MAGNETIC FIELD MERGING IN THE SOLAR WIND *Karl Schindler*

ABSTRACT Magnetic field merging in the solar wind is discussed in terms of steady-state merging, which involves a steady flow field, and of spontaneous merging, which involves an instability such as the tearing instability. Spontaneous merging is found to be much more effective than steady-state merging.

INTRODUCTION

It is generally believed that magnetic field merging takes place in the neutral sheet of the geomagnetospheric tail [*Dungey*, 1961; *Axford et al.*, 1965; *Coppi et al.*, 1966; *Dessler*, 1968]. The question has been raised [*Dessler*, 1970] of whether one would expect similar processes to occur in the interplanetary sector boundaries. Merging of magnetic lines of force is of interest because it governs the topology of the magnetic field and thereby, for instance, electron heat conduction; also, the electric fields involved can accelerate particles inside the neutral sheet.

Available satellite observations seem to be consistent with the concept of an ideal sector structure within 1 AU [*Wilcox and Ness*, 1965]. Therefore, we may state the problem of field line merging in the solar wind in the form of two questions: Why is merging unimportant within 1 AU? Do we or don't we expect more merging at larger heliocentric distances? There is no final answer to these questions as yet. This note approaches the problem from a particular viewpoint and gives some preliminary answers that could perhaps serve as a basis for further discussion.

It seems convenient to distinguish between *steady-state* merging, which involves a steady flow field, and *spontaneous* merging, which involves an instability such as the tearing instability [*Schindler and Soop*, 1968].

Both types of merging have been suggested for the magnetospheric tail, and it is quite possible that both occur under suitable circumstances. For instance, there is increasing evidence that the neutral sheet has a fine structure that can be consistently explained by assuming spontaneous merging [*Schindler and Ness*, 1971].

Similarly, for sector boundaries we cannot *a priori* exclude either type of merging. However, on the basis of the simple estimate given at the end of this note, it seems that steady-state merging is less important than spontaneous merging for the solar wind. We therefore concentrate on processes involving neutral sheet instabilities to see in what way and to what extent they might lead to merging of magnetic field lines across the sector boundaries.

SPONTANEOUS MERGING

Let us consider stability of a static one-dimensional neutral sheet separating regions of homogeneous magnetic field with opposite field directions. It is easy to show that such neutral sheets are stable against perturbations that keep the magnetic field strictly frozen into the plasma—that is, $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$. This implies that at the neutral plane ($B = 0$) the electric field has to vanish. It is a characteristic property of neutral sheet instabilities that this constraint is violated, allowing for a finite electric field in the neutral sheet.

Table 1 gives some properties of a number of neutral sheet instabilities. It is evident that the two tearing instabilities are those with the largest wavelengths and hence they are more likely to give rise to macroscopic effects. Table 2 indicates how tearing can be stabilized.

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Table 1. *Neutral sheet instabilities*

Mode	Driven by	Geometry	Approximate growth rate	Reference
Collision-free tearing	Electrons passing neutral plane	$k_B L < 1$ $k_j L < \left(\frac{a_i}{L}\right)^{1/4} \sqrt{k_B L}$	$k v_{th_e} \left(\frac{a_e}{L}\right)^{3/2}$	[*Laval et al.*, 1966]
Resistive tearing	Finite resistivity	$k_j \approx 0$	$\frac{\eta}{\mu_o L^2} \left(\frac{\mu_o v_A}{k\eta}\right)^{2/5}$ $S^{-1/4} < kL < 1$	[*Furth et al.*, 1963]
Resistive rippling	Spatial variation of resistivity	$k_B = 0$	$\frac{\eta}{\mu_o L^2} \left[\left(\frac{L\eta'}{\eta}\right)^2 \frac{kL^2 \mu_o v_A}{\eta}\right]^{2/5}$ $S^{-2/7} < kL < S^{2/3}$	[*Furth et al.*, 1963]
Resistive gravitational interchange	Finite resistivity and external force	$k_B = 0$	$\frac{\eta}{\mu_o L^2} \left(\frac{L\vert\rho_o'\vert}{\rho_o} \frac{kL^3 \mu_o g}{\eta v_A}\right)^{2/3}$ $S^{-1/4} G^{1/8} < kL < S^{1/2} G^{-1/4}$ $G < 1, (k_L)^2$ $G > (kL)^{-2/5} S^{-2/5}, (kL)^{-8/5} S^{-2/5}$	[*Furth et al.*, 1963]
Fried-Weibel (collision-free)	Temperature anisotropy	$k_B \approx 0$	$\frac{v_{th_e}}{a_e} \left(1 - \frac{T_\perp}{T_\parallel}\right)_e^{5/4}$	[*Fried*, 1959; *Weibel*, 1959]

$S = \frac{\mu_o L v_A}{\eta}$

$G = \frac{L^2 g \vert\rho_o'\vert}{V_A^2 \rho_o}$

μ_o vacuum permeability

v_A Alfvén velocity

v_{th} thermal velocity

ρ_o mass density

L neutral sheet width

η resistivity

g acceleration due to external force

$'$ derivative normal to the sheet

R wave number

a Larmor radius

T temperature

Subscripts e, i, B, j, $\perp$, $\parallel$ refer to electrons, ions, direction of the magnitude field **B**, direction of the electrical current j, direction perpendicular and parallel to **B**.

The average electron temperature anistropy in the solar wind works in the direction of stabilization. However, since the average anistropy is small, there may be times during which the isotropy is reversed, such that the tearing wave can grow. Also it seems possible to vizualize nonlinear perturbations that grow even in the linearly stable regime. Of course, we do not assume a normal field component at the outset, because that should be the result of merging. Nor do we have to worry about boundaries in the solar wind.

The problem becomes more involved when we look at the nonlinear properties of tearing. Quasilinear stabilization is effective for extremely small initial perturbations [*Biskamp et al.*, 1970], and it may not be important if the initial perturbation is larger or if the spectrum is sufficiently narrow. Under suitable conditions a single mode will grow and field loops will start to coalesce by forming larger loops from smaller ones. This process is much faster than the original tearing [*Biskamp and Schindler*, 1971]. When sufficiently large concentrations of the electrical current are formed, it is conceivable that magnetohydrodynamic pinch instabilities will set in,

Table 2. *Properties related to the stabilization of collision-free tearing*

1. Temperature anisotropy (linear theory) [*Coppi and Rosenbluth,* 1968; *Laval and Pellat,* 1968]

$$(T_\parallel - T_\perp/T_\parallel)_e > a_e/L$$

2. Normal magnetic field component (linear theory?)
3. Boundary stabilization:

   Boundary at a few neutral sheet widths [*Furth,* 1968]

4. Nonlinear effects

   a. Quasi-linear stabilization at

$$\mathbf{B}/B_o \sim ka_e(a_e/L)^2$$

   [*Biskamp et al.,* 1970]

   b. Single mode dynamics
   Loop merging [*Biskamp and Schindler,* 1971]

$$\gamma \sim \min \begin{cases} kv_{th_e}\sqrt{a_e/L}\,(\log\sqrt{L/a_e})^{-1/2} \\ kv_{th_i} \end{cases}$$

   MHD pinch-instabilities

$$\gamma \sim kv_{th_i}$$

γ growth rate, $\mathbf{B}$ absolute value of magnetic field perturbation; other quantities as in table 1.

which may stop further loop growth. Because of the large growth rate for coalescence of loops, a firm answer cannot be given at the present time.

Let us therefore explore the consequences of the hypothesis that, in an infinite system, tearing would grow indefinitely. In a finite system with a convective flow superimposed, as there is both in the solar wind and in the magnetosphere, the occurrence of merging would then depend on the ratio of the growth time of the tearing instability and the characteristic time of convective flow across the system.

Figure 1 gives the critical heliocentric radius R^* at which tearing will occur as a function of the width L of the sector boundary. R^* is given by

$$\int^{R^*} \frac{\gamma(R)}{v_{sw}}\, dR = 1$$

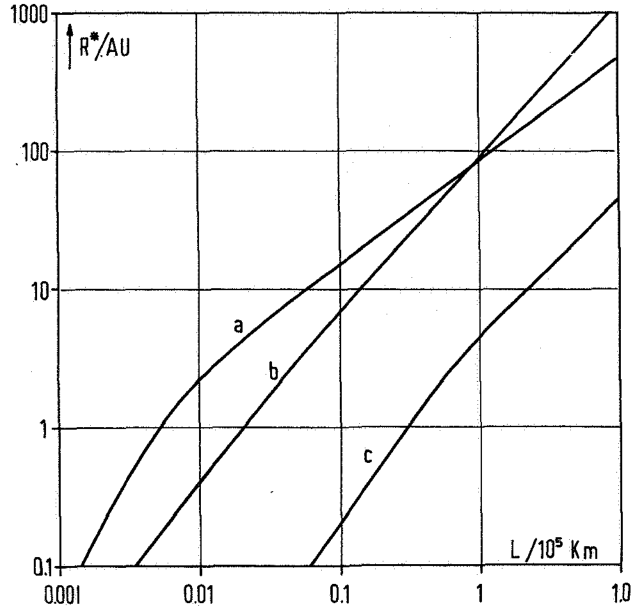

Figure 1 *Heliocentric distance R^* at which tearing may occur as a function of the width L of the sector boundary: (a) resistive, (b) collision-free, and (c) collective-resistive tearing with $\nu_{eff} = 10^{-3}\ \omega_{pi}$ (see also text).*

To evaluate the radial dependence of the growth rate γ a very simple solar wind model is chosen. The velocity v_{sw} is constant (500 km/sec), and the magnetic field lines form Archimedean spirals. For $R < 1$ AU the electron temperature scales as $R^{-2/5}$; going to $T_e \sim R^{-2/7}$ would not change the results significantly [*Forslund,* 1970]. For $R > 1$ AU we assume adiabatic electron cooling with a specific heat ratio of 5/3. Note that L does not vary with distance in an Archimedean spiral field.

The collision-free tearing curve does not use the standard sheet pinch growth rate [*Laval et al.,* 1966], which was derived for a situation where the plasma density drops to zero outside the neutral sheet, but a more general form that applies to situations such as the solar wind:

$$\gamma \sim kv_{th_e} \frac{(c/\omega_{pe})^2}{\sqrt{a_e}\, L^{3/2}}$$

The notation is explained in the captions of tables 1 and 2. For $L = 10^5$ km, which is close to the upper limit $L = 1.5 \times 10^5$ km [*Wilcox and Ness,* 1965], tearing would become important only for $\gtrsim 100$ AU. Smaller widths would lead to tearing closer to the sun. To have tearing at 1 AU, one would need sector boundaries as thin as 2000 km. More experimental information on the width of sector boundaries is necessary before one can cite this fact as a possible reason for the absence of tearing for

$R \lesssim 1$ AU. Within roughly 100 AU, resistive tearing based on electron-ion collisions requires even smaller values of L, for instance, $L \sim 500$ km for $R^* \sim 1$ AU. For $R > 100$ AU, resistive tearing is more effective than the collision-free mode.

Resistive tearing may also be produced if small-scale electrostatic fluctuations are present, leading to an effective collision frequency ν_{eff}. Choosing $\nu_{eff} = \epsilon\, \omega_{pi}$, the curve c in figure 1 corresponds to $\epsilon = 10^{-3}$. The same curve holds for arbitrary values of ϵ if we reinterpret the abscissa as measuring the quantity $10\, L/\epsilon^{1/3}$ instead of L. More experimental observation on the fluctuation level is necessary before the importance of collective-resistive tearing can be firmly evaluated.

Although we are not concerned here with the magnetosphere it may be interesting to note that for a neutral sheet width of 1 R_E, a characteristic length along the tail of 100 R_E, and a convection speed of 5.10^6 cm/sec, the growth time of the (collision-free) tearing instability is about equal to the characteristic time for convection. Therefore, also in the magnetosphere convection may be responsible for limiting the growth of tearing.

It remains to visualize the field configuration a spatially growing tearing mode would give. Figure 2 shows what one qualitatively obtains by replacing time in a typical tearing mode perturbation by the space coordinate (divided by the flow velocity) along the field lines. It is evident that by this process field lines of neighboring sectors merge, thereby more and more decoupling the out-flowing plasma magnetically from the sun.

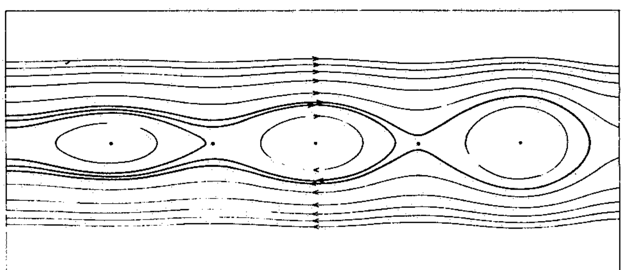

Figure 2 *Qualitative picture of field lines of a neutral sheet configuration with a spatially growing tearing mode. Note field line merging (heavy field lines).*

STEADY-STATE MERGING

To estimate the importance of steady-state merging, we compare the characteristic time τ with the resistive tearing growth time. We obtain τ from

$$\frac{d\psi}{dt} = -\oint_s \eta \mathbf{j} \cdot d\mathbf{s}$$

where ψ is the magnetic flux through a closed integration path s moving with the plasma. With $d\psi/dt \sim \delta\psi/\tau = \kappa\psi/\tau$ (κ being the fraction of the total flux dissipated after time τ) we estimate

$$\tau \sim \kappa \frac{sL\mu_o}{\eta}$$

where s is the characteristic length of a flux tube of one polarity. We compare with resistive tearing (growth rate γ) by estimating

$$\gamma\tau \sim \left(\frac{L\omega_p{}^2 \nu_A}{c^2 \nu}\right)^{1/2} \frac{\kappa s}{L}$$

Both processes are expected to have roughly the same macroscopic effect if $\kappa \sim L/s$. Thus we find for solar wind conditions at 1 AU

$$\gamma\tau \sim 3 \times 10^5$$

CONCLUSIONS

From the above, spontaneous merging clearly is much more effective than the corresponding steady-state process. Note that spontaneous merging is not expected to be as regular as shown in figure 2, which is only given to illustrate the field line topology. In fact, one might rather expect a turbulent structure. The present conclusions must remain tentative, however, until more experimental data (such as the width of sector boundaries) and theoretical information (such as the long-time asymptotic behavior of the tearing instability) are available.

REFERENCES

Axford, W. I.; Petschek, H. E.; and Siscoe, G. L.: Tail of the Magnetosphere. *J. Geophys. Res.*, Vol. 70, 1965, p. 1231.

Biskamp, D.; Sagdeev, R. Z.; and Schindler, K.: Non-Linear Evolution of the Tearing Instability of the Geomagnetic Tail. *Cosmic Electrodyn.*, Vol. 1, 1970, p. 297.

Biskamp, D; and Schindler, K.: Instability of Two-Dimensional Collisionless Plasma with Neutral Points. Submitted to *Plasma Phys.*, 1971.

Coppi, B.; Laval, G.; and Pellat, R.: Dynamics of the Geomagnetic Tail. *Phys. Rev. Letters*, Vol. 16, 1966, p. 1207.

Coppi, B.; and Rosenbluth, M. N.: Model for the Earth's Magnetic Tail, in *Stability of Plane Plasmas*, ESRO SP-36, 1968.

Dessler, A. J.: Correction to a Paper by A. J. Dessler, "Magnetic Merging in the Magnetospheric Tail." *J. Geophys. Res.*, Vol. 73, 1968, p. 1861.

Dessler, A. J.: The Role of the Geomagnetic Tail in Substorms. Invited review presented at the STP Symposium, Leningrad, U.S.S.R., May 11–20, 1970.

Dungey, J. W.: Interplanetary Magnetic Field and the Auroral Zones. *Phys. Rev. Letters*, Vol. 6, 1961, p. 47.

Forslund, D. W.: Instabilities Associated with Heat Conduction in the Solar Wind and Their Consequences. *J. Geophys. Res.*, Vol. 75, 1970, p. 17.

Fried, B. D.: Mechanism for Instability of Transverse Plasma Waves. *Phys. Fluids*, Vol. 2, 1959, p. 337.

Furth, H. P.: The External Region of the Tearing Mode, in *Stability of Plane Plasmas*, ESRO SP-36, 1968.

Furth, H. P.; Killeen, J.; and Rosenbluth, M. N.: Finite-Resistivity Instabilities of a Sheet Pinch. *Phys. Fluids*, Vol. 6, 1963, p. 459.

Laval, G.; and Pellat, R.: Stability of the Plane Neutral Sheet for Oblique Propagation, in *Stability of Plane Plasmas*, ESRO SP-36, 1968.

Laval, G.; Pellat, R.; and Vuillemin, M.: Proceedings of the Conference on Plasma Physics and Controlled Nuclear Fusion, Culham 1965, Vol. II, International Atomic Energy Agency, Vienna, 1966, Paper CN-21/71.

Schindler, K.; and Ness, N. F.: Neutral Sheet Fine Structure. Paper presented at the AGU Meeting, April 12–16, Washington, D.C., 1971.

Schindler, K; and Soop, M.: Stability of Plasma Sheaths. *Phys. Fluids*, Vol. 11, 1968, p. 1192.

Weibel, E. S.: Spontaneously Growing Transverse Waves in a Plasma Due to an Anisotropic Velocity Distribution. *Phys. Rev. Letters*, Vol. 2, 1959, p. 83.

Wilcox, J. M.; and Ness, N. F.: Quasi-Stationary Corotating Structure in the Interplanetary Medium. *J. Geophys. Res.*, Vol. 70, 1965, p. 5793.

DISCUSSION

M. Dryer Karl, did I understand you to say that the merging will take place independent of the boundary conditions but rather depending on the modes that caused the resistivity? The boundary conditions do not matter?

K. Schindler Yes, in a certain sense. You see I was just quoting or trying to quote the existing models for merging. And there are particular classes that depend very highly on boundary conditions, such as the slow fields of this type. We may have this locally somewhere in the solar wind, but this is the exception rather than the rule because the solar wind models don't have these particular situations as they probably have it in the magnetosphere. So tentatively I have excluded these classes for this discussion.