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CALCULATION OF IRROTATIONAL WIND PATTERN WITH APPLICATION TO CLEVELAND TOPOGRAPHY

by Robert Siegel

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Cleveland, Ohio 44135

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SUMMARY

Small perturbation theory is applied to compute the deflection of the wind blowing across land that has an irregular topography. As an illustration, the method is applied first to the flow around a single hill of Gaussian profile. Then calculations are made for the irregular topography on the east side of Cleveland where the elevation changes by several hundred feet. It was found that the topography produced small wind deflections that would not be of practical importance in air pollution dispersion studies. The calculations were for a neutrally stable atmosphere. Although they are not investigated here, other factors such as thermal stratification of the atmosphere, diurnal variations, and convection currents resulting from the proximity of Lake Erie and the city heat island effect are expected to be more significant than the influence of topography.

INTRODUCTION

In order to use the diffusion equation or various dispersion models to calculate the dispersion of pollutants from emission sources in a city, it is necessary to know the local wind velocities. For example in reference 1 the diffusion equation was solved numerically for the Los Angeles area. The wind velocities were interpolated from measurements at 32 stations throughout the city. In the simplified dispersion model devised by Hanna (ref. 2), which is applicable for distributed area sources of pollutants, the local concentration was found to depend inversely on the local wind velocity. Even though the analytical result in reference 2 has a very simple form, it was found that good concentration results were obtained in calculations performed for the Los Angeles and Chicago areas.

One of the factors influencing the local wind velocities and flow pattern is ground topography. In reference 3 computations were made of the smoke dispersion from an electric power plant. Downwind in the direction of the prevailing wind were two hills that influenced the wind pattern. The situation in greater Cleveland is somewhat similar. On the east side of the city there is a rise in the land in the direction of the wind, which is predominantly from the west and southwest. In this region the land also rises to the south away from the lake as shown by the simplified contour map in figure 1. This figure also shows the location of the Cuyahoga river valley in the central portion of the city where heavy industry is concentrated.

The object of this report is to determine whether topography will significantly deflect the wind that is passing over the industrial region and then blowing across the east side of the city. This would have some bearing on the interpretation of information from pollution measuring stations distributed throughout the city and would provide information for use in dispersion computations.

To obtain a first approximation as to whether the irregular topography would significantly deflect the wind, it was felt that boundary-layer effects could be neglected and an

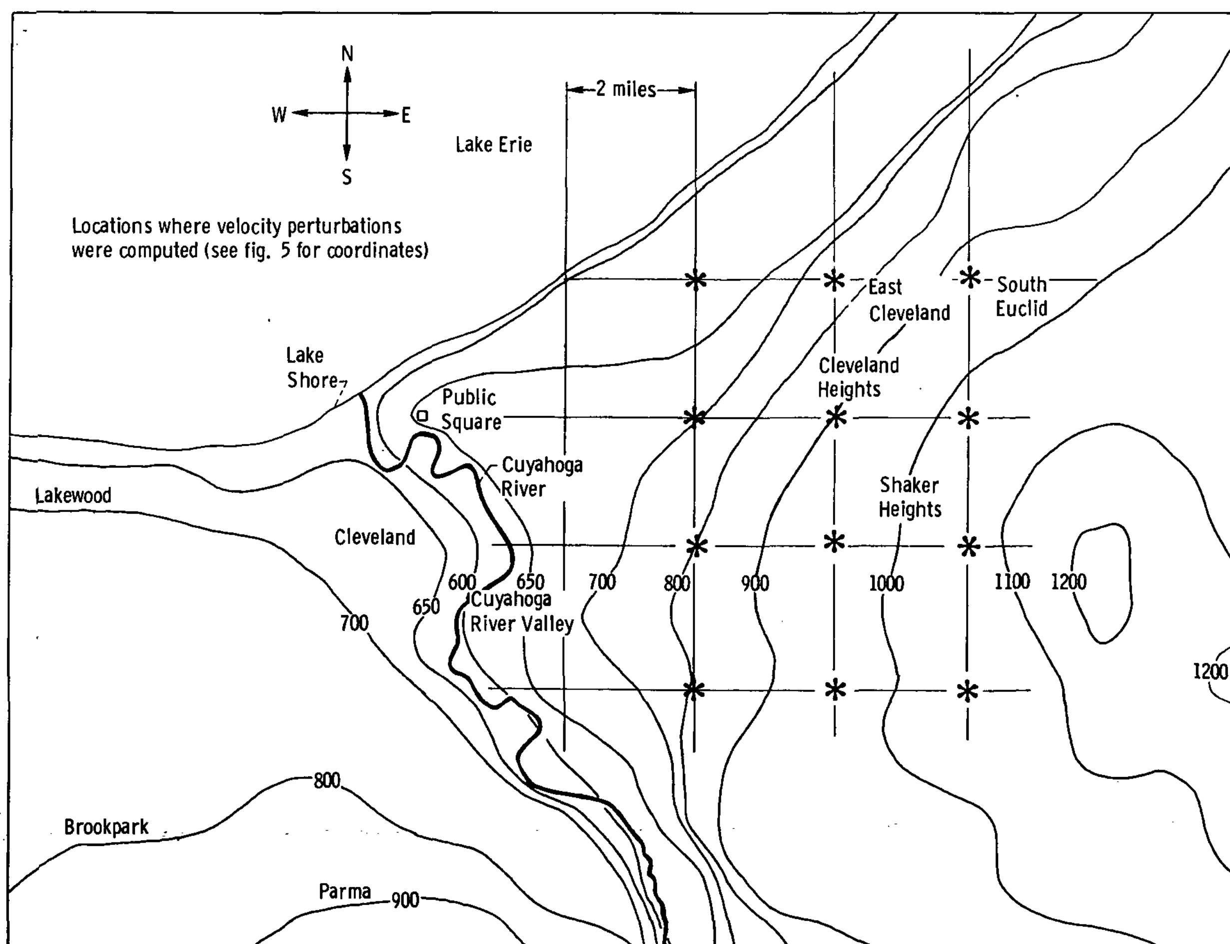


Figure 1. - Simplified contour map of greater Cleveland area showing the increase in elevation in the southeast area. (Contour elevation lines are in feet.)

inviscid irrotational analysis performed for a neutrally stable atmosphere. If the inviscid results show appreciable effects on the flow, then these results can be used as the outer flow and the boundary layer effects can be calculated. An inviscid calculation would not be valid in a region where there is flow separation such as on the downstream side of a hill. However, as discussed in reference 4 (p. 29), inviscid irrotational flow theory provides a reasonable quantitative description of the flow pattern on the windward side of a hill. The prevailing wind direction in Cleveland is directed toward the rising land in the region of interest here, and hence the inviscid analysis should be reasonable for a first approximation.

SYMBOLS

A	height of undisturbed streamline above x-y plane
H	dimensionless height of ground above x-y plane, h/l_r
h	height of ground above x-y plane
h_{max}	maximum height of hill
l_r	reference length
p	pressure
t	intermediate variable, $\xi + i\eta$
u, v, w	velocities in positive x, y, z directions
u_∞	undisturbed wind velocity in positive x direction
W	complex potential function, $\Phi + i\Psi$
X, Y, Z	dimensionless rectangular coordinates, X is parallel to direction of undisturbed wind
X', Y'	dimensionless grid coordinates for Cleveland
x, y, z	rectangular coordinates, x is in direction of undisturbed wind
$\hat{z}$	complex variable, x + iz
α	angle with respect to -x axis of approaching undisturbed wind
ζ	dummy z variable
η	dummy y variable; imaginary part of t
ξ	dummy x variable; real part of t
$\hat{\xi}, \hat{\eta}$	dimensionless variables, ξ/l_r, η/l_r
ρ	density

σ	constant in Gaussian distribution
Φ	potential, real part of W
φ	perturbation potential
Ψ	streamfunction, imaginary part of W
Ψ_n	streamfunction for n^{th} streamline

ANALYSIS

To gain some appreciation of how the flow is affected by topographical features, the flow across two configurations of regular geometry will be briefly examined: a two-dimensional step and a hill with a Gaussian profile. All of the calculations are for a neutrally stable atmosphere and flow separation effects are neglected.

Deflection of Flow by a Two-Dimensional Step

The geometry for the two-dimensional step is shown in figure 2. At a great distance before reaching the step, the undisturbed flow velocity has components $u = -u_\infty$ and $v = v_\infty$ so that the velocity vector is at angle $\alpha = \tan^{-1}(v_\infty/-u_\infty)$ relative to the negative x direction. To show how the flow is deflected, the streamlines will be obtained for this approaching flow. For an inviscid irrotational calculation it is shown in appendix A that the velocity in the y-direction will be v_∞ everywhere, and the component of the velocity vector in the x-z plane can be treated separately from that in the y-direction. In reference 5, the solution is given in the x-z plane for a two-dimensional flow approaching in the -x direction and then passing over a step. Since the effect of the v-velocity component is uncoupled, this solution gives the streamlines projected in the x-z plane for the

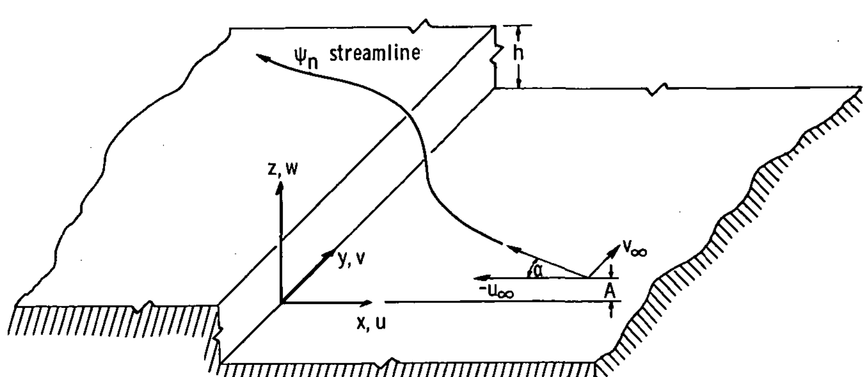


Figure 2. - Streamline deflection by two-dimensional step.

present case. The interest now is in obtaining a streamline path projection in the x-y plane by including the effect of the v-velocity component.

Along a streamline

$$\frac{dy}{dx} = \frac{v}{u}$$

Since v is everywhere equal to v_∞ this equation can be integrated to obtain

$$y(x) = v_\infty \int_{\text{streamline}} \frac{1}{u}\, dx \tag{1}$$

where the integration is along a streamline. To obtain u along a streamline the solution in reference 5 is used. This gives the complex potential W as a function of x and z in terms of an intermediate variable t,

$$W = \Phi + i\Psi = \frac{-u_\infty h}{\pi} \cosh t \tag{2}$$

$$\hat{z} = x + iz = \frac{h}{\pi}(t + \sinh t) \tag{3}$$

where h is the height of the step. Then the complex conjugate velocity in the x-z plane is given by

$$u - iw = -\frac{dW}{d\hat{z}} = -\frac{dW}{dt}\frac{dt}{d\hat{z}} = u_\infty \frac{\sinh t}{1 + \cosh t} \tag{4}$$

If we let $t = \xi + i\eta$, taking the imaginary part of equation (2) gives

$$\Psi = \frac{-u_\infty h}{\pi} \sinh \xi \sin \eta \tag{5}$$

Along the n^{th} streamline, which originates at a distance A above the base plane in figure 2, Ψ_n is a constant. If we let $\Psi_n = 0$ along $z = 0$, then $\Psi_n = -u_\infty A$ since the difference between stream function values for streamlines is equal to the volume flow between them. Hence the relation between ξ and η along a streamline is, from equation (5),

$$\frac{\pi A}{h} = \sinh \xi \sin \eta \tag{6}$$

To obtain u, take the real part of equation (4) to give

$$u = u_\infty \frac{\sinh \xi}{\cosh \xi + \cos \eta} \tag{7}$$

Then u along a streamline is given in terms of η by eliminating ξ in equation (7) by use of equation (6). This yields

$$\left.\frac{u}{u_\infty}\right|_{\Psi_n} = \frac{\pi A}{h} \frac{1}{\sin \eta \cos \eta + \sqrt{\sin^2 \eta + \left(\frac{\pi A}{h}\right)^2}} \tag{8}$$

To carry out the integration in equation (1), the relation between x and η along the streamline must be known. In addition, to be able to plot the streamlines a relation between z and η along a streamline is needed. These relations are obtained by taking the real and imaginary parts of equation (3) which gives,

$$\frac{x}{h} = \frac{1}{\pi}[\xi + \sinh \xi \cos \eta] \tag{9}$$

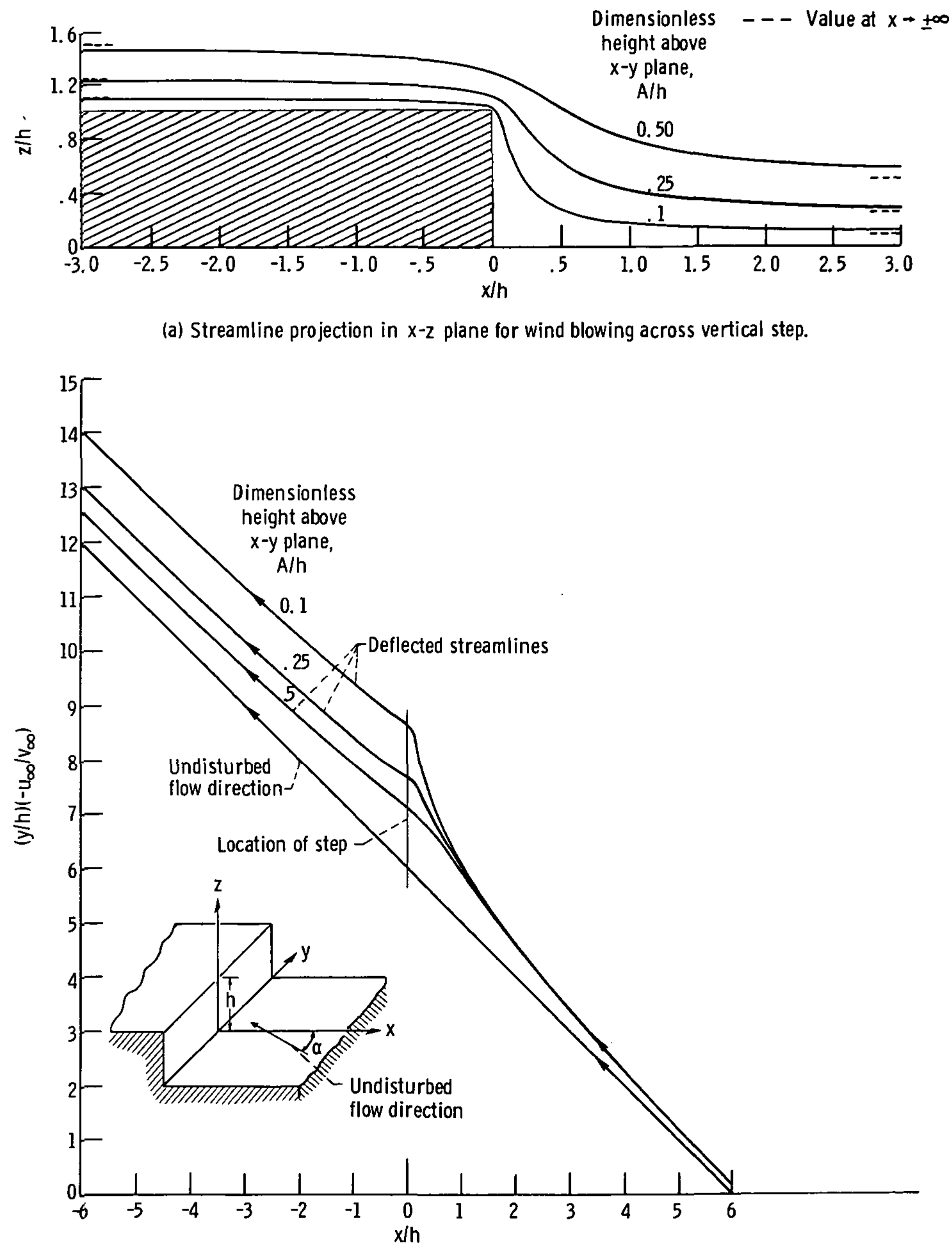
$$\frac{z}{h} = \frac{1}{\pi}[\eta + \cosh \xi \sin \eta] \tag{10}$$

Then eliminate ξ by use of equation (6) to obtain x and z along a streamline in terms of η

$$\left.\frac{x}{h}\right|_{\Psi_n} = \frac{1}{\pi}\left[\sinh^{-1}\left(\frac{\pi A}{h \sin \eta}\right) + \frac{\pi A}{h} \frac{1}{\tan \eta}\right] \tag{11}$$

$$\left.\frac{z}{h}\right|_{\Psi_n} = \frac{1}{\pi}\left\{\eta + \left[\sin^2 \eta + \left(\frac{\pi A}{h}\right)^2\right]^{1/2}\right\} \tag{12}$$

By evaluating equations (11) and (12) for various η, the streamlines projected in the x-z plane are plotted in figure 3(a).



(a) Streamline projection in x-z plane for wind blowing across vertical step.

(b) Deflection of wind by a vertical step for flow approaching at 45° angle (undisturbed horizontal velocity components are $-u_\infty$, v_∞).

Figure 3. - Deflections for vertical step.

The y coordinates of the streamline (as a function of x) are obtained by inserting equation (8) into equation (1) to yield

$$\frac{y}{h} = \frac{v_\infty}{u_\infty} \frac{h}{\pi A} \int_{x_0/h}^{x/h} \left[\sin \eta \cos \eta + \sqrt{\sin^2 \eta + \left(\frac{\pi A}{h}\right)^2} \right] d\left(\frac{x}{h}\right) \tag{13}$$

The integration in x is carried out along a streamline using equation (11) to obtain $\eta(x)$ so that the integrand can be evaluated at any x value. The x_0 in the lower limit of integration is a value taken far enough before the step so that the flow has not yet been deflected ($x_0 \approx 6h$ was reasonable in most instances).

It is evident from equations (13) and (11) that the quantity $(y/h)\ (-u_\infty/v_\infty)$ depends only on the single parameter A/h which is the height of the undisturbed streamline above the x-y plane divided by the height of the step. The quantity $(y/h)\ (-u_\infty/v_\infty)$ is plotted as a function of x/h in figure 3(b) for streamlines with three different A/h. If $-u_\infty = v_\infty$, that is, if the wind approaches the step at $\alpha = 45^\circ$, these curves are the actual streamline paths and show the horizontal wind deflection. For $v_\infty \neq -u_\infty$ the horizontal deflection is found by multiplying the ordinate of figure 3(b) by $v_\infty/-u_\infty$.

Analysis for a Three-Dimensional Topography

For a three-dimensional variation in land elevation, the solution for the Laplace equation governing the velocity potential would usually not be possible analytically and could be obtained numerically by finite differences or by a technique such as in reference 6. In many instances, however, the changes in elevation are rather gradual (several hundred feet per mile) and consequently a small perturbation theory can be applied to obtain deviations in velocity from the free-stream value. Some of the basic aspects of the small perturbation theory are given in reference 7. This theory was applied to a real topography in reference 3 although the details of the analysis are not given there. The small perturbation equations used here are derived in appendix B. The results for the perturbation velocities are

$$\left.\begin{aligned}
\frac{u(X,Y,Z)}{u_\infty} &= 1 + \frac{1}{2\pi}\int_{\hat{\eta}=-\infty}^{\infty}\int_{\hat{\xi}=-\infty}^{\infty} \frac{-2(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2}{\left[(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2\right]^{5/2}} H(\hat{\xi},\hat{\eta})\,d\hat{\xi}\,d\hat{\eta}, \\
\frac{v(X,Y,Z)}{u_\infty} &= -\frac{3}{2\pi}\int_{\hat{\eta}=-\infty}^{\infty}\int_{\hat{\xi}=-\infty}^{\infty} \frac{(X-\hat{\xi})(Y-\hat{\eta})}{\left[(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2\right]^{5/2}} H(\hat{\xi},\hat{\eta})\,d\hat{\xi}\,d\hat{\eta} \\
\frac{w(X,Y,Z)}{u_\infty} &= -\frac{3}{2\pi} Z\int_{\hat{\eta}=-\infty}^{\infty}\int_{\hat{\xi}=-\infty}^{\infty} \frac{X-\hat{\xi}}{\left[(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2\right]^{5/2}} H(\hat{\xi},\hat{\eta})\,d\hat{\xi}\,d\hat{\eta}
\end{aligned}\right\} \qquad (14)$$

The $H(X,Y)$ is the local elevation of the land above the X-Y plane normalized by a characteristic dimension. In the integrals the X, Y, and Z are the coordinates of the location at which the velocity perturbations are being computed. The integration over $\hat{\xi}$ and $\hat{\eta}$ takes into account the effect that the surrounding land contour has on the velocity perturbations.

Deflection of Flow by a Hill

As an illustration of the flow for a three-dimensional topography, the wind pattern around a hill will be considered. For a three-dimensional topography such as a hill, there is a relief effect (ref. 7) resulting from some of the flow being diverted around the hill rather than having to flow over the top of it as in a two-dimensional situation. A simple hill geometry can be specified by using a Gaussian contour of revolution (all lengths are nondimensionalized by the maximum hill height, h_{max})

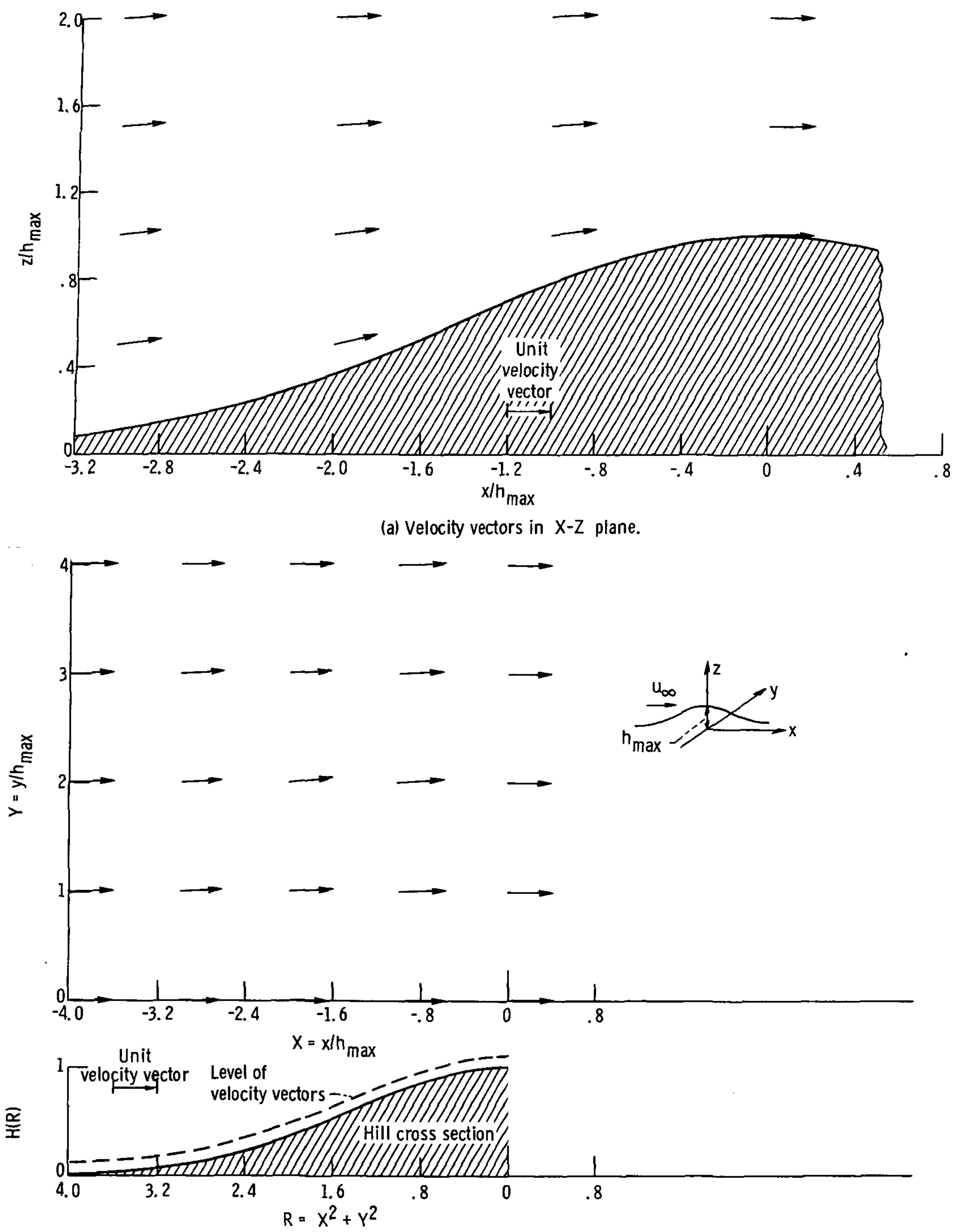
$$\frac{h(x,y)}{h_{max}} = H(X,Y) = e^{-(X^2+Y^2)/\sigma^2} \tag{15}$$

(Another function used in discussing wind flows (ref. 8) is the "mountain" function $\left[1 + (X^2 + Y^2)/\beta^2\right]^{-1}$ where β is a constant.) The σ in equation (15) determines the steepness of the hill. Results were carried out here for $\sigma = 2$ and the hill contour for this value is shown in figure 4. The $H(X,Y)$ was inserted into equations (14), and the integrations carried out numerically for various (X, Y, Z). Some results are tabulated in table I(a) for the first X, Y quadrant. This is sufficient to give the entire flow pattern because the inviscid flow is symmetric around the hill as flow separation effects are not taken into account. The velocity vectors at various heights in the X-Z plane are shown in figure 4(a).

Since measurements of the wind are often at a fixed height above the ground, a set of calculations of this type was also performed where the height above the ground was 0.1 of the maximum hill height. For example, for a hill 500 feet high (typical of Cleveland topography) the results show the wind 50 feet above the ground. For this calculation, the $Z(X,Y)$ of equation (14) was set equal to

$$Z(X,Y) = H(X,Y) + \frac{\text{height above ground}}{h_{max}} = H(X,Y) + 0.1$$

Figure 4(b) shows the horizontal velocity vectors at various X, Y values and illustrates the flow deflection around the hill.



(a) Velocity vectors in X-Z plane.

(b) Horizontal velocity vectors for flow around hill at fixed distance of one-tenth of maximum hill height above the ground Z(X, Y) = H(R) + 0.1.

Figure 4. - Wind velocity vectors for flow over Gaussian hill (σ = 2).

TABLE I. - VELOCITY COMPONENTS

[Each set of three values is u/u_∞, v/u_∞, w/u_∞.]

(a) Velocity components for flow over hill with Gaussian profile ($\sigma = 2$)

Z	Y	Dimensionless rectangular coordinate in direction of undisturbed wind, X			
		0	1	2	3
0.5	0	Inside hill	Inside hill	0.983 0 -.229	0.914 0 -.125
	1	Inside hill	Inside hill	0.989 -.068 -.186	0.929 -.046 -.103
	2	1.143 0 0	1.091 -.068 -.093	1.001 -.083 -.102	0.959 -.059 -.058
	3	1.075 0 0	1.051 -.046 -.034	1.009 -.059 -.039	0.984 -.046 -.024
1	0	1.159 0 0	1.105 0 -.123	1.007 0 -.146	0.955 0 -.095
	1	1.140 0 0	1.093 -.030 -.103	1.008 -.039 -.123	0.962 -.030 -.081
	2	1.097 0 0	1.067 -.039 -.061	1.011 -.052 -.075	0.978 -.041 -.051
	3	1.057 0 0	1.042 -.030 -.027	1.012 -.041 -.034	0.992 -.035 -.025

Z	Y	Dimensionless rectangular coordinate in direction of undisturbed wind, X			
		0	1	2	3
1.5	0	1.102 0 0	1.073 0 -.074	1.015 0 -.095	0.978 0 -.071
	1	1.092 0 0	1.066 -.017 -.064	1.015 -.023 -.082	0.981 -.019 -.062
	2	1.068 0 0	1.050 -.023 -.041	1.014 -.032 -.054	0.989 -.028 -.042
	3	1.043 0 0	1.033 -.019 -.021	1.013 -.028 -.028	0.996 -.026 -.023
2	0	1.068 0 0	1.051 0 -.046	1.016 0 -.063	0.990 0 -.052
	1	1.063 0 0	1.047 -.010 -.041	1.016 -.014 -.056	0.992 -.013 -.047
	2	1.048 0 0	1.037 -.014 -.028	1.014 -.021 -.039	0.996 -.019 -.034
	3	1.033 0 0	1.026 -.013 -.016	1.012 -.019 -.022	1.000 -.019 -.020

TABLE I. - Concluded. VELOCITY COMPONENTS

[Each set of three values is u/u_∞, v/u_∞, w/u_∞.]

(b) Velocity components at height above ground equal to 0.1 of the maximum hill height for flow over a Gaussian hill ($\sigma = 2$)

Y	Dimensionless rectangular coordinate in direction of undisturbed wind, X					
	0	1	2	3	4	5
0	1.145 0 0	1.114 0 -.140	0.980 0 -.236	0.887 0 -.146	0.909 0 -.042	0.955 0 -.007
1	1.156 0 0	1.115 -.044 -.139	0.982 -.077 -.205	0.897 -.060 -.117	0.923 -.027 -.033	0.960 -.011 -.006
2	1.147 0 0	1.097 -.077 -.102	0.991 -.108 -.119	0.940 -.076 -.058	0.952 -.037 -.016	0.972 -.017 -.003
3	1.088 0 0	1.058 -.061 -.039	1.004 -.077 -.038	0.978 -.056 -.018	0.978 -.031 -.005	0.984 -.016 -.001
4	1.041 0 0	1.029 -.029 -.008	1.009 -.038 -.008	0.996 -.031 -.004	0.992 -.021 -.001	0.992 -.013 -.001

Application to the Topography of Greater Cleveland

As illustrated by the results for the Gaussian hill, the small perturbation method is adequate to provide flow deflections for gradual changes in land elevation. To insert the land contour of Cleveland into equation (14), it was decided to use a 1 mile square grid alined east-west and north-south and interpolate intermediate points for purposes of the double integrations. This grid has been designed as X', Y' to distinguish it from the X and Y directions which correspond to the horizontal directions along and normal to the undisturbed wind. The origin of the grid was placed at downtown Public Square. The grid and ground elevations are shown in figure 5. Lake Erie is the region of constant elevation (584 ft) in the upper part of the figure. To normalize the distances, a characteristic length of 528 feet (0.1 mile) was selected; thus each grid spacing was 10 units. The normalized elevation H(X, Y) above lake level, which was taken as the base plane, was then given by

$$H(X, Y) = \frac{\text{elevation at } (X, Y) - 584}{528}$$

The calculations for the wind were made at a level 50 feet above the ground so that the Z in the integrals of equation (14) is given by

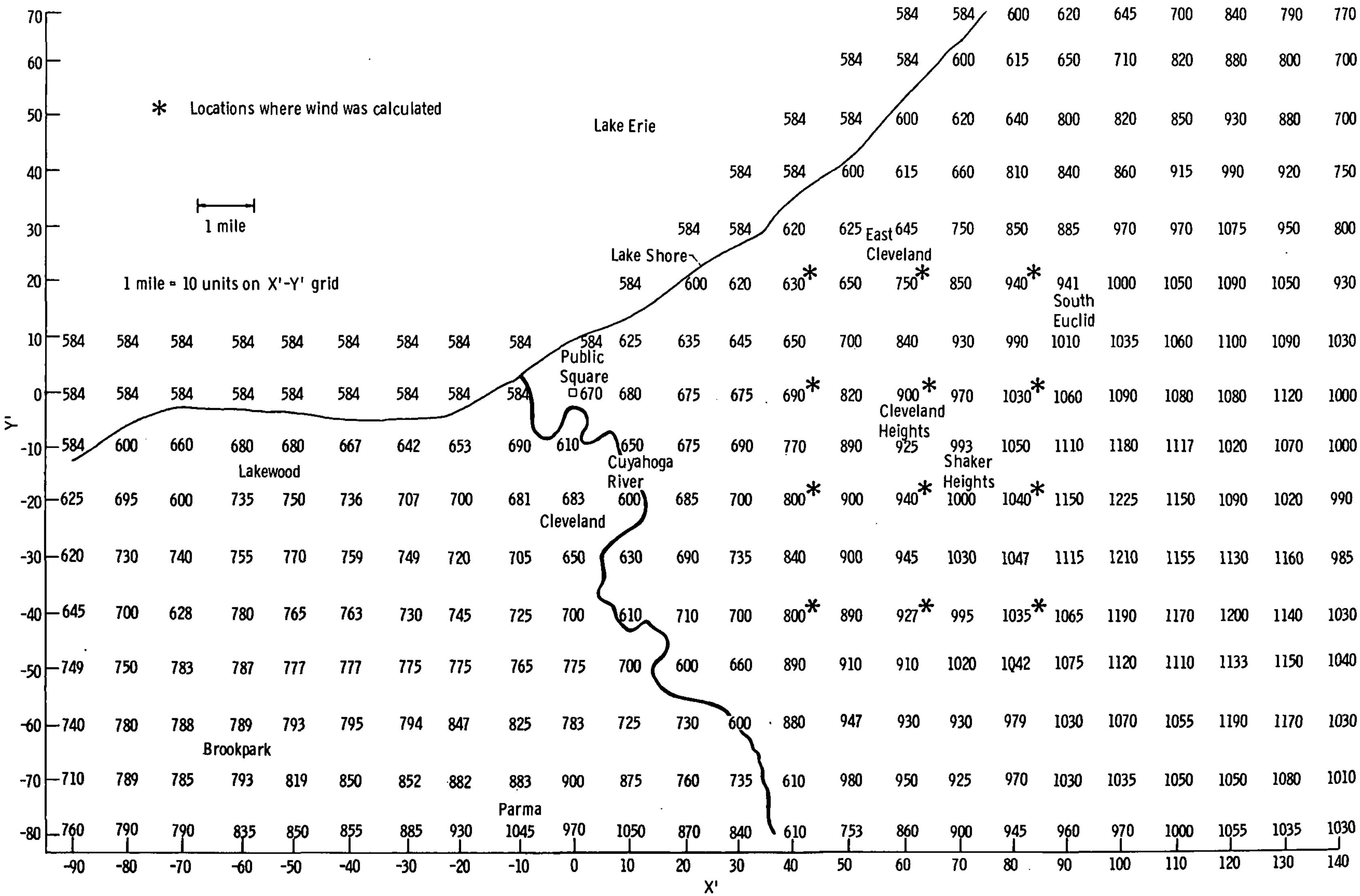


Figure 5. - Land elevations in feet for 1 mile grid of greater Cleveland area (X' and Y' are in units of 1/10 mile).

$$Z = H(X, Y) + \frac{50}{528}$$

In the analysis, the undisturbed flow is in the X direction. This would correspond to a wind from the west when X is alined with the X' on the grid of figure 5. The prevailing wind in Cleveland is from the west or southwest. To obtain results for a southwest wind it is necessary to use a rotation of coordinates to obtain the ground elevations $H(X,Y)$ used in the integration. For a southwest wind the X-Y integration coordinates are rotated counterclockwise 45° from the X'-Y' coordinates in figure 5. Hence, to obtain $H(X,Y)$, the corresponding X', Y' are found from

$$X' = \frac{\sqrt{2}}{2}(X - Y) \qquad Y' = \frac{\sqrt{2}}{2}(X + Y)$$

and then the H is obtained at these X', Y' in figure 5.

The main topographical features evident on figures 1 and 5 is that the west side of greater Cleveland is quite flat. Relative to the surrounding land there is a decrease of elevation of a few hundred feet in the Cuyahoga river valley where heavy industry is located. The main feature is a rise in elevation of several hundred feet on the east side into the "Heights" area (Cleveland Heights and Shaker Heights). The land rises from west to east and from north to south so that an elevated plateau exists in the form of a rounded corner when viewed from above. With regard to the dispersion of pollutants, it is desired to know whether the wind from the west or southwest that passes over the industrial section is significantly deflected when passing into the heights area.

The wind velocity perturbation integrals in equation (14) were carried out at locations $X' = 40$, 60, and 80 for $Y' = -40$, -20, 0, and 20. As shown in figure 5, these locations are in the region of the largest elevation change and hence provide the largest wind deflections. The results for the velocity perturbations are summarized in table II and will be discussed in the next section. It is evident, however, that the velocity perturbations are quite small.

DISCUSSION

To obtain an indication of the flow deflection by topographical features, the streamline deflection was computed for an initially uniform horizontal wind incident at an angle across a step as shown in figures 2 and 3 The u component of velocity decreases as the step is approached as shown by the spreading of the streamlines projected in the x-z plane. The decrease in u component provides more time for the v component to move

TABLE II. - VELOCITY COMPONENTS FOR WIND

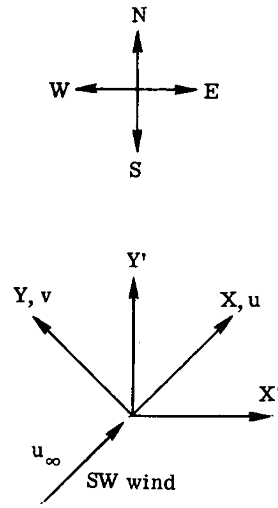
ACROSS CLEVELAND AT LOCATIONS

50 FEET ABOVE GROUND

[Each set of three values is u/u_∞, v/u_∞, w/u_∞.]

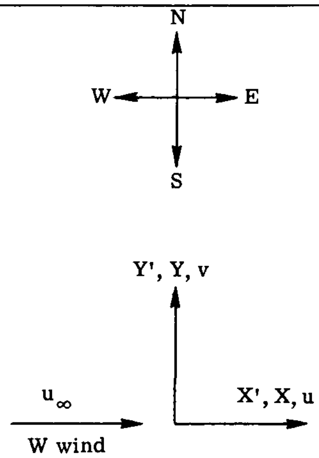
(a) Southwest wind

Y'	X' 40	X' 60	X' 80
-40	0.997	0.998	1.001
	-.007	.002	-.003
	.008	.009	.005
-20	0.999	1.004	1.001
	.005	.004	.004
	.008	.006	.009
0	0.996	1.005	1.011
	.012	.004	-.005
	.001	.004	.002
20	1.000	1.006	1.011
	.002	.002	-.005
	.000	.000	-.003



(b) West wind

Y'	X' 40	X' 60	X' 80
-40	1.004	0.998	0.993
	-.006	-.005	-.003
	.017	.011	.007
-20	0.998	1.001	1.000
	.007	.001	.000
	.020	.010	.013
0	0.980	0.995	1.011
	.004	.003	.002
	.006	.012	.008
20	0.993	0.998	0.985
	.010	.021	.009
	.009	.030	.008



the flow along the y-direction. Consequently, during the approach to the step, the flow is turned along the y-direction. At the crest of the step the opposite effect occurs; the u component increases, and the flow is turned toward its original direction. The ordinate in figure 3(b) can be interpreted directly as the streamline deflection for an incidence angle of $\alpha = 45^{\circ}$ ($v_\infty = -u_\infty$). Streamlines for any other angle can be found by multiplying the ordinate in figure 3(b) by $v_\infty/-u_\infty = \tan \alpha$. The parameter A/h is the fraction of the step height at which the streamline of the incoming undisturbed flow is located above the x-y base plane. The streamlines near the ground are deflected the most as a result of having the largest decrease in u velocity when approaching the step. The results show that the deflections for incidence at 45° are of the order of a few step heights; when passing over a 500-foot cliff, the wind originating 50 feet from the ground would be horizontally displaced about 1000 feet.

For a three-dimensional flow around a hill there is a relief effect with regard to the acceleration of flow over the crest of the hill. This is because the flow is partially diverted around the sides of the hill. The flow is also decelerated somewhat as it approaches the hill in a fashion similar to the approach to a step. These two factors contribute to the decrease in the u velocity (the component in the incident flow direction) for the flow approaching the hill and produce the values of u/u_∞ that are less than 1 in table I. To illustrate the three-dimensional calculation technique, a few results were carried out for a hill of Gaussian cross section (fig. 4). The results in figure 4(b) are for a height above the ground of one-tenth of the maximum hill height. The turning of the flow is always small; the perturbations of the velocity components (table I) are in the range of several percent.

Since the variations in greater Cleveland land elevation are in the range of several hundred feet, the preceding calculations for simple geometries would lead one to expect that the deflections of the wind by the topography would be rather small. Two sets of calculations were carried out, and both substantiate this expectation. The velocity perturbations for flow across the east side of the city were at most only a few percent of the mean velocity in the regions of greatest change in ground elevation. This was true for winds from the west and from the southwest. The w/u_∞ components are approximately equal to the slope of the land in the x-direction as would be expected for gradual changes in contour. Note that u is in the direction of the undeflected wind and v and w are normal to that direction. Since the velocity perturbations are small, it is felt that the topographical features would represent a second-order effect on the wind deflection as compared with other factors. Factors of possible significance are convection currents resulting from temperature inequalities arising from the lake adjacent to the city, heat island effects, diurnal temperature variations, and thermal stratification of the atmosphere. These would provide complicated transient wind variations.

CONCLUSIONS

A small perturbation technique was applied to compute the three-dimensional velocity perturbations for wind blowing across the topography of greater Cleveland. The calculations were irrotational and for a neutrally stable atmosphere. The velocity perturbations were found to be small, indicating that the small perturbation technique used in the analysis was adequate. It is concluded that the topographical features of greater Cleveland provide only a second-order effect on the wind as compared with other factors of possible significance such as diurnal temperature variations and heat island effects.

Lewis Research Center,
National Aeronautics and Space Administration
Cleveland, Ohio, August 21, 1972,
501-24.

APPENDIX A

INVISCID IRROTATIONAL EQUATIONS

For inviscid flow in three dimensions, the Euler equations are

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{aligned}\right\} \tag{A1}$$

For flow across the two-dimensional step in figure 2, there is no variation in the y-direction so that $\partial/\partial y = 0$. Then equations (A1) become

$$\left.\begin{aligned} u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} &= 0 \\ u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z} \end{aligned}\right\} \tag{A2}$$

For irrotational motion

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0 \qquad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

and with $\partial/\partial y = 0$ these yield $\partial v/\partial x = 0$, and $\partial v/\partial z = 0$. Hence, v must be a constant throughout the flow and is v_∞ everywhere. The Euler equations (eqs. (A2)) then further reduce to

$$\left.\begin{aligned} u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\ u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} \end{aligned}\right\} \tag{A3}$$

which are the two-dimensional equations for u and w in the x-z plane. The u and w components are thus uncoupled from the constant $v = v_\infty$ component.

APPENDIX B

SMALL PERTURBATION ANALYSIS FOR THREE-DIMENSIONAL VELOCITY DISTRIBUTION

The quantities u, v, and w are the velocity components, and for the situation considered here $|u - u_\infty| << |u_\infty|$, $|v| << |u_\infty|$, and $|w| << |u_\infty|$. Let the velocity potential be $\Phi(x,y,z) = u_\infty x + \varphi(x,y,z)$ where φ is the perturbation potential. Then $\partial\Phi/\partial x = u_\infty + \partial\varphi/\partial x = u$, $\partial\Phi/\partial y = \partial\varphi/\partial y = v$, and $\partial\Phi/\partial z = \partial\varphi/\partial z = w$. The local elevation of the ground above the x-y base plane is given by $h(x,y)$ (fig. 6). At the surface of the ground the boundary condition requires that the streamlines are tangent to the surface. Hence at the surface

$$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$

Then within the approximation of small perturbation theory

$$w = u\frac{dz}{dx} = \left[u_\infty + (u - u_\infty)\right]\frac{dz}{dx} = u_\infty \left.\frac{dz}{dx}\right|_{surface}$$

Using $w = \partial\varphi/\partial z$ and the fact that $dz/dx|_{surface} = \partial h/\partial x$ gives

$$\frac{\partial\varphi}{\partial z} = u_\infty \frac{\partial h}{\partial x} \tag{B1}$$

Equation (B1) expresses the derivative normal to the x-y plane of the unknown perturbation potential in terms of the known free-stream velocity and slope in the undisturbed flow direction of the land contour. For small perturbation theory this boundary condition at the surface of the ground is applied at the base plane $z = 0$ (fig. 6).

The solution for the perturbation potential requires solving Laplace's equation $\nabla^2\varphi = 0$ in the half space $0 < z < \infty$ subject to the boundary condition that the normal derivative at $z = 0$ is given by equation (B1). The Greens function for this problem is given in reference 9 as

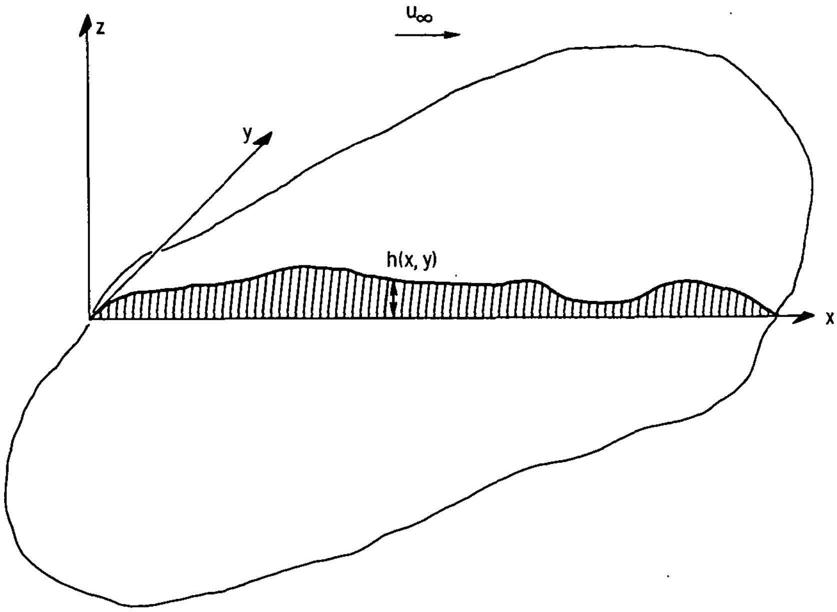


Figure 6. - Geometry for small perturbation theory.

$$g = \lim_{\zeta \to 0} \left\{ \frac{1}{\left[(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\right]^{1/2}} + \frac{1}{\left[(\xi - x)^2 + (\eta - y)^2 + (\zeta + z)^2\right]^{1/2}} \right\}$$

$$= \frac{2}{\left[(\xi - x)^2 + (\eta - y)^2 + z^2\right]^{1/2}} \tag{B2}$$

Then from Greens fundamental solution (ref. 9)

$$\varphi(x, y, z) = - \frac{1}{4\pi} \iint_{\substack{x-y \\ \text{plane}}} g \frac{\partial \varphi}{\partial \zeta} d\xi \, d\eta$$

Substituting equations (B1) and (B2) gives

$$\varphi(x, y, z) = - \frac{u_\infty}{2\pi} \int_{\eta=-\infty}^{\infty} \int_{\xi=-\infty}^{\infty} \frac{1}{\left[(\xi - x)^2 + (\eta - y)^2 + z^2\right]^{1/2}} \frac{\partial h}{\partial \xi} d\xi \, d\eta \tag{B3}$$

Integrate by parts with respect to ξ to obtain

$$\varphi(x,y,z) = -\frac{u_\infty}{2\pi}\int_{\eta=-\infty}^{\infty}\left\{\frac{h}{\left[(\xi - x)^2 + (\eta - y)^2 + z^2\right]^{1/2}}\Bigg|_{\xi=-\infty}^{\infty} - \int_{-\infty}^{\infty}\frac{\left(-\frac{1}{2}\right)2(\xi - x)h}{\left[(\xi - x)^2 + (\eta - y)^2 + z^2\right]^{3/2}}\,d\xi\right\}d\eta$$

$$\varphi(x,y,z) = \frac{u_\infty}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(x - \xi)h(\xi,\eta)}{\left[(x - \xi)^2 + (y - \eta)^2 + z^2\right]^{3/2}}\,d\xi\,d\eta$$

Divide all lengths by a characteristic dimension l_r to obtain the dimensionless form

$$\hat{\varphi}(X,Y,Z) = \frac{\varphi(X,Y,Z)}{u_\infty l_r} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(X - \hat{\xi})H(\hat{\xi},\hat{\eta})}{\left[(X - \hat{\xi})^2 + (Y - \hat{\eta})^2 + Z^2\right]^{3/2}}\,d\hat{\xi}\,d\hat{\eta} \tag{B4}$$

The velocity components are given by

$$\left.\begin{aligned}
\frac{u(x,y,z)}{u_\infty} &= 1 + \frac{1}{u_\infty}\frac{\partial\varphi(x,y,z)}{\partial x} = 1 + \frac{\partial\hat{\varphi}}{\partial X}\\
\frac{v(x,y,z)}{u_\infty} &= \frac{1}{u_\infty}\frac{\partial\varphi(x,y,z)}{\partial y} = \frac{\partial\hat{\varphi}}{\partial Y}\\
\frac{w(x,y,z)}{u_\infty} &= \frac{1}{u_\infty}\frac{\partial\varphi(x,y,z)}{\partial z} = \frac{\partial\hat{\varphi}}{\partial Z}
\end{aligned}\right\} \tag{B5}$$

By differentiating $\hat{\varphi}$ the velocity components are obtained as

$$\left.\begin{aligned}
\frac{u(X,Y,Z)}{u_\infty} &= 1 + \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{-2(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2}{\left[(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2\right]^{5/2}} H(\hat{\xi},\hat{\eta})d\hat{\xi}\, d\hat{\eta} \\
\frac{v(X,Y,Z)}{u_\infty} &= -\frac{3}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{(X-\hat{\xi})(Y-\hat{\eta})}{\left[(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2\right]^{5/2}} H(\hat{\xi},\hat{\eta})d\hat{\xi}\, d\hat{\eta} \\
\frac{w(X,Y,Z)}{u_\infty} &= -\frac{3Z}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{X-\hat{\xi}}{\left[(X-\hat{\xi})^2 + (Y-\hat{\eta})^2 + Z^2\right]^{5/2}} H(\hat{\xi},\hat{\eta})d\hat{\xi}\, d\hat{\eta}
\end{aligned}\right\} \quad (B6)$$

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