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† INVESTIGATION OF RESONANCE INTEGRALS OCCURRING IN COSMIC RAY DIFFUSION THEORY

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We have investigated the method of calculating the pitch angle diffusion coefficient for cosmic-rays in a static random magnetic field using the "resonance integral" method of Hasselman and Wibberenz [1968] and Jokipii [1972]. As elaborated by Jokipii [1972] the pitch angle diffusion coefficient may be derived from the Vlasov equation via ensemble averaging to obtain the result

$$D(\mu) \propto \int_{-\infty}^{\infty} dk P(k) \frac{\sin(k-k_o)z}{k-k_o} \quad (1)$$

where $k_o = \omega_o/\mu w$, $z = \mu wt$, ω_o is the gyrofrequency in the average field $\langle B \rangle$, μ is the cosine of the pitch angle and w is the particle speed. P(k) is the wave number power spectrum of the random part of the magnetic field.

Jokipii presents arguments that for $t \gg \omega_o^{-1}$ the resonant function $\frac{\sin(k-k_o)z}{k-k_o}$ becomes effectively a delta function of k and may be replaced by $\pi\delta(k-k_o)$ to give

$$D(\mu) \propto \pi P\left(\frac{\omega_o}{\mu w}\right) \quad (2)$$

We have investigated the integral in equation (1) by means of contour integration using a power spectrum of the form $P(k) = k_c^{-1}\left[1+\left(\frac{k}{k_c}\right)^2\right]^{-\alpha}$ where $k_c = \ell_c^{-1}$, ℓ_c being the correlation length, and $\frac{1}{2} < \alpha < 1$. We obtain the integral

$$I = \frac{k_c^{2\alpha-1}}{2i} \int_{-\infty}^{\infty} dk \, \frac{e^{i(k-k_o)z} - e^{-i(k-k_o)z}}{(k+ik_c)^{\alpha}(k-ik_c)^{\alpha}(k-k_o)} \qquad (3)$$

Since there is no pole at $k = k_o$ for the total integral we may run the contour above or below the real k axis provided we do so systematically. After this choice has been made the integral may be split in two and the contour for the integral containing $e^{i(k-k_o)z}$ ($e^{-i(k-k_o)z}$) may be moved up, c_1 (down, c_2) deforming about the (now present) pole at $k = k_o$ and the branch points at $k = \pm ik_c$ as in figure 1, where it is assumed that the initial contour ran below $k = k_o$.

The pole term yields the time independent term

$$I_o = \frac{\pi}{k_c} \left[1 + \left(\frac{k_o}{k_c}\right)^2\right]^{-\alpha} = \pi P(k_o) \qquad (4)$$

which is the "resonance" result of Jokipii and Hasselman and Wibberenz. The integrals along the branch cuts may be transformed into real integrals and numerically integrated. The result is given in terms of the ratio of time dependent to time independent terms $R(\varepsilon,\tau)$ where the total integral is given by $I = I_o[1 + R(\varepsilon,\tau)]$ and where $\varepsilon = k_c/k_o$ and $\tau = k_c Z = Z/\ell_c$. Our result is

$$R(\varepsilon,\tau) = \frac{2}{\pi} \sin\pi\alpha \; \varepsilon^{1-2\alpha}(1+\varepsilon^2)^{\alpha} e^{-\tau}$$

$$\times \; [\sin(\tau/\varepsilon)F(\varepsilon,\tau) - \varepsilon\cos(\tau/\varepsilon)H(\varepsilon,\tau)]$$

$$F(\varepsilon,\tau) = \int_0^{\infty} dx \; \frac{e^{-\tau x}}{(x^2+2x)^{\alpha}[1+\varepsilon^2(1+x)^2]} \tag{5}$$

$$H(\varepsilon,\tau) = F(\varepsilon,\tau) - \frac{\partial F(\varepsilon,\tau)}{\partial \tau}$$

$F(\varepsilon,\tau)$ is a slowly varying function of τ of order unity and is plotted for $\varepsilon = 10^{-1}$, 10^{-2} and $\alpha = 3/4$ in figure 2.

We see from equation (5) that the ratio is large ($\varepsilon^{1-2\alpha} >> 1$ for $\alpha > \frac{1}{2}$), oscillating at the gyrofrequency ($\tau/\varepsilon = \omega_o t$), and decays exponentially on the correlation time scale $\tau = Z/\ell_c$ not in a time ω_o^{-1}. It would appear that for this form of the diffusion coefficient the arguments of Hasselman and Wibberenz and of Jokipii do not hold.

These authors also calculate the ad hoc Fokker Planck coefficient $<\Delta\mu^2>/\Delta t$ by calculating $<[\Delta\mu(t)]^2>$ as a function of time, dividing by the time t and demonstrating that this ratio approaches a constant asymptotically as $t\to\infty$. This constant is equal to the asymptotic value of D(u) obtained from the Vlasov equation. The expression for $<[\Delta\mu(t)]^2>$ is

$$<[\Delta\mu(t)]^2> \propto \int_{-\infty}^{\infty} dk \; \frac{P(k)}{\mu w} \; \frac{[1-\cos(k-k_o)z]}{(k-k_o)^2} \tag{6}$$

where all of the symbols have the same meanings as before.

The resonance argument now leads to the replacement of the term $[1-\cos(k-k_o)z]/(k-k_o)^2$ by $\pi z\delta(k-k_o)$ and yields

$$<[\Delta\mu(t)]^2> \propto \pi t P(k_o) \tag{7}$$

Since the remainder terms in $D(\mu,t)$ oscillate at the gyro-frequency it is not surprising that a running time average increases the rate at which such terms damp out.

It is our opinion, however, that since the derivation from Vlasov equation is of a much more fundamental nature than the ad hoc approach the slow decay on the correlation time scale more accurately represents the physics of the situation. Furthermore it should be noted that as $\mu \to o$ the time to reach asymptoticity grows without limit in both approaches.

REFERENCES

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2. Jokipii, J. R. 1972, Ap. J., 172,319.

FIGURE CAPTIONS

Figure 1. Complex k plane showing deformed contours. Original contour ran along real axis and below $k=k_o$. Contours were pushed up, C_1, around pole and upper branch cut, and down, C_2, around lower branch cut.

Figure 2. Plot of $F(\varepsilon,\tau)$ and $\varepsilon H(\varepsilon,\tau)$ vs. τ for $\alpha = 3/4$ and $\varepsilon = 10^{-1}$ and 10^{-2}.

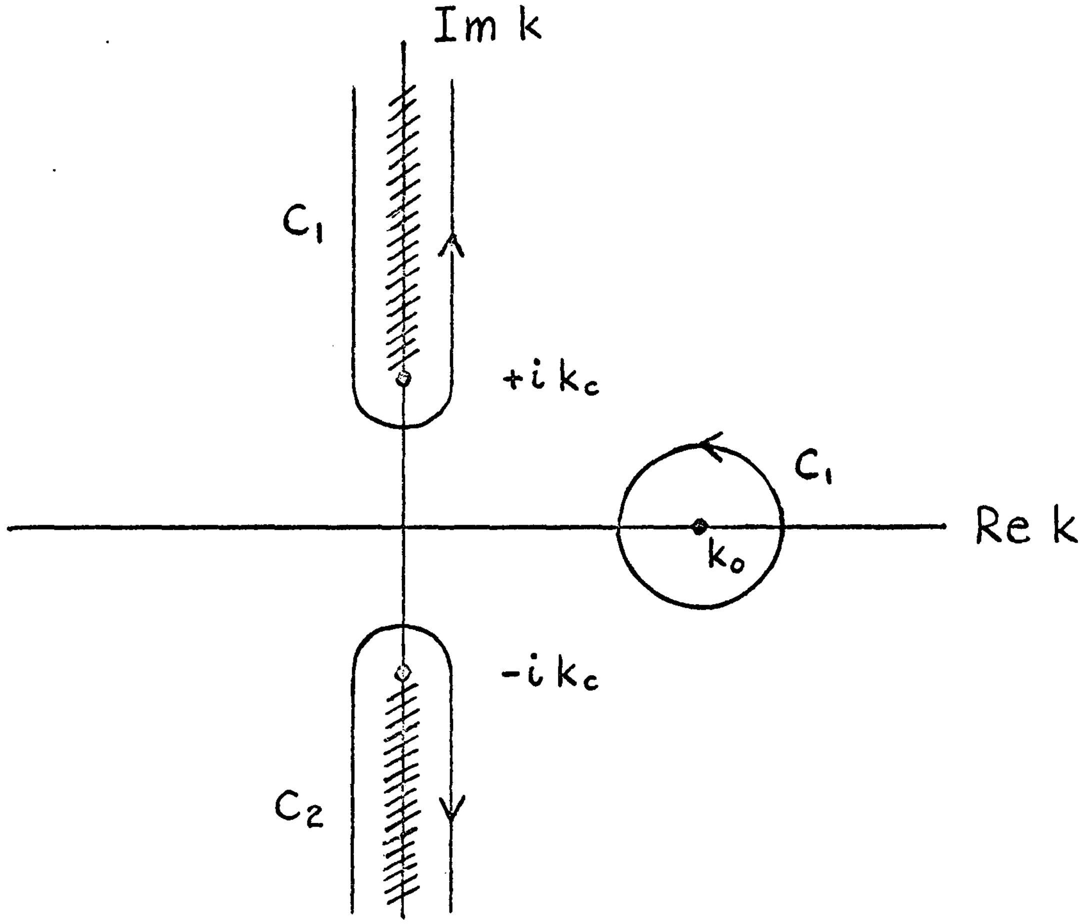

Figure 1

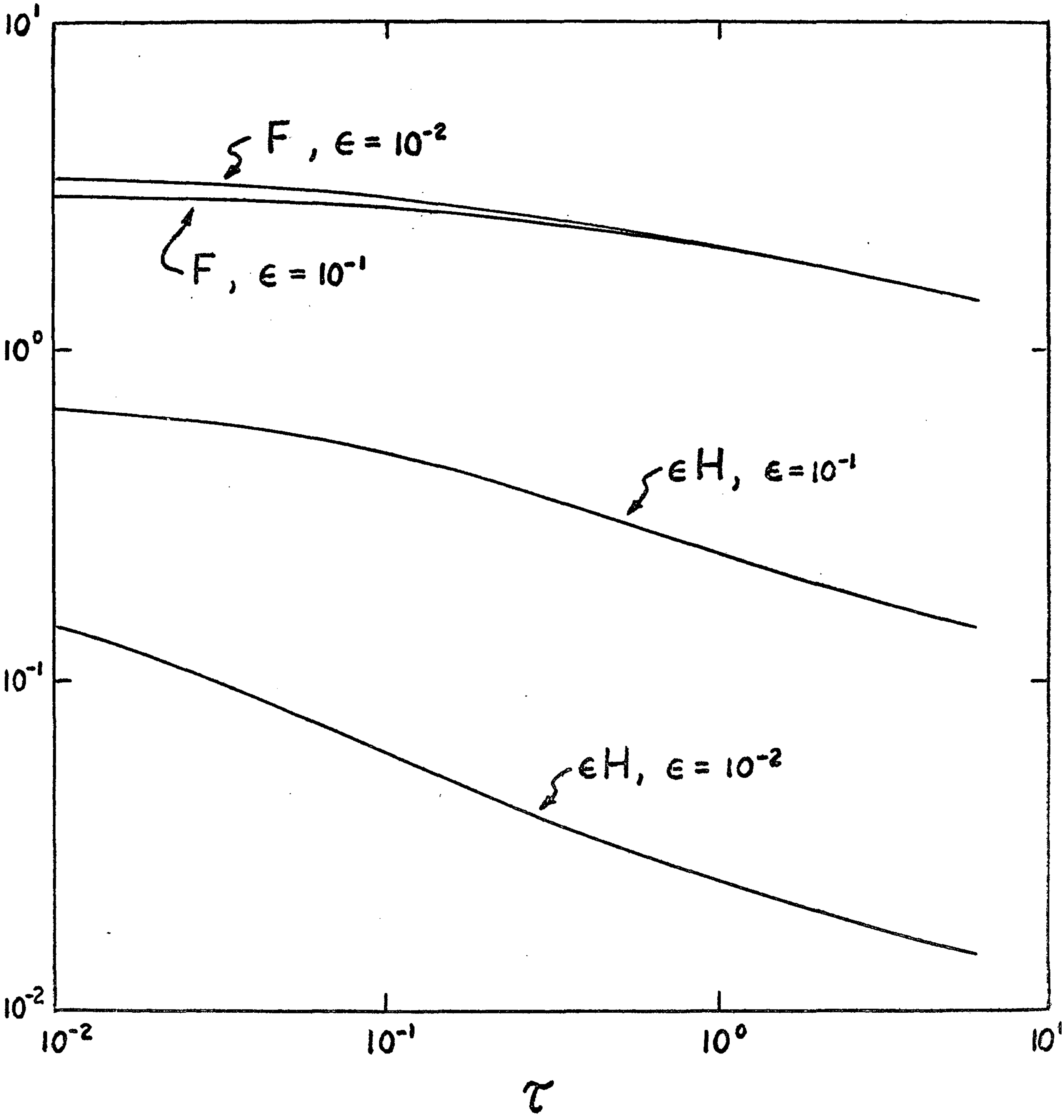

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Figure 2