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GERHARD BÖRNER

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On the Properties of Matter in Neutron Stars*

by

Gerhard Börner**

NASA/Goddard Space Flight Center
Greenbelt, Maryland 20771 U.S.A.

and

Max-Planck Institut fur Physik und Astrophysik***
8 Munchen 23
Fohringer Ring 6

* Submitted to obtain the "venia legendi" at the University of Munich

** This work was performed while the author held a NAS-NRS research-associateship at NASA/Goddard Space Flight Center.

*** Present address

Abstract

A review of recent developments in the description of neutron star matter is presented, and its relevance to pulsar observations is discussed. This review is aimed at the astrophysicist. For a detailed review of the nuclear physics involved see H. A. Bethe (1971) in Ann. Rev. Nuc. Sci., Vol. 21.

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I. Introduction

In 1968 the first source emitting a continuous train of radio pulses was discovered in Cambridge (Hewish et al. (1968)). Subsequently more and more pulsars, as they were called, have been found up to a total of 64 at present. It turned out that their periods were kept very precisely, though not as precisely as those of atomic clocks. So one has for CP1919, the first pulsar discovered, a period of

$$T = 1.337301101618 \pm 7 \times 10^{-13} \text{ (sec)}$$

(see Manchester et al., (1972)). The range of periods is rather large:

$$0.033 \text{ (sec)} \leq T \leq 3.74 \text{ (sec)} \quad ,$$

and all periods seem to increase with time

$$0 \leq \frac{dT}{dt} \leq 4.23 \times 10^{-13} \quad .$$

Let us first of all review briefly the elimination process (Maran and Cameron (1969)) by which one arrives at the conclusion that pulsars have to be rotating neutron stars:

Initially there were essentially two alternatives for explaining the observed periods of pulsars: Pulsations of very dense stars, with mean densities ρ in the range of 10^8 - 10^9 g/cc (white dwarfs, where $\sqrt{G\rho} = 10^{0.5}$ to 10 rad/sec), discussed by a number of authors (cf. Cameron, Maran (1969)). Secondly rotating neutron stars with a frequency ω such that $\omega^2 < < G\rho$ (Gold (1969); Pacini (1968)). The further possibility of dense contact binary systems, which leads essentially to the same relation between the period and the density as pulsation in the fundamental mode, was soon ruled out by the very high stability of the periods. This stability showed that there cannot be an emission of large amounts of gravitational radiation. The white dwarf pulsation hypothesis was ruled out consequently with the discovery of the fine structure of the pulses, with that of the regular rapid change of the angle of polarization during each pulse, and particularly with the discovery of the two young pulsars in supernova remnants with periods of less than 0.1 sec, which cannot be understood at all as stable pulsation of white dwarfs (Crab PSR0531-21: T = 33 ms, Vela PSR0833-45: 89 ms). Finally the slow secular increase of the periods typical for pulsars is another point in favor of the rotating neutron star hypothesis, because one would expect the loss of rotational energy to lead to a slowing down of the rotation. Rotating neutron stars also can account for the energy of the Crab Nebula (Finzi and Wolf (1969); Wheeler (1966)) and in the Vela X remnant (Rees and Trimble (1970); Börner and Cohen (1971a)). For all these reasons the model of a rotating neutron star as an explanation for the pulsar phenomenon has been generally accepted.

Although pulsars have become known only very recently, the concept of neutron stars goes back to the 1930's (Landau (1932); Oppenheimer and Volkoff (1939)) when the equilibrium of a large body consisting of neutrons was considered. The temperature was assumed to be at absolute zero, and since the neutrons follow Fermi statistics, the pressure P of a gas of neutrons is related to the number density ρ by

$$P = \frac{(3\pi^2)^{2/3}}{5} \frac{h^2}{m_n^{8/3}} \rho^{5/3} \qquad (1.1)$$

m_n: neutron mass

By integrating Einstein's equations for a spherically symmetric fluid

$$-\frac{dP}{dr} = G \frac{\left(m + 4\pi r^3 \frac{P}{c^2}\right)\left(\rho + \frac{P}{c^2}\right)}{r^2 \left(1 - \frac{2G}{c^2}\frac{m}{r}\right)} \qquad (1.2)$$

$$m(r) = \int_0^r \rho 4\pi r^2 dr \tag{1.3}$$

(r is a radial coordinate, such that the surface of a sphere of this radius is given by $4\pi r^2$) it was found that there exist stable configurations of neutrons, forming large bodies, so-called neutron stars.

That there must be a maximum stable mass becomes quite clear already from Newtonian considerations: The critical number of baryons A is reached when the addition of one more baryon (of mass m_n) will decrease the gravitational energy by an amount

$$\frac{-Gm_nA}{R} m_n \qquad \text{(R: radius of the body)}$$

larger than the gain in statistical energy

$$\frac{dE}{d\rho} = \frac{2h^2}{3m_n^{8/3}} (3\pi^2)^{2/3} \quad \rho^{-1/3} \quad (\rho \frac{4\pi}{3} R^3 = A) \quad .$$

It turns out that $A \sim 10^{58}$.

Employing Einstein's general theory of relativity decreases the maximum mass, because the pressure gradient is increased on the right hand size of equation (1.2) compared to the Newtonian form of

$$\frac{dP}{dr} = \frac{Gm\rho}{r^2} \tag{1.4}$$

Oppenheimer and Volkoff find for their neutron gas star a maximum mass of

$$M = 0.76 \ m_\odot \quad (m_\odot\text{: solar mass} \cong 2 \times 10^{33} \text{ g}) \quad ,$$

a maximum central density of

$$\rho_c = 3.8 \times 10^{14} \text{ g/cm}^3 \quad ,$$

and a maximum radius of

$$R = 9.42 \text{ km} .$$

This is the qualitatively correct picture of a neutron star: a massive, extremely dense and small object. More involved and more realistic equations of state change these parameters quantitatively, but the qualitative features remain the same.

The formation of such a small and dense object in a supernova event will also lead to a strong magnetic field. Because of the high electron number to be expected in such a star, the electrical conductivity will be very large, and the magnetic flux will be frozen in. Thus if we start with an object of radius $R = 10^{11}$ cm, $M = 1M_{\odot}$, $\rho = 1$ g/cm^3 and $B = 100$ gauss, we will end up with a neutron star of $R = 10^6$ cm, $\rho = 10^{15}$ g/cm^3, $M = 1M_{\odot}$, having a magnetic field of 10^{12} gauss.

These strong magnetic fields provide the link of communication between the rotating neutron star and the observer. All that is observable is the electromagnetic radiation, produced by charged particles accelerated in the strong magnetic fields around the pulsar and reaching us as continuous radiation or in pulsed form. No convincing model of how the pulses are formed has been put forward, but some gross features have been explained quite well. Thus it was shown by Goldreich and Julian (1969) that despite the strong gravitational attraction from a neutron star, there cannot be a vacuum outside the star (they took the magnetic field aligned with the rotation axis; the oblique rotator was treated similarly by Cohen and Toton (1971)).

Assume an interior magnetic field, which will be frozen in and which is consistent with an exterior dipole field. Because of the high conductivity of the neutron star interior, the condition that the electric field vanishes in the rest frame of the star is a good approximation. Consequently in the rest frame of an observer at rest at infinity, the electric field is given by

$$\underline{E} + \underline{V} \times \underline{B} = 0$$

Via div $\underline{E} = \rho/\varepsilon_o$, the charge-density associated with the electric field is given by

$$\rho = -2\, B_o\, \omega\, \varepsilon_o \cos\theta \quad (\theta\text{: angle between B and } \omega)$$

If it is assumed that the neutron star is surrounded by a vacuum, the solution of Maxwell's equations in the vacuum outside has to be matched to the interior solution via continuity of the magnetic field component normal to the surface and of the tangential component of the electrical field. It is found then that the quantity $\underline{E} \cdot \underline{B}$, which is zero inside the star, does not vanish outside. On the contrary

$$\underline{E} \cdot \underline{B} \sim R\, \omega\, B_o^2 \quad .$$

Thus near the surface charge layer of the neutron star the electric force along the magnetic field exceeds the gravitational force by a large factor of the order of 10^{13} for electrons and 10^{10} for protons. These ratios were obtained by using parameters typical of the Crab pulsar PSR0531-21 ($B \sim 10^{12}$ gauss, $\omega \sim 200\ \text{sec}^{-1}$, $R \sim 10$ km, $m = 1m_\theta$). Thus if the surface region is ionized, the surface charge layer cannot be in dynamical equilibrium. A rotating magnetic neutron star must possess a magnetosphere, composed of charged particles traveling along the magnetic field lines. What we observe is the radiation from these charged particles injected into the magnetosphere.

No satisfying quantitative description of the electromagnetic link between the rotating neutron star and the radiation pattern of the pulsar has been given so far. Indeed, not even the case of a magnetosphere of radiating particles, where the axis of the magnetic field coincides with the rotation axis of the neutron star, has been solved. Whereas to explain the pulse producing mechanism one would have to treat the much more complicated case of at least a slight deviation from axial symmetry.

In the absence of a convincing pulsar mechanism theory the observations permit only a few rather crude conclusions on the physical properties of the rotating neutron star. For 22 of the 61 discovered pulsars, both frequency ω, and change of frequency $\dot{\omega} \equiv \frac{d\omega}{dt}$ have been measured. Then by determining their rate of loss of energy

$$E = I \omega \dot{\omega} ,$$

we could in principle find the moment of inertia I of these neutron stars. This in turn would precisely fix mass and density profile of the star according to the equation of state used. Although the observations are not exact enough to permit definite conclusions in this line of investigation, certain limits on the physical parameters of a realistic neutron star can be derived. So as we proceed in the following sections to describe the physics of neutron star matter in the different density regimes, we shall always try to make clear how the different assumptions about the properties of matter affect the models of neutron stars and how these in turn relate to astrophysical observations. We hope then for a subsequent feedback of astronomical information on ideas about the fundamental structure of matter at high densities.

This paper is set up in the form of a review, trying to give an up-to-date survey of part of the work that has been done recently in this field. Although even for the last three years only an incomplete survey of the existing literature could be given, I have tried to incorporate the important ideas. Much of the material covered exists in the form of preprints or has been published quite recently. Several new ideas, speculations and criticisms of my own are incorporated too. These reflect the outcome of numerous inspiring discussions which I have had with many of my distinguished colleagues.

I would like to take this opportunity to express special gratitude to Hans Bethe (Ithaca), Al Cameron (New York), Jeffrey Cohen (Philadelphia), Ludwig Biermann, Peter Kafka, and Friedrich Meyer (Munich).

II. Qualitative Description of the Interior of a Neutron Star

(II.1) Validity of the "isotropic fluid" approximation

When we describe a star which contains superdense matter, we obtain the equation of state (pressure P as a function of density ρ and temperature T), from the local physics (two particle interactions, etc.) without taking into account the gravitational field. We thus separate the influence of "global physics" - the gravitational field produced by this matter configuration via Einstein's equations - from our local physics, although the gravitationally induced binding energy in heavy neutron stars amounts to $\sim$ 250 MeV per particle, or $\sim$ 25% of the rest mass energy. So the influence of the gravitational field seems to be rather strong, and we must ask in how far the separation introduced to obtain nonrelativistically $P(\rho)$ is a good approximation to reality.

Obviously the concept of deriving an equation of state non-relativistically and plugging it into Einstein's equations would lose its validity if the gravitational potential varied strongly over distances of the order of

$$\frac{\hbar}{m_n c} \sim 10^{-13} \text{cm}$$

(G: gravitational constant; h: Planck's constant). Investigations of a system of fermions with gravitational interaction (Bondi (1964); Bonazzola, Ruffini (1969)), which were treated in a certain approximation within the framework of general relativity, showed that the gravitational potential varies strongly over distances of 10^{-8} cm only if the density is well above 10^{42} g/cm^3. But densities in the center of neutron stars are always less than 10^{16} g/cm^3, and therefore one is on safe grounds when employing the usual procedure of finding $P(\rho)$ from local physics and then putting it into Einstein's equations to determine global effects on the star.

(II.2) Qualitative picture of the interior

A cross-section through a neutron star would approximately look like the picture drawn in Fig. 1, where the mass density of the star increases with depth.

In the different approaches for treating a system of nucleons with interaction there is general agreement on the qualitative features, despite some quantitative disagreement. As indicated in Fig. 1 there are several different states of matter (see e.g. Cameron's review article, 1970):

1. Under an atmosphere of a thickness of a few m (whose properties may be drastically influenced by the specific surface structure of the neutron star, see section III) we have

2. a solid crust composed of neutron-rich nuclei arranged in a lattice and a degenerate electron gas. Up to a certain density ρ_1 the charge Z and mass A of the nuclei is determined by equilibrium between β-decay and electron capture.

3. For densities above ρ_1 unbound neutrons diffuse out of the nuclei ("neutron-drip"), and between ρ_1 and a much higher density ρ_2, we have equilibrium between free neutrons, neutron-rich nuclei in a lattice, and electrons.

4. For densities above ρ_2 the nuclei disappear, and we have a small number (a few percent) of protons (and, of course, the same number of electrons) imbedded in the neutron sea. At a still higher density muons appear.

5. Finally at a density ρ_3 and above new baryons probably make their appearance and hyperons like Σ^-, Λ^o, Ξ^-, Σ^o, Ξ^o, . . . etc. will be present together with neutrons, protons, and electrons.

The uncertainties in the quantitative analysis throughout the different regimes 1 to 5 increase monotonically with density.

From computations of the cooling process of neutron stars (Cameron, Tsuruta (1966)) one finds that neutron stars cool very quickly from the initial, very hot state of formation down to about 10^8 ^oK in the interior (the surface will be cooler still, probably $\sim 10^5$ ^oK (Cameron,(1970)). The thermal kinetic energies of $\sim$ 10 keV are therefore negligible compared to the several MeV per particle of nuclear energies or Fermi energies involved. In that respect, therefore, a neutron star is a very cold system. It is a good approximation to neglect the thermal energies and to consider the matter in neutron stars at absolute zero, $T = 0$.

III. Atmosphere and Surface of a Neutron Star

(III.1) The atmosphere

In the introduction we described the picture of Goldreich and Julian, where in the stationary state of a rotating magnetic neutron star ions and electrons continuously stream out along the field lines. Thus near the surface of the star there will be an atmosphere, which is thought to consist mainly of Fe^{56} - the endpoint of nuclear burning - and electrons (detailed models of the composition have been given by Rosen (1968, 1969)). The scale height in the atmosphere is determined from

$$\frac{d\rho}{dr} = -\rho g \frac{dp}{dr} , \tag{3.1}$$

where $g = \frac{GM}{R^2}$ is 10^{11} times the earth's g. If we neglect ionization the pressure is

$$P = \rho \frac{kT}{Am_n} \qquad (A = 56 \text{ for } Fe^{56} , m_n\text{: neutron mass}) ,$$

so the scale height is of the order of a few cm. Hence the density increases rather rapidly, and finally with growing pressure the matter present will be fully ionized.

With increasing density the matter is gradually solidifying and continuously merging into a solid crust. In this region which corresponds to densities in the range $5 \leq \rho \leq 10^4$ g cm^{-3}, an equation of state due to Feynman et al. (1949) has usually been applied. In this approach a Thomas-Fermi model for atoms under pressure with a radius determined by the density is employed, but no effects of the magnetic field are considered.

Recent work (Ruderman (1971); Mueller et al. (1971)), however, has made it clear that because of the extremely strong magnetic fields near the surface of a neutron star, the gradient of the density becomes much steeper than one had previously thought. Also the general validity of the picture of a gaseous atmosphere as drawn above has become rather doubtful. This controversial point will be discussed in the following.

(III.2) The surface

Ruderman (1971) investigates the surface of a neutron star in the presence of a strong magnetic field. He finds that in huge magnetic fields ($B \geq 10^{12}$ g) matter forms a tightly bound, dense ($\sim 10^4$ g cm^{-3}) solid with the properties of a one-dimensional metal and a work function of the order of a keV. This model for atoms in strong magnetic fields is quite simple:

Assuming cylindrical symmetry around a uniform magnetic field B, one finds that the motion of charged particles perpendicular to the field is restrained. The particles can only move in certain quantized orbits (Landau orbitals) of radius

$$\rho_n = (2n + 1)^{1/2} \rho \quad (n = 0, 1, 2, \ldots) \tag{3.2}$$

and

$$\rho = \left(\frac{hc}{eB}\right)^{1/2} = \frac{2.6 \times 10^{-4}}{B^{1/2}} \text{ cm} \tag{3.3}$$

The higher states are excited by integer multiples of

$$\frac{ehB}{mc} \sim 12\ B_{12}\ \text{keV}$$

(where B_{12} is the magnetic field in units of 10^{12} Gauss). This high excitation energy assures that in the description of atoms in the stellar surface only the ground state is important. The lowest energy state of a single atom is realized by having the electrons in Landau orbitals which keep them (in directions perpendicular to B) much closer to their nuclei than the Coulomb field of the nucleus would by itself. The resulting atoms are shortened perpendicularly to B, and elongated along B. The lowest energy state of an assembly of such atoms is reached when they coalesce to form a tightly bound one-dimensional lattice parallel to B surrounded by a cylindrical electron sheath. This can be explained in two qualitatively different ways (the quantitative answer is the same in both cases!). Kaplan and Glasser (1972) consider an electron gas against a uniformly charged positive background in a strong magnetic field. When the Larmor radius for an electron $\frac{eB}{mc}$ becomes smaller than the radius of the (spherical) volume per electron at the density considered, the system is essentially a dilute electron gas normal to the magnetic field and should undergo a "Wigner" transition to an ordered state, because then the quantum mechanical exchange correlation energy will dominate over the free-electron kinetic energy. This ordered state should resemble a two-dimensional lattice of charged rods with each rod behaving as a linear electron gas. The magnetic field B required for such a state is in the range of 2.5×10^{11} to 2.5×10^{13} G. Ruderman (1971), on the other hand, considers the energy of nuclei in a uniform density of electrons. The unscreened Coulomb repulsion between the nuclei is minimized by a bcc type lattice configuration.

A satisfactory quantum mechanical theory amalgamating these two points of view has yet to be done. We can, however, estimate the energy per atom from the classical Coulomb energy of a system of Z-charge nuclei sitting "like pearls on a string" (Ruderman (1971)) surrounded by a uniform density cylinder of electrons (radius $\sim 2\ell$). One finds

$$E_a = -\frac{(Ze)^2}{\ell} \times 1.2 \quad , \tag{3.4}$$

where the lattice constant ℓ is given as

$$\ell = 2.4\, a_o\, Z^{-1}\, \eta^{-4/5} \tag{3.5}$$

in a region (i) where

$$\eta = \left(\frac{B}{4.6 \times 10^9 Z^3}\right)^{1/2} >> 1$$

and

$$\ell \sim a_o\, Z^{-1}\, \eta^{-12/15} \tag{3.6}$$

for $1 >> \eta >> Z^{-3/2}$ (region (ii)). a_o is the Bohr radius.

E_a greatly exceeds the binding energy of an isolated atom (in region (i) and (ii)), and therefore equation (3.4), resp. (3.6), give the binding energy of an atom in the lattice. Thus iron nuclei ($Z = 26$) at the surface will be bound with an energy of - 30 keV per atom, if we assume a magnetic field of 5×10^{12} g.

Adjacent chains will have strong Coulomb attraction when one is displaced half a lattice length along B relative to the other. An array of chains will then cohere so that the nuclei form a body centered orthorhombic lattice which is almost bcc. The mass density will be (Fe^{56}, $B = 5 \times 10^{12}$ g)

$$\rho = 4 \times 10^4 \text{ g cm}^{-3} \quad .$$

The temperature of the surface will be $\sim 10^5$ ^oK and evaporation of ions will be practically impossible. Hence there is a very sharp transition from a diffuse atmosphere, which contains probably mostly electrons, to the solid surface of the neutron star. This surface would then look at close examination rather similar to the skin of a not very smoothly shaved porcupine, with chains of Fe-atoms sticking out in all directions along the field lines.

Let us now discuss the consequences of Ruderman's (1971) model for pulsar observations. If ions cannot get out, then only electron currents would flow in the magnetosphere. The electrons are accelerated away from the surface by the electric field that is produced there (by the rotation of the magnetic field vector),

$$E = \frac{R\,\omega\,B}{c} \tag{3.7}$$

According to Goldreich and Julian (1969) particles stream out continuously from the surface of the neutron star, and a lower limit for the Crab pulsar ($\omega = 200$) seems to be 10^{33} particles/sec. If these particles are all electrons, they would in time build up an electric field opposite to the one given in (3.7). Thus the electric field that drags out charges from the neutron star will be weakened, then nulled, and no longer will charges of any kind flow out. The time scale can be estimated by simply looking at how long it would take for the surface charge of

$$N = \frac{B\omega R}{4\pi c} 4\pi R^2 \tag{3.8}$$

to be dissipated. Putting in numbers for the Crab pulsar we find

$$\frac{N}{10^{33}} = 0.03 \text{ sec} \tag{3.9}$$

Thus the outflow of charges would cease after a time of less than 0.1 sec, and the pulsar would stop, if positively charged ions cannot get away from the surface.

Evidently Ruderman's (1971) description has very drastic consequences for pulsars. These consequences cannot be avoided by the assumption of an atmosphere composed of elements lighter than Fe^{56} which would not solidify by themselves (Mueller et al. (1971)), because such an atmosphere will be transported away very quickly, and then the same problem as before has to be faced. If we do not want to invent a pulsar mechanism which is qualitatively different from the model proposed by Goldreich and Julian (1969), we have to find a way to get ions out of the surface.

Let us try: What energy do we gain in moving a positively charged ion over the lattice distance ℓ? In region (i), i.e., using (3.5), we obtain from (3.7)

$$eE = 10^{-10}\,\omega \left(\frac{B}{B_o}\right)^{-1/5} \text{ (ergs)}, \tag{3.10}$$

where we introduced $B_o = 5 \times 10^{12}$ gauss. This energy has at least to equal the binding energy, which in region (i) is

$$E_a = -10^{-7} \left(\frac{B}{B_o}\right) 2/5 \qquad \text{(ergs)} \quad . \tag{3.11}$$

We find a lower limit for ω

$$\omega \geq 10^3 \left(\frac{B}{B_o}\right)^{-1/5} \text{sec}^{-1} \; . \tag{3.12}$$

This result indicates that for none of the known pulsars ($\omega_{max} = 190$ sec^{-1}) particles can be extracted from its surface, a contradiction to the observations, which directly establish that at least the Crab ($\omega = 190$) and Vela ($\omega = 70$) pulsar send out a flow of charged particles.

If we look at region (ii), the situation gets slightly better, but the validity of this approximation remains dubious because here $\eta \approx 0{,}3$ and for (3.6) to hold we must have $\eta << 1$. This time we get

$$\omega \geq 200 \left(\frac{B}{B_o}\right)^{7/5} \tag{3.13}$$

So the Crab pulsar barely makes it. One can, of course, juggle around with B and try by variation of B to bring ω closer to realistic values. But in (3.12) ω does not vary strongly with B, and in (3.13) B would have to be as low as 10^{11} Gauss, if we want to incorporate the slowest pulsars. This would then invalidate Ruderman's considerations anyhow. Then it would seem that Ruderman's concept cannot be reconciled to the qualitative picture of a pulsar as proposed by Goldreich and Julian.

There is, however, still another method to extract ions from the neutron star. The electric field exerts a pressure $\frac{E^2}{4\pi}$ on each atom, pulling at the interface of 2 atoms in the chain, and if this force is big enough to break the chain at any one point, the whole piece of the chain will be lifted up from the surface of the neutron star. The energy gained by lifting such a piece of a chain of atoms' over one lattice distance is

$$\frac{E^2}{4\pi} \ell^2 \cdot \ell \, , \tag{3.14}$$

and analogously to the foregoing considerations we find for ω in region (i)

$$\omega \geq 200 \left(\frac{B}{B_o}\right)^{-1/5} \tag{3.15}$$

and in region (ii)

$$\omega \geq 30 \left(\frac{B}{B_o}\right)^{7/5} \tag{3.16}$$

Again for region (i) there is, even through strong variations of B, no way to bring ω down to 2 (PSR 0525 + 21 has $\omega = 1.7$). In region (ii) the situation is more favorable; even without varying B the three fastest pulsars have the ω required by (3.16). When we let B go as low as 7×10^{11} gauss, then all the pulsars observed so far could work in the currently accepted way and have a solid surface of the kind predicted by Ruderman (1971). However, estimates of the magnetic field in pulsars give field strengths around 3×10^{12} gauss (assuming dipole fields), and one would expect the actual fields near the neutron star surface to be still larger. Furthermore, there does not seem to be a correlation between magnetic field strength and period of the pulsar, as we would predict it here.

I would like to point out that it is, indeed, still an unsolved problem to reconcile Ruderman and Goldreich and Julian, especially in view of the fact that the approximation of region (ii) is probably not too reliable, and if one relies on the approximation of region (i) the discrepancies are evident.

It may nevertheless be interesting to point out that in the model developed above, electrons would flow out from the pulsar as single charges, whereas the ions would come out in little chunks, each piece consisting of 10 to a few 100 Fe^{56} atoms.

IV. Nuclear and Solid State Physics in the Crust

(IV.1) The range of the densities below the neutron drip line

Below the surface densities are immediately greater than 10^4 g cm^{-3}, and the nonrelativistic Fermi energies of electrons increase quickly beyond 10 keV. Since we expect temperatures in these regions of less than 10^8 ^oK, the electrons form a degenerate plasma. This makes the star optically thick, because photons (w, k) can only propagate if

$$w^2 = c^2 k^2 + w_p^2 \quad , \tag{4.1}$$

where the plasma frequency

$$w_p^2 = \frac{4\pi \; n_e \; e^2}{me} \tag{4.2}$$

increases as $\rho^{\frac{1}{2}}$. Already for $\rho \geq 2 \times 10^5$ g cm^{-3} one has that

$$\frac{\hbar w_p}{k} > 10^8 \; {}^oK$$

and photons can no longer be produced by thermal excitation. So we will find no photons inside a neutron star except in the outermost few meters.

At a density of $\rho \geq 10^7$ g cm^{-3} the electrons become completely relativistic. Nuclei will no longer be screened by clouds of electrons, but rather the negative charges will form a uniform background. The nuclei will then feel their relative Coulomb charges, repel each other, and in trying to minimize their energy, arrange themselves in a lattice, probably of bcc type. So the surface (almost bcc)lattice induced by the magnetic field will be replaced by a Coulomb lattice here. The lattice energy $\sim Z^{2/3} \frac{e^2}{hc}$ is negligible with regard to the energy balance at lower densities, but becomes important in determining the most stable nucleus.

At still higher densities above 8 x 10^6 g cm^{-3} the electron capture process

$$p + \bar{e} \rightarrow n + \nu \tag{4.3}$$

becomes energetically more favorable then the inverse reaction, the β-decay of the neutron. The high Fermi levels of the electrons make the neutron into a stable particle. New equilibrium configurations turn up where Fe^{56} is no longer the most stable nucleus, and more and more neutron-rich nuclei appear. These nuclei would be unstable under laboratory conditions, but here they are stabilized by the high Fermi levels of the electrons.

Recently Baym et al. (1971a) re-determined the most stable nucleus present at a given density under these conditions (Table 1). A more detailed treatment of this region, however, would have to take into account the way in which the species of nuclei was determined at the time it was frozen into the lattice during the cooling down of the initially very hot neutron star (the temperature of the very young star was $\sim 10^{11}$ to 10^{10} °K, enough to melt the lattices considered here). Probably the most stable nucleus was not always realized, and certain defects might be in the lattice giving rise to creep phenomena. Sophisticated solid state physics would have to be employed in describing the crust.

Already, however, the simplified picture given by Bethe et al. (1970) can explain some basic physical properties quite adequately, and we therefore briefly report on that paper. The total energy is a sum of the energy of nucleons in nuclei E_N, the free electron energy, and the lattice energy. The energy per nucleon E_N/A is given by the semiempirical mass-formula (Myers, Swiatecki (1968))

$$E_N/A = -c_1 + c_2 Z^2 A^{-4/3} + c_3 \frac{(N-Z)^2}{A^2} + c_4 A^{-1/3} \qquad (4.4)$$

(N: neutron number; Z: proton number, A = N + Z). Secondly for stable nuclei we have an equilibrium between β-decay and electron capture:

$$E_p + E_e = E_n + (m_n - m_p) c^2 \qquad (4.5)$$

(subscript (p,e,n) for (proton, electron, neutron) resp.).

The Fermi energy, or the chemical potential, as the energy of the highest occupied state, is given by

$$\mu_n = \frac{\partial E_N}{\partial N} \qquad \mu_p = \frac{\partial E_N}{\partial Z} \qquad (4.6)$$

for neutrons and protons respectively. Equation (4.5) must also hold for the Fermi energies

$$\mu_n - \mu_p = \mu_e \tag{4.7}$$

The small neutron-proton mass difference is omitted. It does not influence the results appreciably because μ_e will turn out to be about 20 MeV or more. At higher densities the electrons are a highly relativistic Fermi gas, and therefore

$$\mu_e = c\, \rho_N^{1/3}\, x^{1/3} \tag{4.8}$$

where $x = Z/A$, ρ_N: number density of nucleons bound in nuclei; c, a constant.

When we now determine the minimum of E (neglecting the small lattice energy

$$E_L \sim - \frac{2Z^2e^2}{a} \quad , \; a^3\, \rho_N = 2),$$

we find all quantities as functions of x:

$$A = \frac{c_4}{2c_2}\, x^{-2} \quad , \tag{4.9}$$

a remarkably simple formula. A increases with decreasing x (increasing density); even Z = Ax increases. The neutron Fermi energy is a monotonically decreasing function of x, for $x > 0.04$. μ_n is zero for $x = 0.32$, while the total energy at that point is still negative $E/A = -1.6$ MeV. For x lower than 0.32, μ_n is positive; therefore, free neutrons appear and matter consists no longer only of nuclei, but of nuclei immersed in neutron matter. The density ρ_1 of this neutron drip line was found by Bethe et al. (1970) to be

$$\rho_1 = 2.8 \times 10^{11} \text{ g cm}^{-3} \qquad (4.10)$$

in agreement with other authors (Cohen et al. (1969)). Baym et al. (1971b) using a slightly different mass formula find $\rho_1 = 4 \times 10^{11}$ g cm^{-3}.

(IV.2) Neutron drip line to break-up of nuclei

The number-density of free neutrons increases very rapidly with density, whereas the density of protons and electrons does not change rapidly at first and is then always a few percent of the neutron density in this region. Therefore the revised approach of Leung and Wang (1971) to treat the nuclear matter in this region as consisting purely of neutrons (neutron matter) seems reasonable.

In reality one has to deal with a system of electrons, free neutrons, protons and neutrons bound in nuclei. This is a typical case of nuclear matter and can best be treated by the many-body methods of the Brueckner-Bethe-Goldstone theory (see e.g. Day (1967)). This theory is essentially a sophisticated perturbation technique, adapted to many-body problems. Starting from a two-particle interaction described by a potential the two-particle correlations are computed. The ultimate aim is to describe real nuclei; but so far one has just been working on the reproduction of the properties of nuclear matter (N = Z) of infinite extension. In equilibrium (as one knows from large nuclei) the binding energy should be -16 MeV, the average particle distance $r_o = 1.12$ fm, and the Fermi momentum k_F ($\rho = \frac{2}{3\pi^2} k_F^3$) = 1.36 fm^{-1}. The potential used to describe the interaction of two nucleons is - in most of the cases discussed here - the Reid soft-core potential (Reid (1968)), which fits scattering data very well, and gives reasonably good values for the binding energy of nuclear matter: -9 MeV in the approach of Nemeth and Sprung (1968), -11 MeV by the improved computational methods of Siemens (1970). To reproduce the exact value of -16 MeV corrections are introduced in all interactions of isospin T = 0 and T = 1, as in approximation (1a) of Nemeth and Sprung (1968), where thepotential energy of all two-body states is multiplied by a factor of 1.22, or just in the T = 0 channel, as e.g. in (1b) of Nemeth and Sprung (1968) or Siemens (1970). The second approximation seems to be rather more accepted at present, since many authors feel that the discrepancy lies mainly in the tensor force in the $^3S_1$ - $^3D_1$ state (T = 0 interaction). Matter consisting mainly or purely of neutrons should then be well described by using the nuclear matter calculations with the T = 0 channel switched off.

It should be stressed here that the attempts to find agreement between theory and experiment in the nuclear matter many-body calculations are meaningful only when a reliable method of computation has been established. One has to show explicitly that higher order terms, that have been neglected in a two-body correlation computation, contribute much less than the terms considered. Near the saturation density of $\rho = 2 \times 10^{14}$ g cm^{-3} it has been shown mainly through the work of Bethe that in the framework of the BBG method the third-order correlations are indeed small. (Rajaraman, Bethe (1967)). This has not been done in any other scheme, and, therefore, the BBG method is to date the one reliable method for doing nuclear matter computations. In Figure 2 the results of several such neutron matter computations for the equation of state can be seen (curves (2), (3), (5), (6)). They all agree remarkably well in the region of 10^{12} g cm^{-3} to 10^{14} g cm^{-3}.

The clustering of some nucleons into nuclei does not change the pressure-density relation appreciably in this region (as e.g. shown in Börner and Sato (1971)). The question of the equilibrium species of nuclei coexisting with neutron matter has, however, great interest in itself. Also the lattice formed by nuclei in the outer layers of a neutron star determines the elastic properties of that region. This problem was first treated by Cohen et al. (1969), who took the Levinger-Simmons (1961) potential with constants adjusted by Weiss and Cameron (1969) to fit certain nuclear matter results. They do not use many-body techniques but calculate nuclear matter energies by simply taking first order expectation values of the energy (for a detailed discussion see Bethe et al. (1970)).

Bethe et al. (1970) treated the problem again and employed calculations by Nemeth and Sprung (1968) carried out using full nuclear matter theory and the Reid (1968) soft core potential.

Stable nuclei can exist simultaneously with neutron matter only if the Fermi energies of the neutrons are the same in the nuclei (μ_{nN}) and in the neutron gas (μ_{nG}):

$$\mu_{nN} = \mu_{nG} \tag{4.11}$$

If this equality were not true, e.g., if $\mu_{nN} > \mu_{nG}$, then neutrons would evaporate from the nuclei until (4.11) is fulfilled. Since $\mu_{nG} > 0$, one finds that μ_{nN} must also be positive. This was already realized by Harrison et al. (1964).

C

Nuclei with $\mu_{nN} > 0$ are unfamiliar; however, they are easily interpreted physically. μ_{nN} is the largest energy of any neutron in the nucleus, but it is still only a few MeV, whereas the kinetic energy of the same neutron in the nucleus is still about 30 MeV. Therefore for most of the neutrons in the nucleus, the total energy ϵ is still negative; they are bound in the usual sense. A second class of neutrons will have energy $\epsilon > 0$, but high angular momentum ($\ell > 4$); these neutrons feel a potential barrier preventing them from leaving the nucleus in spite of their positive energy; they may thus still be considered as essentially bound. A third kind of neutron with $\epsilon > 0$ and low angular momentum will essentially be able to go freely between nucleus and neutron gas. Thus for $\mu_{nN} > 0$ the nuclei have the somewhat unusual property that some neutrons can pass freely in and out, but the nucleus still forms a compact structure in the surrounding uniform neutron gas.

The electron Fermi energy is again given by the formula for the ultrarelativistic Fermi gas, and in simple neutron matter (i.e. without imbedded nuclei) there must be β-equilibrium, hence the proton Fermi energy in neutron matter is

$$\mu_{pG} = \mu_{nG} - \mu_{eG} \tag{4.12}$$

In fact, usually the concentration of protons in neutron matter is calculated by determining the proton Fermi energy which is mainly due to the potential energy of a proton in the neutron gas and is therefore strongly negative; then the electron density must be chosen so that (4.12) is fulfilled.

In a mixture of neutron matter and nuclei the density of protons in the neutron gas phase is zero and hence the proton Fermi energy is less than for neutron matter of the same neutron density. However, this difference is very small; it is due to the proton kinetic energy

$$T_p = \frac{2\mu_e}{2M_p c^2} \quad . \tag{4.13}$$

If one takes into account the Coulomb energy of the lattice of nuclei, this difference again almost cancels out (Baym et al. (1971b)).

It was found by Bethe et al. (1970) that at a density of $\rho \sim 5 \times 10^{13}$ g cm^{-3} the proton Fermi energy in the gas (μ_{pG} = -58 MeV) decreased below that of protons in the nuclei. Thus at that density there was a sharp transition, where protons bound in nuclei distributed themselves in the surrounding neutron gas, dissolving the nuclei. Bethe et al. (1970) neglected both the proton kinetic energy and the lattice energy of nuclei. Langer et al. (1969) took the proton kinetic energy into account and found the transition to occur more smoothly but also in a narrow region around the same density. Including also the lattice energy again gives a sharp transition (Baym et al. (1971b).

In both cases the nuclei were described by a semiempirical mass formula, and the dependence of the surface symmetry energy and the Coulomb energy term on the density of the neutrons outside was neglected (Cohen et al. (1969)) or underestimated (Bethe et al. (1970)).

Baym et al. (1971b) carried on this earlier work and found that to determine the nuclear size the Coulomb energy of the lattice formed by the nuclei becomes more and more important. They changed some terms in the semi-empirical mass formula to obtain better fits to measured nuclear radii - which is a dangerous thing to do - and they included besides the Coulomb lattice energy the dependence on the outside neutron density of the Coulomb energy, which tends to favor small A nuclei, and of the nuclear surface energy, which tends to favor nuclei with large A. They use the improved neutron matter calculations of Siemens (1970) and Siemens and Pandharipande (1971). They find that as the density increases the nuclei fill more and more of space. Finally at a density of 2×10^{14} g cm^{-3} they begin to touch. With further increase in density the nuclei disappear discontinuously in a first order phase transition around 3×10^{14} g cm^{-3}. At that point, as can be seen from Table 2, they obtain nuclei with $Z \sim 120$, $A \sim 2500$.

Barkat and Buchler (1971), on the other hand, base their approach on a Thomas-Fermi model for nuclei and on BBG nuclear matter calculations and find the proton number Z to be relatively small ($Z \sim 33$ to 36). They do not determine the much more uncertain values of A. In agreement with Baym et al. (1971b) they find that the nuclei disappear by way of a first order phase transition around normal nuclear density $\rho_2 = 3 \times 10^{14}$g cm^{-3} (both approaches find contrary to Bethe et al. (1970) that always $\mu_{pG} > \mu_{pN}$).

The divergence in the results seems to indicate that the question of which type of nuclei is present near the point where they disappear has not been settled yet, and further investigations will be necessary (Hartree-Fock calculations are being carried out by Negele and coworkers at MIT now - H. A. Bethe, personal communication).

The properties of the equation of state are, however, not appreciably affected by these different results obtained for the shape and size of the nuclei. The nucleon-nucleon interaction is the most important feature. As can be seen from Fig. 2, for densities up to normal nuclear density all the equations of state that are obtained via realistic nuclear matter calculation, where one reliable and well-tested method is the BBG method, agree rather well. Even Leung and Wang (1971) who treat the case of pure neutron matter, with no protons or other particles, do not depart from this general picture. Other equations of state obtained with the same methods but different potentials agree rather well too.

(IV.3) Superfluidity

In the range below normal nuclear densities, the neutrons lying at the top of the Fermi sea have an attractive potential for one another. This attractive force between neutrons arises from the $^1S_0$ interaction, which changes its phase shift from attractive to repulsive at normal nuclear densities. Thus below $\rho_2 = 3 \times 10^{14}$ g cm^{-3} pairs of neutrons will form, very similar to electron pairs forming a superconductor. One expects that the free neutrons in the crust will probably form a superfluid, because of this mutual attraction between pairs of particles. Similarly the protons will form a superfluid at about the same densities (see Cameron (1970) for extensive references).

In the spectrum of single-particle energy states immediately above the Fermi surface an energy gap of about 1 MeV will form in the superfluid. Clark and Yang (1971) basing their computations on the Bogoliubov-Valatin transform find a gap energy of 3.3 MeV at a density of 6×10^{13} g cm^{-3}.

Thus the gap energy is of the same order as the neutron Fermi energy and it may be that pairing effects can considerably influence the properties of neutron matter at those densities.

It has been pointed out that the rotation of the neutron star imposes interesting conditions on the superfluids. The superfluid will normally not partake in the rotation, but quantized vortex lines will be established throughout the interior. It is expected that the separation between quantized vertex lines will be $\sim 10^{-2}$ cm, and the radius of a core of a vertex line $\sim 10^{-12}$ cm (Baym et al. (1969)). Whereas the superfluid by itself could not be expected to contribute to the moment of inertia of the star, the mixture of superfluid and quantized vortex lines will probably rotate as a rigid body.

The crust, with a Coulomb lattice of nuclei through which the superfluid neutrons move, presents many interesting problems to the solid state physicist. There are indications that some of the properties of the crust manifest themselves in events that can be observed in pulsars (see section IV.4).

At normal nuclear density the $^1S_0$ phaseshift changes sign, the neutron-neutron $^1S_0$ interaction becomes repulsive, and this type of superfluidity will disappear.

But then, above 2×10^{14} g cm^{-3} a significant attraction between pairs appears in the $^3P_2$ state. Thus a new type of superfluidity would be present up to very high densities. However, this superfluid state would be anisotropic (i.e., different energy gaps in different directions), a type of superfluidity that has not been found yet in the laboratory.

The energy gap would be of the order of 0.5 MeV (Hoffberg et al. (1970)).

(IV.4) Observational evidence for solid state phenomena in pulsars

Besides the general slowing down of the pulse rate, two pulsars exhibited a sudden increase in frequency. In March 1969 the Vela pulsar PSR0833-45 showed a sudden increase in frequency ('glitch') followed by the usual (though slightly increased) slowing down (Reichley, Downs (1969); Radhakrishnan, Manchester (1969)). Two and a half years later in August 1971 a second glitch of similar magnitude was observed in Vela (Reichley, Downs (1971)).

In September 1969 a glitch in the Crab pulsar PSR0531-21 was observed, smaller but similar in nature to the speed-up inferred for the Vela pulsar (Boynton et al. (1969); Richards et al. (1969)). A second glitch in the Crab of the same magnitude as the first one was found to have occurred in October 1971 (Lohsen (1971)).

A detailed analysis of these second glitches is not yet available, so the following discussion will have to rely on the data evaluated for the 1969 events.

The parameters for these two pulsars are as follows:

	Vela	Crab
ω	70.5	190
$T = \frac{\omega\dot{\omega}}{4\pi^2}$	2.4×10^4	2.4×10^3
$\frac{\Delta\omega}{\omega}$	2.34×10^{-6}	6.9×10^{-9}
$\frac{\Delta\dot{\omega}}{\dot{\omega}}$	6.8×10^{-3}	8.5×10^{-4}

The analysis of the data showed that the post speed-up behavior looked very much like some sort of relaxation phenomenon, where the frequency seems to fall back exponentially to the steady state with a characteristic time of $\sim$ 8 days for the Crab. Thus the pulsars settled down to a long-term frequency increase of

$$\frac{\Delta\omega}{\omega} = 0.3 \times 10^{-9} \quad \text{for Crab}$$

$$\frac{\Delta\omega}{\omega} = 2 \times 10^{-6} \quad \text{for Vela} \quad .$$

This post-glitch behavior can be understood in terms of a simple two-component model (Baym et al. (1969)). One component is the combined crust charged particle system of moment of inertia I_c, rotating uniformly with the angular velocity $\omega(t)$. The second component is the neutron superfluid with moment of inertia I_n rotating uniformly with angular velocity $\Omega_n(t)$. The initial glitch is a sudden change in I_c and $\omega(t)$. Then the neutron superfluid responds in a characteristic time τ_c to the sudden change in the crust's angular velocity. If it were just a normal fluid, the time τ_c, characterizing the coupling between crust and core, would be of the order of 10^{-10} sec, i.e. no relaxation effects could be observed. But τ_c is, as stated above, 8 days for the Crab, 1 year for the Vela pulsar, which very strongly indicates that the interior of these pulsars is superfluid. When both protons and neutrons are superfluid the coupling is via the magnetic moment interaction of the electrons with the 'normal fluid' cores of the vortex lines in the rotating superfluid. This interaction, in fact, has coupling times of the order of 1 year. Thus there is a strong indication that the observation of pulsar glitches allows conclusions on the interior of the neutron star; namely that it contains a crust and a superfluid component.

This is probably a safe conclusion, although there are other theories which try to explain the glitches in a different way:

The Vela glitch is always difficult to account for and most of the theories proposed can be eliminated on the grounds that they offer an explanation only for the Crab pulsar.

The explanation that speed-ups are caused by planets (Rees et al. (1971)) is ruled out by the observed post-glitch longterm frequency increase and by the 'microglitches' (noise component observed in the Crab pulsar (Groth (1971)). Since a critical examination (Börner, Cohen (1972b)) rules out all other explanations, the two-component theory remains the most likely model for the post-glitch behavior.

This model does not account, however, for the sudden initial speed-up and there have been several attempts at explaining that event. The first one proposed was the so-called 'starquake-theory' (Ruderman (1969)): The initially oblate crust, formed when the star was spinning comparatively fast and stressed as the centrifugal force on it decreases, cracks when the external stress exceeds the yield point. This results in a fractional decrease in its moment of inertia, and by conservation of its angular momentum in a speed-up of the crust. However, the explanation of star-quakes as common happenings (every 2 years) meets with difficulties because it is virtually impossible to introduce big enough stresses in the crust of these 2 pulsars in about 2 years. In fact, the Vela pulsar cannot be explained at all, and the Crab pulsar has to be entirely solid, with a mass of less than 0.12 $m_\odot$. This, however, contradicts the observations which establish a lower limit of at least 0.35 $m_\odot$ for the Crab pulsar to supply the necessary energy to the Crab nebula.

The situation seems to have been remedied by the suggestion of an accretion model (Börner and Cohen (1971)). The two-component model for the post-glitch behavior is accepted, but the initial speed-up is attributed to the infall of mass: A massive body falling onto the pulsar would transfer its angular momentum to the crust, and produce the observed initial spin-up. After that the pulsar will settle down to the observed long-term frequency increase as in the two-component model. Börner and Cohen (1971) find that by choosing a specific model of a neutron star all the unknown quantities are determined, even the infalling mass Δm can be found. Assuming a rotating neutron star model of 1.44 $m_\odot$ they find for the Crab pulsar that $\Delta m = 3 \times 10^{-10}$ $m_\odot$, about 10% of the moon's mass. For the Vela pulsar Δm would be 2×10^{-6} $m_\odot$, about 2/3 of the earth's mass. This theory gives a lower limit from glitch observations of the mass of the Crab pulsar of 1 $m_\odot$.

V. The Liquid Interior

In proceeding to higher densities above 3×10^{14} g/cm^3, we find after the dissolution of the nuclei a mixture of neutrons, protons and electrons, where the protons are just a few percent of the number of neutrons. Although nuclear matter theory is really tested only around normal nuclear density, it is generally believed that the method can be extrapolated with reasonably good results in this regime up to a density of 6×10^{14} g/cm^3 or even 10^{15} g/cm^3.

As the density increases above normal nuclear density, the chemical potentials of the electrons and neutrons increase too. Then when the electron chemical potential (μ_{e^-}) exceeds the muon (μ^-) rest mass, it becomes energetically more favorable to replace electrons by muons and to start filling up new Fermi levels. The production of π^- mesons would, of course, be even a bigger advantage, since they are bosons, and one could just pack them all into the lowest energy state, thus keeping the electron Fermi energy μ_{e^-} from ever rising beyond 140 MeV. There are problems, however, because pions and nucleons have a repulsive interaction, which tends to impede to appearance of pions until very high densities are reached.* Finally also the neutron chemical potential (μ_n) will, with increasing density, become so large that it exceeds the rest masses of the lowest mass hyperons (μ_n here includes the neutron rest mass).

The general formalism describing the appearance of various species of particles i, with number density n, baryon number B_i and charge Q_i, has been given by Ambartsumyan and Saakyan (1960). One has total charge density

$$n_Q = \sum_1 n_i Q_i = 0 \qquad (5.1)$$

and total ~~energy~~ baryon density

$$n_B = \sum_1 n_i B_i \qquad (5.2)$$

The energy E (n_1, n_2, . . .) has then to be minimized with fixed n_B, and $n_Q = 0$. Instead one minimizes

$$E^1 = E - \lambda_n n_B + \lambda_e n_Q \qquad (5.3)$$

*Recently (Sawyer (1972)) it has been claimed that such a π-meson condensation phase will form. The influence of this on the equation of state is still unclear.

If a species i is present E^1 will be a minimum for some $n_i \neq 0$

$$0 = \frac{\partial E^1}{\partial n_i} \equiv \frac{\partial E}{\partial n_i} - \lambda_n B_i + \lambda_e Q_i \tag{5.4}$$

The Lagrangian multipliers can be determined from the fact that there always are neutrons and electrons present and therefore

$$\lambda_n = \mu_n \quad \text{and} \quad \lambda_e = \mu_e \tag{5.5}$$

We thus have for the chemical potential of particle i:

$$\mu_i = B_i \mu_n - Q_i \mu_e \tag{5.6}$$

corresponding to the reaction

$$(Q_i)\, e^- + (\text{particle } i) \longleftrightarrow (B_i)\,(\text{neutron}) + \nu\ . \tag{5.7}$$

If a species i is not present, then E^1 always increases as n_i increases, i.e.,

$$\mu_i > B_i \mu_n - Q_i \mu_e \tag{5.8}$$

The density where the right hand side of (5.8) equals μ_i is the threshold for the appearance of particle i. For free particles μ_i would just be $m_i c^2$; but with interacting particles the threshold density may be lowered in the case of attraction or raised in the case of repulsion. It is generally assumed that the first hyperons make their appearance in neutron star matter at densities around $\rho_3 \approx 10^{15}$ g/cm^3. The regime of neutron star matter above this density then poses the interesting problem of describing the properties of a system of interacting hyperons and nucleons at zero temperature.

VI. The Hyperon Core

(VI.1) General remarks

At densities around 8 x 10^{14} g/cm^3 the forces between the nucleons change from attractive to repulsive, and the precise shape of the repulsive core is important; whereas, for nuclear physics the exact knowledge of the repulsive core does not play an equally important role. The fitting of potentials to reproduce scattering data fixes the attractive region of the potential, but, unfortunately, the shape of the repulsive core is not determined to the same accuracy. In addition to these difficulties, various hyperons appear successively in the neutron matter and their interactions with nucleons and between themselves have to be included in a description of matter at these densities, although they are experimentally very badly known. Furthermore it is not clear up to what densities the nonrelativistic treatment of the interactions in the spirit of nuclear physics is still valid to reasonable accuracy. Some authors (Buchler and Ingber (1971)) believe that already at 10^{15} g/cm^3 nonrelativistic many-body calculations break down. It is, of course, extremely important to learn more about the equation of state at these very high densities, since the more massive neutron stars have cores, or consist to a large extent, of very dense matter of roughly 10 times normal nuclear density. Previous estimates (Langer and Rosen,(1970)) indicated that the equation of state is changed by less than 1% in the pressure by the inclusion of hyperons, as compared to an equation of state in which their presence is ignored. The basic properties of the nucleon-nucleon interaction at greater than normal nuclear density therefore seem to be more important than the precise statistical equilibrium composition.

(VI.2) The 'bootstrap' approach

One of the two main ideas leading to a quantitative description of very dense ('ultradense') matter is, in contrast to what we just said above, based on the speculation that there may exist an 'ultradense' region of cold matter, where heavier baryons dominate and where there are so many different types of baryons that only certain statistical features of their distribution and interaction are significant, while the lack of knowledge of the individual interactions is unimportant. It is claimed that by taking into account all the baryon species and their resonances (baryon number B = 1 spectrum) and treating them as free particles, a good description of this region is obtained, which also takes into account, by considering all the resonances (the width of resonances is neglected), those features of the interaction important at ultrahigh densities (Frautschi et al. (1971); Wheeler (1971); Leung and Wang (1971)). The baryon level density is assumed to rise exponentially (Frautschi et al. (1971)) as

$$\frac{d\text{ (number of baryon species)}}{d\text{ (mass interval)}} = \rho_{B=1}(m) \approx m^a e^{bm} \tag{6.1}$$

$$-7/2 \leq a \leq -5/2 \qquad b \sim (160\text{ MeV})^{-1}$$

a form which is suggested by several versions of the 'bootstrap' concept of elementary particle theory (Hagedorn (1968); 'Veneziano'-model - c.f. Leung and Wang (1971)).

The equation of state derived from (6.1) becomes extremely soft, the pressure is kept low, even for Fermi-particles, by the effect that with rising density new kinds of particles and resonances are produced with only slightly higher masses, so that only very few Fermi levels are occupied for one type of particle. The equation of state can be derived in analytic form and reads (Wheeler (1971))

$$p = \frac{\rho c^2}{\ln(\rho/\rho_o)} \qquad \rho_o = 2.5 \times 10^{12} \text{ g/cm}^3 \tag{6.2}$$

The domain of validity of this equation of state does, of course, not necessarily reach down to densities around 10^{15} g/cm^3, where there are just the first few species of known baryons and the statistical formula for the level density is not applicable. Thus Frautschi et al. point out that their formula may only be valid above 10^{17} g/cm^3. This argument then leaves essentially all the problems for neutron star matter unsolved, because stable neutron stars contain matter only up to densities of about 10^{16} g/cm^3 (for the most extreme case of Leung and Wang (1971)). But if one believes that the equation of state behaves asymptotically as predicted by the bootstrap concept, then one could follow Leung and Wang (1971) and apply this concept in the whole region where hyperons are present. The equation of state at lower densities then becomes just the equation of state of a mixture of different noninteracting Fermi gasses, until it is joined smoothly at about 5×10^{14} g/cm^3 to the equation of state derived from nuclear many-body theory. In Fig. 2 we plot two equations of state (5 and 6) derived by Leung and Wang (their number

I and II; their numerically derived relation is $P \sim \rho c^2$ at large densities, but the logarithmic dependence in (6.2) might not show up till still higher densities have been reached), where in both cases the bootstrap concept is employed. In (I) a net attractive baryon-baryon force is assumed up to 10^{17} g/cm^3, and the equation of state is joined to a neutron matter equation of state computed from the potential of the Lomon-Feshbach boundary condition model. In (II) the repulsion given by the Reid soft-core potential in neutron matter calculation is assumed to dominate at intermediate densities. Both cases assume complete compensation between repulsive and attractive effects at high baryon densities ($> 10^{17}$ g/cm^3).

The approach discussed above is quite contrary to the spirit of nuclear physics. If the repulsive core in the nucleon-nucleon interaction, for which there is some evidence, persists to very small interparticle separations, then it will become the dominant effect for a wide range of densities, when Fermi statistics and the Pauli principle lose importance because of the many kinds of particle species available. It might be an interesting problem to investigate whether, even in the case of repulsion, the bootstrap concept might take over in a region of extremely high density. I would expect this density to be unrealistically high, however, maybe 10^{30} g/cm^3 or so. This naturally would rob the bootstrap concept of any validity in the case of cold, dense matter.

Although no experimental evidence exists at very high densities, there are difficulties in arguing the repulsive forces away, and if they stay, then the bootstrap approach is neglecting the dominant feature at high densities and is therefore inadequate.

Moreover, Leung's and Wang's (1971) results for neutron stars are in conflict with astrophysical observations. Börner and Cohen (1972a) considered the energy balance of the Crab nebula. The short lifetime of the electrons producing the optical and X-ray synchrotron radiation puts a lower limit on the continuous supply of energy needed to maintain that radiation output. If this energy $\dot{E}_{min}$ is to be supplied by a rotating neutron star with the frequency ω and rate of change $\dot{\omega}$ of the Crab pulsar, then, because of

$$\dot{E}_{min} \leq I \omega \dot{\omega} \tag{6.3}$$

the pulsar must have at least a moment of inertia of 2×10^{44} g cm^2. Conclusions based on the expanding supernova shell are not as definite. If the currently accepted values for acceleration and snow-plow in the interstellar medium are used, additional energy must be supplied. This energy can be obtained from a neutron star with a moment of inertia of 10^{45} g cm^2.

The value of 2×10^{44} g cm^2 for the moment of inertia of the Crab pulsar, which is obtained directly from the observations, permits us to exclude the equations (I) and (II) of Leung and Wang from the set of realistic candidates, because there the maximum moment of inertia is 1.05×10^{44} g cm^2. The Crab pulsar is therefore definitely not among the stable neutron star models computed by Leung and Wang (1971).

(VI.3) Manybody treatment of a hyperon gas

The many-body treatment of repulsive baryon-baryon interactions suffers from several uncertainties. The shape of the repulsive core is not known very precisely and no reliable method of computation is available so far. Since it seems to be rather difficult to estimate the effects of higher-order, many-body correlations, which become more and more important with increasing density as nuclear-matter calculations indicate, the consideration of only pair correlations in the approaches so far is questionable too. The use of a potential to describe the interaction between the baryons is certainly better justified here than in high energy particle physics, because one is dealing with matter of high pressure, but low, nonrelativistic kinetic energies.

Both in the approach of Pandharipande (1971) and Bethe and Johnson (1972), which are so far the only attempts to deal with repulsive baryon-baryon interactions using many-body methods, the Reid soft-core potential is used and universal repulsion is assumed in all pairs of interacting particles. Different potentials are assumed in the different angular momentum states $\ell = 0$, ℓ odd, ℓ even $\neq 0$. Hyperonic matter is assumed to be electrically neutral and the Coulomb force is neglected. The nonrelativistic Schrödinger equation is then employed (the use of the nonrelativistic equation seems to be justified, because it turns out that the momenta of the particles are always small; v = 1/2 c is the maximum velocity occurring), together with a variational method to numerically minimize the energy, where the trial wave function is a Jastrow-type wave function

$$\chi = A\, e^{i\, k \cdot r} \prod_{\ell,m} f_{\ell m}(r) \tag{6.4}$$

$f_{\ell m}$ describes the correlation between particles. Only pair correlations are considered. Pandharipande (1971) tested the method by applying it in a computation of the groundstate properties of He^3 and He^4 and obtained excellent results. Although this test speaks in favor of the method, the whole scheme depends very sensitively on the assumptions made about the interaction. This is illustrated by Table 3, where the results of models (A) and (C) and of the pure neutron matter calculations of Pandharipande (1971) are compared to Bethe's and Johnson's results.

In his model (A) Pandharipande treats a mixture of the following particles: n, p, Λ, Σ, Δ, μ^-, e^-, assuming universal repulsion and universal intermediate range attraction in all hadron pairs. In model (C) the attraction between hadrons and nucleons is ad hoc lowered by $\sim$ 10 %. Bethe and Johnson differ from Pandharipande only in that they assume identical repulsion in S- and P- states, i.e., they take the S-wave repulsion of the Reid soft-core potential ($\sim$ 6484.2 $e^{-7\mu r/\mu r}$ MeV) for the P-state too, whereas Pandharipande just takes the P-state Reid potential averaged over j (repulsion $\sim$ 4152.2 $e^{-6\mu r/\mu r}$ MeV). Thus at short distances the repulsion in P-states with Bethe and Johnson is doubled compared to Pandharipande. The differences in the results are quite drastic (c.f. Table 3). Bethe and Johnson obtained energies per particle in excess of Pandharipande's for neutron matter; and, therefore, their equation of state is very stiff and close to the pure neutron matter equation of state. This is a rather satisfactory result since it supports the statement made above that for regions where the repulsive forces dominate, nuclear forces determine the equation of state and the nature of the statistical equilibrium composition plays only a secondary role.

Bethe and Johnson find an analytic expression for the dependence of pressure and density:

$$P \sim \rho^{2.54} \tag{6.5}$$

Pandharipande's model (A) gives negative pressures (for baryon number densities $\leq 1\ \mathrm{fm}^{-3}$) in a certain range of densities, so the possibility of a phase transition accompanying the transition from nuclear to hyperonic matter might be envisioned. In Bethe's and Johnson's approach, as well as in Pandharipande's model (C), however, the transition is smooth. These differences clearly illustrate that one is walking on rather unsafe grounds, and that the precise shape of the repulsive core has a big effect on the equation of state.

(VI.4) Model of a lattice of baryons

It has first been suggested by Bethe (1969) that because of their repulsive interaction at high densities nucleons (and possibly hyperons) would tend to minimize their energy by arranging themselves in some kind of lattice. Similar to the Coulomb lattice of nuclei, which comes into existence because of the strong Coulomb repulsion of the nuclei, this lattice would exist when strong repulsion dominates the interaction. The suggestion is especially attractive with regard to the treatment of higher order correlations in a many-body description of a system of nucleons and hyperons. The lattice structure could be expected to take care of all the higher order terms, and only two-body correlations in a lattice would have to be computed.

First approaches to the problem considered a lattice formed only with neutrons, interacting via the Reid potential (Banerjee et al. (1970);

A bcc lattice was then treated as a classical system of oscillators. It was found that the vibrational zero-point energy increased very rapidly with density; and, at 3×10^{15} g/cm^3 the neutrons were already highly relativistic (their vibrational energy exceeded their rest mass). So the classical approach was abandoned. But subsequent quantum-mechanical treatments (Bethe and Johnson (1971); Pandharipande (1971a)) of a lattice of neutrons also ran into difficulties. Even with sophisticated variational techniques the zero-point vibrations became very large. Although the neutrons did not move relativistically here, still their kinetic energies were always at least 40% of the correlation energies that were available to hold them in their lattice positions (at 3×10^{15} g/cm^3 the kinetic energy still exceeds the correlation energy in the calculations of Pandharipande (1971a)). Since it is generally believed that a lattice starts to melt when the kinetic energy of its constituents is 1% of the lattice interaction energy, these results indicate that a lattice will never form at high densities. The nuclear interaction is probably not repulsive enough to force the particles into lattice positions.

There is, however, considerable uncertainty in the microscopic description of matter at these high densities, and many different approximations seem possible. Canuto and Chitre (1972) recently found that a lattice of baryons can exist and will be stable against melting above a density of about 1.5×10^{15} g/cm^3. Their approximation consists of treating only two-particle correlations and cutting off the contributions from all states with angular momentum $J > 2$. Therefore it is possible to have a baryon lattice, if one sticks to a specific approach. The merits of Canuto's and Chitre's work could be judged if the correct description of matter at high densities was known. At the moment, therefore, one can only state that it is an undecided question whether or not a lattice of baryons will exist in the cores of neutron stars.

A microscopic description of a lattice mode of neutrons, protons and other hyperons involves the variation of the number densities of the different particles to find the minimum of the energy. But each time one particle is changed into another species, the symmetry of the lattice will be changed too. In view of these difficulties, Canuto and Chitre (1972) computed only various specific examples with given particle composition and given lattice symmetry. They find that for any given particle composition the fcc lattice always had the lowest energy. They considered as baryon species n, p, Λ, Σ, and found that a lattice consisting purely of lambda particles had the lowest energy. Thus, according to their results, the heavier neutron stars would mainly consist of a core of Λ particles in an fcc lattice, and the name 'lambda' star might be appropriate.

Recently it has been claimed (Anderson and Palmer (1971)); Clark and Chao (1972)) that by extrapolating experimental results on quantum solids, one can show that neutron star matter solidifies at densities around 3×10^{14} g/cm^3. This value throws considerable doubt on the validity of the extrapolation. At 3×10^{14} g/cm^3 nucleon-nucleon interaction is still mostly attractive, so there is no reason at that density for an interacting neutron gas to solidify under pressure; since, a free Fermi gas would not become solid either. Furthermore, Anderson and Palmer (1971) and Clark and Chao (1972) seem to be unfamiliar with work on neutron lattices discussed above, and it has been shown (Canuto and Chitre (1972)) that their lattice is unstable against melting.

(VI.5) Conclusion

In Fig. 2 the various equations of state discussed have been plotted. All equations of state which take into account a varyon-baryon potential which is attractive at intermediate range and repulsive at short distances show a qualitatively similar behavior. They are below the line for the free neutron gas up to slightly above normal nuclear density; then they become much stiffer and increase rather sharply in the region of repulsive nuclear forces. Typical for that are curves 2 and 3 which are derived from the Reid potential using nuclear many-body techniques. The equation of state of Bethe and Johnson (1972) is not plotted. It would run slightly above 2 and below 1.

The equation of state numbered 1 (Cohen et al. (1969)) crosses the free neutron gas line at a lower density, $\rho = 2 \times 10^{14}$ g/cm^3, than all the others, which would correspond to a potential changing from attractive to repulsive at interparticle separations greater than 1.5 fm. This equation of state therefore presents something like an upper limit on the pressure vs. density relation; i.e., one cannot expect a many-body calculation with a realistic soft-core potential to lead to a stiffer equation of state. Indeed, the computation of neutron star models from 1 gives, to good accuracy, the same maximum mass models as an approach where constant density throughout the star was assumed, with central pressure and density related through 1.

The equations of state of Leung and Wang (1971), on the other hand, are always below the free neutron gas and have a smaller slope throughout. The departure from all the other equations of state plotted in Fig. 2 becomes very pronounced in the region of normal nuclear density. However, high density matter that soft cannot supply enough pressure to allow the existence of large enough stable neutron stars to agree with astrophysical observations.

The ultimate equation of state will be much stiffer than 5 and 6, less stiff than 1, probably close to 2. To give some qualitative properties of recently computed neutron star models, we list in Table 4 selected models derived from the equation of state, number 2.

Appendix

Since these notes have been completed the IAU Symposium #53 on "The Physics of Dense Matter" took place at the University of Colorado in Boulder, August 21-26, 1972. Bethe, Canuto and Ruderman in their talks presented their work as it is discussed in the preceding pages. The author could see that this review is not yet outdated, but reflects the present state of research quite well. One new result should be mentioned in addition: Negele (MIT) reported on Hartree-Fock calculations of nuclei in neutron stars. It turns out that nuclei in neutron stars stay small, their charge Z always remaining below 50.

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Table 1. Nuclei in Equilibrium with Electron Fermi Gas (from Baym et al. (1971a)

Density (g/cm^3)	Equilibrium Nuclei
7.8 E6	$_{26}Fe^{56}$
2.8 E8	$_{28}Ni^{62}$
1.2 E9	$_{28}Ni^{64}$
8.0 E9	$_{34}Se^{84}$
2.2 E10	$_{38}Zn^{80}$
1.1 E11	$_{38}Zn^{82}$
1.8 E11	$_{42}Mo^{124}$
2.7 E11	$_{40}Zr^{122}$

5

Table 2. Mass Number A and Charge Z of Nuclei in Neutron Matter as a Function of Matter Density ρ

BBS: Bethe et al. (1970)

BBP: Baym et al. (1971b)

BB: Barkat & Buchler (1971)

ρ	BBS		BBP		BB	
g/cm^3	Z	A	Z	A	Z	A
2.8 E11	39	122	40	122	34	80
7.8 E12	45	159	49	178	35	108
1.5 E13	47	177	54	211	34	
2.2 E13	49	190	58	241	34	
3.8 E13	51	205	67	311	32	
5.1 E13	51	211	74	375	30	
7.7 E13			88	529	27	
1.3 E14			117	947		
2.0 E14			201	2500		

Table 3. High Density Equations of State Derived from the Reid Soft-Core Potential

Listed is the energy per particle in MeV as a function of density.

BJ: Bethe and Johnson (1972)

A,C,N: Pandharipande (1971) Model (A), (C), Neutron Matter

Density (g/cm^3)	1.7 E15	4 E15	1.0 E16
BJ	250	875	2775
A	22	115	1220
C	100	380	1670
N	157	620	2070

Table 4. Properties of Selected Neutron Star Models

Log Central Density (g cm^{-3})	Mass/ 10^{33} (g)	Radius (km)	Moment of Inertia/ 10^{45} (g cm^2)	Central Dragging of Inertial Frames (Ω_c/w)	Edge Dragging of Inertial Frames (Ω_e/w)	T_o (ms)
15.6	3.45	8.7	1.19	0.79	0.26	U
15.5	3.44	9.1	1.26	0.73	0.25	0.79
15.4	3.35	9.5	1.30	0.68	0.22	0.51
15.3	3.17	9.9	1.27	0.61	0.19	0.41
15.2	2.88	10.2	1.16	0.53	0.16	0.35
15.1	2.48	10.5	0.98	0.45	0.13	0.33
15.0	2.01	10.6	0.75	0.36	0.09	0.31
14.9	1.53	10.7	0.53	0.28	0.06	0.31
14.8	1.09	10.7	0.33	0.21	0.04	0.33
14.7	0.73	11.0	0.20	0.14	0.02	0.38
14.6	0.52	11.6	0.13	0.11	0.01	0.45
14.5	0.37	12.5	0.08	0.08	0.007	0.67
14.4	0.25	15.1	0.05	0.06	0.002	1.38
14.3	0.14	47.9	0.04	0.03	0.00005	45.9
14.2	0.95	34.1	76.2	0.03	0.0004	U

Figure Captions

Figure 1. The Interior of a Neutron Star

Figure 2. Pressure as a Function of Density

1. Cohen et al. (1969) - CCLR
2. Bethe et al. (1970), Pandharipande (1971) - BBS
3. Baym et al. (1971a) - BPS
4. Free Neutron Gas
5. Leung and Wang (1971) Model (II)
6. Leung and Wang (1971) Model (I)

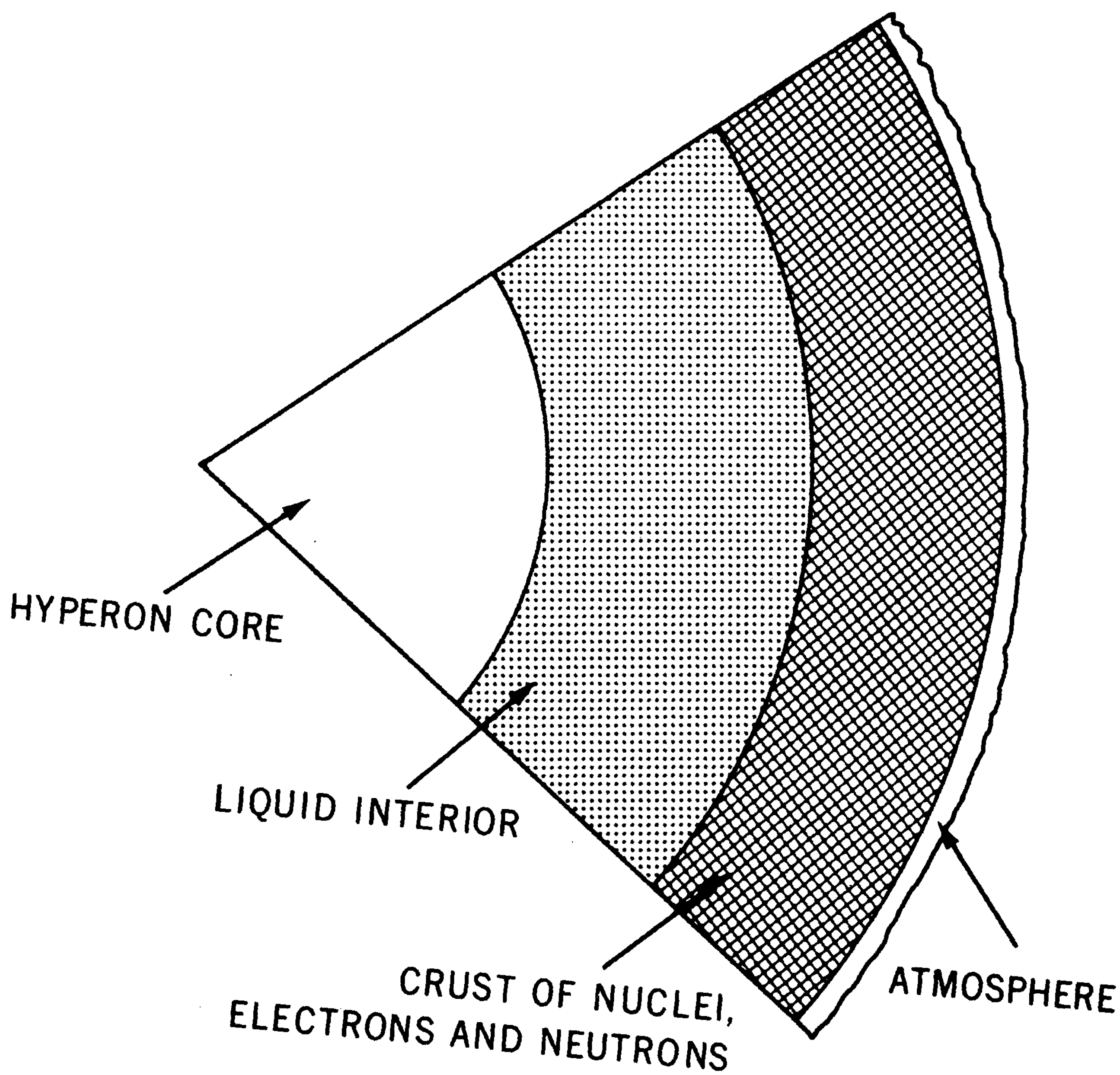
HYPERON CORE
LIQUID INTERIOR
CRUST OF NUCLEI, ELECTRONS AND NEUTRONS
ATMOSPHERE

Figure 1

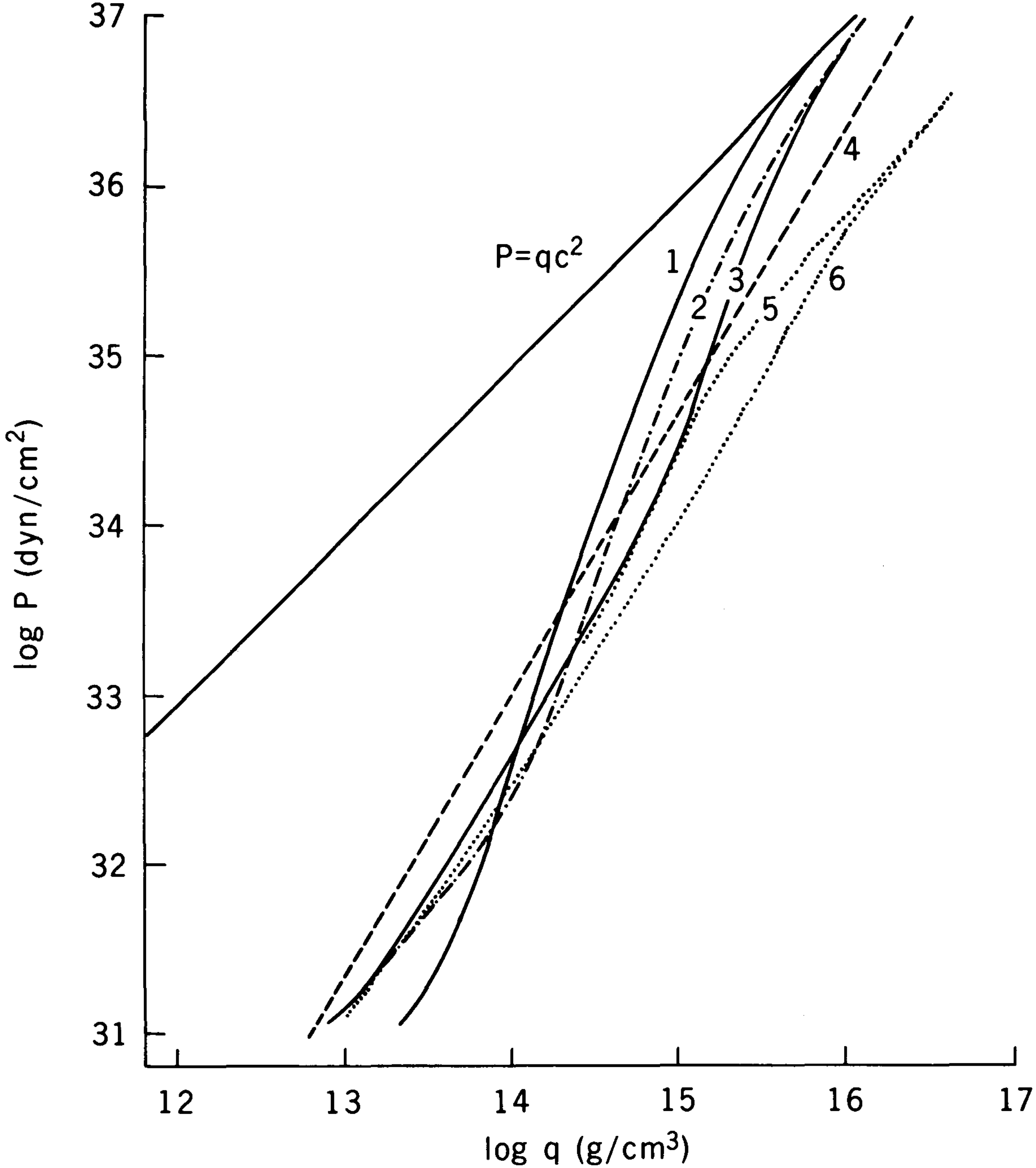
37
36
35
34
33
32
31
log P (dyn/cm²)
P=qc²
1
2
3
4
5
6
12
13
14
15
16
17
log q (g/cm³)

Figure 2