

# EVIDENCE FOR A CONTINUOUS, POWER LAW, ELECTRON DENSITY IRREGULARITY SPECTRUM

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There is a controversy over the spectral form of the irregularities in electron density that cause interplanetary scintillation (IPS) of small angular diameter radio sources. The intensity scintillation technique always yields an "irregularity scale size," which is of the order of the first Fresnel zone for the wavelength at which the observations are taken. This includes not only the radio wavelength measurements of the structure of the interplanetary medium, with which we are most concerned here, but also radio wavelength measurements of the irregularity structure of the ionosphere and interstellar medium, and optical wavelength measurements of the irregularity structure of the atmosphere. The reasons are relatively straightforward. The fundamental question with regard to the interplanetary medium is: Is the scale size we determine from the analysis of IPS data an artifact of Fresnel diffraction, or is it a physically meaningful parameter of the irregularity structure and just coincidentally on the order of the radius of the first Fresnel zone?

## ABSTRACT

## THEORY

We use the "thin screen" model for scattering in the interplanetary medium [Salpeter, 1967]. Because of the inverse-square law dependence of electron density on heliocentric distance, we assume with reasonable confidence that the scattering takes place in a slab located at the distance of closest approach to the sun of the observer-source ray trajectory. We further assume that the effect of the slab, which introduces phase perturbations across the front of an advancing plane wave, is equivalent to a single, thin scattering screen located at the exit plane of the slab. The screen introduces exactly the same phase perturbations as the thick slab. For observations within  $60^\circ$  of the sun, the screen may be taken to be roughly 1 AU from an observer on the earth.

The basic theoretical problem is to relate the electron density irregularity structure as described by, say, a three-dimensional wave number spectrum  $F_{ns}(K_x, K_y, K_z)$ , to the two-dimensional wave number spectrum  $F_\phi(K_x, K_y)$  that describes the phase structure

in the screen. In turn,  $F_\phi$  must be related to the two-dimensional intensity wave number spectrum  $F_{Is}(K_x, K_y)$  in the observer's plane. Finally,  $F_{Is}$  must be related to the temporal spectrum of the intensity fluctuations  $F_{It}(\nu)$  seen by an observer as the two-dimensional intensity pattern is convected across his plane at the velocity of the solar wind.

The relevant equations for the power spectra are as follows:

$$F_\phi(K_x, K_y) = 2\pi r_e^2 \lambda^2 L F_{ns}(K_x, K_y, 0) \quad (1)$$

$$F_{Is}(K_x, K_y) = \mathcal{F}(K_x, K_y) F_\phi(K_x, K_y) \quad (2)$$

$$F_{It}(\nu) = 2\pi U^{-1} \int_0^\infty F_{Is}(2\pi \nu |U, K_y) dK_y \quad (3)$$

where

- $r_e$   $2.8 \times 10^{-18}$  km
- $\lambda$  radio wavelength
- $L$  effective slab thickness
- $U$  solar wind velocity

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and  $\mathcal{F}$  is the Fresnel filtering function:

$$\mathcal{F}(K_x, K_y) = 4 \sin^2 \left[ (K_x^2 + K_y^2) / K_f^2 \right] \quad (4)$$

where

$$K_f = \text{spatial Fresnel wave number} \\ = \sqrt{4\pi/\lambda z} \approx (110 \text{ km})^{-1} \lambda_m^{-1/2}$$

for a scattering screen at  $z = 1 \text{ AU}$  from the observer, with  $\lambda_m = \lambda$  in meters. As a temporal fluctuation frequency,  $K_f$  appears as  $\nu_f$ .

$$\nu_f = 0.5 (U/350 \text{ km} \cdot \text{sec}^{-1}) \lambda_m^{-1/2} \quad (5)$$

where again the irregularities are assumed to be 1 AU from the observer.

For  $K_r^2 = K_x^2 + K_y^2 < K_f^2$ ,  $\mathcal{F}$  imposes fourth-power filtering on the density irregularity spectrum as  $\mathcal{F} \approx [2^{1/2} K_r / K_f]^4$ ,  $K_r < K_f$ . Thus for typical meter wavelength observations, one would not expect to see significant intensity fluctuations arising from structure larger than  $\sim 110 \text{ km}$ . In terms of the temporal spectrum  $F_I(\nu)$ , even if the spectrum were power law at frequencies higher than  $\nu_f$ , with an index  $\beta + 1$  so that  $F_I(\nu) \propto \nu^{-\beta-1}$ , at temporal frequencies lower than  $\nu_f$  one would expect to see a rather flat spectrum; the scale of the spectrum would therefore be on the order of  $\nu_f$ .

## OBSERVATIONS

Unfortunately, very few measurements of the spectral index have been made for scintillation spectra. Lovelace *et al.* [1970] computed spectra for observations taken with the Arecibo dish; see Lovelace [1970] for significant refinements of the power-law spectral index. The signal-to-noise ratio was sufficient to establish the spectral form as a power law (for  $\nu > \nu_f$ ) rather than gaussian. Lovelace's estimate of  $\beta$  is 1.6 for observations from 0.2 to 0.5 AU. The spectra are rather flat at frequencies below  $\nu_f$ .

As a further indication that Fresnel effects are determining the scale of the intensity spectrum, table 1 summarizes the scale of intensity spectra at various wavelengths and solar elongations; the scale is defined as  $\nu_{2I}$ , the square-root second moment of the spectrum—that is,

$$\nu_{2I} = \sqrt{\int_0^\infty \nu^2 F_{It}(\nu) d\nu}$$

The Fresnel frequency, also is given; note that  $\nu_{2I}$  and  $\nu_f$  differ by factors of less than 2. A detailed explanation of the difference would require information about the power law index for  $\nu > \nu_f$ , the axial ratio of the

irregularities, and various other parameters about which we unfortunately have little or no information. As has been suggested [Cronyn, 1970], it is important to evaluate the spectral index of scintillation spectra, in addition to the more usual parameters of scale and scintillation index.

Another indication that we are dealing with a continuous power law spectrum of irregularities is provided by a comparison of the electron density spectrum deduced from IPS spectra with the proton density spectrum from space-probe measurements. For a power law irregularity spectrum, the relationship between an intensity scintillation spectrum  $F_{It}(\nu)$  and a space probe density spectrum  $F_{nt}(\nu)$  may be shown [Cronyn, 1971] to be given by:

$$F_{nt}(\nu) = \left. \frac{3.7 \nu F_I(\nu) \Gamma(\beta/2+1)}{\beta \rho (U/350 \text{ km} \cdot \text{sec}^{-1}) \lambda_m^2 \Gamma(\beta+1/2)} \right\} \quad (6) \\ \nu > \nu_f$$

where

- $\beta$  spectral index for space-probe spectrum;  $\beta + 1$  is spectral index for temporal intensity spectrum  $F_I(\nu)$
- $\Gamma$  the gamma function
- $\rho$  observer – source ray trajectory distance of closest approach to the sun in AU

Table 1. IPS spectral scales and estimated Fresnel frequencies

$\rho$ , AU	$\lambda$ , m	$\nu_{2I}$	$\nu_f^*$	Reference
0.06 – 0.14	0.11	2.2 – 1.3	1.5	Cohen and Gundermann [1969]
0.04 – 0.14	0.21	2.5 – 1.2	1.1	Cohen and Gundermann [1969]
0.14 – 0.60	0.70	1.2 – 0.7	0.6	Cohen and Gundermann [1969]
>0.34	1.54	0.6	0.4	Cohen et al. [1967]
>0.55	3.70	0.5	0.3	Dennison [1969]

\*Computed according to equation (5) with  $U = 350 \text{ km/sec}$

Taking  $\beta = 1.6$ ,  $U = 350 \text{ km}\cdot\text{sec}^{-1}$ , fixing the absolute power spectral density of  $F_f(\nu)$  from the scintillation index as given by Hewish [1971], and assuming that  $F_{nt}(\nu)$  may be extrapolated to much lower frequencies than  $\nu_f$  gives:

$$F_{nt}(\nu) = 1.4 \times 10^{-3} \nu^{-1.6} (\text{electrons/cm}^3)^2 \text{Hz}^{-1} \quad (7)$$

at 1 AU from the sun; for a detailed discussion, see Cronyn [1971]. This estimate of the power spectral density for electron density irregularities agrees remarkably well with  $F_{nt}(\nu)$  measured for proton density irregularities [Intriligator and Wolfe, 1970]: at  $\nu = 10^{-3} \text{ Hz}$ , equation (7) gives  $F_{nt}(\nu) = 100$ , while  $I$  and  $W$  measure 250; at  $\nu = 10^{-4} \text{ Hz}$ ,  $F_{nt} = 5.1 \times 10^3$ , while  $I$  and  $W$  measure  $4.6 \times 10^3$ .

The only evidence that contradicts the continuous power law irregularity model is the wavelength dependence of the scintillation index, which Hewish has discussed at this conference and elsewhere [Hewish, 1971]. The linear wavelength dependence which Hewish argues for would rule out the power law spectrum. However, as Hollweg and Jokipii have pointed out (p. 495), the dependence can be  $m \propto \lambda^\alpha$ , where  $\alpha$  could be at least as high as 1.25 because the data have substantial scatter. For  $\beta = 1.6$ ,  $\alpha$  would theoretically have to be 1.4.

## CONCLUSIONS

There is evidence that for  $\nu > \nu_f$  the intensity spectrum is a power law indicating a power law electron density spectrum. At frequencies less than  $\nu_f$  the spectra flatten out, as one would expect because of Fresnel filtering. It would therefore appear to be unreasonably fortuitous for the flattening to represent the intrinsic irregularity spectrum, especially since the plasma density irregularities tend to again be described by a power law spectrum at still lower values of  $\nu$ . In any case, the relatively small difference between  $\nu_{2I}$  and  $\nu_f$  is an indication that Fresnel filtering is having a pronounced effect. Finally, there is the very interesting indication that extrapolation to large scales of the very small-scale electron density structure actually agrees very well with measured proton density spectra. Thus, the conclusion we must draw is that the small-scale electron density structure is power law, that it may be smoothly extrapolated to much larger scale sizes, and that the wavelength dependence of the scintillation index can probably accommodate such a spectrum.

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DISCUSSION    *G. Newkirk* I would like to ask a question about the physics of all of this, which seems not to have been discussed very thoroughly at the present time. We have given a great deal of attention as to whether or not the spectral index at long wavelength can be carried down to short wavelength. Whether or not it does, the significance of this is what escapes me, and I wonder if someone could say a couple of words as to what we might expect if, for example, we have a medium dominated by turbulence, what we might expect if we have a medium dominated by waves.

*F. W. Perkins* I would like to answer Dr. Newkirk's question to some extent. I think what the spacecraft are observing apparently is outward going Alfvén waves that are probably left over from the processes that are heating the solar wind. Meanwhile, what is being measured in the interplanetary scintillations are smaller scale fluctuations that are characteristic of the microturbulence processes that are playing the role of cushions in the plasma. From what we know about these microprocesses they seem to occur most readily in the sense of velocity deviations from Maxwellian velocity distributions that are less for waves around the ion cyclotron frequency and for wavelengths in the order of the ion gyroradius. These waves have one characteristic signature that could possibly be found in the data, that is, their phase velocities go inward along the magnetic field whereas the waves observed by spacecraft seem to have an outward going phase velocity.