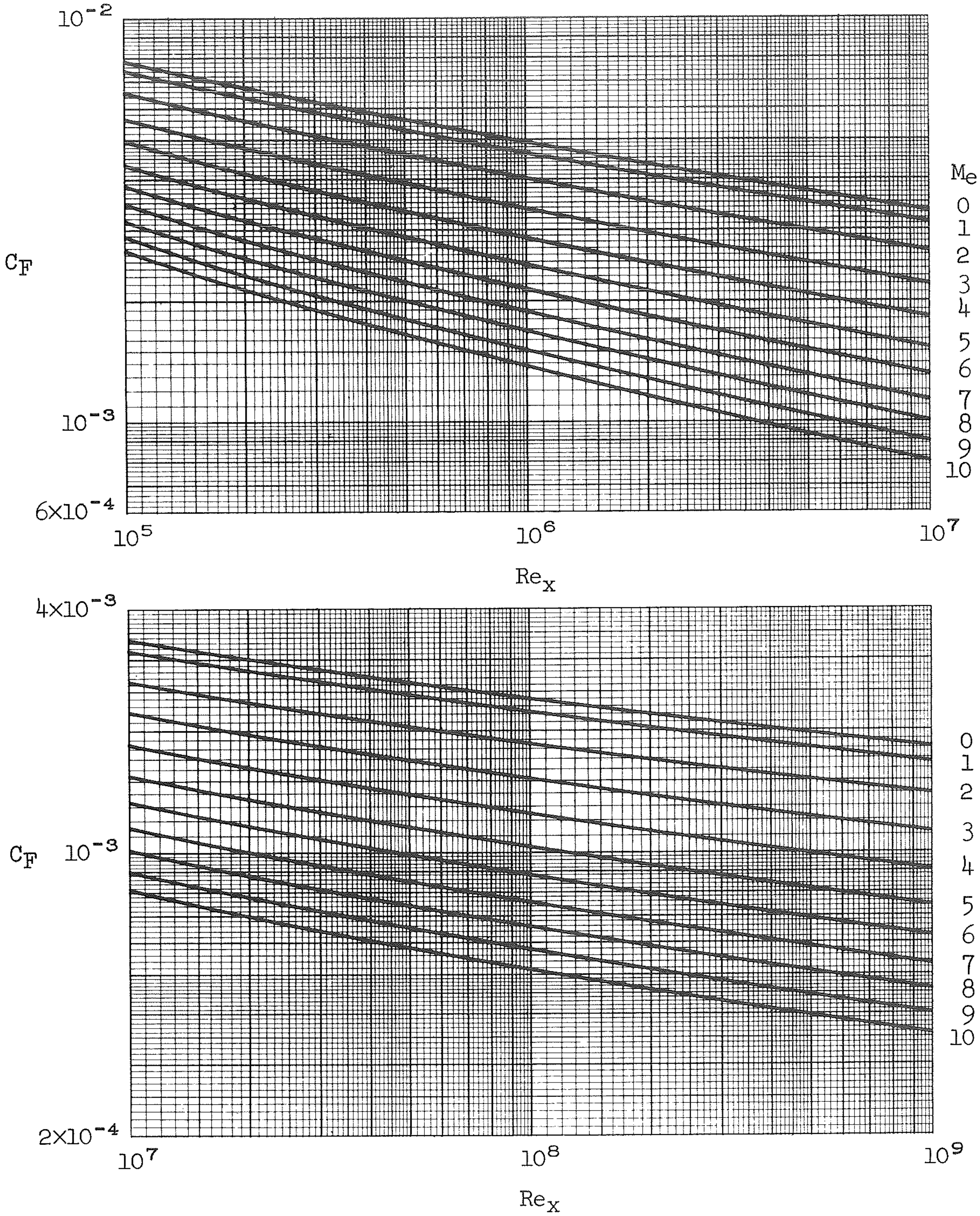
(d) C_F vs Re_x ($T_w/T_{aw} = 0.4$)

Figure 4.- Continued.



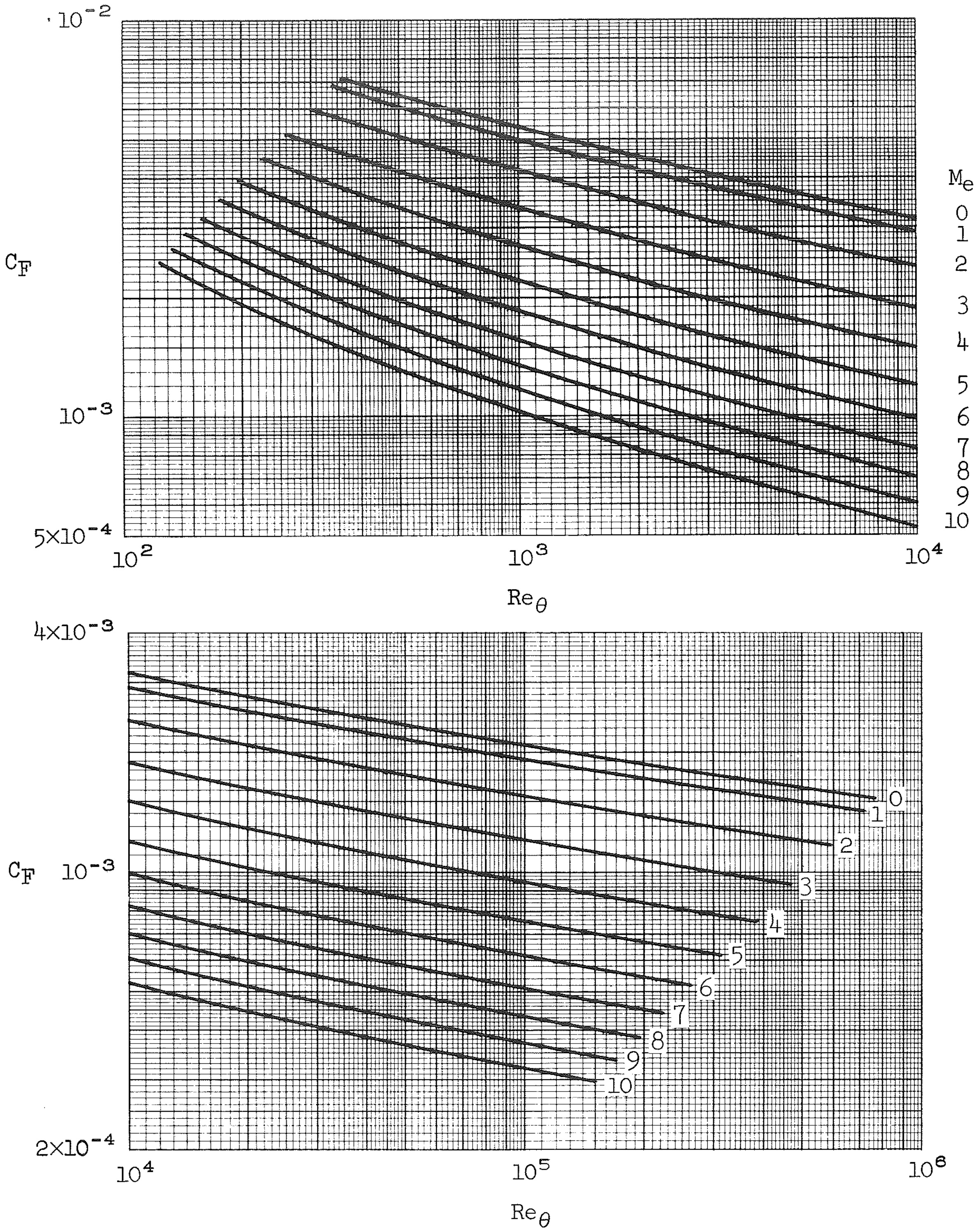
(e) C_F vs Re_θ $(T_w/T_{aw} = 0.6)$

Figure 4.- Continued.



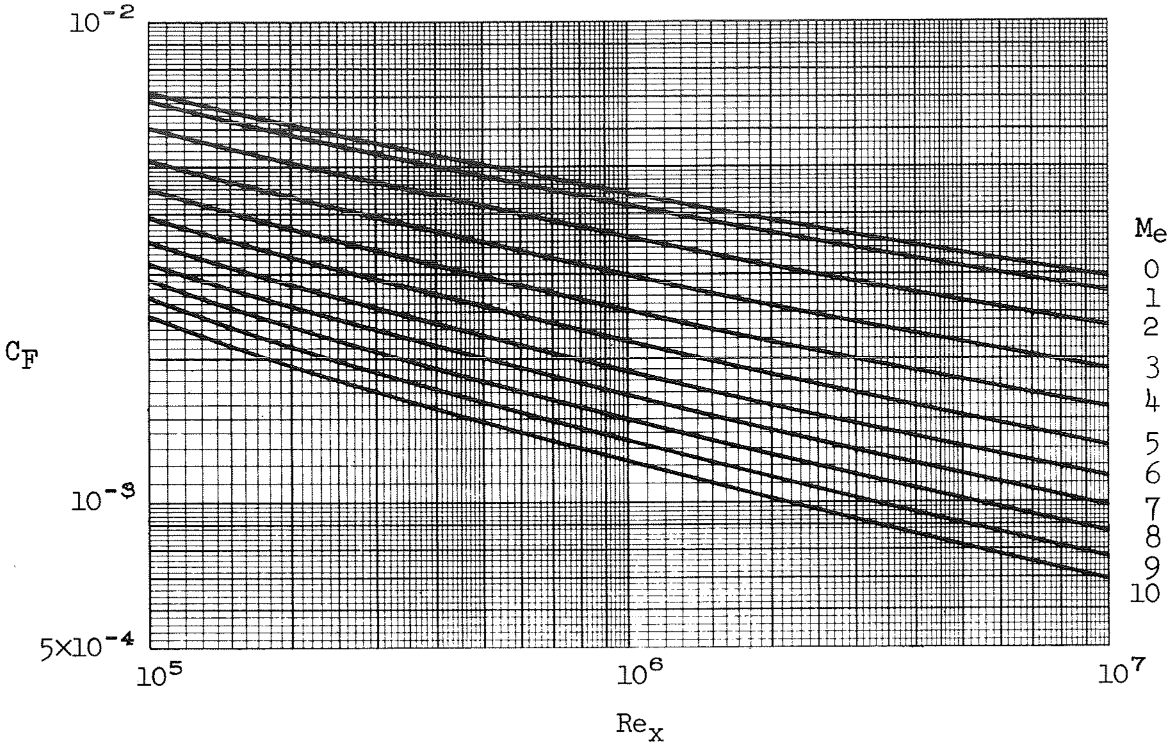
(f) C_F vs Re_x ($T_w/T_{aw} = 0.6$)

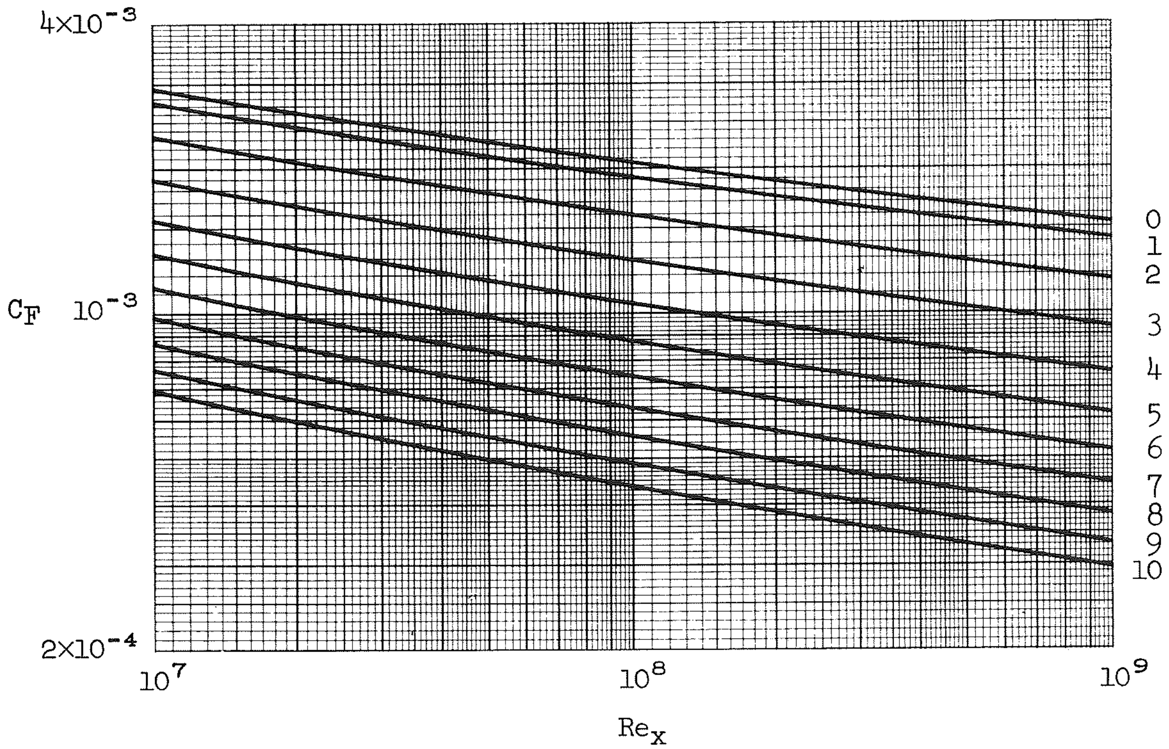
Figure 4.- Continued.



(g) C_F vs Re_θ (T_w/T_{aw} = 1.0)

Figure 4.- Continued.





(h) C_F vs Re_x (T_w/T_{aw} = 1.0)

Figure 4.- Concluded.

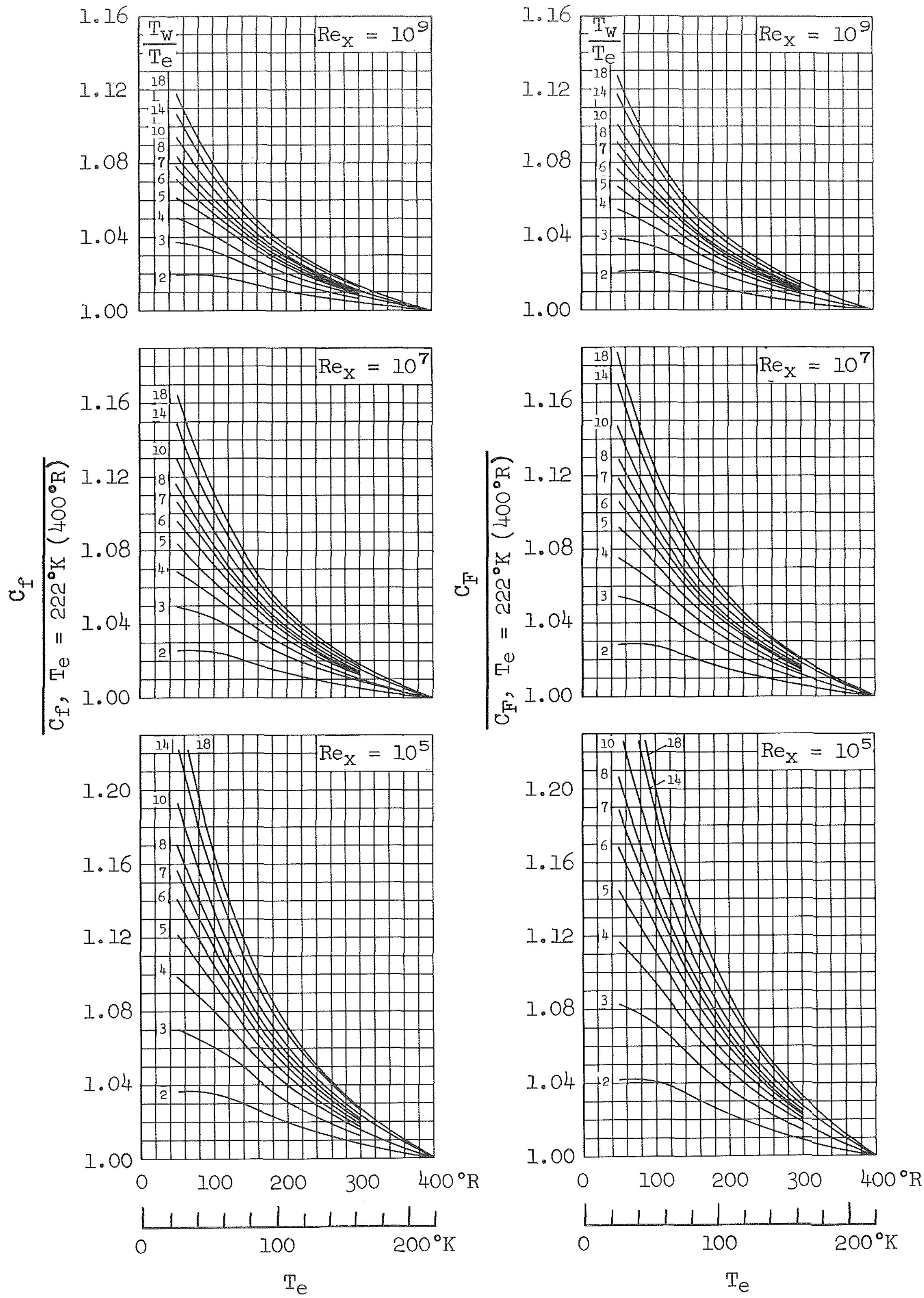
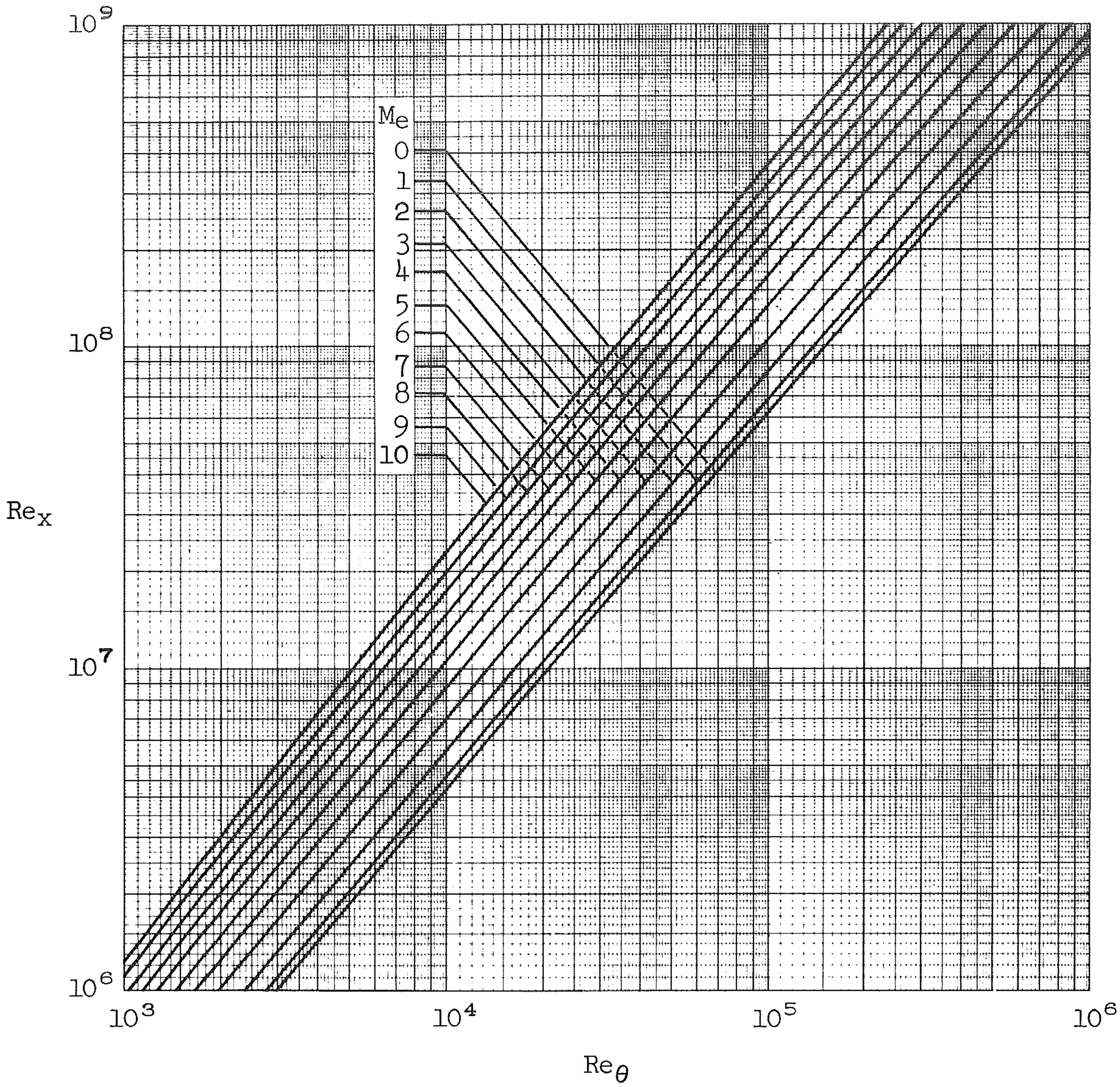
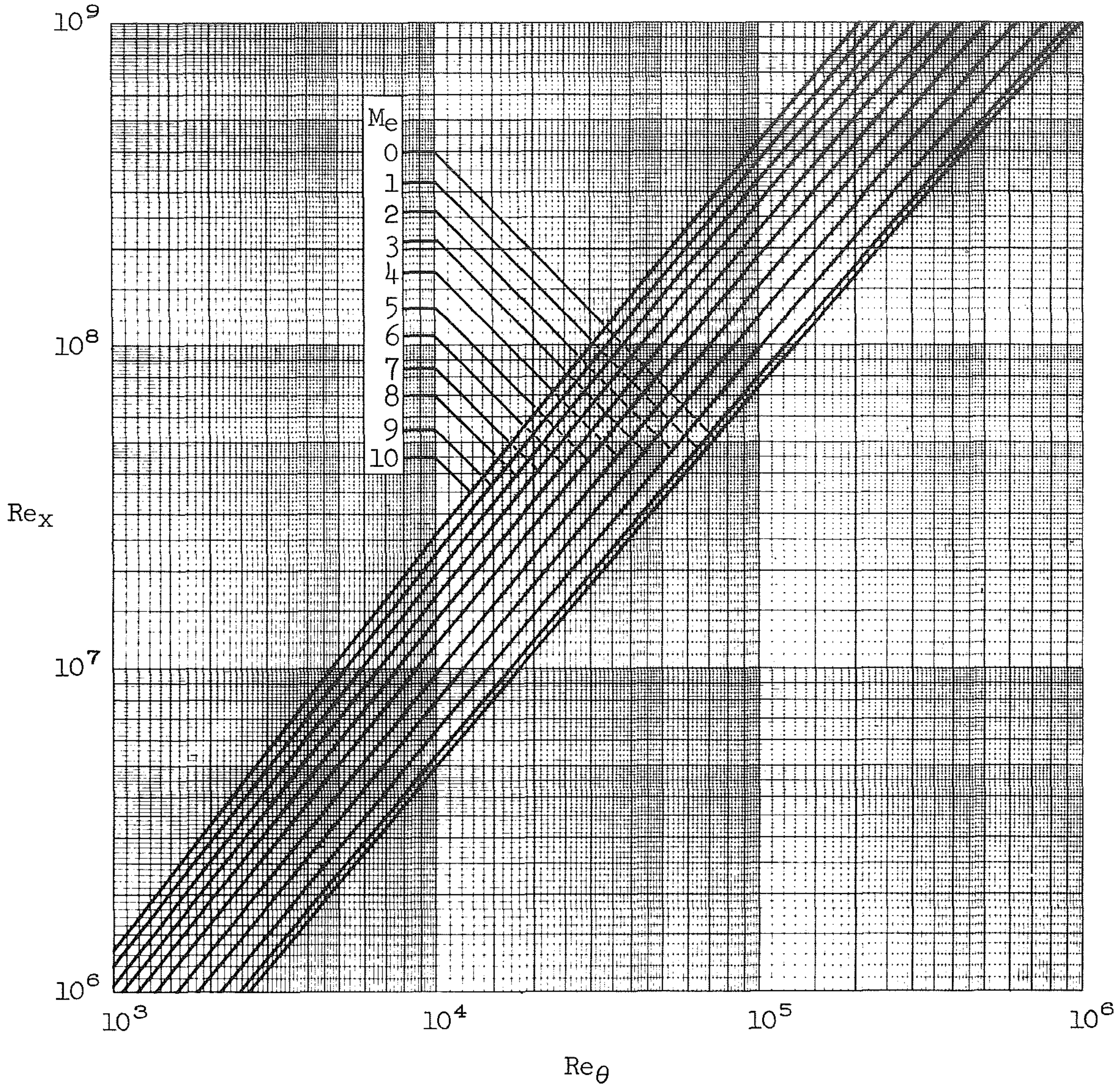


Figure 5.- Skin-friction correction factors, M_e = 0 - 10.



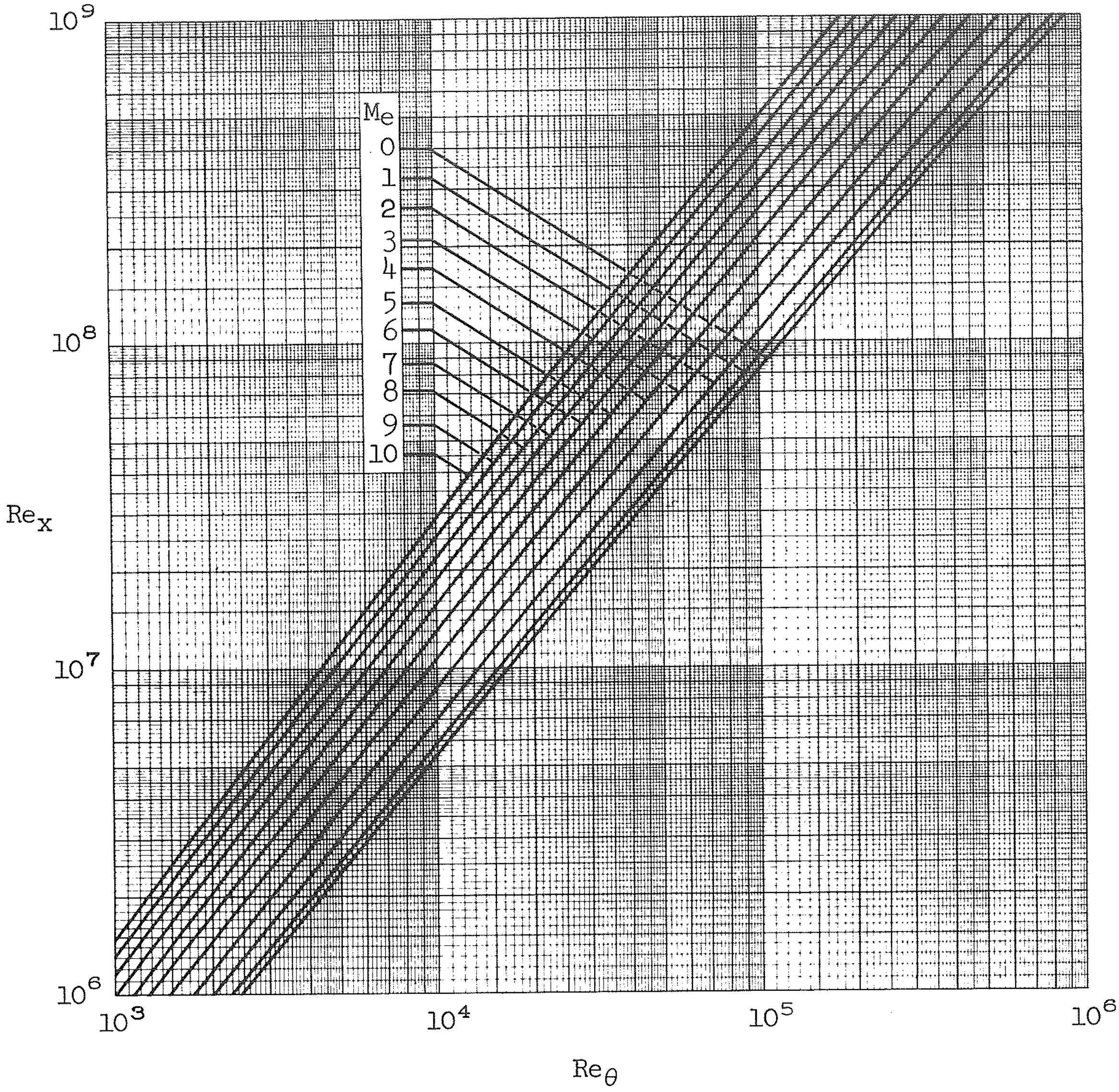
(a) $T_w/T_{aw} = 0.2$

Figure 6.- Relationship between Re_x and Re_θ predicted by the Van Driest method (II); T_e = 55.6°K (100°R).



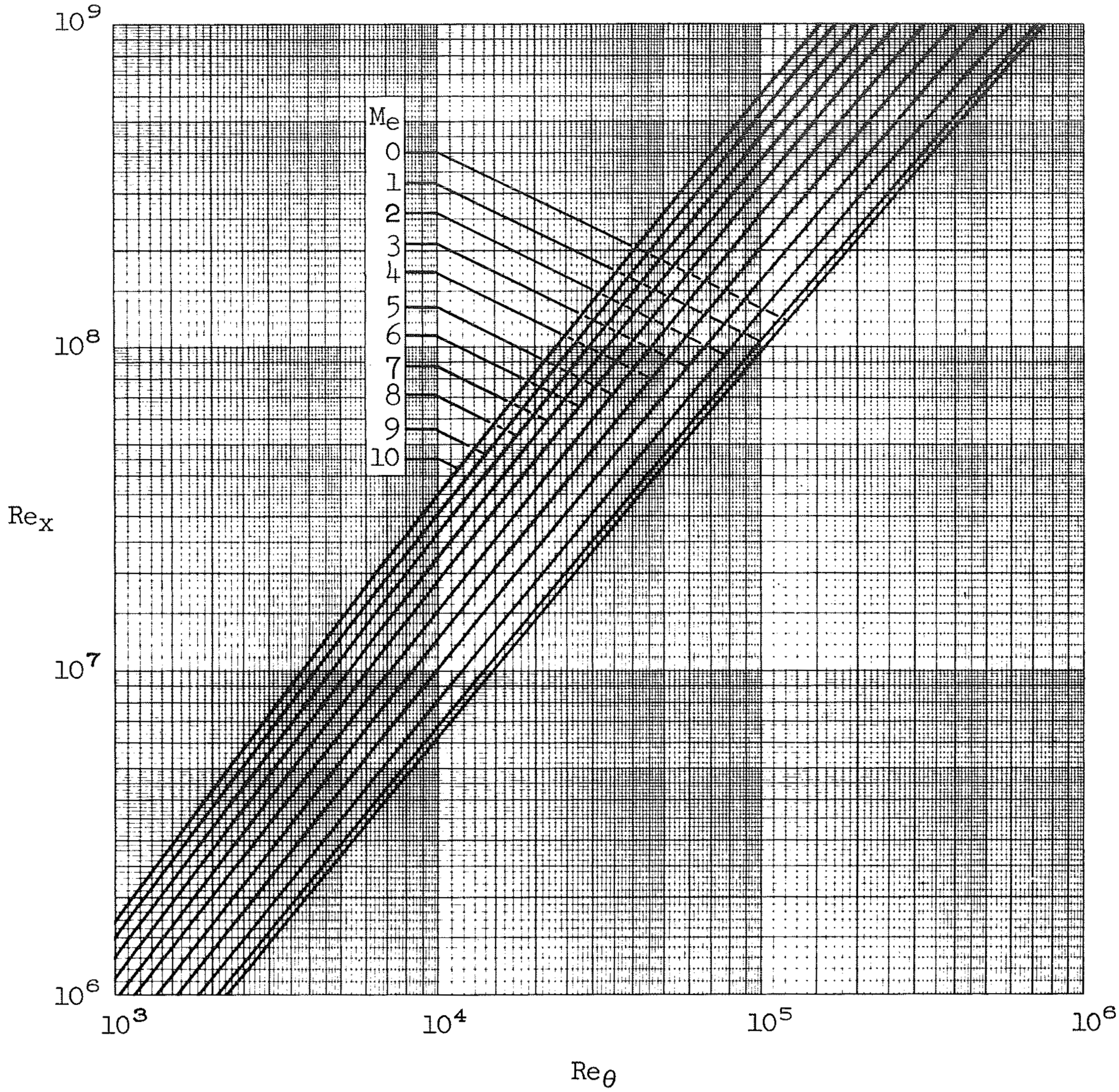
(b) $T_w/T_{aw} = 0.4$

Figure 6.- Continued.



(c) $T_w/T_{aw} = 0.6$

Figure 6.- Continued.



(d) $T_w/T_{aw} = 1.0$

Figure 6.- Concluded.

NASA-Langley, 1972 —— 12 A 4488

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546