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THE INFLUENCE OF A CROSSFLOW ON JET NOISE

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16. Abstract

The acoustic field which is generated by a circular jet in the presence of a low speed crossflow is calculated. The solution is obtained for a quadrupole source located in a uniformly moving acoustic medium. Ribner's work on straight jets is used to model the flow in the turbulent crossflow jet. The movement of the acoustic medium results in the acoustic intensity being greater upstream of the jet flow than at an equal distance downstream. A summary is also presented of the analysis for the acoustic intensity of a crossflow jet which is in motion through a stationary acoustic medium. This analysis is applicable to the noise generated by the lift jet of a V/STOL aircraft under take-off or landing conditions. The results of these two configurations provide a comparision of the directional distribution of acoustic intensity measured in a wind tunnel test of a crossflow jet and in a corresponding flight test.

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Summary

In this report, an analysis is presented of the acoustic field which is generated by a circular jet in a weak crossflow. The main portion of this report contains the analysis for the acoustic intensity which would be measured from the jet flow in an ideal wind tunnel (i.e., a wind tunnel with no reflecting walls). For this situation, the movement of the acoustic medium results in the acoustic intensity being greater upstream of the jet flow than at an equal distance downstream. The Appendix of this report contains a summary of the analysis of the acoustic intensity of a crossflow jet in motion such as would be generated by the lift jet of a V/STOL aircraft under take-off conditions. The result of this work is a comparison of the directional distribution of acoustic intensity for an ideal wind tunnel test of a jet and a corresponding flight test.

SYMBOLS

a_o		speed of sound in the acoustic medium
d	=	$[x_1^{\ 2} + x_3^{\ 2}]^{\frac{1}{2}}$
d_o		diameter of jet aperture
$f(\lambda)$		function of separation distance
I		acoustic intensity
i		acoustic intensity per unit volume of turbulence
L		characteristic length in the turbulence
M		convection Mach number ($a_o \underline{M}$ is the corresponding velocity vector)
M_j	=	$\frac{V_j}{a_o}$ - Jet Mach number
N	=	$\frac{V_2}{a_o}$ - crossflow Mach number or aircraft Mach number ($a_o\underline{N}$ is the corresponding velocity vector)
$P_{ijkl}(\underline{y},\underline{\lambda},\tau)$		stress correlation function
P		thermodynamic pressure
P_{ij}	=	$P\delta_{ij} + \mu\,[- \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} + \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij}]$
$R_{ijkl}(\underline{y},\underline{z},\tau)$		stress correlation function
R	=	$\frac{[(1 - N^2)\lvert\underline{x} - \underline{y}\rvert^2 + N^2(x_2 - y_2)^2]^{\frac{1}{2}} - N(x_2 - y_2)}{1 - N^2}$
R_α	=	$r_\alpha - N\ (x_2 - y_2)$
R_β	=	$r_\beta - N\ (x_2 - z_2)$
R_{xx}	=	$\overline{\hat{u}_x(\underline{y})\hat{u}_x(\underline{y} + \underline{\lambda})}$ second-order velocity correlation function
r	=	$\lvert\underline{x}\rvert$
$\vec{r}$	=	$\lvert\underline{\vec{x}}\rvert$
r_α	=	$\lvert\underline{x} - \underline{y}\rvert$

r_β		$\lvert \underline{x} - \underline{z} \rvert$
r_*	=	$[(1 - N^2)\lvert \underline{x} - \underline{y} \rvert^2 + N^2 (x_2 - y_2)^2]^{\frac{1}{2}}$
T_{ij}	=	$P_{ij} + \rho u_i u_j - a_o^2 \rho \delta_{ij}$
t		time (in different coordinate systems - $\hat{t}$, $\tilde{t}$, $\vec{t}$)
t_α	=	$t - \frac{\lvert \underline{x} - \underline{y} \rvert}{a_o}$
t_β	=	$t - \frac{\lvert \underline{x} - \underline{z} \rvert}{a_o}$
U_i		mean velocity component
u_i	=	$U_i + \hat{u}_i$ - velocity component in turbulent flow
$\hat{u}_i$		turbulent velocity component
u_x	=	$\frac{u_i x_i}{r}$ - component of velocity in the direction of the observer
V_i	=	$u_i + V_i \delta_{i2}$ - total velocity component
V_j		jet exit velocity
V_2		crossflow velocity
x_i		spatial coordinate (in different coordinate systems - $\hat{x}_i$, $\tilde{x}_i$, $\vec{x}_i$)
$\underline{x}$		position vector of the observer
y_i		spatial coordinate in the turbulent flow
$\underline{y}$		position vector in the turbulent flow
z_i		spatial coordinate in the turbulent flow
$\underline{z}$		position vector in the turbulent flow
γ	=	$\frac{1}{[1 - N^2]^{\frac{1}{2}}}$
$\delta(x_i)$		Dirac Delta function ($\delta(x_i) = o, x_i \neq o; \int_a^b \delta(x_i) dx_i = 1, a<o<b$)

δ_{ij}	=	$\{\begin{matrix} 1 - i = j \\ 0 - i \neq j \end{matrix}$ - the Kronecker Delta function
$\underline{\Delta}$	=	$\underline{z} - \underline{y}$ - separation vector
ε		distance between elements of the quadrupole source
$\underline{\eta}$		position vector in the turbulent flow (in a moving coordinate system)
θ		polar angle between the observer and the x_1-axis
λ	=	$\lvert\underline{\lambda}\rvert$
$\underline{\lambda}$	=	$\underline{\Delta} - a_o \underline{M}\tau$ - separation vector
μ		dynamic viscosity
$\underline{\xi}$		position vector in the turbulent flow (in a moving coordinate system)
ρ		total density
ρ_o		equilibrium density
σ		parameter related to eddy size
τ	=	$(\tau_\beta - \tau_\alpha)$ or $(t_\beta - t_\alpha)$ - separation time
τ'	=	$t - \frac{R}{a_o}$
τ_α	=	$t - R_\alpha/a_o$ - time of emission of signal
τ_β	=	$t - R_\beta/a_o$ - time of emission of signal
ϕ		polar angle between the crossflow direction and the direction of the observer
ϕ_{ij}		function related to ρ such that $\rho - \rho_o = \frac{\partial^2 \phi_{ij}}{\partial x_i \partial x_j}$
ω_f		characteristic frequency in the turbulence

Subscripts

i,j,k,ℓ	indices taking on values of 1,2,3

I. Introduction

The acoustic performance of aircraft components has become an important consideration. Recently, the wind tunnel has been used as a test facility to evaluate the acoustic properties of aircraft components under simulated flight conditions (ref. 1). In particular, lift fans for use in V/STOL (vertical/short take-off and landing) aircraft have been tested in the 12.3 meter by 24.6 meter (40 feet by 80 feet) wind tunnel of the Ames Research Center of the National Aeronautics and Space Administration. The lift fan is placed so that it exhausts in a direction perpendicular to the direction of flow in the wind tunnel. This arrangement simulates the take-off or landing configuration of a V/STOL aircraft in flight. The acoustic measurements are made from microphones which are placed at various positions around the lift fan.

While tests in a wind tunnel of aerodynamic properties can be used directly to infer the properties under flight conditions, the acoustic data gathered in wind tunnel tests do not completely simulate the acoustic conditions obtained in flight tests. One source of noise from the lift fan may be considered to be the turbulent exhaust flow. When the noise from this source is measured in the wind tunnel, both the source and the observer (i.e., the microphone) are at rest, but the acoustic medium (i.e., the flow in the wind tunnel) is in motion. Signals which are emitted by the source must propagate through a uniformly moving acoustic medium in order to reach the observer. On the other hand, the type of acoustic data which is normally wanted from flight tests is the noise level from an aircraft in flight which is measured by an observer at rest on the ground. In this case, signals which are emitted by the turbulent flow from the engines of a moving aircraft propagate to the observer through an acoustic medium at rest (assuming calm atmospheric conditions). This is a different acoustic configuration than that of the wind tunnel test.

In this report an analysis is presented of the sound field which is generated by the turbulent flow from a lift fan in a wind tunnel with no reflecting walls (i.e., an "ideal" wind tunnel). The flow from the lift fan is modeled by a cold jet flow from a circular aperture. The jet exit velocity is subsonic, and the jet exhausts into a uniform crossflow. The directional distribution (or directivity) of the acoustic intensity which is calculated is compared with the corresponding directivity for a flight test. The results provide a comparison between the acoustic data taken in a wind tunnel and the corresponding data taken in a flight test.

Characteristics of a Circular Jet in a Crossflow

The model for the actual flow from a V/STOL lift fan is the turbulent jet flow from a circular aperture. In order to simulate take-off conditions, the jet will be considered to exhaust into a uniformly moving crossflow with velocity V_2 in the positive x_2- direction (fig. 1). The exit velocity from the jet, V_j, is subsonic, and the jet flow is considered to have the same density as the crossflow. The resulting flow differs in several respects from that of the circular jet with no crossflow. There are three differences which influence the acoustic analysis.

The most obvious feature of the crossflow jet is the deflection of the turbulent flow in the direction of the crossflow. The severity of the deflection is related to the ratio of the jet velocity to the crossflow velocity. The position of the centerline of the jet, as determined by measurements of the maximum velocity in the jet, is indicated on fig. 2 for several values of V_j/V_2 (ref. 2). It should be noticed that for V_j/V_2 as low as 8, the centerline of the jet is only deflected two jet diameters in the first eight jet diameters downstream from the aperture. In the following, a crossflow for which $V_j/V_2 \geq 8$ is referred to as a weak crossflow.

Another prominent feature of the crossflow jet is the distortion of the initial circular cross-section of the jet as the flow proceeds downstream. The shear produced by the crossflow initially distorts the circular flow into one with a cross-section of "kidney" shape (see fig. 1). As the flow proceeds further downstream, a typical cross-section contains two counter-rotating vortex-like structures (see ref. 3).

Finally, in the immediate vicinity of the jet flow, the crossflow is deflected by the jet. In this regard, the jet flow has been likened to a porous cylinder which acts as an obstacle to the crossflow (see ref. 3).

Acoustic Analysis

The conventional use of a wind tunnel is to conduct tests which simulate flight conditions. In order to do this, the air in the wind tunnel is moved uniformly past a test model. When acoustic measurements are made in a wind tunnel, signals which are emitted from a source must propagate through the moving medium to reach an observer (e.g., a microphone). Acoustic tests performed outdoors, however, generally involve measurements made on the ground of the noise generated by the engines of an aircraft either in flight or on the ground. A fundamental difference between acoustic measurements made in a wind tunnel and outdoors is that in the wind tunnel the acoustic medium is in motion relative to the observer while outdoors the medium can normally be considered to be at rest.

The analysis of acoustic propagation in a moving medium begins from a consideration of the continuity and momentum equations. The acoustic medium is assumed to be uniformly moving with a velocity of V_2 in the positive x_2- direction (see fig. 1). The total velocity $\mathcal{V}_i$ at any point in the medium is the resultant of the convection velocity V_2 and other velocities u_i which might result from an acoustic disturbance, turbulence, etc.; thus, $\mathcal{V}_i = u_i + V_2\,\delta_{i2}$ where δ_{i2} is the Kronecker Delta ($\delta_{i2} = \{ {0 \; i \neq 2 \atop 1 \; i = 2}$).
The continuity equation is written

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \mathcal{V}_i}{\partial x_i} = 0 \quad , \text{ or,}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} + V_2 \frac{\partial \rho}{\partial x_2} = 0 \qquad (1)$$

since V_2 is uniform. The momentum equation is written

$$\frac{\partial \rho \mathcal{V}_i}{\partial t} + \frac{\partial \rho \mathcal{V}_i \mathcal{V}_j}{\partial x_j} = - \frac{\partial P_{ij}}{\partial x_j}$$

where, for a Newtonian fluid,

$$P_{ij} = P\delta_{ij} + \mu \left[- \frac{\partial \mathcal{V}_i}{\partial x_j} - \frac{\partial \mathcal{V}_j}{\partial x_i} + \frac{2}{3} \frac{\partial \mathcal{V}_k}{\partial x_k} \delta_{ij}\right] .$$

Upon substitution for V_i and use of the continuity equation, the momentum equation is obtained in the following form:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + V_2 \frac{\partial \rho u_j}{\partial x_2} = - \frac{\partial P_{ij}}{\partial x_j} . \tag{2}$$

The continuity and momentum equations are combined by taking the partial derivative with respect to time of eq. 1 and subtracting from it the derivative with respect of x_i of eq. 2. Use of the continuity equation and subtraction of the term $a_o^2 \frac{\partial^2 \rho}{\partial x_i^2}$ from both sides of the resulting equation gives the following governing equation:

$$\frac{\partial^2 \rho}{\partial t^2} + 2V_2 \frac{\partial^2 \rho}{\partial t \partial x_2} + V_2^2 \frac{\partial^2 \rho}{\partial x_2^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{3}$$

where $T_{ij} = P_{ij} + \rho u_i u_j - a_o^2 \rho \delta_{ij}$. The left-hand side of eq. 3 describes the propagation of acoustic waves in the uniformly moving medium. The right-hand side is a quadrupole source involving the stress T_{ij} which results from pressure, viscous stresses, and momentum flux. In classical acoustics this stress is negligible. In a region of turbulence, however, T_{ij} is non-negligible primarily because of the momentum flux contribution.

The solution of eq. 3 is obtained by using a method of coordinate transformations. This technique consists of finding a system of coordinates in which both the medium and the source are at rest with respect to the observer. In order to begin, a new function ϕ_{ij} is defined which is related to the density as follows:

$$\rho(\underline{x},t) - \rho_o = \frac{\partial^2 \phi_{ij}}{\partial x_i \partial x_j} \tag{4}$$

where ρ_o is the undisturbed density of the acoustic medium. When eq. 4 is substituted into eq. 3, the following equation for ϕ_{ij} is obtained:

$$\frac{\partial^2 \phi_{ij}}{\partial t^2} + 2V_2 \frac{\partial^2 \phi_{ij}}{\partial t \partial x_2} + V_2^2 \frac{\partial^2 \phi_{ij}}{\partial x_2^2} - a_o^2 \frac{\partial^2 \phi_{ij}}{\partial x_i^2} = T_{ij} . \tag{5}$$

In general, the stress T_{ij} is a function of position in a region of turbulence, $\underline{y}$, and of time, t. The stress can be expressed using the Dirac Delta function as $T_{ij}(t)\delta(x_1 - y_1)\delta(x_2 - y_2)\delta(x_3 - y_3)$ where $\delta(x_1 - y_1)\delta(x_2 - y_2)\delta(x_3 - y_3)$ is equal to zero at all values of position (x_1, x_2, x_3) except at the location of the stress (y_1, y_2, y_3). A solution to eq. 5 is first obtained for a stress located at the position (y_1, y_2, y_3).

The equation to be solved is the following:

$$\frac{\partial^2\phi_{ij}}{\partial t^2} + 2V_2 \frac{\partial^2\phi_{ij}}{\partial t\partial x_2} + V_2{}^2 \frac{\partial^2\phi_{ij}}{\partial x_2{}^2} - a_o{}^2 \frac{\partial^2\phi_{ij}}{\partial x_i{}^2} =$$

$$T_{ij}(t)\delta(x_1 - y_1)\delta(x_2 - y_2)\delta(x_3 - y_3) \ . \tag{6}$$

The left-hand side of this equation can be transformed into the normal wave operator on ϕ_{ij} by the following Galilean coordinate transformation:

$$\hat{x}_1 = x_1 - y_1 \ ; \quad \hat{x}_2 = x_2 - y_2 - V_2 t \quad ; \quad \hat{x}_3 = x_3 - y_3; \ \hat{t} = t.$$

The relationship between derivatives is the following:

$$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial \hat{x}_1} \ ; \quad \frac{\partial}{\partial x_2} = \frac{\partial}{\partial \hat{x}_2} \ ; \ \frac{\partial}{\partial x_3} = \frac{\partial}{\partial \hat{x}_3} \ ; \ \frac{\partial}{\partial t} = \frac{\partial}{\partial \hat{t}} - V_2 \frac{\partial}{\partial \hat{x}_2} \ .$$

The above relationship between coordinates is used to transform eq. 6 into the following equation:

$$\frac{\partial^2\phi_{ij}}{\partial \hat{t}^2} - a_o{}^2 \frac{\partial^2\phi_{ij}}{\partial \hat{x}_i{}^2} = T_{ij} \ (\hat{t})\delta(\hat{x}_1)\delta(\hat{x}_2+V_2\hat{t})\delta(\hat{x}_3) \ . \tag{7}$$

This transformation has a simple interpretation. In the new coordinate system moving with velocity V_2 in the positive x_2-direction, an observer finds the acoustic medium to be stationary. Relative to the observer, however, the source moves with a velocity V_2 in the negative x_2-direction.

The series of coordinate transformations by which eq. 7 may be solved is outlined in Morse and Ingard (ref. 4). The first of these, a Lorentz-type of coordinate transformation, has the property of leaving the left-hand side of eq. 7 in the same form; however, the source term on the right-hand side becomes stationary. This coordinate transformation is given as follows:

$$\tilde{x}_1 = \hat{x}_1 \ ; \quad \tilde{x}_2 = \gamma(\hat{x}_2 + V_2\hat{t}) \ ; \quad \tilde{x}_3 = \hat{x}_3 \ ; \quad \tilde{t} = \gamma(\hat{t} + \frac{N}{a_o}\hat{x}_2) \ ;$$

where

$$\gamma = (1-N^2)^{-\frac{1}{2}} \quad \text{and} \quad N = V_2/a_o \ .$$

Thus,

$$\frac{\partial}{\partial \hat{x}_1} = \frac{\partial}{\partial \tilde{x}_1} \ ; \quad \frac{\partial}{\partial \hat{x}_2} = \gamma\left(\frac{\partial}{\partial \tilde{x}_2} + \frac{N}{a_o}\frac{\partial}{\partial \tilde{t}}\right); \quad \frac{\partial}{\partial \hat{x}_3} = \frac{\partial}{\partial \tilde{x}_3}; \quad \frac{\partial}{\partial \hat{t}} = \gamma\left(\frac{\partial}{\partial \tilde{t}} + V_2\frac{\partial}{\partial \tilde{x}_2}\right) \ .$$

In terms of the new coordinates, eq. 7 becomes

$$\frac{\partial^2 \phi_{ij}}{\partial \tilde{t}^2} - a_o^{\,2}\frac{\partial^2 \phi_{ij}}{\partial \tilde{x}_i^{\,2}} = \gamma T_{ij}(\gamma\tilde{t})\delta(\tilde{x}_1)\delta(\tilde{x}_2)\delta(\tilde{x}_3) \ . \tag{8}$$

The above form of the equation results from use of the property of the Dirac Delta function that $\delta(\tilde{x}_2/\gamma) = \gamma\delta(\tilde{x}_2)$.

One final coordinate transformation is needed to put the governing equation into a form for which the solution is immediate. This transformation is the following:

$$\vec{x}_1 = \gamma\tilde{x}_1 \ ; \quad \vec{x}_2 = \gamma\tilde{x}_2 \ ; \quad \vec{x}_3 = \gamma\tilde{x}_3 \ ; \quad \vec{t} = \gamma\tilde{t} \ .$$

In terms of these coordinates, eq. 8 becomes

$$\frac{\partial^2 \phi_{ij}}{\partial \vec{t}^2} - a_o^{\,2}\frac{\partial^2 \phi_{ij}}{\partial \vec{x}_i^{\,2}} = \gamma^2 \, T_{ij}(\vec{t})\delta(\vec{x}_1)\delta(\vec{x}_2)\delta(\vec{x}_3) \ . \tag{9}$$

This equation represents wave propagation in three dimensions through a stationary uniform acoustic medium resulting from a stationary source of strength $\gamma^2 T_{ij}(\vec{t})$ located at the origin. The solution to eq. 9 is the following:

$$\phi_{ij}(\vec{r},t) = \frac{\gamma^2 T_{ij}(\vec{t} - \frac{\vec{r}}{a_o})}{4\pi a_o^{\,2}\,\vec{r}} \quad , \tag{10}$$

where

$$\vec{r} = [\vec{x}_1^{\,2} + \vec{x}_2^{\,2} + \vec{x}_3^{\,2}]^{\frac{1}{2}} .$$

It should be noted that in order to determine the function ϕ_{ij} at any position $\vec{r}$ and time $\vec{t}$, T_{ij} must be evaluated at the time of emission of the acoustic signal, $\vec{t} - \frac{\vec{r}}{a_o}$.

The independent variables $(\vec{x}_i, \vec{t})$ in the final system of coordinates are related through the coordinate transformations to the independent variables (x_i, y_i, t) in the original coordinate system; thus

$$r = \frac{[(1-N^2)|\underline{x}-\underline{y}|^2 + N^2(x_2-y_2)^2]^{\frac{1}{2}}}{1 - N^2} ,$$

and

$$\vec{t} - \frac{\vec{r}}{a_o} = t - \frac{R}{a_o} ,$$

where

$$R = \frac{[(1-N^2)|\underline{x}-\underline{y}|^2 + N^2(x_2-y_2)^2]^{\frac{1}{2}} - N(x_2-y_2)}{1 - N^2} . \qquad (11)$$

In terms of the original coordinates the solution for ϕ_{ij} is the following:

$$\phi_{ij}(\underline{x},t) = \frac{T_{ij}(t - \frac{R}{a_o})}{4\pi a_o^{\,2}[(1-N^2)|\underline{x}-\underline{y}|^2 + N^2(x_2-y_2)^2]^{\frac{1}{2}}} \qquad (12)$$

This equation determines the value of ϕ_{ij} which results from a single source T_{ij} located at the position (y_1, y_2, y_3). When a region of turbulence is present, the solution for ϕ_{ij} is generalized by integrating the result from each source in the region. Thus, for a distributed source,

$$\phi_{ij}(\underline{x},t) = \frac{1}{4\pi a_o^{\,2}} \int \frac{T_{ij}(\underline{y},t - \frac{R}{a_o})}{[(1-N^2)|\underline{x}-\underline{y}|^2 + N^2(x_2-y_2)^2]^{\frac{1}{2}}} \, d^3\underline{y} \qquad (13)$$

where $\underline{y}$ is the position in the region of turbulent flow, and the integral extends over the entire region of turbulence.

The acoustic density disturbance at a position $\underline{x}$ and time t is related to ϕ_{ij} by eq. 4 and is therefore determined by the following equation:

$$\rho(\underline{x},t) - \rho_o = \frac{1}{4\pi a_o^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \frac{T_{ij}\ (\underline{y}, t - \frac{R}{a_o})}{[(1-N^2)|\underline{x}-\underline{y}|^2 + N^2 (x_2-y_2)^2]^{\frac{1}{2}}}\, d^3\underline{y}. \tag{14}$$

In this equation the retarded time (i.e. the time at which the signal is emitted) is $t - \frac{R}{a_o}$ where R is defined in eq. 11. The quantity R represents the distance that the signal propagates with speed a_o through the moving medium from the source to the observer. This interpretation can be demonstrated with the aid of fig. 3. The plane indicated in fig. 3 contains both the observer located at position (x_1, x_2, x_3) and the source located at the origin. The direction of the observer from the direction of the crossflow is given by angle ϕ . In order for a signal to be received by an observer, it must propagate through distance $R = a_o \Delta t$ where Δt is the time interval between emission and reception of the signal. During the time interval Δt, the signal is convected a distance $V_2 \Delta t = NR$ by the acoustic medium. These relationships allow R to be evaluated in terms of $r = |\underline{x}|$ and ϕ as follows:

$$R^2 = (r\sin\phi)^2 + (r\cos\phi - NR)^2 ,$$

and therefore,

$$R = \frac{-N\ r\cos\phi \pm [(1-N^2)r^2 + N^2 r^2 \cos^2\phi]^{\frac{1}{2}}}{1-N^2}$$

When N is less than unity, the positive sign is taken, and the relationship $r\cos\phi = x_2$ is used to obtain the value of R in eq. 11 for a source located at the origin.

Equation 14 provides the formal solution for the acoustic density disturbance when the acoustic medium is moving uniformly with velocity V_2 in the positive x_2-direction. When the Mach number of the medium is such that $N^2 \ll 1$, the denominator of eq. 14 is approximately equal to the distance between the source and the observer. In this limit, however, the retarded time is not given by $t - \frac{|\underline{x}-\underline{y}|}{a_o}$ as is the case for a stationary acoustic medium. At small Mach numbers, the primary effect of the movement of the acoustic medium is therefore the alteration of the time between emission and reception of acoustic signals. The net acoustic signal which is received at a point from sources located near to each other is strongly dependent upon the time interval between emission and reception of the signal from each source. The alteration in the retarded time by the movement of the acoustic medium therefore influences the acoustic field generated by nearby sources (e.g., the acoustic field which is generated by a region of turbulent flow). This effect can be put in a quantitative form by carrying out the differentiation of the integrand of eq. 14 as follows:

$$\frac{\partial}{\partial x_j}\left(\frac{T_{ij}(\underline{y},\tau)}{r_*}\right) = -\frac{1}{r_*}\left[\frac{T_{ij}}{r_*}\frac{\partial r_*}{\partial x_j} + \frac{1}{a_o}\frac{\partial T_{ij}}{\partial \tau'}\frac{\partial R}{\partial x_j}\right],$$

where

$$r_* = [(1 - N^2)|\underline{x}-\underline{y}|^2 + N^2(x_2-y_2)^2]^{\frac{1}{2}}$$

and
$$\tau' = t - \frac{R}{a_o}.$$

Therefore,

$$\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{T_{ij}(\underline{y},\tau')}{r_*}\right) = \frac{1}{a_o^2 r_*}\frac{\partial R}{\partial x_i}\frac{\partial R}{\partial x_j}\frac{\partial^2 T_{ij}}{\partial \tau'^2} + \frac{1}{a_o r_*}\left(\frac{1}{r_*}\frac{\partial R}{\partial x_i}\frac{\partial r_*}{\partial x_j} + \frac{1}{r_*}\frac{\partial r_*}{\partial x_i}\frac{\partial R}{\partial x_j} - \frac{\partial^2 R}{\partial x_i \partial x_j}\right)\frac{\partial T_{ij}}{\partial \tau'} + \frac{1}{r_*}\left(\frac{2}{r_*}\frac{\partial r_*}{\partial x_i}\frac{\partial r_*}{\partial x_j} - \frac{\partial^2 r_*}{\partial x_i \partial x_j}\right)T_{ij}.$$

When the solution is restricted to the acoustic far field (i.e., to distances from the source which are large compared to $(2\pi)^{-1}$ times a typical acoustic wavelength), the dominant term of the integrand is the one that decays most slowly. The coefficient of $\frac{\partial^2 T_{ij}}{\partial \tau'^2}$ is of order r_*^{-1} while the coefficients of $\frac{\partial T_{ij}}{\partial \tau'}$ and T_{ij} can be shown to be of order r_*^{-2} and r_*^{-3}, respectively; therefore,

$$\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{T_{ij}(\underline{y},\tau')}{r_*}\right) = \frac{1}{a_o^2 r_*}\frac{\partial R}{\partial x_i}\frac{\partial R}{\partial x_j}\frac{\partial^2 T_{ij}}{\partial \tau'^2}.$$

The derivatives of R are easily calculated with the result that

$$\frac{\partial R}{\partial x_i} = \frac{1}{(1-N^2)} \left[\frac{(x_i-y_i)(1-N^2) + (x_i-y_i)N^2\delta_{i2}}{r_*} - N\delta_{i2}\right] .$$

The density disturbance in the acoustic far field is therefore given as follows:

$$\rho(r_*,t)-\rho_o = \frac{1}{4\pi a_o^4(1-N^2)^2} \int \frac{1}{r_*} \left[\frac{(x_i-y_i)(1-N^2) + (x_i-y_i)N^2\delta_{i2}}{r_*} - N\delta_{i2}\right] \tag{15}$$

$$\left[\frac{(x_j-y_j)(1-N^2) + (x_j-y_j)\delta_{j2}}{r_*} - N\delta_{j2}\right] \frac{\partial^2 T_{ij}(\underline{y},\tau')}{\partial\tau'^2} d^3\underline{y} .$$

A somewhat unintuitive result is contained in eq. 15. When the source is a quadrupole with either axis in the direction of the crossflow (i.e., with either i or j equal to 2), a greater density disturbance is calculated upstream of the source than at an equal distance downstream for all observer positions except along the x_2-axis. This result can be demonstrated by considering the source to be a single lateral quadrupole T_{12} located at the origin. This type of source represents a shear-stress on the x_1-plane in the x_2-direction or vice versa. For this example the square of the crossflow Mach number is considered to be small compared to unity (i.e., $N^2 << 1$). The density disturbance is given by eq. 15 to be the following:

$$\rho(r,t) - \rho_o = \frac{1}{4\pi a_o^4 r} \left(\frac{x_1}{r}\right)\left(\frac{x_2}{r} - N\right) \frac{\partial^2 T_{12}}{\partial\tau'^2} , \text{ where } r = |\underline{x}| . \tag{16}$$

A stress is thought of as resulting from two equal and opposite forces acting on a fluid element. Each force can be represented by a dipole (i.e., an adjacent source and sink of equal strengths which emit simultaneously). Therefore, the quadrupole source T_{12} is represented as shown on fig. 4 where the + indicates a source and the - indicates a sink of equal strength. In the absence of a crossflow (i.e., with N = 0), the directional distribution of the density disturbance at a fixed distance r which is generated by the source is $\left(\frac{x_1}{r}\right)\left(\frac{x_2}{r}\right)$. This pattern is the symmetric "rose" shown on fig. 4a.

The reason that an observer along the x_1- and x_2- axes finds zero density disturbance is the interference of the signals which are simultaneously emitted from the sources and sinks comprising the quadrupole. The observer on either axis is equidistant from source-sink pairs of the same strength; thus, the signals which are simultaneously emitted from each source-sink pair of the quadrupole propagate through the medium with velocity a_o and arrive at the observer position simultaneously. The result of a signal from a source arriving in phase with a signal from a corresponding sink is complete destructive interference and zero net density disturbance. For the observer located off-axis, however, the corresponding source-sink elements of the quadrupole are not equidistant from the observer and the interference is not complete. When the acoustic medium is moving with a velocity $a_o N$, signals which propagate in any direction are simultaneously convected with the medium as they propagate. Convection by the acoustic medium has the effect of altering the time required for the signals from each element of the quadrupole to reach the observer. This gives rise to a distorted distribution of the acoustic density disturbance. This directional dependence is given by eq. 16 to be

$$\left(\frac{x_1}{r}\right)\left(\frac{x_2}{r} - N\right) .$$

Figure 4b gives the directional dependence of the density disturbance for the lateral quadrupole T_{12} which results when the medium is moving in the positive x_2-direction with a Mach number of $N = 0.25$. Two features distinquish this pattern from the symmetric pattern of fig. 4a. First, zero net signal is found not along the x_1-axis but rather along a ray downstream of the x_1-axis. This is explained directly by considering the effect of convection of the medium on the propagating signals. The signals which would have destructively interfered with each other along the x_1-axis when there was no movement of the medium are now convected downstream. The second feature of the distorted "rose" pattern is the enhanced signal received upstream of the source compared to that received at corresponding positions downstream. This feature can be obtained by considering the propagation of signals from each element of the quadrupole to an observer located at an angle θ from the x_1-axis (measured counter-clockwise) and distance r from the origin. The separation distance between adjacent sources and sinks in the quadrupole is

ε, where ε is small such that $(\frac{\varepsilon}{r}) \ll 1$. Each element of the quadrupole (i.e., each source or sink) is then located a distance ℓ_i from the observer, where the index i refers to the quadrant on which the element is located (e.g., the source on the first quadrant is located a distance ℓ_1 from the observer at position (r,θ). The observer distance from the source elements of the quadrupole can be calculated to be the following: $\ell_{1,3} = r\,[1 \mp \frac{\varepsilon}{2r}(\sin\theta + \cos\theta)]$. Similarly, the observer distances from the sink elements of the quadrupole are $\ell_{2,4} = r\,[1 \mp \frac{\varepsilon}{2r}(\sin\theta - \cos\theta)]$. Since the velocity of the medium is $a_o N$ on the x_2-direction, the resultant propagation velocity in any direction θ is $a_o(1 + N\sin\theta)$. The time required for a signal to traverse a distance ℓ_i is therefore given approximately by $t_i = \frac{\ell_i}{a_o(1 + N\sin\theta)}$. (Note, the approximation is that the signal from each element propagates on the direction θ.) The source elements of the quadrupole may be considered to fluctuate with a frequency ω as $\sin\omega t$ while the sink elements fluctuate as $-\sin\omega t$. A signal which is emitted by any element of the quadrupole at time t arrives at the observer at a later time $t + t_i$; for instance, the signal which is received by the observer from the sink located on the second quadrant is $-\sin[\omega(t + t_2)]$. The net signal received (NSR) by the observer is then calculated by summing the contributions from each element of the quadrupole as follows:

$$\begin{aligned} NSR = {} & \sin[\omega(t + t_1)] - \sin[\omega(t + t_2)] \\ & + \sin[\omega(t + t_3)] - \sin[\omega(t + t_4)] \,. \end{aligned}$$

This calculation for NSR is simplified by using the smallness of $(\frac{\varepsilon}{r})$ with the result that

$$NSR = \frac{\sin\theta\,\cos\theta}{(1 + N\sin\theta)^2}\left(\frac{\omega\varepsilon}{a_o}\right)^2 \sin\left[\omega\left(t + \frac{r}{a_o(1 + N\sin\theta)}\right)\right] .$$

The amplitude of NSR is given by the factor $\frac{\sin\theta\,\cos\theta}{(1 + N\sin\theta)^2}\left(\frac{\omega\varepsilon}{a_o}\right)^2$ which provides the approximate directional properties of the quadrupole source. The symmetric "rose" pattern (i.e., $\sin\theta\,\cos\theta$) is modified by the convection factor $(1 + N\sin\theta)^{-2}$ to provide an enhanced upstream NSR compared with that at corresponding downstream positions. This modification occurs because the destructive interference of the signals from each element of the quadrupole source is less complete for observer positions upstream of the source owing to the reduced propagation velocity in that direction. As a result, the upstream "petals" of the distorted "rose" pattern are larger than the downstream "petals". It should be noted that the greater effectiveness of quadrupole sources upstream than downstream also applies to simple sources (monopoles) and to dipoles when the axis of the dipole is aligned with the x_2-direction.

Human response to a sound field is determined more by the acoustic intensity than by the acoustic density disturbance. In the acoustic far field, the intensity is defined to be the average rate of flow of acoustic energy through a unit area normal to the direction of wave propagation. When the intensity is integrated over a large surface which encloses an acoustic source, the total power output of the source is obtained. The relationship between the acoustic density distrubance and the acoustic intensity is given as follows:

$$I = \frac{a_o^3}{\rho_o} \overline{(\rho(\underline{x},t)-\rho_o)^2}$$

where the overbar indicates the time-average of the quantity underneath. (This is an approximate relationship for the acoustic intensity for small crossflow Mach numbers. It is a reasonable form for the example calculations which are included in this report. The exact expression for the intensity is given in ref. 14.) The acoustic intensity in the far field is determined by substituting eq. 15 into the above relationship. In order to simplify the results that follow, it will be assumed that the square of the crossflow Mach number is small compared to unity (i.e., $N^2 \ll 1$). With this assumption, the acoustic intensity is given as follows:

$$I = \frac{1}{16\pi^2 a_o^5 \rho_o} \int\int \frac{1}{r_\alpha r_\beta} \left[\frac{x_i - y_i}{r_\alpha} - N\delta_{i2}\right] \left[\frac{x_j - y_j}{r_\alpha} - N\delta_{j2}\right]$$

$$\left[\frac{x_k - z_k}{r_\beta} - N\delta_{k2}\right] \left[\frac{x_\ell - z_\ell}{r_\beta} - N\delta_{\ell 2}\right] \overline{\frac{\partial^2 T_{ij}(\underline{y},\tau_\alpha)}{\partial \tau_\alpha^2} \frac{\partial^2 T_{k\ell}(\underline{z},\tau_\beta)}{\partial \tau_\beta^2}} d^3\underline{y}\, d^3\underline{z} \qquad (17)$$

where $\underline{y}$ and $\underline{z}$ are the positions in the turbulent flow, $r_\alpha = |\underline{x} - \underline{y}|$, $r_\beta = |\underline{x} - \underline{z}|$, $\tau_\alpha = t - \frac{R_\alpha}{a_o}$, $\tau_\beta = t - \frac{R_\beta}{a_o}$, $R_\alpha = r_\alpha - N(x_2-y_2)$, and $R_\beta = r_\beta - N(x_2-z_2)$.

The integrand of eq. 17 contains the product of direction factors and a time-averaged quantity which depends on the properties of the turbulence at different points in the flow field. It is useful to recast the integrand into a form which recognizes the eddy structure of the turbulence. This procedure is developed by Ffowcs Williams (ref. 6). First, a separation time τ is defined such that $\tau = \tau_\beta - \tau_\alpha$. The turbulence is specified to be a stationary function of time which allows the following relationship to be proven:

$$\overline{\frac{\partial^2 T_{ij}(\underline{y},\tau_\alpha)}{\partial \tau_\alpha{}^2} \frac{\partial^2 T_{k\ell}(\underline{z},\tau_\beta)}{\partial \tau_\beta{}^2}} = \frac{\partial^4}{\partial \tau^4} R^*_{ijk\ell}(\underline{y},\underline{z},\tau) \tag{18}$$

where the correlation function $R^*_{ijk\ell}(\underline{y},\underline{z},\tau) \doteq \overline{T_{ij}(\underline{y},\tau_\alpha) T_{k\ell}(\underline{z},\tau_\alpha+\tau)}$. The above correlation function is determined by taking the time-average of the stress in the turbulence at two positions $\underline{y}$ and $\underline{z}$ and at corresponding times τ_α and $\tau_\alpha + \tau$. It is expected that the correlation function will have a maximum for positions $\underline{y}$ and $\underline{z}$ which are close together and zero for positions which are widely separated. The region of high correlation corresponds to a turbulent eddy.

The independent variable $\underline{z}$ is eliminated in favor of a separation vector $\underline{\Delta}$ which is related to $\underline{y}$ and $\underline{z}$ as follows: $\underline{\Delta} = \underline{z}-\underline{y}$; $d^3\underline{z} = d^3\underline{\Delta}$. A new correlation function is defined by expressing $R^*_{ijk\ell}(\underline{y},\underline{z},\tau)$ in terms of the variables $\underline{y}$, $\underline{\Delta}$, and τ; thus, $R_{ijk\ell}(\underline{y},\underline{\Delta},\tau) = R^*_{ijk\ell}(\underline{y},\underline{z},\tau)$. The separation time τ is dependent on the observer position $\underline{x}$ and the positions in the turbulence. This relationship is $\tau = \tau_\beta - \tau_\alpha = \frac{R_\alpha - R_\beta}{a_o}$. The distance R_α is a function of $\underline{x}$ and $\underline{y}$ while R_β is a function of $\underline{x}$ and $\underline{z}$. The definition of the separation vector $\underline{\Delta}$ allows R_β to be expressed as a function of $\underline{x}$, $\underline{y}$, and Δ. Since the correlation function $R_{ijk\ell}(\underline{y},\underline{\Delta},\tau)$ is expected to be non-negligible only for positions in the turbulence which are relatively close to each other, the expression for τ need only be determined for values of $\underline{\Delta}$ which are small compared to $(\underline{x} - \underline{y})$. Thus, for an observer far from the turbulent field, there is only a small difference in $(\underline{x} - \underline{y})$ and $(\underline{x} - \underline{z})$ over the dimension of a turbulent eddy, and these differences can be neglected in the calculation of the acoustic intensity. The expression for τ is then $\tau = \frac{\underline{\Delta}\cdot(\underline{x}-\underline{y})}{a_o|\underline{x}-\underline{y}|} - \frac{N\Delta_2}{a_o}$. This expression gives the time difference between the emission of signals from points located a small distance $\underline{\Delta}$ apart.

In a turbulent flow there is, in general, a velocity associated with the convection of a turbulent eddy in the flow. For instance, in the mixing region of a cold subsonic jet, the convection velocity of a turbulent eddy is approximately 0.65 times the jet exit velocity (see ref. 7, pp. 355-357). In order to estimate accurately the acoustic intensity, it is important to minimize the changes in $R_{ijk\ell}(\underline{y},\underline{\Delta},\tau)$ with respect to τ which result merely from eddy convection. This is possible by defining a new separation vector $\underline{\lambda}$ in a coordinate system which moves with the convection velocity $a_o\underline{M}$. The relationship between $\underline{\Delta}$ and $\underline{\lambda}$ is $\underline{\Delta} = \underline{\lambda} + a_o\underline{M}\tau$. The separation time τ is then determined as a function of $\underline{\lambda}$. In order to facilitate this transformation and the following results, it will be assumed that the convection velocity is of magnitude a_oM_1 in the positive x_1-direction only. This corresponds to the assumption of a weak crossflow. The resulting expression for τ is then

$$\tau = \frac{\underline{\lambda}\cdot(\underline{x}-\underline{y}) - N\lambda_2|\underline{x}-\underline{y}|}{a_o\{|\underline{x}-\underline{y}| - M_1(x_1-y_1)\}} .$$

A new correlation function $P_{ijk\ell}(\underline{y},\underline{\lambda},\tau)$ is defined by expressing $\underline{\Delta}$ in terms of $\underline{\lambda}$ and τ; therefore, $P_{ijk\ell}(\underline{y},\underline{\lambda},\tau) = R_{ijk\ell}(\underline{y},\underline{\Delta},\tau)$. Finally, the relationships between $\underline{\lambda}$, $\underline{\Delta}$, and τ are used to calculate the changes with respect to τ of $R_{ijk\ell}(\underline{y},\underline{\Delta},\tau)$ and $P_{ijk\ell}(\underline{y},\underline{\lambda},\tau)$. As shown by Ffowcs Williams (ref. 6) these two derivatives are not simply related because of the dependence of τ on $\underline{\lambda}$ and $\underline{\Delta}$. The relationship can be shown to be given by the following:

$$\frac{\partial}{\partial\tau} R_{ijk\ell}(\underline{y},\underline{\Delta},\tau) = \left\{\frac{|\underline{x}-\underline{y}|}{[|\underline{x}-\underline{y}|-M_1(x_1-y_1)]}\frac{\partial}{\partial\tau} - a_oM_1\frac{\partial}{\partial\lambda_1}\right\} P_{ijk\ell}(\underline{y},\underline{\lambda},\tau).$$

When integrated over $\underline{\lambda}$, the second term on the right-hand side can be shown to give a negligible contribution, and it is therefore neglected. The differential volumes $d^3\underline{\Delta}$ and $d^3\underline{\lambda}$ are related by the Jacobian of the coordinate transformation as follows:

$$d^3\underline{\Delta} = d^3\underline{\lambda}\,[1 - \frac{M_1(x_1-y_1)}{|\underline{x}-\underline{y}|}]^{-1} \ .$$

The acoustic intensity (eq. 17) can now be expressed in terms of the correlation function $P_{ijk}\ (\underline{y},\underline{\lambda},\tau)$ as follows:

$$I = \frac{1}{16\pi^2 a_o^5 \rho_o} \int\int \frac{1}{r_\alpha^{\ 2}[1 - \frac{M_1(x_1-y_1)}{|\underline{x}-\underline{y}|}]^5} [\frac{x_i-y_i}{r_\alpha} - N\delta_{i2}]\,[\frac{x_j-y_j}{r_\alpha} - N\delta_{j2}] \tag{19}$$

$$[\frac{x_k-y_k}{r_\alpha} - N\delta_{k2}]\,[\frac{x_\ell-y_\ell}{r_\alpha} - N\delta_{\ell 2}]\,\frac{\partial^4}{\partial\tau^4} P_{ijk\ell}(\underline{y},\underline{\lambda},\tau)\ d^3\underline{\lambda}\ d^3\underline{y}\ .$$

This integral is simplified by restricting the observer position $\underline{x}$ to be far from the turbulent field compared to the dimensions of the flow field. Under this limitation, $|\underline{x}-\underline{y}| \cong |\underline{x}| = r$ and $x_i-y_i \cong x_i$, and direction factors become independent of the integration. Equation 19 is made consistent with the assumption that $N^2 << 1$ by expanding the four direction factors appearing in the equation to give the following terms:

$$[\frac{x_i}{r} - N\delta_{i2}]\,[\frac{x_j}{r} - N\delta_{j2}]\,[\frac{x_k}{r} - N\delta_{k2}]\,[\frac{x_\ell}{r} - N\delta_{\ell 2}] =$$

$$\frac{x_i x_j x_k x_\ell}{r^4} - \frac{N}{r^3}\,[\delta_{i2}x_j x_k x_\ell + x_i\delta_{j2}x_k x_\ell + x_i x_j \delta_{k2} x_\ell + x_i x_j x_k \delta_{\ell 2}]\ . \tag{20}$$

The final expression for the acoustic intensity is in the following very useful form:

$$I(r) = \frac{1}{16\pi^2 a_o^5 \rho_o r^2 [1 - \frac{M_1 x_1}{r}]^5} \int\int \{\frac{x_i x_j x_k x_\ell}{r^4} - \frac{N}{r^3}\,[\delta_{i2}x_j x_k x_\ell + x_i\delta_{j2}x_k x_\ell \tag{21}$$

$$+ x_i x_j \delta_{k2} x_\ell + x_i x_j x_k \delta_{\ell 2}]\}\ \frac{\partial^4}{\partial\tau^4} P_{ijk\ell}(\underline{y},\underline{\lambda},\tau)\ d^3\underline{\lambda}\ d^3\underline{y}\ .$$

The effect of eddy convection is given by the Doppler factor $[1 - \frac{M_1 x_1}{r}]^{-5}$ while the influence of the medium velocity $a_o N$ appears in the numerator of eq. 21.

Even with the assumptions made to date to simplify the calculation, the solution depends upon the knowledge of the correlation function $P_{ijk\ell}(\underline{y},\underline{\lambda},\tau)$ at each point in the turbulence. This implies knowing the stress at one spatial point and one time in the turbulence and at all other points at the proper separation time. In order to proceed beyond this point, it is necessary to use very simplified models of the jet turbulence.

MODELING THE TURBULENCE

In principle, the acoustic intensity can be calculated from eq. 21 if the correlation function $P_{ijk\ell}(\underline{y},\underline{\lambda},\tau)$ is known in the turbulent jet flow. At best, this function is very difficult to determine either experimentally or analytically for any turbulent flow. As a result, a very simplified model for the correlation function is used to calculate the qualitative aspects of the acoustic intensity which is generated by the crossflow jet. The method which is followed is given by Ribner (ref. 8) who calculates the acoustic intensity which results from a turbulent jet with no crossflow.

For turbulent flows of low Mach number, the stress T_{ij} is approximated as follows: $T_{ij} \cong \rho_o u_i u_j$, where ρ_o is the density of the surrounding acoustic medium. This approximation is reasonable for those flows in which heating or cooling is caused only by compression or expansion of the flow (see ref. 5). In this approximation, viscous effects have been neglected; the only contribution of viscosity would be a slight damping of the acoustic field outside the turbulent flow and a small contribution to the stress inside the turbulent flow. The correlation function in eq. 21 is then given by the following:

$$P_{ijk\ell}(\underline{y},\underline{\lambda},\tau) = \rho_o^{\,2}\ \overline{u_i(\underline{y},\tau_\alpha)u_j(\underline{y},\tau_\alpha)u_k(\underline{y}+\underline{\lambda},\tau_\alpha+\tau)u_\ell(\underline{y}+\underline{\lambda},\tau_\alpha+\tau)}\ . \tag{22}$$

This correlation function is substituted into eq. 21 to give the following equation for the acoustic intensity:

$$I(r) = \frac{\rho_o}{16\pi^2 a_o^5 r^2 (1 - \frac{M_1 x_1}{r})^5} \frac{\partial^4}{\partial\tau^4} \int\int [\overline{u_x u_x u_x' u_x'} - 2N \{\overline{u_2 u_x u_x' u_x'} + \overline{u_x u_x u_2' u_x'}\}] d^3\underline{\lambda}\, d^3\underline{y} \,, \tag{23}$$

where $u_x = \frac{u_i x_i}{r}$ is the component of fluid velocity in the direction of the observer, u_2 is the velocity in the positive x_2-direction and the prime indicates the velocity evaluated at $(\underline{y} + \underline{\lambda},\ \tau_\alpha + \tau)$ while the unprimed velocities indicate evaluation at $(\underline{y}, \tau_\alpha)$. The velocity u_x represents the total velocity in the turbulent flow and can therefore be expressed as the sum of the mean velocity and the fluctuating component as follows: $u_x = U_x + \hat{u}_x$ where U_x is the mean velocity and $\hat{u}_x$ is the fluctuating component which is defined such that $\overline{\hat{u}_x} = 0$. This and similar expressions for u'_x, u_2 and u'_2 are substituted into the terms such as $\overline{u_x u_x u_x' u_x'}$ which appear in eq. 23, and the indicated multiplication is performed. Several terms in the resulting expression can be neglected. Since the correlation function is to be operated on by the derivative with respect to τ, those terms which contain no dependence on τ are disregarded. Also, terms involving triple velocity correlations (e.g., $\overline{\hat{u}_x \hat{u}_x \hat{u}_x'}$) are neglected because they are either zero or small depending on how the turbulence is modeled (see ref. 8). The result of these simplifications is given as follows:

$$\overline{\hat{u}_x \hat{u}_x \hat{u}_x' \hat{u}_x'} = 4U_x U_x' \ \overline{\hat{u}_x \hat{u}_x'} + \overline{\hat{u}_x \hat{u}_x \hat{u}_x' \hat{u}_x'}$$

When the other terms in eq. 23 are similarly considered, the integrand becomes the following:

$$\overline{\hat{u}_x \hat{u}_x \hat{u}_x' \hat{u}_x'} + 4U_x U_x' \ \overline{\hat{u}_x \hat{u}_x'} - 2N[2(U_2 U_x' + U_x U_2') \ \overline{\hat{u}_x \hat{u}_x'} + 2U_x U_x' \ (\overline{\hat{u}_2 \hat{u}_x'} + \overline{\hat{u}_x \hat{u}_2'}) + \overline{\hat{u}_2 \hat{u}_x \hat{u}_x' \hat{u}_x'} + \overline{\hat{u}_x \hat{u}_x \hat{u}_x' \hat{u}_2'} \ ] \ . \tag{24}$$

At this point, the assumption is introduced that the turbulence in the jet is isotropic. For this assumption to be strictly valid, the intensity of the turbulence (i.e., $[\overline{\hat{u}^2}]^{\frac{1}{2}}$) at every point in the flow must be the same in all directions. This state of affairs is not found in the circular jet with no crossflow (see ref. 9), and there is no reason to believe that the assumption will be strictly valid for the circular jet in the presence of a crossflow. The assumption however, does allow the calculation of acoustic intensity to be greatly facilitated, and it is expected that the result will provide a reasonable first approximation.

In isotropic turbulence, it can be shown (see ref. 10) that the second-order two-point correlations of the velocity in the same direction (e.g., $\overline{\hat{u}_1\hat{u}_1{}'}$) are non-zero while the correlations of a velocity at one point with a velocity directed perpendicularly at another point (e.g., $\overline{\hat{u}_1\hat{u}_2{}'}$) are equal to zero. Therefore, when the direction of the observer is the x_2-direction, the terms $\overline{\hat{u}_2\hat{u}_x{}'}$ and $\overline{\hat{u}_x\hat{u}_2{}'}$ are equal to $\overline{\hat{u}_x\hat{u}_x{}'}$. When the observer is in the x_1- or x_3-directions, however, the terms $\overline{\hat{u}_2\hat{u}_x{}'}$ and $\overline{\hat{u}_x\hat{u}_2{}'}$ are identically equal to zero. Thus, for any observer position,

$$\overline{\hat{u}_2\hat{u}_x{}'} = \overline{\hat{u}_x\hat{u}_2{}'} = \overline{\hat{u}_x\hat{u}_x{}'} \cos\phi$$

where ϕ is the polar angle between the crossflow direction (x_2-axis) and the observer direction (see fig. 5). Substitution of these relationships into eq. 24 gives the following expression:

$$\overline{\hat{u}_x\hat{u}_x\hat{u}_x{}'\hat{u}_x{}'} + 4U_xU_x{}' \overline{\hat{u}_x\hat{u}_x{}'} - 2N[2(U_2U_x{}' + U_xU_2{}' + 2U_xU_x{}' \cos\phi) \overline{\hat{u}_x\hat{u}_x{}'} + \overline{\hat{u}_2\hat{u}_x\hat{u}_x{}'\hat{u}_x{}'} + \overline{\hat{u}_x\hat{u}_x\hat{u}_2{}'\hat{u}'_x}] . \tag{25}$$

The expression given by eq. 25 contains terms which are second-order velocity correlations and fourth-order velocity correlations. While much is known about the behavior of the second-order velocity correlation in isotropic turbulence, little information is avilable concerning the fourth-order correlation. As a result, the assumption is made that the joint-probability distribution of $\hat{u}_x$ and $\hat{u}_x{}'$ is normal. This distribution gives the probability of finding a particular velocity $\hat{u}_x$ at one point along with a particular velocity $\hat{u}_x'$ at another point. While this assumption is known not to be strictly accurate (see ref. 11) it does allow a relationship to be made between the fourth-order velocity correlation $\overline{\hat{u}_x\hat{u}_x\hat{u}_x'\hat{u}_x'}$ and the second-order correlation $\overline{u_x u_x'}$. This relationship is the following (see ref. 11):

$$\overline{\hat{u}_i\hat{u}_j\hat{u}_k'\hat{u}_\ell'} = \overline{\hat{u}_i\hat{u}_j} \cdot \overline{\hat{u}_k'\hat{u}_\ell'} + \overline{\hat{u}_i\hat{u}_k'} \cdot \overline{\hat{u}_j\hat{u}_\ell'} + \overline{\hat{u}_i\hat{u}_\ell'} \cdot \overline{\hat{u}_j\hat{u}_k'} \ .$$

The fourth-order velocity correlations in eq. 25 are written, using this relationship, as follows:

$$\overline{\hat{u}_x\hat{u}_x\hat{u}_x'\hat{u}_x'} = 2(\overline{\hat{u}_x\hat{u}_x'})^2$$

and

$$\overline{\hat{u}_2\hat{u}_x\hat{u}_x'\hat{u}_x'} = \overline{\hat{u}_x\hat{u}_x\hat{u}_2'\hat{u}_x'} = 2(\overline{\hat{u}_x\hat{u}_x'})^2 \cos\phi \ . \tag{26}$$

The terms which have no dependence on τ have been neglected in the above relationships. The second-order velocity correlation $\overline{\hat{u}_x\hat{u}_x'}$ is a function not only of the separation distance $\underline{\lambda}$ but also of the separation time τ. It is reasonable to expect that the dependence on τ is exponential so that the correlation can be written as $\overline{\hat{u}_x\hat{u}_x'}(\underline{\lambda},\tau) = R_{xx}(\underline{\lambda})\exp(-\omega_f\tau)$ (27) where $R_{xx}(\underline{\lambda}) = \overline{\hat{u}_x(\underline{y})\ \hat{u}_x(\underline{y}+\underline{\lambda})}$ is the simultaneous second-order velocity correlation, and ω_f is a characteristic frequency of the turbulence. Then eqs. 23, 26 and 27 combine to give the following equation for acoustic intensity:

$$I(r) = \frac{\rho_o\omega_f^4}{\pi^2 a_o^5 r^2 (1 - \frac{M_1 x_1}{r})^5} \int\int \{2R_{xx}^2 (1 - 4N\cos\phi) + \frac{1}{4}R_{xx}[U_xU_x'(1 - 2N\cos\phi) - \frac{N}{4}(U_xU_2' + U_2U_x')]\} d^3\underline{\lambda}\ d^3\underline{y}. \tag{28}$$

In this equation, τ has been equated to zero under the argument that the separation time τ is negligible over the volume of a turbulent eddy. For this argument to be reasonable, the convection Mach number M_1 should be small (see ref. 12, p. 9). The acoustic intensity is seen to consist of two primary contributions. The first involves R_{xx}^2 which gives the intensity which results from the interaction of turbulence with turbulence.

This contribution is identified as "self-noise". The second contribution results from the interaction of mean shear (i.e., spatial variation in the mean velocity) and turbulence. This contribution is called "shear-noise" and can be shown to be non-negligible only in the presence of mean shear.

The functional behavior of the second-order velocity correlation is known from the kinematics of isotropic turbulence (see ref. 10) to be the following:

$$R_{xx} = \overline{\hat{u}^2}[f(\lambda) + \frac{\lambda^2 - \lambda_x^{\ 2}}{2\lambda} \frac{\partial f(\lambda)}{\partial \lambda}] \tag{29}$$

where $\lambda = |\underline{\lambda}|$ is the distance between point P (where the turbulent velocity in the x-direction is $\hat{u}_x(\underline{y})$ and point P' (where the turbulent velocity in the x-direction is $\hat{u}_x(\underline{y} + \underline{\lambda})$, λ_x is the component of $\underline{\lambda}$ in the x-direction (see fig. 6), and $f(\lambda)$ is a spherically symmetric function which vanishes for large values of λ. A simple function which has the required properties for $f(\lambda)$ is the following (see ref. 8):

$$f(\lambda) = \exp\ (-\pi\lambda^2/L^2) \tag{30}$$

where L is a characteristic length for the turbulence. With this form for $f(\lambda)$, the velocity correlation is given by

$$R_{xx} = \overline{\hat{u}^2}\ [1 - \frac{\pi}{L^2}\ (\lambda^2 - \lambda_x^{\ 2})]\exp\ (- \frac{\pi\lambda^2}{L^2})\ . \tag{31}$$

The acoustic intensity in eq. 28 depends on the integral of functions of R_{xx} over all values of the separation vector $\underline{\lambda}$. This integration is easily performed using spherical coordinates. The self-noise contribution to the intensity from a unit volume of turbulence is calculated using eq. 31 to be

$$8(1 - 4N\cos\phi) \int R_{xx}^{\ 2}(\underline{\lambda})d^3\underline{\lambda} = 2^{\frac{1}{2}}(\overline{\hat{u}^2})^2 L^3(1 - 4N\cos\phi)\ . \tag{32}$$

In order to calculate the shear-noise contribution to the acoustic intensity, the variation of the mean velocity in the jet must be known. For weak crossflows the mean velocity is directed primarily in the direction of the jet at the exit plane (i.e., in a direction normal to that of the crossflow) for the first eight jet diameters downstream. It is therefore assumed that the mean velocity is directed in the x_1-direction only. It should be noted that this assumption has previously been made in conjunction with the eddy convection velocity. In its present context the assumption implies the neglect of the mean shear which results from the mean velocity in the crossflow direction. It is this mean shear which is responsible for rolling the edges of the jet into vortex-like structures. With this assumption, the mean velocity in the direction of the observer can be expressed in terms of the mean velocity in the x_1-direction (i.e., U_1) as follows:

$$U_x = U_1 \cos\theta$$

where θ is the polar angle between the observer and the x_1-axis (see fig. 5). This assumption implies the neglect of the terms involving $U_x U_2{}'$ and $U_2 U'_x$ in eq. 28. The data of Patrick (ref. 13) indicates that the mean velocity in the crossflow jet, as in the jet with no crossflow, varies rapidly across the jet but slowly along the jet. Ribner's model (see ref. 8) for the variation of mean velocity in the jet with no crossflow can therefore be used for the crossflow jet. In this model the mean velocity is considered to vary rapidly across the jet but no variation is included along the jet. The specific form for the mean velocity variation is the following:

$$U_1 U_1{}' = U_1{}^2 \exp\left(-\frac{\sigma\pi x_2{}^2}{L^2}\right)$$

where σ is a parameter which is related to the eddy size of the turbulence. This approximation is only expected to be valid over the dimension of a typical turbulent eddy. The above model for the mean shear gives the following shear-noise contribution to the acoustic intensity from a unit volume of turbulence:

$$(1 - 2N\cos\phi)\cos^2\theta \int R_{xx}\, U_1 U_1{}' d^3\underline{\lambda} = \frac{U_1{}^2\, \overline{\hat{u}^2}\, L^3 \sigma}{2(1 + \sigma)^{3/2}} (1 - 2N\cos\phi) \cos^4\theta \, . \qquad (33)$$

This contribution to the acoustic intensity is equal to zero when σ equals zero which corresponds to the case of a uniform flow; therefore, this contribution is closely associated with the presence of mean shear.

The results given in eqs. 32 and 33 combine in eq. 28 to give the intensity per unit volume of turbulence as follows:

$$i(r,\theta,\phi) = \frac{\rho_o \omega_f^4 (\overline{\hat{u}^2})^2 L^3}{2^{3/2}\pi^2 a_o^5 r^2 (1-M_1\cos\theta)^5} \left[(1-4N\cos\phi) + \frac{U_1^2 \sigma}{2^{3/2}(1+\sigma)^{3/2}\, \overline{\hat{u}^2}} \times (1-2N\cos\phi)\cos^4\theta\right]$$

where $M_1 \cos\theta$ has replaced $M_1 x_1/r$. The factor multiplying the shear-noise contribution contains σ which is related to the eddy size. As stated by Ribner, this factor is difficult to determine but is estimated to be equal to unity in the mixing region of the jet. With this estimate, the acoustic intensity per unit volume of turbulence is given by

$$i(r,\theta,\phi) \simeq \frac{\rho_o \omega_f^4 (\overline{\hat{u}^2})^2 L^3}{2^{3/2} \pi^2 a_o^5 r^2} \frac{[(1-4N\cos\phi) + (1-2N\cos\phi)\cos^4\theta]}{(1 - M_1\cos\theta)^5} . \tag{34}$$

This equation is the desired result for acoustic intensity in the wind tunnel configuration.

RESULTS

The acoustic intensity in a moving medium from a unit volume of turbulence in a jet flow is given by eq. 34. The medium has a velocity $a_o N$ in the positive x_2-direction, and the convection velocity of a turbulent eddy is $a_o M_1$ in the positive x_1-direction. The directional distribution of the acoustic intensity at a constant distance from the jet is given by the following expression:

$$\frac{(1 + \cos^4\theta) - 2N\cos\phi(2 + \cos^4\theta)}{(1 - M_1\cos\theta)^5} \tag{35}$$

where ϕ is the polar angle between the wind direction and the observer, and θ is the polar angle between the jet exit direction and the observer (see fig. 5). This pattern is plotted on fig. 7 for an observer in the plane of the jet where $\phi = \frac{\pi}{2} - \theta$. The reference intensity is the pattern given by $(1 + \cos^4\theta)$ which is referred to by Ribner as the "basic jet directivity". The curve of zero dB therefore indicates the intensity pattern which would be emitted by a "jet" with $M_1 = N = 0$. The intensity patterns from a jet with $M_j = 0.54$ are presented on fig. 7 for two values of N. The pattern given by $N = 0$ represents the distribution of acoustic intensity from a straight jet and is symmetric about the x_1-axis. The Doppler factor $(1 - M_1\cos\theta)^{-5}$ increases the intensity by a large amount in front of the jet and decreases the intensity behind the jet. The second pattern shown on fig. 7 is the acoustic intensity from a jet in a weak crossflow $(V_j/V_2 \cong 8)$. The crossflow Mach number of $N = 0.07$ produces an asymmetric pattern with a greater intensity in the upstream direction and a reduced intensity in the downstream direction. Even for this weak crossflow, the difference in acoustic intensity measured at corresponding positions upstream and downstream is as much as 1.5 dB (at $\theta = 315^{\circ}$ and $= 45^{\circ}$). As discussed previously, this upstream shift results from the convection of signals by the acoustic medium.

Under the assumption of a weak crossflow, the acoustic field which is generated by a crossflow jet in motion (i.e., the flight configuration) can be calculated directly using the analyses of Lighthill, Ffowcs Williams, and Ribner (refs. 5, 6, and 8, respectively). A summary of this calculation is presented in the Appendix of this report. In the calculation the velocity of the aircraft is a_oN in the negative x_2-direction while the turbulent flow is characterized by an eddy convection velocity of a_oM_1 in the positive x_1-direction. The directional distribution of the acoustic intensity at a constant distance from the aircraft is obtained from eq. A9 to be the following:

$$\frac{(1 + \cos^4\theta)}{(1 + N\cos\phi)(1 - M_1\cos\theta)^5} \qquad (36)$$

where θ is the polar angle between the jet axis and the observer position and ϕ is the polar angle between the positive x_2-direction and the observer.

The distribution of acoustic intensity for an observer in the plane of the jet (i.e., the plane in which $\phi = \frac{\pi}{2} - \theta$) is presented on fig. 8. The reference intensity is again taken to be the basic jet directivity $(1 + \cos^4\theta)$. With no aircraft velocity (i.e., N = 0), the intensity pattern for the same jet as that on fig. 7 (i.e., $M_j = 0.54$) is identical with that described for the wind tunnel configuration with N = 0. The aircraft Mach number of 0.07 shifts the pattern slightly in the direction of motion of the aircraft.

The difference in intensity between the two patterns shown on fig. 7 is greater than the corresponding difference on fig. 8. That is, a cross-flow of small Mach number in the wind tunnel configuration produces a greater change in acoustic intensity than an equal aircraft Mach number in the corresponding flight configuration. This can be demonstrated quantitatively when N is small compared to unity (i.e., $N \ll 1$) by expanding the denominator of eq. 36 to give the following:

$$\frac{(1 + \cos^4\theta) - N\cos\phi(1 + \cos^4\theta)}{(1 - M_1\cos\theta)^5} \tag{37}$$

The difference between eq. 37 and eq. 35 is the difference between the directional distribution of intensity for the wind tunnel configuration and the flight configuration. In the plane of the jet this difference is

$$-\frac{N\sin\theta(3 + \cos^4\theta)}{(1 - M_1\cos\theta)^5} \quad . \tag{38}$$

This expression is zero for $N = 0$, $\theta = 0$, and $\theta = \pi$. For values of $0<\theta<\pi$ the intensity in the wind tunnel is less than that for a flight test while for $\pi<\theta<2\pi$ the intensity is greater for the wind tunnel test. That is, the asymmetry of the intensity pattern for a given value of N is greater for the wind tunnel configuration than for the flight configuration. This results provides a comparison between wind tunnel measurements and measurements in a flight test for small values of N.

CONCLUDING REMARKS

The result of the analysis for the directivity of the acoustic intensity from a crossflow jet in the wind tunnel configuration is given by eq. 35. This equation can be compared with the corresponding result for the flight configuration given by eq. 36. It has been necessary to make several assumptions in deriving these equations which are known to be not strictly valid for the crossflow jet. Equations 35 and 36 therefore must be considered to provide only a qualitative prediction of the acoustic intensity. The values of the constants and parameters which multiply the directional distribution of the intensity in eqs. 34 and A9 give at best an order of magnitude estimate for the acoustic intensity. It is felt that the directional factor gives a reasonable prediction of the directivity from the crossflow jet in the absence of reflecting surfaces. This belief in the basic correctness of the directivity patterns is based on Ribner's comparisons (see ref. 8, p. 130) of the directivity of the straight jet (i.e., no crossflow) with experimental data. In this comparison, the patterns of acoustic intensity which are calculated using the directional distribution of eq. 35 with $N = 0$ give reasonable agreement over most values of θ with experimental data taken on straight jets. For angles near the jet axis, however, the theoretical distribution predicts much larger values for acoustic intensity than are observed. Ribner attributes this discrepancy to refraction of sound by the mean flow of the jet. In this report, refractive effects are not considered by the acoustic theory, and similar discrepancies are expected to occur.

Improvement of this analysis is possible when a more complete picture of the crossflow jet is available. One area for improvement is the possible refraction of noise emitted by the jet which results from the deviation of the crossflow around the jet. Since the jet flow acts as a partial obstacle to the crossflow, there exist gradients in the crossflow velocity near the jet flow. This variation in the crossflow may result in a refraction of the sound generated in the jet and provide another mechanism for the distortion of the sound field.

It should be recalled that in the calculations of the results, curvature of the jet by the wind has been neglected. This has been justified by restricting the analysis to weak crossflows and postulating that the noisy regions of the jet should be the first eight jet diameters downstream of the exit. Curvature of the jet flow, however, could interfere with the propagation of signals by shielding some observer positions from signals emitted from the jet.

The results of this analysis for the directional distribution of acoustic intensity from a jet in a weak crossflow provide a comparison for corresponding wind tunnel and flight measurements. As demonstrated on figs. 7 and 8 and eq. 38, the wind tunnel configuration is more "efficient" at shifting the intensity distribution than the flight configuration for a given value of N. The effect of the convection of signals by the acoustic medium in the wind tunnel is therefore an important consideration in interpreting acoustic data taken in the wind tunnel tests.

APPENDIX

Acoustic Analysis for the Flight Configuration

The acoustic analysis presented here is a summary of the contributions of Lighthill, Ffowcs Williams, and Ribner. The observer (e.g., a microphone) is at rest in a uniform acoustic medium which is also at rest. The source of noise is a turbulent flow which can be thought of as being generated by an engine of a moving aircraft. This source will be modeled by the flow from a cold subsonic jet.

The analysis of the acoustic field generated by turbulent flows has been formally developed by Lighthill (refs. 5, 6). In his development, the continuity equation and the momentum equation are combined to give the following governing equation:

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{A1}$$

where a_o is the speed of sound in the acoustic medium, $T_{ij} = P_{ij} + \rho u_i u_j - a_o^2 \rho \delta_{ij}$ and for a Newtonian fluid,

$$P_{ij} = P\delta_{ij} + \mu \left[- \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} + \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij}\right] .$$

The left-hand side of eq. A1 represents the propagation of acoustic waves through a stationary-uniform medium. The right-hand side has the form of a quadrupole source which is at rest. The quantity T_{ij} is a stress produced by pressure, viscous shear stresses, and momentum flux.

The solution to eq. A1 is given as follows:

$$\rho(\underline{x},t) - \rho_o = \frac{1}{4\pi a_o^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \frac{T_{ij}\left(\underline{y}, t - \frac{|\underline{x}-\underline{y}|}{a_o}\right)}{|\underline{x}-\underline{y}|} d^3\underline{y} \tag{A2}$$

where ρ_o is the undisturbed density of the medium, $\underline{x}$ is the position of the observer, and $\underline{y}$ is the position in the turbulent region (see fig. 9). The integral is evaluated over the entire region of turbulence.

The solution given by eq. A2 expresses the density disturbance at point $\underline{x}$ and time t which results from a region of turbulence which is at rest relative to the observer. If the turbulence is being generated by the engines of an airplane which is in motion relative to the observer, the solution must be altered. This situation is considered by Ffowcs Williams (ref. 6). A new coordinate system is defined which moves with the aircraft at velocity $-a_o\underline{N}$. In the new coordinate system, a point in the turbulence which is moving along with the aircraft is identified by the position vector $\underline{\eta}$. The relationship between the coordinate systems is $\underline{y} = \underline{\eta} - a_o\underline{N}t$ when both specify the same position in the turbulence. Signals which are received by the observer at time t were emitted at time $t - \frac{|\underline{x}-\underline{y}|}{a_o}$. At this time, the relationship between the coordinates is $\underline{y} = \underline{\eta} - a_o\underline{N}\,t + \underline{N}\,|\underline{x}-\underline{y}|$ as indicated on fig. 9. In terms of the new coordinate system, T_{ij} becomes a function of position $\underline{\eta}$ and the retarded time $t - \frac{|\underline{x}-\underline{y}|}{a_o}$. The coordinate transformation has the property of changing the volume element in the turbulent region with the relationship between $d^3\underline{y}$ and $d^3\underline{\eta}$ given by the Jacobian of the coordinate transformation as follows:

$$d^3\underline{\eta} = d^3\underline{y}\,\left[1 + \frac{\underline{N}\cdot(\underline{x}-\underline{y})}{|\underline{x}-\underline{y}|}\right] .$$

The solution for the density disturbance which results from a turbulent region of fluid moving with the aircraft is

$$\rho(\underline{x},t) - \rho_o = \frac{1}{4\pi a_o^2}\frac{\partial^2}{\partial x_i \partial x_j}\int \frac{T_{ij}\left(\underline{\eta}, t - \frac{|\underline{x}-\underline{y}|}{a_o}\right)}{\left[|\underline{x}-\underline{y}| + \underline{N}\cdot(\underline{x}-\underline{y})\right]} d^3\underline{\eta} \quad . \tag{A3}$$

This solution can be given in a different form by carrying out the differentiation of the integrand. If the observer position is restricted to the acoustic far field of the source, the solution is given by the following:

$$\rho(\underline{x},t) - \rho_o = \frac{1}{4\pi a_o^4} \int \frac{(x_i-y_i)(x_j-y_j)}{[|\underline{x}-\underline{y}| + \underline{N}\cdot(\underline{x}-\underline{y})]^3} \frac{\partial^2 T_{ij}}{\partial t_\alpha^2} (\underline{\eta},t_\alpha)\, d^3\underline{\eta} \tag{A4}$$

where $t_\alpha = t - \frac{|\underline{x}-\underline{y}|}{a_o}$. The Doppler factor $[1 + \frac{\underline{N}.(\underline{x}-\underline{y})}{|\underline{x}-\underline{y}|}]^{-3}$ represents the influence of the movement of the sources of the sound field.

The expression for the acoustic intensity is obtained from eq. A4 to be the following:

$$I = \frac{1}{16\pi^2 a_o^5 \rho_o} \int\int \frac{(x_i-y_i)(x_j-y_j)(x_k-z_k)(x_\ell-z_\ell)}{[|\underline{x}-\underline{y}| + \underline{N}\cdot(\underline{x}-\underline{y})]^3[|\underline{x}-\underline{z}| + \underline{N}\cdot(\underline{x}-\underline{z})]^3} \overline{\frac{\partial^2 T_{ij}(\underline{\eta},t_\alpha)}{\partial t_\alpha^2} \frac{\partial^2 T_{k\ell}(\underline{\xi},t_\beta)}{\partial t_\beta^2}}\, d^3\underline{\eta}\, d^3\underline{\xi} \tag{A5}$$

where $t_\beta = t - \frac{|\underline{x}-\underline{z}|}{a_o}$ and $\underline{z}$ and $\underline{\xi}$ have the same relationship as $\underline{y}$ and $\underline{\eta}$. The equation for the acoustic intensity can be put into a more useful form following the development of Ffowcs Williams. The procedure is identical to that discussed previously in this report in conjunction with the wind tunnel configuration. A time difference $\tau = t_\beta - t_\alpha$ is defined and the random turbulence variables are assumed to be stationary functions of time in the $\underline{\eta}$ and $\underline{\xi}$ coordinates. A separation vector $\underline{\Delta}$ is introduced which is related to $\underline{\eta}$ and $\underline{\xi}$ as follows: $\underline{\Delta} = \underline{\xi} - \underline{\eta}$. The acoustic intensity is then expressed in terms of the correlation function given by $R_{ijk\ell}(\underline{\eta},\underline{\Delta},\tau) = \overline{T_{ij}(\underline{\eta},t_\alpha)T_{k\ell}(\underline{\eta}+\underline{\Delta},t_\alpha+\tau)}$ as follows:

$$I = \frac{1}{16\pi^2 a_o^5 \rho_o} \int\int \frac{(x_i-y_i)(x_j-y_j)(x_k-z_k)(x_\ell-z_\ell)}{[|\underline{x}-\underline{y}| + \underline{N}\cdot(\underline{x}-\underline{y})]^3[|\underline{x}-\underline{z}| + \underline{N}\cdot(\underline{x}-\underline{z})]^3} \frac{\partial^4}{\partial\tau^4} R_{ijk\ell}(\underline{\eta},\underline{\Delta},\tau)\, d^3\underline{\eta}\, d^3\underline{\Delta}. \tag{A6}$$

As pointed out by Ffowcs Williams, the solution in this form is not suited for the calculation of the acoustic intensity generated by turbulence which is convected relative to the aircraft. The effect of eddy convection is included by defining a new separation vector $\underline{\lambda}$ in terms of a coordinate system which moves relative to the aircraft at the velocity of convection of a turbulent eddy, $a_o\underline{M}$. A new correlation function $P_{ijk\ell}(\underline{\eta},\underline{\lambda},\tau)$ is defined by expressing $\underline{\Delta}$ in terms of $\underline{\lambda}$ and τ with the simplification that the separation $\underline{\Delta}$ should be small compared to $(\underline{x}-\underline{y})$. The result of this calculation is given by Ffowcs Williams to be the following equation for the acoustic intensity:

$$I = \frac{1}{16\pi^2 a_o^5 \rho_o} \int\int \frac{(x_i-y_i)(x_j-y_j)(x_k-y_k)(x_\ell-y_\ell)}{[|\underline{x}-\underline{y}|+\underline{N}\cdot(\underline{x}-\underline{y})][|\underline{x}-\underline{y}|-\underline{M}\cdot(\underline{x}-\underline{y})]^5} \frac{\partial^4}{\partial\tau^4} P_{ijk\ell}(\underline{\eta},\underline{\lambda},\tau)\,d^3\underline{\eta}\,d^3\underline{\lambda}\,. \tag{A7}$$

This equation is simplified further by restricting the observer position to distances from the turbulence which are large compared to the dimensions of the turbulent flow field. With this restriction, the acoustic intensity is given by

$$I(r) = \frac{1}{16\pi^2 a_o^5 \rho_o r^6} \frac{x_i x_j x_k x_\ell}{\left(1 + \frac{\underline{N}\cdot\underline{x}}{r}\right)\left(1 - \frac{\underline{M}\cdot\underline{x}}{r}\right)^5} \int\int \frac{\partial^4}{\partial\tau^4} P_{ijk\ell}(\underline{\eta},\underline{\lambda},\tau)\;d^3\underline{\eta}\;d^3\underline{\lambda} \tag{A8}$$

where $r = |\underline{x}|$. The acoustic intensity which is calculated from eq. A8 is then applicable to the acoustic field generated by the turbulent flow from the engines of an aircraft which is traveling at a velocity $-a_o\underline{N}$ through a uniform stationary medium. The calculated intensity would be measured by an observer at rest in the uniform medium.

In order to use Ribner's method of modeling the correlation function $P_{ijk\ell}(\underline{\eta},\underline{\lambda},\tau)$ (ref. 8), the turbulence in the flow from the jet is assumed to be convected with a velocity a_oM_1 only in the x_1-direction. Again, this assumption implies that the curvature of the jet flow resulting from the crossflow is small (i.e., the crossflow is weak). The velocity of the aircraft motion is given by a_oN in the negative x_2-direction. With the assumption of a weak crossflow, Ribner's model for the correlation function is applicable to the crossflow jet. The arguments which lead to this model have been outlined previously. The result is the following equation for the acoustic intensity which would be emitted by a unit volume of turbulence in the mixing region of the jet:

$$I(r,\theta,\phi) \simeq \frac{\rho_o \omega_f^{\,4} (\overline{\hat{u}^2})^2 L^3}{2^{3/2} \pi^2 a_o^{\,5} r^2} \; \frac{(1 + \cos^4\theta)}{(1 + N\cos\phi)(1 - M_1 \cos\theta)^5} \tag{A9}$$

where ϕ is the polar angle between the observer and the x_2-axis and θ is the polar angle between the observer and the direction of the convection of the turbulent flow (i.e., the x_1-axis). This result provides the acoustic intensity per unit volume of turbulence in a jet under flight conditions. It provides the comparison to eq. 34 for the intensity from a jet in the wind tunnel configuration.

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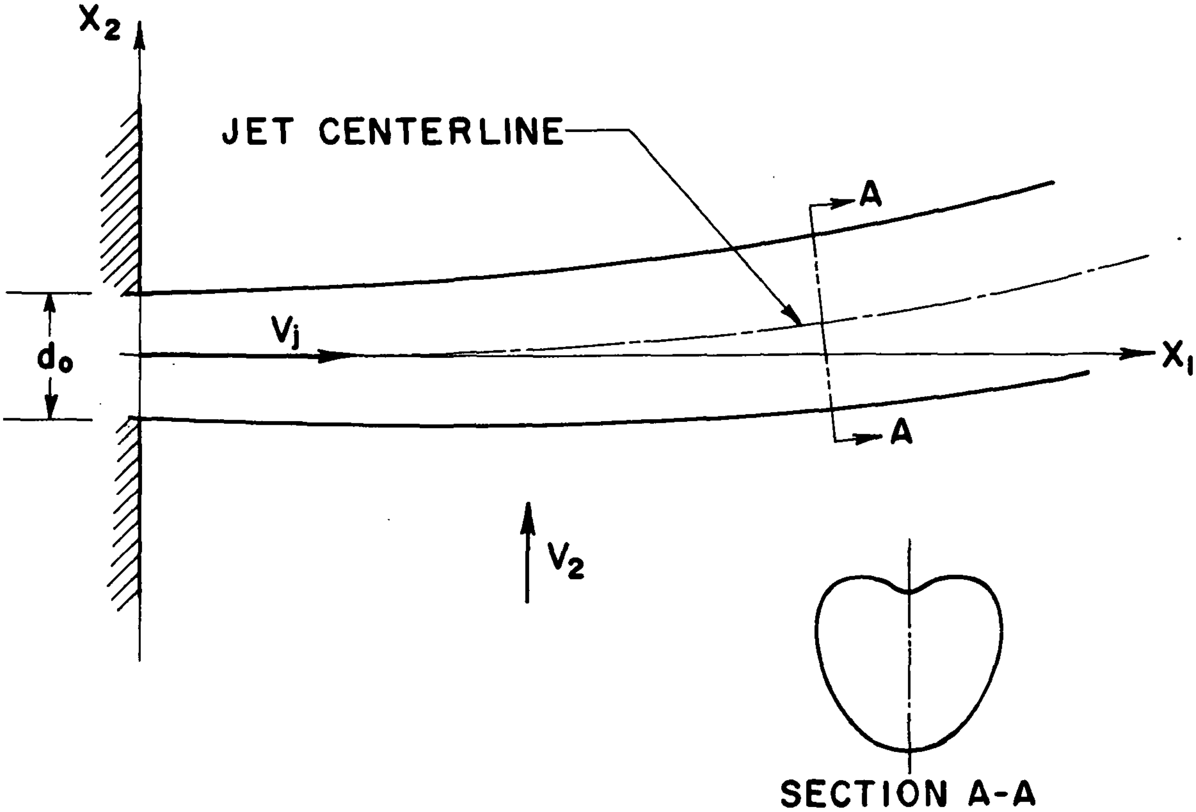

FIGURE 1. Diagram of a circular jet exhausting into a crossflow.

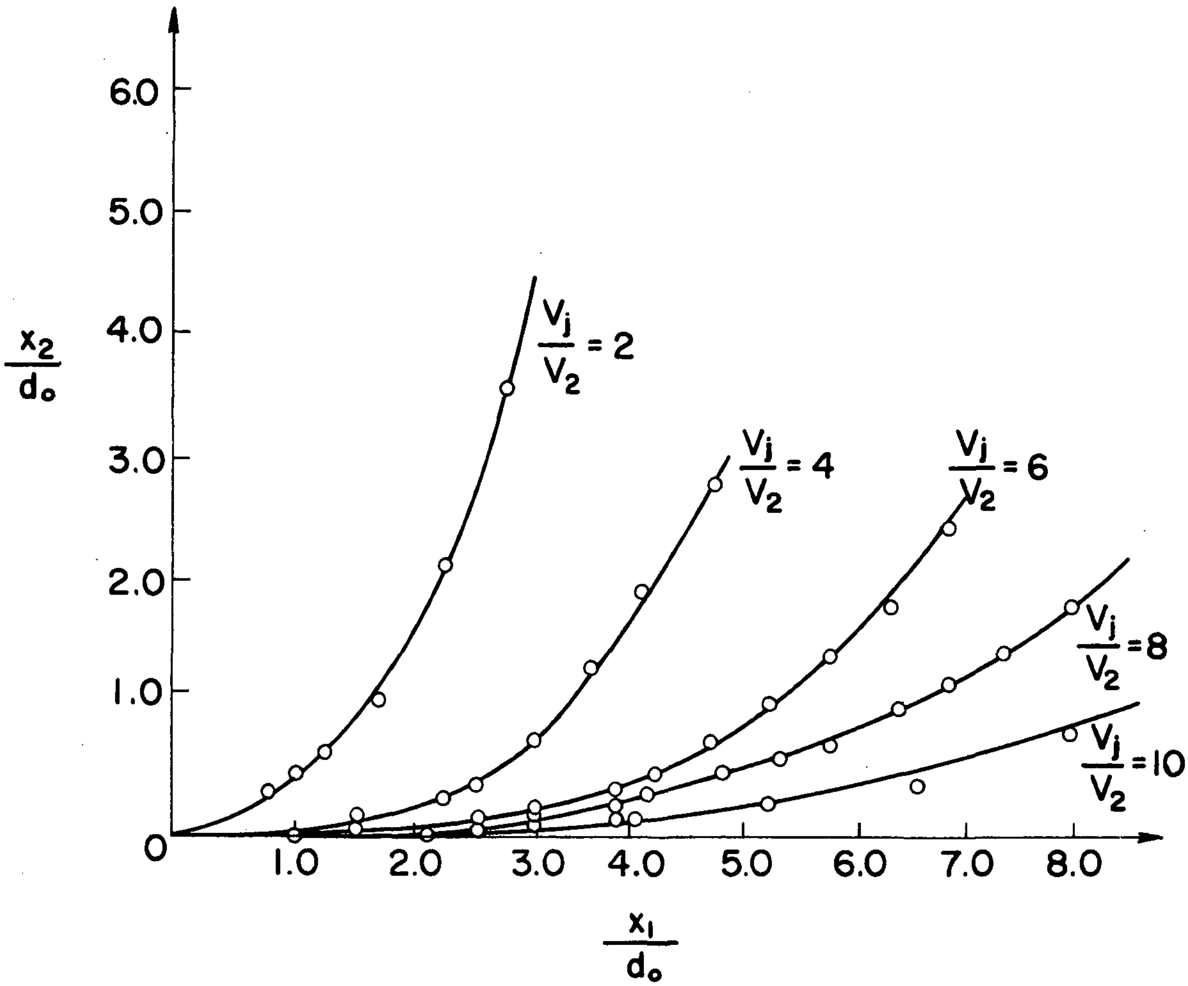

FIGURE 2. Jet centerline as determined by measurements of maximum velocity for several values of V_j/V_2.

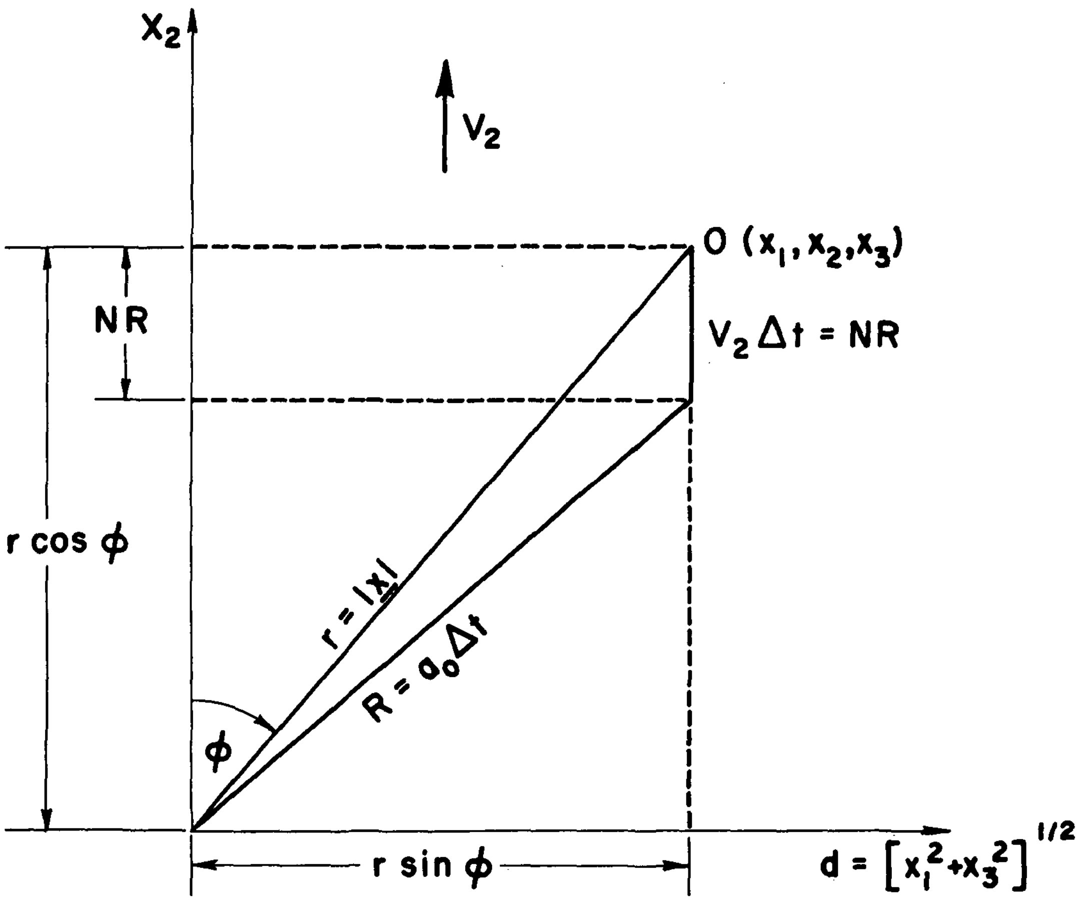

FIGURE 3. Relationship between R, r, and ϕ for wave propagation in a uniformly moving acoustic medium.

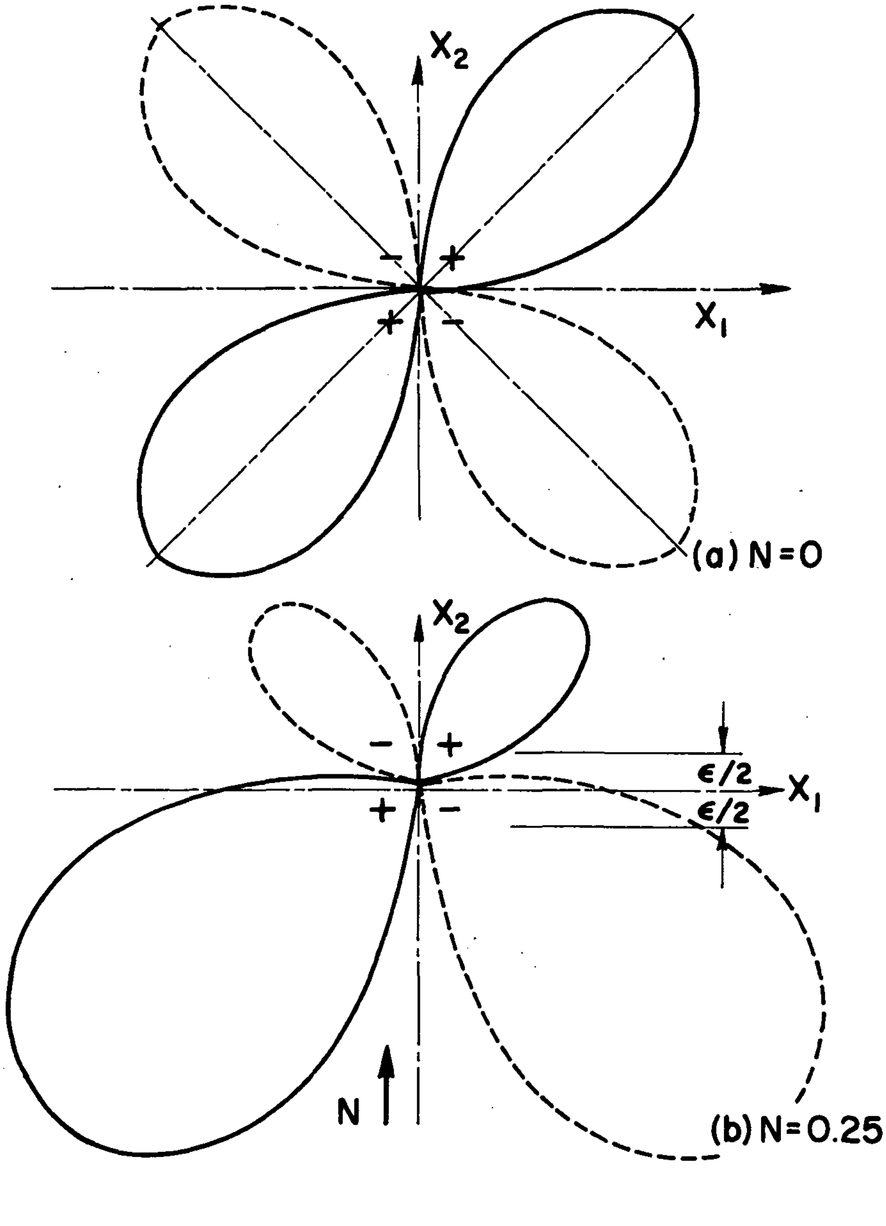

FIGURE 4. Distribution of acoustic density disturbance in the far field for the lateral quadrupole T_{12} located at the origin.
(———— $\rho - \rho_o > 0$; ---- $\rho - \rho_o < 0$)

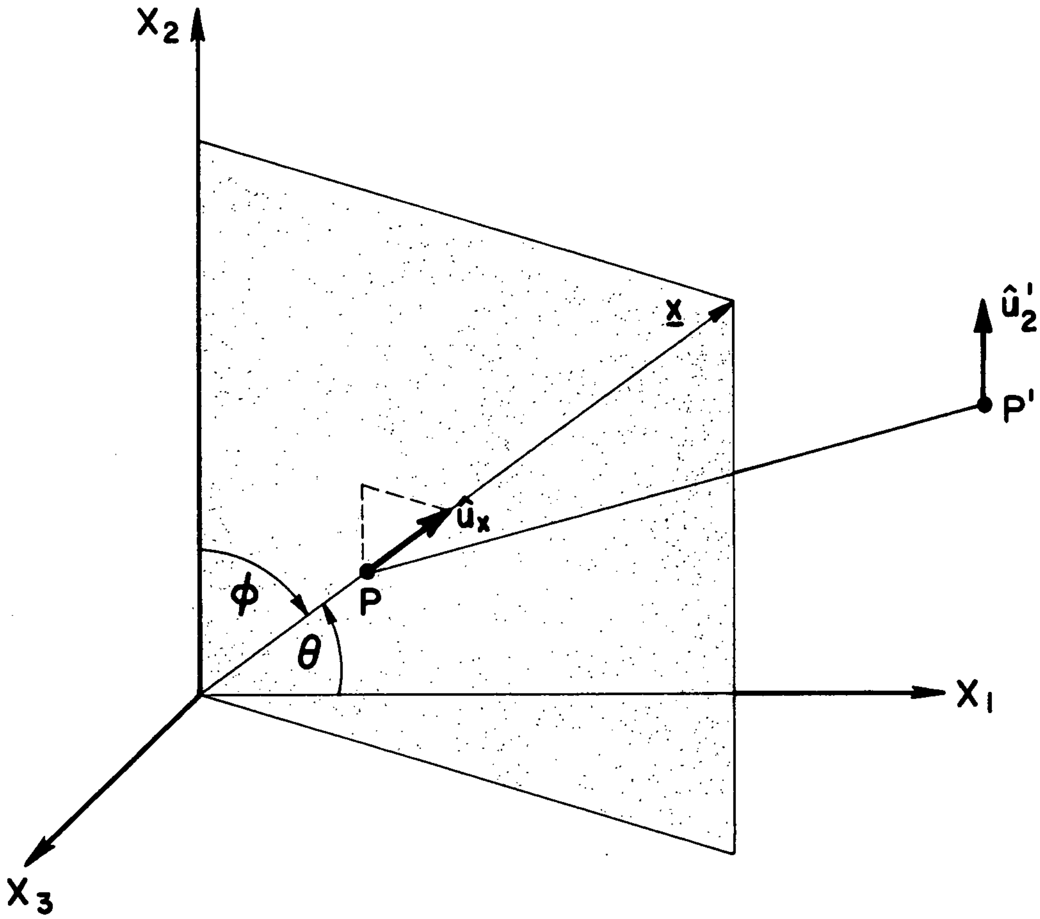

FIGURE 5. Velocity correlation in isotropic turbulence.

$(\overline{\hat{u}_x \hat{u}'_2} = \overline{\hat{u}_2 \hat{u}'_x} = \overline{\hat{u}_x \hat{u}'_x} \cos\phi)$

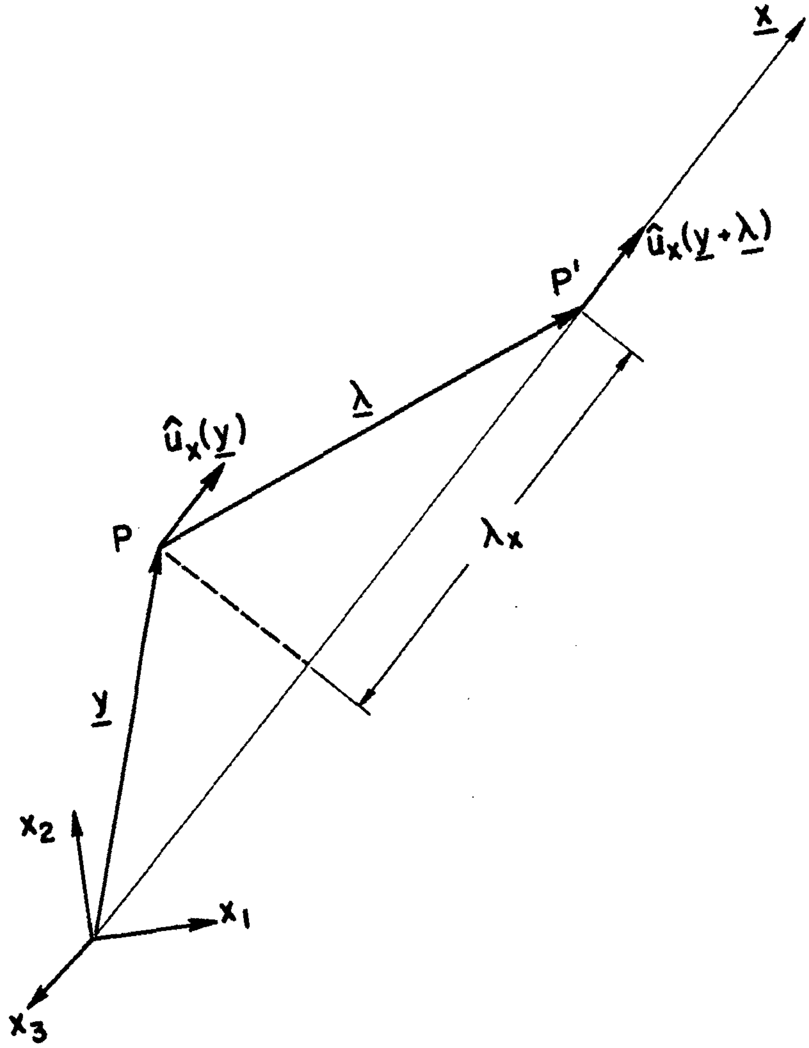

FIGURE 6. Description of the second-order velocity correlation R_{xx}.

$$\left(R_{xx} = \overline{\hat{u}_x(\underline{y})\ \hat{u}_x(\underline{y} + \underline{\lambda})} = \overline{\hat{u}_x{}^2}\ \left[f(\lambda) + \frac{\lambda^2 - \lambda_x{}^2}{2\lambda}\ \frac{\partial f(\lambda)}{\partial\lambda}\right]\right)$$

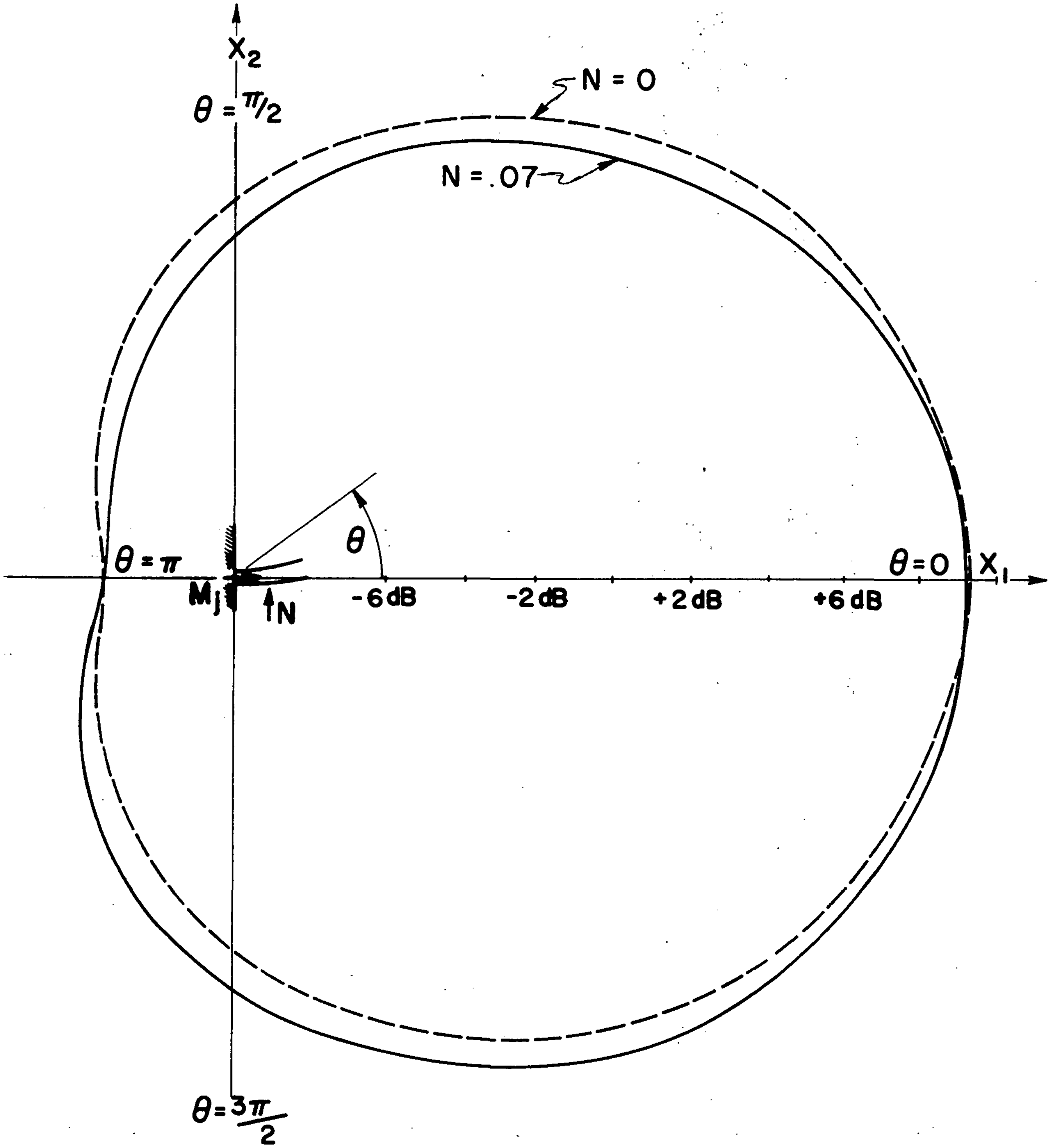

FIGURE 7. Directional distribution of acoustic intensity in the plane of a crossflow jet in a wind tunnel - $\dfrac{(1+\cos^4\theta) - 2\ N\sin\theta\ (2+\cos^4\theta)}{(1 - M_1\cos\theta)^5}$. ($M_j = 0.54$; $M_1 = 0.65M_j$)

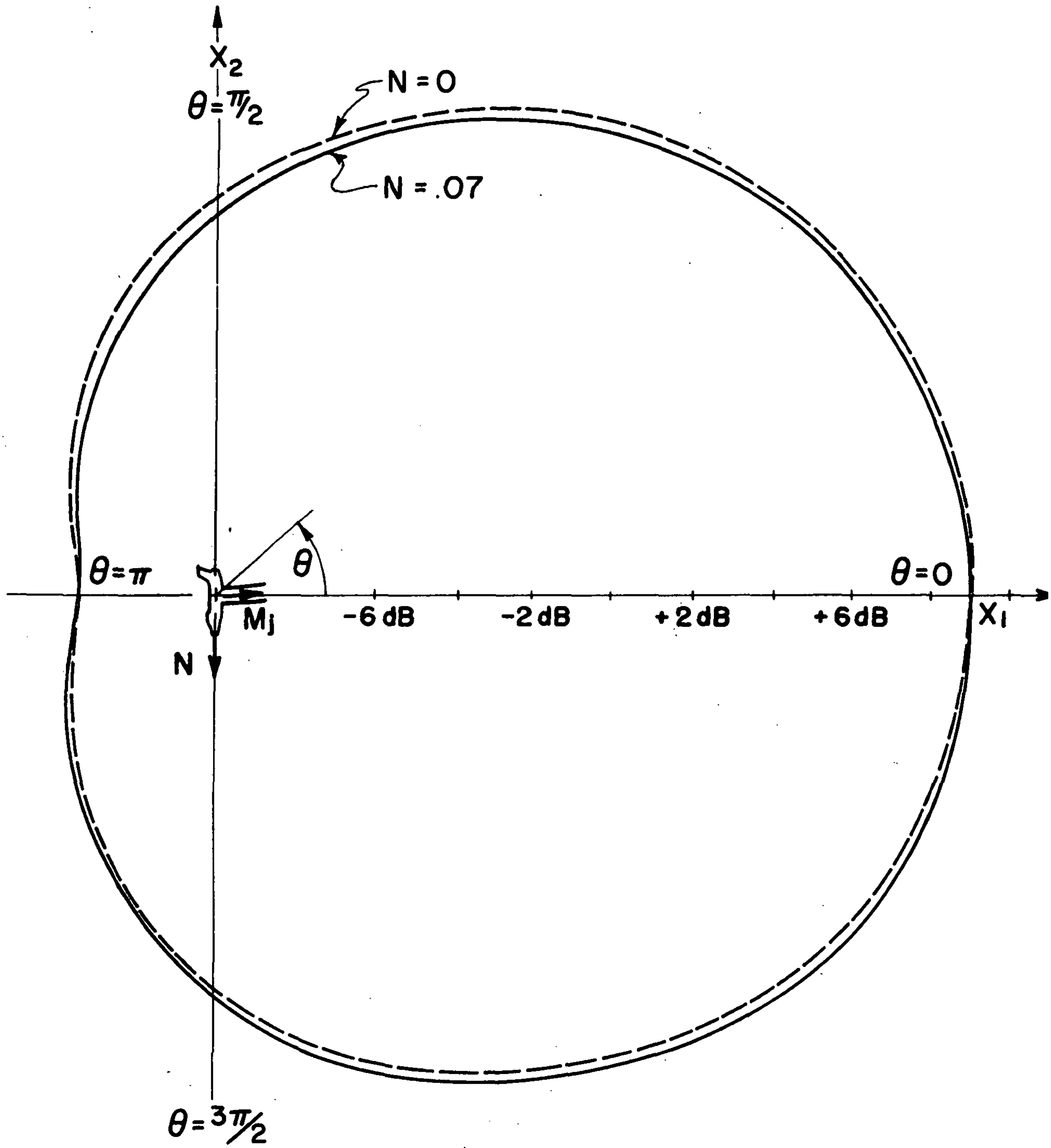

FIGURE 8. Directional distribution of acoustic intensity in the plane of a crossflow jet under flight conditions - $\dfrac{(1 + \cos^4\theta)}{(1+N \sin\theta)(1-M_1 \cos\theta)^5}$ $(M_j = 0.54;\ M_1 = 0.65\ M_j)$

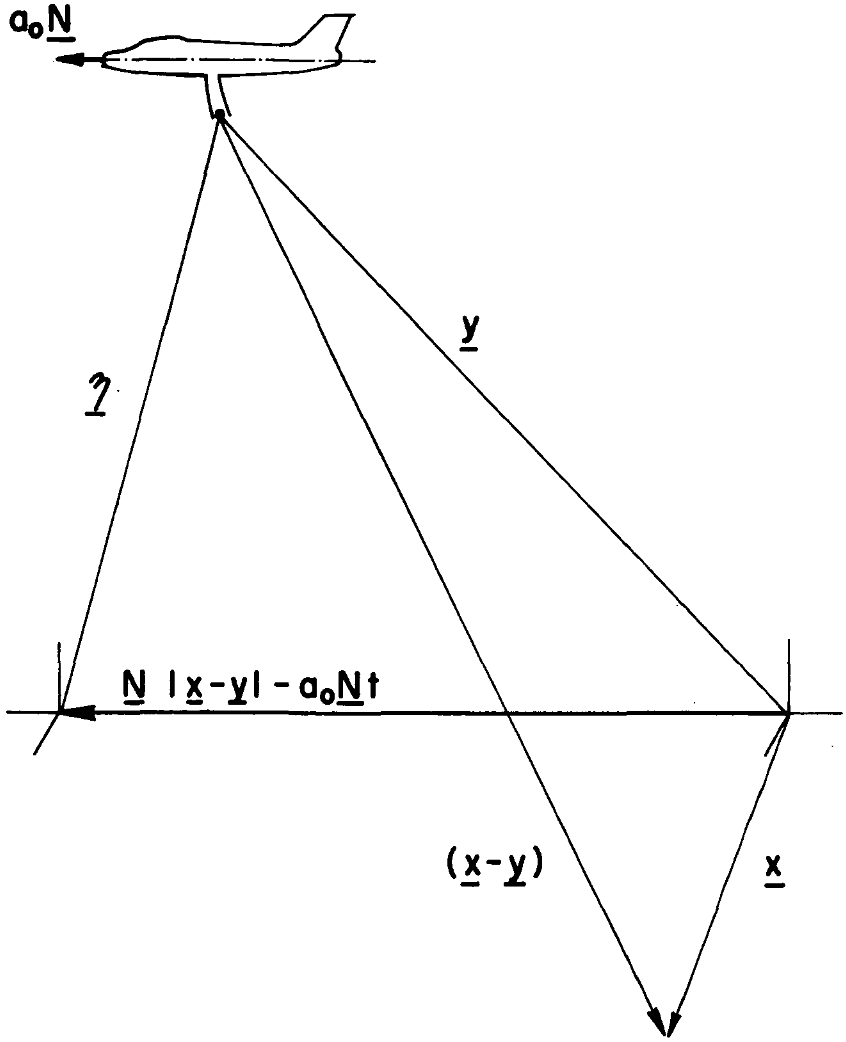

FIGURE 9. Diagram showing the relationship between the coordinate systems for $\underline{y}$ and $\underline{\eta}$ at the time when the signal which arrives at the observer position $\underline{x}$ was emitted.

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