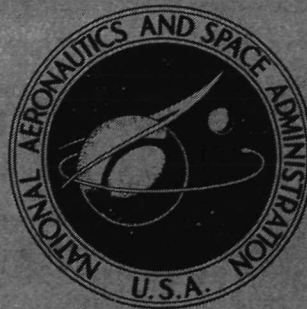


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**AN ANALYTICAL SOLUTION FOR
THE SQUEEZE FILM BETWEEN A
NONDEFORMABLE SPHERE AND GROOVE**

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16. Abstract An analysis is presented to compute the film thickness, pressure and load relations between a rigid ball and rigid groove in normal approach when lubricated by a fluid with an exponential pressure-viscosity relationship. The geometry of the ball-groove system is reduced to the equivalent system of a paraboloid approaching a flat plate. Exact and approximate solutions are presented for the load and pressure relations. There is found to be a limiting load for a given geometry and lubricant regardless of the rate of approach.			
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SUMMARY

An analysis is presented to compute the film thickness, pressure and load relations between a rigid ball and rigid groove in normal approach when lubricated by a fluid with an exponential pressure-viscosity relationship. The geometry of the ball-groove system is reduced to the equivalent system of a paraboloid approaching a flat plate.

Exact and approximate solutions are presented for the load and pressure relations. There is found to be a limiting load for a given geometry and lubricant regardless of the rate of approach.

INTRODUCTION

Comparatively little work has been done on squeeze films between curved surfaces. In reference 1, the squeeze film between a ball and spherical seat was investigated for the case of a constant viscosity lubricant. In reference 2 the elastohydrodynamic lubrication of spheres in normal approach was investigated using an exponential pressure-viscosity relationship. In reference 2, an analytical solution for the case of rigid surfaces and a numerical solution for deformable surfaces up to moderate pressures and for large film thicknesses are presented. For the range of pressures and film thicknesses considered in reference 2 it was found that the rigid surface solution yields results close to the elastic solution.

The work reported herein extends the analysis of reference 2 to the case of a sphere approaching a cylindrical groove. The direction of approach is normal to the two surfaces.

ANALYSIS

The system under consideration together with the coordinate system used is shown in figure 1. In accordance with standard notation, the radius of curvature is considered positive when the center of curvature lies within the body.

If the film thickness is much smaller than the radii of curvatures, then the customary parabolic approximation may be made and the system is approximately equivalent to a paraboloid approaching a flat plate as shown in figure 2. The equivalent radii of curvature in the x-z and y-z planes are given by

$$R_x = \frac{R R_{Gx}}{R + R_{Gx}}$$

$$R_y = \frac{R R_{Gy}}{R + R_{Gy}}$$

where the signs of R_{Gx} and R_{Gy} are taken as defined previously. Using the parabolic representation, the film thickness at x y is given by

$$\begin{aligned} h &= h_0 + \frac{x^2}{2R_x} + \frac{y^2}{2R_y} \\ &= h_0 + Ax^2 + By^2 \end{aligned} \tag{1}$$

The Reynolds equation for the lubricant film is given in reference 3.

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{h^3}{\mu} \frac{\partial P}{\partial y} \right) = 12 \frac{\partial h}{\partial t} \quad (2)$$

The pressure-viscosity relationship will be considered to follow the exponential law:

$$\mu = \mu_0 e^{\alpha P} \quad (3)$$

This relationship has been shown in reference 4 to be invalid at high shear when the pressure exceeds $3.8 \cdot 10^8 \text{ N/m}^2$ (55 000 psi). However, in reference 2 it is found that elastic considerations become important as the central pressure increases beyond a value equivalent to $\alpha P_0 = 5$

For a synthetic paraffinic lubricant ($\alpha = 1.3 \cdot 10^{-8} \text{ m}^2/\text{N}$) this corresponds to a central pressure of $3.86 \cdot 10^8 \text{ N/m}^2$ (56 000 psi). Thus, the breakdown of the exponential pressure-viscosity relationship and the rigid surface approximation occur at about the same pressure.

Substituting the pressure-viscosity relationship (3) in the Reynold's equation (2) and considering the lubricant to be isothermal, the following partial differential equation is obtained

$$\frac{\partial}{\partial x} \left(\frac{h^3}{e^{\alpha P}} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{h^3}{e^{\alpha P}} \frac{\partial P}{\partial y} \right) = 12 \mu_0 \dot{h} \quad (4)$$

The particular solution to this equation is

$$p = -\frac{1}{\alpha} \ln \left(1 + \frac{3\mu_0 \dot{h} \alpha}{(A+B)h^2} \right) \quad (5)$$

letting

$$\frac{3\mu_0 \dot{h} \alpha}{(A+B)} = -k^2$$

this equation becomes

$$p = -\frac{1}{\alpha} \ln \left(1 - \frac{k^2}{h^2} \right) \quad (6)$$

In accordance with the arguments presented in reference 5 and reference 6 that the pressure distribution is only weakly dependent upon the shape of the boundaries, equation (6) will be considered as the complete solution.

The normal load is obtained by integrating over the area and since the pressure drops off rapidly with distance from the origin, the upper limits on x and y will be taken as infinity. Therefore the load W is given by:

$$W = 4 \int_0^{\infty} \int_0^{\infty} p dx dy \quad (7)$$

Integrating (7), the expression for W is found to be

$$W = \frac{\pi}{\alpha \sqrt{AB}} \left[h_0 \ln \left(\frac{h_0^2 - k^2}{h_0^2} \right) + k \ln \left(\frac{h_0 + k}{h_0 - k} \right) \right] \quad (8)$$

A simpler approximation for the pressure and load may be obtained by writing the pressure as a series:

$$p = -\frac{1}{\alpha} \ln \left(1 - \frac{k^2}{h^2} \right) = -\frac{1}{\alpha} \left[-\frac{k^2}{h^2} - \frac{k^4}{2h^4} - \frac{k^6}{3h^6} - \dots \right]$$

For small values of k/h , higher order terms can be neglected and therefore:

$$p \approx \frac{k^2}{\alpha h^2} \quad (9)$$

Integrating as before:

$$\begin{aligned} W &= 4 \frac{k^2}{\alpha} \int_0^{\infty} \int_0^{\infty} \frac{1}{h^2} dx dy \\ &= \frac{\pi k^2}{\alpha h_0 \sqrt{AB}} \end{aligned} \quad (10)$$

The solutions may be written in dimensionless form as follows:

$$\frac{W \alpha \sqrt{AB}}{\pi h_0} = \ln \left(1 - \frac{k^2}{h_0^2} \right) + \frac{k}{h_0} \ln \left(\frac{1 + k/h_0}{1 - k/h_0} \right)$$

$$\frac{W\alpha\sqrt{AB}}{\pi h_0} = \frac{k}{h_0} \ln\left(\frac{1 + k/h_0}{1 - k/h_0}\right) - \alpha p_0 \quad (11)$$

and for the approximate expression:

$$\frac{W\alpha\sqrt{AB}}{\pi h_0} = \left(\frac{k}{h_0}\right)^2 \quad (12)$$

A comparison of the exact and approximate solutions for the load parameter versus central film thickness is shown in figure 3.

The limiting case for the exact expression as $h_0 \rightarrow k$ is:

$$\frac{W\alpha}{2\pi h_0 \sqrt{R_x R_y}} = \ln 4 = 1.386$$

This is the same value obtained by Christenson (ref. 2) for the case of a sphere and plate.

As in the case of the sphere on a flat plate this implies a limiting load for a given fluid and geometry which is given by the expression

$$W = \frac{2.772 \pi \sqrt{R_x R_y} h_0}{\alpha}$$

The pressure distribution may also be written in dimensionless form by use of the variables:

$$H = h/h_0$$

$$\dot{H} = \dot{h}/h_0$$

$$K = k/h_0$$

$$P = \alpha p$$

$$X = x/\sqrt{2R_x h_0^0}$$

$$Y = y/\sqrt{2R_y h_0^0}$$

then

$$H = 1 + X^2 + Y^2$$

and for the exact solution

$$P = -\ln \left(1 - K^2/H^2 \right)$$

and further approximate solution

$$P = K^2/H^2$$

Typical pressure distribution along the major axis for the exact and approximate solutions for $K = 0.7$ are shown in figure 4.

CONCLUDING REMARKS

The pressure distribution follows the same trend as for rigid spheres in normal approach and differs considerably from the Hertzian pressure distribution. As in the case of spheres, there is a limiting load which is reached when the central pressure approaches infinity. In reality, of course, significant elastic deformation will occur before this load is attained and the pressure distribution will be altered.

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APPENDIX - SYMBOLS

A	$1/2R_x, m^{-1}$ (in)
B	$1/2R_y, m^{-1}$ (in)
H	dimensionless film thickness = h/h_0
h	film thickness, m (in)
h_0	central film thickness, m (in)
K	k/h_0
k	$\sqrt{-3\mu_0 \dot{h}\alpha / (A+B)}, m$ (in)
P	dimensionless pressure
p	pressure, N/m^2 (psi)
p_0	central pressure, N/m^2 (psi)
R	radius of ball, m (in)
R_{Gx}	radius of curvature of groove in x-z plane, m (in)
R_{Gy}	radius of curvature of groove in y-z plane, m (in)
R_x	radius of curvature of paraboloid in x-z plane, m (in)
R_y	radius of curvature of paraboloid in y-z plane, m (in)
W	normal load, N (lb)
X	dimensionless coordinate = $x/\sqrt{2h_0R_x}$
Y	dimensionless coordinate = $y/\sqrt{2h_0R_y}$
x,y,z,	Cartesian coordinate, m (in)
α	pressure-viscosity exponent m^2/N (psi^{-1})
μ	absolute viscosity, $N\text{-sec}/m^2$ ($lb\text{-sec}/in^2$)
μ_0	ambient viscosity, $N\text{-sec}/m^2$ ($lb\text{-sec}/in^2$)

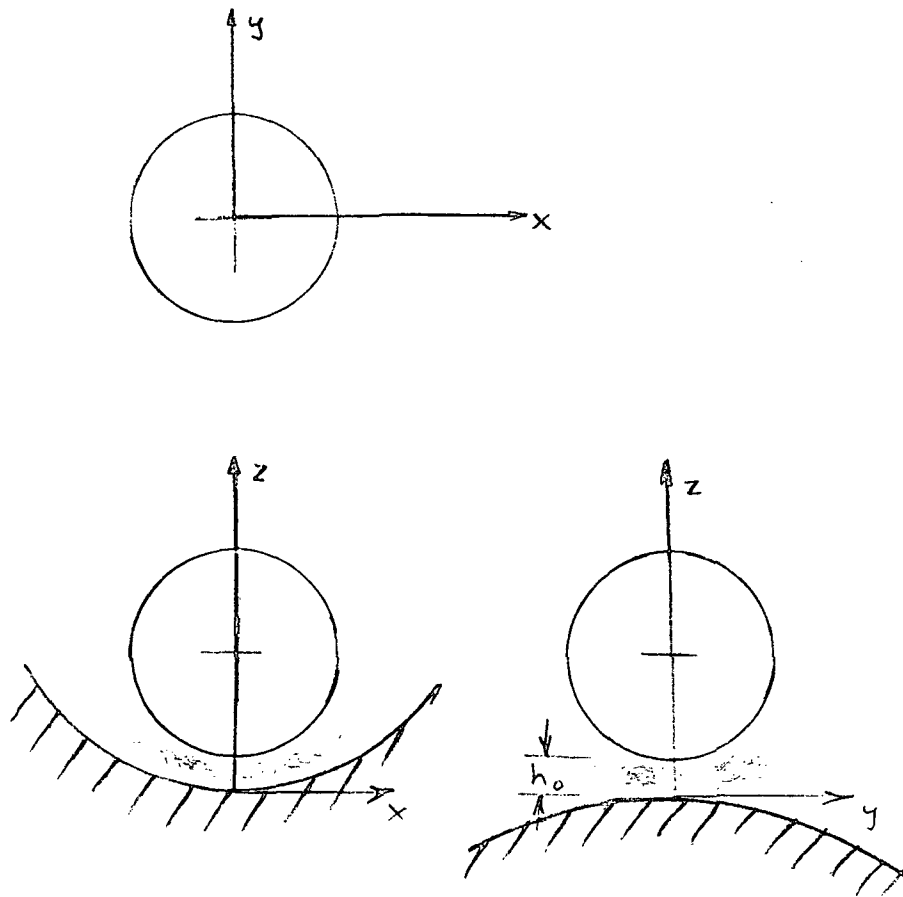


Fig. 1 BALL APPROACHING GROOVE SEPARATED BY LUBRICANT FILM

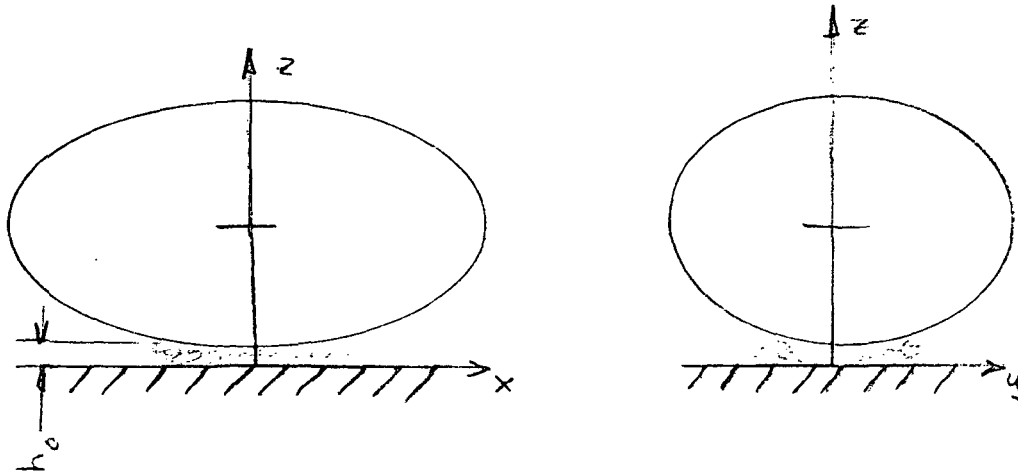


Fig. 2 EQUIVALENT SYSTEM - PARABOLOID APPROACHING
FLAT SURFACE

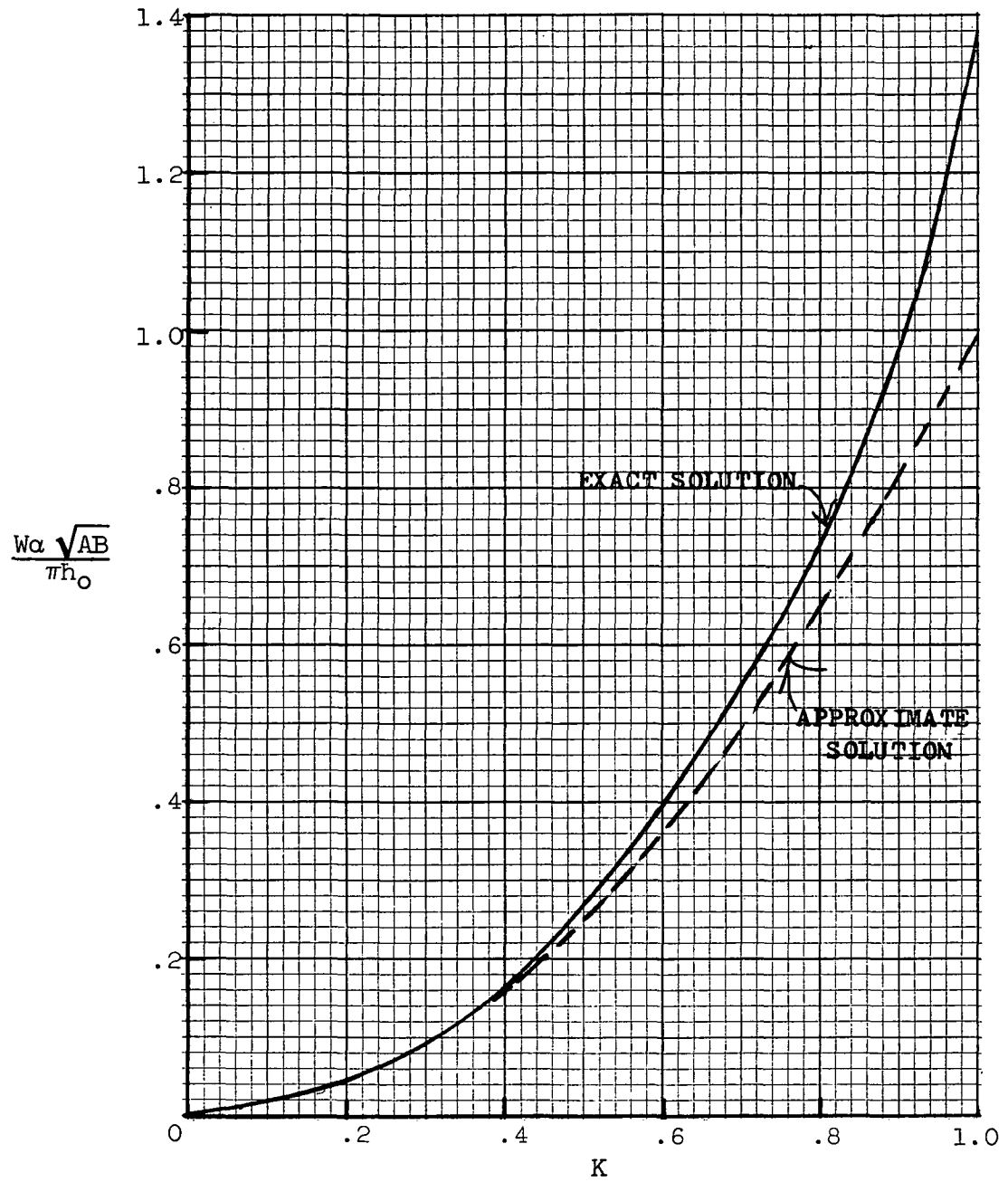


Fig. 3 COMPARISON OF APPROXIMATE AND EXACT SOLUTIONS FOR LOAD VARIABLE

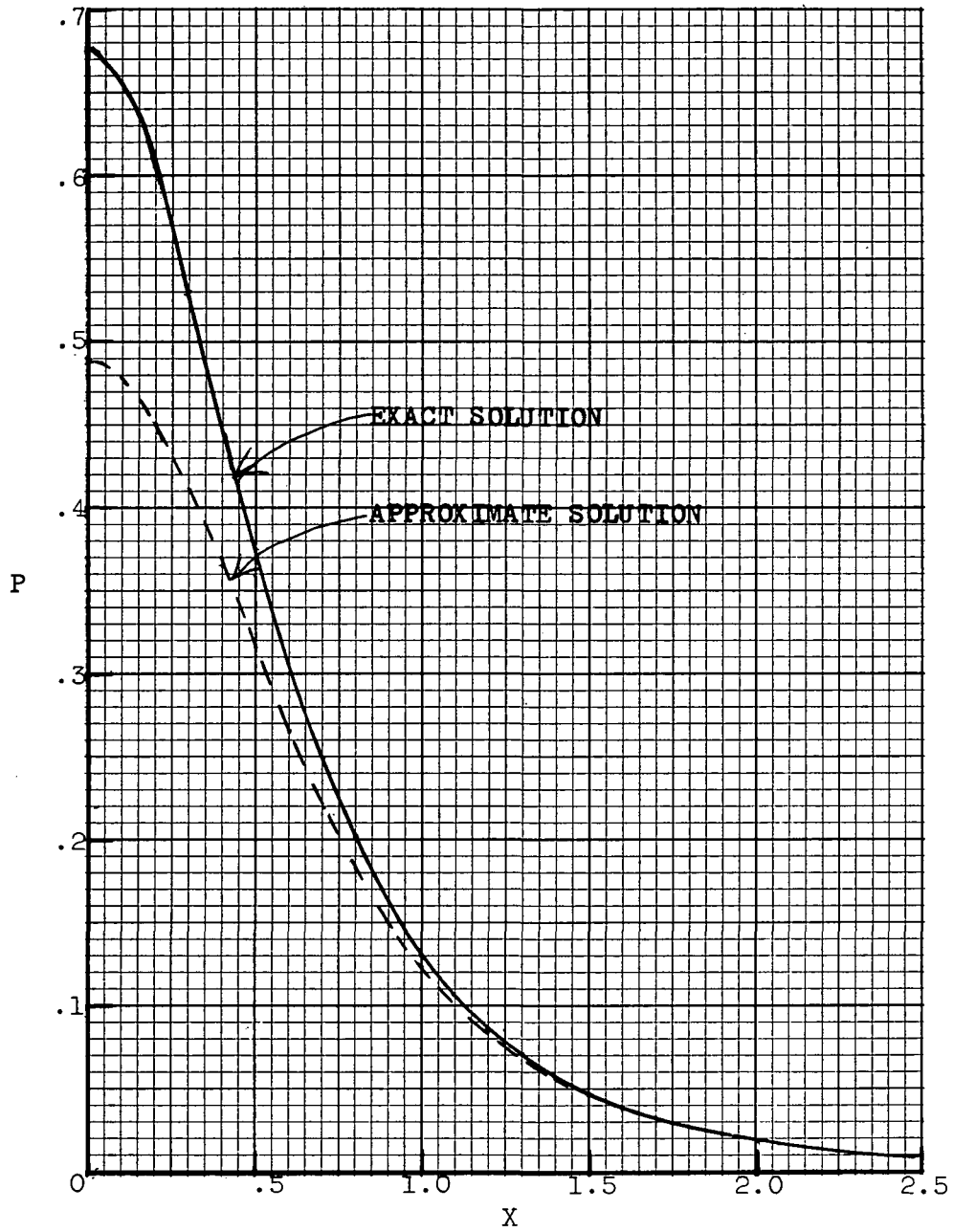


Fig. 4 EXACT AND APPROXIMATE PRESSURE DISTRIBUTIONS
FOR $k = 0.7$



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