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by
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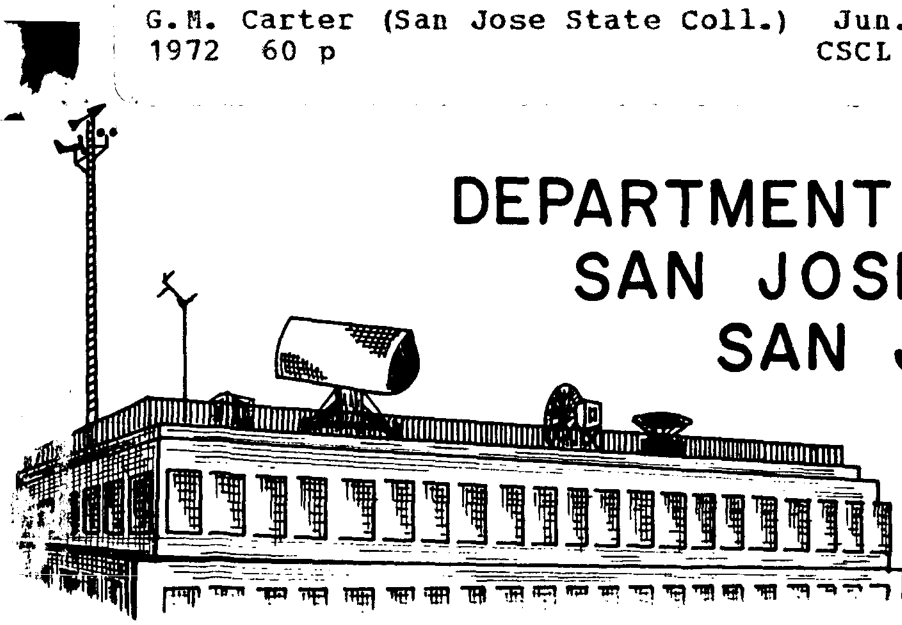

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CHAPTER I

INTRODUCTION

During recent years, a number of investigators (e.g., Borgman, 1960; Russo, 1966; Thompson, 1968) have devoted attention to methods of improving the utility and economic value of weather forecasts. As a corollary to such studies, procedures for making useful decisions in the face of uncertainty have become of increasing interest to the meteorologist, and many concepts of mathematical decision theory have been applied to this problem (Gleeson, 1960; Glahn, 1964; Thompson, 1966).

In general, the purpose of such studies has been to provide methods for optimizing the use of weather information, and to consider the economic or operational consequences of a certain decision tactic when applied to a specific operation. In this study, it is proposed to examine how differing decision models and operational characteristics affect the economic expenses (i.e., the costs of protection and losses suffered if no protective measures have been taken) associated with the use of predictive weather information.

CHAPTER II

METHOD OF APPROACH

Weather Information and Decision Making

Inherent in the nature of weather information is the characteristic of uncertainty. This arises from the lack of totally accurate knowledge of the initial state of the atmosphere, as well as from the inability to formulate and solve in complete detail the equations which describe how one state of the atmosphere develops from another. Even though better observing and forecasting methods will gradually alleviate these difficulties, it is likely that this factor of uncertainty will continue to be of importance throughout the forseeable future. Therefore, when the expenses resulting from the use of weather information are evaluated quantitatively, the nature and magnitude of meteorological uncertainty must be considered.

The uncertainty in weather forecasts may be expressed conveniently through the language of probability. While some theoretical differences exist concerning the interpretation of this language, this study will take the practical point of view that the probability of occurrence of an event is equivalent to its relative frequency of occurrence. In making use of this concept, it is proposed to consider first the simple dichotomous decision problem: Whether or not to protect against the occurrence of adverse weather.

Dichotomous Decisions--Theory

It has been shown by Thompson and Brier (1955) that optimum long-run decisions will be obtained for repetitive dichotomous operations, if protective measures are taken according to the following criterion:

$$P \begin{array}{c} > \\ = \\ < \end{array} C/L \quad \begin{array}{l} \text{Protect} \\ \text{Either course} \\ \text{Do not protect} \end{array} \qquad (1)$$

where P is the probability of occurrence of adverse weather,

C is the cost involved on each occasion that protective measures are taken, and

L is the loss suffered on each occasion that adverse weather occurs and no protection is provided.

This decision procedure is designed to minimize the long-run expense (i.e., cost of protection, plus loss suffered if no protection is provided and adverse weather occurs) incurred by the forecast user. Thus, the decision is specified once the value of the operational risk ratio, C/L, is ascertained. For a series of such decisions, the results may be represented as shown in Table I.

TABLE I. Generalized contingency table relating decisions for protective measures and observed frequencies (a, b, c, d) of the occurrence (W) and non-occurrence (No W) of adverse weather.

		Decision		
		Do not protect	Protect	Total
Observed weather	No W	a	b	a + b
	W	c	d	c + d
	Total	a + c	b + d	N = a+b+c+d

Assuming that Table I is based on equation (1), the total expense for protecting against adverse weather will be due to the cost of protection, C, multiplied by the frequency with which such protective measures are taken, plus the loss, L, multiplied by the frequency with which adverse weather occurs and protective measures are not provided. Thus, E_{opt}, the total weather expense for optimum long-run decisions, is given by

$$E_{opt} = C\,(b + d) + L\,c \quad . \qquad (2)$$

As a matter of convenience, E_{opt}, as well as all other expenses, are represented in this study in "non-dimensional" form (i.e., as the expense per unit forecast per unit of loss). Therefore,

$$E_{opt}' = (C/L)\ (b + d)/N + c/N \quad . \qquad (2a)$$

However, if operationally "perfect" forecasts were attainable, then the only expense would be due to the cost of protection, multiplied by the number of occurrences of adverse weather. The non-dimensional form of such expenses, E_{per}', is thus:

$$E_{per}' = (C/L)\ (c + d)/N \quad . \qquad (3)$$

For the most part, probability information is not made available to the decision maker (forecast user), but instead categorical predictions of the occurrence or non-occurrence of adverse weather are issued. Since these forecasts must be used by the entire community, they are based on a quasi "average" operational risk ratio.[1]

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[1]In this study, it is assumed that the categorically predicted weather event is that which is most likely to occur (i.e., the modal value of the probability distribution). For a dichotomous problem, this is the weather event which is predicted with a probability exceeding 0.5; thus inferentially, the "average" operational risk ratio (C/L) is assumed by the forecaster to have this value.

Thus the expense associated with the use of such predictions, E_{cat}', is:

$$E_{cat}' = (C/L)\ (b_a + d_a)/N + c_a/N \quad , \qquad (4)$$

where the subscript "a" denotes the fact that the categorical forecast is made at an "average" decision level.

A different expense is incurred by the operator with limited capital, who wishes to minimize the likelihood of experiencing an undesirable sequence of weather-caused losses. An appropriate strategy for this situation would, in general, involve taking protective measures more often than with the optimum long-run strategy. As noted by Gleeson (1960) and Thompson (1966), this alternate strategy may be carried out by the use of a second-order estimate of the meteorological uncertainty, in which a lower limit of confidence is applied to each decision. Such a strategy produces the effect of minimizing the likelihood of incurring maximum losses for short-run periods. In decision theory terminology, this is a form of "mini-max" strategy. The expense associated with this strategy, E_{min}', is:

$$E_{min}' = (C/L)\ (b_m + d_m) + c_m/N \quad , \qquad (5)$$

where the subscript "m" denotes the use of the "mini-max" tactic.

In similar manner, a procedure in which protective measures are taken less often than is the case for the optimum long-run strategy, may be specified. Here upper confidence limits are determined, and protective measures are implemented only when the forecast probability is greater than, or equal to, the upper limit at the appropriate operational risk ratio. Such a procedure maximizes the likelihood of incurring minimum economic expenses, and is called in this study a "maxi-min" strategy. The expense associated with this strategy is computed as in equation (5), but with the respective frequencies represented by the occurrence of weather events prescribed by decisions made at the upper confidence limits.

Before examining a more complex decision problem, it should be noted that the equations (2)-(5), which express the expenses associated with the various dichotomous decision strategies, involve only the use of the forecast verification frequencies and the operational risk ratios. Therefore, these equations are quite general and can be applied to weather dependent operations in all aspects of the economy, where simple dichotomous decisions are involved.

Multiple Category Decisions--Theory

Of greater complexity than the dichotomous situation are those problems which involve decisions concerning whether or not to take greater protective measures as the severity of the predicted adverse weather increases. Such problems require that the user provide a "utility matrix", in which the economic expenses (i.e., costs and losses representing a decrease in profits) associated with the decisions and resulting weather events, are specified. Using a notation similar to that of Gleeson (1960), the general structure of such a matrix is shown in Table II, where the expenses, a_{ij}, are expressed, as before, in "non-dimensional" units.

TABLE II. Economic expense, a_{ij}, for various decisions, D_i, and observed weather events, X_j.

		Weather Events		
		X_1	X_j	X_k
	D_1	a_{11}	a_{1j}	a_{1k}
	.			
	.			
Decisions	D_i	a_{i1}	a_{ij}	a_{ik}
	.			
	.			
	D_m	a_{m1}	a_{mj}	a_{mk}

Various decision strategies may be evaluated by applying the methods of mathematical decision theory to the values of a_{ij} in Table II. For each of these strategies, the theoretical expense corresponding to each decision, D_i, is computed. Then, if desired, the decision which corresponds to the lowest expense may be selected. (These theoretical expenses are equivalent to the actual economic expenses only if the assumed probability distributions are realized.)

As an illustration of this concept, a Bayes strategy (Chernoff and Moses, 1959), which, in decision theory terminology, is equivalent to an optimum long-run strategy, may be applied to the expense values in Table II. Here, the Bayes expense, E_i', associated with each decision is defined by the following equation:

$$E_i' = \sum_{j=1}^{k} a_{ij}\, p_j \quad , \tag{6}$$

where p_j is the forecast probability of occurrence of observed weather event, X_j. A decision which is based on a minimum value of E_i', assures the forecast user that if over a sufficient time span the predicted probability distribution is realized, then his actual economic expense will be a minimum.

In similar fashion, the decisions corresponding to perfect forecasts can be determined. However, the probability, p_j, takes on a value of unity for observed events and zero for all others.

In order to provide a rational basis for making a "categorical prediction" (see page 5), the weather event which is most likely to occur (i.e., the modal value of the probability distribution) is again considered as being the categorically predicted event.

A decision resulting from the application of a Bayesian "mini-max" tactic, can be determined by a procedure proposed by Gleeson (1960).[2] Upper and lower confidence limits, p_j'' and p_j', are evaluated for the forecast probabilities, p_j, associated with each weather event, X_j, subject to the restriction,

$$1 \geq (p_j'' - p_j') \geq 0 \quad . \tag{7}$$

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[2]Since Gleeson was concerned with the economic expectation, instead of the economic expense, the notation used in this study for the multi-category "mini-max" and "maxi-min" strategies is the reverse of that which appears in Gleeson's article.

In order to implement the "mini-max" tactic, the maximum expense associated with each decision, $E_i'^{(max)}$, may be determined by using the following equation:

$$E_i'^{(max)} = \sum_{j=1}^{k} a_{ij} \, p_{ij}^{(max)} \quad , \tag{8}$$

where $p_{ij}^{(max)}$ is a probability which is derived for each weather event. This derived probability is a maximum or minimum when a_{ij} is maximum or minimum, respectively, such that,

$$p_j'' \geqq p_{ij}^{(max)} \geqq p_j' \quad . \tag{9}$$

The "mini-max" decision is the one for which $E_i'^{(max)}$ is minimum.

In analogous fashion, a "maxi-min" strategy (Thompson, 1966) may be considered. Selecting the appropriate decision for this strategy involves the evaluation of the minimum economic expense, $E_i'^{(min)}$. Here,

$$E_i'^{(min)} = \sum_{j=1}^{k'} a_{ij} \, p_{ij}^{(min)} \quad , \tag{10}$$

where $p_{ij}^{(min)}$ is a maximum or minimum when a_{ij} is a minimum or maximum, respectively. The decision for which $E_i'^{(min)}$ is lowest is selected as the "maxi-min" decision.

The final expense which will be considered in connection with the multi-category decision problem, is produced by either a quasi "mini-max" (play it safe) tactic or a quasi "maxi-min" (gambling) tactic in those situations when the user is provided only with categorical predictions. Here, several alternatives are available for selecting appropriate decisions. A "play it safe" procedure, which would seem to be adequate for the purposes of this analysis, is one involving the implementation of protective measures as if adverse weather of one category more severe than is predicted by the categorical prediction will occur. In similar fashion, a suitable "gambling" tactic would involve the taking of protective measures as if adverse weather of one category less severe than is predicted by the categorical prediction will occur. Thus, once the categorical forecast is obtained, the appropriate decisions may be specified.[3]

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[3]The "play it safe" and "gambling" strategies were not considered for the dichotomous decision problem, since comparable procedures would involve the implementation of trivial courses of action (i.e., either protecting against adverse weather all of the time or not protecting at all).

Of interest is the mean economic expense, E_s', which is associated with each of the multi-category strategies for which the decision procedures have been outlined. In order to evaluate this expense, it is necessary to compute,

$$E_s' = (1/N) \sum_{k=1}^{N} (a_{dw})_k \quad , \tag{11}$$

where "N" represents the total number of forecasts, "a_{dw}" is the "non-dimensional" expense (Table II, page 8), and the subscripts are defined as follows: "s" denotes the particular decision strategy, "d" refers to the decision which was implemented, and "w" indicates the weather event which actually occurred.

In summary, it has been shown that the methods for determining the economic expenses corresponding to multiple option and simple dichotomous decision problems are different. In particular, while the dichotomous situation involves the use of forecast verification data and the users' operational risk ratios, the multi-category strategies require that the expense associated with each individual forecast be determined in relation to specific operational requirements.

CHAPTER III

ECONOMIC EXPENSES--SPECIFIC EXAMPLES

Dichotomous Decisions

It is of interest to evaluate the expenses associated with the various dichotomous decision strategies. In order to accomplish this task, U.S. National Weather Service forecast verification data were collected and analyzed. Data included: (1) short-period (3-, 5-, and 7-hour) predictions of aviation terminal weather for 8 major airports across the United States (Allen, 1969), (2) medium-range (12-, 24-, and 36-hour) precipitation forecasts at 22 cities throughout the nation (Russo, et al., 1967), and (3) extended-period (5-, 30-, and 90-day) temperature and precipitation predictions for the entire United States (Namias, 1964).

Based on the dichotomous meteorologic-economic model which has been presented (pages 3-7), the raw data were processed with the aid of an electronic computer. Examples of the results which were obtained are presented in Figs. 1 and 2. In each case, the economic expense is plotted against the operational risk ratio.[4] The

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[4]The expenses corresponding to the implementation of the dichotomous "maxi-min" strategy are not presented

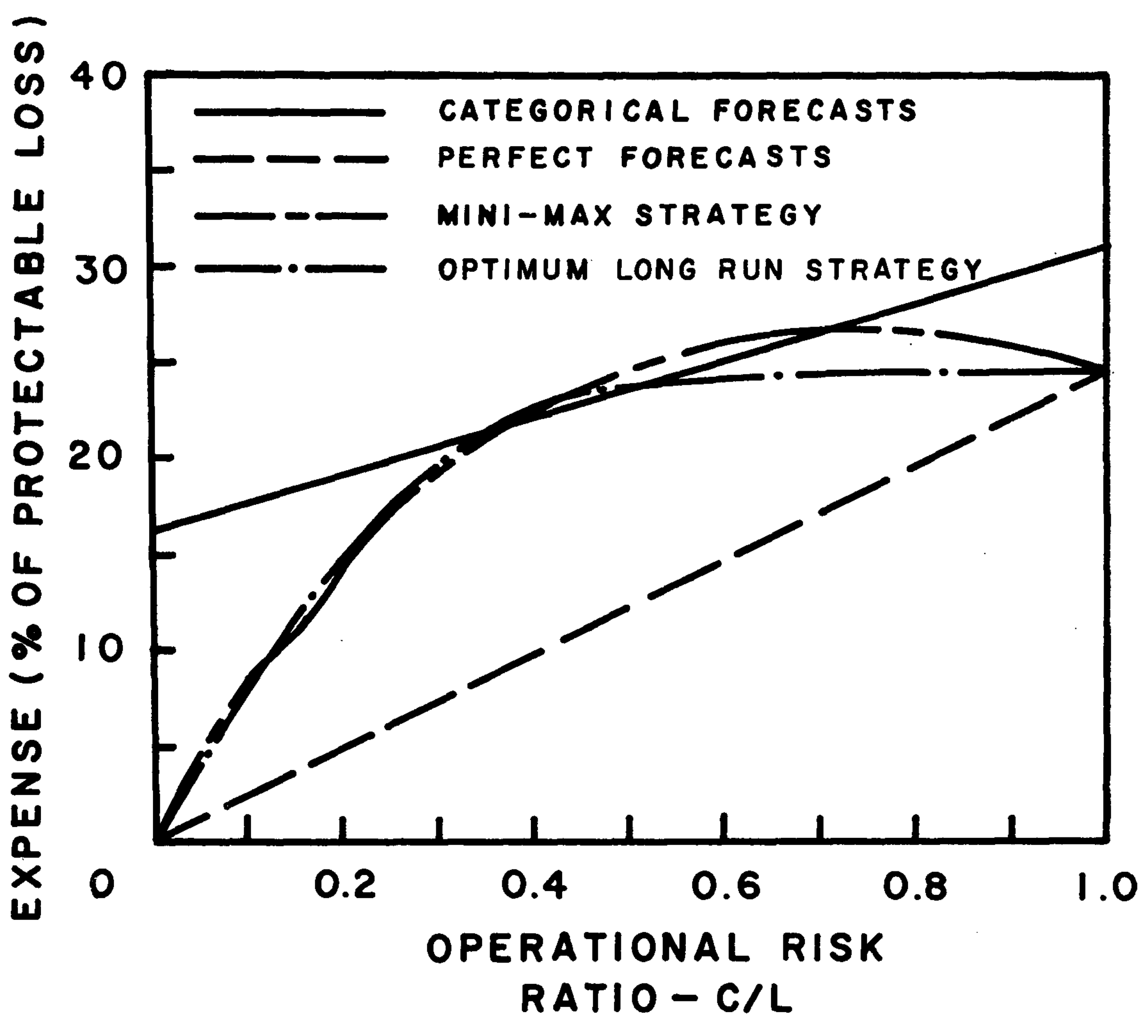

FIG. 1. Expenses associated with differing dichotomous decision strategies for 24-hour precipitation forecasts (Boston, Massachusetts, April-September, 1967, inclusive).

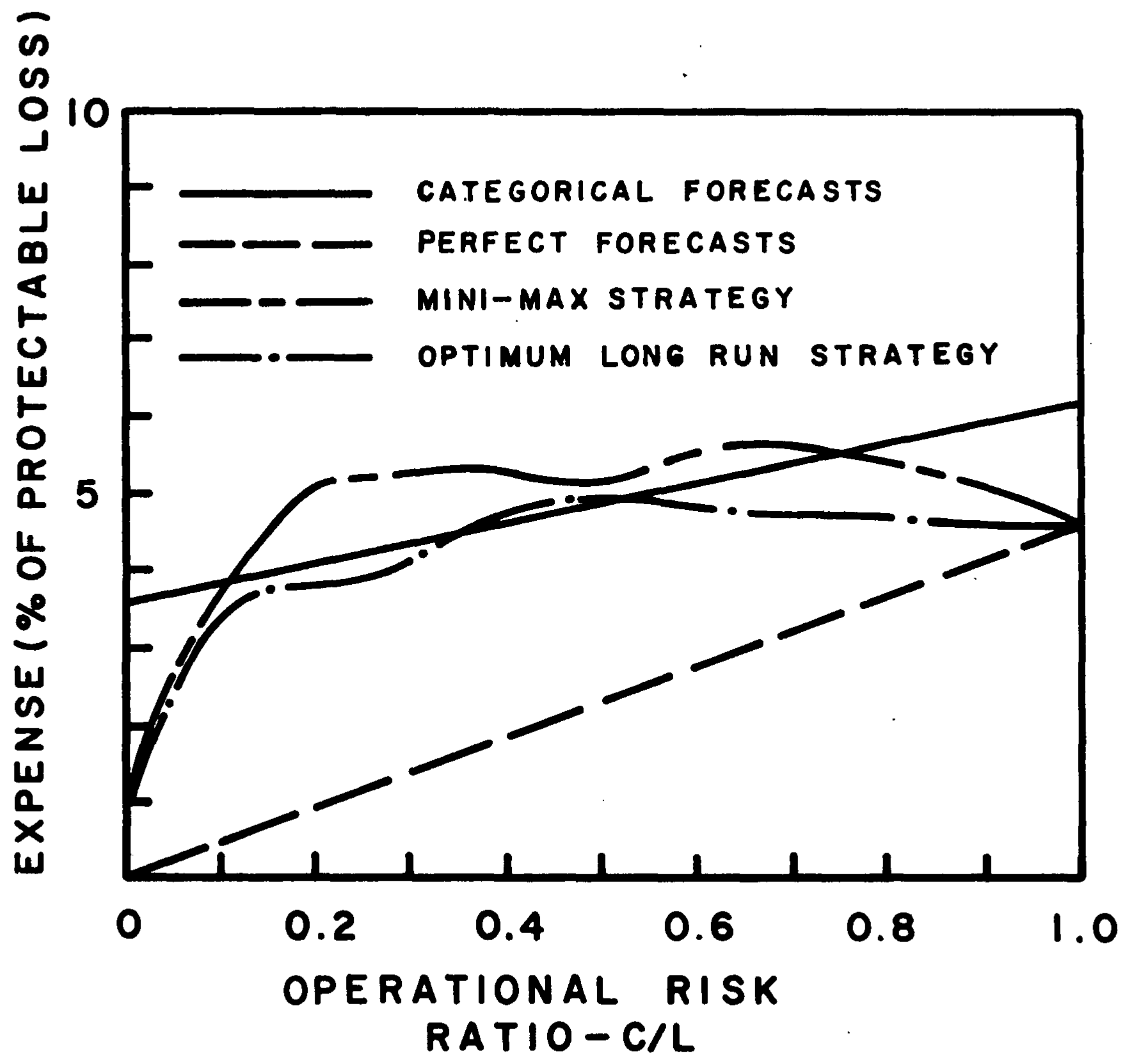

FIG. 2. Expenses associated with differing dichotomous decision strategies for 5-hour ceiling forecasts (San Francisco, California, September, 1965-March, 1966, inclusive).

confidence limits, necessary for the evaluation of the "mini-max" strategy expense (page 6), were obtained by a method outlined in Appendix A.

The expenses which result from applying the various dichotomous decision strategies to a series of precipitation forecasts at Boston, Massachusetts, are depicted in Fig. 1. The expense associated with the use of perfect forecasts is included for reference. The optimum long-run strategy appears to produce the least expense for the majority of the operational risk ratios (C/L). It is also apparent that the expenses produced by categorical and perfect forecasts, which both do not require the use of a decision strategy, are linear functions of the ratio C/L. This is because the frequencies (a, b, c, d, Table I, page 4) are constant for all values of C/L [see equations (3) and (4)] when categorical or perfect forecasts are used in this manner. However, the equations [(2a) and (5)] which correspond to the other strategies (optimum long-run and "mini-max") make use of differing frequencies with each

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since: (1) this extremely hazardous strategy is seldom used in practice, (2) excessive amounts of computer time are involved in the evaluation of the expenses, and (3) several examples of the expenses associated with this strategy are considered in regard to the more interesting multiple option decision problem.

value of C/L, and the associated expenses are non-linear. In addition, the observation that the optimum and "mini-max" strategies produce the same expense as the perfect forecasts at C/L = 0 is fortuitous (see Fig. 2). In practice, of course, no forecasts would be needed at the C/L limits, since at C/L = 0 there is no cost of protection and at C/L = 1 the cost of protection is the same as the loss.

Fig. 2 depicts an analysis of short-range ceiling forecasts at San Francisco, California. In general, this example exhibits characteristics which are similar to those observed in Fig. 1. However, the magnitude of all expenses are much lower in Fig. 2, since these forecasts cover a shorter time span (5 hours) than those used for Fig. 1 (24 hours).

Multiple Category Decisions

As a means of illustrating how the multi-category decision strategies are used in a practical manner, two utility matrices, Tables III and V, were developed for weather dependent operations. In each matrix the numerical value of the economic expense corresponds to a given forecast and observed weather event. Thus, once the forecast is specified, a decision can be determined by some appropriate method. The economic expenses in each matrix are based on an arbitrary scale ranging from zero to unity. The categories of weather events had previously been selected by the U.S. National Weather Service, which also provided the forecast verification data.

TABLE III. Relative economic expense due to the prediction and/or occurrence of indicated categories of ceiling and visibility.

Forecast Category	Observed Category 1	2	3	4	5
1	.70	.60	.65	.70	.75
2	.90	.40	.30	.25	.10
3	.95	.40	.30	.25	.05
4	.95	.45	.35	.20	.05
5	1.00	.50	.40	.30	0

Explanation of categories:

Category	Ceiling (feet)	Visibility (miles)
1	≤ 100	$\leq 3/8$
2	200-400	1/2-1 3/8
3	500-900	1 1/2-2 1/2
4	1000-2900	3-4
5	≥ 3000	≥ 5

A utility matrix for a generalized aviation landing and take-off operation is presented in Table III.[5] In this matrix the values of the economic expenses represent "mean" estimates for a number of varied operations. Therefore, some adjustment in these values may be expected for differing individual circumstances. A list of the operational basis for the various combinations of forecast and observed weather event for this matrix is contained in Appendix B.

By using short-period (3-, 5-, and 7-hour) forecasts of ceiling and visibility in conjunction with the utility matrix in Table 3, and implementing the methodology which has been described (pages 8-13), an analysis of the multiple category decision strategies was conducted. An example of the results is presented in Table IV.

An examination of Table IV reveals the same general expense pattern as was observed for the dichotomous decision strategies. The expenses associated with the use of perfect forecasts are lowest, while the other strategies produce expenses of similar magnitudes; except for the relatively

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[5]The matrix was developed by Professor Gerald Shreve of the Aeronautics Department, California State University, San Jose, in consultation with a number of private, business, and commercial aircraft operators.

TABLE IV. Multi-category decision expenses (per cent of protectable losses) associated with the use of differing strategies for 5-hour ceiling and visibility forecasts (Seattle, Washington, September, 1965-March, 1966, inclusive).

	Strategies						
	Perfect Forecasts	Optimum Long-run	Mini-max	Maxi-min	Categorical Forecasts	Play it Safe	Gambling
Ceiling	7.8	10.3	11.2	10.6	10.9	13.2	11.2
Visibility	5.8	7.7	8.4	7.7	8.1	11.7	7.8

large values associated with the "play it safe" strategy. Of additional interest, is the fact that the expenses produced by the ceiling forecasts are of greater magnitude than those associated with visibility predictions. This arose because low ceilings occurred more often than adverse visibilities during this time period.

In analogous fashion, Table V depicts a utility matrix which was used in connection with 24-hour quantitative precipitation forecasts.[6] An examination of Table V reveals that for the construction operation to which this matrix applies, the losses accumulate rapidly as the rainfall rate increases.

The expenses which result when a precipitation forecasting system (Thompson, 1950) is applied to this matrix are listed in Table VI. Here the economic expenses are, in general, of small magnitude (i.e., 1 to 5 per cent of the mean protectable losses). This is to be expected, since the majority of the expense values in the utility matrix (Table V) are relatively small. Thus, for

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[6]The matrix was developed by Dr. R. Robert Rapp, Certified Consulting Meteorologist, in regard to a construction operation in the Los Angeles area.

the construction operation under consideration, this collection of precipitation forecasts appears to be quite useful, since the undesirable situation where little or no precipitation is forecast and heavy precipitation obtains, happens infrequently.

TABLE V. Relative economic expense due to the prediction and/or occurrence of indicated categories of precipitation.

Forecast Category	Observed Category 1	2	3	4	5
1	0	.024	.119	.476	1
2	.008	.008	.103	.460	.974
3	.040	.040	.040	.396	.920
4	.159	.159	.159	.159	.682
5	.333	.333	.333	.333	.333

Explanation of categories:

Category	Precipitation (inches)
1	<.01
2	.01-.15
3	.16-.49
4	.50-1.50
5	>1.50

TABLE VI. Multi-category decision expenses (per cent of protectable losses) associated with the use of differing strategies for 24-hour quantitative precipitation forecasts (Los Angeles, California, October-March, 1961-1970, inclusive).

Strategies						
Perfect Forecasts	Optimum Long-run	Mini-max	Maxi-min	Categorical Forecasts	Play it Safe	Gambling
1.3	3.0	3.0	3.2	3.3	4.9	3.5

CHAPTER IV

DISCUSSION OF RESULTS

Dichotomous Decisions

Because of the generality of the dichotomous decision strategies, an analysis of the economic expenses for the entire economy may be carried out. If it is assumed that all operations are equally likely and equally important, simple average values of the economic expense may be computed, thus providing a summary of the results. Such mean values are shown in this chapter.

Of interest, is the manner in which the expenses associated with the various strategies fluctuate in response to changes in weather elements and length of forecast period. Since the prediction of low visibility is of increasing importance to modern aircraft operations, the results of the expense computations for 5-hour forecasts of visibility, where the "adverse" weather is visibility 2 1/2 miles or lower, are presented in Fig. 3. Complete data for 5-hour ceiling and visibility forecasts are listed in Table XII (Appendix C, page 56). The curves in Fig. 3 show that average expenses associated with the differing strategies at the 8 major air terminals under consideration,

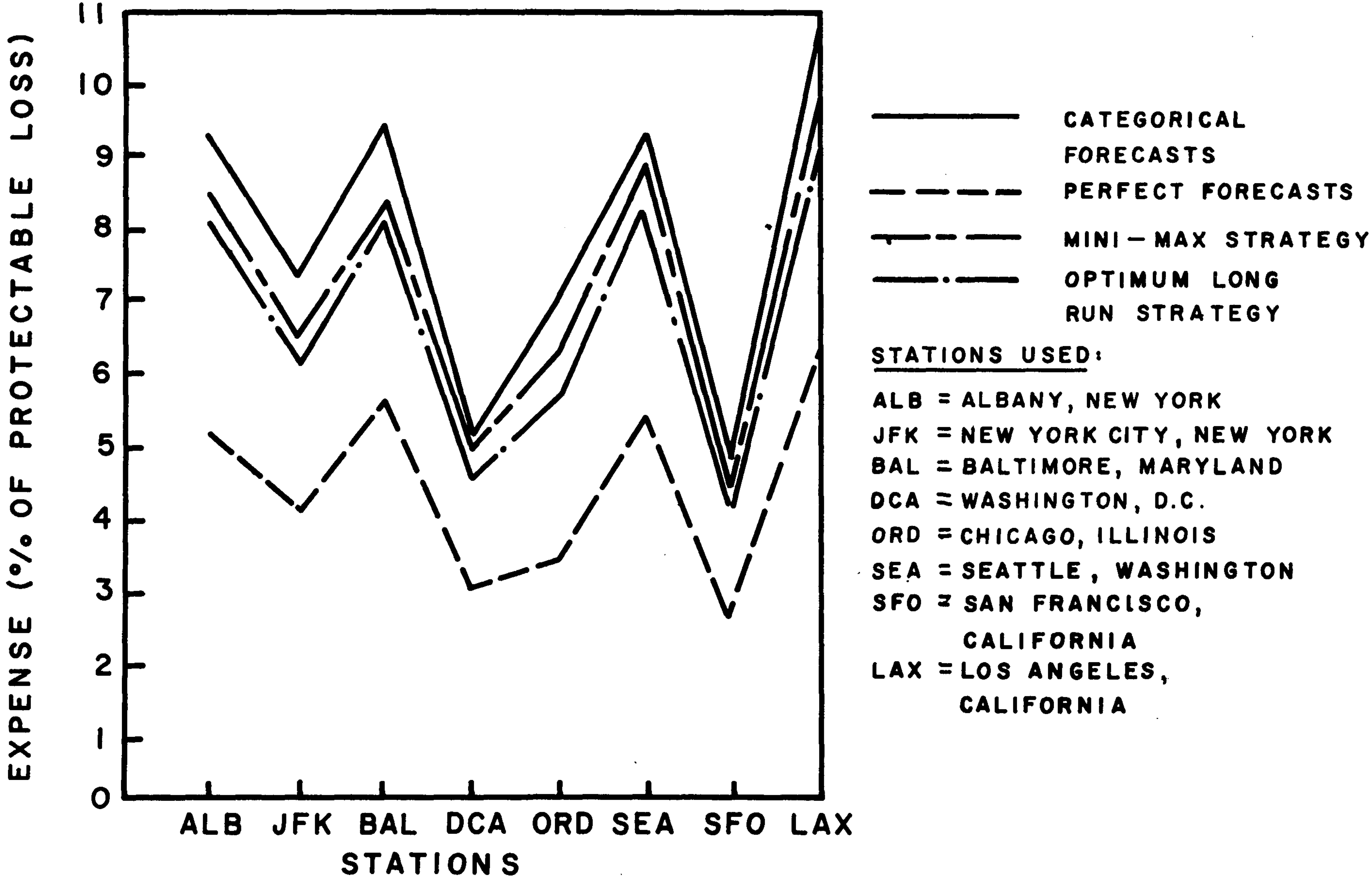

FIG. 3. Expenses associated with differing dichotomous decision strategies for 5-hour visibility forecasts (September, 1965-March, 1966, inclusive)

range from 3 to 11 per cent of the protectable losses. Thus, in percentage terms, these expenses are relatively small, which is presumably due to the short-range nature of the forecasts.

Regardless of the decision strategy, it may be noted that the expenses in Fig. 3 at Los Angeles International Airport are the largest of all airports shown, while those at San Francisco International Airport are the smallest. In part at least, this is due to the fact that Los Angeles had over twice as many occurrences of low visibility as did San Francisco. Possible causes for this difference in low visibility frequency are: (1) airport location (i.e., proximity to the ocean), (2) trajectory of the prevailing winds, (3) local sources of pollution, and/or (4) the advection of inland nocturnal radiation fogs.

Medium-range predictions were also examined and Figs. 4-9 depict the manner in which the economic expenses associated with 24-hour precipitation forecasts are influenced by seasonal weather variations.

Fig. 4 illustrates the "winter" season pattern of precipitation frequency for the United States. It is apparent that the greatest frequencies occur in the Pacific Northwest, while a secondary maximum is present near the Great Lakes. In contrast, an area of minimum precipitation frequency is located in the Southwest.

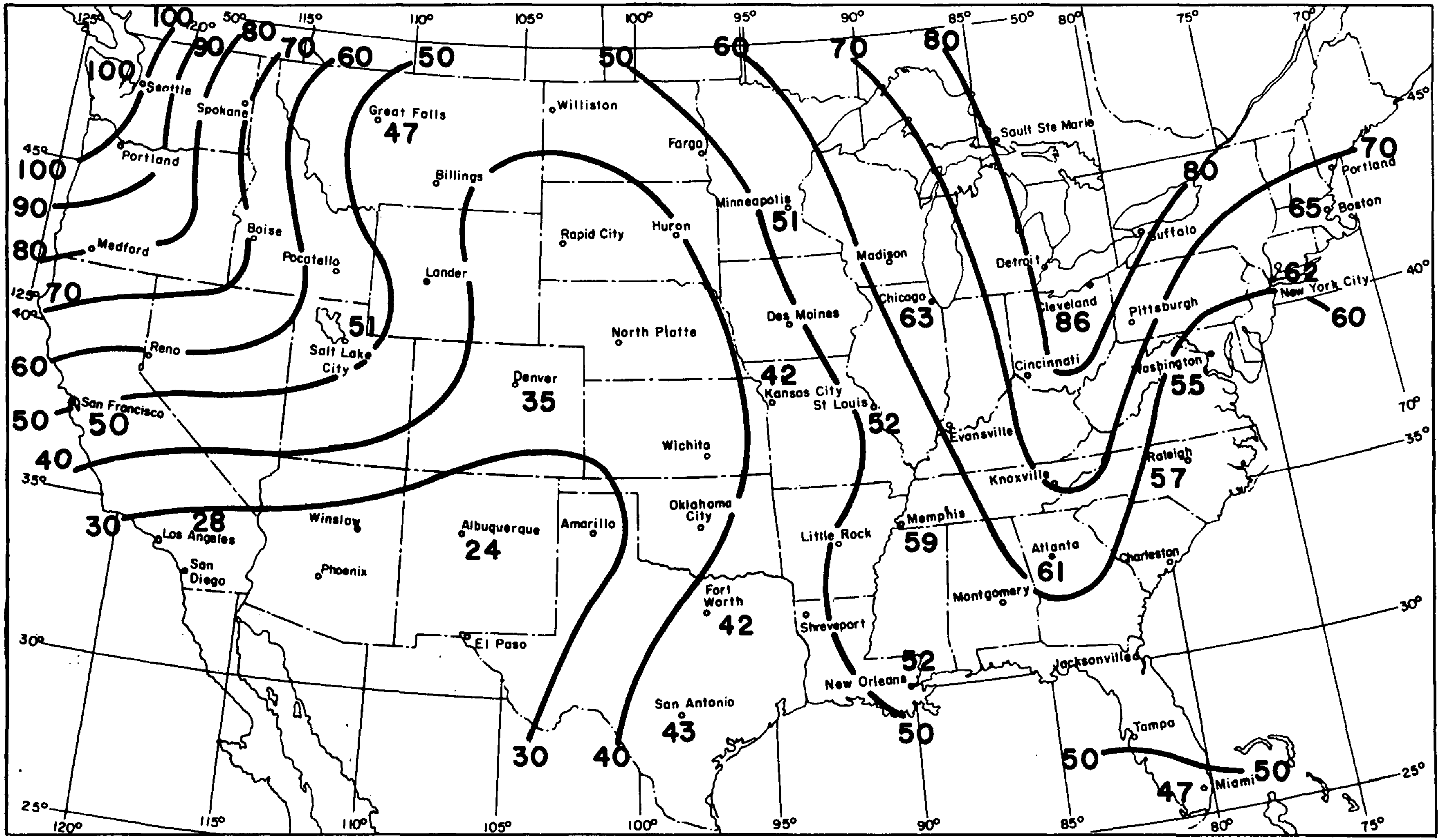

FIG. 4. Number of days with measurable precipitation ($\geq$ 0.01 inch) for "winter" season (October, 1967-March, 1968, inclusive).

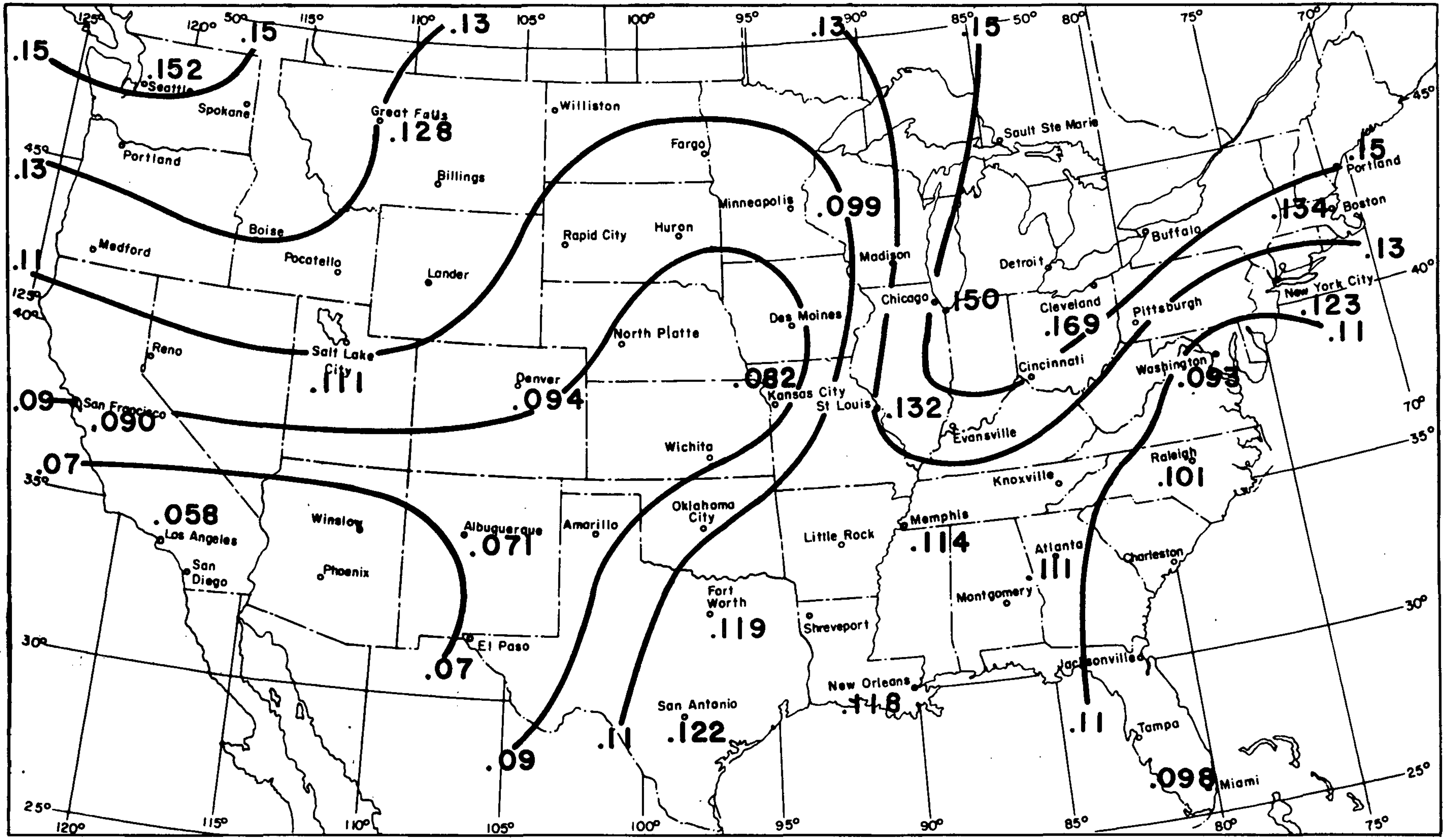

FIG. 5. Mean probability scores for "winter" season (October, 1967-March, 1968, inclusive) for 24-hour precipitation forecasts.

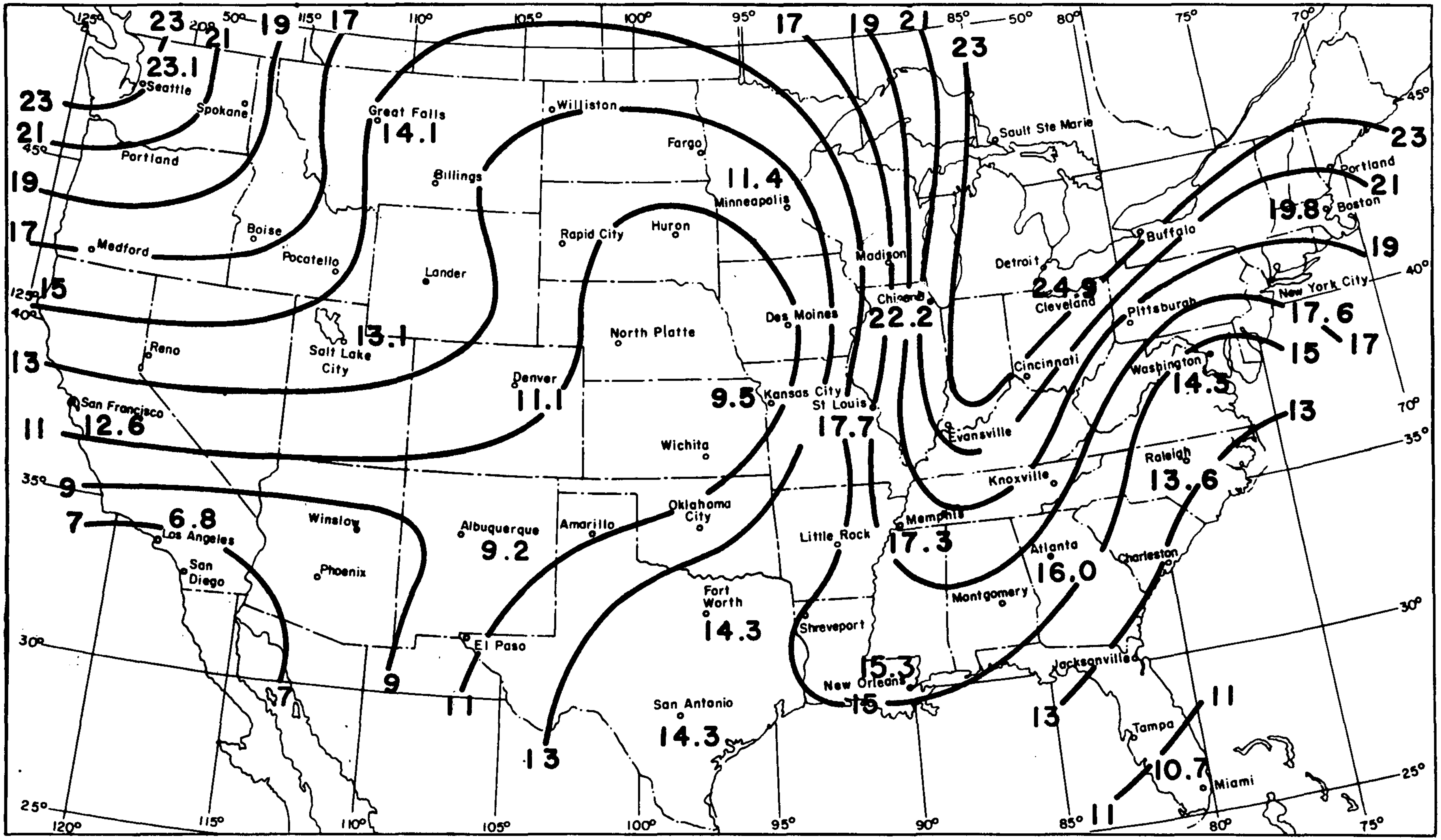

FIG. 6. Dichotomous decisions--optimum long-run strategy mean expense values for "winter" season (October, 1967-March, 1968, inclusive) using 24-hour precipitation forecasts. Units are per cent of protectable loss.

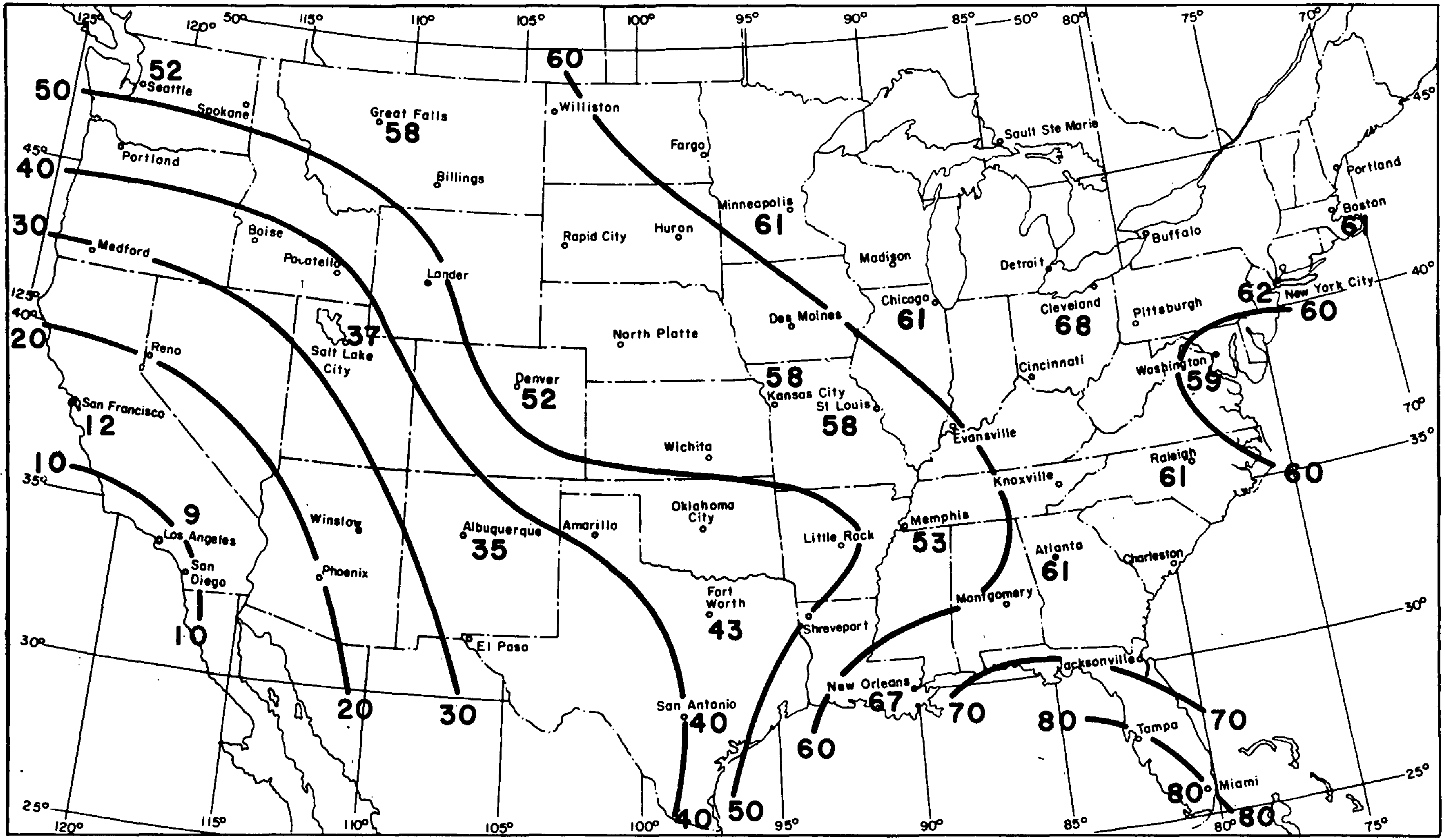

FIG. 7. Mean number of days with measurable precipitation ($\geq$ 0.01 inch) for "summer" season (April-September, inclusive, 1967 and 1968).

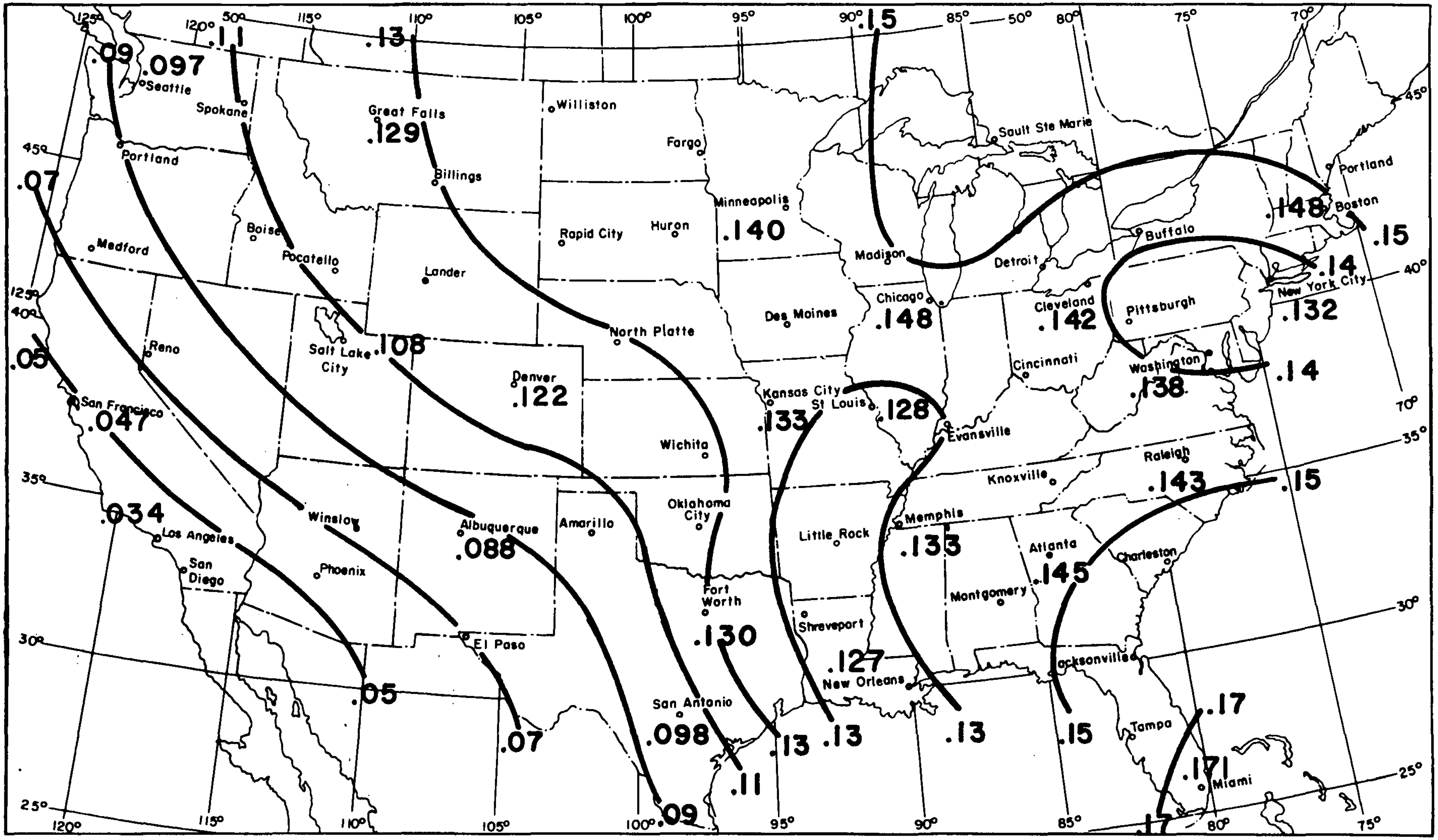

FIG. 8. Mean probability scores for "summer" season (April-September, inclusive, 1967 and 1968) for 24-hour precipitation forecasts.

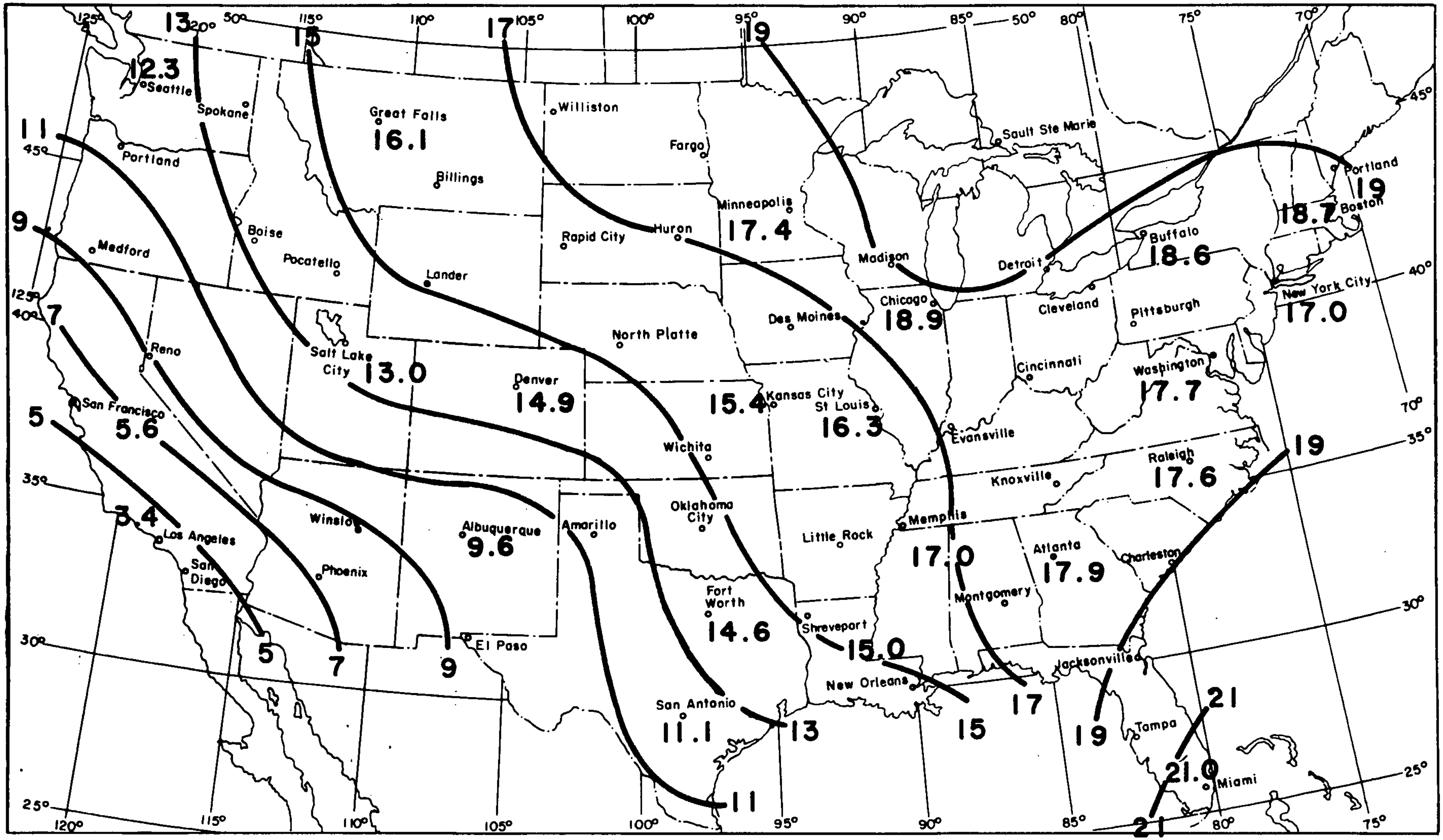

FIG. 9. Dichotomous decisions--optimum long-run strategy mean expense values for "summer" season (April-September, inclusive, 1967 and 1968) using 24-hour precipitation forecasts. Units are per cent of protectable loss.

In similar fashion, mean probability scores (Brier, 1950), which gives an indication of the general "accuracy" of the forecasts, are shown in Fig. 5.[7] A comparison of Fig. 5 with Fig. 4, indicates that the forecasts are less accurate in areas of high precipitation frequency.

Fig. 6 reveals the pattern of the mean expenses which are associated with the use of the optimum long-run decision strategy. Here, the isopleths of expense produce patterns which are similar to those presented in Figs. 4 and 5. This is reasonable, since the expense is dependent upon both the occurrence of adverse weather and the accuracy of the forecasts. In addition, the mean expenses for the "winter" season corresponding to the implementation of the other dichotomous decision strategies (i.e., perfect forecasts, "mini-max", categorical forecasts) exhibit the same general characteristics as the optimum strategy (Fig. 6).

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[7]The accuracy in this case is a function of both the "reliability" of the probability estimates (i.e., the relationship between each probability statement and the subsequent relative frequency of occurrence), and their "resolution" (i.e., the closeness to zero or unity of each probability statement).

The "summer" season pattern of precipitation frequency, Fig. 7, demonstrates the manner in which seasonal weather variations influence the economic expense values. The primary maximum is located in the southeastern United States, where random showers and thunderstorms are a frequent phenomenon. Consequently, the mean probability scores, shown in Fig. 8, reach the largest values (i.e., the forecasts are least "accurate") in this region.

An examination of the "summer" season mean expenses produced by the optimum long-run strategy, Fig. 9, indicates the similarity between the pattern of expense and the precipitation frequency. As is the case with the data for the "winter" season, further analysis shows that the other dichotomous decision strategies produce expense patterns which are similar to those associated with the optimum strategy.

Confirmation of the supposition that the average expenses are closely related to both the precipitation frequency and accuracy of the forecasts, was obtained by computing the respective linear correlation coefficients. The results of this analysis are listed in Table VII. All of the correlation coefficients are quite large and all are significant at the 1 per cent level, according to an analysis of variance which was conducted.

TABLE VII. Linear correlation coefficients between dichotomous decision strategy expenses, and the frequency of precipitation or mean probability scores, for 24-hour precipitation forecasts.

	Frequency of Precipitation		Mean Probability Scores	
	Winter	Summer	Winter	Summer
Perfect Forecasts	1.000	1.000	.902	.977
Optimum Long-run	.995	.998	.941	.989
Mini-max	.991	.997	.948	.989
Categorical Forecasts	.992	.998	.946	.984

The final application of the various dichotomous decision strategies involved the use of extended-range categorical predictions of temperature and precipitation for the United States as a whole. Since a priori (conditional) probability statements were not available, the probability information for these forecasts had to be obtained from a posteriori (verification) data. As a consequence, these latter probabilities may be less "sharp" in the statistical sense, than the individual conditional probability statements associated with the other forecasts which have been considered.

For these extended predictions, Fig. 10 illustrates the variation in the expense with increase in the length of the forecast period. It is evident that the expenses associated with the various strategies become larger as the forecast period increases. In particular, the expenses associated with the seasonal (90-day) predictions range from approximately 15 to 35 per cent of the mean protectable losses. In addition, Fig. 10 shows that the expenses for the differing strategies demonstrate large deviations in magnitude for the extended-range predictions.

In connection with the categorical statement of seasonal predictions, Namias (1964) has shown that forecasts of this nature possess marginal positive skill

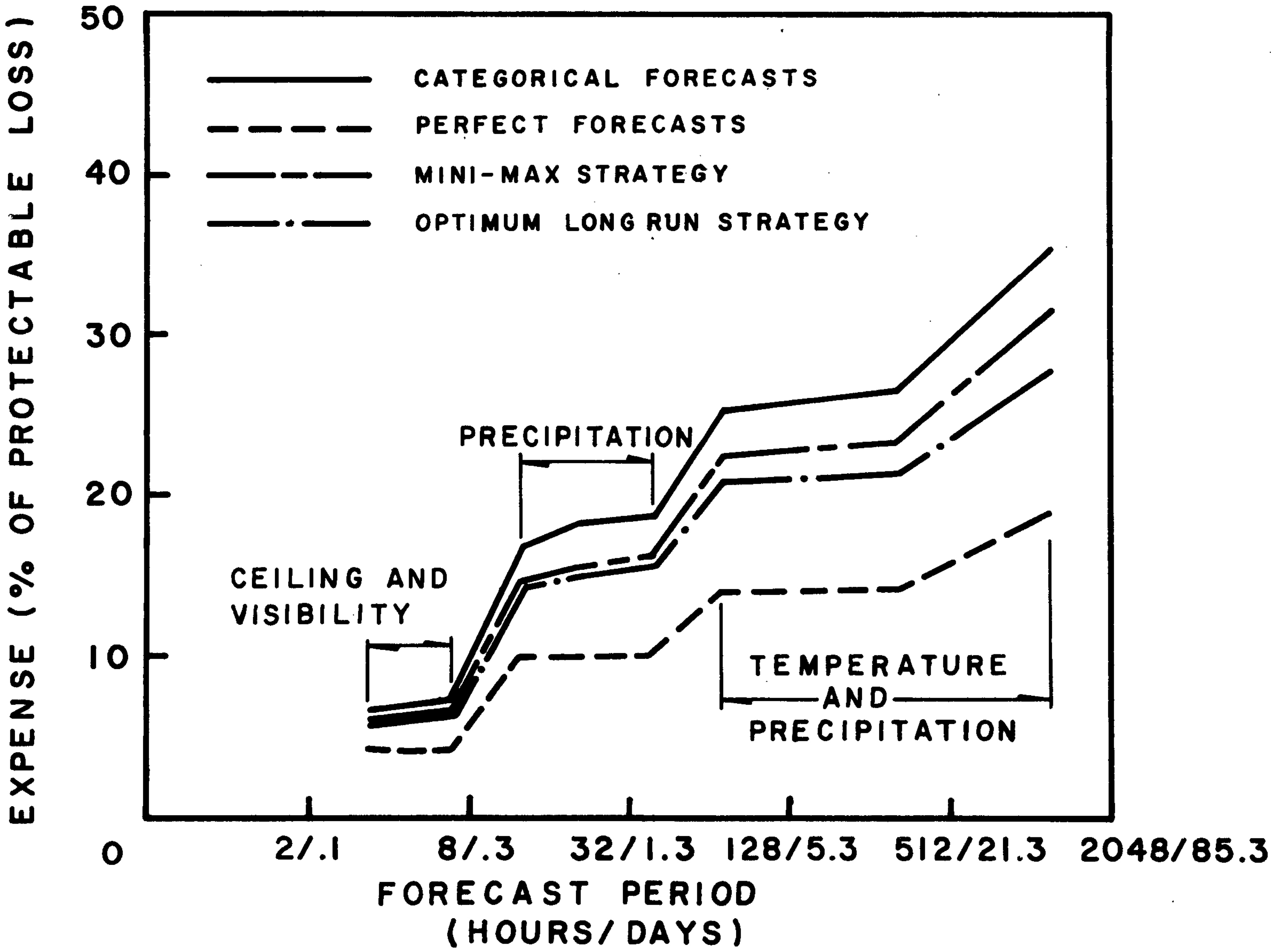

FIG. 10. Expenses associated with various dichotomous decision strategies in relation to the length of forecast period.

with respect to "climatological" forecasts (i.e., predicting the weather in the same ratio as would be obtained by chance, using the climatological expectancy). In order to investigate the contrast in economic utility between the *a posteriori* seasonal forecasts and climatological forecasts, an analysis of the expenses was conducted and the results are shown in Table VIII. Furthermore, the respective skill scores and the results of the statistical "chi-square" test are listed in Table IX.

TABLE VIII. Dichotomous decision expense values (per cent of protectable losses) associated with *a posteriori* seasonal predictions of temperature and precipitation for the nation as a whole (1959-1968), compared with the expenses produced by using climatological forecasts.

	Strategies			
	Perfect Forecasts	Optimum Long-run	Mini-max	Categorical Forecasts
A Posteriori Seasonal Forecasts	18.61	27.77	31.74	35.17
Climatological Forecasts	18.03	27.22	30.50	33.33

TABLE IX. Skill scores and values of "chi-square" for categorical seasonal predictions of temperature and precipitation (1959-1968) as compared with climatological forecasts.

	Skill Score (%)	Chi-square	Chi-square required for significance: 1%	5%	10%
Categorical Seasonal Temperature Forecasts Vs. Climatological Forecasts	9	15.74	13.28		
Categorical Seasonal Precipitation Forecasts Vs. Climatological Forecasts	2.5	1.02	13.28	9.49	7.78

Table VIII shows that the expenses produced by using climatological expectancy are slightly less than those associated with the seasonal predictions. Thus, it would appear that the particular set of *a posteriori* forecasts which were used in this analysis are actually of little economic value, compared to climatological predictions, when applied to the average of all operations for the entire economy.

The skill scores in Table IX, which were evaluated for the categorical predictions in comparison with climatological predictions, are in general agreement with the results of Namias (1964), i.e., 9 per cent for the temperature forecasts and 2.5 per cent for the precipitation forecasts. The values of "chi-square" show that while the categorical predictions of temperature are, at 1 per cent significance level, different than climatological forecasts, the predictions of precipitation are not. This result is also in agreement with the findings of Namias.

Multiple Category Decisions

While dichotomous decision problems clearly exist, it is apparent that certain operations require the consideration of multiple options. Therefore, in order to determine if the expenses for this type of problem differ markedly from those examined in the previous section, an analysis of the application of short-period ceiling and visibility forecasts to the generalized aviation matrix (page 19) was undertaken. The results of this study are presented in Figs. 11 and 12.

The economic expenses corresponding to the various multi-category decision strategies for a series of 5-hour visibility forecasts are depicted in Fig. 11. Here, the expenses are comparable in magnitude (i.e., 3 to 13 per

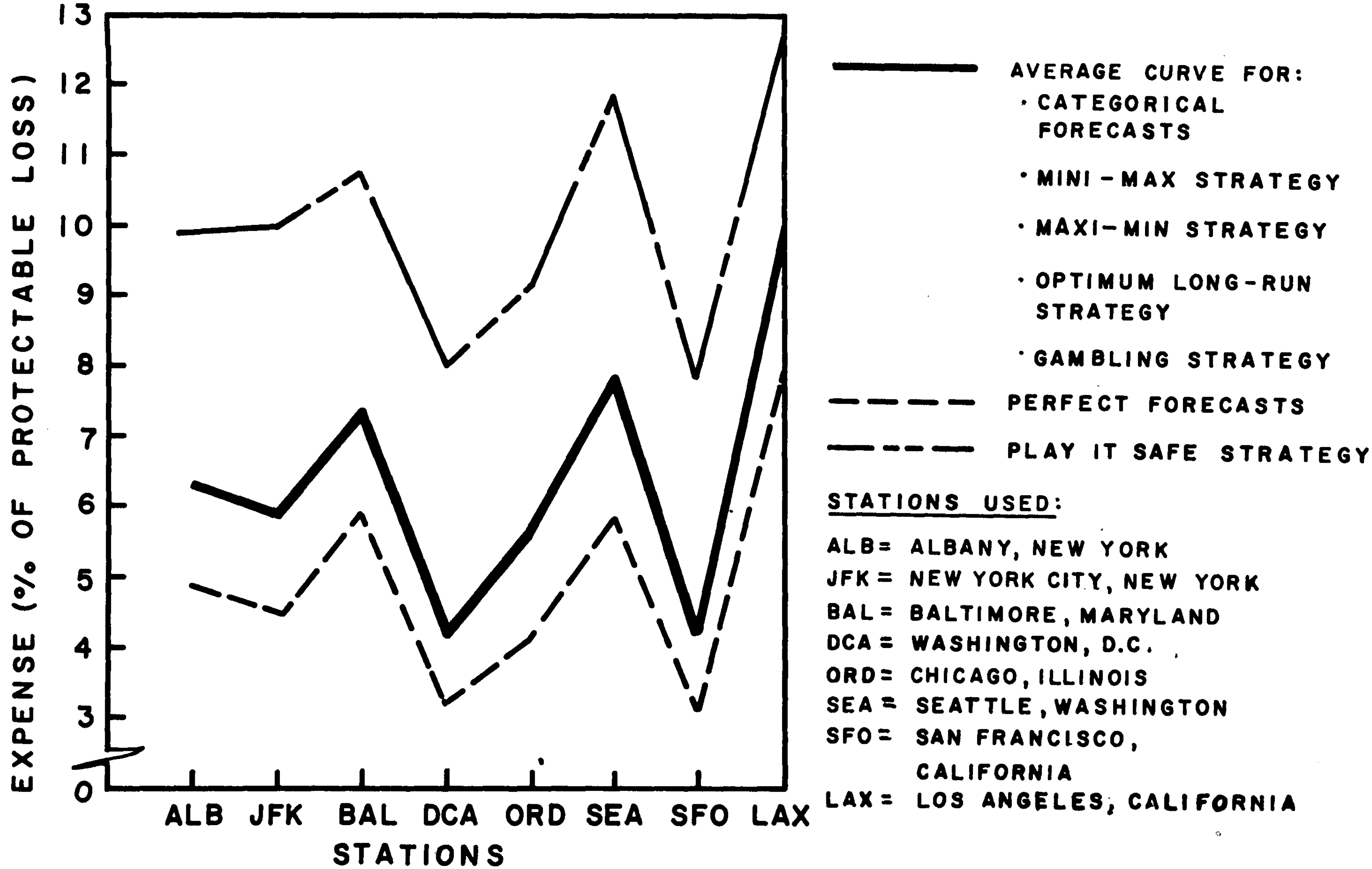

FIG. 11. Expenses associated with differing multi-category decision strategies for 5-hour visibility forecasts (September, 1965-March, 1966, inclusive).

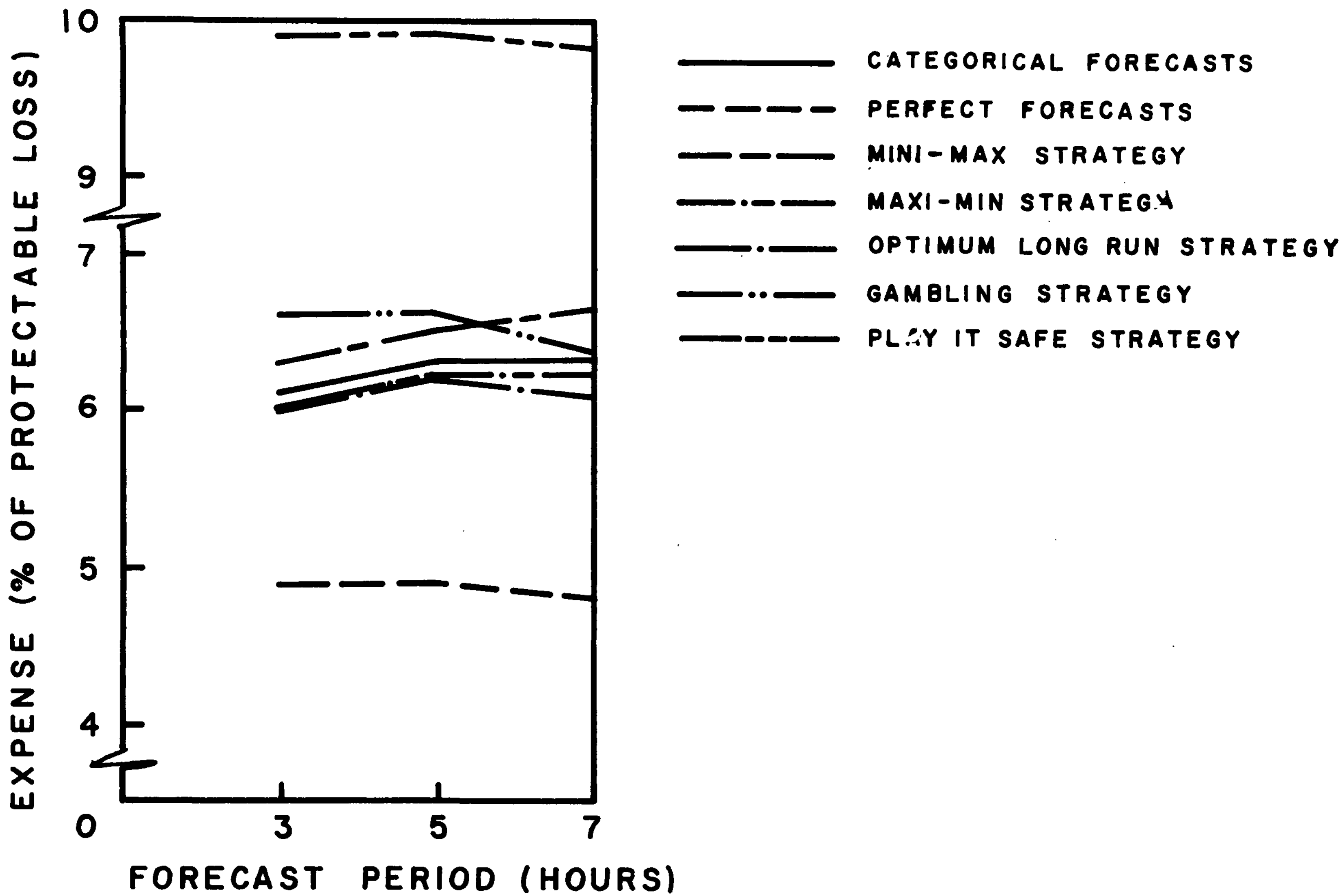

FIG. 12. Expenses associated with differing multi-category decision strategies for 3-, 5-, and 7-hour ceiling and visibility forecasts (September, 1965-March, 1966, inclusive).

cent of the protectable losses) to those which were presented in connection with the congruous dichotomous decision model (Fig. 3, page 26). A complete listing of the expenses produced by the multi-category decision strategies for the 5-hour predictions of ceiling and visibility is given in Table XIII (Appendix C, page 57).

The heavy line on Fig. 11 shows that the magnitudes of the expenses produced by the various strategies differ very little. An exception is the "play it safe" strategy, for which the expense values are quite large. This suggests that the procedure used to implement this strategy (see page 12), while most likely a practical tactic when probability information is not available, may tend to produce considerably larger economic expenses than would be the case if such information were provided. Conversely, the "gambling" strategy, although introducing the possibility of substantial short-run losses, produces long-run expenses which are similar to those associated with the optimum strategy. However, it should be noted that these observations concerning the "play it safe" and "gambling" strategies are highly dependent on the structure of the particular utility matrix which was used to evaluate the expenses.

As a means of summarizing the expenses associated with the multiple option decision problem under consideration, Fig. 12 depicts the manner in which the expenses vary as the length of the forecast period changes. Here, the expenses produced by the differing strategies remain relatively constant as the forecast period ranges from 3 to 7 hours. Once again the expense corresponding to the use of perfect forecasts is lowest, while the other strategies, with the exception of the "play it safe", produce expenses of similar magnitude (approximately 6 per cent of the protectable losses).

Total Dollar Losses

Based on a meteorologic-economic model and implementing the concepts of mathematical decision theory, economic expenses have been computed for variations in users' decision options and strategies. In general, these variations appear to produce only small differences in the results for prediction periods of less than two days, but the differences increase to operationally significant values for monthly and seasonal prognoses.

However, even small variations in economic expenses can be of importance when applied to the total protectable losses which are sustained by large segments of the economy. For example, the approximate magnitude of the annual total protectable losses for major activities in the United States are listed in Table X (Thompson, 1972). As shown in Table X, the total protectable weather-caused losses are substantial (i.e., over 5 billion dollars). Therefore, the expenses produced even by short-period predictions (i.e., 5 to 10 per cent of the total protectable losses), when expressed in actual dollar amounts, represent sizeable quantities of capital. For this reason, small differences between the economic expenses associated with various decision strategies can be of significance when applied to the economy of the United States.

TABLE X. Summary of annual total protectable weather losses for major activities in the United States

Activity	Losses (millions of dollars) due to adverse weather which could be protected against if adequate warnings for the appropriate period in advance could be provided.
Agriculture	3,554.2
Aviation (commercial)	56.9
Construction	328.6
Communications	6.4
Electric Power	13.9
Energy (e.g., fossil) Fuels	1.0
Manufacturing (provisional estimate)	238.0
Transportation (rail, highway, and water)	45.8
Other (general public, government, etc.)	1,057.8
Totals	5,302.6

CHAPTER V

CONCLUSIONS

The economic expenses which are produced when contrasting decision models and operational requirements are applied to various types of predictive weather information have been shown to exhibit several interesting characteristics.

(1) The mean values for the dichotomous decision strategy expenses (per cent of protectable losses for the average of all operations) increase as the length of the forecast period increases (e.g., less than 10 per cent for predictions of up to 7 hours; 10 to 20 per cent for predictions of up to 36 hours; 15 to 35 per cent for predictions of up to 90 days.

(2) While the differences between the mean expenses associated with various dichotomous decision strategies are of small magnitude (less than 5 per cent) for predictions of less than two days, the variations for monthly and seasonal forecasts are much larger (5 to 15 per cent).

(3) The geographic variations in the mean expenses for dichotomous decision problems are closely related to the frequency of adverse weather and the accuracy of the predictions.

(4) Dichotomous and multiple option decision problems produce mean expenses of similar magnitude (less than 10 per cent) for short period predictions.

(5) Due to the large monetary value (over 5 billion dollars) of the total protectable weather-connected losses throughout the United States, even small variations in the economic expense values are of importance.

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APPENDIX A

CONFIDENCE LIMITS--METHOD OF DETERMINATION

Lower and upper confidence limits are necessary for the implementation of the "mini-max" and "maxi-min" strategies for both dichotomous and multiple category decision problems. These limits were obtained by first computing the relative frequency of occurrence of adverse weather (e.g., precipitation, low airport ceiling and visibility) for a sample series of occurrences as a function of each class of predicted probability. Then the relative frequencies which included 90 per cent of the sample data were subjectively evaluated for each such class. These relative frequencies represent the 90 per cent lower and upper limits on the "confidence" associated with the probability estimates issued by the forecaster and/or forecasting system.

A more sophisticated procedure was used to check the accuracy of the method outlined above. Assuming that the errors in the predicted probabilities are normally distributed, a normal curve was fitted to the sample data. The lower and upper confidence limits were then determined by standard statistical procedures (e.g., Panofsky and Brier, 1968). Analysis of the results revealed little difference between the confidence limits obtained by the two methods.

APPENDIX B

EXPLANATION OF AVIATION UTILITY MATRIX

TABLE XI. Summary of basis for relative values of the economic expense given in Table III.

Forecast Category	Observed Category	Economic Expense	Comment
1	1	.70	Probable trip cancellation; if attempted, would require diversion.
1	2	.60	Probable trip cancellation; if attempted, subject to ILS approach delays.
1	3	.65	If trip cancelled, loss of revenue and intangible dissatisfaction since flight could be completed; if attempted, subject to some delays.
1	4	.70	Same as previous comment, except if attempted, little or no delay.
1	5	.75	Same as previous comment, except if attempted, no delay.
2	1	.90	Diversion or holding probably anticipated; alternate planned and holding fuel carried, but serious delay encountered.
2	2	.40	Same as previous comment, except no diversion necessary; delay due to ILS approaches.

TABLE XI. (continued)

Forecast Category	Observed Category	Economic Expense	Comment
2	3	.30	Some traffic delay, but weather better than forecast; holding fuel carried.
2	4	.25	Same as previous comment, except less traffic delay.
2	5	.10	Traffic flow optimum, but holding fuel carried.
3	1	.95	Alternate probably not named; insufficient fuel for holding; must divert or land short of destination, but pilot might expect need for holding.
3	2	.40	Traffic flow reduced with ILS approaches; pilot may anticipate need for holding.
3	3	.30	Weather observed as predicted, but considerable delay likely.
3	4	.25	Slight traffic delay, but holding fuel carried due to adverse weather forecast.
3	5	.05	Traffic movement optimum, but holding fuel carried.
4	1	.95	Alternate probably not named; insufficient fuel for holding; must divert or land short of destination, but pilot might expect need for holding.
4	2	.45	Traffic flow delay with unexpected ILS approaches; holding fuel necessary, but probably no diversion.

TABLE XI. (continued)

Forecast Category	Observed Category	Economic Expense	Comment
4	3	.35	Same as previous comment, but probably less traffic delay.
4	4	.20	Weather observed as predicted, but some traffic delay.
4	5	.05	Traffic movement optimum, but extra fuel carried due to slightly adverse forecast.
5	1	1.00	Alternate probably not named; insufficient fuel for holding; must divert or land short of destination.
5	2	.50	Same as previous comment, except may be able to make destination after ILS delays.
5	3	.40	Same as previous comment, but may land with only general approach delays.
5	4	.30	Same as previous comment, but traffic moving well with visual approach used.
5	5	0	Traffic movement optimum.

APPENDIX C

ECONOMIC EXPENSES FOR 5-HOUR CEILING AND VISIBILITY FORECASTS

TABLE XII. Dichotomous decisions--expenses associated with differing strategies for 5-hour ceiling and visibility forecasts (September, 1965-March, 1966, inclusive). Units per cent of protectable loss. (C = Ceiling Forecasts, V = Visibility Forecasts)

	Strategies							
	Perfect Forecasts		Optimum Long-run		Mini-max		Categorical Forecasts	
Stations	C	V	C	V	C	V	C	V
ALB	2.9	5.2	4.6	8.1	5.3	8.5	5.5	9.3
JFK	5.1	4.2	6.4	6.2	6.6	6.6	6.9	7.4
BAL	4.2	5.6	5.4	8.1	5.5	8.3	6.4	9.4
DCA	3.3	3.1	4.5	4.6	4.8	5.0	5.3	5.2
ORD	2.1	3.5	3.7	5.7	4.3	6.3	4.8	7.1
SEA	4.8	5.5	8.0	8.2	9.0	8.9	9.5	9.3
SFO	2.5	2.8	4.2	4.3	4.7	4.6	5.0	4.9
LAX	4.0	6.3	5.6	9.0	6.3	9.7	6.4	10.6

Stations used:

ALB = Albany, New York
JFK = New York City, New York
BAL = Baltimore, Maryland
DCA = Washington, D.C.
ORD = Chicago, Illinois
SEA = Seattle, Washington
SFO = San Francisco, California
LAX = Los Angeles, California

TABLE XIII. Multiple category decisions--expenses associated with differing strategies for 5-hour ceiling and visibility forecasts (September, 1965-March, 1966, inclusive). Units are per cent of protectable loss. (C = Ceiling Forecasts, V = Visibility Forecasts)

	Strategies													
	Perfect Forecasts		Optimum Long-run		Mini-max		Maxi-min		Categorical Forecasts		Play it Safe		Gambling	
Stations	C	V	C	V	C	V	C	V	C	V	C	V	C	V
ALB	4.9	4.9	6.3	6.2	6.6	6.4	6.2	6.2	6.3	6.2	9.5	9.9	6.6	6.5
JFK	4.9	4.5	5.9	5.8	6.0	6.0	5.9	5.7	5.9	5.8	10.1	10.0	6.3	6.0
BAL	3.8	5.9	4.6	7.2	4.6	7.4	4.5	6.4	4.6	7.4	8.8	10.8	4.9	7.8
DCA	3.3	3.2	4.0	4.1	4.1	4.2	4.0	4.2	4.1	4.1	8.0	8.0	4.4	4.3
ORD	4.7	4.1	5.9	5.5	6.3	5.8	6.1	5.4	6.0	5.4	9.5	9.1	6.6	5.6
SEA	7.8	5.8	10.3	7.7	11.2	8.4	10.6	7.7	10.9	8.1	13.2	11.7	11.2	7.8
SFO	4.1	3.1	5.3	4.1	5.9	4.4	5.5	4.1	5.4	4.1	9.0	7.8	5.8	4.3
LAX	4.9	7.7	6.0	9.5	6.5	9.8	6.1	9.8	6.1	9.6	9.9	12.4	6.7	10.3