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REPORT

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METHODOLOGY FOR THE SYSTEMS
ENGINEERING PROCESS
Volume III: Operational Availability

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FOREWORD _____

This report is submitted in accordance with the requirements of
Contract NAS8-27567. Martin Marietta Corporation submits this
report in three volumes as follows:

Volume I--System Functional Activities (NASA CR-61380)

Volume II--Technical Parameters   (NASA CR-61381)

Volume III--Operational Availability (NASA CR-61382)

CONTENTS ---

SUMMARY
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Volume III is a detailed description and explanation of the
operational availability parameter. The fundamental mathematical
basis for operational availability is developed, and its relation-
ship to a system's overall performance effectiveness is illustrated
within the context of identifying specific availability require-
ments. Thus, in attempting to provide a general methodology for
treating both hypothetical and existing availability requirements,
the concept of an "availability state", in conjunction with the
more conventional probability-time capability, is investigated.
In this respect, emphasis is focused upon a balanced analytical
and pragramatic treatment of operational availability within the
system design process. For example, several applications of
operational availability to typical aerospace systems are presented,
encompassing the techniques of Monte Carlo Simulation, System
Performance Availability Trade-Off Studies, Analytical Modeling of
specific scenarios, as well as the determination of launch-on-Time
probabilities. Finally, an extensive bibliography is provided to
indicate further levels of depth and detail of the operational
availability parameter.

I. INTRODUCTION _____

Operational availability is a measure of the extent to which a
system can be expected to be in a state or condition to perform
its assigned function within an established time frame and
under given environmental conditions. As such, a system's
operational availability includes both a detailed description of
the performance characteristics of individual system elements, as
well as the specification of the overall system states. In this
respect, the resultant complexity of large aerospace systems
involves a broad set of requirements that express the multitude of
mission objectives. This complexity brings with it the need for
a quantitative means to measure the total effectiveness of a system,
particularly where alternative approaches are to be considered.
One fundamental approach to the measurement of total systems
effectiveness (SE) is to formulate its effectiveness in terms
of a figure of merit, such as:

$$SE = \sum P(A) \cdot P(C),$$

where,

    $P(A)$ = probability that the system will be operational during
    a specified time interval, or at a particular instant of time;

    $P(C)$ = probability of achieving the mission objectives, given
    the system is available and dependable.

The development of analytic models for total SE evaluation is
treated in detail in Appendix A. The basic characteristics of an
effectiveness model, along with the equations for specifying
significant effectiveness functions are also given. The real
value of the resultant SE index lies in the definition/design of
systems where choices between alternatives are made, and the
primary objective is to select that particular candidate having
the greatest overall benefit. In such cases, indices of
effectiveness aid the decision maker in terms of augmenting his
experience and skill with a quantifiable measure of overall
systems performance.

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A.   APPROACH AND DEFINITION

In this study, the specification and definition of operational
availability is approached from two distinct points of view:

1) The contribution of operational availability to the deter-
    mination of a total SE index or figure of merit within the
    overall systems context;

2) The quantification of the availability parameter in terms
    of   specific performance characteristics, such as instant-
    aneous availability, interval availability, steady-state
    availability, or a time-dependent probability.

Thus, in the real world, the availability parameter addresses
the time usage of a system and deals with problems of failure of
system elements (people, equipment, facilities, etc), as well
as with the areas of support requirements needed to correct and
prevent failures. As such, the measure of availability is a
determination of the degree to which a system can be expected
to perform some particular function. It is a probabilistic-time
capability of the system. The origin of these requirements lie
in the prior mission analyses that are usually performed to
quantify the overall objectives and that will ultimately drive
the definition of a particular availability index. In the early
concept phase, some of these factors are:

1) What is needed to satisfy the objectives?

2) What is achievable with the resources available?

3) What are the acceptable risks of success and failure?

The mission analyses in the concept phase examine these factors
in detail for the major performance objectives and formulate
specifications (initial) that will subsequently be used to select
the "design to" availability parameters for the concept that best
meets these total mission requirements. The degree of sophistication
and complexity of such analyses can vary from very complex simulations
of the mission scenarios to a cursory examination of only the
principal factors. Much depends on the criticality of the particular
problem being considered.

The scope of such studies depends on how much is known about the
mission and types of systems that can accomplish it. It is obvious,
for example, that a contemplated mission never before attempted,
requiring systems never before produced, will be much more uncertain
(in terms of a probability measure) than a modification to an

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existing system for a previously performed mission. The former case is of greater significance, because it poses the problem of establishing availability requirements for new and untried systems. In attempting to provide a general methodology for treating both hypothetical and existing availability requirements, the concept of an "availability state" is introduced. This formulation of availability is entirely consistent with the probability-time capability previously discussed and further includes the dimension of states. The definition previously given for operational availability, i.e., a measure of the extent a system can be expected to be in a state or condition to perform its assigned function, is general; however, it does imply that a probability, a state, and a time capability make up the quantification of this parameter. Availability requirements can, therefore, be expressed in several ways--determined by the system and the specific manner in which it is used. The availability requirement may be expressed as:

1) A specific schedule time;

2) A time period or interval;

3) A random time within a time period;

4) A set of time frames, i.e., states;

5) An instantaneous time;

6) A steady-state condition;

7) A probability-time-state use;

8) Combinations of the above.

B.   ANALYTIC DEVELOPMENT OF AVAILABILITY PARAMETERS

A complex systems usage may include one or more of these requirements distributed among several elements of the system and applying to various usage states (launch, payload delivery, orbit operation, recovery, etc). The probabilities associated with each state requirement represent the degree of assurance or probability of success desired. Thus, operational availability may be expressed as a probability of time requirement for any system state. The relationships are illustrated in Fig. 1, and bring together the concepts of probability-time-state to quantify the availability parameters.
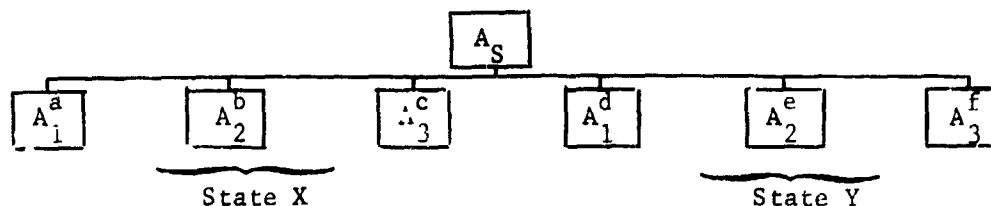
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Fig. 1 Availability States

In Fig. 1, A is the availability requirement, subscripts 1, 2, 3...indicate system elements, and superscripts a, b, c...indicate specific availability requirements for the system elements. Thus, the total operational availability for all states and all system elements may be written symbolically from Fig. 1 as:

$$A_S = F \left\{ A_1^a, \ A_2^b, \ A_3^c; \ A_1^d, \ A_2^e, \ A_3^f; \ t \ (A) \right\},$$

where t denotes the time dependency for each particular state.

One may, for the purpose of this illustrative example, consider X as the launch state, and Y as the orbit mission state. The elements of X are the launch vehicle, support equipment, launch site facilities, etc, while the elements of Y are the payload, experiment package, telemetry, data link, etc. Then, some typical availability specifications for these elements would be:

1) Launch on time probability (launch state);

2) Probability of payload availability (orbit state).

For each system element, its operational availability would be expressed symbolically as:

$$
\left.
\begin{aligned}
A_1 &= \sum A_1^a \cdot A_1^d \\[6pt]
A_2 &= \sum A_2^b \cdot A_2^e \\[6pt]
A_3 &= \sum A_3^c \cdot A_3^f
\end{aligned}
\right\}
\quad \text{System element level operational availability}
$$

For each system state, the operational availability is:

$$
\left.
\begin{aligned}
A_{\substack{\text{Launch} \\ \text{State}}} &= \sum A_1^a \cdot A_2^b \cdot A_3^c \\[10pt]
A_{\substack{\text{Orbit} \\ \text{State}}} &= \sum A_1^d \cdot A_2^c \cdot A_3^f
\end{aligned}
\right\}
\quad
\begin{aligned}
&\text{System state level operational} \\
&\text{availability}
\end{aligned}
$$

The total system availability is given by:

$$A_{System} = \sum A_{\substack{Launch \\ State}} \cdot A_{\substack{Orbit \\ State}}$$

where the availability of the orbit state is conditional on the launch state availabilities.

These symbolic equations illustrate the technique of dealing both with the system element and system state level of availabilities.

C. DETAIL BREAKDOWN OF AVAILABILITY

For each specific element and subelement, the measure of operational availability may be broken down still further as follows:

1) Instantaneous availability - The probability that the element will be available at any random time t;

2) Interval availability - The proportion of time in an interval that the element is available for use;

3) Steady-state availability - The proportion of time that the element is available for use when the time interval considered is very large.

In the limit, these three measures of availability approach the steady-state availability. Which measure is most applicable depends on the element state and its conditions of use. For elements or subelements that are to be operated in continuous systems, e.g., a detection radar system, steady-state availability may be the satisfactory measure. For elements whose usage is defined by a duty cycle, e.g., a tracking radar system that is called on only after an object has been detected and is expected to track continuously during a given time period, interval availability may be the most satisfactory measure. Finally, for elements that are required to perform a function at any random time, e.g., a data processing system as part of a telemetry system that is to be employed to process orbital data and then remain idle for a length of time, instantaneous availability may be the most satisfactory measure. In general, the duty cycle of each element would determine the form of availability most appropriate as the measure of performance.

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The expression for instantaneous availability can be determined by describing the failure and repair process in a system of linear differential and/or difference equations as with reliability, with the exception that one allows for repairs out of the failed state of the system. These equations would describe the transition from one operability state to another. The solution of the differential equations is the probability that the system is available at any random time t.

1. Instantaneous Availability

For a single equipment (single element), the measure of instantaneous availability is given by:

$$A(t) = \left(\frac{\mu}{\mu + \lambda}\right) + \left\{\frac{\exp -(\mu + \lambda)t}{\mu + \lambda}\right\},$$

where

μ = equipment repair rate;

λ = equipment failure rate;

t = a random time.

This result and the two- and three-equipment parallel and standby redundant system instantaneous availability equations are shown in Table 1. Both parallel and standby redundancy and single and multiple repair capability formulas are given for comparative purposes.

2. Interval Availability

The proportion of time that the element is available in an interval t_1 to t_2 can be computed by taking the average value of instantaneous availability over the time of usage.

$$A_m(t) = 1/(t_2 - t_1) \int_{t_1}^{t_2} A(t)dt,$$

where

t_1 = start of mission time (usually $t_1 = 0$);

t_2 = end of mission time.

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Table 1 Availability of Some Redundant Systems Based on Exponential Failure and Repair Distributions

No. of Repair units	Type Redundancy	Repair	Instantaneous Availability Model	Definitions of Constants for Instantaneous Availability Model	Steady-State Availability Model	Av. for $\lambda = 0.01$ $\mu = 6.2$
1	—	—	$A(t) = \dfrac{\mu}{\mu+\lambda} + \dfrac{\lambda}{\mu+\lambda}e^{-(\mu+\lambda)t}$	—	$\dfrac{\mu}{\mu+\lambda}$	0.95
	Standby	Single		$s_1 = -(\lambda+\mu) - \sqrt{\lambda\mu}$ $s_2 = -(\lambda+\mu) + \sqrt{\lambda\mu}$	$\dfrac{\mu^2+\mu\lambda}{\mu^2+\mu\lambda+\lambda^2}$	0.998
2		Multiple		$s_1 = -\tfrac{1}{2}[(2\lambda+3\mu)+\sqrt{\mu^2+4\mu\lambda}]$ $s_2 = -\tfrac{1}{2}[(2\lambda+3\mu)-\sqrt{\mu^2+4\mu\lambda}]$	$\dfrac{2\mu^2+2\mu\lambda}{2\mu^2+2\mu\lambda+\lambda^2}$	0.999
	Parallel	Single		$s_1 = -\tfrac{1}{2}[(3\lambda+2\mu)+\sqrt{\lambda^2+4\mu\lambda}]$ $s_2 = -\tfrac{1}{2}[(3\lambda+2\mu)-\sqrt{\lambda^2+4\mu\lambda}]$	$\dfrac{\mu^2+2\mu\lambda}{\mu^2+2\mu\lambda+2\lambda^2}$	0.996
		Multiple		$s_1 = -2(\mu+\lambda)$ $s_2 = -(\mu+\lambda)$	$\dfrac{\mu^2+2\mu\lambda}{\mu^2+2\mu\lambda+\lambda^2}$	0.998
	Standby	Single		s_1, s_2 and s_3 correspond to the three roots of	$\dfrac{\mu^3+\mu^2\lambda+\lambda^2\mu}{\mu^3+\mu^2\lambda+\lambda^2\mu+\lambda^3}$	0.9999
3		Multiple		s_1, s_2 and s_3 correspond to the three roots of	$\dfrac{6\mu^3+6\mu^2\lambda+3\mu\lambda^2}{6\mu^3+6\mu^2\lambda+3\mu\lambda^2+\lambda^3}$	0.99998
	Parallel	Single		s_1, s_2 and s_3 correspond to the three roots of	$\dfrac{\mu^3+3\mu^2\lambda+6\mu\lambda^2}{\mu^3+3\mu^2\lambda+6\mu\lambda^2+6\lambda^3}$	0.9993
		Multiple		s_1, s_2 and s_3 correspond to the three roots of	$\dfrac{\mu^3+3\mu^2\lambda+3\mu\lambda^2}{\mu^3+3\mu^2\lambda+3\mu\lambda^2+\lambda^3}$	0.9999

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3. ## Steady-State Availability

The proportion of time that the element is available in an interval (0, t) as t becomes very large can be arrived at by taking the limit as $t \to \infty$ of the above equation.

$$A_{ss}(\infty) = \lim_{t \to \infty} \frac{1}{t} \left\{ \int_{0}^{t} A(t)dt \right\}.$$

Note that integrating the right-hand terms of the instantaneous availability equations of Table 1 over the interval (0, ∞) results in a solution leaving the first term on the right side of these equations. Thus, for example, the limiting expression for a single element (single equipment) system is:

$$A_{ss}(\infty) = \mu / (\mu + \lambda).$$

For most elements, the measure of steady-state availability is satisfactory. Note also that because the steady-state value is always less than the values of instantaneous and interval availability, the use of the former measure allows for conservation.

If one assumes that repairs cannot be made until complete element failure, the expression for a single element steady-state availability can be approximated by:

$$A_{ss} = \frac{MTTF}{MTTF + MRT} \text{ ,}$$

where

$$MRT = \text{mean repair time} = 1/\mu \text{ ;}$$

$$MTTF \text{ is given by } 1/\lambda \text{ .}$$

This rather familiar expression may be further expanded to illustrate in greater detail the pragmatic subelement considerations. For example, the above formula may be reduced to:

$$A_{ss} = \frac{\text{Total Up-Time for a Period}}{\text{Total Up-Time + Total Downtime for this Period}} \text{ ,}$$

$$= \frac{\text{Mean Up-Time for a Period}}{\text{Mean Up-Time + Mean Downtime for this period}}.$$

The above can be written:

$$A_{ss} = \frac{MTBM}{MTBM + MDT} = \frac{MTBM}{MTBM + (\overline{Mct} + \overline{Mpt})} \text{ ,}$$

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where

MTBM = mean time between maintenance actions involving both corrective and preventive maintenance;

MDT = mean downtime;

$\overline{(Mct + Mpt)}$ = mean of the time for the aggregate of corrective and preventive maintenance actions;

Mct = corrective maintenance downtime caused by failures and other causes such as accidents, human-induced maintenance, etc;

Mpt = preventive maintenance that is scheduled (usually includes daily, weekly, and monthly actions), which may include replacement of scheduled wearout parts, recycling, etc., depending on customer project rules.

Note: Operational administrative delay of maintenance is not considered.

The above expression can be reduced to:

$$A_{ss} = \frac{MTBF}{MTBF + \overline{Mct}},$$

where

$\overline{Mpt}$ does not cause system downtime. This is the case when Mpt is eliminated or its need is restricted by design to off-duty periods such as during Mct or excused periods such as recycling or depot overhaul.

$\overline{Mct}$ considered is limited to maintenance correction of random hardware failures and does not include accidents, human induced failures, etc.

$$\overline{Mct} = \overline{(Mct_{Inherent} + Delay_{Supply} + Delay_{Maintenance\ Administrative})},$$

where

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$\overline{Mct}_{Inherent}$ = the mea. of the controlled degree of
ma' tainability designed into hardware. It is the
responsibility of maintenance to define (by
participation in system analysis and trade studies),
specify, predict, review, assist designers,
determine design corrections, and evaluate results
in hardware for maintenance time and design character-
istics that will enable economical maintenance on time
to achieve A_{ss} assuming that the facilities, environ-
ment, manpower, skills, procedures, tools, test equip-
ment, spares, and supplies, obligated by the maintenance
concept are used.

$\overline{Delay}_{Supply}$ = the mean of the lost time while a
maintenance task is suspended awaiting spares or
supplies. It starts on official supply demand by
maintenance at the designated supply point. Such
delay is a function of stock identification, planned
stock layout, purchase and delivery, and receipt/
storage/issue by launch sites. It is aggravated
when sites do not follow the planned system.

$\overline{Delay}_{Maint\ Admin}$ = mean of the lost time while a mainten-
ance task is delayed awaiting the arrival of skills, tools,
test equipment, and time out for personnel and official
reasons. This can be aggravated when launch sites do
not follow the plans identified for facilities, mainten-
ance, environment, manpower, skills, procedures, tools,
and test equipment.

D.   AVAILABILITY SPECIFICATION

The detailed illustrative samples presented in the preceding
sections furnish a general framework within which the specifi-
cation of availability parameters may be carried out. In
particular, these methods include modeling of the mission to
provide the means for examining the given scenarios and con-
tingencies for each mission state and apportioning the system
availability parameters to the major elements and subelements.
The techniques and procedures in this initial phase of activity
are systems analyses, in wh _h modeling and simulations based on
estimates, similarities to previous systems, etc are developed.
The definition/design phase proceeds with specific requirements
for operational availability allocated to these major modules.
The task faced in the definition/design phase is to establish
system availability specifications.

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1.   Illustrative Example of Availability Specification

The above methods are illustrated in the following example, which
specifies the availability requirements as they relate to an
extended mission of a manned orbital vehicle. In particular, the
analysis deals with the maintainability tradeoffs in terms of
maximizing the total systems effectiveness and minimizing such
cost factors as maintenance times and the weight and volume of
spares. The methodology makes use of a computer simulation to
determine:

1)   Whether to use redundant parts, carry spare modules for easy
     replacement, make repairs in flight, or how best to combine
     these approaches;

2)   Whether to use built-in automatic failure location devices
     or auxiliary test gear;

3)   What the effect is of replacement times and the required
     weight and volume of spare modules on the equipment
     availability;

4)   The applications for which commonality of parts can be used
     effectively;

5)   The techniques that can be used to improve the maintainability
     per unit weight, volume, or other associated cost factors.

Almost every aspect of reliability/availability/maintainability
design is a matte of compromise. If every conceivable require-
ment in the way of spare parts, tools, test equipment, and
trained repairmen is foreseen and provided, the space vehicle
would become a combination warehouse and factory and never get
off the ground. Because of the long mission times and the large
number of system parts involved, it is practically impossible to
achieve acceptable dependability/availability through reliability
alone, but it would be equally impossible to maintain an unreliable
system. Furthermore, these factors must be considered from the
standpoint of the value of the mission because, if dependability/
availability were the primary mission goal, man would stay on the
ground. The following discussion does not include all factors,
but does indicate the scope of the problem.

Initially, there are the tradeoffs between reliability,
maintainability and availability. The increase of reliability
through the use of more reliable components, of parallel and
standby redund ncy, of higher standards of quality control, of more
exacting requirements for assembly and checkout, etc, is limited
by the law of diminishing returns, and, in the case of manned
space mission with extended lifetimes, the value of maintainability
is easily seen.   11

Next, there are the tradeoffs associated with "indenture" level or the size or amount of circuitry or equipment included in each replacement module. The lower the indenture level, the smaller the replacement modules and the less replacement of properly operating equipment; but locating the failure, gaining access to the failed part, and replacing it would be expected to take longer. The penalty, in terms of failure effect on the mission objectives, would depend on the importance of the objectives involved, the criticalness of their scheduling, etc. Also, the optimum indenture level would depend on whether the replaced modules are to be serviced. Using a higher indenture level has the advantages of shorter failure location and module replacement times, and, if the replaced module can be repaired, this would greatly reduce the required number of spares. There would, however, be a costly penalty associated with the test and repair equipment needed and providing a trained repairman for the crew.

Further tradeoffs are associated with the completeness of the failure location and test equipment, and the required crew training. Presumably, failure location equipment provided could be so complete that the module containing the failed part is indicated automatically without any further checking. However, this would involve a rather extensive system that would, itself, be subject to failure. For each type of mission and vehicle, there is some optimum level of completeness of the failure location equipment and the associated training of the crew members, etc.

To obtain the optimum compromise in these and other aspects, it is necessary to compare costs and the accomplishment of various mission objectives. This suggests the use of the concept of system cost effectiveness. Cost effectiveness is defined as the per unit cost of value received and system effectiveness as the probability of achieving mission objectives. If some commensurable value can be assigned to each mission objective, then it is more useful to define system effectiveness as the expected mission accomplishment in these value units and system cost effectiveness as the per unit cost of this expected accomplishment. This provides a single quantity for comparison and reduces the reliability, maintainability and availability, design problem to finding the approach that yields the maximum system cost effectiveness.

The approach is to use a computer program to simulate the performance of a set of mission objectives along with a history of the failure and maintenance actions performed on the various system elements based on failure and repair or module replacement times chosen randomly from specified probability distributions. The purpose of the simulation program, however, is not to show what can happen in one particular case, but to indicate the distribution of events along with their probabilities.

2.   ## Application to a Sample Program

To gain a better understanding of some of the relations involved, the simulation program was used to study indenture level tradeoffs in a typical attitude control computer as it might be incorporated in the attitude control system of a manned orbital space station. This piece of equipment was selected because it is typical of spaceborne equipment and because reasonable estimates can be made of the various pertinent parameters such as the mean time to failure and the fault location and repair times. A mission time of 1000 hr was selected, during which time a set of idealized experiments were assigned a representative range of performance times, critical times, availability coefficients, etc. The importance of each branch to each mission is given in terms of a set of <u>a priori</u> objective coefficients.

Five representative mission objectives were selected. The aim was to have mission objectives with a range of performance times, allowed failure effects, critical downtimes, and availability coefficients. To prevent complication of the results, the availability coefficients were purposely kept low but were varied to have a basis for determining the effects and possible flexibility of this variation. The first objective has an objective performance time as long as the total mission. The equipment must operate for the entire time to have no failure effect (the allowed failure effect is zero), and a downtime longer than 1/2 hr would result in the failure of this objective (the critical downtime is 0.5). The second objective also has a performance time as long as the mission, but can be achieved in one-tenth of this time (the allowed failure effect is 0.9 of the performance time without taking away from the success of this objective). The critical downtime is also as long as the mission, which means that any operation of the equipment during the objective performance time will result in at least partial success in meeting the objective. The remaining objectives have performance times of 100, 10, and 1 hr, respectively, and a variety of allowed failure effects and critical downtimes to evaluate the effects of these quantities on the mission achievement. In this study, the mission achievement was either 1.000 or 0.999 for every run, and no further analysis of the failure effect was made.

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Four different indenture levels or module arrangements (where modules are the parts to be replaced) were investigated with the attitude control system. It was assumed that each module was positioned with holddown screws and connected into the remainder of the system with connectors. The four indenture levels are:

1) Single-circuit modules (total, 35 modules);

2) Dual-circuit modules (total, 19 modules);

3) Multicurcuit modules (total, 4 modules);

4) Compl e system module (total, 1 module).

Twenty-five runs were made for each indenture level where a run is the simulation of a complete mission. Each run has a different sequence of random numbers so it represents a different possible mission history.

The mean time between failures (MTBF) for each circuit in the attitude control system was computed by summing the mean failure rates for each component. The mean times for maintenance operations were estimated for each indenture level on the basis of the module size and composition.

The computer simulation results for a typical indenture level is shown in Table 2. Listed for each run are the weight and volume of spares required, the combined weight of computer and spare modules, and the system cost effectiveness (computed using a weight cost factor of 10^{-4} to put the values in a convenient range, a volume cost factor of 0, and based on the combined weight of computer systems and spares). These runs are arranged in order of decreasing cost effectiveness.

The contribution of each run to the system cost effectiveness is given by the last column in Table 2. From these results, it is evident that, for this case, the optimum indenture level depends on the confidence required in meeting the mission objectives (although the relation between percentage of runs and the confidence of achieving a given ystems cost effectiveness is not discussed here).

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Table 2 Computer Simulation Run Results for Indenture Level A

Run	Mission Achievement	Spares Weight g	Spares Volume in. 3	Combined Weight, g	SCE
24	0.999	90.10	3.02	3394.00	2.943
21	0.999	270.20	9.04	3574.10	2.795
11	0.999	368.30	13.06	3672.20	2.720
4	1.000	458.40	15.08	3762.30	2.653
5	0.999	458.40	15.08	3762.30	2.655
18	0.999	458.40	15.08	3762.30	2.655
2	0.999	548.50	18.09	3852.40	2.593
1	0.999	639.40	21.10	3943.30	2.533
6	0.999	639.40	21.10	3943.30	2.533
8	0.999	639.40	21.10	3943.30	2.533
16	0.999	639.40	21.10	3943.30	2.533
20	0.999	639.40	21.10	3943.30	2.533
7	0.999	729.50	24.12	4033.40	2.477
23	0.999	729.50	24.12	4033.40	2.477
14	0.999	909.70	30.15	4213.60	2.371
15	0.999	909.70	30.15	4213.60	2.371
3	0.999	1007 80	33.16	4311.70	2.317
12	0.999	100 .80	33.16	4311.70	2.317
17	0.999	10 7.80	33.16	4311.70	2.317
19	0.999	1C 7.80	33.16	4311.70	2.317
10	0.999	11 1.70	39.20	4435.60	2.252
13	0.999	1131.70	39.20	4435.60	2.252
25	0.999	1131.70	39.20	4435.60	2.252
22	0.999	1861.10	63.32	5165.00	1.934
9	0.999	2984.80	102.51	6288.70	1.588

The tradeoff between indenture level and downtime is not apparent because the allowed failure effects, critical downtimes, and the objective coefficients assigned to the various mission objectives were such that the mission achievement was equally high for all indenture levels. For more critical mission objectives, this would not be the case, and the additional time required to locate the failures and replace the failed modules at the lower indenture level would result in a significant decrease in the mission achievement; this would have to be weighed against the advantage of the small weight of spares.

Another factor contributing to the lower weight and volume of spares required for the lowest indenture level (A) is the high commonality. The 35 modules are of five types, which reduces the spares requirement considerably. For higher indenture levels, the ratio of the number of modules to the number of types is shown as follows:

1) Indenture Level A, 35 modules,
   5 types, commonality ratio = 7.0;

2) Indenture Level B, 19 modules,
   5 types, commonality ratio = 3.8;

3) Indenture Level C, 4 modules,
   2 types, commonality ratio = 1.0;

4) Indenture Level D, 1 module,
   1 type, commonality ratio = 1.0.

In the attitude control computer selected, this commonality occurs because many of the elements and branches perform similar tasks. In many other systems, the different elements would perform distinctive tasks, and commonality could be increased only at the expense of the efficiency or performance of the system.

3.   Conclusion

The problem of determining the expected range of failure/maintenance/availability histories, spare parts and maintenance equipment requirements, and the resulting effects on the system cost effectiveness for manned space vehicles with extended mission times is ideally suited to the simulation approach. However, developing a simulation program is only part of the problem; an effective study of the maintainability design problem must also include an effort to determine the significant factors and how to incorporate these factors in the program to improve the program as a design tool to specify given requirements for maintainability, reliability, and availability.

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In the following chapters, the analytical basis for the
formulation of operational availability and examples of practical
problems are discussed. Although the treatment of this subject
is specifically aimed at the definition/design phase, the
theoretical considerations and parameters are also present in the
concept analyses areas.

SCOPE AND CRITERIA

The problem of specifying operational availability requirements,
either at the system state, or at the system element level is a
complex of analytical and design actions that involve a wide
range of design variables and disciplines. As in the design of
'aunch vehicle system to achieve a given performance capability,
the design for operational availability is a series of iterative
actions in which sizing of system elements permits comparing
what is obtainable versus what is needed. At each stage of design,
choices exist between both design alternatives and the system
performance parameters involved in meeting the mission require-
ments. The decision criteria must, therefore, be provided at each
design maturity, and these criteria must be consistent with each
level of detail. The respective design interaction, and the
correlations existing among some of the availability parameters,
provide both a rationale and a baseline for refining the design
solution.

The subsequent implementation of a design protocol for the
determination of system operational availability involves the
delineation of those functional areas and associated analytical
techniques that permit quantification of the availability
parameters. However, within each given functional area, detailed
analyses are required to determine, in depth, the respective
parametric performance characteristics and their interactions on
related subsystems. In this respect, the discipline of systems
analysis furnishes the necessary methodologies to assess various
alternatives as well as conduct optimization and suboptimization
studies.

Design Approach - As in most design problems, sizing and optimization
of design is an endeavor to strike a balance between several
variables that are often conflicting. Sizing to achieve an opera-
tional availability is no exception. In a maintainable system, the
balance is achieved between reliability of the system and the
maintenance provisions. This balance is usually established on
the basis of a criterion that describes the value of the system and
program. The value criteria in an aerospace program will usually
include the following factors:

1) Cost;

2) Development risk;

3) Weight;

4) Safety;

5) Size;

6) Schedule.

The implementation of a balanced design takes the form of a detailed examination of the requirements to achieve the objective (in this case, the degree and amount of operational availability) as well as the postulation of alternatives that will satisfy the program/system requirements. The closing on a compatible design configuration of operational and support elements is essentially a progressive process.

Figure 2 illustrates the interaction between the respective system design disciplines and the basic elements comprising each functional area. The analytical tools required, as well as the parameters generated at each point in the design process, yield an operational policy for implementing the design decisions in accordance with a set of criteria. The overall analysis uses simulation models and hypothetical scenarios for estimating the system performance characteristics. Subsequent verification of these system concepts provides a baseline from which further refinements are made. This iterative process converges rapidly in terms of satisfying the original require- ments and objectives. Figure 3 is an example of some of the elements that enter into the maintainability/reliability design process. The resultant design solutions (limited by technology available) include an assessment of the critical or most sensitive design or performance parameters. The analytic formulation of some typical design problems is treated in the following chapters. The basic areas of redundancy, wearout, maintenance policies, failure and repair rates, as well as operational factors are considered. Both analytical approxi- mations and simulation techniques are demonstrated as viable tools in the solution of large-scale system problems.
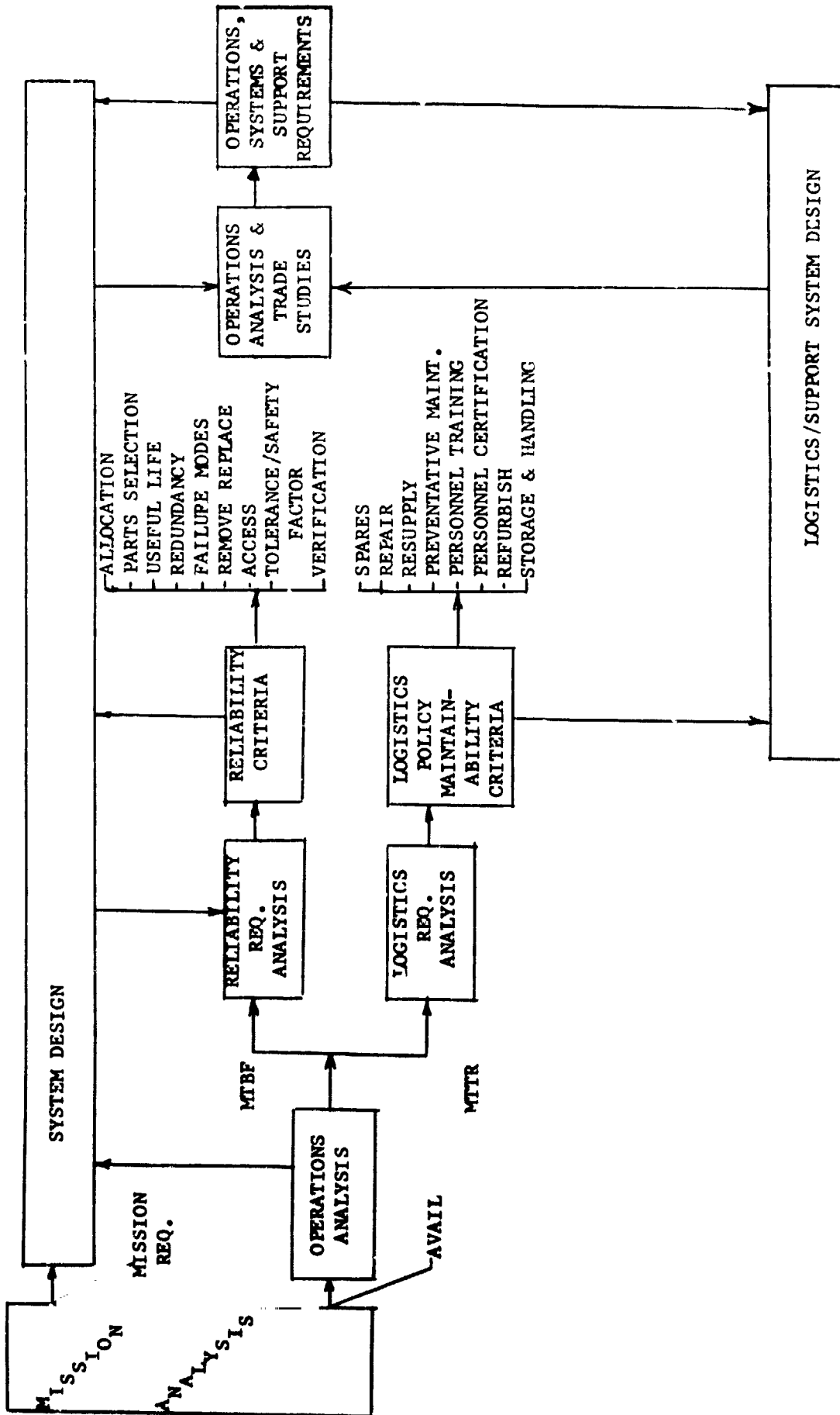
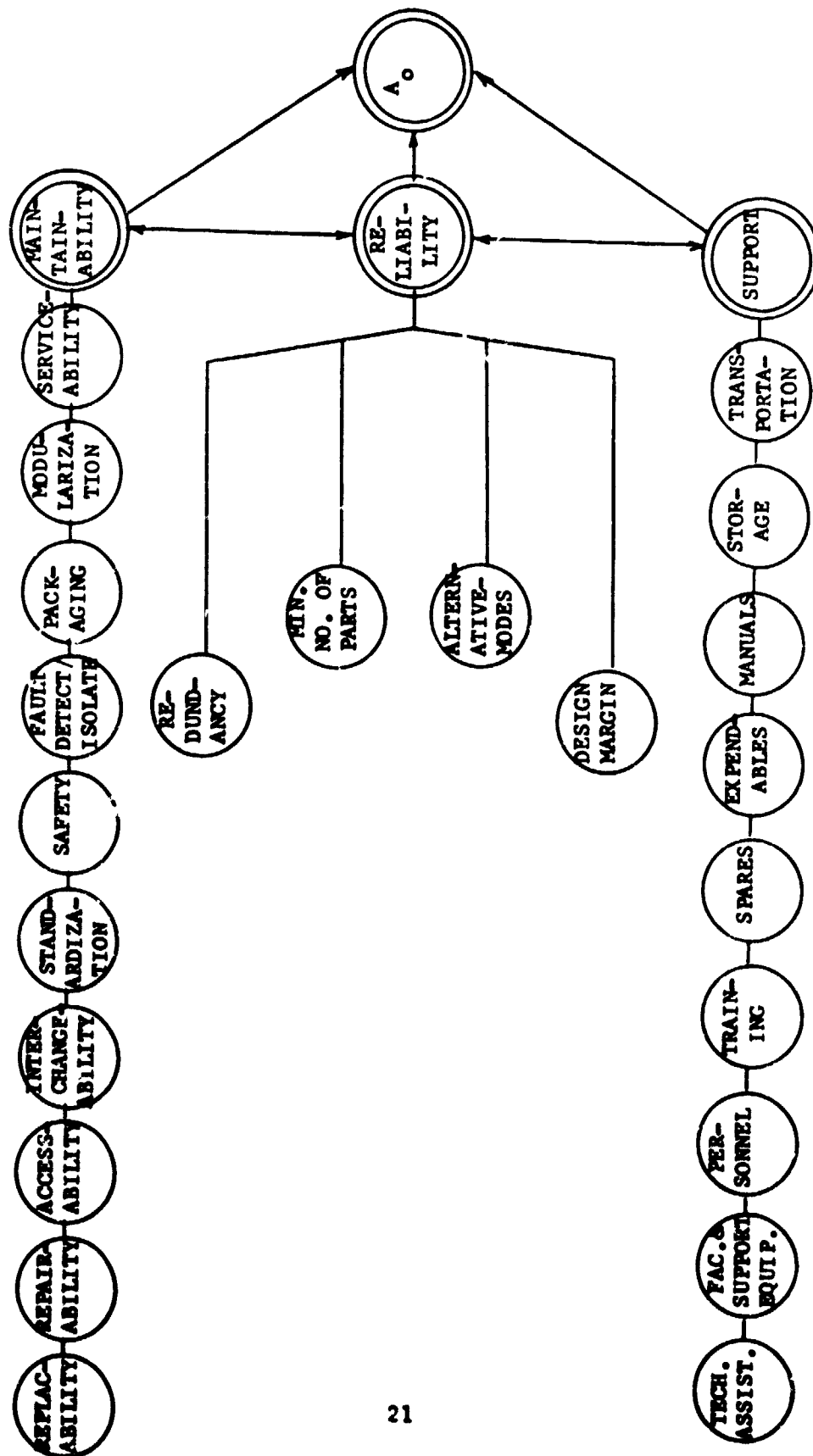Fig. 2 -- System Design Disciplines, Criteria, and Functional Elements

Fig. 3 -- Maintainability/Reliability Support Parameters

III.   ANALYTICAL DESCRIPTION OF OPERATIONAL AVAILABILITY

A full description of the availability function and the associated operational practice of a given system that can be maintained requires the specification of:

1)   The equipment failure and repair processes;

2)   The system configuration;

3)   The repair and maintenance policy;

4)   The state in which the system is to be defined as failed.

Within the past few years, much attention has been paid to an analytical technique known as Markov Processes. Mathematical models based on Markov Processes have found wide applicability in the fields of biology, engineering, physics, and the social sciences. In general, this technique uses the essential concept that the probability of obtaining a particular outcome on any trial (given a sequence of independent trials) depends only on the outcome of the directly proceeding trial. This implies a knowledge of the conditional probability associated with every pair of outcomes. In addition, the space of all possible admissible states and how transitions are made over a sequence of trials must be adequately defined. Thus, for example, one may define the states of a piece of equipment as operating or failed, and then consider how transitions are made back and forth from each of the possible states. It is essentially this formulation that is applicable to the analytical description of availability. Furthermore, if the conditional transition probability is constant, the resultant Markov Process is stationary. In the discussion that follows, the development of an analytical model using the Markov Process is developed under these conditions.

To employ a Markov representation, it is assumed that the individual equipment fails in accordance with the negative exponential distribution, and the times-to-repair are also exponentially distributed.

If an item of equipment is designed so that those items that are expected to fail frequently have a relatively short repair time compared with those items that fail infrequently, an exponential distribution of repair times is observed. On the other hand, if every part in an item of equipment has the same failure rate, and each takes equally as long to repair, a rectangular or uniform distribution of repair times results. The induced

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distribution of equipment repair times can be more easily controlled by the designer than can the distribution of times to failure. For example, if a few parts contribute the greatest percentage of equipment failures, the system designer can make sure that they are easily accessible for rapid replacement.

It is assumed that the equipment failure cumulative distribution is described by a negative exponential distribution, $F(t) = 1 - e^{-\mu t}$ so that the conditional failure probability in the interval t, $t + dt$ is $\lambda\, dt$. It is also assumed that the major portion of failures can be repaired in a short time, while those items that fail infrequently take a long time to repair. Therefore, the equipment repair cumulative distributions are exponentially distributed, and given by $G(t) = 1 - e^{-\mu t}$. Also, the probability of completing a repair in the interval t, $t + dt$, given that it was not completed at time t, is $\mu\, dt$.

In this formulation for maintained systems, it is necessary to develop the forward and backward differential equations that describe how transitions are made back and forth from state to state. If it is assumed that when an item of equipment fails it is immediately detected and repair is begun, and the times-to-failure and times-to-repair are each independently exponentially distributed, the resulting Markov Process is referred to as a "Birth and Death Process." The Birth and Death Process describes a system's availability in terms of transient and steady-state components. For systems that are to be operating continuously for a long time, the steady-state solutions are usually sufficient. The basic equations for this simple formulation are developed in Appendix B, as well as some significant variations, such as partial repair, the case of n equipments with r repair men ($r = n$, and $r < n$), as well as a two-equipment items redundant system. In addition, the problem of wearout, which involves a non-Markovian Process, is decomposed into a sequence of Markovian Processes by segmenting the input distribution into several exponential phases. This technique is also discussed in detail in Appendix B.

For systems that can be maintained, there are two figures of merit that are usually of interest. The extent to which a system can be expected to be in a state or condition to perform its assigned function within an established time frame and under given environmental conditions is referred to as the system's "availability." This approach has been treated in detail in the previous chapters. Still another figure of merit that is relevant to maintained systems is the "Mean Recurrence Time." This is the length of time to return to an acceptable state from a failed state. Sometimes this

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figure of merit is called Mean Single Downtime. The importance of the Mean Recurrence Time is obvious, because availability is concerned with the total time the system spends in acceptable states and does not indicate how this time is distributed. For example, in a 10,000-hr period, the system may fail once and be down for a 10-hr duration, yielding an availability of 0.999. On the other hand, the system may fail 10 times in the same period and be down for 1 hr each time, also yielding an availability of 0.999.

Durations of single downtimes can also be related to a penalty cost that must be paid for each consecutive unit time the system is unavailable. An air traffic control system that establishes flight plans and directs the landing and takeoff of aircraft is an example. Long durations of single downtimes may mean queuing at takeoff, thereby delaying schedules, lost flights, and so on. In this sense, it would be preferable to have a higher system failure rate with shorter durations of single downtime incidences than a low system failure rate with long durations of single downtime incidences.

During the preliminary design phases, consideration is usually given to the expected repair time of all items in the system and which failures would require the system to be down. A good example is the case of bearing supports in a radar pedestal.

Although the failure rate of bearings may be relatively low, it may take several hours to replace one, and the radar set would have to be "off the air." Therefore, although the steady-state availability may not be significantly affected, the eventuality of a bearing failure may cause serious consequences--the radar would be unavailable for several hours. At the preliminary design phase, this effect must be considered and alternative approaches that would reduce the duration of single downtime without significantly increasing system cost evaluated. One approach is to use high-level redundancy and duplex the radar pedestal. However, this alternative is costly. Another approach is a lower level of redundancy where only the bearing gear supports are duplexed to permit repair of a single bearing failure without affecting system downtime.

It may be well at this point to distinguish between the concept of statistical expectation and the steady-state, or state of statistical equilibrium. The expected value is simply the average value of the availability function over all possible values of the variable. If we had a large number of equipment items that had been operating for some time, then at any particular time we would expect the number of equipment items that are in state 0 (available) to be NP_o. Thus, the ratio of the number of equipment items available to the total number of equipment items is simply $NP_o/N = P_o$.

When we are concerned with steady-state solutions, we are postulating the existence of limits, i.e., the steady-state distribution will maintain itself ideally in an infinitely large ensemble. The Markovian formulation does not consider fluctuations in the individual items of equipment. For example, if a particular equipment item fails on the average every 100 hr of operation and takes 1 hr to repair, its steady-state availability is $P_o = 100/101$.

However, the range of the time of equipment failure is $(0, \infty)$. Thus, the steady-state availability 100/101 says nothing about the fluctuations of an individual equipment item's availabilities. It tells us that, in an infinitely large ensemble of equipment items, for each item that never fails, there is one that fails the instant it is "put-on." For a single-equipment item system (including some significant variations) it is not too difficult to develop the availability distribution and its moments. However, for complex configurations the analytic effort is intractable and Monte Carlo simulation techniques usually have to be employed. In this respect, it is essential to conduct a statistical analysis of the simulation results to obtain confidence limits and assess the effects of sampling. The question of how many samples to run is usually a compromise between computer costs involved and the accuracy of the results desired.

IV.   ILLUSTRATIVE EXAMPLES OF OPERATIONAL AVAILABILITY

A.   MINIMUM COST CRITERIA FOR A GIVEN AVAILABILITY

When operational availability is used to address a problem of
meeting a time constraint, i.e., a specific schedule time, the
analysis is concerned with the selection of operational modes
in terms of time, spares, and expected failure rates. When
availability is defined to mean the probability that the system
will be in an acceptable condition in any given state, the
analysis is concerned with either a particular solution
(evaluation of initial conditions) or long-term (steady-state)
solution of the basic availability equation. In this case,
the sizing of MTBF and MTTF serve as goals or allocations from
which system concept and definition design can be performed.

One example of the latter case is d scribed in Appendix C. In
this design problem, the steady-state, long-term availability
of a system in a given mission state is of prime concern. This
example describes a particular problem in which the given
availability and minimum cost are the criteria for selection.
In addition to the specification of the factors that contribute
to MTBF and MTTF, this example problem also shows that the
sensitivity of figures of merit require testing to verify that the
indicated results are valid. In this case, a simple joint
probability computation verifies the selection.

The analytical development of this technique for a uniform
distribution is given in Appendix D. The results are quite
straightforward, and the derivation may be applied to various
other overlapping probability densities.

B.   PROBABILITY OF LAUNCH ON TIME

The launch-on-time concept of availability can be addressed in a
manner similar to the example given in Appendix C. The decision
as to how to achieve a given level of probability of readiness
is concerned with apportioning between MTBF and MTTF and then
allocating these values to the technical parameters of the system.

There is, however, yet another approach to solving this same
generic type of problem. This is illustrated in Appendix E.
This example is an operational analysis aimed at selecting
mission state duration, as well as the generation of a concept
to meet a given probability of two successful launches. In this
case, the MTBF of the system is assumed fixed. This is a
reasonable condition because the vehicle design leads the ground
system design in the development process. This analysis has as
its objective to establish the launch operations strategy and the
reliability and logistics requirements that yield an acceptable
operational availability. The exact nature of this analysis
is described below:

It is assumed that the system concept is made up of a launch
vehicle and a mission payload module. The problem is then to
launch two payloads within a specified launch period. The
operational analysis provides a solution to establish the launch
operations strategy and the reliability and support requirements
that best meet the availability objectives. The variables in
this analysis are:

1) The probability of holds during operations on the launch pad
   due to,

   a) Launch vehicle malfunctions,

   b) Spacecraft malfunctions,

   c) All other causes;

2) Recycle of the launch vehicle in the event of a malfunction
   on the launch pad;

3) Recycle of the spacecraft in the event of a malfunction on
   the launch pad;

4) Spares provisioning and replacement time for,

   a) Launch vehicle,

   b) Spacecraft;

5) The duration of the launch period;

6) Turnaround time between launch attempts.

Dealing at this level of design, major system decisions are made such as:

1) What is the optimum time for launch operations as a function of reliability of the major system elements?

2) What are the system element spares that yield the greatest probability of mission success?

To illustrate this operational analysis, a hypothetical case is developed in which it is desired to perform two launches of a vehicle and its payload. In this example, the analyses resulted in:

1) Estimates of reliability for the system elements (launch vehicle and payload);

2) Recycle time for the system elements in the event of malfunction;

3) Limitation on the number of malfunctions.

Furthermore, previously determined mission analysis studies have selected three operational concepts for analysis:

1) Candidate 1,

   a) 22-day launch period,

   b) 10-day turnaround (launch-to-launch),

   c) Spare launch vehicle and payload;

2) Candidate 2,

   a) 30-day launch period,

   b) 16-day turnaround,

   c) No launch vehicle with spare payload;

3) Candidate 3,

   a) 30-day launch period,

   b) 10-day turnaround (launch-to-launch),

   c) No launch vehicle with spare load.

Table 3 shows the initial conditions that apply to these three cases.

Table 3 Conditions and Assumptions

1. Probability of a malfunction or delay during any one launch attempt (can reoccur):

Launch vehicle	0.05
Payload	0.05
All other causes	0.05

2. Recycle Times (malfunction-to-launch):

Payload recycle	19 days
Launch vehicle recycle	14 days
Replace integrated launch vehicle and payload	5 days
Remove payload from launch vehicle and replace	9 days
All other causes	3 days

3. The payload is limited to a total of 2 recycles.

4. The probability of the payload requiring recycle before launch pad operations = 0.05 (can reoccur). This applies to both prelaunch and recycle operations.

5. The probability of the launch vehicle sustaining a malfunction during recycle that would preclude launching on time = 0.05.

6. The program requirement for probability of two launches within a specified time period is 0.015.

The detailed analysis of these three candidates is included in Appendix E. For this sample problem, the analytical approach is relatively simple and straightforward. It would have required more complex methods and techniques had the numbers of variables entering into the decision been larger.

Table 4 compares the three approaches in terms of percentage of launch-on-time risk. The second candidate differs from the first in that the launch period was longer (22 to 30 days) and the turnaround time was longer (10 to 16 days). These changes essentially counteracted each other. The significant difference is that a launch vehicle and payload were considered in Candidate 1. Deletion of the spare set increased the launch vehicle risk from 0.30 to 5.46%.

The third candidate differs from the second in that the turnaround time has been reduced from 16 to 10 days. This reduces the total risk from 5.98 to 0.85%. Most of the risk in No. 3 is due to the launch vehicle, because no spare is provided. This candidate compares favorably with No. 1, although it is not quite as good.

Table 5 compares the approaches in terms of the number of malfunctions that can be tolerated on the launch pad during the launch period. The No. 2 approach is very restrictive; only one malfunction could occur in the launch vehicle and that had to occur in the first vehicle. The third candidate can tolerate more malfunctions than No. 1, depending on where they occur; however, there is an increased risk of a malfunction occurring during launch vehicle recycle. The first approach allows for one launch vehicle recycle, while No. 3 allows a maximum of three. The more recycles, the higher the risk.

The following conclusions were drawn from this study:

1) The launch-on-time risk for No. 3 is substantially less than for No. 2 and compares favorably with No. 1. This was achieved by reducing the turnaround time from 16 to 10 days;

2) Candidate No. 1 meets the allocated launch-on-time risk of 1.5%;

3) Most of the risk is caused by the launch vehicle because a spare is not available;

4) The launch-on-time risk cannot be reduced by further reduction of the turnaround time or extension of the launch period;

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Table 4 Comparison of Launch-on-Time Risks

| | Launch-on-Time Risk, % | | |
	Candidate 1	Candidate 2	Candidate 3
Payload	0.32	0.54	0.09
Launch Vehicle	0.30	5.46	0.76
All Other Causes	0.05	0.01	0.00
Total Risk, %	0.66	5.98	0.85

No. 1 - 22-day launch period
        10-day turanround
        Spare launch vehicle and payload

No. 2 - 30-day launch period
        16-day turnaround
        No spare launch vehicle
        With spare payload

No. 3 - 30-day launch period
        10-day turnaround
        No spare launch vehicle
        With spare payload

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Table 5 Maximum Number of Malfunctions That Can Be Tolerated
(Two Launches)

	Malfunction		
	Candidate 1	Candidate 2	Candidate 3
Payload	2	2*	2 or 3
Launch Vehicle	2	1	2 or 3
All Other Causes	3	4	6

*Both malfunctions must occur in the first
vehicle; otherwise, we can tolerate one pay-
load malfunction in Vehicle 1 OR one payload
malfunction in Vehicle 2.

Must occur in Vehicle 1. We cannot tolerate
any launch vehicle malfunctions in Vehicle 2.

5) An increase in turnaround time or a decrease in the launch period would result in a significant increase in the launch-on time risk;

6) The launch-on-time risk could be significantly reduced by providing a standby launch vehicle and payload. This would also provide flexibility to changes in turnaround time and launch period. However, a spare launch vehicle and payload is not warranted by this analysis. A more sophisticated analysis would have to be conducted to determine if such a selection should be made;

7) On the basis of this analysis, Candidate 1 is selected because it provides the most margin with respect to the system risk requirement.

C. MONTE CARLO AVAILABILITY SIMULATION

The Monte Carlo method may briefly be described as the device
of studying a real-world phenomena by means of a simulated
mathematical process, using random sampling to structure the
solution and establish statistical inferences.   The device
is certainly not new. Moreover, the theory of mathematical
simulation has been a subject of study for quite some time,
and the novelty of the Monte Carlo method does not lie here.
The novelty lies rather in the suggestion that where complex
situations exist that demand numerical solutions not readily
obtainable by standard analytical methods, there may exist a
mathematical simulation process with statistical distributions
or parameters that adequately approximate the real-world
circumstances. In these instances it may actually be more
efficient and economical to construct such a process and compute
the statistics, rather than attempt to use deterministic methods
to describe the system.

Historically, the original von Neumann-Ulam concept seems to
have been that Monte Carlo specifically designated the use
of random sampling procedures for treating complex deterministic
mathematical problems not easily amenable to numerical solution.
However, some define Monte Carlo to be the exclusive use of
random sampling procedures to treat problems, whether of a
deterministic or probabilistic sort. Others demand that the
sampling be sophisticated (involve the use of some variance
reducing technique) to qualify as Monte Carlo; they reserve
the names straightforward sampling, experimental sampling, or
model sampling for the cases where purely random sampling is
used. However, the economics of computing changes so rapidly
with the advent of faster and faster machines that Monte Carlo
methods are being successfully applied to more and more
sophisticated and complex problems. The situation now is that
in common usage, Monte Carlo is synonymous with any use of
random sampling in treatment of either deterministic or
probabilistic problems.

The application of the Monte Carlo simulation technique to
operational availability is given in Appendix F. This model
computes the system availability of a launch vehicle, which
involves a sequence of tests, and storage of specified system
and subsystems. Figure 4 illustrates in schematic form the
launch vehicle storage and test cycles.

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CONDITION
VECTOR
DEL. FROM FACTORY

$$[\ 1,0,0,0\ ] \times \begin{bmatrix} \text{LAUNCH VEHICLE} \\ \text{INITIAL} \\ \text{STORAGE} \end{bmatrix} \times \begin{bmatrix} \text{LAUNCH VEHICLE} \\ \text{INTEGRATED} \\ \text{TEST} \end{bmatrix} \times \begin{bmatrix} \text{LONG-TERM} \\ \text{VEHICLE} \\ \text{STORAGE} \end{bmatrix}$$

D=PROB. NOT CAUSING DET. FAIL

N=PROB. NOT CAUSING UNDET. FAIL

P=PROB. DET. A DET. FAILURE

$$\begin{bmatrix} DN & (1-D)N & (1-N)D & 1-E \\ DNP & N(1-PD) & PD(1-N) & 1-E \\ 0 & 0 & D & 1-D \\ 0 & 0 & PD & 1-PD \end{bmatrix}$$

NWS TRANSITION
MATRIX

γ = FAILURE RATE
δ = UNTESTABLE PARTS
ζ = TESTABLE PARTS
τ = TIME OF STORAGE

$$\begin{bmatrix} R_1R_2 & R_1Q_2 & R_2Q_1 & 1-E \\ 0 & R_1 & 0 & Q_1 \\ 0 & 0 & R_2 & Q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

STORAGE TRANSITION MATRIX

$$R_1 = \epsilon^{-\delta\gamma\tau}$$

$$R_2 = \epsilon^{-\zeta\gamma\tau}$$

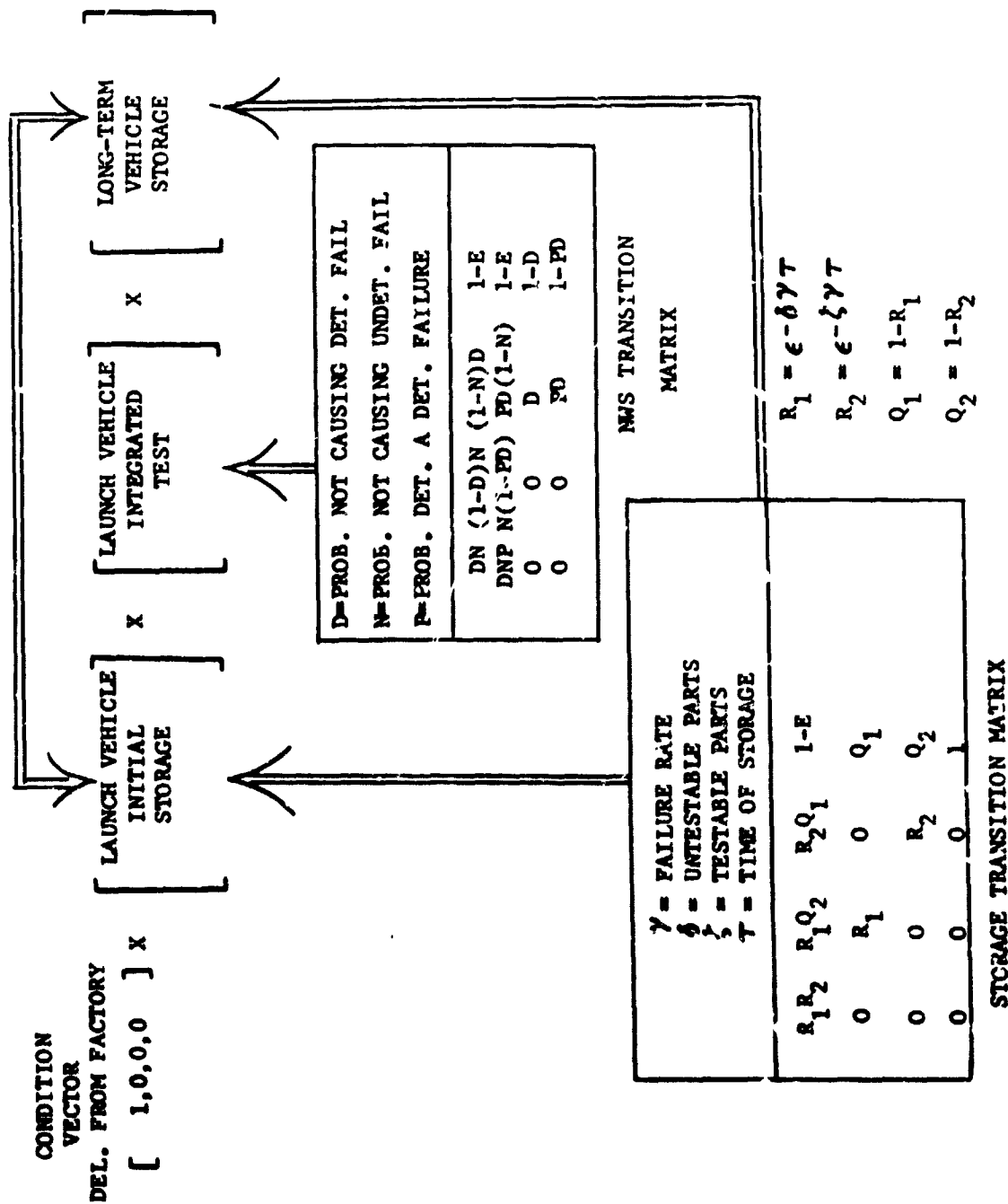$$Q_1 = 1-R_1$$

$$Q_2 = 1-R_2$$

Figure 4 -- Simplified Transition Model of Storage and Test Cycles

The development of the storage and test cycles is illustrated in Appendix F, along with the simulation results. It may be concluded that:

1) The most important factor affecting the availability of the launch vehicle is failure rate of parts;

2) Availability after each storage and test cycle drops off continuously;

3) Probability of detecting a failure is relatively (within normal ranges) insensitive to availability;

4) Length of launch vehicle storage (for example, 12 months or 36 months) will cause availability to fall off sharply.

In running this Monte Carlo simulation, the basic data were accumulated over a 36-month period, using Markov transition matrices for storage and test operations. Thus, the sample size was large, so that statistical variations were minimized.

The illustrative examples presented focus attention on the various applications and different types of problems en   ..¹ ³d in determining operational availability. The analytic·l concepts developed provide a framework for structuring a broad class of problems, and furnish insight into the subtle mechanisms of obtaining real-world solutions that are useful to the engineer and the designer. The subsequent interpretation and integration of these results into a comprehensive methodological design tool is a continuing process, combining both the skill and analytical ability of the design team. Note that a total system approach involves a complete effectiveness analysis of the essential parameters of operational availability in terms of the performance characteristics of the overall system. Thus, although operational availability has been treated as a separate topic, its effective impact on related subsystems and other functional requirements must also be examined. The end result is to yield a design concept that is analytically sound, economically feasible, and capable of being implemented in a real-world application.

In summary then, operational availability may be approached from several apparently different and divergent viewpoints. However, it should be observed that this basic parameter must be analyzed both on a systems and individual basis to achieve an integrated and balanced design.

A.   ANALYTIC MODELS FOR EFFECTIVENESS EVALUATION

Any meaningful application of the systems performance effective-
ness concept to a particular project requires a quantitative
methodology to evaluate the effectiveness of a proposed or actual
system in terms of selected measures, requirements, and decision
criteria. Until this is done, the concept of systems performance
effectiveness for a project has little use--except perhaps as a
rallying point for arguments about the advantages of System X
over System Y. The need for a systems performance effectiveness
evaluation methodology begins at the inception of the system
life cycle and continues through the succeeding design, develop-
ment, production, test, and operational phases. Despite the
obvious differences in the depth of the analysis applicable to
these phases, the need for a quantitative metho ogy applies
throughout.

Evaluation methodologies for systems performance effectiveness
characteristics can be broadly characteriz d in terms of two
approaches--the empirical and the analytic (Ref 1).

An empirical methodology is one devoted to data collection and
evaluation of existing systems. Thus, it is possible to evaluate
systems performance effectiveness by means of performance obser-
vations of systems in the field. While this approach is undoubt-
edly the most accurate, it is feasible only for systems or pro-
jects that are very far advanced in their life cycles.

An analytic methodology, on the other hand, is o: that derives
its results by inference, and uses a set of assumptions and pro-
cedures as a framework to compute an effectiveness description
of the system in question. This descriptive system frame work
is called an analytic model, and the description of System X in
these terms is called the analytic model of System X.

Purely empirical or purely analytic methodologies are, of course,
not very useful. The former yields highly authoritative data
too late to be useful, while the latter yields answers unsup-
ported by facts. In practice, a balance is sought. This bal-
ance will normally change during the life cycle of a system. As

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data about the behavior of the system become more available, the analytic model gradually merges into am empirical model; as the data become more available and as confidence in their value increases, statistical sample data supplant the assumptions.

Analytic models, moreover, usually remain useful even with re ard to the empirical data obtained from system samples ta en during acceptance tests. Also, these data often require interpretation simply because it is impractical to conduct tests that e suffi-ciently elaborate to yield statistically significant effective-ness data directly.

The need for analytic models to predict systems performance effectiveness thus emerges from the need to evaluate the effec-tiveness before the system has been in use for many years. The following discussion considers the analytic models--with he understanding that empirical methods will always be required to provide inputs for the analysis.

B.   CHARACTERISTICS OF AN EFFECTIVENESS MODEL (REF 2)

There are certain general characteristics that any mathematical model should have to be a useful tool to predict effectiveness. These are:

1) Independence from design assumptions - If the concept of effectiveness is to be applied as a technical management tool, there is a demand that the effectiveness-analysis technique, and consequently the analytic model, be capable of evaluating alternative (or modified) system designs with respect to a fixed set of mission models and variables. To whatever extent the analytic model presupposes system design configurations or characteristics, the model is not able to evaluate alternative designs that are not within these con-straints, and hence it may not provide a basis for compari-son or optimization. For example, if the analytic model is built in terms of a given system-design configuration, other design configurations may be inequitably treated if subjected to the same analysis.

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2) Usefulness throughout the system life cycle - The analytic model should be one that can be used throughout the system life cycle. In the early stages of the cycle, relatively few data are available on the statistical or performance capability of the system, and a substantial number of assumptions must be made to permit the analysis. As the life cycle progresses through design, development, test, and implementation, additional design and sample test data ordinarily become available. The analytic model, therefore, should be designed to accommodate these changes in inputs, and yield successive systems performance effectiveness predictions throughout the life cycle, with increased confidence in the results.

3) Realism in the analytic assumptions - The physical and mathematical assumptions on which the model is founded must be realistic with respect to the expected characteristics of the mission and system operations. There is a great temptation to construct analytic models based more on mathematical elegance than on realism.

4) Tractability of the evaluation - For the model to be usable, it must give numerical answers when exercised. This implies the model must be quantitative even in the face of limited data and it must be amenable to computation. Clearly, this model characteristic must be traded off against the characteristic of realism. The art of modeling consists, in large measure, of establishing this balance.

C.   SELECTING AN EFFECTIVENESS MODEL

There appear to be three fundamental classes of considerations that enter into the selection of an appropriate effectiveness model--the outputs required for system management and optimization, the nature of the systems to be analyzed, and the mission characteristics to be employed.

1.   Output Considerations

The definitions of the variables, requirements, and decision criteria influence the selection of an appropriate effectiveness model. The following questions are typical of those that must be answered:

1) Can the system-oriented performance variables be identified with specific hardware, or are they more closely tied to overall system behavior, including software?

2) Was an iterative methodology employed to establish the requirements and decision criteria? (The requirements on the model themselves could change during the iterative procedure, and these changes must be incorporated.)

3) Is there one principal performance variable that corresponds to one principal system function, or is the system called upon to do many things?

4) Will the utility and tradeoff data permit the results of effectiveness analysis to be expressed in terms of discrete quantities, or will probability distributions be required to describe systems performance effectiveness adequately?

5) Are the variables binary (success/failure) or multivalued?

2. System Considerations

System considerations concerning the choice of an effectiveness model have their greatest effect on the statistical and logical assumptions that underlie the model. In a given system, it may be uniquely possible to identify subsystems with their corresponding functions, and in such a case the effectiveness evaluation is simplified. On the other hand, if interaction of subsystem functions is expected, particularly with degraded modes of operation, the model must incorporate this flexibility of interaction. Additionally, the analytic model often incorporates assumptions concerning the statistical behavior of the system. These assumptions may be valid for the system in question, and they may be consistent with the available data. Finally, the scope and complexity of the system must be considered. The delicate balance between tractability and realism discussed above must be resolved in terms of anticipated system size (size being expressed in such terms as the number of components).

3. Mission Considerations

In addition to mission effects, described in Subsection 1, a series of representative mission profiles also must be examined as part of the model-selection process.

The results of the studies discussed elsewhere in this report
are closely involved in this examination. For example, is the
system operating in a steady-state environment, or are the
missions short compared with other statistical time parameters?
In the former case, an equilibrium or steady-state model may be
employed. In the latter case, a mission-sensitive model in
whole or in part is required.

Again, is the mission function carried out over a time segment,
or is a point mission involved? Are there one or several critical
mission segments? Do the requirements and decision criteria for
systems performance effectiveness change with mission mode? Do
the reliability and maintainability characteristics of the sub-
systems change as a function of a mission segment?

a. **Mission Analysis (Ref 3)** - Before analyzing a well-defined
system, let alone developing a new one, it is necessary to know
what the system is supposed to do, i.e., what missions it must
perform. It is relatively easy to establish performance envelopes
for various subsystems that, in turn, enable the system to per-
form one task in one environment. The problem, however, becomes
much more difficult if one must consider many tasks under many
environments.

One scheme often used is to develop figures of merit. This
scheme weights the effectiveness figure for each task by the
frequency with which the system may be called upon to perform
each task. However, systems are developed to satisfy specific
mission requirements that have been formulated on the basis of
specific tools. The best system selected in accordance with
such a scheme may not, therefore, be capable of responding to a
specific parameter value that of itself may be extremely serious,
but also may occur very often. Thus, a system may not be satis-
factory from the standpoint of a specific subtask.

It is possible, however, to alter the method by weighting the
parameter by their severity and expected frequency of occurence,
and thus design the system for some other weighted average of
values. However, this does not solve the problem either, be-
cause optimizing an answer to any average value does not optimize
the answer to each particular value. Perhaps the solution is to
develop a procedure to optimize the effectiveness of answering a
suitable, chosen, average that is subject to the achievement of
a given effectiveness figure against each specific value.

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On one hand, it makes little sense to speak about the average effectiveness, or the average environment. On the other hand, one cannot optimize with respect to a single mission unless the system has only one task to perform. Such a procedure, therefore, leads to all the difficulties associated with suboptimization, which results when one tries to optimize a system by optimizing each subsystem.

b. <u>Figure of Merit</u> - Perhaps the difficulty associated with the formulation of an appropriate figure of merit can be best explained by a discussion of the analogous problem encountered with the parameters of a frequency distribution. For example, with a distribution a population can be charaterized by its mean; but this does not indicate the variability about this mean. A measure of variability, called the standard deviation can also be added to this characterization.

Yet there will be other properties of the population that are not pictured by any of these characterizations, e.g., the lack of symmetry. If the 10th and 90th percentiles are also given, however, a more complete picture of the population begins to take shape. Nevertheless, no finite set of parameters can ever completely describe a real population or its frequency distribution.

Similarly, if one wants to know the system-effectiveness figures in all situations, something analogous to the frequency distribution is needed. Although there may be a figure of merit analogous to the mean in the frequency-distribution example, it can never give all the information about the system.

c. <u>Degraded Performance</u> - At the component level, degradation can be thought of as measured by the number of failed components. At the system level, however, degraded performance may refer to the probability of performing the mission. For example, the system might be designed to be capable of performing its mission 95% of the time in one environment. In another environment, however, it may have only a 90% capability, and this may be considered degraded performance.

On the other hand, consider a radar-weapons system. Its design performance consists of being able to pick up an object at a certain distance, and then being able to assign appropriate weapons once the object has been correctly classified. A probability is associated with each subsystem performance and, hence, also with the system's performance. If the probability associated with any subsystem is reduced, however, the probability of system performance will also be reduced; this, too, can be considered degraded performance.

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Models of degraded performance can also be modified so that they are time-variant. There are, moreover, many other possible definitions of degraded performance. The one to use, of course, is the one that is useful in assessing a system. Much current work is misunderstood because one group does not use the same concept of system degradation as another.

d. __Human Factors__ - Concern for human factors falls into three major areas--life support, personnel and training, and human engineering. It is the goal of life support to maintain and protect the human by controlling his environment; the goal of personnel and training is to select, train, and assign the human for operational tasks; and human engineering provides the design engineer with the basis for the most effective use of the human component of the system. In short, the discipline of human factors in systems performance effectiveness requires that the man module be considered just as a hardware component--to be evaluated for cost, reliability, maintainability, availability, and operability. In addition, the man module must be considered for trainability.

To achieve these goals in a disciplined fashion, certain procedures must be followed. These procedures are not one-time events to be accomplished early in the development phase, but rather are iterative procedures that must be reviewed and changed where necessary just as design is reviewed and changed during an equipment's development.

D.   ANALYTIC FRAME WORKS FOR EFFECTIVENESS MODELS

The development of a model that satisfied the conditions cited in the preceding section is at best a complicated task. However, even under the assumption that such a model is realizable, its range of application without some types of modification would be restricted. This is due to the nature of the requirements, diverse operating conditions, and use factors that generally are an integral part of the specific system. To help solve this problem, an approach that uses a system effectiveness framework has been developed. In this approach, the basic system performance effectiveness elements remain constant for different system missions and use functions, although the more detailed factors underlying the basic elements are subject to change, depending on the particular problem analyzed.

Several systems performance effectiveness models have been developed. Table 1 lists equations for the more general models used. All these equations concern systems performance effectiveness, but each approaches the subject in a different manner, reflecting the needs of the individual user.

The following general equations are derived (Ref 4):

$$E_S = E_A \approx E,$$

$$f(F,A,U) = f(P_C, P_T) \approx f(A,D,C).$$

Note that properly constructed models of the same system carrying out the same mission will give the same evaluation and may even be mathematically identical. The basic equations given in Table 1 can only be used for simple systems in simple missions, even if the equations are assumed to be in matrix form.

Several systems performance effectiveness models are given below. Emphasis has been placed on describing the framework of each model rather than on providing a detailed description.

1. ## The Effectiveness Model (Ref 5)

The first term, performance (P), in the effectiveness model (PAU) can be expressed within several frames of reference. In the single-mission system, the expression is derived from a variety of measurements, e.g., area destroyed, tons of cargo or number of passengers delivered, emitters located and identified. Two important conditons apply: (1) the measurement standard used must be applicable to the parameter used to determine the performance level, and (2) the answer derived from exercising the expression must be used with caution because, with other than extremely simple systems, the achieved performance capability is almost always less than the theoretical performance capability. This circumstance occurs because the design-optimization process requires that some tradeoffs be made to achieve optimization of the overall system. As a result, even for the relatively simple, single-mission system, P is expressed as an index representing the ratio of the achieved performance level to the theoretical desired level. In essence, it is a figure of merit even under the assumption of absolute availability and absolute use.

Table 1 - Equations for Systems Performance Effectiveness Models

TITLE	EQUATION	TERM	EXPLANATION OF TERMS
Systems Performance Effectiveness	$E_S = (PAU)$ $E_S = f(P,A,U)$	E_S	Index of Systems Performance Effectiveness
		P	Index of System Performance - a numerical index expressing system capability assuring a hypothetical 100% availability and utilization of performance capability in actual operation
		A	Index of System Availability - a numerical index of extent to which a system is ready and capable of fully performing its assigned mission(s)
		U	Index of System Utilization - a numerical index of the extent to which the performance capability of the system is utilized during the mission
Analytic Systems Performance Effectiveness	$E_A = (P_C P_T)$ $E_A = f(P_C P_T)$	E_A	Systems Performance Effectiveness
		P_C	Measure of Performance Capability - a measure of adequacy of design and system degradation
		P_T	Measure of Detailed Time Dependency - a measure of availability with a given utilization

TITLE	EQUATION	TERM	EXPLANATION OF TERMS
System Effectiveness	$E = (ADC)$	E	Quantitative measure of systems performance effectiveness
	$E = f(A,D,C)$	A	Measure of Availability - a measure of the condition of a system at the start of a mission when the mission is called for at unknown (random) point in time
		D	Measure of Dependability - a measure of the system condition during the performance of the mission given its condition (availability) at the start of the mission
		C	Measure of Capability - a measure of the results of the mission given the condition of the system during the mission (dependability)
Cost Effectiveness	$E_C = \dfrac{E_S}{C_a + C_o} = \dfrac{f(P,A,U)}{C_a + C_o}$	E_c	Index of Cost Effectiveness
		C_a	Cost of Acquisition - the aggregated costs of acquiring the system, including prorated development costs
		C_o	Cost of Ownership - the aggregated costs of operating and maintaining the system

TITLE	EQUATION	TERM	EXPLANATION OF TERMS
Defense Effectiveness	$E_d = \dfrac{W}{E_t}$	E_d	Index of Defense Effectiveness
	$E_c = \dfrac{W \ell (P,A,U)}{E_t(C_a + C_o)}$	W	Index of Military Worth
		E_t	Index of Time Effectiveness
Cost of Acquisition	$C_a = f(C_{at}, C_{am}, C_{af}, C_{as})$	C_{at}	Acquisition Time Costs - the calendar time required to acquire the system
		C_{am}	Acquisition Manpower Costs - the manpower and skill levels required to acquire the system
		C_{af}	Acquisition Financial Costs (Dollar) - the dollar outlays required to acquire the system, including the dollar costs associated with manpower and its acquisition and supporting facilities
		C_{as}	Acquisition Supporting Facilities Costs - the penalty cost to other systems through use of support facilities by the system during acquisition of the system

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TITLE	EQUATION	TERM	EXPLANATION OF TERMS
Cost of Ownership	$C_o = f(C_{om}, C_{of}, C_{os})$	C_{om}	Ownership Manpower Costs – the manpower and skill levels required to operate and maintain the system
		C_{of}	Ownership Financial Costs (Dollar) – the dollar outlays required to operate and maintain the system, including the dollar costs associated with manpower and its acquisition and supporting facilities
		C_{os}	Ownership Supporting forces and Facilities Costs – the penalty costs to other systems through requirement for and use of supporting facilities and forces in the operation and maintenance of the system

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In the case of the multimission system, consideration must also be given to the interaction of two judgments. The first comprises the assignment of weighting (importance) factors to the several mission modes of the system in such a way that their sum is 1.0. The second comprises the determination of the fraction of the system's total mission time that will be devoted to each of the several mission modes.

It is in the multimission system that the compelling reason for using indexes becomes most apparent. Many such systems have completely disparate standards for measuring the performance of their various mission modes. A comparison or aggregation of performance indexes that use different measurement standards cannot be attempted validly. For example, tons of cargo delivered, area destroyed, personnel transported, and enemy radar sites located and identified cannot logically be compared or aggregated.

The second term, availability (A), is more complex than the first. Overall availability is relatively easy to measure, but separating the overall value into factors of reliability, maintainability, operability, and supportability remains a difficult task--particularly in regard to prediction of the effect of up-time or downtime.

The third systems performance effectiveness term, utilization (U), accounts for factors that are introduced by the tactical, functional, logistical, and environmental use of the system; all four are a function of the operational doctrine of the system.

Utilization factors represent the degradation in system performance caused by mission conditions. The following are some examples:
1) Loss of accuracy;
2) Increase in part failure rate due to high ambient temperature;
3) Reduction in repair-part availability due to remoteness of location from supply depot;
4) Infrequent use of search radar for security reasons.

The utilization factors, except for analytic exercise of the model, are relatively constant. However, the assigned values will change whenever operational goals and criteria are modified in the process of achieving consonance between them and technical goals and criteria.

The real significance of the utilization factors lies in their ability to be varied in both sensitivity and tradeoff analyses for optimizing the entire system and its use. The systems performance effectiveness model thus becomes a tool for bringing operational and technical goals and criteria into agreement with each other.

If the goals and criteria are not in agreement, technical managers can use the models to demonstrate to their operational counterparts the desirability of changing the operational goals and criteria. In such a demonstration, the utilization factors are varied to show the impact of the variances on the index of the system's performance effectiveness. If this exercise does not demonstrate the desirability of changing the operational goals and criteria, the technical manager can readily understand why he must revise his goals and criteria to coincide with those of the operational manager. In most cases it will become clear to both that revisions are necessary on both sides to achieve an optimum system.

As with the performance and availability indexes, the variances in utilization indexes must be evaluated in terms of cost considerations and overall worth considerations. Each variance of a factor affects the other factors and is, in turn, affected by variances in other factors. At the same time, each variance of a factor has an associated cost that must be considered. Only when all factors have been considered in terms of mission accomplishment will true performance effectiveness be achieved for the system.

2.   **Mission-Oriented System-Effectiveness Model**

This subsection summarizes the generalized mission-oriented system-effectiveness model that was developed by Task Group II of WSEIAC.* In the simple case in which the system can only be in either a working state or a failed state, the measures of availability, dependability, and capability concern the following fundamental questions:

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*Weapons System Effectiveness Industry Advisory Committee(WSEIAC), Final Report of Task Group II, Prediction Measurement. AFSC-TR-65-2. System-Effectiveness Division Air Force Systems Command; January 1965. (AD-458454, 48455, and 45856)

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1) Is the system working at the start of the mission?

2) If the system is working at the start of the mission will it
    continue to work throughout the mission?

3) If the system worked throughout the mission, will it achieve
    mission success?

Although these questions represent the fundamental approach to
be used in evaluating effectiveness on a mission-oriented basis,
they are too simplified for purposes of model construction.
Moreover, as the systems considered become more complex (e.g.,
there are more than two possible system states) such elements as
degraded modes of operation, multimission requirements, enemy
countermeasures, and natural environment must also be quantified
in the model.

The basic effectiveness model can be divided into two major ele-
ments--the probability that the system will be in a particular
state at mission-performance time, and the effectiveness of the
system when it is in that state. Thus, if effectiveness is
quantified by a probability that the system will successfully
meet the mission objectives, each term in the product $\underline{A} . \underline{D}$
represents the probability that the system will be in a particu-
lar state, and the corresponding term in the $\underline{C}$ vector is the
effectiveness of the system, given that state. For example,

$$E = \underline{A} . \underline{D} . \underline{C},$$

$$= \sum \left\{ P \left[\text{system is in state i} \right] . P \left[\text{mission objectives are met, given state i} \right] \right\}.$$

For some types of systems and missions, it may be more desirable
to quantify effectiveness by some performance parameter other
than a probability. For example, the expected miss distance for
a missile might be a more meaningful performance parameter than
the probability of the hitting within a specified area. For a
reconnaissance system, the average amount of usable information
might be appropriate. Figures of merit for these forms are
readily usable by the appropriate quantification of the $\underline{C}$ vector.

The mission model proposed by the WSEIAC Task Group II is, in
essence, more a model framework for effectiveness evaluation
than a directly applicable set of equations. This generality
is necessary because the range of possible systems, missions,
and depth of analysis precludes the specification of any single
model.

The model framework, based on the availability, dependability, and capability factors, allows for flexibility in application by an appropriate combination of the associated elements. In Volume 3 of the Task Group II report, detailed examples are presented for an airborne avionics system, an intercontinental missile system, a radar surveillance system, and a spacecraft system.

The level of detail at which an analysis is performed will depend on the information and data available and on the purpose of the evaluation. For one study, a mean repair time may be sufficient input for the availability evaluation, while for another study such factors as queuing theory, spare parts availability, maintenance efficiency, and periodic-checkout procedures may have to be incorporated.

There are still many different areas that will require further research. One major problem is to develop improved techniques to convert available data into the appropriate vector and matrix elements of $\underline{A}$, $\underline{D}$, and $\underline{C}$. Better analytic and computational techniques are required to incorporate state changes and those associated capabilities that can occur over a continuous interval. Such factors as state occupancy times and steady-state behavior may be involved in such analyses. Study also is recommended on a means to obtain some measure of "confidence" in the results of the effectiveness evaluation, both in the probabilistic combination of estimates and in guiding the decision process associated with the evaluation. Computerized analytic and simulation methods are needed for complex systems that generate a very large number of system states.

The WSEIAC model frame work, or similar approach, has been applied to several systems, and has generally been found to be a reasonable method for evaluating effectiveness on a mission-oriented basis. Because of the impetus provided by WSEIAC, a great deal of research is being sponsored by the military and private agencies in order to improve this first effort.

3.   Data Problems

The quality of the data used to perform calculations during the course of a systems performance effectiveness analysis will have a significant effect on the accuracy and utility of the results. Unfortunately, the mathematical model that describes a system configuration and behavior often is far more precise than the input data available. If effectiveness values--obtained from an exercise of the system model--are used as relative rather

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than absolute values, the quality of the data is usually found
to be adequate. As a general rule, such values are satisfactory
when the analysis is performed to obtain comparisons between
alternate designs, or to determine the effect of changes on a
specific configuration. If absolute values are required, however,
extreme care must be used in selecting the input data, and cau-
tion should be observed in interpreting the results.

E.   REFERENCES

1. Systems Effectiveness. Compiled by Systems Effectiveness
    Branch, Office of Naval Material, January 1965. (AD 659-520)

2. Proceedings of the NMSE Systems Performance Effectiveness
    Conference. The NMSE Systems Performance Effectiveness
    Steering Committee, April 1965. (AD 629-145)

3. Proceedings of the Second NMSE Systems Performance Effective-
    ness Conference. The NMSE Systems Performance Effectiveness
    Steering Committee, April 1966. (AD 651-819)

4. Proceedings of the NMC Third System Performance Effective-
    ness Conference. The NMC System Performance Effectiveness
    Steering Committee, May 1967. (AD 660-422)

5. L. D. Whitelock and P. J. Giordano: The Navy's Systems Per-
    formance Effectiveness Program. Aeronautic and Space Engi-
    neering Manufacturing Meeting, Society of Automotive Engi-
    neers, October 1966.

APPENDIX B ANALYTIC DERIVATION OF THE AVAILABILITY FUNCTION

A rather definitive treatment of the rationale and equations
leading to the derivation and specification of an availability
function is given in G. H. Sandler's text, System Reliability
Engineering, Prenctice Hall, 1964. In particular, Chapter A5
presents several interesting and appropriate variations, such
as multiple repairmen, series and parallel redundancy, as well
as n equipment items that have failed. The intent of this
appendix is to highlight the method for obtaining the basic
equations for the simplest case of a single equipment system
and interpret the results in terms of system design approaches.
Also, the problem of wearout is treated from a design viewpoint,
and a rather simple reduction to a Markovian Process is illus-
trated for the assumptions given.

For the simple single equipment system, we designate two states--
State 0 (the system is operating) and State 1 (the system is
failed and under repair). Now because the conditional prob-
ability of failure in t, t + dt is dt, and the conditional
probability of completing a repair in t, t + dt is dt, we have
the following transition matrix:

$$P = \begin{bmatrix} 1-\lambda & \lambda \\ \mu & 1-\mu \end{bmatrix}.$$

The differential equations describing the stochastic behavior
of this system can be formed by considering the following:
the probability that the system is in State 0 at time t + dt
is derived from the probability that it was in State 0 at time
t and did not fail in t, t + dt, or that it was in State 1 at
time t and returned to State 0 in t, t + dt. Thus, we have

$$P_o \ (t+dt) = P_o(t) \ (1-\lambda dt) + P_1(t) \mu dt + 0 \ dt.$$

Similarly, the probability of being in State 1 at time t + dt
is derived from the probability that the system was in State 0
at time t and failed in t, t + dt, or it was in State 1 at time
t, and the repair was not completed in t, t + dt. Therefore,

$$P_1(t + dt) = P_0(t)\lambda\,dt + P_1(t)(1 - \mu\,dt) + O(dt).$$

The term $O(dt)$ in both equations represents the probability of two events taking place in $t, t + dt$, which is negligible. Note that the coefficients of these equations represent the rows of the transition matrix. As before, we find the differential equations by defining the limit of the ratio:

$$\frac{P_i(t + dt) - P_i(t)}{dt},$$

which yields,

$$\left.\begin{array}{l} P_0'(t) = -\lambda P_0(t) + \mu P_1(t) \\[2mm] P_1'(t) = \lambda P_0(t) - \mu P_1(t) \end{array}\right\} . \tag{1}$$

If we say that at time $t = 0$ the system was in operation, the initial conditions are $P_0(0) = 1, P_1(0) = 0$. It is also of interest to consider the case where we begin when the system is down and under repair. In this case, the initial conditions are $P_0(0) = 0, P_1(0) = 1$.

Transforming Equations (1) into Laplace transforms under the initial conditions that $P_0(0) = 1, P_1(0) = 0$ we have,

$$sP_0(s) - 1 + \lambda P_0(s) - \mu P_1(s) = 0,$$

$$sP_1(s) \qquad - \lambda P_0(s) + \mu P_1(s) = 0,$$

and simplifying,

$$(s + \lambda)P_0(s) - \mu P_1(s) = 1,$$

$$-\lambda P_0(s) + (s + \mu)P_1(s) = 0.$$

Although the solution for $P_0(s)$ and $P_1(s)$ can be found easily in this case, we shall apply Cramer's rule because it will be useful in later examples. To solve this system of equations, we

introduce the determinant D, whose elements are the coefficients of $P_i(s)$'s. We also introduce the determinant D_i, which is formed by substituting the solution vector for the ith coefficient column. Then the solution is $P_i(s) = D_i/D$. Therefore:

$$P_o(s) = \frac{\begin{vmatrix} 1 & -\mu \\ 0 & s+\mu \end{vmatrix}}{\begin{vmatrix} s+\lambda & -\mu \\ -\lambda & s+\mu \end{vmatrix}} \quad ,$$

and

$$P_o(s) = \frac{s+\mu}{s(s+\lambda+\mu)} \quad .$$

Now the availability function that shall designate at $A(t)$ will be the inverse transform of $P_o(s)$, that is, $A(t) = \,^{-1}P_o(s)$. Solving

$$A(t) = P_o(t) = \frac{\mu}{\lambda+\mu} + \left[\frac{\lambda}{\lambda+\mu}\right] e^{-(\lambda+\mu)t} \quad ,$$

and (2)

$$\{1 - A(t)\} = P_1(t) = \frac{\lambda}{\lambda+\mu} - \left[\frac{\lambda}{\lambda+\mu}\right] e^{-(\lambda+\mu)t} \quad .$$

If the system was initially failed, the initial conditions are $P_o(0) = 0, P_1(0) = 1$, and the solutions are

$$A(t) = P_o(t) = \frac{\mu}{\mu+\lambda} - \left[\frac{\lambda}{\mu+\lambda}\right] e^{-(\lambda+\mu)t} \quad ,$$

and (3)

$$\{1 - A(t)\} = P_1(t) = \frac{\lambda}{\lambda+\mu} + \left[\frac{\lambda}{\lambda+\mu}\right] e^{-(\lambda+\mu)t} \quad .$$

We note that t becomes very large, Equations (2 and (3) become equivalent. This indicates that after the system has been operating for some time, its behavior becomes independent of its starting state.

The availability function A(t) can be interpreted as the probability that at any time t the system is in an operating state. In many cases, we are interested in the average uptime for some definite period. This can be found simply by summing A(t) over the time interval of interest and dividing by the total time.

$$A(T) = \frac{1}{T} \int_0^t A(t)dt \quad .$$

(4)

In this instance we have

$$A(T) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda+\mu)^2 T} - \frac{\lambda}{(\lambda+\mu)^2 T} e^{-(\lambda+\mu)T} \quad .$$

(5)

If we are interested in the long-term availability of the system we can let $t \to \infty$ and find,

$$A(\infty) = \left(\frac{\mu}{\mu + \lambda} \right) \quad .$$

(6)

This condition is usually referred to as the steady-state availability. Essentially, it implies that for a large ensemble of equipment items, the process will maintain itself in a state of statistical equilibrium.

It is this analytical expression that is commonly used to describe availability in the form:

$$\text{Operational Availability} = \frac{\text{Mean Time Between Failures (MTBF)}}{\text{(MTBF)} + \text{Mean Time to Repair (MTTR)}} \quad .$$

This parameter is a measure of an attribute of the system and, as such, is subject to two conditions:

1) The value that is attainable within the resources available;
2) The value that is needed to meet the objectives of the mission.

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The initial design analysis objective is to establish that the attainable probability of system availability meets the mission needs. The mission requirement depends on the criticality of the mission and risk that is acceptable to management. It may be a rigorous requirement or flexible, i.e.g., the highest probability attainable within a given resource allocation. In either case, the analytical problem becomes one of optimizing the two operational availability parameters MTBF and MTTR for the operational modes of the system.

The above analysis is for a single item of equipment with constant repair and failure rates. Several variations of this basic approach to more complex situations are of interest. For example, consider the problem where the equipment is subjected to two types of repair. Thus, when the equipment fails for the first time, partial repair is performed, which restores the system to operation. However, this increased the probability of failure. After the equipment fails for the second time, a second repair is performed. Thus, the analytical procedure is as follows:

1) Let λ_1 designate the failure rate when the equipment has been through a complete repair;

2) Let λ_2 designate the failure rate when the equipment has been through a partial repair ($\lambda_2 > \lambda_1$);

3) Let μ_1 be the repair rate for a partial repair;

4) Let μ_2 be the repair rate for a complete repair, i.e., $\mu_2 < \mu_1$.

Consider the four states of the system as:

1) State 0 - System is failed and a partial repair is being performed;

2) State 1 - System is failed and a partial repair is being performed;

3) State 2 - System is operating after completion of partial repair;

4) State 3 - System is failed and a complete repair is being performed.

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The transition matrix becomes:

$$
\begin{bmatrix}
(1 - \lambda_1) & \lambda_1 & 0 & 0 \\
0 & (1 - \mu_1) & 1 & 0 \\
0 & 0 & (1 - \lambda_2) & \lambda_2 \\
\mu_2 & 0 & 0 & (1 - \mu_2)
\end{bmatrix}
$$

The resulting proportion of time spent in an acceptable state,

$A(\infty) = {}^{'}P_0 + P_2$ is given by:

$$
A(\infty) = \frac{\lambda_1 \, \mu_1 \, \mu_2}{\lambda_1 \lambda_2 \mu_1 + \lambda_2 \mu_2 \mu_1 + \lambda_1 \lambda_2 \mu_2 + \lambda_2 \mu_1 \mu_2}. \tag{7}
$$

Thus, if $\lambda_1 = \lambda_2$, and $\mu_1 = \mu_2$, the above equation reduces to Equation (6).

There are several additional interesting variations of the repair or maintenance policies that can be considered analytically. The available function is given for each of these formulations. The respective derivations may be found in <u>System Reliability Engineering</u> by G. H. Sandler.

1) n equipment with r = n repairment:

$$
A(\infty) = \frac{\mu^n}{(\lambda + \mu)^n} \tag{8}
$$

2) Two-equipment item series system with two repairman:

$$
A(\infty) = \frac{3\mu}{3\mu + \mu 4\lambda + 2\lambda^2}. \tag{9}
$$

3) n equipment items with $R < n$ repairmen working independently:

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$$A(\infty) = \sum_{K=0}^{n-m-1} P_k \quad , \tag{10a}$$

where

$$P_k = \frac{n! \; \theta^k}{(n-k)! \; k!} \; \left\{ P_o \right\} \qquad \text{for } k < r \tag{10B}$$

$$P_k = \frac{n!}{(n-k)!} \; \left\{ \frac{\theta}{r} \right\}^{k-r} \left[P_o \right] \qquad \text{for } k \geq r \;\; , \tag{10c}$$

$$P_o = \left\{ \sum_{k=o}^{r-1} \frac{n! \; \theta^k}{(n-k)! \, k!} + \sum_{k=r}^{n} \frac{n! \; \theta^r}{(n-k)! \, k!} \left[\frac{\theta}{r} \right]^{k-1} \right\}^{-1} \tag{10d}$$

$$\theta = \frac{\lambda}{\mu} \quad . \tag{10e}$$

4) Two-equipment item redundant system operating in parallel:

$$A(\infty) = \frac{(\mu^2 + \mu\lambda 2)}{[\mu + \lambda]^2} \; . \tag{11}$$

5) Two-equipment item redundant system operating in parallel in which it is not possible to service a failed item of equipment until the complete system fails:

$$A(\infty) = \frac{3\mu^2 + \mu\lambda 2}{3\mu^2 + \mu3\lambda + \lambda^2} \; . \tag{12}$$

A.   DESIGN FOR WEAROUT

The expression of an availability probability in terms of
variables MTBF and MTTR serves the purpose of establishing
system objectives that can be assessed for feasibility and
guidelines for conceptual design.

The total mission time can be compared to MTBF to assure
compatibility. If the mission time in the order of 1/10
of the MTBF, the failure characteristic can be treated as
an exponential decay function. If this is not the case,
and the mission time is a larger percentage of the MTBF,
then the problem of wearout (useful life) enters into the
problem. Wearout is the case where the failure rate starts
to increase after the steady-state failure performance has
been reached, as in Fig. 1.
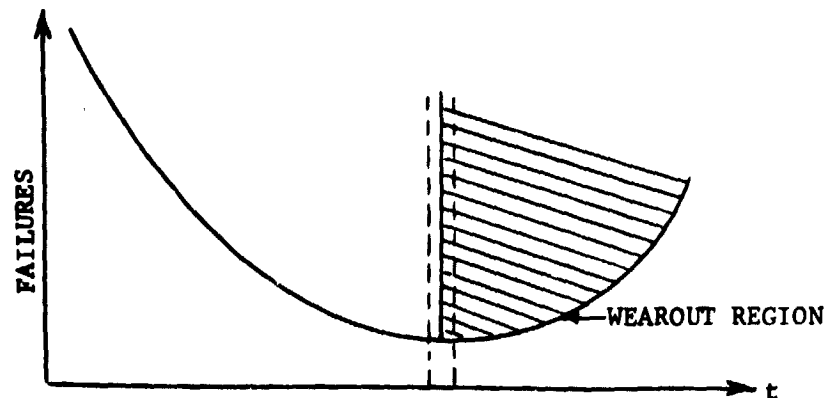
Fig. 1 -- Wearout Characteristic

The analytical expression for operational availability
comes into use in the concept and definition phases. In
the case of wearout, this involves the formulation of a
non-Markovian Process to obtain the steady-state avail-
ability function. Initially, nonlinear stochastic equa-
tions result, which, in many cases (but the simplest),
prove to be intractable. However, there are many cases in
which the equipment failure distributions are other than
exponential, but the transition process can be treated as a

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Markov Process by increasing the number of states, each
being described by a constant transition rate. For example,
consider the single equipment case in which $F(t) =$
the gamma distribution (see Fig. 1). It is assumed that
the equipment goes through two exponential phases, each of
length $1/\lambda$. Three states are defined:

1) State 0 - System is operating in the first phase;

2) State 1 - System is operating in the second phase;

3) State 2 - System is failed.

The transition matrix becomes:

$$\begin{bmatrix} (1-\lambda) & & 0 \\ 0 & (1-\lambda) & \\ \mu & 0 & (1-\mu) \end{bmatrix}$$

The steady-state equations are:

$$-\lambda P_o + \mu P_2 = 0$$

$$\lambda P_o - \lambda P_1 = 0$$

$$\lambda P_1 - \mu P_2 = 0$$

$$P_o + P_1 + P_2 = 1$$

$$A(oo) = 2P_o = \frac{2\mu}{2\mu + \lambda}$$

This simple example illustrates the technique for solving
these types of problems in terms of the Markov formula-
tion. In general, the application to a specific situation
involves the following steps:

1) On the mission level, the equipment performance
   characteristics are compared against the mission
   profile to provide an hypothetical operational scenario,
   for the purposes of sizing and to determine critical
   equipment usages. The respective snesitivities in mar-
   gins as to the allocation of failure rates and repair
   rates are assessed to determine overall availability
   requirements and equipment compatibilities;

2) The identification of a subsequent maintenenac model,
   and repair regimen is delineated in accordance with
   the projected equipment use profile, and provision for
   wearout is made by the specification of a regimen in
   the operational cycle at which wearout results in a
   catastrophic failure;

3) The analysis procedure includes the formulation of
   either a simulation or, at first, a simple analytical
   model to indicate the appropriate solution to the avail-
   ability function. This implies the estimation of failure
   rates, and the knowledge of the state of the system be-
   fore wearout;

4) Refinement of the availability function to include the
   effects of interrupted states may be accomplished by
   the specification of successive transition matrices
   and solving for the composite availability function.

In terms of the design methodology, and its implications
concerning the systems engineering process, each new state
corresponds to an alternative solution in terms of specify-
ing transition probabilities and failure and repair rates.
The optimal combination of each transition matrix at each
point in the design process involves a multistage decision
process, which can be formulated as a dynamic programming
problem. In a great many instances intuition and empirical
experience are substitutes for formal analytical procedures.
However, where a large number of cost tradeoffs are in-
volved, and a great number of transition matrices need be
computed, the automation of the decision process would prove
useful.

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B. SINGLE ELEMENT WEAROUT DESIGN FOR MAINTAINED SYSTEMS

In general, it has been observed that a large class of
electronic and mechanical equipment and components ex-
hibit failure distributions similar to those shown in
Fig. 2 (Ref 1). This distribution can be considered most
conveniently as the sum of three elementary distributions,
i.e.,

$$f(t) = a_\alpha f_\alpha(t) + a_\beta f_\beta(t) + a_\gamma f_\gamma(t) ,$$

where

a_α, a_β, and a_γ = weights for combining the distri-
bution so that $f(t)$ will satisfy
the conditions of a probability
density distribution function. For
the purpose here, $a_\alpha + a_\beta + a_\gamma = 1$

$f_\zeta(t)$ = failure distribution which dominates
in period ζ
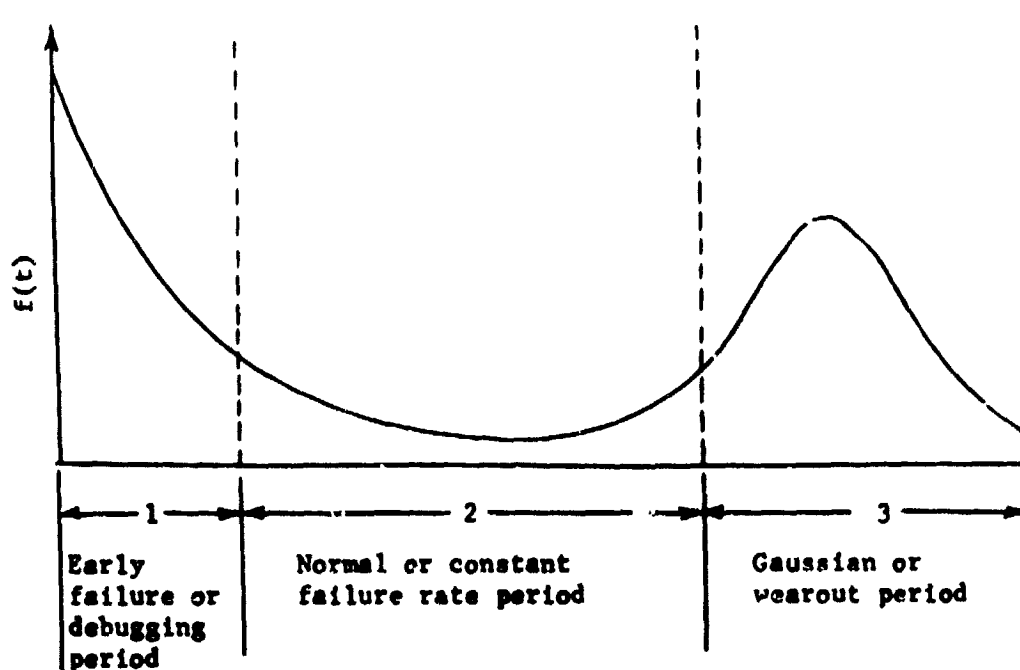
Fig. 2 -- General Failure Distribution for Electronic and
Mechanical Equipment and Components

Period 1 is called the early failure or debugging period where high failure rate items are uncovered and defects that escaped quality control inspection are found. The number of failures is expected to decrease rapidly with time in this period. A gamma distribution may be used to approximate the distribution that dominates in this period. Figure 3 illustrates the shapes of some common failure distributions and related functions.

Period 2 is called the normal failure or constant failure rate (to be more exact, constant hazard rate) period. It is the period in which equipment reliability is usually considered. In this period, the exponential law of failure dominates.

Period 3 is called the Gaussian or wearout failure period where some elements of the equipment fail from wear. The normal (Gaussian) distribution may be used to approximate the dominating distribution for this period. To the extent that such a period is known to exist during the useful life of the equipment, it is necessary to establish overhaul and maintenance policies. It is, therefore, this level or amount of equipment survivability that is the key to providing the designer with a methodology for achieving the required performance. The concept is that there exists some degree of survivability, p, between 0 and 1, which the designer can manipulate early in the design cycle to obtain alternative and meaningful solutions to the reliability and maintainability problems. This degree of survivability can be thought of in two ways:

1) It is a compilation of experience with respect to similar equipment in similar operational environments--the percentage of failures that have been found to occur and are reparable.

2) It is a rough measure of maintainability--defined by spares availability, level of checkout, and system repair capability.

Thus, in case of a failure, a subsystem can be repaired with a certain probability. Generally, this probability will be a complex function of the above maintainability factors, availability of spares, the capability of the repairman, and the efficiency of the fault isolation equipment.
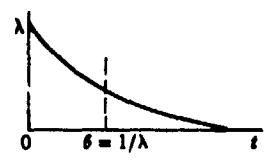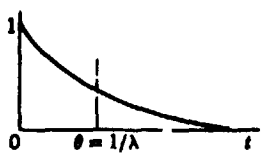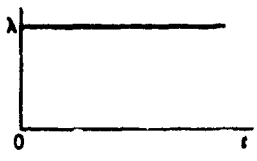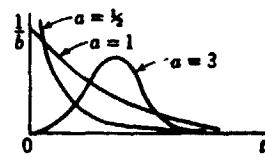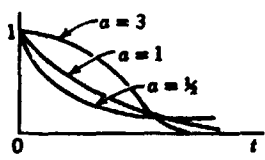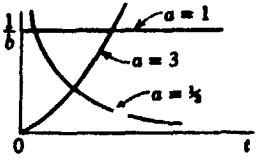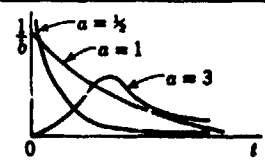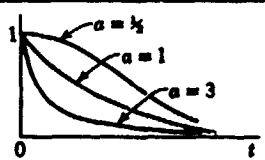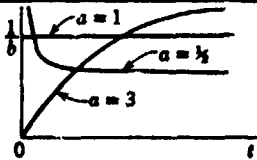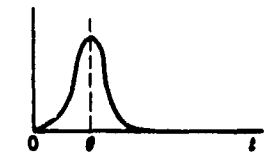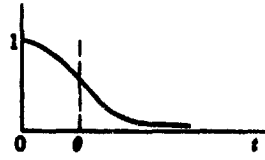
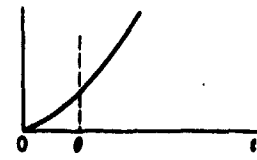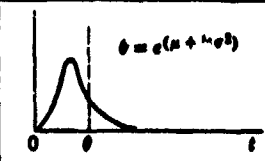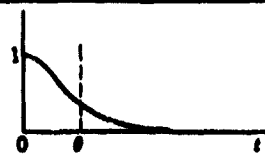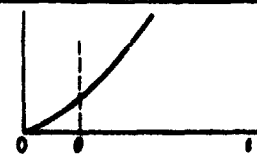| TYPE OF DISTRIBUTION	FAILURE DISTRIBUTION $f(t)$	RELIABILITY FUNCTION $R(t)$	HAZARD RATE $z(t) = \frac{f(t)}{R(t)}$
Exponential	$\lambda e^{-\lambda t}$	$e^{-\lambda t}$	λ
Weibull	$\frac{a}{b} t^{a-1} e^{-t^a/b}$	$e^{-t^a/b}$	$\frac{a}{b} t^{a-1}$
Gamma	$\frac{1}{(a-1)! \, b} \left(\frac{t}{b}\right)^{a-1} e^{-t/b}$	$\frac{1}{(a-1)! \, b^a} \int_t^\infty \tau^{a-1} e^{-\tau/b} \, d\tau$	$\dfrac{t^{a-1} e^{-t/b}}{\int_t^\infty \tau^{a-1} e^{-\tau/b} \, d\tau}$
Normal	$\frac{1}{\sigma \sqrt{2\pi}} e^{-(t-\theta)^2/2\sigma^2}$	$\frac{1}{\sigma \sqrt{2\pi}} \int_t^\infty e^{-(\tau-\theta)^2/2\sigma^2} \, d\tau$	$\dfrac{e^{-(t-\theta)^2/2\sigma^2}}{\int_t^\infty e^{-(\tau-\theta)^2/2\sigma^2} \, d\tau}$
Log normal	$\frac{1}{\sigma t \sqrt{2\pi}} e^{-(\ln t - \mu)^2/2\sigma^2}$	$\frac{1}{\sigma \sqrt{2\pi}} \int_t^\infty \frac{1}{\tau} e^{-(\ln \tau - \mu)^2/2\sigma^2} \, d\tau$	$\dfrac{\frac{1}{t} e^{-(\ln t - \mu)^2/2\sigma^2}}{\int_t^\infty \frac{1}{\tau} e^{-(\ln \tau - \mu)^2/2\sigma^2} \, d\tau}$

Fig. 3   Shapes of Common Failure Distributions/and Related Functions

What follows is neither a very formalistic nor a mathematically rigorous approach. Rather, the emphasis is on how the designer plays his hand through the proper specification of the survivability factor, p. In this, he is using p as a convenient and useful rule of thumb--a designer's index--which, however, can have a powerful effect on the design of other elements of the system. The basic premise is that the reliability of a maintained system is a function of four parameters, ψ, Ω, t, and p. These are defined as follows.

t is the desired operating time for the equipment. It is assumed that repairs can be accomplished at any point within this time--the model ignores the periods during the mission where repair is impossible. Examples of such periods are rendezvous operations where the crew is fully occupied with the rendezvous maneuver; midcourse correction where the crew members are strapped to their seats and are unable to move to the areas of the spacecraft where repairs may be necessary; and the descent or ascent phases of the mission. Usually, these time periods account only for a very small fraction of the total equipment operating time, and, in this case, no significant error is introduced by ignoring them.

ψ is the failure rate, the reciprocal of the mean-time-between failures, for the system. The widely accepted Weibull failure distribution is assumed here. This means that it is possible to approximate failures by varying the distribution parameters during any of the three time periods delineated in Fig. 2.

Ω is the repair rate, the reciprocal of the average time to restore the system to operating condition. Depending on the equipment design and maintenance concept, the repair-time distribtuion may take many forms. It is assumed to be exponentially distributed. The reason for this assumption is that since the model is insensitive to the exact form of the distribution (see Ref 1), a distribution that simplifies the mathematics as much as possible will be used.

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The factor, p, is the keystone of this approach.   It
defines the probability of wearout, i.e., survivability.
In effect, p is equivalent in many respects to some of
the current definitions of maintainability, at least
with respect to a system in an operating environment.
The functional relationship of p to the factors pre-
viously given (spares availability, level of checkout,
and system repair capability) is a complicated one, but
since the emphasis here is toward the application of
the designer's knowledge, experience,** or the intui-
tive judgment of p, it is assumed that each of these
factors is probabilistically independent. This is not
entirely accurate.   Indeed, much of the work being done
today in maintainability is directed toward trying to
establish a more formal relationship between, for ex-
ample, test equipment and training, in order to struc-
ture and understand their interaction.

It is clear that if a piece of equipment is repaired and
put back into operation, it will have a new failure dis-
tribution.   If a group of equipment of various ages are
kept operating by means of immediate repair after any
failure, ultimately this mixed-age population will appear
to have an exponential failure distribution.

1.   MATHEMATICAL MODEL FOR A SINGLE ELEMENT MAINTAINED SYSTEM

Figure 4 illustrates a typical single element maintained
configuration for which the designer desires to achieve a
given level of survivability.   There are many functions

**If the equipment being designed is similar to other equipment
already in use, and if the operational environments are compar-
able, p can be obtained from experience, i.e., the percentage of
those failures that have occurred over a period of time that were
found to be repairable.   Furthermore, if spares are consumed dur-
ing a mission, p is really a function of time.   However, it is
assumed that spares consumption is low, thereby allowing the time
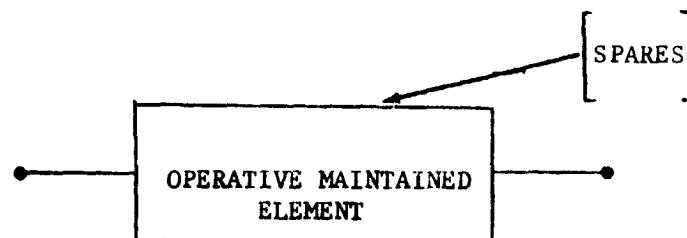dependence of p to be ignored.

Fig. 4 TYPICAL SINGLE ELEMENT MAINTAINED CONFIGURATION

aboard a spacecraft that can be fulfilled by such a system where a backup is not required. For example, the VHF communication system may very well be without backup, since being "down" for repairs does not necessarily imply a catastrophic event. For these systems one is primarily interested in availability; i.e., what percentage of the total mission time the communication system is in operating condition. The basic assumptions for this model are:

1) During standby operation or while undergoing repair, the failure rate of the element is zero. During operation, the failure distribution is of the Weibull type, i.e., the probability that a failure occurs between t and t + dt is ψdt, where ψ is the failure rate.

2) The repair distribution is also of the   bull type, with repair rate Ω.

3) The probability of survival is p (wearout condition).

4) Each failure is repairable.

5) Failures are detected immediately, and repair action starts as soon as the failure is discovered.

At any one time, the system may be in one of three mutually exclusive states.

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State 1 - operating

State 2 - down and in repair

State 3 - down and nonrepairable

Following the usual methods (Ref 2) one can relate the state probabilities at time "t" with those a short time "dt" later by the equations:

$$P_1(t + dt) = P_1(t) - \psi P_1(t)dt + \Omega P_2(t)dt$$

$$P_2(t + dt) = p\psi P_1(t)dt + P_2(t) - \Omega P_2(t)dt \qquad (13)$$

$$P_3(t + dt) = P_1(t)(1 - p)\psi dt + P_3(t)$$

These equations can be solved by first obtaining the Laplace transform, where $P_1(o) = 1$, and $P_2(o) = P_3(o) = 0$, and then finding the inverse. Thus, the differential transition matrix is given by:

$$\begin{bmatrix} -\psi & p\psi & (1-p)\psi \\ \Omega & -\Omega & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (14)$$

The probability that the system is operational at time t is given by

$$P_1(t) = \pounds^{-1} \left[\frac{S(S + \Omega)}{S(S + \psi)(S + \Omega) - Sp\Omega\psi} \right] \qquad (15)$$

$$P_2(t) = \frac{1}{a-b} \left\{ (\Omega + a) \epsilon^{at} - (\Omega + b) \epsilon^{bt} \right\} \qquad (16)$$

where

$$a = \frac{-(\psi+\Omega) + \left\{ (\psi-\Omega)^2 + 4\psi\Omega(1 - p) \right\}^{1/2}}{2}$$

$$\left. \right\} \qquad (17)$$

$$b = \frac{-(\psi+\Omega) - \left\{ (\psi-\Omega)^2 + 4\psi\Omega(1 - p) \right\}^{1/2}}{2}$$

The availability of the system, or the average time the system is operational, during an interval t, is

$$A(T) = \frac{1}{t} \left\{ \int_0^t P_1(S)ds \right\} \tag{18}$$

$$= \frac{1}{(a-b)t} \left\{ \left[\frac{\Omega + a}{a}\right](\epsilon^{at}-1) - \left[\frac{\Omega + b}{b}\right](\epsilon^{bt}-1) \right\} \tag{19}$$

The percentage availability improvement over a similar nonmaintained (i.e., nonsurvivable) system is defined as

$$\left[\begin{array}{c}\% \text{ availability} \\ \text{improvement}\end{array}\right] = \frac{\left(\begin{array}{c}\text{availability of a} \\ \text{maintained system}\end{array}\right) - \left(\begin{array}{c}\text{availability of non-} \\ \text{maintained system}\end{array}\right)}{\text{availability of nonmaintained system}} \times 100$$

This function is plotted in Fig. 5 for given parametric values of Ψ and Ω . The use of this tool by the designer is illustrated in the following section.

2.   A DESIGN APPLICATION

Consider the designer who is in the prototype design stages of, say, the VHF communication system of a spacecraft. He has been told that the availability required of this particular system is 0.928. However, now that he has finished his prototype design, he does a simple and acceptable analysis by means of a parts count, and finds that his availability is 0.800. How does he get the 16% improvement to meet the requirement?

There are a few standard approaches he can use. First, he can reexamine his circuit design with the aid of a reliability expert and perhaps uncover some marginal component applications. He can, with the aid of a component parts expert, examine the parts in the equipment and determine whether he has, in fact, used the highest reliability components available to him. If, at the conclusion of this exercise he has improved the availability of the
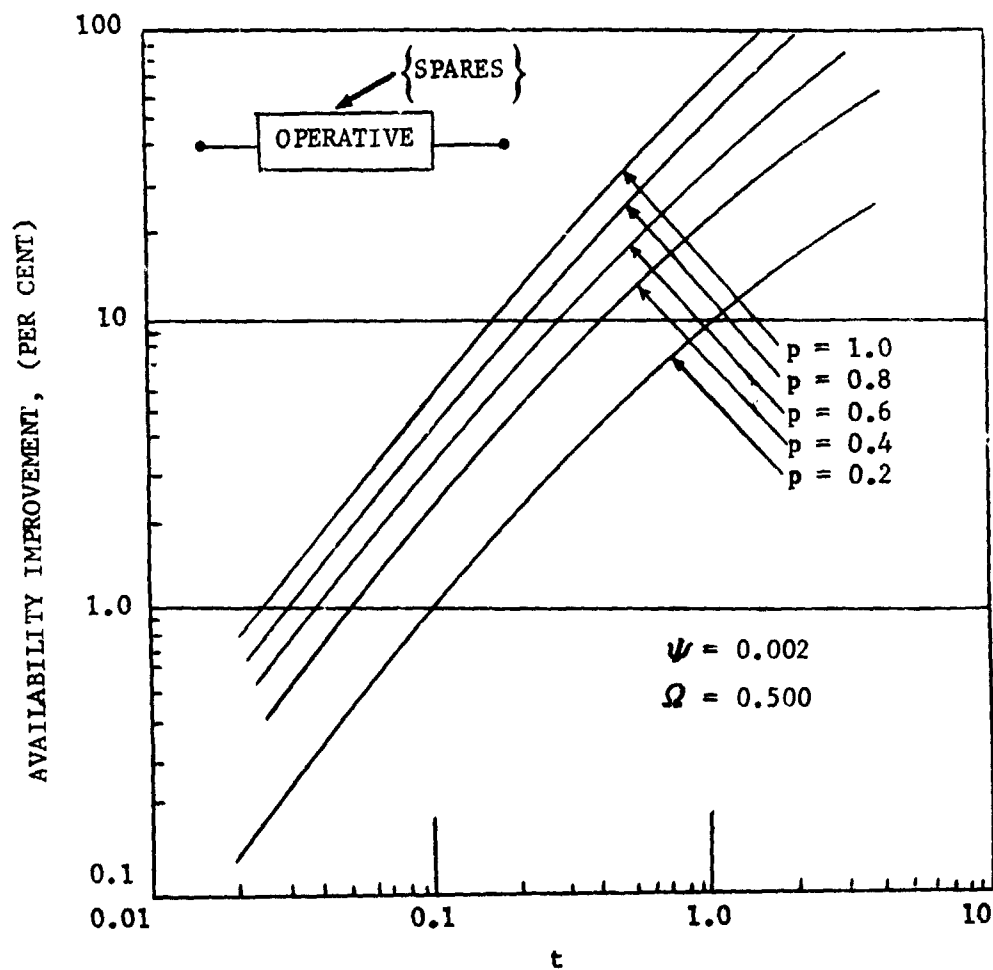
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Fig. 5 -- Availability Improvement of a Single-Element Maintained System

system from 0.800 to 0.844, or by about 5.5%, he is still
faced with the problem of achieving an additional 10% im-
provement in availability. What is his next step?

If he is guided by p૮ ૮ experience, the designer will
examine the possibility of incorporating some form of re-
dundancy into his design. The equations that will give
him additional availability for a given degree of redun-
dancy are fairly straightforward. He applies them and
notes that he can indeed achieve the additional avail-
ability in this manner. This, of course, means that he
is not making the entire VHF system redundant but, rather,
he is making redundant only those parts of the system that
he has found to be the least reliable and/or most amenable
to such redundancy. In so doing, however, he has probably
increased the number of interconnections and als⁻ the system
weight; and, since this is a space system and weight is ex-
tremely important, he may be in trouble.

This search for the proper level of redundancy is a valid
one. In many·cases the use of redundancy will solve his
problem, especially if the system has a relatively short
desired operating time as compared to its mean-time-between-
failures (MTBF). However, it is at this point that the de-
signer, if he has not yet achieved the required design
availability, frequently shows his lack of appreciation for
the role of maintainability. He will attempt a wholesale
circuit redesign in order to achieve his reliability goal,
rather than examine the logical next step--enhancing the
survivability of the system by making it maintainable.

Making a system maintainable is not easy. For one thing
the system design must be such that components or sub-
systems can be removed and replaced and, furthermore, spares
must be made available for replacement. The designer must
determine just how much and what kind of repair should be
included in his system. He must do this in an environment
of considerable uncertainty, for at this early stage of the
project he does not know the characteristics of the checkout
or fault-isolation equipment; he does not know what spares
he will be allowed, and he has only a very general appre-
ciation of the maintainance requirements.

But this air of uncertainty has some advantages for the
designer. It allows him to place additional requirements
on the system design, because, as the example given below
will show, he can determine just how much improvement in
system reliability a given level of survivability will give.

For the system shown in Fig. 5, let the MTBF be 500 hr, and
assume the desired operating time is 250 hr (about 11 days).
The availability of this system is 0.844. This number is
obtained from:

$$A(T) = (1 + \psi t)e^{-\psi t}$$

Suppose the designer's availability goal, arrived at
through apportionment, is 0.928. This implies that avail-
ability improvement of at least 10% is required. By re-
ferring to Fig. 5, we see that for a value of
$t = 0.002 \times 250 = 0.5$, a p factor of approximately 0.6
gives an availability improvement of 10%. So a survivability
of 0.6 will yield the desired boost in availability-the de-
signer must now find ways of obtaining the 0.6 p factor.

Let us assume that the designer concludes that the system
repair capability is 75%. Then, based on his experience
with fault-isolation systems and knowledge of the fault
isolation task inherent in his design, and from consultations
with designers of in-flight test systems, he may reasonably
require the fault isolation system to be 90% effective in
locating the faulty part within his system. Then, there re-
mains the problem of assessing the spares availability that
can be obtained from a certain allowance of spares weight.
Fortunately, there are techniques (Ref 3) for solving this
problem but, for the moment, we will just apply the rule of
thumb that for electronic equipment more than 90% of the
failures are attributable to 10% of the total number of
parts in the equipment. Further, assume that the designer
can, in fact, stock this 10% of the parts and stay within
the weight constraints. He can then consider his spares
availability to be equal to 90%. Multiplying these three
factors, the designer obtains a value

$p = 0.75 \times 0.90 \times 0.90 = 0.6075$

and has achieved the p factor required for the desired de-
gree of availability improvement.

It is at this point that the designer should verify that
this system does indeed meet the availability specificat ons
more efficiently (say at a lower overall system weight) than
that of his other alternatives.

REFERENCES

1. Molina, E. C., Poisson's Exponential Binomial Limit
    Tables, Van Nostrand, Princeton, New Jersey, 1942.

2. Bazovsky, I., Reliability Theory and Practice,
    Prentice Hall, Englewood Cliffs, New Jersey, 1961.

3. Ryerson, C. M., "Reliability and Product Assurance",
    in D. W. Chorafas, Statistical Processes and Reli-
    ability Engineering, Van Nostrand, Princeton, New
    Jersey, 1960.

This example illustrates an approach to conducting a Reliability (MTBF) and Maintainability (MTTR) tradeoff when given a specified Inherent Availability. The desired alternative is based on initial and sustaining costs.

A.   Design Problem

A requirement exists to design a radar receiver that will meet an Inherent Availability of 0.990, a minimum MTBF of 200 hours, and a MTTR not to exceed 4.0 hours. Existing design with the use of Military Standard parts meets an Availability of 0.97, a MTBF of 150 hours, and a MTTR of 4.64 hours.

B.   Possible Solutions

Three different alternative design configurations are being considered to satisfy availability requirements:

Design Configuration	A	MTBF*	MTTR (hr)+
1. R - derating of military standard parts M - modularization and automatic testing	0.990	200	2.02
2. R - design includes high reliability parts/components M - limited modularization and semiautomatic testing	0.990	300	3.03
3. R - design includes partial redundancy M - manual testing and limited modularization	0.990	350	3.54

Design Configuration 1 emphasizes the Maintainability aspects in the design while Design Configuration 3 emphasizes Reliability improvement. The Reliability-Maintainability relationship is derived through the equation:

$$\text{Inherent Availability } (A_i) = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

* Conservative Estimate - Lower 2σ Bound

+ Conservative Estimate - Upper 2σ Bound

The area for reliability-maintainability tradeoff is illustrated in Figure 1.

MEAN-TIME-BETWEEN-FAILURE (MTPT)--HR
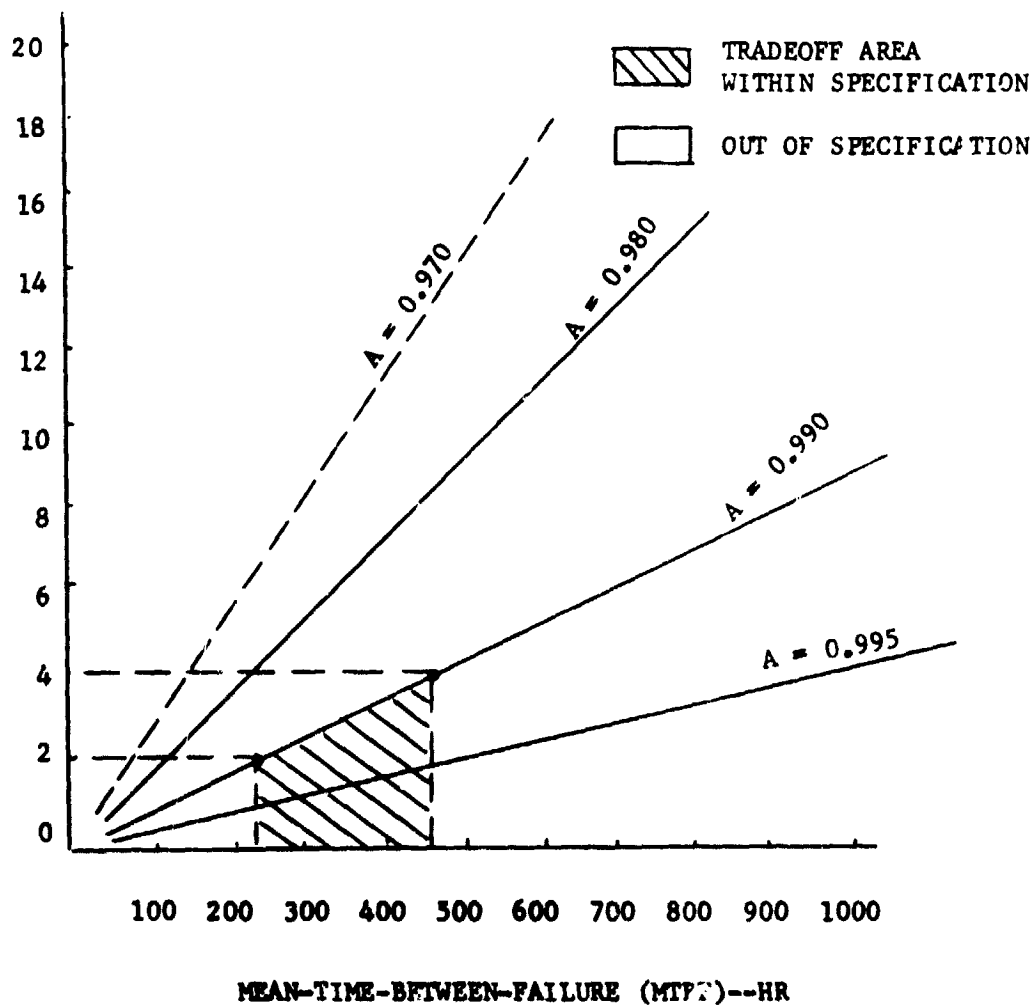
REQUIREMENT

A   = 99%
MTBF = 200 HR. MIN.
MTTR =   4 HR. MAX.

FIGURE I -- Reliability-Maintainability Tradeoff

C.   Tradeoff Approach

1) Cost data are developed against each configuration. Such
data include both initial costs (those associated with design
and manufacture of the equipment) and sustaining costs (those
associated with field operations - manpower, test equipment,
spare parts, facilities, etc).

2) Initial cost factors are as follows (the values used are
estimated and not necessarily representative of actual
experience).

Item	Existing Configuration (dollars)	Configuration 1 (dollars)	Configuration 2 (dollars)	Configuration 3 (dollars)
RDT & E Cost	300,000	324,937	319,125	321,500
Reliability	-	1,187	6,625	16,750
Maintainability	-	23,750	12,500	4,750
Manufacture Cost*	4,500,000	4,534,250	4,524,700	4,530,250
Reliability	-	1,750	9,200	22,500
Maintainability	-	32,500	15,500	7,750

* Manufacture Cost is total based on 300 units.

The following tabulation presents the net incremental value (cost)
of Reliability and Maintainability in equipment design as derived
from the above figures.

Item	Configuration 1 (dollars)	Configuration 2 (dollars)	Configuration 3 (dollars)
Reliability	2,937	15,825	39,250
Maintainability	56,250	28,000	12,500
Total	59,187	43,825	51,750

3) Sustaining cost factors represent **estimated** cost to support 300
units for 10 years.

SUSTAINING COST FACTORS

Item	Existing Configuration (dollars)	Configuration 1 (dollars)	Configuration 2 (dollars)	Configuration 3 (dollars)	Remarks
Cost of Spares	210,000 (14@15,000)	150,970 (10@15,097)	105,420 (7@15,060)	90,402 (6@15,067)	Frequency of maintenance/10 yr. Existing Conf--139: Conf. 1 --104; Conf. 2-- 69; Conf. 3-- 59; Assume 10% spares
Cost of Repair					
Administrative	385,065	125,370	124,773	124,176	Assume $1.99/direct labor hour: One technician/ maintenance action
Active Repair	700,470	151,200	188,200	212,160	Existing Conf-E6 $3.62 per hour Conf. 1 - E3 $2.40/hr. Conf. 2 - E4 $3.00/hr. Conf. 3 - E5 $3.40/hr.
Test Equip/Spares	174,150	56,700	56,430	56,160	Assume 3 items-standard equipment (burden-$0.90/ direct labor hr)
Spares (piece parts)	1,390	1,040	690	590	Assume $10/repair action
Facilities	36,765	11,970	11,913	11,856	Use existing facilities (burden-$0.19/direct labor hour)
Cost of training	140	80	100	120	Based on $20/day

SUSTAINING COST FACTORS (CONTINUED)

Item	Existing Configuration (dollars)	Configuration 1 (dollars)	Configuration 2 (dollars)	Configuration 3 (dollars)	Remarks
Cost of Technical Manual	20,000	14,000	16,000	18,000	Assume $200/page
Cost of Provisioning					
Provisioning	--	30,000 (30 mods.)	15,000 (15 mods.)	5,000 (5 mods.)	Provisioning-assume cost of $1000/line item.
Handling	475,000 (2500 parts)	494,950 (2605 parts)	487,450 (2565 parts)	499,700 (2630 parts)	Handling - assume $19/line item/year
Total Cost	2,902,980	1,036,280	1,005,876	1,018,164	

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Referring to the sustaining cost table, a net sustaining cost savings is realized for each proposed design configuration. This saving is derived through the improvement in equipment availability, resulting in a reduction in required maintenance support. The following computation illustrates the cost savings against each design configuration when compared against existing design:

          Configuration 1 - $966,700
          Configuration 2 - $997,104
          Configuration 3 - $984,016

4) Subsequent to deriving both initial and sustaining cost factors, the net effect is obtained through subtracting the initial added costs for Reliability and Maintainability from the overall sustaining cost savings:

          Configuration 1 $966,700 - $59,187 = $907,513
          Configuration 2 $997,104 - $43,825 = $953,279
          Configuration 3 $984,816 - $51,750 = $933,066

Because the sustaining cost factors are estimated values, an uncertainty of + 20% in the total costs yields the following overall sustaining cost savings:

+ 20%   Configuration 1 $759,444 - $59,187 = $700,257
        Configuration 2 $795,928.80 - $43,825 = $752,103.80
        Configuration 3 $781,182.20 - $51,750 = $729,433.20

- 20%   Configuration 1 $1,173,956   - $59,187 = $1,114,769
        Configuration 2 $1,198,279.80 - $43,825 = $1,154,454.80
   Configuration 3 $1,188,448.80 - $51,750 = $1,136,698.80

D.   Design Decision

The intent of this tradeoff is to generate and evaluate the alternative Reliability and Maintainability design features required to meet a specified Availability. In doing so, the basic evaluation criterion is cost. Referring to the above cost factors, Configuration 2 satisfies the required equipment Availability with maximum cost savings or minimum initial cost expenditure, even under the conditions of a + 20% uncertainty in the total estimated costs. This lack of sensitivity indicates that the total sustaining cost savings has a broad maximum, over which perturbations in such factors as spares, facilities, and test equipment have little influence in changing the decision relative to the remaining alternatives. The reason for this is that

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Configuration 2, which represents the most cost/effective condition, is a conservative position between the two extremes of too little reliability (R), and too much modularization (M) and testing. The combined effect is to produce a cost savings figure of merit based on the product of M and R that, as one factor is increased, the other factor decreases in approximately the same proportion. Thus, the essential tradeoff (for the same amount of availability) emphasizes a design solution that is a compromise and does not incorporate one overriding critical cost element that would result in a reversal of the sensitivity figures for Configurations 1 or 3.

APPENDIX D   SOME PROBABILITY BACKGROUND - JOINT PROBABILITIES

In the study of probability theory, it is shown that for two random continuous variables, x and y, which may or may not be independent random variables, one can define a joint probability density function, $f(xy)$. The important property of this function is that if one takes a random sample from the sample space of xy, then the probability of finding a value of x that is somewhere in the region Γ and, at the same time, finding a value of y that is in in the same region Γ is given by
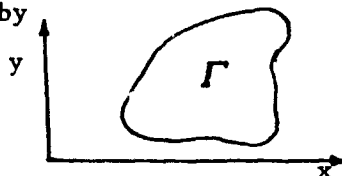
where the integration is made over the region Γ .

$$P\left\{ xy;\ x,\ y \epsilon \Gamma \right\} = \int \int_{\Gamma} f(xy)\quad d\,x\,dy$$

More specifically, the probability of finding x in the region

$x_1 \leq x \leq x_2$ and, at the same time, finding y in the region

$y_1 \leq y \leq y_2$ is

$$P\left\{ xy;\ x_1 \leq x \leq x_2,\ y_1 \leq y \leq y_2 \right\} = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f(xy)\, d\,x\,dy$$

Furthermore, if x and y are independent random variables, then the joint probability density function $f(xy)$ is simply the product of the probability density functions for each of the variables individually.

In particular, if x and y are assumed to be independent random variables, the joint probability density function for these variables is:
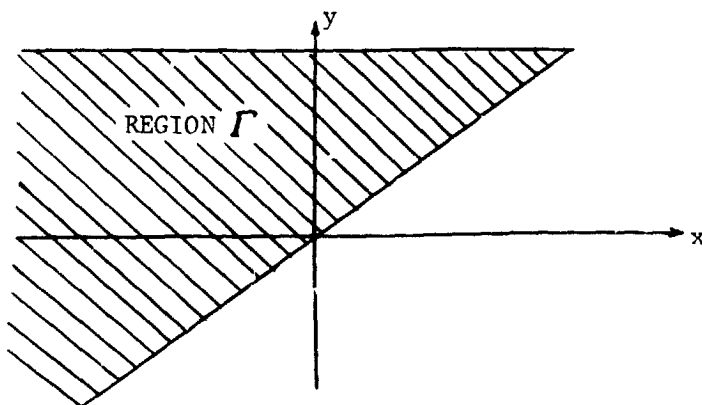
$$f(xy) = p_x(x)\ p_y(y)$$

where $p_x(x)$ is the probability density function for x and $p_y(y)$ is the probability density function for y.

A.   PROBABILITY OF EVENT $\quad y > x$

Let A be the probabilistic event that the random variable y
exceeds x. Then the probability for the occurrence of event A
is the probability that, in any random sample taken from x and y,
one will find $y \geq x$ and that x will be found any place at all,

$$P \left\{ A \right\} = P \left\{ xy; \; y \geq x, \; -\infty \leq x \leq \infty \right\}$$

To calculate $P \left\{ A \right\}$, the joint probability density function
$f(xy)$ must be integrated over the shaded region Γ of the x, y plane
because that area is the region where $y \geq x$ and where $-\infty \leq x \leq \infty$.

This gives:

$$P \left\{ A \right\} = \int \int p_x(x) \; p_y(y) \; dx \; dy$$

or

$$P \left\{ A \right\} = \int_{x \to -\infty}^{x \to \infty} \int_{y = x}^{y \to \infty} p_x(x) \; p_y(y) \; dx \; dy$$

B.   CALCULATION PROCEDURE

To evaluate this integral, perform the y integration first.   Then,
x will appear in the result when the limits are substituted.   Then
perform the x integration.   This procedure can be shown in the
following formula.   Let

$$J(x) = \int_{y=x}^{y \to \infty} p_y(y) \, dy$$

Then

$$P\{A\} = \int_{x \to -\infty}^{x \to \infty} p_x(x) \, J(x) \, dx$$

C.   ALTERNATIVE INTEGRAL

Alternatively, we could cover the same shaded region Γ by another
integration,

$$P\{A\} = \int_{y \to -\infty}^{y \to \infty} \int_{x \to -\infty}^{x=y} p_x(x) \, p_y(y) \, dx \, dy \qquad (7)$$

Again to evaluate this integral, perform the x integration first,
and y will appear as a parameter in the result.   Then perform the
y integration.   The procedure can be shown by the following.   Let

$$I(y) = \int_{x \to -\infty}^{x=y} p_x(x) \, dx \qquad (8)$$

Then

$$P\{A\} = \int_{y \to -\infty}^{y \to \infty} p_y(y) \, I(y) \, dy \qquad (9)$$

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D.   STUDY OF SOME SPECIAL CASES

To check the validity of these formulae, we shall use them to calculate $P\{A\}$ for some simple cases where the computation is not difficult and the results can be checked with intuitive expectation.

1.   Case 1 -- Two Nonoverlapping Rectangular Distributions
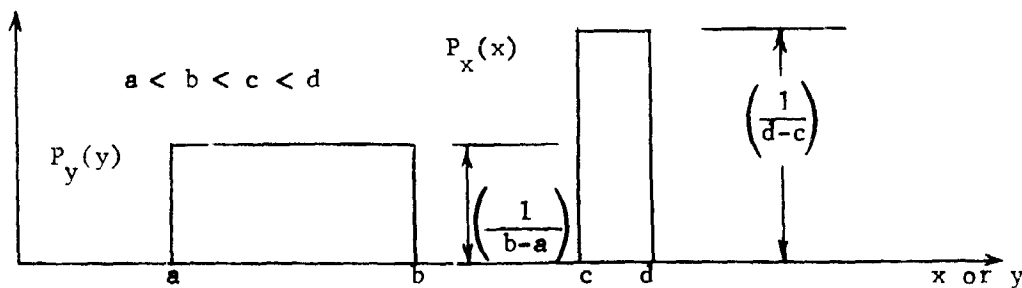
Consider two rectangular distributions as shown above.

$$P_x(x) = \begin{cases} \left(\dfrac{1}{d-c}\right) & \text{for } c \leq x \leq d \\ 0 & \text{otherwise} \end{cases} \qquad P_y(y) = \begin{cases} \left(\dfrac{1}{b-a}\right) & \text{for } a \leq y \leq b \\ 0 & \text{otherwise} \end{cases}$$

Using eq (9), we calculate directly that

$$I(y) = \int_{x \to -\infty}^{x=y} P_x(x)\, dx = \begin{cases} \prime & \text{for } y < c \\ \dfrac{y-c}{d-c} & \text{for } c \leq y \leq d \\ 1 & \text{for } y > d \end{cases}$$

and

$$P\{A\} = \int_{y \to -\infty}^{y \to \infty} P_y(y)\, I(y)\, dy$$

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But $I(y) = 0$ for $y < c$

and $P_y(y) = 0$ for $y < b$

and also $b < c$.

Therefore, $P\left\{A\right\} = 0$ as expected.

2.   Case 2 -- Two Rectangular Distributions, Completely Overlapping
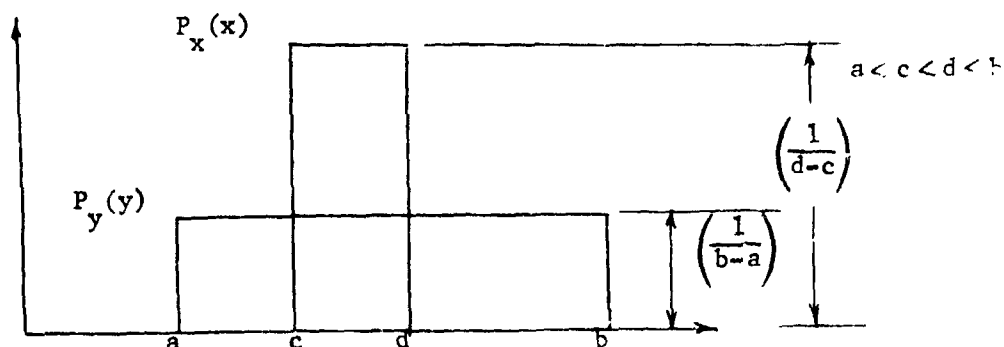
Consider:

$$P_y(y) = \begin{cases} \left(\dfrac{1}{b-a}\right) a \leq y \leq b \\[2ex] 0 \quad \text{otherwise} \end{cases} \qquad P_x(x) = \begin{cases} \left(\dfrac{1}{d-c}\right) c \leq x \leq d \\[2ex] 0 \quad \text{otherwise} \end{cases}$$

Using Eq (9) we calculate directly that:

$$I(y) = \int_{x \to -\infty}^{x=y} P_x(x) \, dx = \begin{cases} 0 & \text{for } y < c \\[1ex] \left(\dfrac{y-c}{d-c}\right) & \text{for } c < y < d \\[1ex] 1 & \text{for } y > d \end{cases}$$

and by a straightforward calculation,

$$P\left\{A\right\} = \int_{x_y \to -\infty}^{x_y \to \infty} P_y(y) \, I(y) \, dy = \left\{ \frac{(b-d)}{(b-a)} + \frac{1}{2}\left(\frac{(d-c)}{(b-a)}\right) \right\}$$

3.   <u>_ cial Case -- Identical Distributions</u>

Suppose that d=b and c=a. Then the result above reduces to:

$$P\left\{A\right\} = \frac{1}{2}$$

4.   **Special Case -- Symmetrically Located Distributions**

Suppose that the two rectangular distributions are symmetrically located relative to each other. That is, $b-d = c-a$. Then the result for $P\{A\}$ in Case 2, again, reduces to

$$P\left\{A\right\} = \frac{1}{2}$$

5.   **Case 3 -- Two Rectangular Distributions, Partial Overlapping**

$$a < c < b < d$$
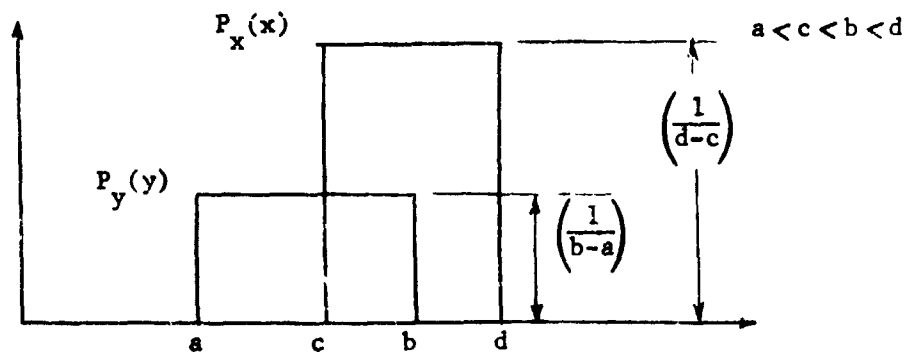
Consider:

$$P_x(x) = \begin{cases} \left(\dfrac{1}{d-c}\right) & \text{for } c \leq x \leq d \\[2em] 0 & \text{otherwise} \end{cases} \qquad P_y(y) = \begin{cases} \left(\dfrac{1}{b-a}\right) & \text{for } a \leq y \leq b \\[2em] 0 & \text{otherwise} \end{cases}$$

Again, using Eq (9)

$$I(y) = \int_{x \to -\infty}^{x=y} P_x(x)\, dx = \begin{cases} 0 & y < c \\[1em] \dfrac{y-c}{d-c} & c \leq y \leq d \\[1em] 1 & y > d \end{cases}$$

and

$$P\,A = \int_{-\infty}^{\infty} P_y(y)\, I(y) = \left\{ \frac{(b-c)^2}{2\,(b-a)\,(d-c)} \right\}$$

or

$$P\left\{A\right\} = \frac{1}{2}\left(\frac{b-c}{b-a}\right)\left(\frac{b-c}{d-c}\right) = \frac{1}{2}\begin{pmatrix}\text{Length of Overlap}\\ \text{Region Relative to}\\ \text{Length of b-a}\end{pmatrix}\begin{pmatrix}\text{Length of Overlap}\\ \text{Region Relative to}\\ \text{Length of d-c}\end{pmatrix}$$
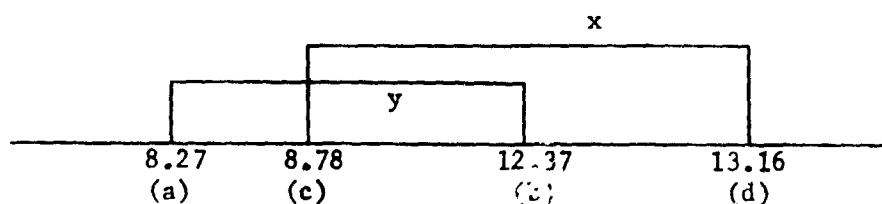
Using the approach described in the preceding section and assuming a fixed probability over the tolerance interval, we can obtain expressions for the mean values for the two alternate systems x and y, and determine the probability that the central value of $x > y$. The following figure describes this case.

$$P\left\{A\right\} = \int_{y=-\infty}^{y=\infty}\int_{x=-\infty}^{x=y} P(y)\ P(x)\ dx\ dy$$

evaluating x first.

Integrating by parts

$$x < c \quad \left.\begin{matrix} \end{matrix}\right| P(x) = 0$$
$$c < x < d \quad \left.\begin{matrix} \end{matrix}\right\} P(x) = \left(\frac{1}{d-c}\right)$$
$$x > d \quad \left.\begin{matrix} \end{matrix}\right| P(x) = 0$$

for y

$$y < a \quad \left.\begin{matrix} \end{matrix}\right| P(y) = 0$$
$$a < y < b \quad \left| P(y) = \left(\frac{1}{b-a}\right)\right.$$
$$b < y < d \quad \left| P(y) = 0\right.$$
$$y > d \quad P(y) = 0$$

Now

$$I(y) = \int_{x=-\infty}^{x=y} P(x)\ dx = \frac{y-c}{d-c}$$

$$P\left\{A\right\} = \int_{y=-\infty}^{y=+\infty} \frac{y-c}{d-c} \times \frac{1}{b-a}\ dy$$

From the evaluation of $P(x)$ and $P(y)$ by parts, the only interval in which both are real values other than zero, are b to c

$$P\left\{A\right\} = \int_{y=c}^{y=b} \frac{y-c}{(d-c)\ (b-a)}\ dy = \left\{\frac{(b-c)^2}{2\ (b-a)\ (d-c)}\right\}$$

An evaluation of this expression using the values in the figure
$p\left\{A\right\} = 0.35$

It is therefore concluded that because there is only a 35% chance that System x has a larger central value than System y, we must treat them as essentially equal.

APPENDIX E LAUNCH-ON-TIME ANALYSES

A. POST DELIVERY REQUIREMENTS

The postdelivery requirements are based on two recycles
during prelaunch operations. Therefore, the probability
of exceeding 2 recycles is considered in the launch-on-
time analysis. This includes the following recycle con-
ditions.

1) P_N = Probability of the payload surviving prelaunch
   operations with N recycles or less.

   P_{N-1} = Probability of the payload surviving prelaunch
   operations with N - 1 recycles or less (allows
   one recycle from launch pad).

   P_0 = Probability of no malfunctions in the payload
   during prelaunch and marriage tests during any
   one cycle (can recur) or recycle.

2) $P_N = P_0 + (1 - P_0) P_0 + (1 - P_0)^2 P_0 + \ldots + (1 - P_0)^N P_0$

   $= 1 - (1 - P_0)^{N+1}$

3) For N = 2, P_0 = 0.95:

   $P_N = 1 - (0.05)^3 = 0.999875$

   $P_{N-1} = 1 - (0.05)^2 = 0.9975$

4) Parametric variations of P_N are presented in Fig. 1.
   The above conditions are used in the following cal-
   culations for launch operations.

B. LAUNCH PAD OPERATIONS - CONFIGURATION 1

   22-Day Launch Period

   10-Day Turnaround

   With Spare Spacecraft

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N = Number of Recycles
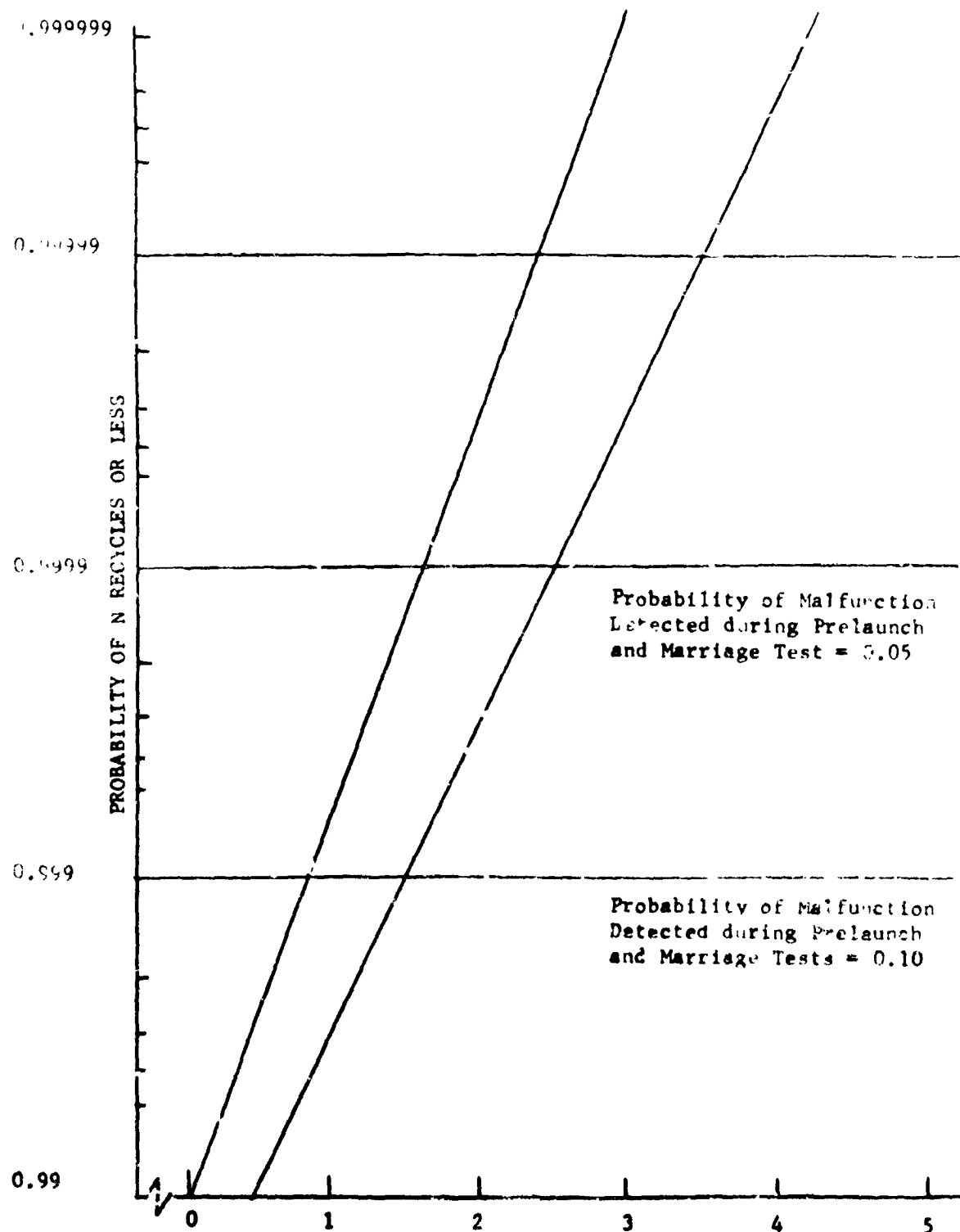
Fig. 1 -- Probability of Recycle during Prelaunch and Marriage Tests

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1. <u>Payload</u>

Figure 2 presents the timeline showing all possible out-
comes of the payload resulting in two launches within the
launch period. Figure 3 presents the corresponding success/
failure diagram. Let

P = the probability of achieving a launch on any one
attempt (can recur)

and

$(PLOT)_2$ = the probability of achieving two launches in
the launch period

The probability of achieving two launches because of the
payload is the sum of the probabilities of the outcome
shown in Fig. 2 and 3 as follows.

Outcome No.	Probability of Outcome
1	$P_N^2 \ P^2$
2	$P_N^3 \ P^2 \ (1 - P)$
3	$P_N^3 \ P^2 \ (1 - P)$
4	$P_N^3 \ P^2 \ (1 - P)^2 \ P_{N-1} \ P_0$
5	$P_N^3 \ P^2 \ (1 - P)^2 \ P_{N-1} \ P_0$

$$\sum = (PLOT)_2 = P_N^2 \ P^2 \left\{ 1 + 2P_N \ (1 - P) + 2P_N \ P_{N-1} \ P_0 \ (1 - P)^2 \right\}$$

Let N = 2, P_0 = 0.95, and P = 0.95.

Then

P_N = 0.999875

and   } See Section A

P_{N-1} = 0.9975

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NOTE: 22-Day Launch Period,
10-Day Turnaround,
With Spare Launch Vehicle and Payload (LP)

Outcome Payload
No.

1   1
     2

2   1
     2
     3

3   1
     2
     3

4   1
     2
     3

5   1
     2
     3

TURNAROUND

REPLACE LP

RECYCLE

LAUNCH

MALFUNCTION

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
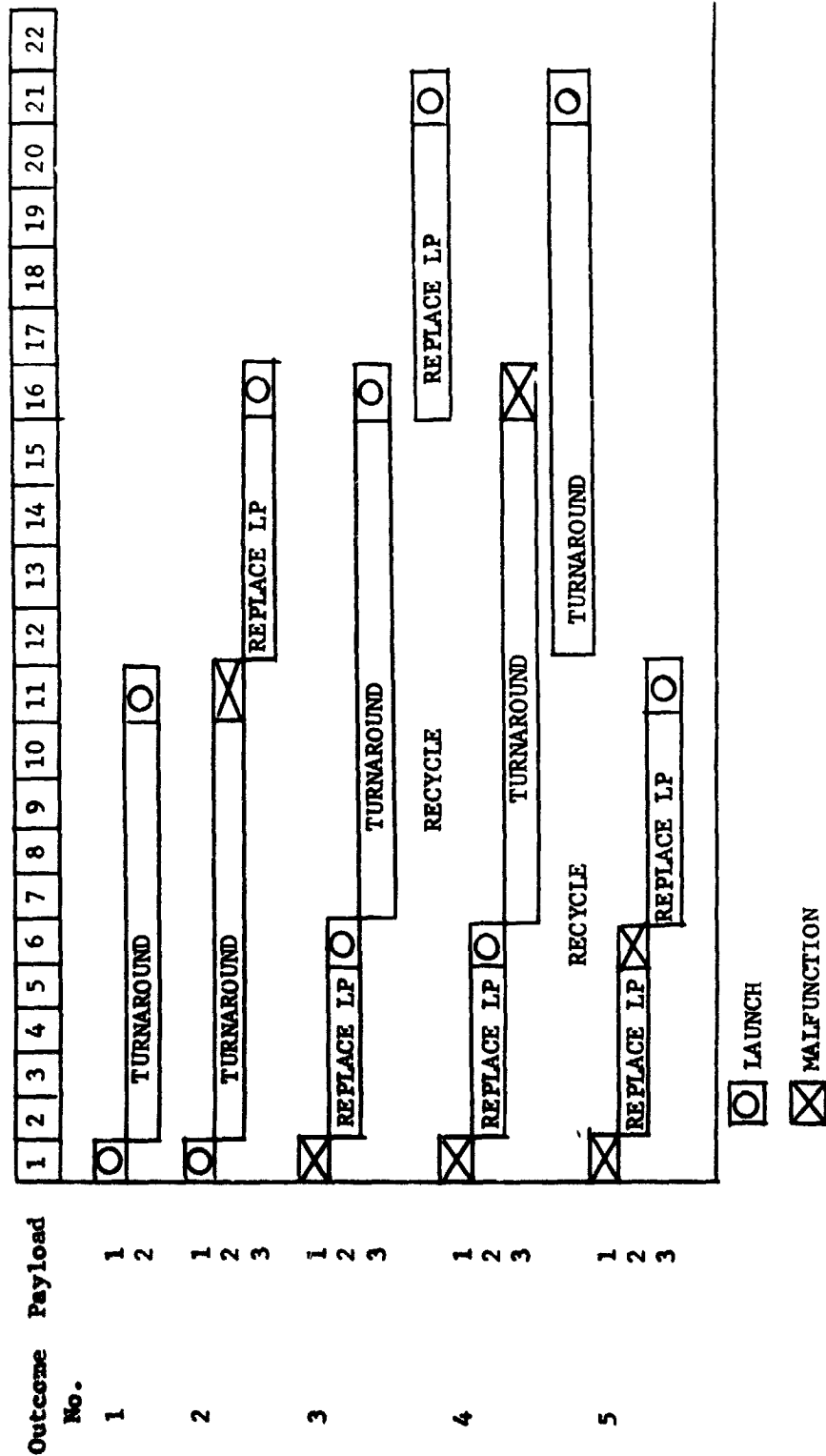
Figure 2 -- Timeline for Configuration 1, Payload and Launch Vehicle

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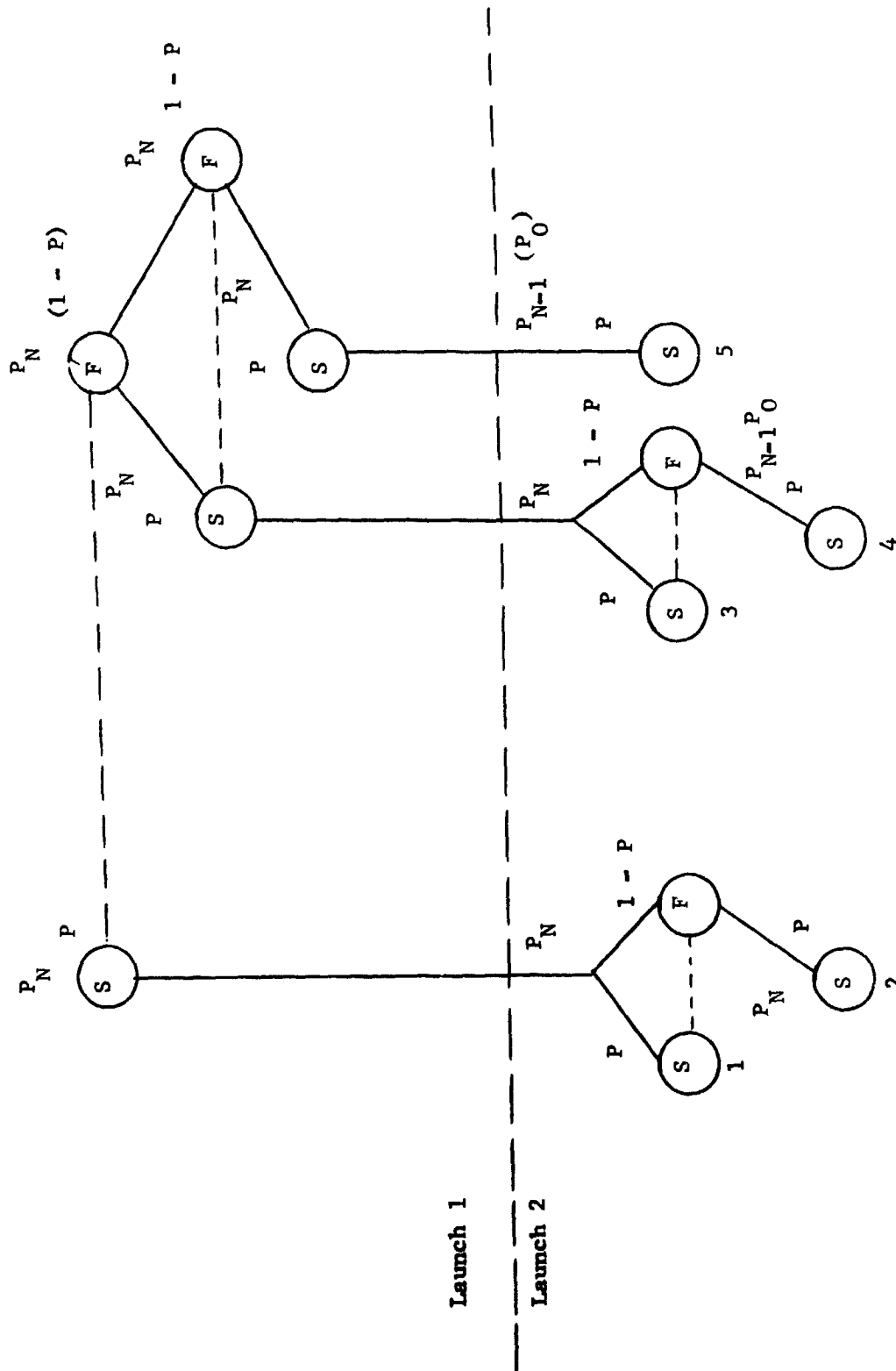
Figure 3 -- Configuration 1, Success/Failure Diagram, Payload and Launch Vehicle

$$(\text{PLOT})_2 = (0.999875)^2 \, (0.95)^2 \quad 1 + 2(0.999875) \, (0.05) +$$
$$2(0.999875) \, (0.9975)$$

$$(0.95) \, (0.0025)$$

$$(\text{PLOT})_2 = 0.996765$$

2. Launch Vehicle

Figures 2 and 3 also apply to the launch vehicle, except
that there is no limitation on the number of recycles
(P_N and P_{N-1} do not apply).

Outcome No.	Probability of Outcome
1	P^2
2	$P^2 \, (1 - P)$
3	$P^2 \, (1 - P)$
4	$P^2 \, (1 - P)^2 \, P_0$
5	$P^2 \, (1 - P)^2 \, P_0$

$$(\text{PLOT})_2 = P^2 \left\{ 1 + 2(1 - P) + 2P_0 \, (1 - P)^2 \right\}$$

for $P = 0.95$, $P_0 = 0.95$

$$(\text{PLOT})_2 = 0.997037$$

3. All Other Causes

Let P = probability of launching on any one attempt (can recur)

N_1 = number of attempts required for the first launch

N_2 = number of attempts available for the second launch

$(\text{PLOT})_2$ = probability of achieving 2 launches in the launch period

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The number of attempts available for the second launch depends on the number of attempts used for the first launch. Therefore, the probability of achieving the second launch in N_2 attempts or less depends on the probability of the first launch being achieved in exactly N_1 attempts.

$$P_N = P_{N_1} \quad P_{N_2}$$

$$P_{N_L} = P(1 - P)^{N_1 - 1}$$

$$P_{N_2} = 1 - (1 - P)^{N_2}$$

$$N_2 = N_T - N_1$$

where N_T = total attempts available.

The probability of launching two on time is the sum of the probabilities of all outcomes

$$(PLOT)_2 = \sum P_N$$

The calculations are presented in Table 1 for $P = 0.90$.

From Table 1

$$(PLOT)_2 = 0.999540$$

Table 1

Calculations for Probability of Two Launches, All Other Causes,

Configuration 1

Attempts at First Launch N_L	Probability of Achieving First Launch in N_1 Attempts	Attempts Available for Second Launch N_2	Probability of Achieving Second Launch in N_2 Attempts or Less	Probability of Two Launches in N_1 and N_2 Attempts
1	0.85	4	0.9999	0.899910
2	0.09	3	0.999	0.089910
3	0.009	2	0.99	0.008910
4	0.0009	1	0.9	0.000810
			$(PLOT)_2 =$	0.999540

Note: 22-Day Launch Period; 10-Day Turnaround

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4. Results - Configuration #1

    22-Day Launch Period

    10-Day Turnarour_ (launch-to-launch)

    With both Spare Launch Vehicle and Payload

	Probability of Launch-on-Time	Launch-on-Time Risk
Payload	0.996765	0.003235
Launch Vehicle	0.997037	0.002963
All Other Causes	0.999540	0.000460
Total	0.993353	0.006647

C.   Launch Pad Operations, Configuration 2

                    30-Day Launch Period
                    16-Day Turnaround
                    No Spare Vehicle with Spare Payload

1.   Payload

Figure 4 presents the timeline for all outcomes of the payload resulting in two launches. Figure 5 presents the corresponding success/failure diagram. Calculations are as follows.

Outcome	Probability of Outcome
1	$P^2 P_N^2$
2	$P^2 P_N^3 (1 - P)$
3	$P^2 P_N^3 (1 - P)$
4	$P^2 P_N^3 P_{N-1} P_0 (1 - P)^2$

$$(PLOT)_2 = P^2 P_N^2 \left\{ 1 + 2P_N(1 - P) + P_N P_{N-1} P_0 (1 - P)^2 \right\}$$

For $P_0 = 0.95$, $P = 0.95$, $P_N = 0.999875$, $P_{N-1} = 0.9975$,

$$(PLOT)_2 = 0.994627$$

2.   Launch Vehicle

Figure 6 presents the timeline and the success/failure diagram for the launch vehicle using Configuration 2 data. Calculations are as follows.

NOTE: 30-Day Launch Period
16-Day Turnaround
No Spare Launch Vehicle with Spare Payload (P)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |

TURNAROUND

TURNAROUND

REMOVE/REPLACE P

REPLACE LP

TURNAROUND

If Malf - No launch
until Day 31

RECYCLE P

REPLACE LP

ORB   REPLACE LP

TURNAROUND

◎ LAUNCH

⊠ MALFUNCTION

Figure 4 -- Timeline for Configuration 2, Payload

Figure 5 — Success/Failure Diagram, Payload Configuration 2, Payload
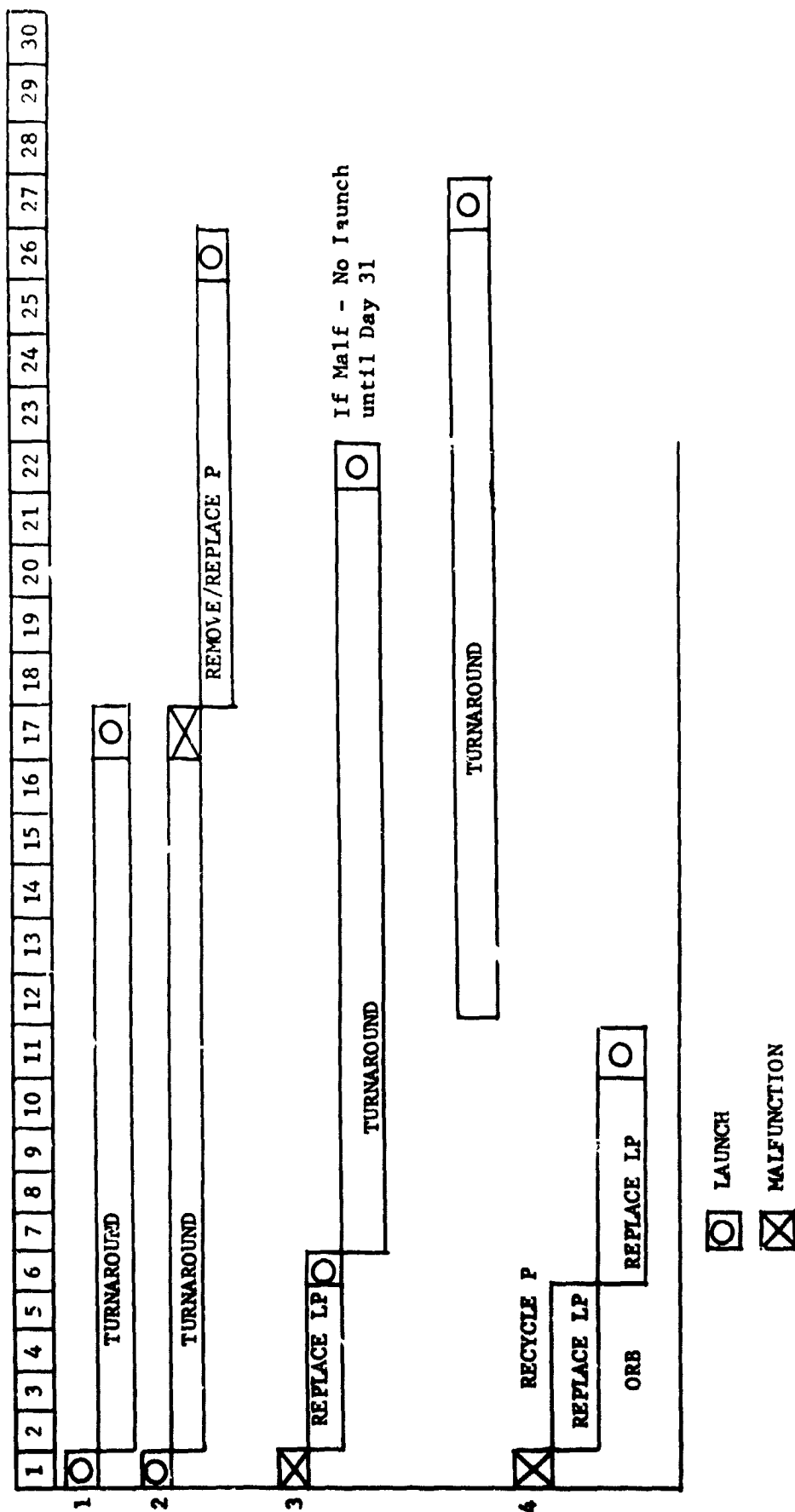
NOTE: 30-Day Launch Period
16-Day Turnaround
No Spare Launch Vehicle with Spare Payload
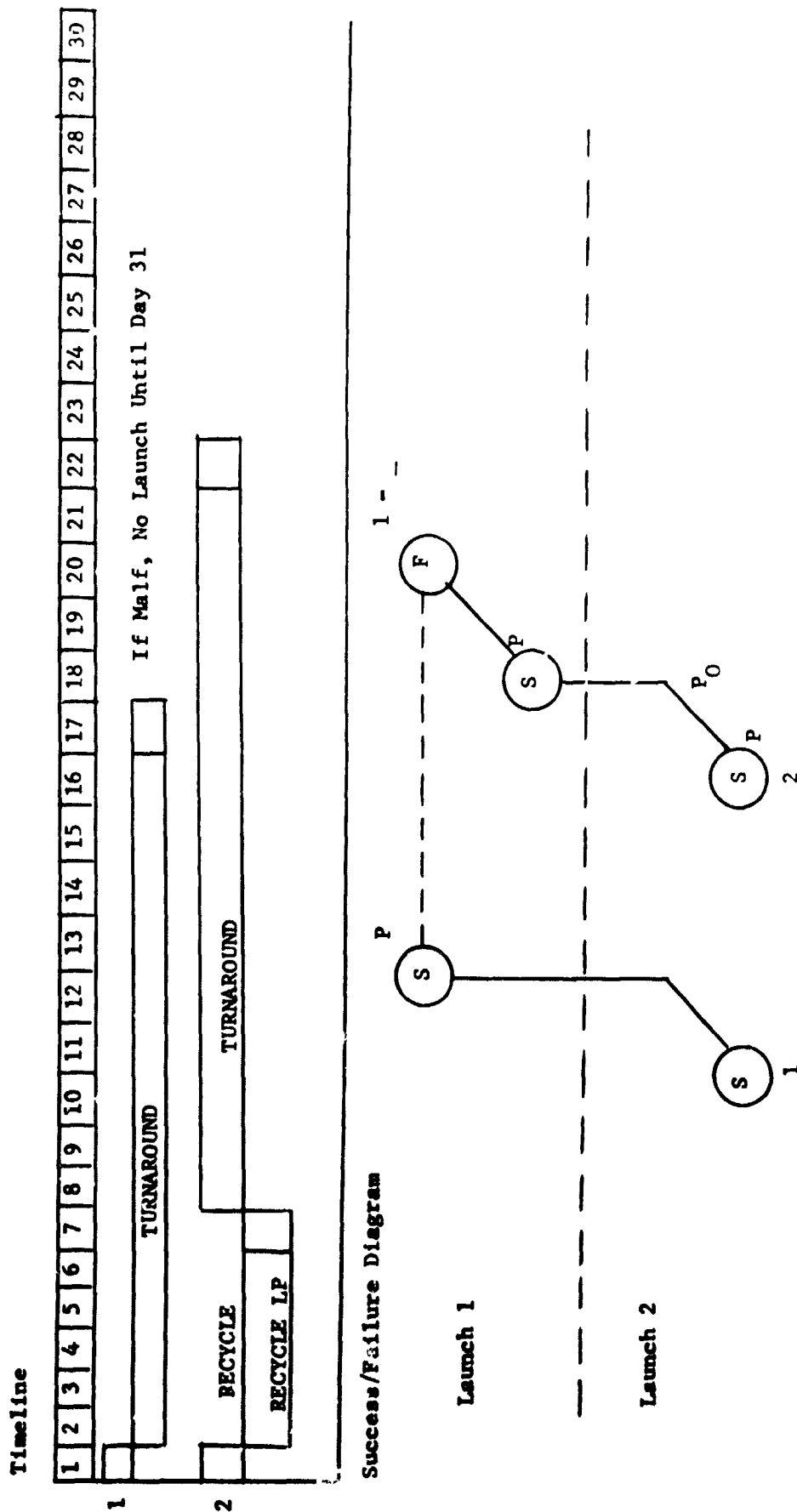
Figure 6 -- Timeline and Success/Failure Diagram, Launch Vehicle

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Outcome	Probability of Outcome
1	P^2
2	$P^2\left\{P_0(1-P)\right\}$

$$(PLOT)_2 = P^2\left\{1 + P_0(1-P)\right\}$$

For $P = 0.95$ and $P_0 = 0.95$,

$$(PLOT)_2 = 0.945369$$

3. All Other Causes

Table 2 shows the number of launches available for the second launch as a function of the number of attempts used for the first launch and presents the calculations. From Table 2,

$$(PLOT)_2 = 0.999945$$

4. Results, Configuration 2

30-Day Launch Period
16-Day Turnaround
No Spare Launch Vehicle with Spare Payload

	Probability of Launch-on-Time	Launch-on-Time Risk
Payload	0.994627	0.005373
Launch Vehicle	0.945369	0.054631
All Other Causes	0.999945	0.000055
TOTAL	0.94038	0.059762

Table 2

Calculations for Probability of Two Launches, All Other Causes, Configuration 2

Attempts at First Launch (N_1)	Probability of Achieving First Launch in Exactly N_1 Attempts	Attempts Available for Second Launch (N_2)	Probability of Achieving Second Launch in N_2 Attempts or Less	Probability of Two Launches in N_1 and N_2 Attempts
1	0.9	5	0.99999	0.899991
2	0.09	4	0.9999	0.089991
3	0.009	3	0.999	0.008991
4	0.0009	2	0.99	0.000891
5	0.00009	1	0.9	0.000081

$$(PLOT)_2 = 0.99994$$

NOTE: 30-Day Launch Period, 16-Day Turnaround.

D.   Launch Pad Operations, Configuration 3

                    30-Day Launch Period
                    10-Day Turnaround
                    No Spare Launch Vehicle with Spare Payload

1.   Payload

Figure 7 presents the time line for Configuration 3.   Figure 8
presents the corresponding success/failure diagram.   The
calculations are as follows.

Outcome No.	Probability of Outcome
1	$P^2 P_N^2$
2	$P^2 P_N^3 (1 - P)$
3	$P^2 P_N^3 (P_{N-1} P_0) (1 - P)^2$
4	$P^2 P_N^3 (1 - P)$
5	$P^2 P_N^3 (P_{N-1} P_0) (1 - P)^2$
6	$P^2 P_N^3 (P_{N-1} P_0) (1 - P)^2$
7	$P^2 P_N^3 (P_{N-1} P_0)^2 (1 - P)^3$
8	$P^2 P_N^3 (P_{N-1} P_0)^2 (1 - P)^3$

NOTE: 30-Day Launch Period
        10-Day Turnaround
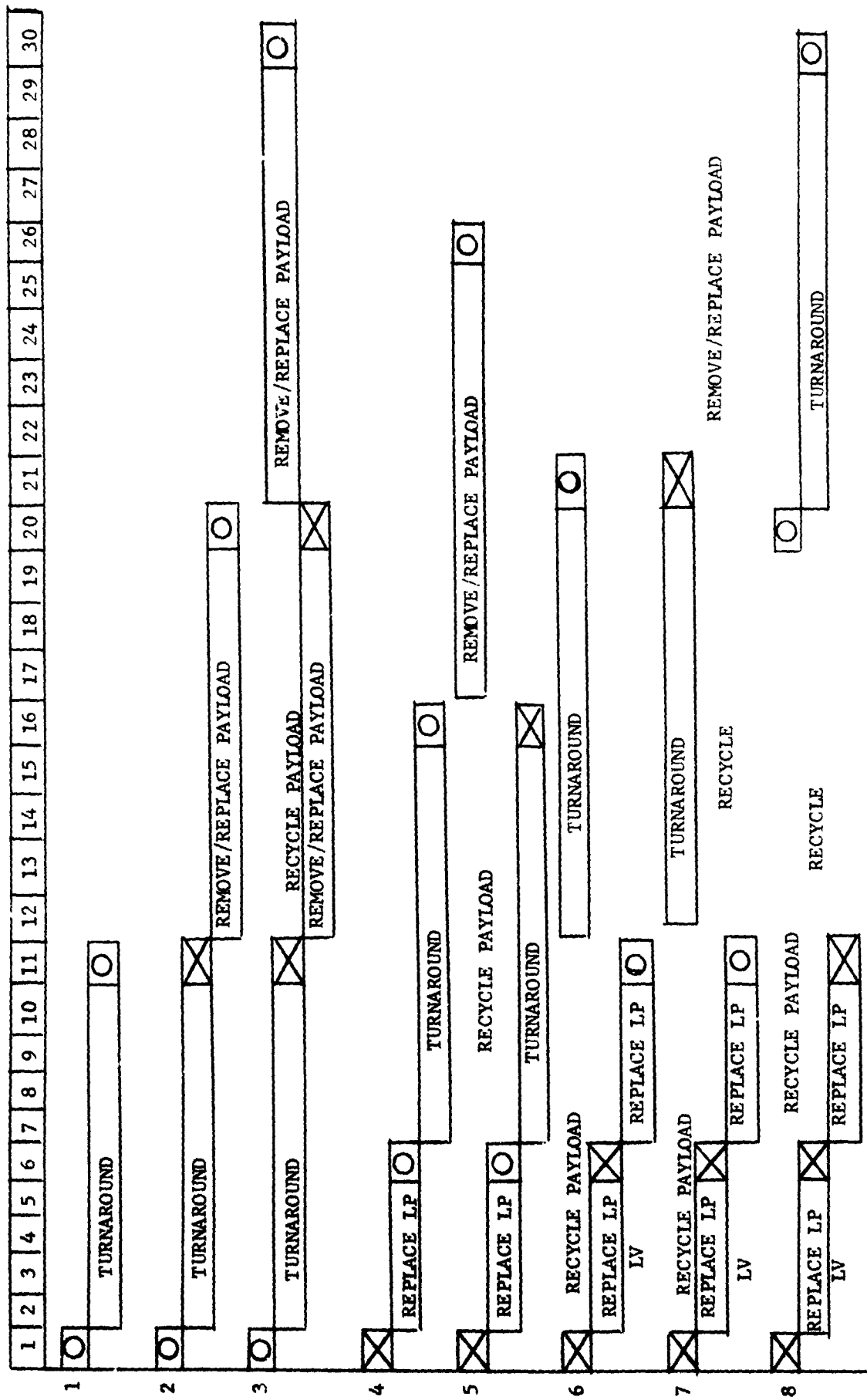   No Spare Launch Vehicle with Spare Payload

Figure 7 -- Timeline for Configuration 3, Payload

NOTE: 30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle with Spare Payload
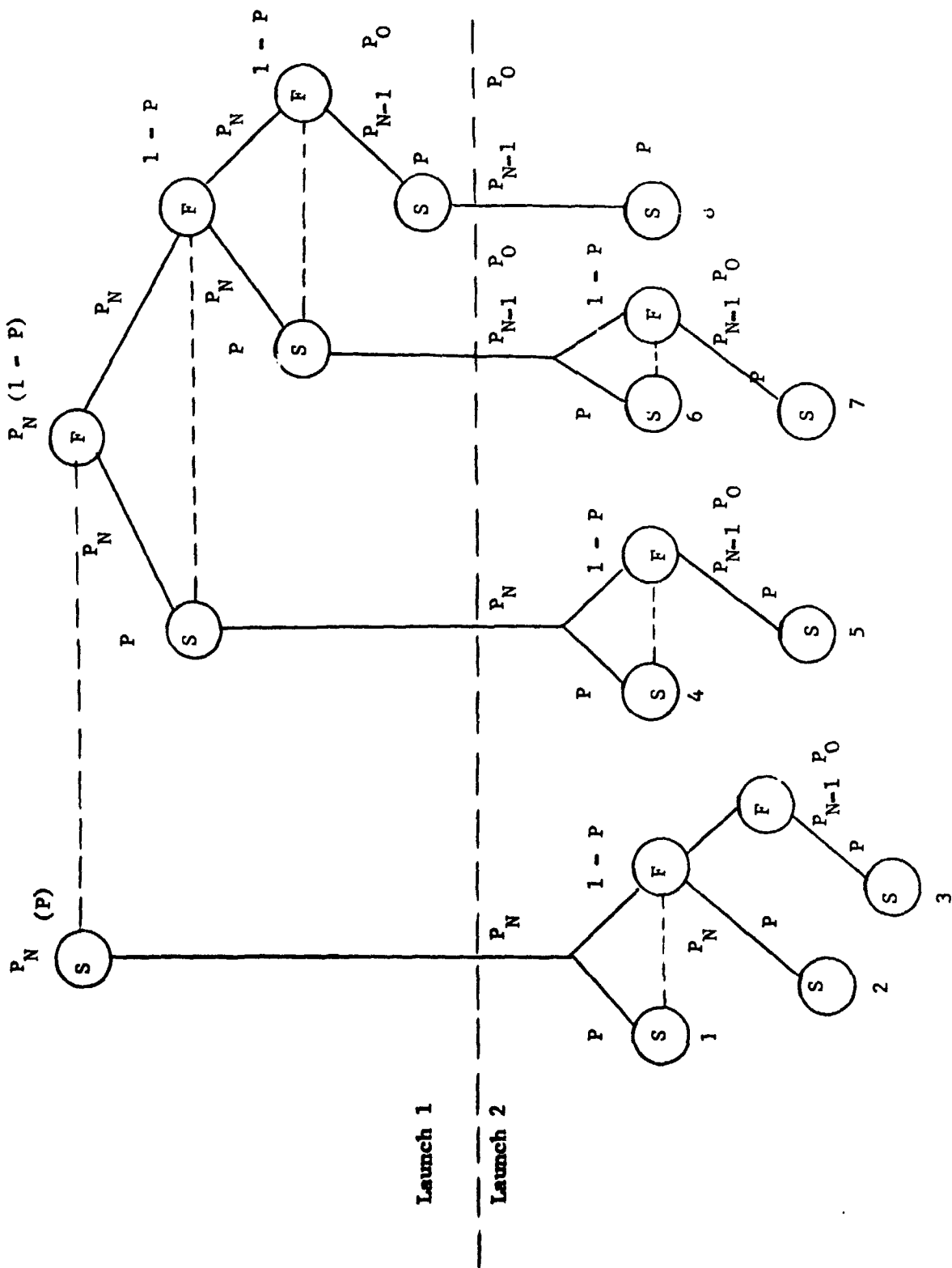
Launch 1

Launch 2

Figure 8 -- Success/Failure Diagram, Payload

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$$(\text{PLOT})_2 = P^2 \, P_N{}^2 \left\{ 1 + 2 \, P_N \, (1 - P) + 3 \, P_N \, (P_{N-1} \, P_O) \, (1 - P)^2 \right.$$

$$\left. + 2 \, P_N \, (P_{N-1} \, P_O)^2 \, (1 - P)^3 \right\}$$

For $P = 0.95$, $P_N = 0.999875$, $P_{N-1} = 0.9975$, and $P_O = 0.95$,

$$(\text{PLOT})_2 = 0.999104$$

2. Launch Vehicle

Figure 9 presents the timeline for Configuration 3. Figure 10 presents the corresponding success/failure diagram. Calculations are as follows.

Outcome No.	Probability of Outcome
1	P^2
2	$P^2 \, P_O \, (1 - P)$
3	$P^2 \, P_O \, (1 - P)$
4	$P^2 \, P_O{}^2 \, (1 - P)^2$
5	$P^2 \, P_O{}^2 \, (1 - P)^2$
6	$P^2 \, P_O{}^3 \, (1 - P)^3$

$$(\text{PLOT})_2 = P^2 \left\{ 1 + 2 P_O \, (1 - P) + 2 P_O{}^2 \, (1 - P)^2 + P_O{}^3 \, (1 - P)^3 \right\}$$

For $P = 0.95$ and $P_O = 0.95$,

$$(\text{PLOT})_2 = 0.992407$$

NOTE: 30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle
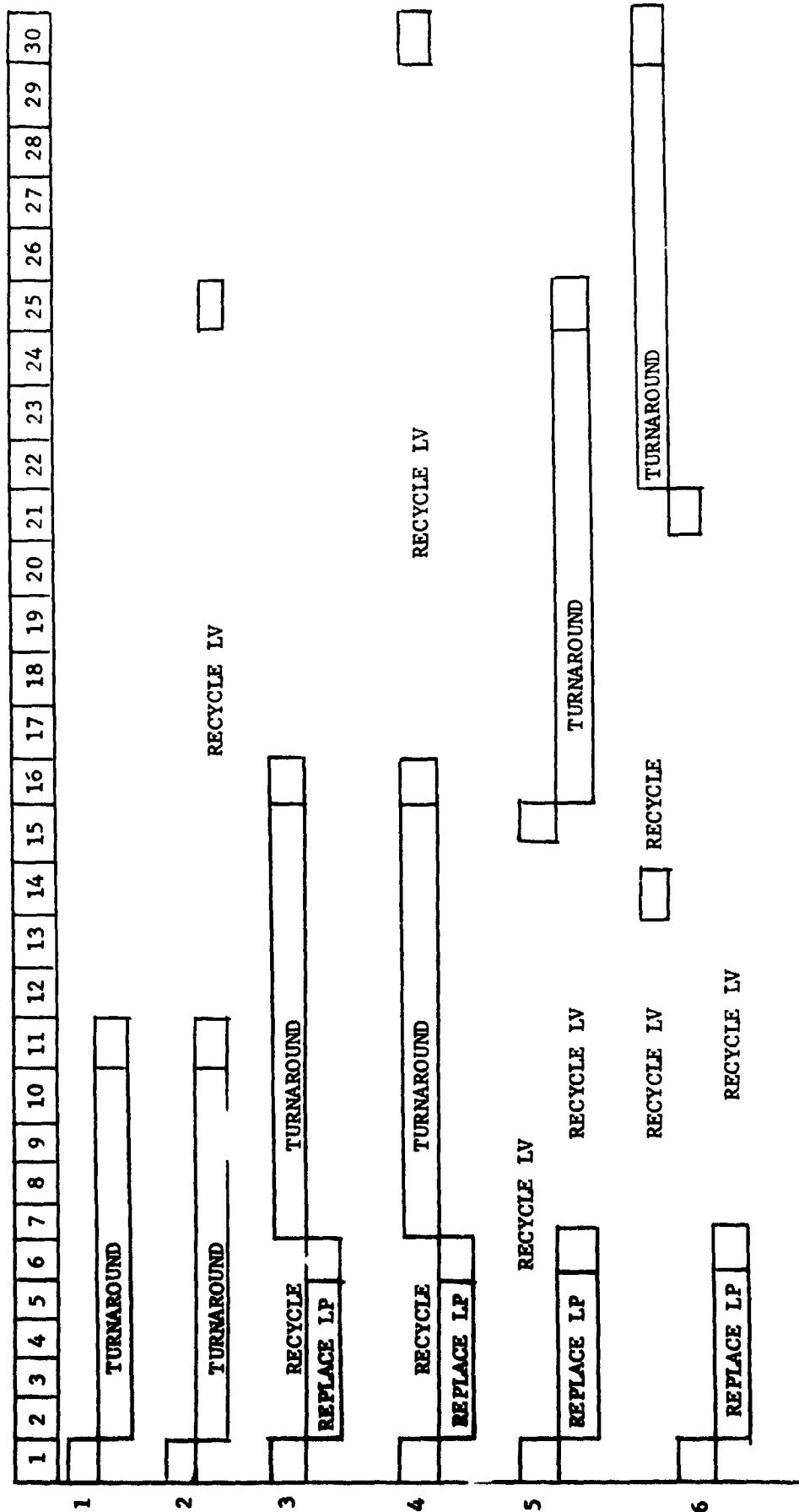
Figure 9 -- Timeline for Configuration 3, Launch Vehicle

NOTE: 30-Day Launch Period
10-Day Turnaround
No Spare Launch Vehicle
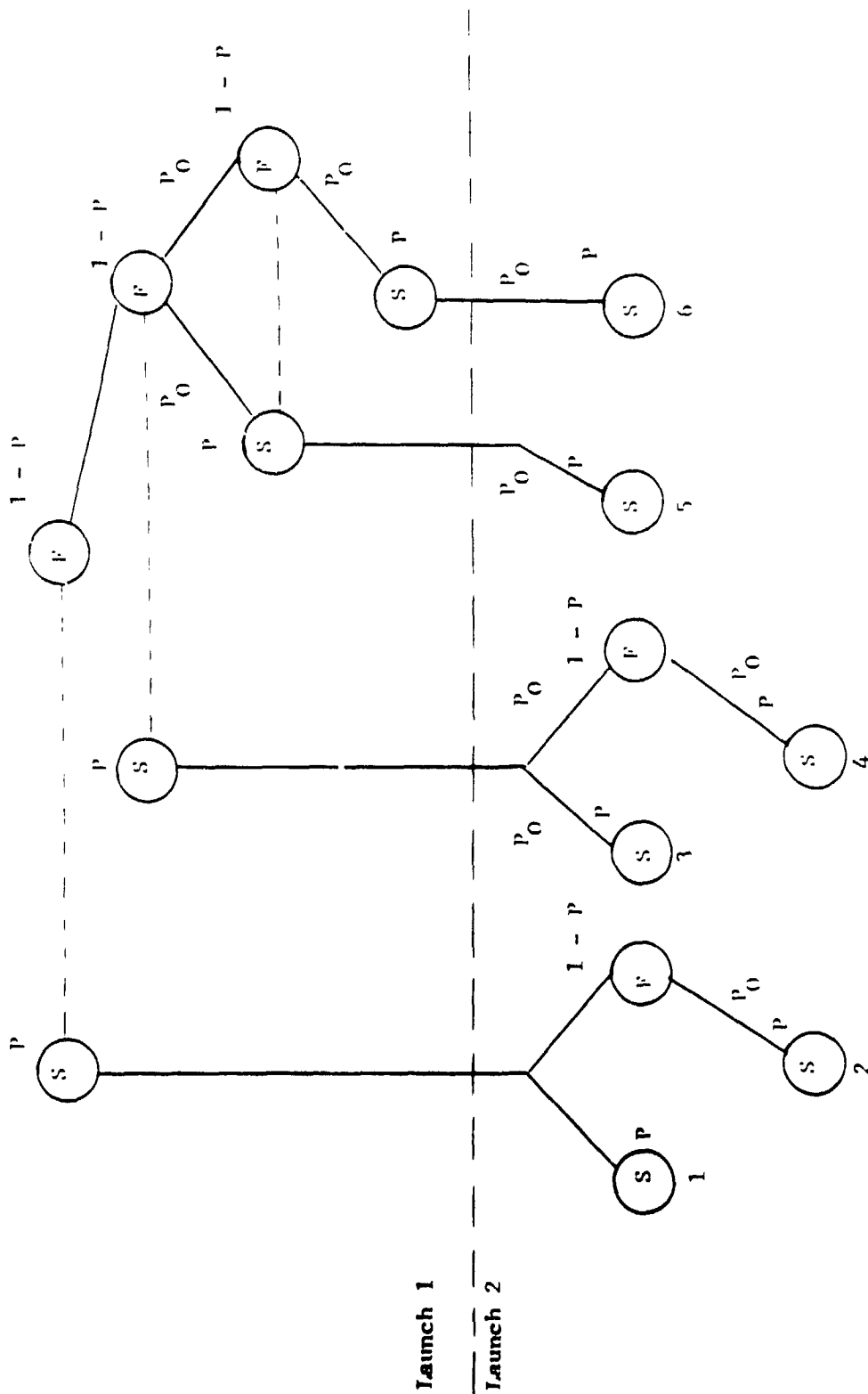
Launch 1
Launch 2

Figure 10 -- Success/Failure Diagram, Launch Vehicle

3. All Other Causes

    Table 3 shows the number of launch attempts available for the
    second launch as a function of the number used for the first
    launch. From Table 3,

    $$(PLOT)_2 = 0.999999$$

4. Results, Current Baseline

    30-Day Launch Period
    10-Day Turnaround
    No Spare Launch Vehicle with Spare Payload

	Probability of Launch-on-Time	Launch-on-Time Risk
Payload	0.999104	0.000896
Launch Vehicle	0.992407	0.007593
All Other Causes	0.999999	0.0000001
TOTAL	0.991516	0.008484

Table 3

Calculation for All Other Causes

Number of Attempts Used for First Launch (N_1)	Probability of Achieving First Launch in Exactly N_1 Attempts	Number of Attempts Available for Second Launch (N_2)	Probability of Achieving Second Launch in N_2 Attempts or Less	Probability of Achieving Two Launches in Exactly $N_1 + N_2$ or Less
1	0.9	7	0.9999999	0.89999991
2	0.09	6	0.999999	0.08999991
3	0.009	5	0.99999	0.00899991
4	0.0009	4	0.9999	0.00089991
5	0.00009	3	0.999	0.00008991
6	0.000009	2	0.99	0.00000891
7	0.0000009	1	0.9	0.00000081
			$(PLOT)_2 =$	0.99999927

NOTE: 30-Day Launch Period, 10-Day Turnaround.

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APPENDIX F   MONTE CARLO SIMULATION MODEL

DEVELOPMENT OF STORAGE MATRIX

		UNTESTABLE PARTS a	TESTABLE PARTS b	
STATE	1	1	1	BOTH GOOD
	2	1	0	a GOOD
	3	0	1	b GOOD
	4	0	0	NEITHER GOOD

$$R_1 \text{ (UNTESTABLE PARTS)} = \epsilon^{-\delta T a}$$

$$R_2 \text{ (TESTABLE PARTS)} = \epsilon^{-\zeta T b}$$

		FINISH IN STATE			
		1	2	3	4
START IN STATE	4	$R_1 R_2 = \epsilon^{-(a+b)\gamma T}$	$R_1 Q_2 = \epsilon^{-\gamma T a}(1-\epsilon)$	$Q_1 R_2 = (1-\epsilon^{-\gamma T a})$	$1 - \Sigma$
	3	0	$R_1 = \epsilon^{-\delta T a}$	0	$Q_1 = (1-\epsilon^{-\gamma T a})$
	2	0	0	$R_2 = \epsilon^{-\zeta T b}$	$Q_2 = 1 - \epsilon^{-\zeta T b}$
	1	0	0	0	1

γ = STORAGE FAILURE RATE
δ = UNTESTABLE PARTS
ζ = TESTABLE PARTS
T = TIME OF STORAGE

DEVELOPMENT OF TEST MATRIX

D = PROBABILITY OF NOT CAUSING A
    DETECTABLE FAILURE

N = PROBABILITY OF NOT CAUSING AN
    UNDETECTABLE FAILURE

P = PROBABILITY OF DETECTING A
    DETECTABLE FAILURE

		FINISH IN STATE			
		1 GOOD	2 DETECTABLE	3 UNDETECTABLE	4 DET. & UNDET.
START IN STATE	1 GOOD	DN	$(1-D)\,N$	$(1-N)\,D$	$(1-\sum) =$ $1-D-N+ND$
	2 DETECT.	DNP	$N(1-PD)$	$PD(1-N)$	$(1-\sum) =$ $1-N-DP-DPN$
	3 UNDET.	0	0	D	$1-D$
	4 DET. & UNDET.	0	0	PD	$1-PD$

EXAMPLES OF MATRIX CONSTRUCTION

START IN STATE 1, FINISH IN STATE 3, OR

GOOD AT START, UNDETECTABLE FAILURE AT FINISH

$$= D\,(1-N)$$
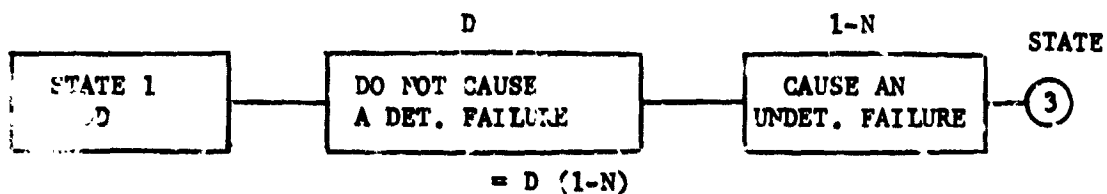
START IN STATE 2, FINISH IN STATE 1 OR

DETECTABLE FAILURE AT START, GOOD AT FINISH.

STATE 2	P	N	D
DETECTABLE FAILURE	DETECT DET. FAILURE	DO NOT CAUSE UNDET. FAILURE	DO NOT CAUSE DET. FAILURE

STATE ①

$$= PND$$

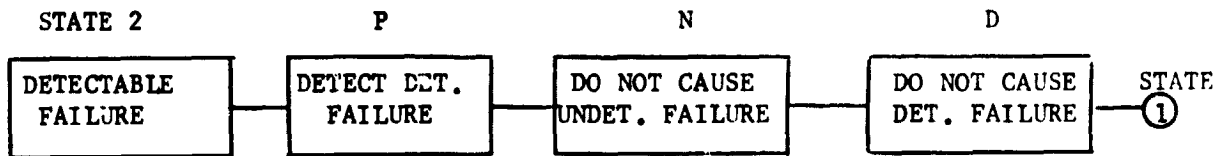START IN STATE 2, FINISH IN STATE 2 OR

DETECTABLE FAILURE AT START, DETECTABLE

FAILURE AT FINISH.

STATE 2 DETECTABLE FAILURE

1-P DO NOT DET. DET. FAILURE — N DO NOT CAUSE UNDET. FAILURE — STATE ②

P DETECT. DET. FAILURE — N DO NOT CAUSE UNDET. FAILURE — 1-D CAUSE NEW DET. FAIL. — STATE ②
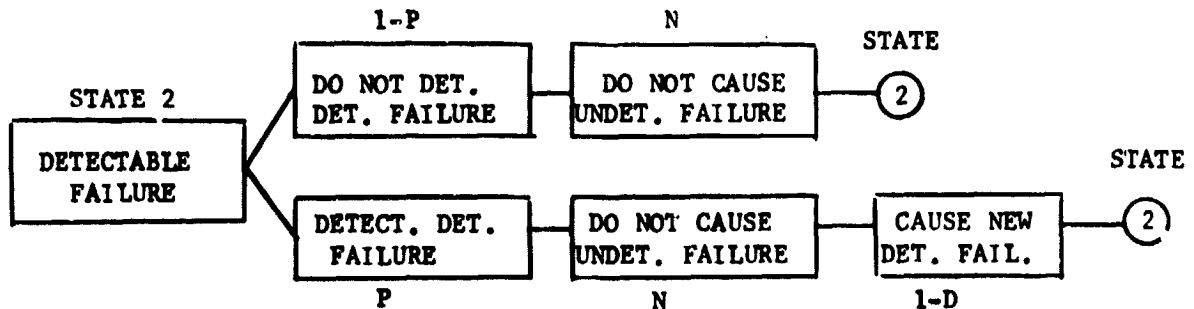
$$= (1-P)N + PN(1-D) = N(1-PD)$$

NOTE: SINCE ALL ROW SUMS IN A STOCHASTIC MATRIX = 1,

THE VALUE OF THE ENTRY IN COLUMN 4 OF EACH ROW

IS CALCULATED BY SUBTRACTING THE SUM OF ENTRIES

1, 2, and 3 OF EACH ROW FROM ONE.

FAILURE RATE 0.1×10^{-9}

.994502 AFTER TEST #1
.993756 AFTER TEST #2
.993429 AFTER STORAGE
.992342 AFTER TEST #3
.991825 AFTER TEST #4
.991326 AFTER TEST #5

AVAILABILITY

ACCUMULATED TIME — MONTHS

118
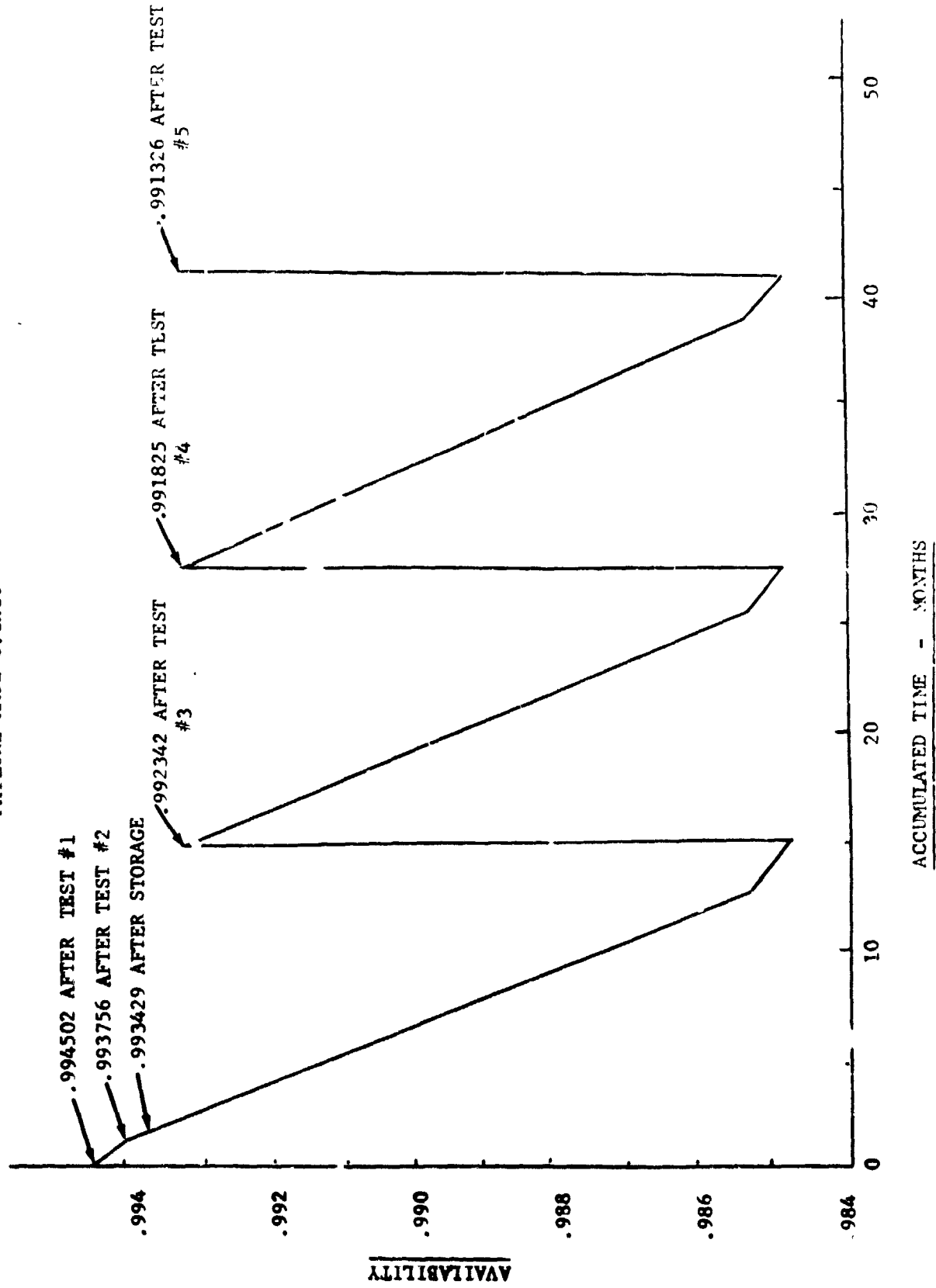
BIBLIOGRAPHY

I.   Systems Reliability, Systems Engineering, and Measures of
     Reliability Effectiveness

     "Reliability of Military Electronic Equipment," AGREE Report,
     U. S. Government Printing Office, Wash., D.C., June, 1957

     "Reliability Monitoring Program for Use in the Design Development,
     and Production of Air Weapons Systems and Support Systems," U.S.
     Air Force Specification Bulletin No. 506, May 11, 1959

     Gryna, F. M., McAfie, N. J., Ryerson, C. M., and Zwerling, S.
     (editors), Reliability Training Text, IRE-ASQC, March, 1960

     Henney, K. (editor), Reliability Factors For Ground Electronic
     Equipment, McGraw-Hill Book Co., Inc., New York, 1956

     Hall, A. D., A Methodology For Systems Engineering, D. Van
     Nostrand Co., Inc., Princeton, N.J., 1962

     Goode, H. H. and Machol, R. E., System Engineering, McGraw-Hill
     Book Co., Inc., New York, 1957

     Flagle, C. D., Huggins, W. H., and Ray, R. H. (editors), Systems
     Engineering and Operations Research, Johns Hopkins Press,
     Baltimore, Md., 1960

     Karr, H. W. and Geisler, M. A., "The Design of Military Supply
     Tables for Space Parts," Operations Research, Vol. 4, 1956

     Kahn, H. and Mann I., "Techniques of System Analysis," Rand
     Corp., Research Memorandum RM-1829-1, June, 1957

     Sterling, L. J., "Decision Making in Weapons Development,"
     Harvard Business Review, Jan.-Feb., 1958

     Hitch, C. J. and McKean, R., The Economics of Defense in the
     Nuclear Age, Harvard University Press, Cambridge, Mass., 1960

     Novick, D., "System and Total Force Cost Analysis," Rand Corp.,
     Research Memo. RM-2695, April 1961

Luce, D. R. and Raiffa, H., Games and Decisions: Introduction
and Critical Survey, John Wiley & Sons, Inc., New York, 1957

Welker, E. L. and Horne, R. C., "Concepts Associated with
System Effectiveness," ARINC Monograph No. 9, ARINC Research
Corp., July, 1960

Reliability Notebook, Rome Air Development Center, Supplement 2,
December 31, 1961

Myers, R. H., "Mathematical and Engineering Aspects of Satellite
Redundancy," Hughes Aircraft Company Report R-60-3, September 1,
1960, p. 24

Mace, A. E., "Guide to Statistical Test Design," Guidance
Document Number 2, Volume 1, Battelle Memorial Institute.
February 15, 1961, pp. 170-174.

Hald, A., Statistical Tables and Formulas, John Wiley and Sons,
New York, 1958

Mace, A. E., "Guide to Statistical Test Design," Guidance
Document Number 2, Volume 1, Battelle Memorial Institute,
February 15, 1961, p. A-3

II. Markov Processes, Probability Theory Optimization of Reliability Parameters, Availability, and Allocation of Redundancy

Feller, W., An Introduction to Probability Theory and Its Applications, Vol. 1, Second Edition, John Wiley & Sons, Inc., New York, 1957

Parzan, E., Modern Probability Theory and Its Applications, John Wiley & Sons, Inc., New York, 1960

Bharucha-Ried, A. T., Elements of the Theory of Markov-Processes and Their Applications, McGraw-Hill Book Co., Inc., New York, 1953

Doob, J. L., Stochastic Processes, John Wiley & Sons, Inc., New York, 1953

Loeve, M., Probability Theory, D. Van Nostrand Co., Inc., Princeton, N. J., 1955

Tables of the Binomial Probability Distribution, National Bureau of Standards, U.S. Government Printing Office, Wash., D.C., 1949

Cramer, H., Mathematical Methods of Statistics, Princeton University Press, Princeton, N. J., 1946

Mood, A. M., Introduction to the Theory of Statistics, McGraw-Hill Book Co., Inc., New York, 1950

Dynkin, E. B., Theory of Markov Processes translated from Russian by Brown, D. D., Prentice-Hall, Inc., Englewood Cliffs, N. J., 1961

Kemeny, J. G. and Snell, J. L., Finite Markov Chains, D. Van Nostrand Co., Inc., Princeton, N. J., 1960

Bellman, R., Introduction to Matrix Analysis, McGraw-Hill Book Co., Inc., New York, 1960

Frazer, W. A., Duncan, N. J., and Collar, A. R., Elementary Matrices and Some Applications to Dynamics and Differential Equations, Cambridge University Press, London, 1938

Chernoff H. and Moses L. E., Elementary Decision Theory, John Wiley & Sons, Inc., New York, 1959

C

Sittler, R. W., "Systems Analyses of Discrete Markov Processes,"
IRE Transactions CT-3, No. 4, Dec., 1956

Weiss, L., Statistical Decision Theory, McGraw-Hill Book Co.,
Inc., New York, 1961

Cahart, R. R., "A Survey of the Current Status of the Reliability
Problem," Rand Corp., Research Memo. RM-1131, August 14, 1953

Boodman, D. M., "The Reliability Airborne Radar Equipment,"
Journal of the Operational Research Society of America, Vol. I,
No. 2, Feb., 1953

Cox, D. R. and Smith, W. L., "On the Superposition of Renewal
Processes," Biometrika, Vol. 41, 1954

Billingsley, P., "Statistical Methods in Markov Chains," The
Annals of Mathematical Statistics, Vol. 32, No. 1, March, 1961

Flehinger, B. J. and Lewis, P. A. "Two-Parameter Lifetime
Distributions for Reliability Studies of Renewal Processes,"
IBM Journal of Research and Development, Vol. 3, No. 1, Jan.,
1959

Widder, D. V., The Laplace Transformer, Princeton University Press,
Princeton, N. J., 1946

Takacs, L., Stochastic Processes, Methuen & Co., Ltd., London,
and John Wiley & Sons, Inc., New York, 1960

Moore, E. F. and Shannon, C. E., "Reliable Circuits Using Less
Reliable Relays," Journal of the Franklin Institute, Vol. 262,
Sept., 1956

Von Newmann, J., "Probabilistic Logics and Synthesis of Reliable
Organisms from Unreliable Components," in Annals of Mathematical
Studies, No. 34, Princeton University Press, Princeton, N. J.,
1956

Birnbaum, Z. W., Esary, D. J., and Saunders, S. C., "Multi-
Component Systems and Structures and their Reliability,"
Technometrics, Vol. 3, No. 1, Feb., 1961

Cox, D. R., "The Analysis of Non-Markovian Stochastic Processes
by the Inclusion of upplementary Variables," Proceedings of
the Cambridge Philosophical Society, 51, Part 3, July, 1955

Lipp, J. P., "Topology of Switching Elements Versus Reliability," IRE Transactions on Reliability and Quality Control, PGRQC-10, June, 1957

Moskowitz, F., "The Analysis of Redundant Networks," Communications and Electronics, No. 39, Nov., 1958

Mine, H., "Reliability of Physical Systems," Transactions of the 1959 Internation Symposium on Circuit and Information Theory, IT-5, Special Supplement, May, 1959

Lofgren, L., "Automata of High Complexity and Methods for Increasing Their Reliability by Redundancy," Information and Control, Vol. 1, No. 2, 1958

Maitra, K. K., "Synthesis of Reliable Automata and Stable Neutral Nets," Proceedings of the Bionics Symposium, WADD Tech. Report 60-600, Sept., 1960

Flehinger, B. J., "Reliability Improvement Through Redundancy at Various System Levels," IBM Journal of Research and Development, Vol. 2, April, 1958

Balaban, H., "Some Effects of Redundancy on System Reliability," Sixth National Symposium on Reliability and Quality Control in Electronics, Wash., D. C., Jan., 1960

Herd, G. R., "Estimation of Reliability Functions," ARINC Monograph No. 3, ARINC Research Corp., May, 1956

Barlow, R., "Applications of Semi-Markov Processes to Counter and Reliability Problems," Stanford Technical Report No. 57 Contact No. N6onr25140, 1960.

Barlow, R. E., and Hunter, L. C., "Reliability Analysis of a One Unit System," Journal of the Operations Research Society of America, Vol. 9, No. 2, March-April, 1961

Ryswick, R. and Weiss, G., "Tables of the Incomplete Gama Function of Integral Order," NAVWEPS Report 7292, U.S. Naval Ordnance Lab., White Oak, Md., Nov., 1960

Karlin, S. and McGregor, J., "Coincident Properties of Birth and Death Processes," Stanford Technical Report No. 9, Contract Nonr-255(28), 1958

Kneale, S. G., "Reliability of Parallel Systems with Repair and Switching," Seventh National Symposium on Reliability and Quality Control in Electronics, Philadelphia, Pa., Jan., 1961

Derman, C., "Some Asymptotic Distribution Theory for Markov Chains with a Denumerable Number of States," Biometrika, Vol. 43, 1956

Whittle, P., "Some Distributions and Moment Formulae for the Markov Chain," Journal of the Royal Statistical Society, Series B, Vol. 17, 1955

Syski, R., Introduction to Congestion Theory in Telephone Systems, Oliver and Boyd, London, 1960

Barlow, R. and Hunter, L., "Mathematical Models for System Reliability," The Sylvania Technologist, Vol. XIII, Nos. 1 and 2, Jan, and April, 1960

Drenick, R. F., "Mathematical Aspects of the Reliability Problem," Journal of the Society for Industrial and Applied Mathematics, Vol. 8, 1960

Epstein, B. and Hosford, J., "Reliability of Some Two-Unit Redundant Systems," Sixth National Symposium on Reliability and Quality Control in Electronics, Wash., D. C., Jan., 1960

Smith, W. L., "Renewal Theory and Its Ramifications," Journal of the Royal Statistical Society, Series B, Vol. 20, No. 2, 1958

Cox, D. R. and Smith, W. L., Queues, Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1961

Morse, P. M., "Dynamics of Operational Systems: Markov and Queuing Processes," in Ackoff, R. L. (editor), Progress in Operations Research, Vol. 1, John Wiley & Sons, Inc., New York, 1961

Karlin, S. and McGregor, J., "The Differential Equations of Birth and Death Processes, and the Stieltjes Moment Problem," Transactions of the American Mathematical Society, Vol. 85, 1957

Morse, P. M. M., Queues, Inventories and Maintenance, John Wiley and Sons, Inc., New York, 1958

Saaty, T. L., "Resume of Useful Formulas in Queuing Theory," Journal of the Operations Research Society of America, Vol. 5, No. 2, 1957

Huggins, W. H., "Representation and Analysis of Signals, Part V: Flow-Graph Representation of System Dynamics," Johns Hopkins University Report TN-59-971, Contract AF 19(604)-1941, Nov., 1959

Weiss, G. H., "On a Semi-Markovian Process with a Particular Application to Reliability Theory," NAVORD Report 435, U.S. Naval Ordnance Laboratory, White Oak, Md., Oct., 1956

Howard, R., Dynamic Programming and Markov Processes, MIT Technology Press, Cambridge, Mass., 1960

Bellman, R., Dynamic Programming, Princeton University Press, Princeton, N. J., 1957

Rutenberg, Y. H., "Sequential Decision Models" Case Institute of Technology, April, 1961

Kettelle, J., "Least Cost Allocation of Redundancy," Seventeenth National Meeting of the Operations Research Society of America

Manne, A. S., "Linear Programming and Sequential Decisions," Management Science, Vol. 6, 1960

Dennis, J. E., Mathematical Programming and Electrical Networks, John Wiley and Sons, 1960

Riley, V., and Gass, S. I., Linear Programming and Associated Techniques: A comprehensive Bibliography on Linear, Non-Linear and Dynamic Programming, The John Hopkins Press, Baltimore, Md., 1958

Vajda, S., Mathematical Programming, Addison-Wesley Pub. Co., Inc., Reading, Mass., 1961

Bellman, R., Holland, J., and Kalaba, R., "On the Application of Dynamic Programming to the Synthesis of Logical Systems," Rand Corp., Paper P-1551, 1958

Dreyfus, S., "Dynamic Programming" in Ackoff, R. (editor), Progress in Operations Research, Vol. 1, John Wiley & Sons, Inc., New York, 1961

Crow, E. L., F. A. Davis, and M. W. Maxfield, Statistics Manual, Dover Publications, New York, 1960

Dixon, W. J., and F. J. Massey, Jr., Introduction to Statistical Analysis, McGraw-Hill Book Co., New York, 1951

Frazer, D. A. S., Statistics, An Introduction, John Wiley and Sons, New York, 1958

Hoel, P. G., Introduction to Mathematical Statistics, John Wiley and Sons, New York, 1952

Sprowls, R. C., Elementary Statistics for Students of Social Science and Business, McGraw-Hill Book Co., New York, 1955

Shewhart, W. A., Economic Control of Quality of Manufactured Product, Van Nostrand Co., Princeton, New Jersey, 1931

Cramer, H., Mathematical Methods of Statistics, Princeton University Press, Princeton, New Jersey, 1946

Feller, W., An Introduction to Probability Theory and Its Applications, 2nd Ed., John Wiley and Sons, New York, 1957

Sandler, G. H., System Reliability Engineering, Prentice-Hall, Englewood Cliffs, New Jersey, 1963

Reliability Notebook, Rome Air Development Center, Supplement 2, December 31, 1961

Bazovsky, I., Reliability Theory and Practice, Prentice-Hall, Englewood Cliffs, New Jersey, 1961

Aroian, L. A., "Maximization of the Reliability of a Complex System by the Use of Items in Sequence," Space Technology Laboratories Report STL/TM/60-0000-00034, 1960

Wong, K., "Use of Redundancy,". Hughes Aircraft Company Reliability Guide, Section 7.0, August 18, 1960

Myers, R. H., "Which Road to Satellite Reliability," Hughes Aircraft Company Report R-60-2, September 1, 1960

Hasford, J., "Measures of Dependability," J. Operations Research Soc. Amer., January-February 1960

Peck, L. G., and R. N. Hazelwood, Finite Queuing Tables, John Wiley and Sons, New York, 1958

Ryerson, C. M., "Reliability and Product Assurance" in D. W. Chorafas, Statistical Processes and Reliability Engineering, Van Nostrand Co., Princeton, New Jersey, 1960

III.   Reliability Models of Maintained and Non-Maintained Systems, and System Maintenance Policies

Moore, E. F. and Shannon, C. E., "Reliable Circuits Using Less Reliable Relays," Journal of the Franklin Institute, Vol. 262, September 1956

Von Neumann, J., "Probabilistic Logics and Syntheses of Reliable Organisms from Unreliable Components," in Annals of Mathematical Studies No. 34, Princeton University Press, Princeton, New Jersey, 1956

Moskowitz, F. and McLean, J. B., "Some Reliability Aspects of System Design," IRE Transactions on Reliability and Quality Control, PGRQC-8, September 1956

Bellman, R. and Dreyfus, S., "Dynamic Programming and the Reliability of Multi-Component Devices," Journal of the Operations Research Society of America, Vol. 6, No. 2, March-April, 1958

Sasaki, M., "A Simplified Method of Obtaining Highest System Reliability," Proceedings of the Eighth National Symposium on Reliability and Quality Control in Electronics, Wash., D. C., January, 1962

Gordon, R., "Optimum Component Redundancy for Maximum System Reliability," Journal of the Operations Research Society of America, Vol. 5, No. 2, April, 1957

Flehinger, B. J., "Reliability Improvement Through Redundancy at Various System Levels," IRE National Convention Record, Part 6, March, 1958

Barlow, R. E. and Hunter, L. C., "Mathematical Models For System Reliability," The Sylvania Technologist, Vol. XIII, Nos. 1 and 2, January and April 1960

Hall, K. M. and McDonald, R. H., "Improving System Reliability," Proceedings of the Seventh National Symposium on Reliability and Quality Control in Electronics, Philadelphia, Pa., January, 1961

Kamins, M., "Determining Checkout Intervals for Systems Subject to Random Failures," Rand Corp. Research Memorandum, RM-2578, June 1960

Firstman, S. I. and Voosen, B. J., "Missle Pre-launch Confidence
Checkout: Content and Equipment Design Criteria," Rand Corp.
Research Memorandum, RM-2485, February, 1960

Morse, P. M. M., Queues, Inventories and Maintenance, John
Wiley & Sons, Inc., New York, 1958

Barlow, R. and Hunter L., "Optimum Preventive Maintenance
Policies," Journal of the Operations Research Society of
America, Vol. 8, 1960

Welker, E. L. and Bradley, C. E., "The Dollar Value of Improved
Reliability," Proceedings of the Seventh National Symposium on
Reliability and Quality Control in Electronics, Philadelphia,
Pa., January, 1961

Brender, D. M., "The Statistical Dynamics of Preventive Replace-
ments," Convention Record, WESCON, August, 1958

Barlow, R. E. and Proschan, F., "Planned Replacement," D1-82-0102,
Boeing Scientific Research Laboratories, April, 1961

Truelove, A. J., "Strategic Reliability and Preventive Maintenance,"
Journal of the Operations Research Society of America, Vol. 9,
No. 1, January-February, 1961

Hall, K. M., "System Maintainability," Proceedings of the Eighth
National Symposium on Reliability and Quality Control in
Electronics, Wash., D. C., January, 1962

"Reliability Stress. Analysis for Electronic Equipment,"
Technical Report TR-59-416-1, R.C.A., Camden, New Jersey,
January, 1959

Earles, D. R., "Reliability Application and Analysis Guide,"
The Martin Company, Denver, Colorado, September 1960

"Handbook for the Prediction of Shipboard and Shore Electronic
Equipment Reliability", TR-133, NAVSHIP 93820, Vitro Laboratories,
April 1961

Bazovsky, I., Reliability: Theory and Practice, Prentice-Hall,
Inc., Englewood Cliffs, New Jersey, 1961

Loyd, D. K. and Lipow, M., Reliability: Management, Methods, and
Mathematics, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1961

IV.   Military Reliability and Availability Requirements

"Reliability of Military Electronic Equipment," Report by
Advisory Group on Reliability of Electronic Equipment, Office
of the Assistant Secretary of Defense (Research and Engineering)
June 4, 1957

Connor, J. A., "Compo nt Application Stress Analysis,"
Proceedings, Electronics Components Conference, 1958

Ryerson, C. M., "RCA Reliability Program and Long Range
Objectives," RCA, Camden, New Jersey, March 15, 1955

Bryson, H. C., "The Training Aspects of Design Reviews,"
Proceedings Fourth National Symposium on Reliability and
Quality Control, January 6-8, 1958

RADC Reliability Notebook, Section 8, Rome Air Development
Command, December 31, 1961

Mil Handbook 217 (WEPS), December 31, 1961. ARINC Research
Corporation, Publication No. 101-17-145, p. 16

H. Smith et al, "A Prediction of AN/GRC-27 Reliability," Rome
Air Development Center Report TN-58-18, August 26, 1957

Military Standard 756 (WEPS), "Reliability of Weapons Systems,
Procedures for Prediction and Reporting Prediction of,"
October 3, 1961

Myers, R. H., "Mathematical and Engineering Aspects of Satellite
Redundancy," Hughes Aircraft Company Report R-60-3, September 1,
1960

Jerencsik, A. P. and W. T. Sackett, Jr., "The Step-Stress Method
of Obtaining Short-Term Life Ratings on Electronic Components,"
Preceedings, National Electronics Conference, Vol. 8, 1952

Reeves, T. C., "A Reliability Approach to Thermal Design and
Evaluation," Electrical Manufacturing, February, 1957

Taylor, N. H., "Designing for Reliability," Proceeding IRE,
June, 1957

Connor, J. A., "Prediction of Reliability," Proceedings, Sixth
National Symposium on Reliability and Quality Control, January, 1960

130

Department of Defense Technical Report, TR-3, "Sampling
Procedures and Tables for Life and Reliability Testing Based
on the Weibull Distribution," Washington, D. C., September 10,
1961

Davidson, K. W., and E. E. Johnson, "Reliability Evaluation
by Application Derating," Proceedings of the National
Symposium on Reliability and Quality Control, January, 1962,
p. 215.

Morrison, H. E., and G. W. Wood, "Application of Linear
Discriminant Analysis as a Screening Technique," Hughes
Aircraft Company, Unpublished Report

Thomas, R. E., A. R. Fish, H. M. Braner, and W. E. Chapin,
Development of a Methodology for Screening Electronic Parts
by Using Linear Discriminants," Battelle Memorial Institute
Electronic Component Reliability Center

Enquist, R. D., R. B. Feuchtbaum, F. Z. Keister, "The Folklore
of Encapsulation," Insulation Magazine, May 1961, Lake Publishing
Corp., Libertyville, Illinois

Myers, R. H., "Which Road to Satellite Reliability," Hughes
Aircraft Company Report R-60-2, September 1, 1960, p. 12

Raytheon Company Industrial Components Division Report,
"Reliability Assurance Program," Figure 1, p. 19

Daniels, A. N., "Micro-Bearing Abstract," New Hampshire Ball
Bearings, Inc., Teterboro, New Hampshire, September, 1958

Hyatt Roller Bearings, Catalog 150, Hyatt Bearings Division,
General Motors Corporation, Harrison, New Jersey

Luebbert, Captain W. F., "Literature Guide on Failure Control
and Reliability," Stanford Electronic Laboratories, December,
1956

"Reliability of Military Electronic Equipment," AGREE Report,
Washington, D. C., U. S. Government Printing Office, June,
1957

Exhibit 60-11A, AFBM, General Reliability Specification for
WS107A-2, Titan 2, October 10, 1960

Handbook H-108 (DOD), "Sampling Procedures and Tables for Life
and Reliability Testing," April 29, 1960

MIL-STD-441, "Reliability of Military Electronic Equipment,"
June 20, 1958

Specification Bulletin 506 (USAF), "Reliability Monitoring
Program for Use in the Design, Development, and Production
of Air Weapons Systems and Support Systems," May 11, 1959

MIL-STD-756, "Procedures for Prediction and Reporting Prediction
of Reliability of Weapon Systems," October 3, 1961

MIL-R-19610 (AER), "Reliability of Production Electronic
Equipment, General Specification for," September 15, 1956

MIL-R-22256 (AER), "Reliability Requirements for Design of
Electronic Equipment or Systems," November 5, 1959

MIL-R-22732 (Ships), "Reliability Requirements for Shipboard
and Ground Electronic Equipment," March 10, 1961

MIL-R-26474 (USAF), "Reliability Requirements for Production
Ground Electronic Equipment," June 10, 1959

MIL-R-26484 (USAF), "Reliability Requirements for Development
of Electronic Subsystems or Equipment," June 2, 1958

MIL-R-26667A (USAF), "Reliability and Longevity Requirements,
Electronic Equip.. nt, General Specification for," June 2, 1959

MIL-R-27070 (USAF), "Reliability Requirements for Development
of Ground Electronic Equipment," March 25, 1960

MIL-R-27173 (USAF), "Reliability Requirements for Electronic
Ground Checkout Equipment," July 6, 1959

MIL-R-27542 (USAF), "Reliability Program Requirements for
Aerospace Systems, Subsystems, and Equipments," June 28, 1961

NPC 250-1 (NASA), "Reliability Program Provisions for Space
System Contractors," May 13, 1963

Sylvania, Design Review Manual, Electronic Systems Division, Mountain View, California. Note: The bulk of the material in this manual was developed at Sylvania by R. S. Cazanjian with the invaluable support of David Ehrenpreis, Consulting Engineer, who provided much of the original thought and effort.

MIL-STD-439B, "The Electronic Circuits," May 15, 1961, Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

NAVWEPS 16-1-519, "Handbook of Preferred Circuits, Navy Aeronautical Electronic Equipment," National Bureau of Standards, Department of Commerce for Bureau of NavWeps, Department of the Navy, September 1, 1955, Supplements 1, 2 and 3

Hurley, R., Junction Transistor Electronics, John Wiley and Sons, New York, 1958

MacDonald, F. R., "Component-Board Vibration Nomograph," Electronics Equipment Engineering, Vol. 6, No. 5 (May 1958), p. 58

Timoshenko, R., Strength of Materials, 3rd ed., Van Nostrand, New York, 1955

MIL-Handbook-5, "Strength of Metal, Aircraft Elements," March, 1959, Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

Navy Reliability Design Handbook, U. S. Navy, Bureau of Ships, Washington 25, D. C., July 15, 1961

Toark, R. J., Formulas for Stress and Strain, 3rd ed., McGraw-Hill, New York, 1954

MIL-I-6181D, "Interference Control Requirements, Aircraft Equipment, November 25, 1959, U. S. Government Printing Office

Reliability Notebook, Rome Air Development Center, Supplement 2, December 31, 1961, Section 7

MIL-R-27542 (USAF), Reliability Program Requirements for Aerospace Systems, Subsystems and Equipments, June 28, 1961

MIL-STD-105C, Sampling Procedures and Tables for Inspection by Attributes, July 18, 1961

MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective, July 11, 1957

Herd, G. R. et al, "Electronics Reliability," ARINC Monograph No. 2, May 25, 1955, pp. 36-37

Doob, J. L., "Renewal Theory from the Point of View of the Theory of Probability," Trans. Amer. Math. Soc., Vol. 23, 1948

Epstein, B., "Statistical Techniques In Life Testing," U. S. Department of Commerce, Office of Technical Services Report PB 171580, pp. 1-102

"Reliability Training Text," Institute of Radio Engineers, 2nd Ed., March, 1960, p. 147

"Air Force Reliability Assurance Program Progress Report No. 1," Aeronautical Radio, Inc., February 15, 1956

Molina, E. C., Poisson's Exponential Binomial Limit Tables, D. Van Nostrand Company, New York, 1942

Brownlee, K. A., Industrial Experimentation, Chemical Publishing Company, Inc., New York, 1949, p. 47

Brownlee, K. A., Industrial Experimentation, Chemical Publishing Company, Inc., New York, 1949, p. 40

Department of Defense Technical Report, TR-3 "Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution," Washington, D. C., September 30, 1961

Department of Defense Handbook H108, "Sampling Procedures and Tables for Life and Reliability Testing," Washington, D. C., April 29, 1960

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, p. 2.3

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, p. 2.17

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, pp. 2.16 and 2.17

U. S. National Bureau of Standards, "Tables of the Binomial Probability Distribution," U. S. Government Printing Office, Washington, D. C., 1949

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, pp. 2.24 - 2.26

Burington, R. S., Handbook of Mathematical Tables and Formulas, 2nd Ed., Handbook Publishers, Inc., Sandusky, Ohio, 1940

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, pp. 2.1 - 2.3

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, pp. 2.13 and 2.14

Epstein, B., "Statistical Techniques in Life Testing," U. S. Dept. of Commerce, Office of Technical Services Report PB 171580, pp. 2.17 - 2.19

Document Number 2, Vol. 1, Battelle Memorial Institute, February 15, 1961, pp. 191 - 194

"ASTM Manual on Quality Control of Materials," American Society for Testing Materials, 1951, p. 28

Ashby, R. M., "A Research and Development View of High Reliability," Transactions of the 1961 Reliability Symposium, Los Angeles, California, Sponsored by the American Society for Quality Control

Shwop, J. E., and H. J. Sullivan, Editors, Semiconductor Reliability, Engineering Publishers, Elizabeth, New Jersey, 1961, p. 20

Handbook of Methods of Cooling Air Force Ground Electronic Equipment, Cornell Aeronautical Laboratories, RADC-TR-58-128, ASTIA AD-148907, 1958

Cazanjian, R. S., "A Unique Approach to Reliability Design Reviews," 6th Joint Military-Industry Guided Missile Reliability Symposium, Vol. 1 (February 1960), pp. 1-94

D

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, pp. 2.34 - 2.69

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, pp. 2.24 - 2.27

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, pp. 2.15 - 2.17

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, p. 2.26

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, p. 2.3

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, p. 2.17

Epstein, B., "Statistical Techniques in Life Testing," U. S.
Dept. of Commerce, Office of Technical Services Report
PB 171580, pp. 2.26 - 2.27

Myers, R. H., "Reliability Testing Sampling Procedures and
Tables Based on Acceptable Reliability Levels," Hughes
Aircraft Company Report R-56-5A, October 10, 1956

MIL-R-26667A (USAF), Reliability and Longevity Requirements
Electronic Equipment, General Specification, June 2, 1959

Department of Defense, "Reliability of Military Electronic
Equipment" (AGREE Report), U. S. Government Printing Office,
Washington, D. C., June, 1958

Golde, P. J., "How RCA Meets AGREE Test Requirements,"
Proceedings 15th American Society for Quality Control
Convention, Philadelphia, Pa., June 5-7, 1961, p. 204

Holohan, J., "Quantitative Approach Gets Reliability Results,"
Space/Aeronautics, April 1961, p. 125

AMCP 74-1A, Reliability Evaluation Procedures for Pilot
Production and Production, February 1, 1961

V.   <u>Reliability Practices, Reliability Production and Testing</u>

Department of Defense, Military Specification MIL-STD-210A, "Climatic Extremes for Military Equipment," August 2, 1957

Fletcher, R. D., "A Relation between Maximum Observed Point and Areal Rainfall Values," Transactions, American Geophysical Union, Vol. 31, pp. 344-348, June 1950

U. S. Air Force Specification Bulletin No. 523, "Space Environmental Criteria for Aerospace Vehicles," November 28, 1960

Tong, K. N., Theory of Mechanical Vibrations, John Wiley and Sons, 1960

Thomson, W. T., Mechanical Vibrations, 2nd ed., Prentice-Hall, Englewood Cliffs, New Jersey. 1953

Den Hartog, J. P., Mechanical Vibrations, 4th ed., McGraw-Hill, New York, 1956

Office of Secretary of Defense, Bulletins of Symposium on Shock, Vibration and Associated Environments, Bulletins February 24, 1957; December 25, 1957; June 27, 1958; August 28, 1960; June 29, 1961

Crandall, S. H. (editor), Random Vibration, Massachusetts Institute of Technology, The Technology Press, 1958

Harris, C. M., and C. E. Crede, Shock and Vibration Handbook, Vols. 1, 2 and 3, McGraw-Hill, New York, 1961

Lehr, S. N., V. J. Tronolone, and P. V. Horton, "Equipment Design Considerations for Space Environment," Space Technology Laboratories Technical Report STL/TR-60-0000-09224, September, 1960

Minzer, R. A., and W. S. Ripley, "The ARDC Model Atmosphere, 1956," Air Force Cambridge Research Center, ARDC, December, 1956

Minzer, R. A., K. S. W., Champion, and H. L. Pond, "The ARDC Model Atmosphere, 1959," Air Force Cambridge Research Center, ARDC, August 1959

Brueschke, E. E., "The Effects of the Space Environment on
Electronic Components and Materials, Parts I and II," Hughes
Aircraft Company Special Research Study, SRS-369, June 15, 1960

Yarwood, J., High Vacuum Technique, John Wiley and Sons, 2nd ed.,
revised 1955

Scott, R. B., Cryogenic Engineering, Van Nostrand, Princeton,
New Jersey, 1959

Valley, G. E., Jr., and H. Wallman, "Vacuum Tube Amplifiers,"
Vol. 18, M.I.T. Radiation Laboratory Series, McGraw-Hill,
New York, 1948

U. S. Air Force Specification MIL-E-5272C, "General Specification
for Environmental Test, Aeronautical and Assoicated Equipment,"
1959

U. S. Department of Commerce, "Statistical Abstract of the U. S.,"
Annual Editions

Greathouse, G. A., and C. J. Wessel, Deterioration of Materials,
Reinhold, New York, 1954

Sissenwine, N. and A. Court, "Climatic Extremes for Military
Equipment," Report No. 146 OQMG, Department of Army, November, 1951

Riley, F. C., and P. F. Walker, and F. W. Schwartz, "General
Bibliography and Index of Environmental Criteria for Aircraft
and Airborne Equipment" (unclassified title), WADC Technical
Report, June 1955

Editorial Report, "Environment," Ground Support Engineering,
April/May 1961, p. 32

Frederic, M., and E. Fornario, "Correlation of Temperature-
Humidity Tests," WADC Technical Report 53-107, Part 6 Summary
Report, December 1953

Carrier Corporation, "Psychrometric Chart, Normal Temperatures,"
Copyright 1946

Frederick Research Corporation, Bethesda, Maryland, Contract
AF 33(038)23341, "Design Techniques for Interference-Free
Operation of Airborne Electronic Equipment," WADC Technical
Report, 1952

MIL-STD-441, "Reliability of Military Electronic Equipment,"
June 20, 1958

MIL-R-27542 (USAF), "Reliability Program Requirements for
Aerospace Systems, Subsystems, and Equipments," June 28, 1961

MIL-R-22256 (AER), "Reliability Requirements for Design of
Electronic Equipment or Systems," November 20, 1959

MIL-R-22732 (SHIPS), "Reliability Requirements for Shipboard
and Ground Electronic Equipment," March 10, 1961

MIL-R-26474 (USAF), "Reliability Requirements for Production
Ground Electronic Equipment," June 10, 1959

MIL-R-26484 (USAF), "Reliability Requirements for Development
of Electronic Subsystems or Equipment," April 18, 1960

MIL-R-26667A, "Reliability and Longevity Requirements,
Electronic Equipment, General Specification For, June 2,
1959

MIL-R-27173 (USAF), "Reliability Requirements for Electronic
Ground Checkout Equipment," July 6, 1959

Air Force Manual 66-1, "Maintenance Management," July 1, 1961

Hughes Aircraft Company Report R-60-4, "Department of Defense
Deficiency and Failure Reporting Code Review Panel Final Report,"
October 15, 1960

Ryerson, C. M., "Reliability Testing Theory Based on the
Poisson Distribution," Proceedings 4th National Symposium
Reliability and Quality Control, Washington, D. C., January
6-8, 1958

Molina, E. C., Poisson's Exponential Binomial Limit Tables,
Van Nostrand, Princeton, New Jersey, 1942

RETMA Report, "A General Guide for Technical Reporting of
Electronic Systems Reliability Measurements," December, 1956