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SYSTEMS SIMULATION FOR AN AIRPORT TRAILING VORTEX WARNING SYSTEM

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George C. Marshall Space Flight Center
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16. ABSTRACT

This report documents the approach, development, and limited system studies associated with a system simulation for an Airport Trailing Vortex Warning System (ATVWS). It attempts to show the usefulness of a systems engineering approach to the problem of developing a system, as dictated by aircraft vortices, which will increase air-traffic flow in the takeoff/landing corridors of busy airports while maintaining the required safety factor for each operation.

It is felt that the development and integration of a total system simulation computer program are essential to provide the system designers a way to develop proper and realistic ATVWS requirements to meet the objectives of decreasing aircraft spacings on takeoff and landing while maintaining an adequate safety margin.

This report documents the capabilities (assumptions and limitations) provided by the Total System Simulation Model. The simulation provides the capability of investigating potential problem areas at a fraction of the cost that would be involved in hardware tests only.

The simulation program has been developed in a modular form which permits new, more sophisticated component models, when they become available and are required, to be incorporated into the program with a minimum of program modifications. This report documents a limited system study that has been performed using this Total System Simulation Model. The resulting preliminary system requirements, conclusions, and recommendations are given in Section V.

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SYSTEMS SIMULATION FOR AN AIRPORT TRAILING VORTEX WARNING SYSTEM

I. INTRODUCTION

During recent years, there has been a continuously increasing number of reports [1, 2, 3, 4] by pilots who have encountered severe turbulence in the wake of other aircraft, even when their aircraft were separated from the generating aircraft by several miles. There has also been an increase in accidents resulting in damage to aircraft and fatality to passengers that are attributable to encounters with high-velocity vortices on or near the ground.

The vortex problems in an airport terminal area are magnified many times compared to cross-country operations. There are many more types of aircraft operations taking place in a much more confined area. High-velocity turbulence near the ground makes landings and takeoffs especially hazardous because: (1) the aircraft are operating at low forward speeds resulting in slowed aircraft response which makes it more difficult to recover from the rolling or settling motions caused by vortex encounters; (2) the aircraft are operating near stall speeds; (3) and, presently, there is no way to detect and interpret these vortex systems.

Several investigations have been made to assess the problems associated with vortex systems in the terminal area [5, 6 (Appendix A), 7, 8, 9, 10, 11, 12, 13]. The present Federal Aviation Administration (FAA) operational procedures of providing specified minimum spacing between aircraft of different sizes for the various modes of operation (enroute, landings, and takeoffs) have been successful in minimizing accidents from all causes.

There are two factors that significantly affect the air-traffic control problem, especially in the terminal area. One is that the continual increase in air traffic calls for the absolute minimum spacing between aircraft for both landing and takeoff operations in the busier airports. The other is the coming of larger jetliners (the required mixing of large with small aircraft) and the supersonic transports.

It is believed that the spacing between aircraft could be decreased in the terminal area (on takeoffs and landings), thus increasing the flow of traffic, if accurate information were available as to the presence, location, and intensity of turbulence in those areas. Also, it is believed that such information would lead to fewer accidents.

The laser doppler techniques developed by MSFC scientists during recent years to measure airflow about models in wind tunnels and wind velocities in the atmosphere are believed to be directly applicable to the measurement of vortex system location, transport, velocity, structure, and decay. It is for this reason and the complexity of the associated problem that a total systems simulation model was developed and a preliminary system study was performed to determine if the laser doppler technique could be employed in an Airport Trailing Vortex Warning System (ATVWS).

The same systems simulation model, now developed, could be used to evaluate other remote sensors, provided theoretical models were available, for potential use in an ATVWS.

II. SYSTEMS ENGINEERING APPROACH TO THE PROBLEM

In considering the problem of defining requirements for the ATVWS, it was considered important to approach it with a normal systems engineering approach. Rather than approaching the problem from the viewpoint of some type of sensor, how can we build a system, it is more appropriate to take a "top down" view. Given the problem of protecting against the hazards of aircraft trailing vortices in the airport terminal area, what are the specifications that should be levied on the system and its subsystems or components?

The systems engineering approach taken for this study is as follows:

a. Defining in quantitative terms the need to be met by the system.

b. Defining criteria to judge the effectiveness of the system.

c. Defining potential subsystems to be treated as candidate solutions.

d. Modeling the proposed systems.

e. Conducting simulations with the model.

f. Evaluating system performance against the effectiveness criteria.

g. Selecting the system concept.

h. Defining the system/subsystem specifications and requirements.

The advantage of this systems engineering approach is that by using general-purpose computer simulations (digital and/or analog), it is possible to conceive of a system and to develop its requirements in a systematic and rational way with less cost before being committed to specific hardware for the system. Obviously, there are certain assumptions and judgements that must be inserted into these simulations before the actual hardware has been built and/or combined into a candidate system. However, the care with which the problem is approached can minimize the risk of these uncertainties.

As indicated above, the systems engineering approach involves development of a model to test alternatives. The application of this principle to the ATVWS problem is illustrated in Figure 1. The overall layout of the systems study simulation tool is shown in Figure 2. It was felt that the development and integration of a total systems simulation ("top down" approach) computer program was essential to provide the systems designer with a way to develop proper and realistic ATVWS requirements to meet the objectives of decreasing aircraft spacing on takeoff and landing while maintaining proper safety. The simulation provides the capability of investigating potential problem areas and determining their significance as related to other areas (within the limits of the simulation) at a fraction of the cost that would be involved in hardware only tests.

By employing the systems simulation tools, a number of candidate systems can be tested and traded off in a relatively short period of time. Significant insight can also be developed into just what factors are important, what is the sensitivity to subsystem characteristics, and what is the effect of parameter variations on the effectiveness and capability of the overall system. This would allow more time for additional conventional tests with hardware if desired and would also focus on the characteristics most important and critical for which improved test data would be needed.

The systems study for the ATVWS [14], which could be used to monitor and predict the location, strength, and transport of aircraft trailing vortices in the airport terminal area, has required that the problem be divided into a number of smaller segments.

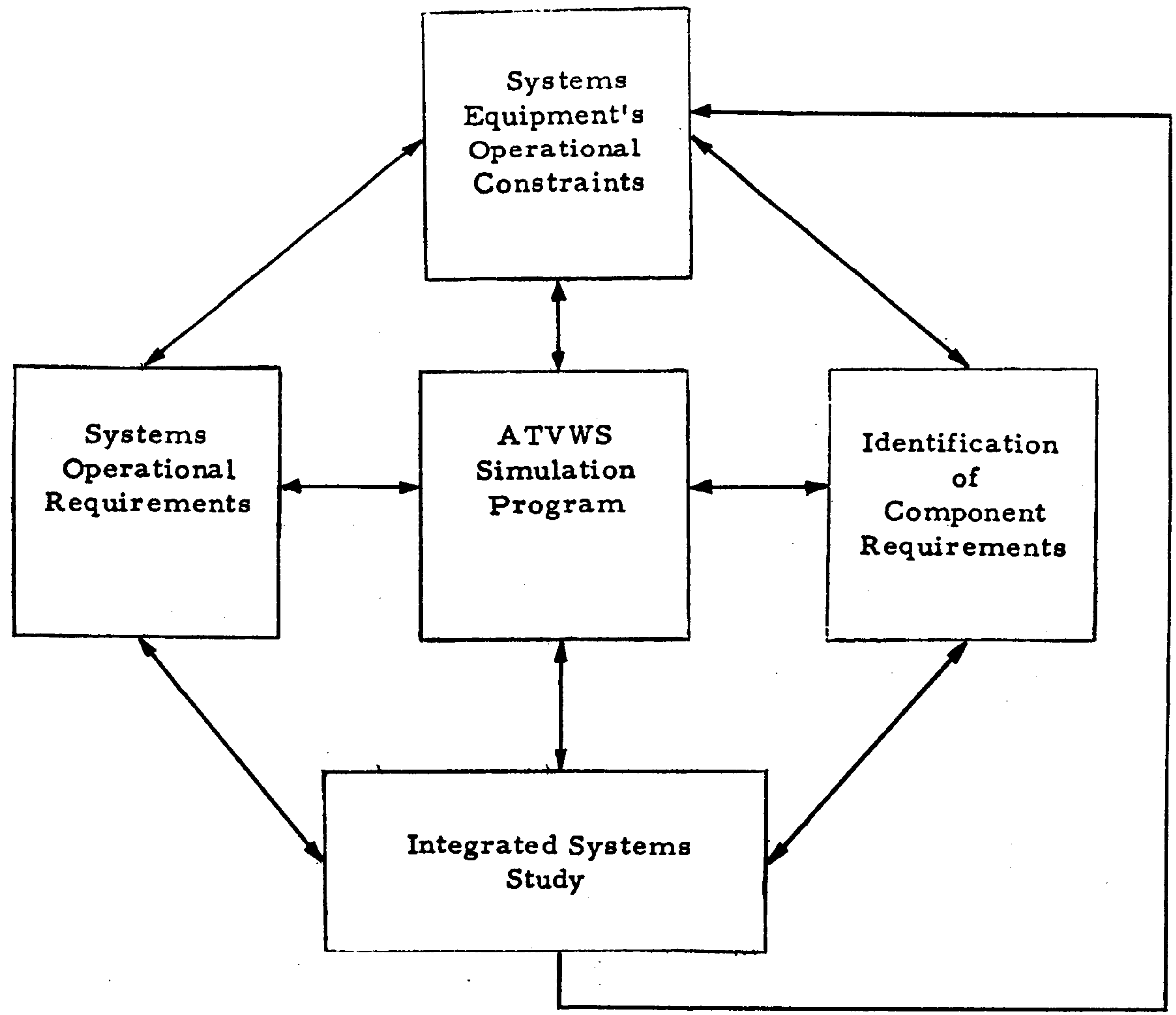


Figure 1. ATVWS systems study.

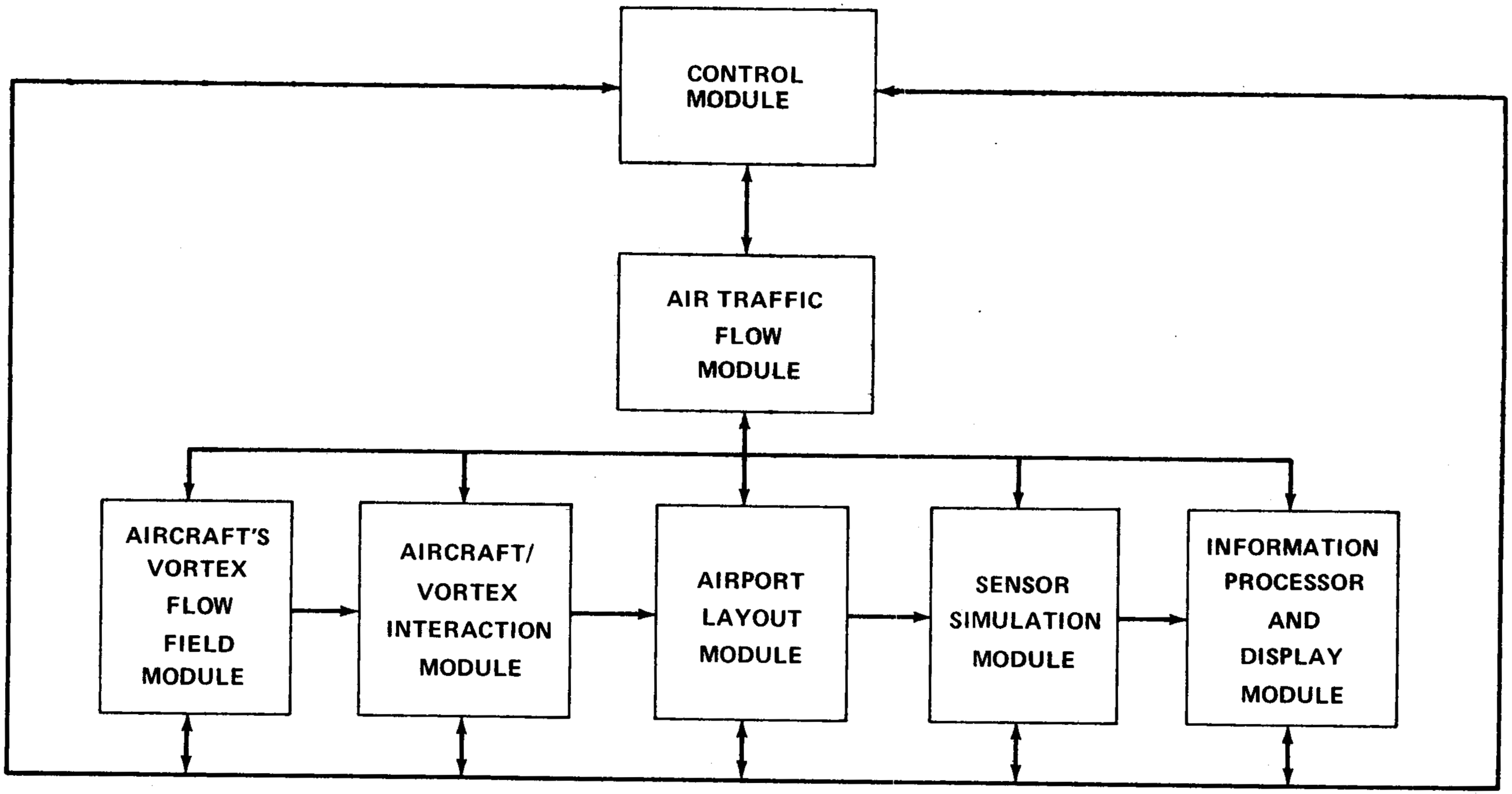


Figure 2. ATVWS simulation computer model.

It is believed that by formulating the problem into a limited number of separate but interrelated segments, the system simulation has been successfully developed and a meaningful preliminary systems study has been performed.

This section attempts to provide the blueprint of the systematic approach employed in formulating the systems approach and performing this ATVWS systems study.

The simulation program has been developed in a modular form with the idea that when better, more sophisticated component models become available and are required, they can be inserted into the system simulation program with a minimum of effort.

Figure 1 is a schematic of the major separate, but interrelated, segments of the ATVWS study and the information, constraints, and requirements flow paths between these segments.

Figure 2 is a schematic diagram of the ATVWS system simulation computer model and the information flow connecting the separate modules which comprise the system.

The modules employed in this system simulation are presented in Section III along with their definition, a detailed description of each module's purpose, and how its output is used in subsequent modules as well as in the determination of the design of an ATVWS.

III. SYSTEMS SIMULATION COMPUTER MODEL

A necessary component of the ATVWS systems study (Fig. 1) consisted of a system simulation computer model (Fig. 2). This simulation was developed to be employed in performing theoretical systems studies to be discussed in Section IV.

The systems simulation computer model is developed in a modular form with the idea that when better, more sophisticated component modules become available and are required, they can be inserted into the systems simulation model with a minimum effort.

As seen from Figure 2, the systems simulation model consists of seven basic modules. Each module generates and shares information used by other modules in the simulation model as dictated by the control module. The control module allows specific parameters of the total system to be investigated and permits their relative degree of dependence on other parameters to be determined within the limits of the simulation.

The modules, their definitions, a detailed description of each module, its purpose, and how its output is used in subsequent modules as well as in the determination of the preliminary system requirements for an ATVWS are to be presented in this section.

A. Aircraft's Vortex Module

This module of the overall ATVWS simulation program is used to simulate aircraft trailing vortices generated by various types of aircraft (from the Cessna 150 to the C-5A) in the takeoff and landing corridors. The theoretical model employed in this simulation is presented in Reference 15 and describes the vortex velocity flowfield as a function of aircraft parameters. This model calculates the tangential, axial, and radial velocities of the vortex generated by an aircraft as a function of aircraft weight, aircraft velocity, wing span, atmospheric density, eddy viscosity, radius in the vortex, and distance behind the vortex generating aircraft (age of the vortex). Figure 3 shows a 747 aircraft's vortex tangential velocity profile as a function of distance behind the generating aircraft (age) as given by Newman's model [15] for a moderately loaded 747 in the takeoff corridor.

The tangential velocity of the vortex is given by:

$$\text{Tangential velocity} = \frac{V \cdot \Gamma}{4\pi} \cdot \sqrt{V_{ac}/Z \cdot (\text{visc})} \tag{1}$$

where

$$V = [1.0 - \exp(-R^2)]/R \tag{2}$$

$$R = \frac{1}{2}\,\text{radius} \cdot \sqrt{V_{ac}/Z \cdot (\text{visc})} \tag{3}$$

$$\Gamma = (4 \cdot \text{weight of aircraft})/(\pi\rho\, V_{ac} \cdot \text{span}) \tag{4}$$

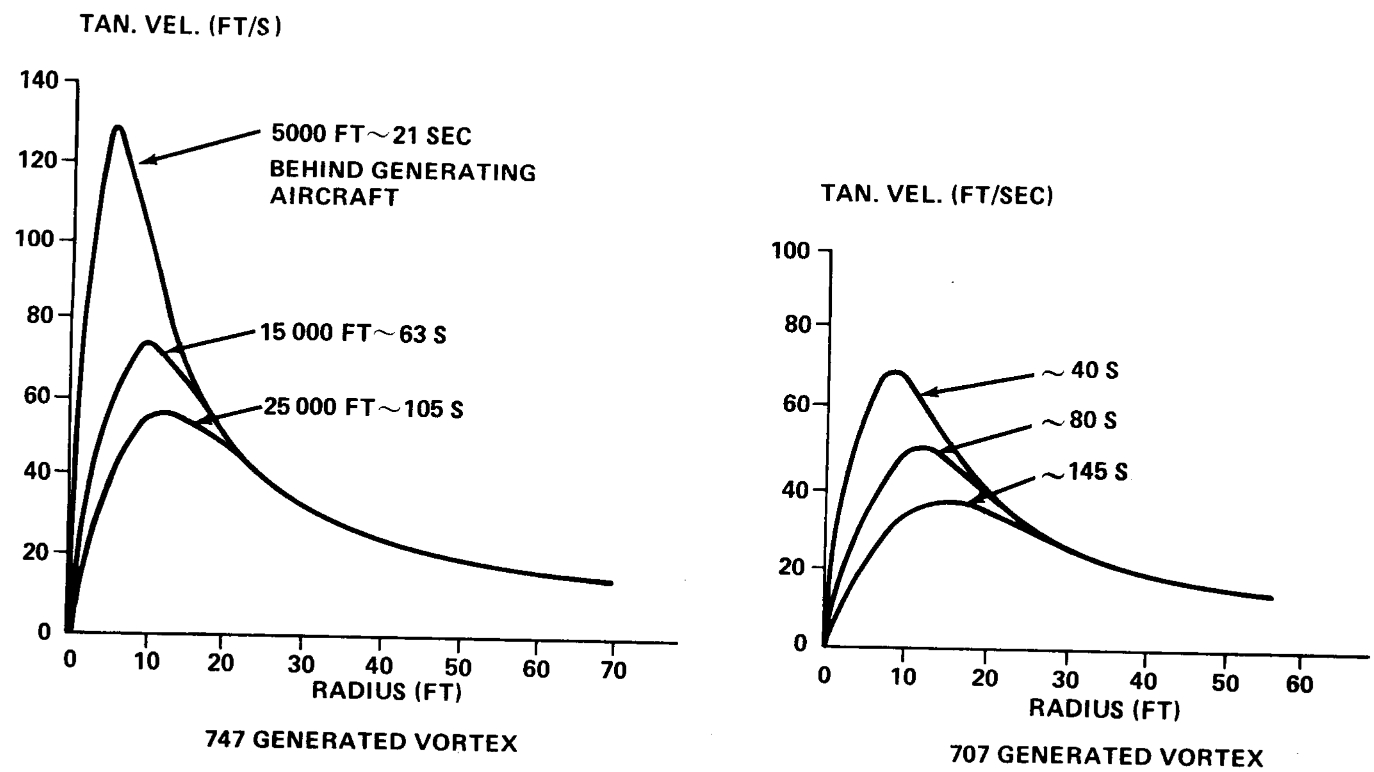


Figure 3. Vortex tangential velocity profile for a 747 and 707 as simulated by Newman's model.

and where

V_{ac} = velocity of the aircraft (vortex generator) (ft/s)

Z = the distance from the vortex generating aircraft to the vortex velocity calculation point (ft)

visc = eddy viscosity of the turbulent medium (ft^2/s)

ρ = air density ($slug/ft^3$)

Γ = circulation strength of the vortex (ft^2/s)

span = wing span of the aircraft generating the vortex (ft)

radius = the radius in the vortex for which the tangential velocity is being calculated (ft).

The module also calculates the theoretical axial and radial velocities of the vortex. However, because of the lack of experimental measurements of these parameters and the uncertainty in the validity of the model in predicting these parameters, their effects are not included in the analysis of the vortex problem.

The horizontal distance (B) between the centers of the rolled-up vortex pair is given by

$$B \cong 0.736 \cdot b \quad \text{(see Reference 16)} \tag{5}$$

b = wing span of the vortex generating aircraft

and is used in this module to initially locate the vortex pair with respect to each other in such a way as to allow the total vortex velocity flowfield to be calculated (Fig. 4).

The vertical movement of the vortex pair is simulated using the model presented by Spreiter and Sacks [16] for self-induced settling of the vortices. It is calculated as a function of aircraft weight, aircraft wing surface area, aircraft wing aspect ratio, air density, and aircraft velocity.

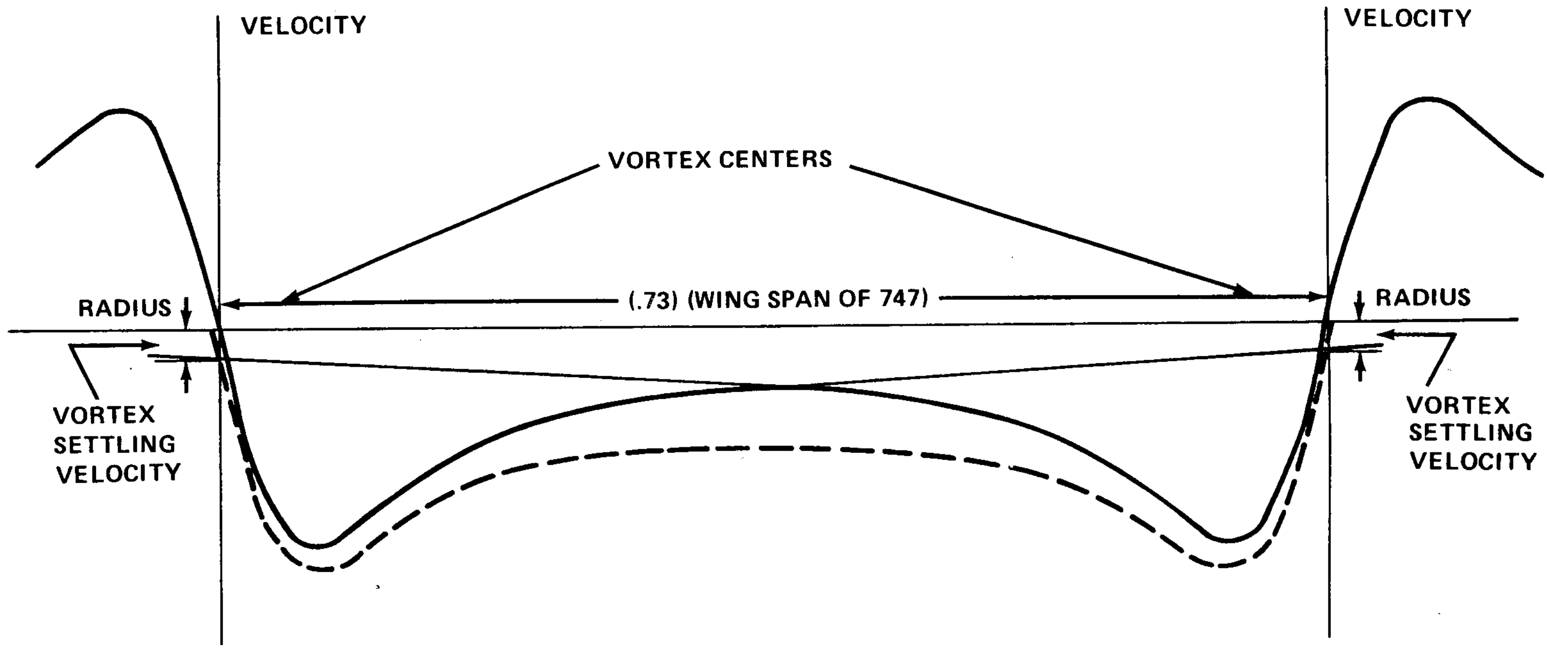


Figure 4. 747 vortex system tangential velocity profile (~ 100 s after generation).

The vortex settling rate (VSR) is given by:

$$VSR = [8 \cdot \text{weight}/S]/(\pi^3 AR \rho V_{ac}) \qquad (6)$$

where

weight = weight of the generating aircraft

S = surface area of the wings

AR = aspect ratio of the wings

ρ = air density

V_{ac} = aircraft velocity.

For most aircraft in the 250 000- to 600 000-lb category, the settling rate is approximately 7 ft/s.

The horizontal movement of the vortex pair is simulated by assuming the vortices move with the wind (headwind and crosswind) until they are a distance one-fourth the wingspan of their generator aircraft above the ground. This is assumed to be the point at which the vortices encounter ground interaction. At that time, the vortices are assumed to pick up horizontal velocities (in opposite directions) of 5 ft/s caused by ground interaction (Fig. 5). At altitudes of less than one-fourth wingspan, the horizontal movement of each vortex is simulated as the resultant velocity of the headwind, crosswind, and ground-interaction-induced velocity.

The output of this module is used in the Airport Layout Module for calculating the vortex velocity flowfield and its movement with respect to the airport runway (takeoff and landing corridors). The velocity flowfield is used in the Aircraft/Vortex Interaction Module to simulate aircraft encounters with vortex systems. Theoretically, this allows the relative degree of hazard associated with vortex encounters to be determined as a function of relative aircraft sizes, separation time, relative position on points of encounter, etc. It is used in combination with the Airport Layout Module and the Aircraft/Vortex Interaction Module to calculate the minimum separation times (on takeoffs and landings) as a function of aircraft, wind condition, etc., for which safe takeoffs and landings can be executed. It provides the Sensor

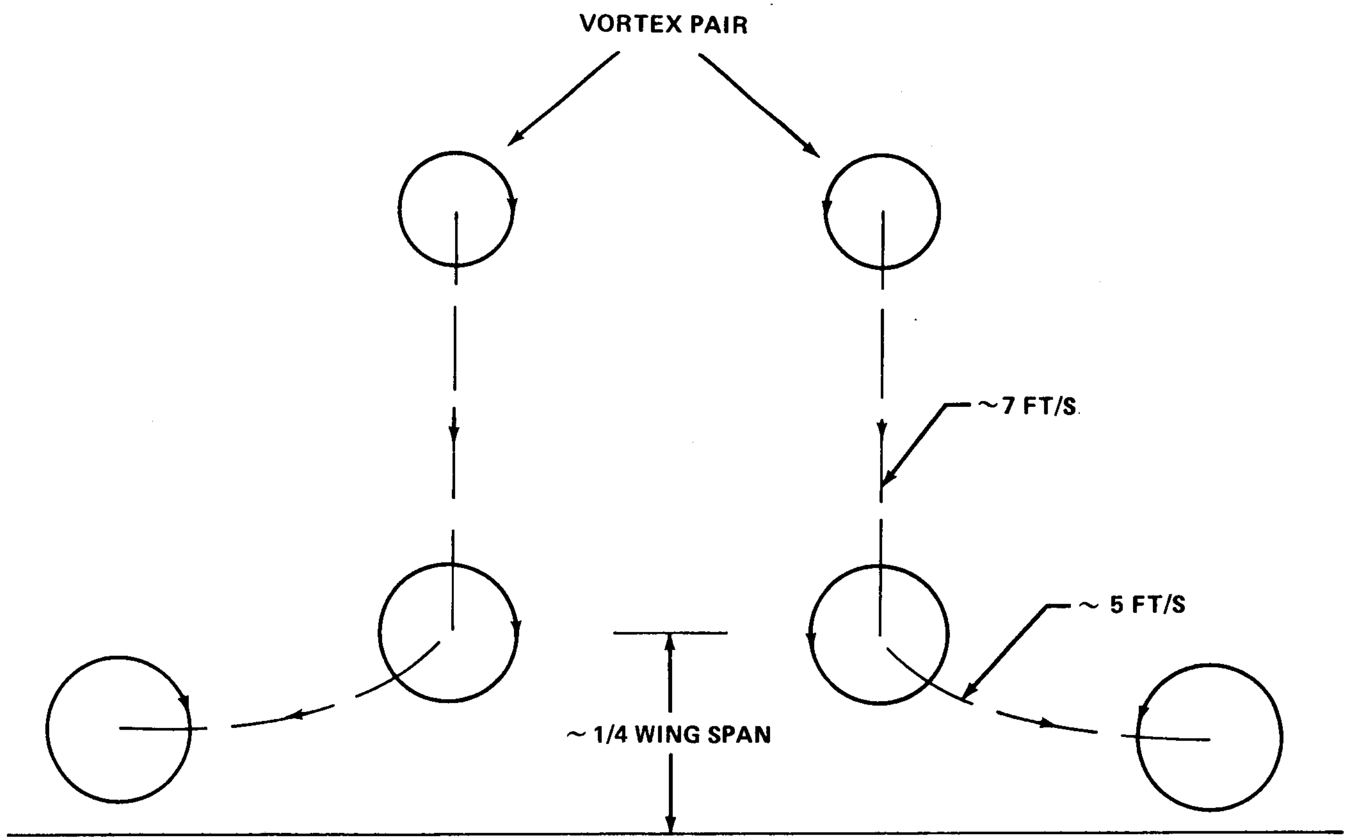


Figure 5. Approximate vertical and horizontal movement of a vortex system with no atmospheric wind.

Simulation Module with a velocity flowfield in the Airport Layout Module. By locating the remote sensing systems at various locations in the Layout Module, the performance (velocity resolution, spatial resolution, and ability to detect vortex systems) of various sensor configurations can be investigated.

B. Aircraft/Vortex Interaction Module

The Aircraft/Vortex Interaction Module is the component of the total system simulation that provides the theoretical tool for investigating interactions between aircraft of various sizes (Cessna 150 to C-5A) and vortices of various aircraft (as provided by the Aircraft Vortex Module).

The aircraft encounters with vortex systems are designed (with the aid of the Airport Layout Module) to simulate the type encounters one would expect to experience in the takeoff and landing corridors of busy airports [17, 18, 19, 20]. The module uses strip theory integration across the wing of an encountering aircraft in the presence of the vortex velocity flowfield (provided by the Aircraft Vortex Module) to calculate the peak roll rate and stall/no-stall condition the encountering aircraft theoretically experiences.

These calculated peak roll rates and stall/no-stall conditions are assumed to serve as an indicator of the potential hazard that the encountering aircraft may experience as a result of the vortex flowfield. The theoretical model employed for calculation of the peak roll rates assumes straight taper wings with no twist, no roll accelerations, and no lateral control inputs of the encountering aircraft (the rolling moment coefficient equal zero).

The methods of calculating the peak roll rate and stall/no-stall condition of the encountering aircraft at a given location and time with respect to a vortex system are given in Appendix B. A computer listing of the Aircraft/Vortex Interaction Module and examples of the output (including resultant peak roll rate) for a DC-9 aircraft following a 747 aircraft in a takeoff and a landing operation are also given in Appendix B.

The Aircraft/Vortex Interaction Module calculates the resultant peak roll rate and stall/no-stall condition for encountering aircraft as a function of the separation distance between the center of the encountering aircraft and the center of the vortex system. Figure 6 shows the hazardous roll rate contours associated with a DC-9 encounter of a 747 vortex system.

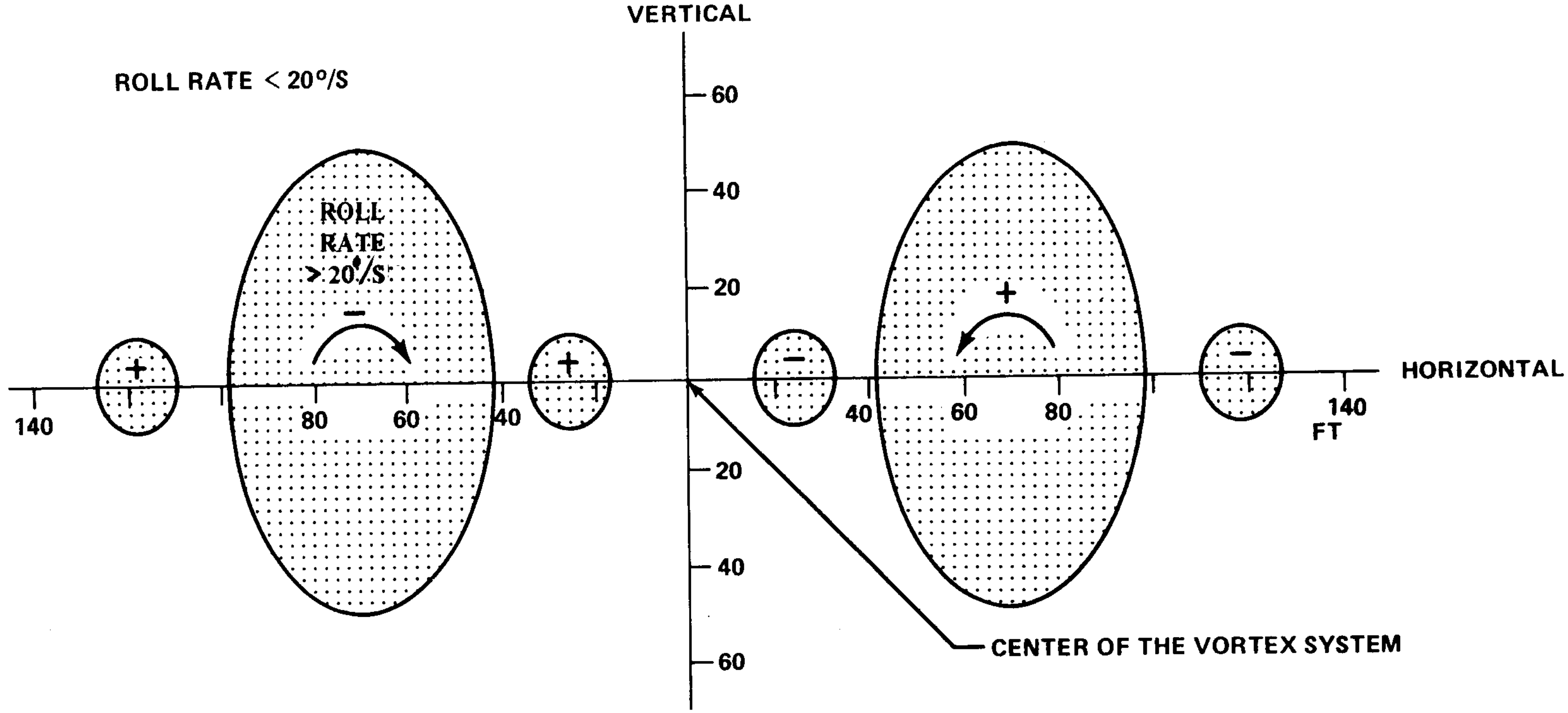


Figure 6. Peak roll rate contours for a DC-9 aircraft in the vicinity of a 747 generated vortex system.

The trajectories of aircraft on takeoff and landing as simulated in the Airport Layout Module give the initial location of the vortex flowfield. The Aircraft Vortex Module allows the position and intensity of the vortex flowfield to be calculated as a function of time and position. This allows the relative positions of vortex velocity flowfields and encountering aircraft to be calculated as a function of time, wind conditions, aircraft, and aircraft/vortex geometry. It is assumed that the aircraft velocity (encountering aircraft) along its flight path (the flight paths, liftoff points, touchdown points of aircraft are variable) is constant, V_{ac}, and that the tangential component of the vortex velocity, V_{vv}, is perpendicular to the encountering aircraft's flight path. This introduces some error into the modules output if the axis of the vortex system is not parallel to the flight trajectory of the encountering aircraft. For cases where the angular difference is less than 10 deg, the error is less than 1.5 percent.

This module, using the theory presented in Appendix B, is used to evaluate the degree of hazard (peak roll rate and stall) associated with vortex systems of various generating aircraft as a function of encountering aircraft, separation time, relative positions, etc. From this module, the parameters of a vortex systems that best correlate with a hazard to an encountering aircraft can theoretically be determined. Indications as to the parameters that a remote sensing system must monitor are provided by studies from this module.

C. Airport Layout Module

The Airport Layout Module provides the simulation with a reference coordinate system in which aircraft trajectories, vortices, vortex encounters, vortex sensor locations, runway, takeoff/landing corridors, and atmospheric winds are defined. In this module, the vortex-generating and vortex-encountering aircraft trajectories are computed as a function of liftoff point/touchdown point, flight path angles, aircraft velocities, and time.

The wind conditions (crosswind and headwind) are input with respect to the runway in this module. This, in conjunction with the Aircraft Vortex Module, allows vortex transport to be simulated with time.

The Airport Layout Module's coordinate system (Fig. 7) is defined as a right-hand orthogonal coordinate system in which the positive Z axis points along the runway in the direction of takeoffs and landings. The Y axis is

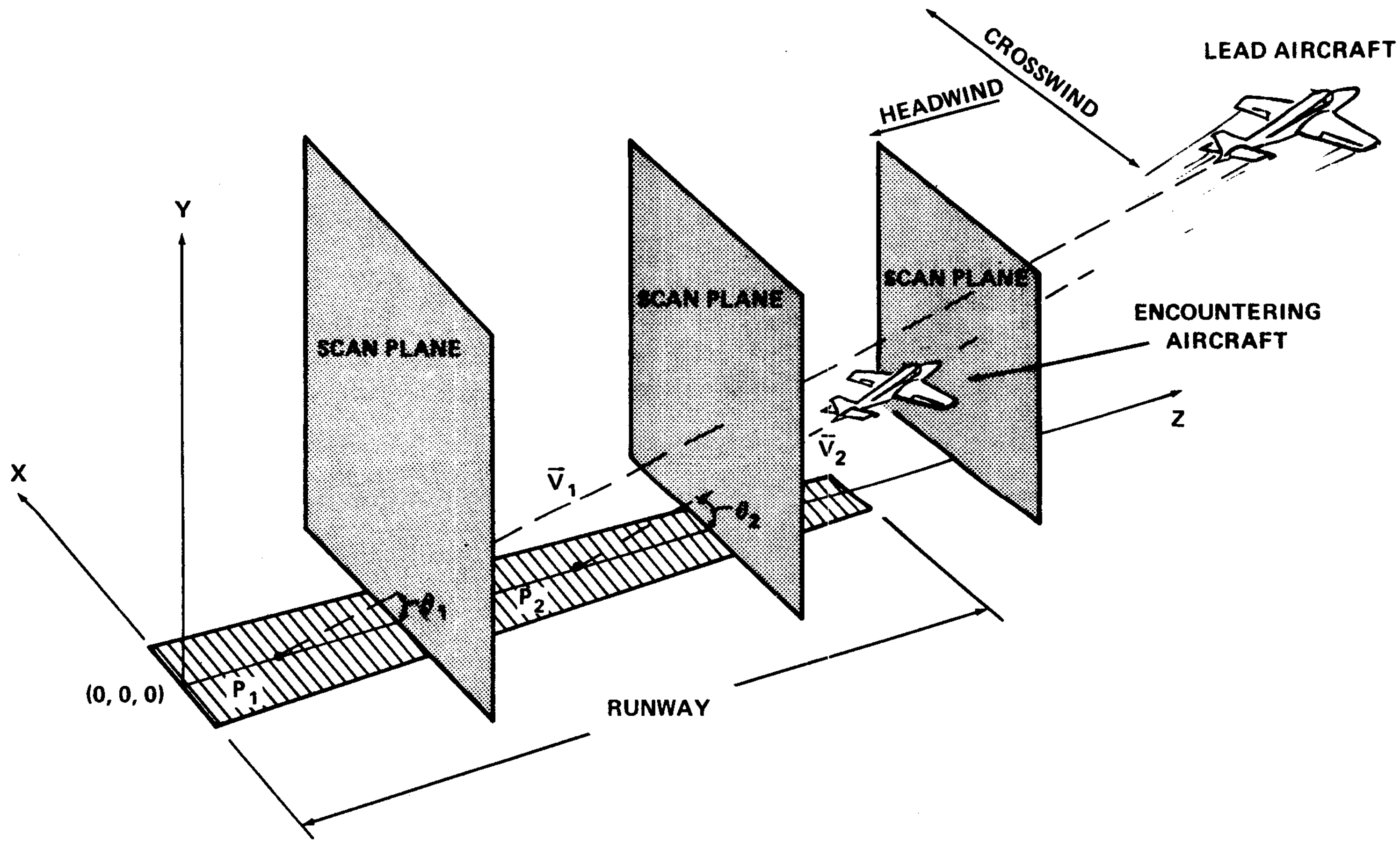


Figure 7. Airport Layout Module's reference coordinate system.

defined as vertical and perpendicular to the Z axis. The X axis completes a right-hand system and is in the horizontal plane. The origin of the coordinate system is defined as the center of the runway at the landing end.

Vertical scan planes along the runway/takeoff-landing corridors are employed to reduce the data required to describe the activities in the airport area. These vertical scan planes are chosen by input controls and are used to store the aircraft penetration points and vortex velocity flowfield as a function of time, wind condition, flight path angle, aircraft type, etc.

The position of the encountering aircraft with respect to the vortex velocity flowfield is calculated in each scan plane. (Any arbitrary number of scan planes can be employed for any given simulation. There is an increase in simulation run time associated with each increase in number of scan planes.) Using the Aircraft/Vortex Interaction Module, the peak roll rate and stall/no-stall conditions are calculated as a function of aircraft separation time/distance, scan plane location, aircraft sizes, and time.

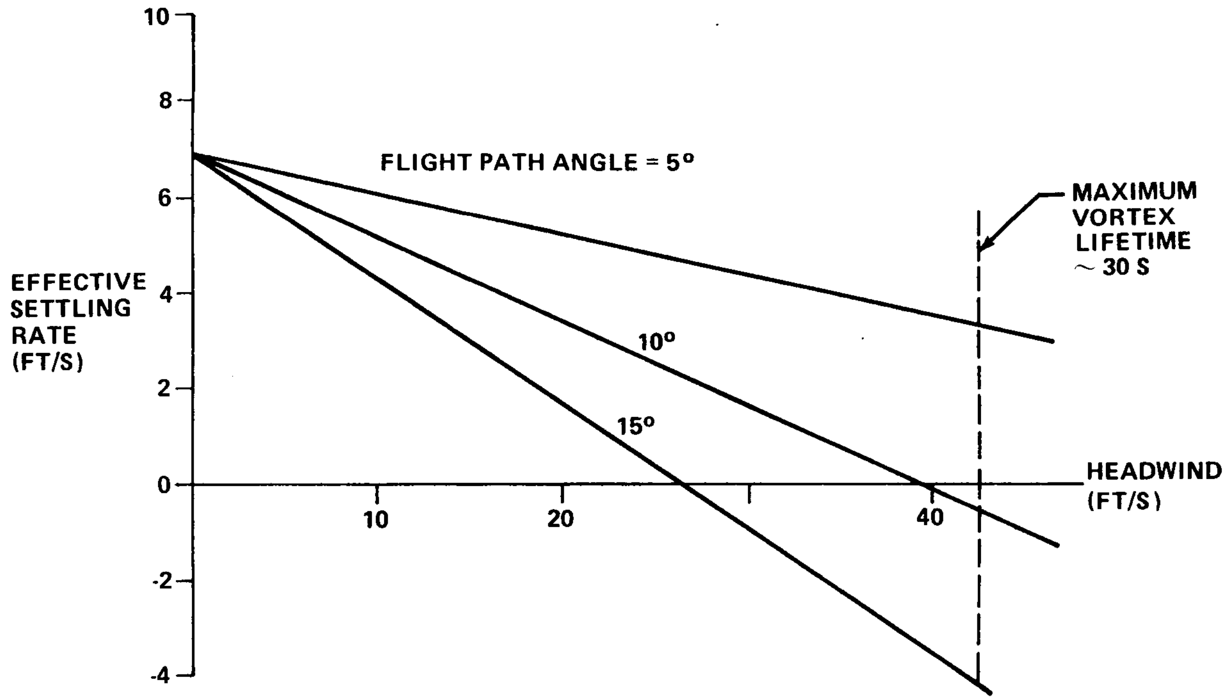
For calculation of the vortex velocity flowfield in the vertical scan plane as a function of time, the headwind/crosswind magnitude and the vortex settling rates are considered.

The horizontal transport of the vortex system is assumed to be equal to the crosswind (except when in ground interaction as described in Section III.A).

The vertical transport of the vortex system is also described in Section III.A; however, the effective vertical transport of the vortex system in a vertical scan plane is described in the following paragraph.

From the Aircraft Vortex Module, the vortex settling rate (VSR) is given by equation (6). The vertical movement of the vortex system in the vertical scan plane is a function of not only the generating aircraft, but also the aircraft's flight path angle and the magnitude of the headwind. Figure 8 shows the effective settling rate (ESR) of a vortex system in a vertical plane as a function of headwind (HW) and flight path angle (FPA) for VSR = 7 ft/sec

$$\mathrm{ESR} = \mathrm{VSR} - [(\mathrm{HW}) \cdot \mathrm{TAN}\,(\mathrm{FPA})] \quad . \tag{7}$$



$$\text{SETTLING RATE} = 8\left(\frac{\text{WEIGHT}}{\text{SPAN}}\right) / (\pi^3 \cdot \text{ASPECT RATIO} \cdot \text{DENSITY AIR} \cdot \text{VELOCITY})$$

$$\text{EFFECTIVE SETTLING RATE} = \text{SETTLING RATE} - \text{HEADWIND} \cdot \text{TAN (FLIGHT PATH ANGLE)}$$

Figure 8. Effective settling rate versus headwind velocity/flight path angle.

The resultant of the horizontal and vertical movement of the vortex system in the vertical scan plane gives the total transport of the vortex system as a function of time.

The choice of the number of scan planes and their location may be influenced by requirements for remote sensing of the vortex velocity flowfield and requirements in determining hazardous roll-rate contours and stall/no-stall conditions. The vertical scan planes of the Airport Layout Module offer a vortex velocity flowfield for the Sensor Simulation Module to use in determining the theoretical performance of Laser Doppler Velocimeter (LDV) models at various locations along the runway/takeoff-landing corridors.

This module of the System Simulation Model presently accommodates one runway — two aircraft for any given period of time (one vortex-generating aircraft and one vortex-encountering aircraft). It approximates the aircraft's climbout/landing trajectory along a straight flight path, and it assumes the aircraft wings are parallel to the ground during takeoff/landing operations (no banks are simulated). The vortex pair is initially located with respect to the generator aircraft's trajectory as described in Appendix B.

By using this module, theoretically safe separation times/distances between aircraft of various sizes, flight geometries, and wind conditions can be investigated and determined. Also, various configurations and locations of remote sensing systems (such as a Laser Doppler Velocimeter) can be investigated to determine their relative performance in detecting and identifying areas of potential hazard to encountering aircraft.

D. Sensor Simulation Module

The LDV Sensor Simulation Module was developed by the Lockheed Missile and Space Company (LMSC), Huntsville, Alabama. The development of this module was completed under NASA Contract NAS8-26668, "Conceptual Design Study of Laser Doppler Systems for Monitoring Aircraft Trailing Vortices in the Terminal Area." After LMSC's completion of this module in April 1972, it was incorporated into the Total System Simulation Model. This module has been used to evaluate the different LDV system design abilities for detecting and monitoring aircraft trailing vortices in the runway/takeoff-landing corridors.

A detailed description of this simulation module is given in Appendix C and with modifications in Reference 21.

The simulation includes the location of various LDV systems in various scan planes along the corridors of interest. The effectiveness of each system is evaluated as a function of its design, location, spatial resolution, velocity resolution, range capability, and angle of observation with respect to the vortex velocity flowfield.

The LDV system designs that are simulated by this module are the pulsed-unfocused system, the continuous-wattage bistatic system, the continuous-wattage coxial focused system, and a theoretically perfect velocity detection system on which to judge the absolute performance of the LDV systems.

The pulsed-unfocused LDV system simulated in this module is a theoretical model of the MSFC-developed Clear Air Turbulence (CAT) detection system [22] that has just finished the first series of flight tests out of the Ames Research Center on the NASA Convair 990 research aircraft. The velocity and spatial resolution of this system is given by the following equations:

$$\Delta V = \frac{\sqrt{2}\lambda}{2t}$$

$$\Delta R = \frac{ct}{2}$$

where

ΔV = velocity resolution

ΔR = spatial resolution along the line of sight of the LDV system

λ = wavelength of the transmitted radiation

c = speed of light

t = pulse length (time) of the transmitted radiation.

The continuous wattage bistatic and coaxial LDV systems simulated in this module are theoretical models of existing systems that have been developed and are being tested at MSFC [23, 24, 25]. Figure 9 gives the theoretical range resolutions versus range for various bistatic and coaxial LDV systems operating with a CO_2 laser (10.6 μ radiation).

An operational LDV system does not make point measurements of the velocity flowfield. It effectively samples a finite volume of space along its line of sight and consequently observes a variety of velocities depending on the inhomogenuity of the flowfield. The reflected signal is the strongest from the focal point or center of the pulse of the transmitted radiation and decreases as the distance of the elemental volume considered increases from the center of the focus or center of the transmitted pulse. The way this signal decreases relates to the systems design, optics diameters, range, and wavelength of the transmitted radiation, and determines the velocity resolution and spatial resolution of the candidate system. A discussion of the signal-weighting functions used in this module for simulating the various LDV system designs is given in Appendix C.

The theoretically perfect velocity detection system simulated in this module assumes the tangential line-of-sight velocity at the focal point/center of the transmitted radiation, with no weighting factor (assumes perfect spatial resolution and velocity resolution) is the velocity detected by this system.

This module simulates observations that are made by one LDV system (one-dimensional information). Other simulations (not total system simulations) have been performed using two and three sensor systems per scan plane (two- and three-dimensional information, respectively). Briefly, the two- and three-dimensional LDV systems simulations indicate that they offer better velocity and spatial resolution with longer range capabilities. This is offered at the expense of increasing the number of sensors required to monitor an airport terminal area. The present Sensor Simulation Module would require considerable modification for the two- and three-dimensional simulations to be incorporated into the Total Systems Simulation Model.

This module has been used to determine the feasibility of employing a LDV system in an ATVWS for the purpose of monitoring aircraft trailing vortices. In conjunction with the Aircraft Vortex Module, the Aircraft/Vortex Interaction Module, and the Airport Layout Module, this module has been used to perform tradeoffs between the LDV system design, LDV system locations, number of sensors required, and performance of the sensors in detecting vortices in the corridors of interest. The result of these studies will be given in Section IV.

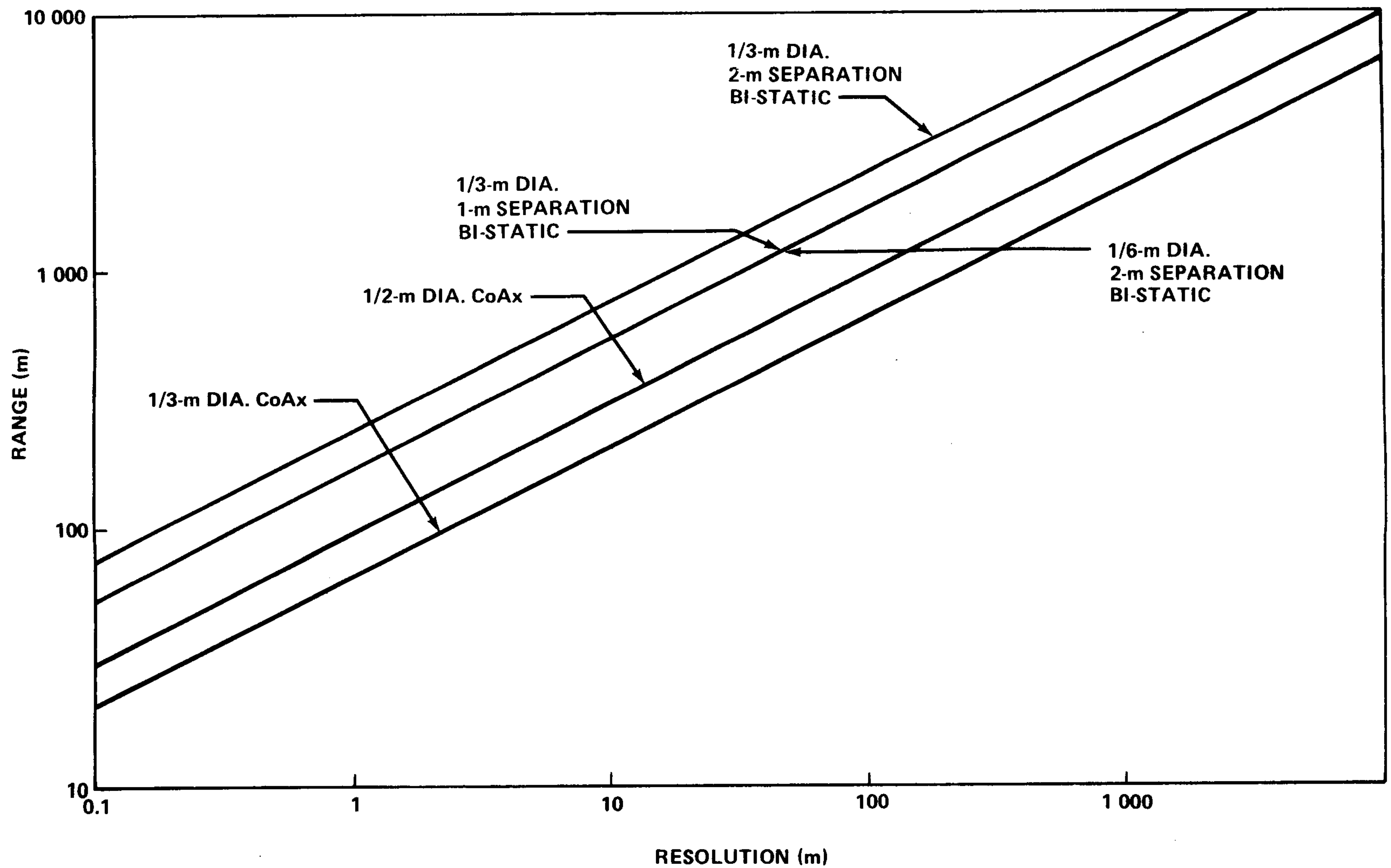


Figure 9. Range resolutions (50 percent return) at 10.6 μ wavelength.

E. Information Processor and Display Module

The Information Processor and Display Module was developed by LMSC, Huntsville, under NASA Contract NAS8-26668, "Conceptual Design Study of Laser Doppler Systems for Monitoring Aircraft Trailing Vortices in the Terminal Area."

In conjunction with the Sensor Simulation Module, LMSC developed this module for the purpose of investigating (through simulation) various types of information processing for the sensors and to allow the processed information to be displayed as printout and SC4020 plots from the computer.

The information processing for this module is described in Appendix C and consists of (1) computing the line-of-sight velocity at the point of interest (focal point of the LDV system), (2) computing the maximum and minimum line-of-sight velocities in the focal volume of the simulated LDV system, (3) computing the centroid of velocity for the focal volume of the simulated LDV system, and (4) computing the absolute maximum line-of-sight velocity.

This module is designed to permit any or all of this information to be displayed as a function of sensor scan angle (azimuth or elevation), range, or through connections with other system modules as a function of any other system parameter.

The output from this module has made visual evaluations of the different LDV system's outputs in various simulated operating simulations easier. It has also served to indicate the type of information displays that will potentially be required of an ATVWS.

The flow chart for the Information Processor and Display Module is shown in Figure C-3 of Appendix C. It is accented and enclosed by the dashed line.

F. Air Traffic Flow Module

The Air Traffic Flow Module is designed to simulate the air traffic flow that is typical of today's busy airports on one runway. It allows specific investigations to be performed relating to the safety or hazard associated with sequence landings or takeoffs between aircraft of different sizes.

The aircraft that are presently used in this simulation module include the Boeing 747, 707, 727, the Lockheed C-5A, the Douglas DC-10, DC-9, DC-3, the Queen Air 60, the Jet Commander 1121, and the Cessna 150 [26, 27]. This module chooses the separation times between aircraft on takeoffs or landings for various flight geometries and automatically increases the separation time in increments of 5 s until a safe flight operation is simulated. A safe flight operation is defined as a takeoff or landing for which the encountering aircraft's roll-rate limit is not exceeded and the aircraft does not experience a stall condition as defined in Appendix B. The roll-rate limit/capability (RRLIM) of the encountering aircraft is calculated as a function of aircraft parameters as follows:

$$\mathrm{RRLIM} = [0.14 \cdot V_{ac}/b] \cdot 57.29 \quad (\mathrm{deg/s}). \tag{8}$$

See Reference 28 for further discussions on aircraft design of its minimum rolling power.

G. Control Module

The Control Module is designed to permit specialized investigations to be performed employing individual modules of the Total System Simulation Model.

The Control Module is an assemblage of several computer subroutines including the main subroutine, the input subroutine, the parameter standardization subroutine, and the printout subroutine.

The Control Module calculates and compares the maximum controllable roll rate of given aircraft with the peak roll rate induced on the encountering aircraft by the vortex systems. If the induced roll rate is greater than the aircraft's controllable roll rate, then the situation is termed hazardous. Also, the Control Module calculates and compares the angle of attack of the total wing with the stall angle of attack for aircraft. If the stall angle is equalled or surpassed, then a stall type 1 situation exists. If the angle of attack of the wing is decreased significantly, potentially causing the aircraft to climb out on takeoff or to land prematurely, the situation is termed a stall type 2. This has been discussed in Appendix B. In either case, the separation time between air operation is step wise increased until the operation is safe.

Through the Control Module, the choices for takeoff or landing simulation, aircraft types, flight geometries, wind conditions, scan plane locations, sensor design, sensor location, data processing, and display are made. These choices are made through input flags and parameter values.

Through the Control Module, it is possible not only to simulate the total systems, but it is also possible to investigate the relative significance of various systems parameters to other system parameters. This aspect of the Control Module allows specific studies or tradeoffs to be performed in much less computer time than would ordinarily be required using the total system simulation. These, in turn, can be used to influence the type of total system simulation investigations and the design of flight test.

Discussions of these system studies and their significance will be given in Section IV.

IV. SYSTEM STUDIES

This section attempts to describe system studies that have been performed with the Total System Simulation Model and gives the results of these studies. This section is not meant to represent a complete system study, but only a preliminary systems study which gives preliminary results. Ideally, this section is expected to answer some of the systems engineering questions relating to an ATVWS and to exemplify the contributions that the Total System Simulation Model can make toward its design.

The system studies were started with the completion of the Aircraft Vortex Module and continued through the completion of the last module (Control Module). It is believed that the studies performed with the individual modules not only gave valuable information to assist in the ATVWS design but also gave a better understanding of how these modules should be integrated to form the Total System Simulation Model.

Given the purpose of an ATVWS, the system studies that have been performed were designed to determine: (1) the requirements of an ATVWS (the criteria by which potential ATVWS are to be evaluated); (2) the feasibility of employing LDV's in an ATVWS, and (3) a possible design of an ATVWS.

The assumed purpose of the ATVWS is to increase the efficiency of an airport by increasing the air-traffic flow in its runway takeoff-and-landing corridors while maintaining or increasing the air-traffic safety factor in the terminal area. The safety factor for air operation is discussed in Appendix A.

There appears to be one obvious way of accomplishing this goal; i.e., design an aircraft wing that will induce a very rapid decay or breakup of its vortex system (30 to 60 s lifetime). While research toward this end is being pursued at other NASA centers, it appears that with its success there would be a number of years required to get the wing design on all operating aircraft requiring such a wing. Thus, the following system studies and potential design of an ATVWS will be presented in the following paragraphs.

The first study was to determine the size and persistence of the object causing the difficulties (aircraft vortices). From experimental observations, it was ascertained that the vortex model presented by Newman [15] represented a reasonable approximation of the vortex's tangential velocity as a function of the aircraft parameters and time (Fig. 3) and the vortex size as a function of time under calm, stable atmospheric condition. Experimental observations from various flight test [29] indicate that the lifetime and decay of the vortex system is very much dependent on the wind and other atmospheric conditions. Reference 16 gives the separation between vortex centers as a function of aircraft wingspan. From this, the size of various vortex systems can be approximated as a function of time. Figure 10 gives the theoretical sizes of the vortex systems for heavy aircraft (C-5A-747 types) and for light aircraft (707 and 727 types) at 30 to 60 s after vortex generation under calm wind conditions and above the ground interaction level. This represents the time period for which the vortex system must be monitored and indicates typical sizes of vortex systems which must be detected.

The time period that the vortex system must be monitored is dictated by the purpose of the ATVWS; i.e., an increase in the air-traffic flow in the runway takeoff-and-landing corridors requires a decrease in the separation time between aircraft on takeoff or landing. The presently required FAA separation times are as follows:

a. Heavy aircraft followed by light aircraft — takeoff minimum separation is 2 min, and landing minimum separation is 5 miles (~ 100 s)

b. Light aircraft followed by light aircraft — takeoff or landing minimum separation is 3 miles (~ 60 s).

This means that an ATVWS must be able to make a decision for a safe air operation in a significantly less time than is presently required (60 to 120 s)

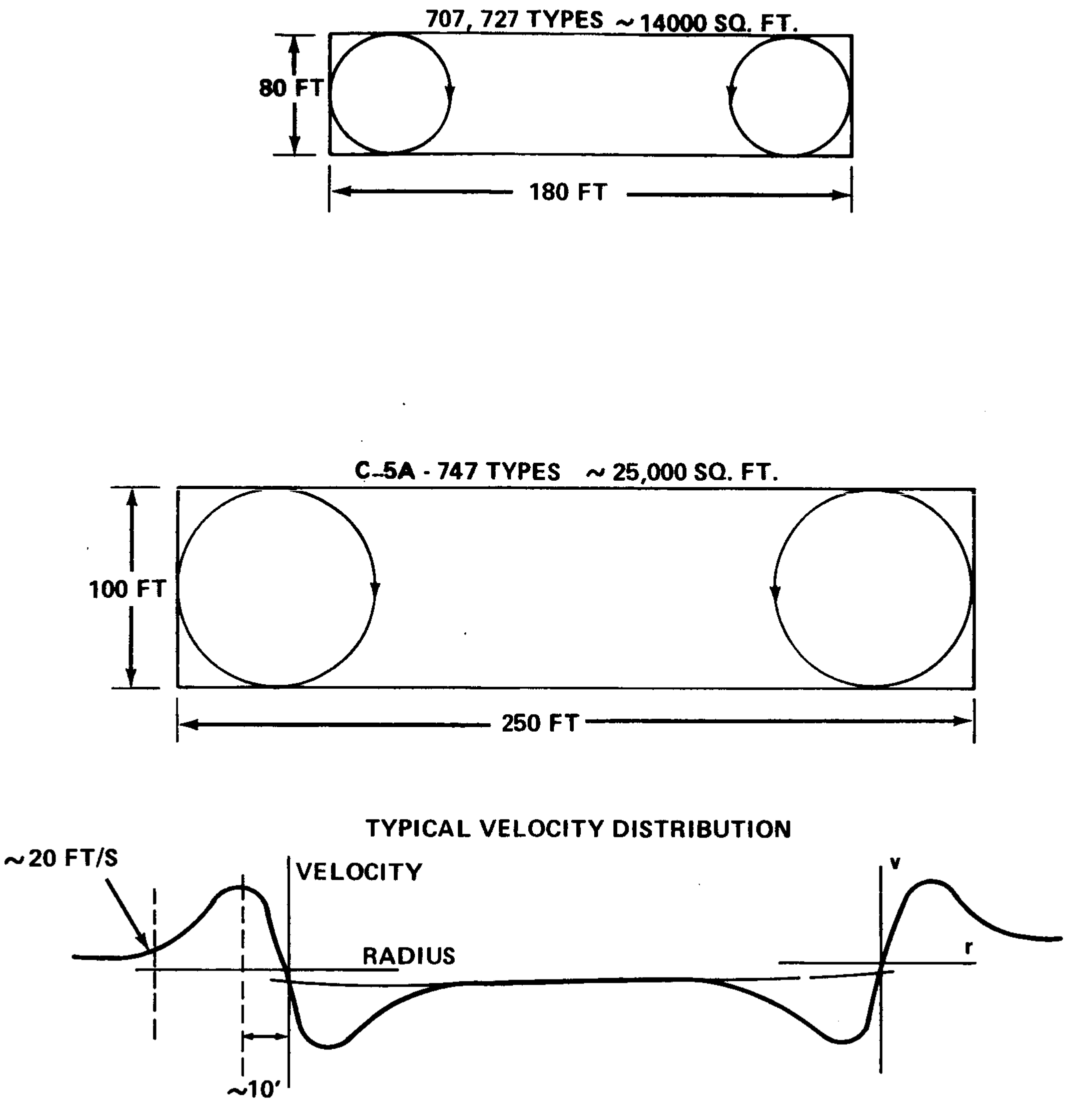


Figure 10. Theoretical sizes of vortex systems at 30 to 60 s after generation.

or with a significantly greater understanding of the potential hazards involved. However, it must be remembered that a monitoring system (ATVWS) cannot guarantee a decrease in the separation time. It should guarantee the minimum separation time for the particular weather condition and aircraft involved that will maintain the required safety factor for the air operation.

The parameter that separates the vortex system from the rest of the atmosphere and thus reveals its location is its velocity flowfield and most likely its tangential velocity component (unless future experiments increase the knowledge of the vortices' axial velocity component to the extent that it proves to be the better informer) or perturbations caused by its tangential velocity.

Another question that must be answered is, "What parameter of the vortex system best correlates with the potential hazard it represents to an encountering aircraft?" In an attempt to answer this question, several studies employing the simulation model were performed.

This is an example of the system simulations advantage over conventional testing methods. Not only is the simulation faster and less expensive, but it is also safer from the pilots point of view in that it will at least result in fewer required flight tests.

The simulation first studied the situation where the aircraft encountered a vortex system parallel to and in the center of one of the vortices from the leading aircraft. Figure 11 contains the results of these studies with DC-9, 707, and 747 aircraft encountering a vortex generated by a 747 aircraft. In this study, the peak roll rate of the encountering aircraft is calculated with the assumption that there are no roll accelerations and no lateral control inputs of the encountering aircraft. Here, the peak roll rate is considered to serve as an indicator of the relative magnitude of the hazard associated with a given vortex system. From this study, it can be seen that the peak roll rate of the encountering aircraft is more dependent on its relative size to the vortex generating aircraft than on age of the vortex when encountered. The solid lines in Figure 11 represent the theoretically expected peak roll rates, with no wind and constant circulation of the vortex, as a function of peak tangential velocity in the vortex, which is a function of time. The maximum vortex lifetime of 180 s is taken from Reference 7. It is known that there are exceptions to this maximum lifetime depending on atmospheric conditions. The dashed lines in Figure 11 represent what is thought to be a reasonable approximation to the aircraft peak roll rates as the tangential velocity of the vortex decreases to zero and the vortice's circulation decreases.

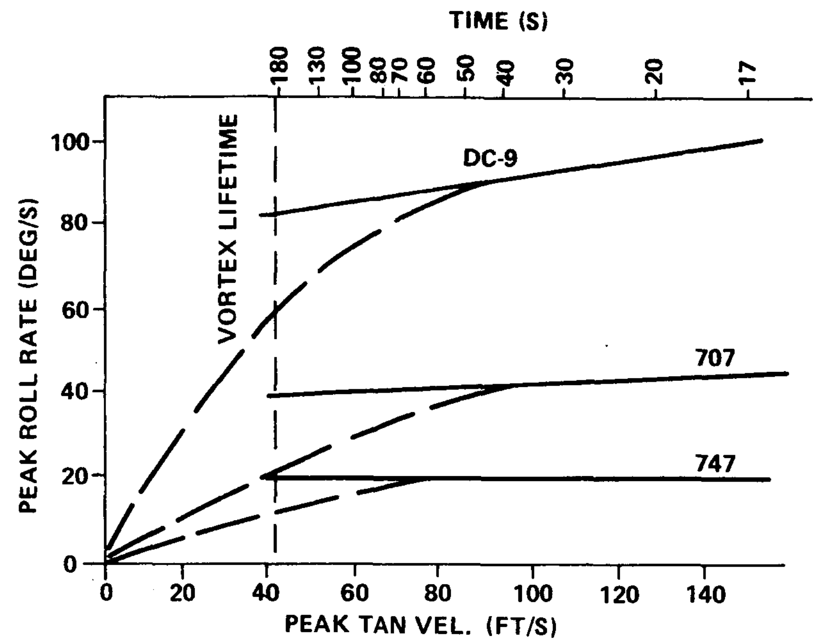


Figure 11. 747-generated vortices encountered by DC-9, 707, and 747 aircraft (the aircraft are simulating center of vortex encounters).

From the results of the preceding study, another study was suggested. The objective of this study was to determine the encountering aircraft's peak roll rate as a function of relative circulation strength [equation (4)] of the generating aircraft. The results of this study are presented in Figure 12. In Figure 12, it can be seen that the peak roll rate of an encountering DC-9 in the wake of a 747 may be as high as 100 deg/s, whereas its roll rate in the wake of another DC-9 is never greater than approximately 25 deg/s. It is also noted that one 747 following another 747 will theoretically experience peak roll rates of $\sim$ 20 deg/s. This limited study implies that the relative circulation strength (circulation strength of the generator aircraft/circulation strength of the encounter aircraft - $\Gamma g/\Gamma e$) is an indicator of the potential hazard that an aircraft may experience when encountering a vortex system. For ($\Gamma g/\Gamma e$) $\geq$ 1.2, the potential hazard is greater than for ($\Gamma g/\Gamma e$)$<$ 1.2 and, from the Simulation Model, it appears this is true in most cases for separation times as great as the present FAA-required separations.

This being the case and realizing that present FAA separation standards were providing an acceptable safety for today's air operation, the question arises, "What makes it safe?"

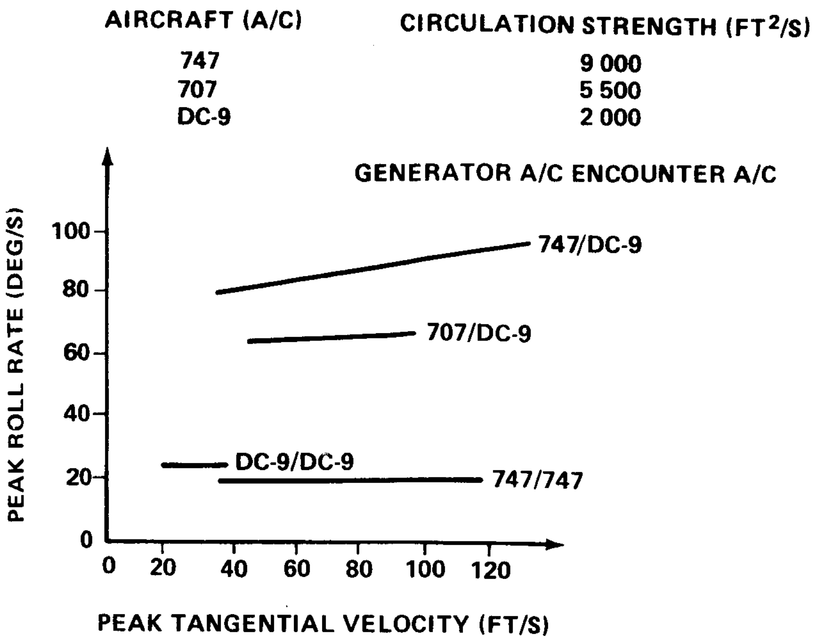
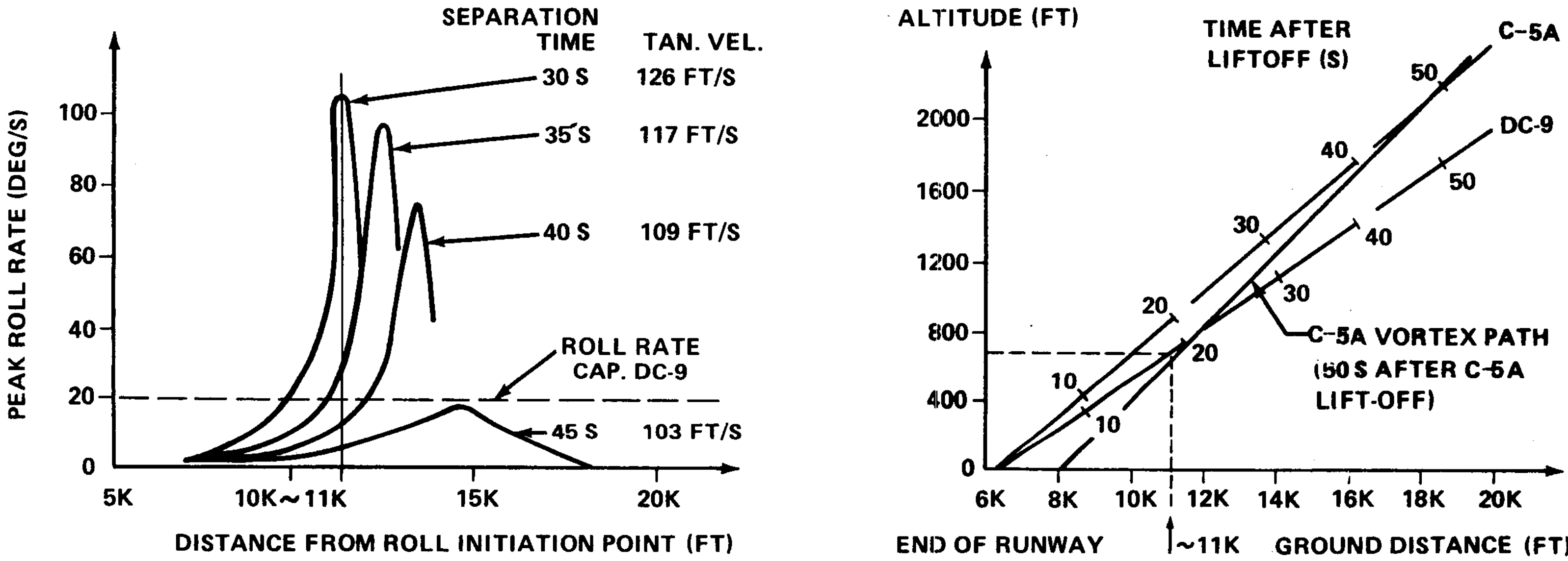


Figure 12. Peak roll rates of encounter aircraft as a function of generator aircraft and peak tangential vortex velocity (aircraft simulating center of vortex encounters).

The system simulation was used to simulate a C-5A and DC-9 aircraft in sequence on a typical takeoff pattern. The initial separation time was taken to be 30 s (Fig. 13). The crosswind was assumed to be 3 ft/s with no headwind. The climbout velocities of the C-5A and DC-9 were assumed to be 236 and 243 ft/s, respectively. Assumed flight path angles were 10 and 8 deg, respectively, while the liftoff point for the C-5A was 6400 ft down the runway and 6500 ft for the DC-9.

With the 30 s separation, the DC-9 experienced a peak roll rate of greater than 100 deg/s at an altitude of ~ 700 and 1000 ft past the end of a 10 000-ft runway. The peak tangential velocity of the vortex system at that time was ~ 126 ft/s. The separation time between the two aircraft was incrementally increased to a separation time of 45 s at which the DC-9 was able to depart from the terminal area without experiencing a roll rate greater than its roll-rate limit capability [equation (8)]. Yet, at this time, the peak tangential velocity of the vortex system was still at a potentially dangerous level of ~ 103 ft/s.

This indicated an answer to the question. Upon observation of the relative locations of the vortex system and the encountering aircraft, it was found that the vortex system had settled to an altitude such that for a 45 s



AIRCRAFT RELATED DATA

	C-5A	DC-9
VELOCITY (FT/S)	236	243
FLIGHT PATH ANGLE (DEG)	10	8
LIFTOFF POINT (FT)	6400	6500
VORTEX SETTLING RATE – ~7 FT/S		
CROSSWIND – ~3 FT/S		

Figure 13. Representative example of C-5A followed by DC-9 aircraft on takeoff.

separation, the encountering aircraft did not fly within some hazard distance (Appendix B) of it. It appears that for the timeframe dictated by the situation (less than 120 s), the location of the aircraft with respect to the vortex system is the most important parameter in determining the potential hazard to the encountering aircraft. This called for a specialized study between several aircraft to determine the hazard distance associated with a given vortex system and an encountering aircraft. Studies were performed with the System Simulation Model to determine the hazard radii associated with a DC-9 and 707 aircraft encountering the vortex system generated by a 747 (Figs. 6 and 14, respectively). It is also noted from the simulation that the areas of stall for encountering aircraft (as defined in Appendix B) are located inside the hazard radii for hazardous roll rates. The results of these specialized studies indicate that the hazard radius for a DC-9 encounter with 747 generated vortex is ~ 130 ft and the hazard radius for the 707 encounter of the 747 vortex is ~ 150 ft. Other such investigations lead to the conclusion that the hazard radius is primarily a function of the wingspan of the generating and encountering aircraft; an example: the hazard radius for a Lear Jet encountering the vortex system of a 747 is less than the hazard radius for a DC-9 encounter of a similar vortex system. However, the degree of hazard (magnitude of peak roll rate) is much greater for the Lear Jet than for the DC-9.

As a result of the use of the System Simulation Model, the determination of hazard radii for particular aircraft combinations was possible. From this and the considerations of flight geometry in the Airport Layout Module, assumptions on spatial distributions of aircraft and vortex systems made it possible to extend the system study by performing the probability study given in Appendix A.

From the Airport Layout Module and partial consideration of the situation existing in the runway takeoff-and-landing corridor, it was possible to define the areas that potentially offer the most information pertaining to probable vortex hazards. These areas should represent the areas to be monitored by the ATVWS's remote sensors. The areas that potentially offer the most vortex hazard information are determined as follows: On departing, the aircraft is cleared for takeoff, begins its ground roll (from velocity zero to velocity lift-off), lifts off, and climbs out at some flight path angle. The next air operation on that runway is determined by the previously mentioned FAA-required separation times (dependent on aircraft size) of from 60 to 120 s. If the ATVWS is to be an active measurement system, it must make a measurement and judgment or prediction of the generated vortex systems location and hazard before the next aircraft is cleared for takeoff. From the time considerations of the situation, this necessitates a measurement of the

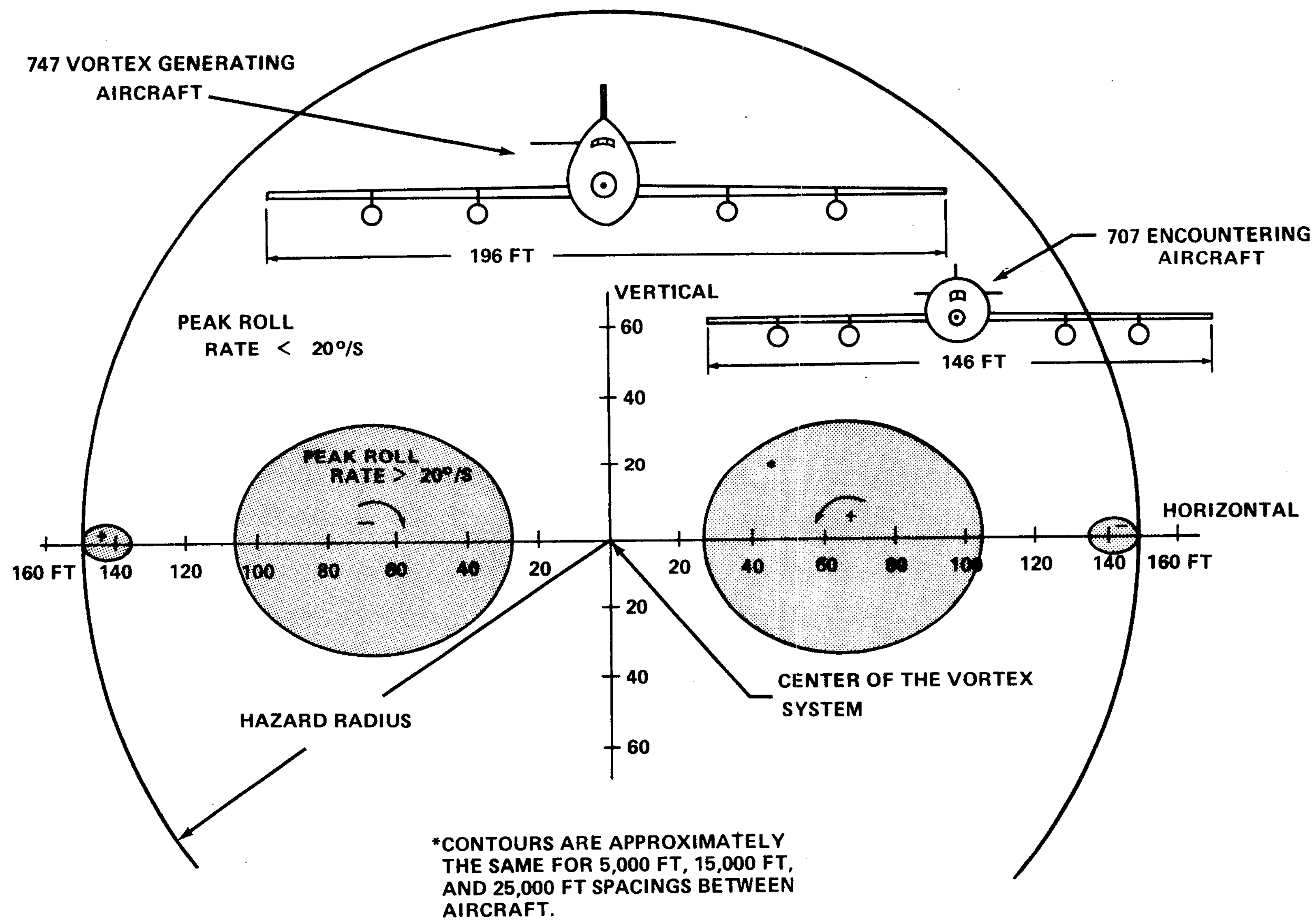


Figure 14. Peak roll-rate contours for 707 aircraft in the vicinity of 747-generated vortices.

vortex system at some time less than 60 s (in some cases) from the time the vortex generating aircraft began its ground roll; i.e., the ATVWS must be a combination measurement-predictive system to be able to satisfy the requirement of increasing air-traffic flow in the takeoff and landing corridors. As seen in Figures 15 and 16, monitoring the vortex systems until they are clear of the areas of concern before releasing or clearing the next aircraft for takeoff or landing can require separation times greater than today's requirements. The scan plane area must permit penetration of all aircraft vortex systems on takeoff. This means the scan plane area must be located past the takeoff end of the runway. For unbiased vortex transport information, the vortices should be observed at altitudes above ground interaction. These considerations lead to the conclusion that at least one scan plane for aircraft on takeoff is required and that this scan plane should be located approximately 1000 ft past the takeoff end of the runway. If the luxury of several sensors can be afforded, then the determination of other scan planes for the sensors is a much-reduced problem. The size and location of a possible takeoff scan plane is given in Figure 17.

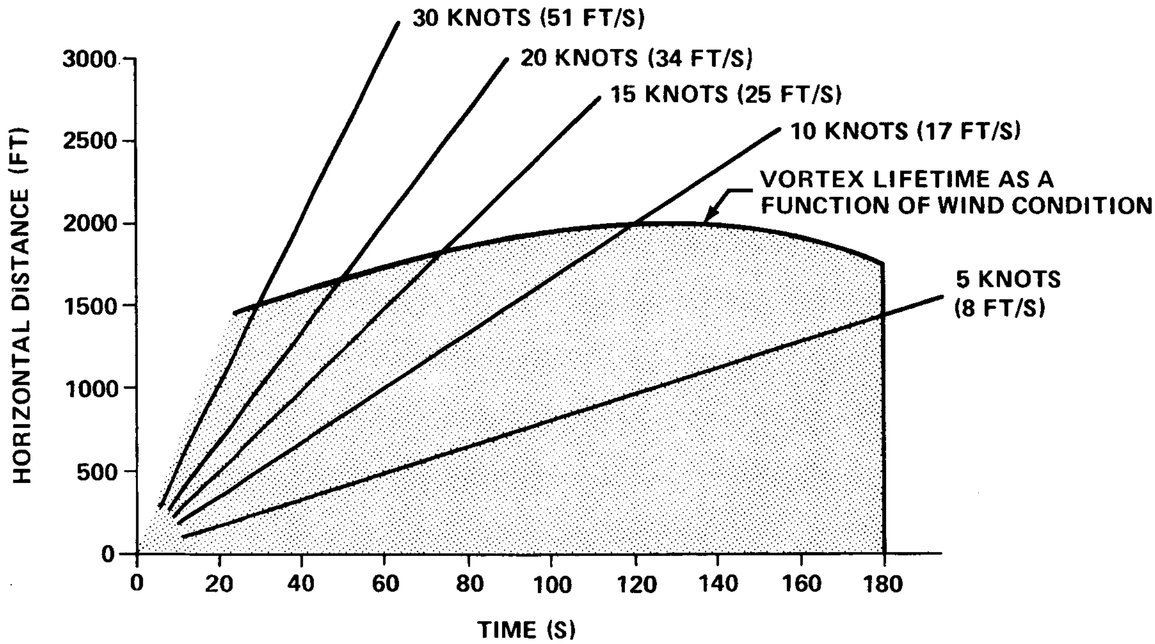


Figure 15. Horizontal movement of vortices as a function of wind velocity, vortex lifetime, and time.

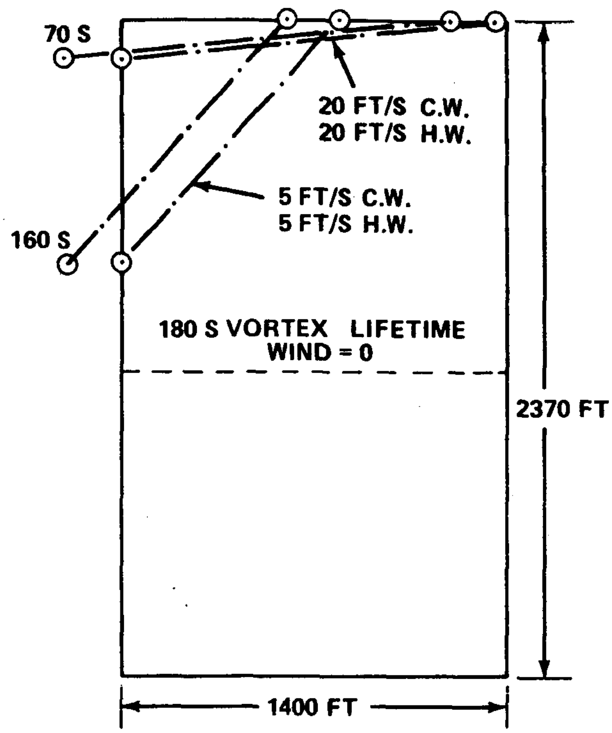


Figure 16. Horizontal and vertical transport of a vortex system as a function of crosswind and headwind.

The arguments for the optimum location of the landing scan plane are not so concrete as those for the takeoff scan plane. It is necessary to choose a location far enough from the landing end of the runway to permit measurements to effect the actions of the following aircraft, if necessary. The scan plane should be in an area where there is a relatively high probability that the aircraft will penetrate it and that penetration will permit measurements to be made on the vortex system unperturbed by ground interaction.

A reasonable choice for this scan plane location is the ~ 3-mile (middle) marker for the instrument landing system. The size and location of this scan plane is given in Figure 17.

From the size and location of these sensor scan planes, the range requirement of potential sensors can be specified as a function of sensor location with respect to the scan planes. If the sensors are located at the center of the scan plane (ground level), then the range requirements for landing and takeoff are ~ 2700 and ~ 2500 ft, respectively (Fig. 17) to be able to monitor all aircraft. These scan planes were used in Appendix A for the probability study.

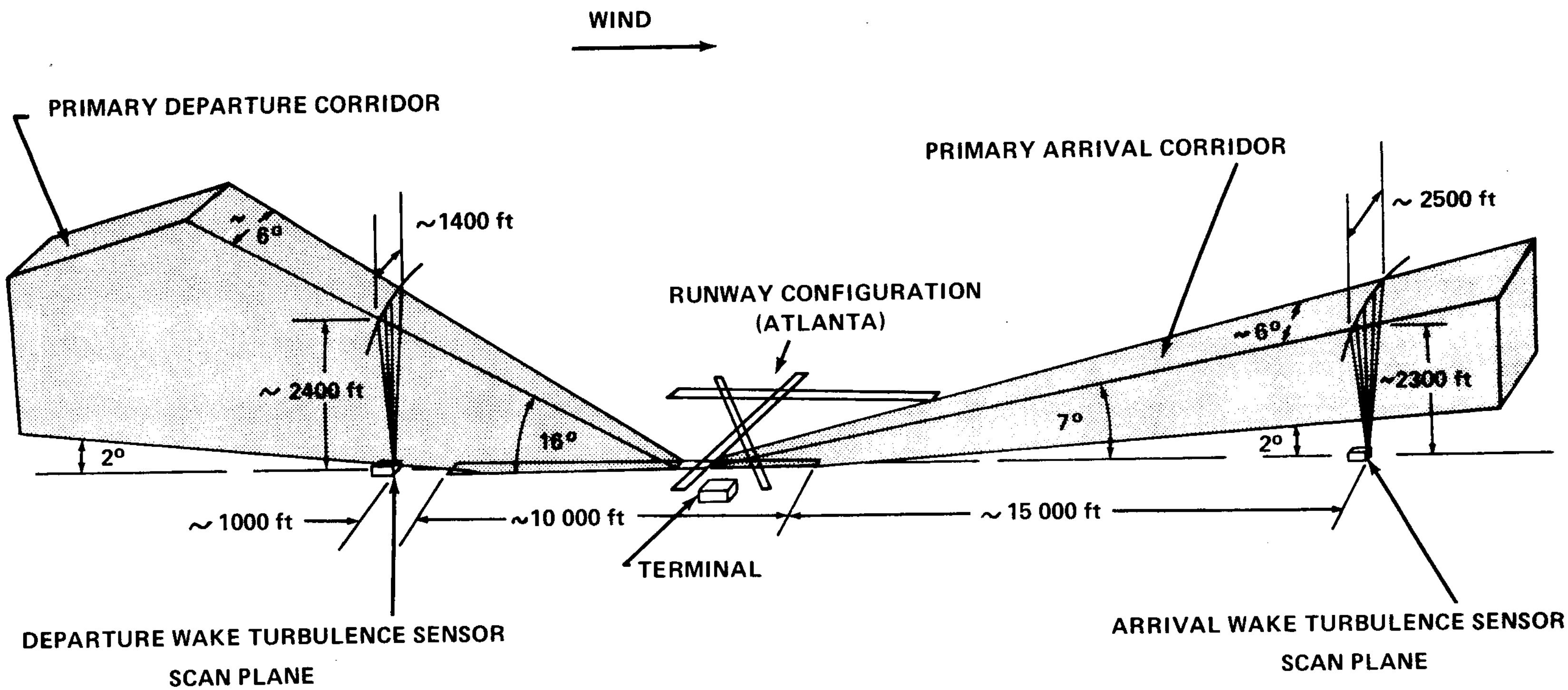


Figure 17. Typical physical layout of an airport with possible takeoff and landing scan planes.

The Aircraft Vortex Module, the Airport Layout Module, and the Control Module make it possible to simulate the horizontal and vertical transport of the vortex system in these scan planes. Figures 5, 8, 15, and 16 present the transport of vortex systems in these scan planes as a function of wind conditions and flight path angles. They also give indications as to the maximum transport and time required for the vortex systems to clear the scan planes and thus the corridors of concern.

From the Total System Simulation Model and the study developed in Appendix A, the preliminary spatial resolution requirements of the vortex sensors for takeoffs and for landings can be specified. The spatial resolution required of the takeoff scan plane vortex sensor is calculated in Appendix A to be ~ 105 ft. The spatial resolution required of the landing scan plane vortex sensor is also calculated in Appendix A and is estimated to be ~ 120 ft.

Also, from Figures A-1 and A-2 of Appendix A, the assumed scan plane penetration distributions for aircraft on takeoff and landings permit the rationale for specifying scan angle requirements for the remote sensors to be presented. Figures 18 and 19 give the scan angle requirements for the vortex sensors in the takeoff and landing plane, respectively.

Figure 18 indicates that for the assumed scan plane penetration distribution and location of a sensor unit at ground level in the center of the departure corridor a scan angle of 25 deg, either side of the vertical, will permit acquisition of at least 98.864 percent of all aircraft penetrations on takeoff. The distribution used to determine this scan angle may be in error, but the rationale for determining the scan angle can be retained for use when better distributions are available.

Figure 19 similarly indicates a scan angle requirement of 75 deg either side of the vertical for landings.

Indications from the system simulations study imply that the sensor units will not be required to scan on a continuous basis. Sequences of scans will be required to determine vortex location and transport in the scan plane. This information can then be used to predict the earliest time the area of concern will be clear of the vortex system. Periodic monitoring of the vortex system can be employed to upgrade the ATVWS's predictions.

The preliminary velocity resolution requirement of the ATVWS's sensor is best stated as the resolution that differentiates the vortex system from the rest of the atmosphere and allows the spatial resolution requirements

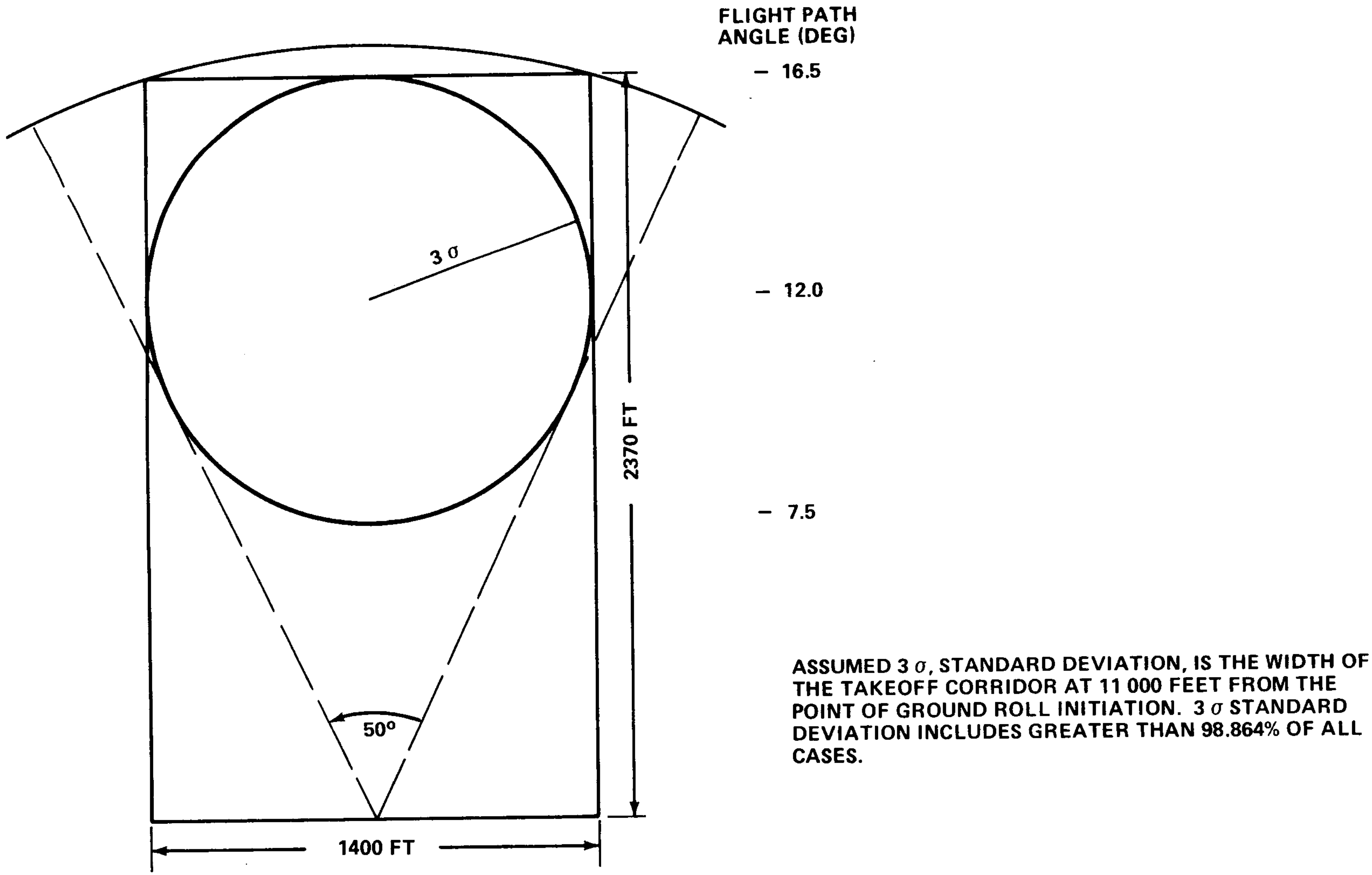
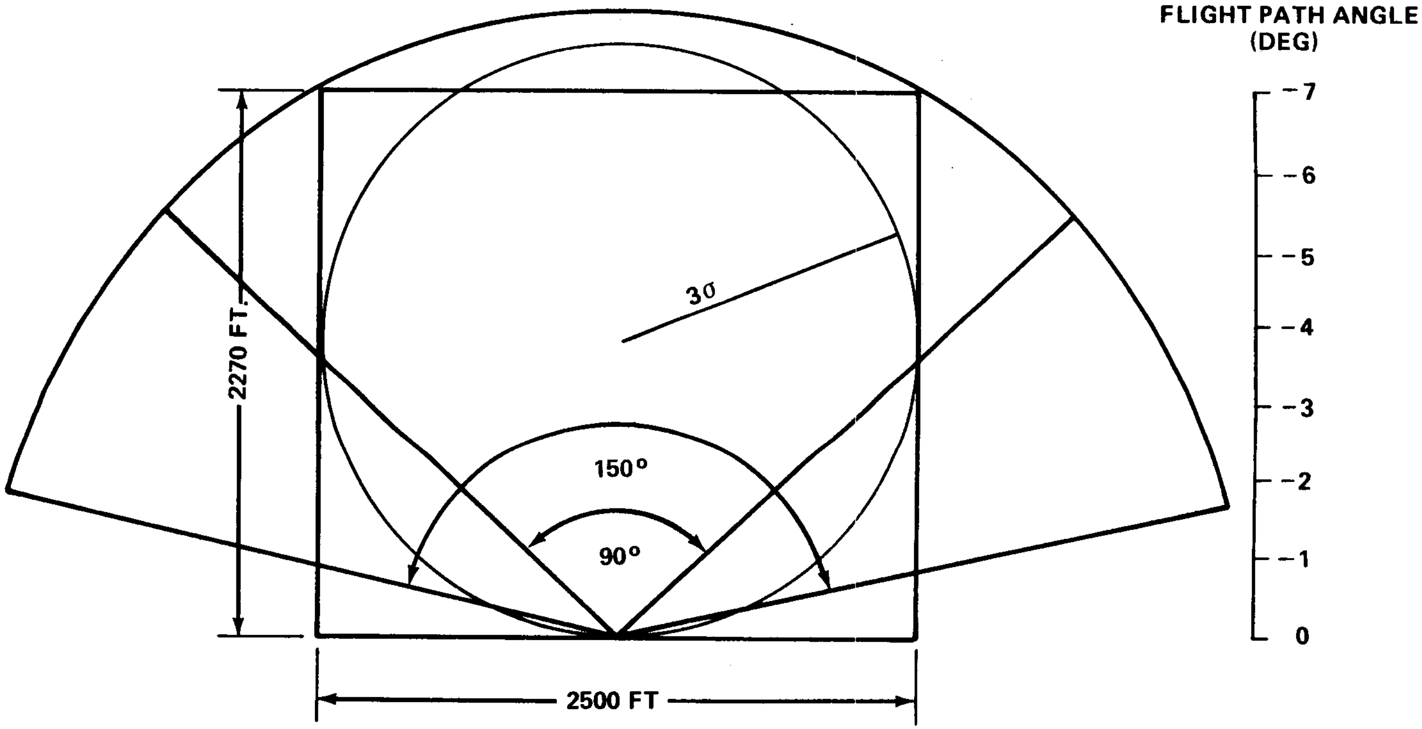


Figure 18. Scan angle as a function of takeoff plane.



ASSUMED 3 σ STANDARD DEVIATION IS THE WIDTH OF THE LANDING CORRIDOR AT 15 000 FEET FROM THE END OF THE RUNWAY. 3 σ STANDARD DEVIATION INCLUDES GREATER THAN 98.864 % OF ALL CASES.

Figure 19. Scan angle as a function of landing plane.

to be met. The preliminary conclusions of the study indicate that a velocity resolution of the vortex sensing system of 15 ft/s would be sufficient. This conclusion is partly based on the results of simulation studies, indicating that vortex velocities that persist for times greater than the present separation times can be hazardous to encountering aircraft and also the fact that the vortex system location with respect to the encountering aircraft, not vortex tangential velocity, is the better indicator of the potential hazard.

The Sensor Simulation Module and the Information Processing and Display Module indicate that a bistatic-focused LDV configuration can satisfy the range, spatial resolution, velocity resolution, and scanning requirements (preliminary) of an ATVWS. Early results from the Information Processing and Display Module indicate that the average weighted tangential velocity will be sufficient to specify the location of the vortex system in the scan plane. Tracking of the vortex system will give the vortex transport which may require 10 to 20 s of tracking data depending on wind condition and vortex settling rates.

The information processor should process the vortex weighted velocity, vortex system position, vortex system transport, and, possibly, vortex lifetime, and predict the time required for the vortex system to clear the corridor of concern with the prevailing atmospheric conditions.

The Data Display can potentially be as simple as indicating the time of takeoff for the next departing aircraft or confirming the required time separation between the next landing aircraft.

From the Total System Simulation, it appears that for best results with a single, one-dimensional sensor system, the best sensor placement will allow scanning in a plane approximately perpendicular to the axis of the vortex systems; i.e., approximately perpendicular to the aircraft flight path angle. This will permit better line-of-sight detection of the vortices' tangential velocities and will result in easier vortex acquisition and tracking.

The requirements of an ATVWS can ideally be more easily met with two- and three-dimensional LDV systems; however, there would naturally be more operational difficulties and more maintenance associated with more sensors.

The summary of system requirements, conclusions, and recommendations derived from this partial systems study will be given in Section V.

V. SYSTEM REQUIREMENTS

The preliminary system requirements derived from the partial systems study performed by the Mission Planning and Analysis Division at MSFC are as follows:

1. The system concept must consist of a combination measurement and predictive system.

2. The range requirement of the remote sensors are approximately 2500 ft for the takeoff corridor and approximately 2700 ft for the landing corridor.

3. The spatial resolution requirements of the remote sensors are approximately 105 ft for the takeoff corridor and approximately 120 ft for the landing corridor.

4. The velocity resolution requirement of the remote sensors is approximately 15 ft/s. The requirement is that velocity resolution which differentiates the vortex system from atmospheric winds and permits the spatial resolution requirement to be satisfied.

5. Scan plane location requirements consist of a takeoff corridor scan plane and a landing corridor scan plane. Because of changes in the wind direction (changes in the takeoff/landing direction), the takeoff and landing scan planes must change, thus resulting in a requirement of four scan planes per runway.

6. The required direction of scan is in a vertical plane, perpendicular to the runway and with angular scans of 75 deg either side of the vertical for the landing corridor and 25 deg either side of the vertical for the takeoff corridor.

7. The preliminary scan rate requirement is that the system must make periodic sequential scans of the scan plane to determine vortex location and transport. Naturally, the scan rate of each scan plane will be dictated by the size of the scan plane and should be of such a rate to avoid unnecessary vortex information degradation.

8. The detection parameter requirement is that parameter which gives the location and transport of the vortex system. At present, this parameter is thought to be the vortex system's tangential velocity.

9. The preliminary display requirement for an ATVWS is an indicator of the minimum required separation time between aircraft on landing and an indicator of the earliest time the takeoff corridor will be clear of vortices, thus permitting the next aircraft to takeoff safely.

VI. CONCLUSIONS AND RECOMMENDATIONS

The conclusions of this partial system study are as follows:

a. A complete system study using the Total System Simulation Model would yield many useful results in specifying the design and evaluation of designs for an ATVWS.

b. From Figure 9, it can be seen that theoretically the 1/3 m diameter, 1 m base leg bistatic LDV system can more than satisfy the preliminary range and range resolution requirements of an ATVWS. Experiments with MSFC's LDV system have shown their capability to satisfy the preliminary velocity resolution requirement. A bistatic LDV configuration can theoretically be designed to satisfy the preliminary system requirements.

c. The preliminary system requirement presented in this study can be used to evaluate the feasibility of employing any remote sensing instrument system in an ATVWS.

As a result of this partial systems study, the following recommendations are presented:

a. A complete systems study should be performed to assist in insuring the successful design of an ATVWS.

b. Because of the importance of vortex system transport, a series of experiments/flight test should be performed. These experiments should be designed to yield a better understanding of vortex transport as a function of aircraft and atmospheric conditions.

APPENDICES

APPENDIX A. PROBABILITY OF HAZARDS IN THE AIRPORT VICINITY BECAUSE OF WINGTIP VORTICES

Present operating procedures require a minimum separation between heavy aircraft (greater than 300 000 lb) and light aircraft (less than 300 000 lb) of 5 miles for landings and 120 s separation for takeoffs because of the hazards associated with wingtip vortices. Separation requirements between two light aircraft call for a minimum of 3 miles ($\approx$ 60 s). This latter minimum separation appears to be due more to old line practices than to the wingtip vortex problem.

The above guidelines in addition to assuming circular bivariate normal distributions (binormal distributions) for the generated vortices and the encountering aircraft's location about some mean position that coincides with typical real world values will be used in this study. It is realized that the assumption of circular binormal distributions is a simplifying assumption and that the results of the study may represent a conservative estimate of the real hazard probability under some circumstances. However, these results should be representative and should provide some perspective to defining the problem and relative risks.

The form of the probability function [$P(R, \phi)$] for the circular binormal distribution [30] is given by:

$$P(R, \phi) = \frac{1}{2\pi} \int_R \int_\phi e^{-K} R dR d\phi \qquad \text{(A-1)}$$

where

$$K = 1/2(R^2 + \overline{R}^2 - 2R\overline{R} \cos \phi) \qquad \text{(A-2)}$$

and

(X, Y) = mean location of the encountering aircraft

$(\overline{X}, \overline{Y})$ = mean location of the vortex system at some time, t

$(\bar{r})$ = the separation distance between the locations (X, Y) and $(\bar{X}, \bar{Y})$ at some time, t

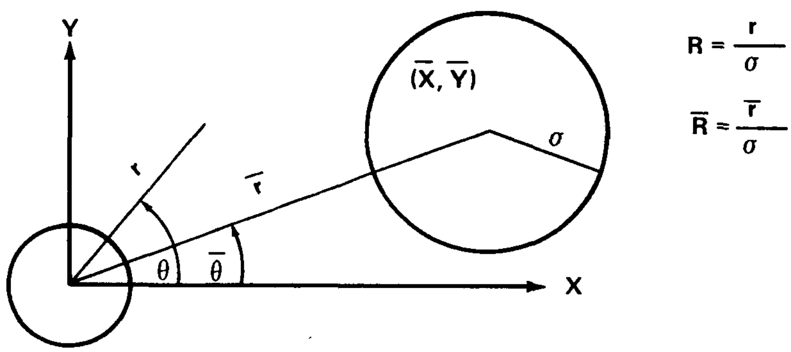
r = the hazard distance as defined for a particular pair of aircraft (vortex-generating and vortex-encountering aircraft)

σ = the standard deviation for the distributions representing the aircraft location and the vortex location.

$$R = \frac{r}{\sigma} \tag{A-3}$$

$$\bar{R} = \frac{\bar{r}}{\sigma} \tag{A-4}$$

For the special case, circular function, $\sigma_x = \sigma_y = \sigma$. Schematically, this can be represented as follows:



$\phi = 0 \rightarrow 2\pi$

To analyze the probability of an aircraft/vortex encounter, two geometric scan planes will be selected. For a takeoff situation, a scan plane perpendicular to the runway and located beyond the takeoff end of the runway will be selected as a representative area of concern. Similarly, for a landing situation, another scan plane located much farther from the landing end of the runway will be selected as a representative area of concern.

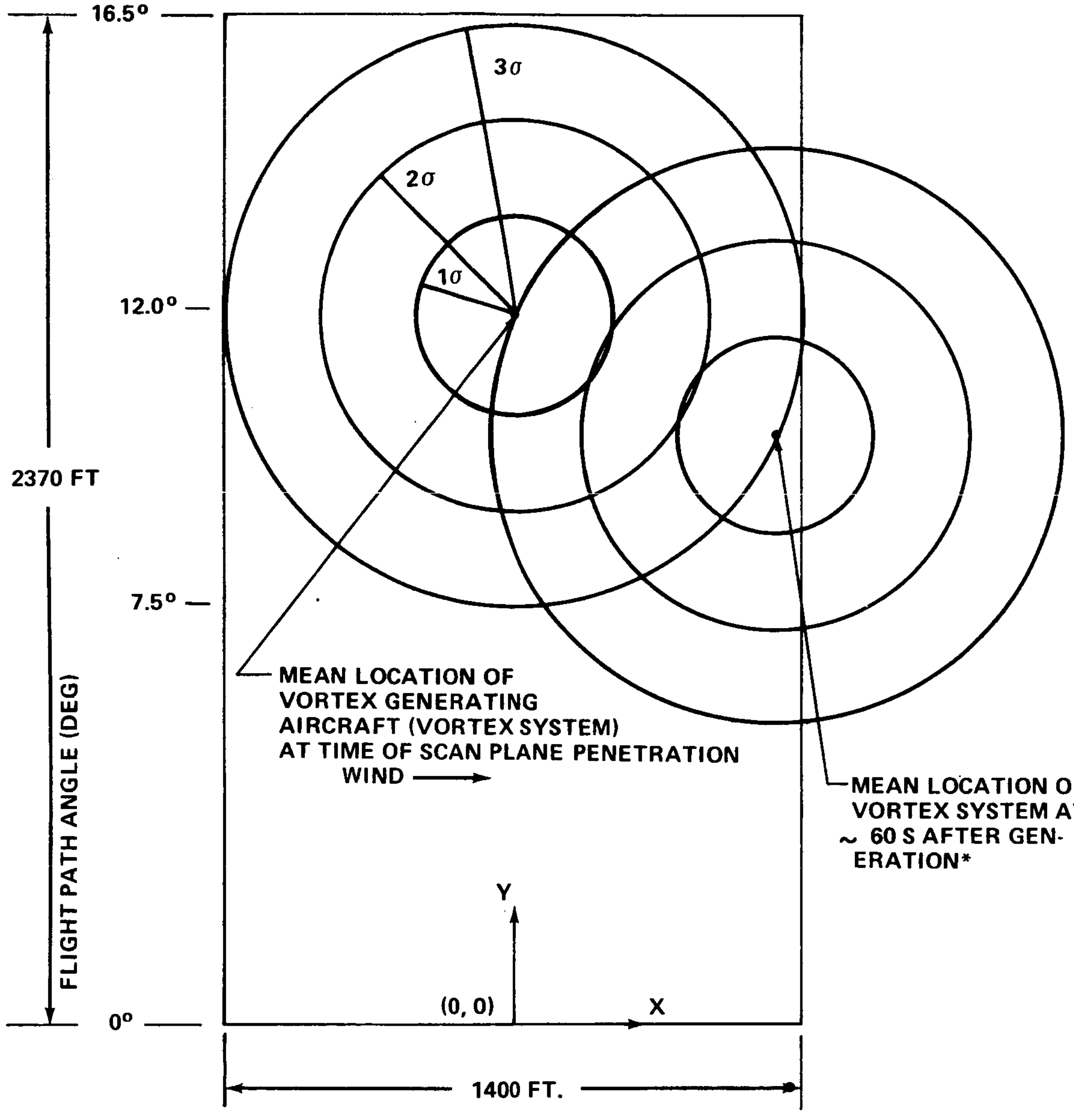
The size of the takeoff "area of concern," which is the takeoff corridor for departing aircraft, is given by a scan plane 1400 ft wide, 2370 ft high, and located approximately 1000 ft beyond the end of a 10 000-ft runway (Fig. 17).

The size of the "landing" area of concern, which is the landing corridor for arriving aircraft, is given by a scan plan 2500 ft wide and 2300 ft high, and located approximately 15 000 ft beyond the end of a 10 000-ft runway (Fig. 17).

The widths of the corridors are used to determine the 3 σ standard deviations for the binormal distributions used to describe the penetration points of departing and arriving aircraft in these planes. From Figure 17, it can be seen that the 3 σ standard deviation for the position of a departing aircraft's vortex system is approximately 700 ft. This assumes that 98.864 percent of all departing aircraft penetrate the takeoff "area of concern" on departing. The 700-ft, 3 σ standard deviation implies that the 1 σ standard deviation is approximately 233 ft. Similarly, the 1 σ standard deviation for an encountering aircraft on departing is also $\sim$ 233 ft. The means for the two binormal distributions used to describe the penetration points of the aircraft generating the vortex system (the vortex system location) and the aircraft encountering the vortex system are considered to be the same at the time of penetration in the scan plane. However, since the vortex-generating aircraft penetrates the scan plane of concern at some time prior to the encountering aircraft, it is assumed that the mean location of the vortex system will change with time as a function of wind conditions and vortex system settling rates.

The mean location of the vortex-generating aircraft (vortex system) at the time of penetration into the plane of concern along with the possible movement in time of the resultant vortex system (origin) is described in Figure A-1. From Figure 17, using the same arguments for landing aircraft, it can be seen that the standard deviation for the location of landing aircraft, both vortex generating and vortex encountering is approximately 416 ft. Again, it is assumed that the mean location of the aircraft's penetration point into the scan plane of concern is the same for both the vortex-generating and vortex-encountering aircraft.

Figure A-2 describes the mean location of vortex systems (for landing aircraft) at the time of scan plane penetration and how its mean location may change as a function of time, wind conditions, and vortex settling rates (Fig. 8).



*ASSUME 10 FT/S HEADWIND, 10 FT/S CROSSWIND, VORTEX SETTLING RATE 5 FT/S .

Figure A-1. Mean and standard deviation for aircraft and vortex systems in the takeoff corridor.

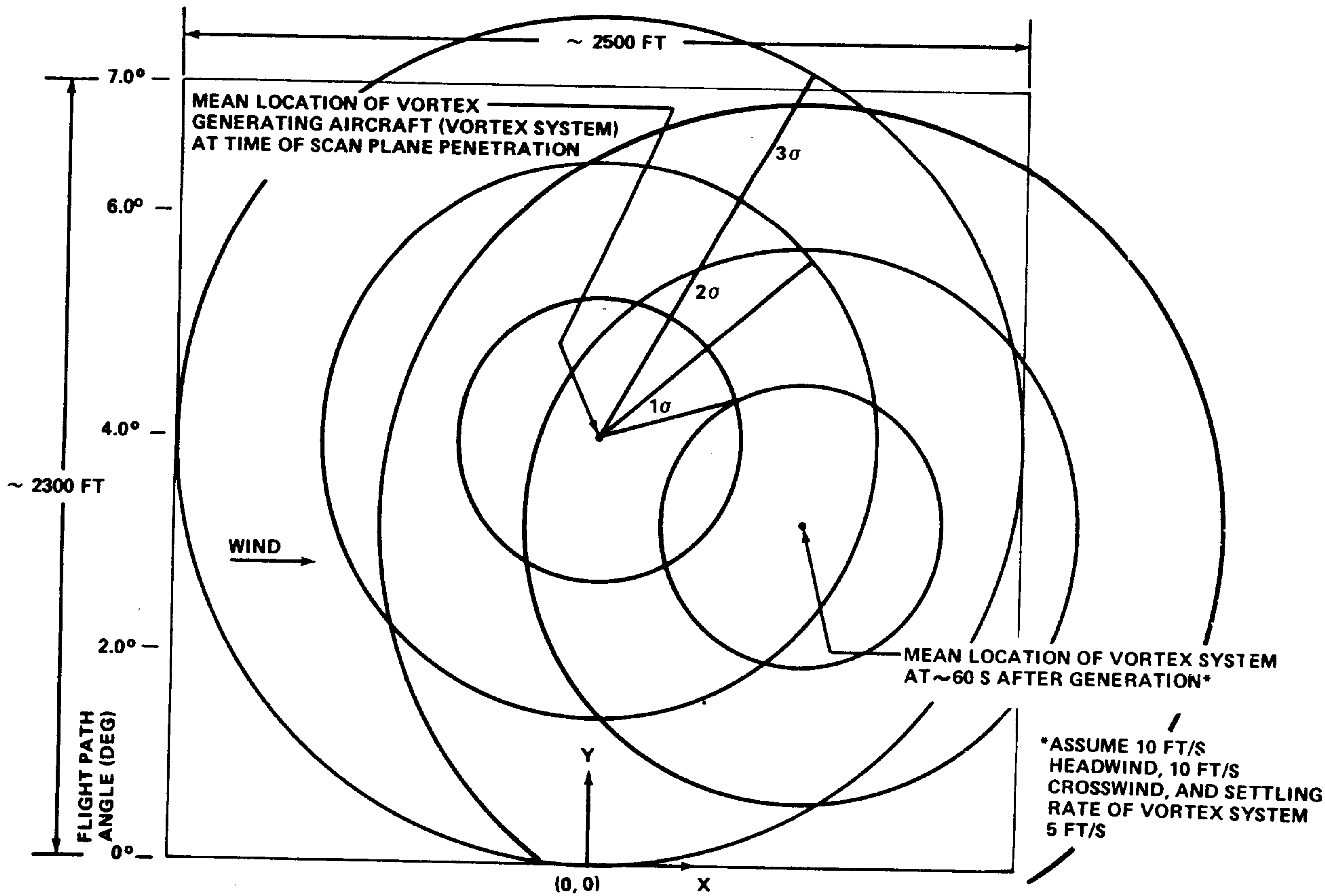


Figure A-2. Mean and standard deviation for aircraft and vortex system in the landing corridor.

This study is performed assuming that the hazard distance (radius) is 150 ft. The hazard radius is defined as the distance, measured from the center of the encountering aircraft's wingspan to the center of the hazardous vortex system, for which a vortex system can induce a maximum roll rate greater than the roll-rate capability of the encountering aircraft (Fig. 14). This hazard distance exists for a 747 generated vortex system with a 707 aircraft encountering the vortex system and would be less for smaller encountering aircraft even though the degree of hazard will potentially be greater.

The maximum probability of a hazard condition occurrence is given by the product of the following probabilities:

1. The generating aircraft's vortices will be located within 150 ft of the mean location of the encountering aircraft's bivariate normal distribution; i.e., the probability that the vortex system will be within a distance for which it can induce uncontrolled roll rates on the encountering aircraft.

2. The encountering aircraft will be within 150 ft of its assumed mean location.

The product of these two probabilities yields the maximum probability of the hazardous vortex system and the encountering aircraft occupying the same area (hazard radius) at the same time (the requirements for a hazard condition).

The probability of a hazard condition occurrence for both takeoffs and landings is theoretically decreased by consideration of the theoretical hazardous roll-rate contour areas inside the hazard radius (Fig. 14). However, it must be remembered that roll is not the only hazard in the vicinity of vortices; there is also the danger of stalls. From the area consideration of the roll-rate contour and the stall zones inside the hazard distance, the probability of a hazard condition inside the hazard radius is approximately 3.0×10^{-1}. This means that the product of the two previously mentioned probabilities can be reduced by 3.0×10^{-1} and still represent a reasonable estimate of the probability of hazard condition occurrence. These probabilities are plotted in Figures A-3 and A-4.

Figure A-3 is a plot of the maximum probability of a hazard condition occurrence (not necessarily an accident) in the takeoff corridor (at the previously mentioned plane) as a function of separation time between aircraft on takeoff for various wind conditions. It is assumed that the mean flight path angle of departing aircraft is 12 deg.

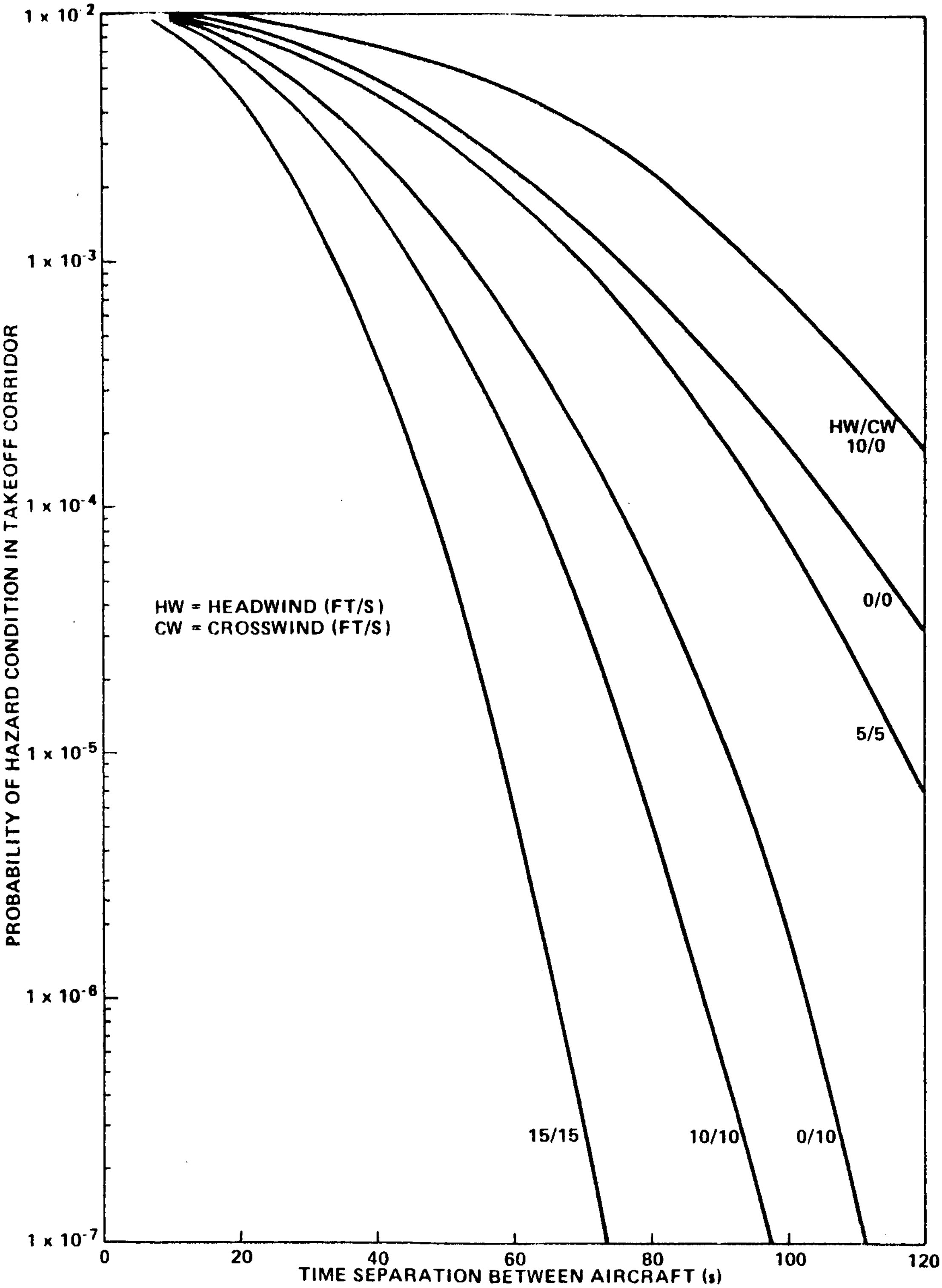


Figure A-3. Maximum probability of a hazard condition occurrence in the takeoff corridor as a function of separation time between aircraft on takeoff for various wind conditions.

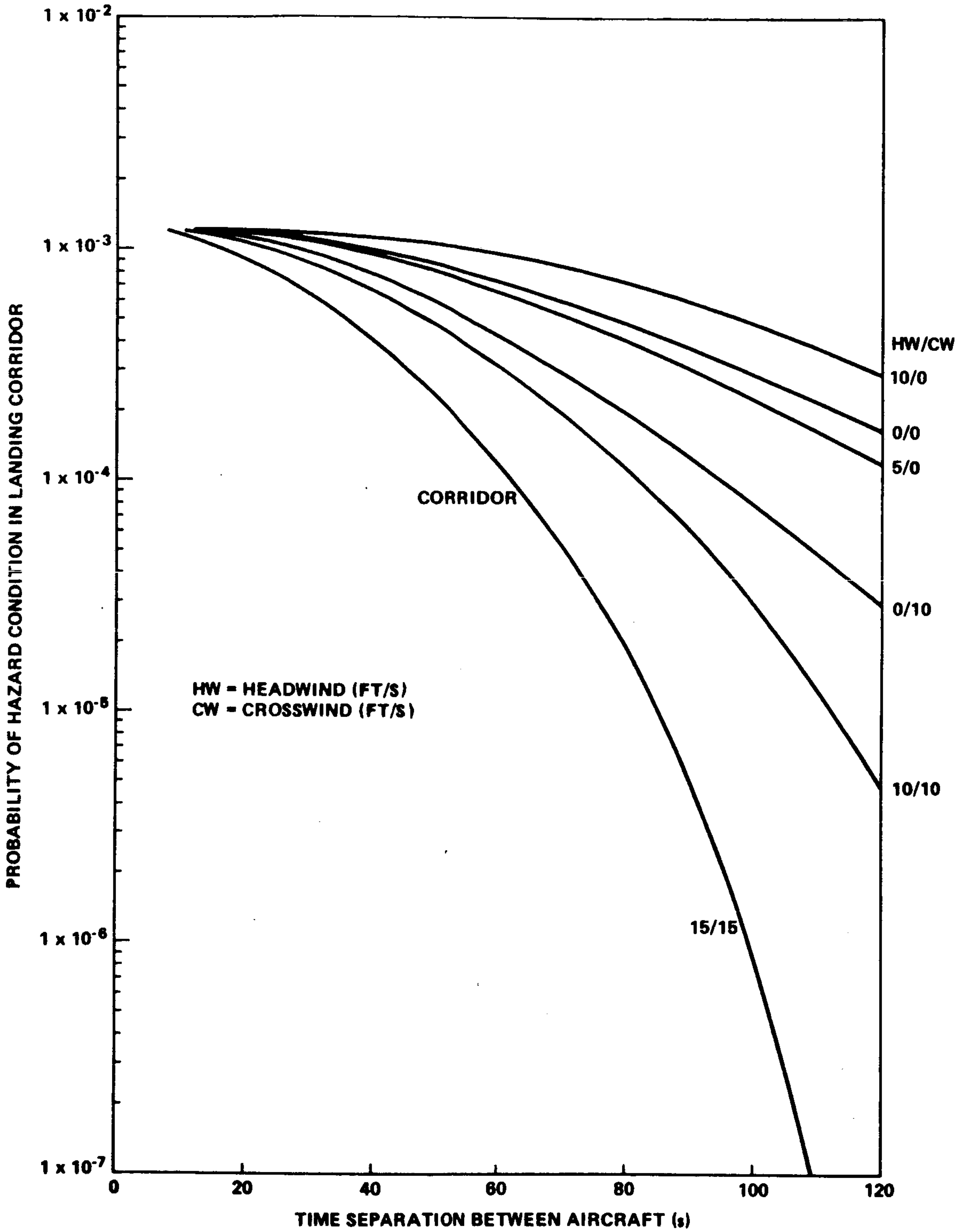


Figure A-4. Resulting probability of hazard condition occurrence for aircraft in the landing corridor.

The information in Figure A-3 implies that the vortex system is blown away from the mean location of the encountering aircraft's binormal distribution, that the separation distance is a function of time, and that the probability of hazard condition occurrence decreases as the separation distance between the two mean locations of the binormal distribution increases.

Figure A-4 is a similar plot for the resulting probability of hazard condition occurrence for aircraft in the landing corridor. Again, the results are similar because the vortex system is moved farther from the mean location of the penetration point into the previously mentioned scan plane (in the landing corridor), and the probability for a hazard condition occurrence decreases.

Remember that this probability reflects only one set of aircraft types on takeoff or landing; i.e., a heavy aircraft followed by a light aircraft (e.g., 747 followed by 707) such that the hazard radius equals 150 ft. For smaller aircraft, this hazard distance will be smaller, thus the probability of a hazard condition occurrence will be different.

It is of interest to note that the present rate of hazard incidents for commercial air operations is approximately 10 incidents per year for approximately 10 million air operations per year [31].

If the objective of the FAA/TSC is to decrease the separation time between departing aircraft or landing aircraft to approximately 30 s, then either (1) the aircraft vortex must be eliminated, (2) a new vortex monitoring and precise aircraft flying capabilities must be employed to maintain present safety standards, or (3) safety standards must be reduced and hazard incidents will undoubtedly rise.

For the average wind condition at Atlanta's Municipal Airport, ~ 9 mph at an angle of 45 deg to the main east-west [32] runways (this corresponds to ~ 9 ft/s headwind and 9 ft/s crosswind), and for the present separation times between large aircraft followed by small aircraft, this study predicts that the probability of hazard condition occurrence should be of the order of 1×10^{-7} for takeoffs and 1×10^{-5} for landings.

Results of this study indicate that a reduction in the separation time between aircraft to 30 s would increase the probability of a hazard condition occurrence to approximately 3.7×10^{-3} for takeoff and 8.7×10^{-4} for landings (using the assumed standard deviations of the binormal distribution) under moderate (10 ft/s headwind, 10 ft/s crosswind) wind conditions.

It appears that for certain wind conditions (high crosswind components), the standard separation time between aircraft can be significantly decreased while maintaining a desired safety factor. In other wind conditions (moderate headwinds and low crosswinds), it appears that the separation time between aircraft (takeoffs and landings) should be increased above standard separation times to maintain a desired safety factor.

From theoretical considerations, it appears that the probability of a hazard condition occurrence can be maintained at a safe operating level ($\sim 1 \times 10^{-6}$) while decreasing the separation time (depending on wind condition) between departing or arriving aircraft if a vortex monitoring and warning system is employed to predict the earliest time the takeoff or landing corridor will be clear of the preceding aircraft's vortex system. Naturally, the accuracy and reliability of such a system will be dependent on the real-time information supplied to the system by remote sensors of such things as vortex locations, atmospheric winds, vortex transport, etc.

The ATVWS could vary in degrees of sophistication from a totally predictive system employing a minimum number of remote sensors (wind sensor) to a combination vortex measurement and predictive system (hybrid system) that would incorporate the use of several remote sensors (vortex location/velocity sensor, atmospheric wind sensor, aircraft-type sensor, aircraft liftoff point sensor, aircraft touchdown point sensor, etc.).

By extending the use of statistics and probability theory and using the distribution assumed in the preceding section of this study, the spatial resolution required of a vortex monitoring system can be specified.

A procedure for determining the spatial resolution required of an ATVWS for departing aircraft is as follows: Assume the width of the departing corridor scan plane is given by Figure A-2 (1400 ft) and the distribution of departing aircraft's penetration into the scan plane is given by Figure A-1.

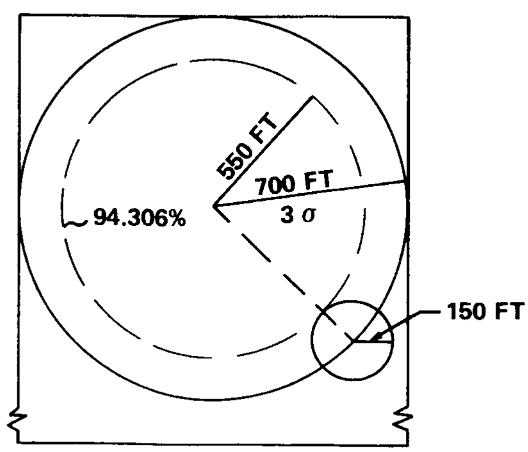
Assume that a requirement for safe air operations on takeoff is that the 3 σ area of the corridor be clear of the center of the vortex system shown in Figure A-1.

Figure 14 shows that the hazard radius for a Boeing 747 generated vortex system with a Boeing 707 encountering aircraft is approximately 150 ft. The 3 σ standard deviation for the distribution describing aircraft penetration points into the scan plane is assumed to be 700 ft or 1/2 the width of the specified corridor at that location. The 3 σ probability that the aircraft on takeoff will penetrate this scan plane is ~ 98.8640 percent. The probability that the 707 aircraft will penetrate the scan plane and be clear of the 747 vortex system centered at the outer boundary of the 3 σ area (that is the probability the 707 will be within 550 ft of its mean location at scan plane penetration) is 94.306 percent.

Now, the question may be asked, "What spatial resolution is required of the vortex monitoring system, when the vortex system is at a distance of 700 ft from the aircraft's mean location, which will ensure that the probability of having an aircraft in the vicinity of the vortex system is less than ~ 1×10^{-6}?"

The standard deviation of the distribution describing the aircraft's penetration points into the scan plane cannot be altered unless accurate takeoff flight paths are maintained by the pilots. Likewise, since it is the aircraft's position that initially determines the location of the vortex system, the vortex system has a probability distribution similar to the aircraft's distribution that describes its location. Since the standard deviation of the aircraft's location cannot easily be changed and thus the initial location of the vortex system generated by the preceding aircraft cannot be changed, it becomes obvious that a measure of the vortex's location by some remote sensing system must be employed to lower the standard deviation of the distribution describing the location of the vortex system. The 3 σ standard deviation of this distribution that allows for a safety factor of 1×10^{-6} for air operation on takeoff represents the spatial resolution required of a remote sensing system that could be employed to improve airport safety.

The following calculations indicate that the 3 σ standard deviation spatial resolution requirements on takeoffs and landings are of the order of 105 and 120 ft, respectively. To arrive at these values, we consider the following:



Let

σ_1 = standard deviation of the distribution, D_1, describing the location of the penetration points of the departing aircraft into the takeoff scan plane = 233 ft.

r_1 = the radius of integration over D_1 = 550 ft.

$\bar{r}_1$ = the radius from the center of the reference coordinate system to the mean location of $D_1 \cong 0$ ft.

Then

$$R_1 = \frac{r_1}{\sigma_1} = \frac{550}{233} \sim 2.36 \qquad \text{(A-5)}$$

and

$$\bar{R}_1 = \frac{\bar{r}_1}{\sigma_1} = \frac{0}{233} \approx 0 \quad . \qquad \text{(A-6)}$$

Using Table I given in Reference 1

$$P_1(R_1, \bar{R}_1) = P_1(2.36, 0) \sim 0.94306 \quad . \qquad \text{(A-7)}$$

This implies the probability of the departing aircraft being within 550 ft of the assumed mean is $P_1 = 0.94306$ or 94.306 percent of all departing aircraft will penetrate the scan plane within 550 ft of the assumed mean penetration point shown in Figure 4.

The standard deviation of the probability distribution, D_2, describing the location of the vortex system is to be determined in the following manner:

Let

σ_2 = standard deviation of the distribution D_2, describing the location of the generated vortex system (which is the parameter we wish to determine).

r_2 = the radius of integration over D_2 (same area as integrated over in D_1) $\cong$ 550 ft.

$\bar{r}_2$ = the radius from the center of the reference coordinate system (from which r_2 originates) to the mean location of D_2 = 700 ft.

Then

$$R_2 = \frac{r_2}{\sigma_2} = \frac{550}{\sigma_2} \tag{A-8}$$

and

$$\bar{R}_2 = \frac{\bar{r}_2}{\sigma_2} = \frac{700}{\sigma_2} \quad . \tag{A-9}$$

It is assumed that the probability of a hazard condition P_T is given by $P_T = P_1P_2$ and it is assumed that P_T be required to be $\cong 1 \times 10^{-6}$ or

$$P_T = 1 \times 10^{-6} = P_1P_2 = 0.94306\ P_2 \quad . \tag{A-10}$$

Thus

$$P_2 = \frac{1 \times 10^{-6}}{0.94306} \cong 1 \times 10^{-6} \quad . \tag{A-11}$$

Now

$$\frac{R_2}{\bar{R}_2} = \frac{\frac{550}{\sigma_2}}{\frac{700}{\sigma_2}} = \frac{550}{700} \quad , \tag{A-12}$$

which yields

$$R_2 = 0.786\,\overline{R}_2 \tag{A-13}$$

and

$$P_2(R_2, \overline{R}_2) = P_2(0.786\,\overline{R}_2, \overline{R}_2) \cong 1 \times 10^{-6} \quad . \tag{A-14}$$

By using Table I in Reference 1, it can be seen that

$$P_2(0.786\,\overline{R}_2, \overline{R}_2) \cong 1 \times 10^{-6} \tag{A-15}$$

only when $\overline{R}_2 = 20$ or $R_2 = 15.72$. Thus,

$$\sigma_2 = \frac{\overline{r}_2}{\overline{R}_2} = \frac{700}{20} = 35 \text{ ft} \quad . \tag{A-16}$$

For the 3 σ_2 standard deviation which includes 98.640 percent of all cases, the spatial resolution required of the monitoring system is ≅ 105 ft. For landing, similar calculations indicate that the required 3 σ_2 spatial resolution of a vortex monitoring system is ≅ 120 ft. It is recognized that this calculation has been made for a system that monitors the vortex system at the edge of the 3 σ area and that a real system must make its measurement after vortex generation but before the vortex system leaves the 3 σ area. This means that the ATVWS must be able to predict when the vortex system center will clear the 3 σ area by considering the vortex transport (settling rate and wind conditions). There will be uncertainties associated with vortex transport that must be added to the standard deviation of the vortex system location when it leaves the 3 σ area. But, these uncertainties will be dependent upon the accuracy of the wind-monitoring equipment, the models for vortex transport, and the time period over which the prediction is made. This will have to be determined as the ATVWS components are better defined.

The results of this study indicate the need for accumulation of statistical distributions describing the trajectories of aircraft on landings and takeoffs, and their liftoff and touchdown points. These distributions will be influenced by atmospheric conditions (winds, visibility, temperature, etc.) which should be studied.

APPENDIX B. THEORETICAL DEVELOPMENT OF THE AIRCRAFT/ VORTEX INTERACTION MODULE

The Aircraft/Vortex Interaction Module is designed to predict the roll rate which an aircraft experiences when it encounters a vortex system. It calculates the resultant peak roll rate and indicates if the aircraft experiences a stall for general cases where the vortex-generating and encountering aircraft are flying approximately in the same directions as occurs in the takeoff or landing corridors. The module calculates the resultant peak roll rate and stall or no-stall conditions for the encountering aircraft no matter how large the separation distance between the center of the encountering aircraft and the center of the vortex system (Fig. B-1).

The trajectories of aircraft on takeoff and landing as simulated in the Airport Layout Module give the initial location of the vortex flowfield (from the Aircraft Vortex Module). This allows the relative positions of vortex velocity flowfields and encountering aircraft to be simulated as a function of time, wind conditions, and aircraft and aircraft-vortex geometry. Using this information, the component of vortex velocity perpendicular to the aircraft wing can be calculated. Assume the aircraft velocity (encountering aircraft) along its flight path (the flight paths, liftoff points, and touchdown points of aircraft are variable) is constant, V_{ac}, and the tangential component of the vortex velocity, V_{vv}, is perpendicular to the encountering aircraft's flight path. This introduces some error into the module's output if the axis of the vortices is not parallel to the flight trajectory of the encountering aircraft. In most practical cases, this error is small; e.g., if the vortex axis is not parallel to the trajectory of the encountering aircraft by an angle θ, then the error in calculation of the vortex velocity perpendicular to the aircraft wing is given by $(1.0 - \cos\theta)$. For $\theta = 10$ deg, the error is $\sim(1.0 - 0.9848) = 0.0152 = 1.52$ percent. Recall from aircraft dynamics that the incremental lift, l, is given by

$$l = qc(a_o\alpha + C_{Lo}) \qquad \text{(B-1)}$$

where

$$q = 1/2\,\rho\,V_{ac}^2 = \text{dynamic pressure} \qquad \text{(B-2)}$$

ρ = air density

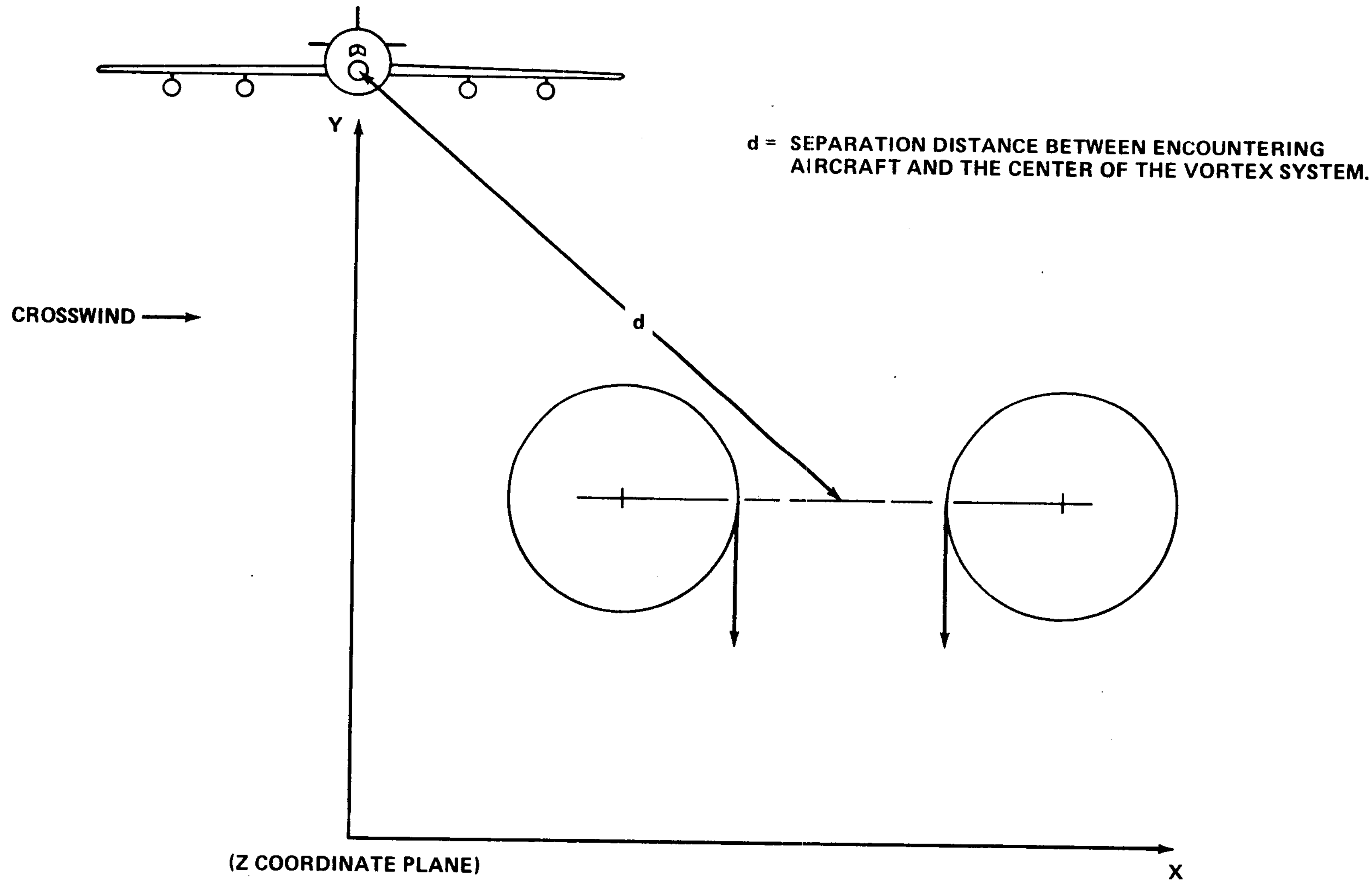


Figure B-1. Separation distance (d) between encountering aircraft and vortex system in reference coordinate system's Z-plane.

α = angle of attack for finite wing

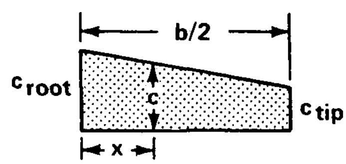
$$a_o = \frac{dCL}{d\alpha} = 0.07775 + 0.0104\alpha - 0.0015\alpha^2 + 0.00004\alpha^3 \qquad \text{(B-3)}$$

(See Reference 33.)

C_{L_o} = coefficient of lift at zero angle of attack due to flap setting

c = wing chord.

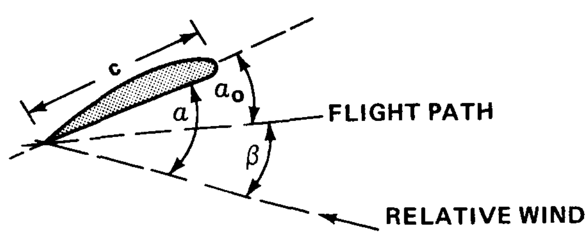
The wing chord dimension, c , depends on the taper ratio, λ , in a linear fashion for a straight taper wing as follows:



Taper ratio, $\lambda = c_{tip}/c_{root}$

Average chord, $\bar{c} = \frac{c_{tip} + c_{root}}{2}$

Chord, $c(x) = \frac{4\bar{c}}{b(\lambda + 1)} \; b/2 + (\lambda - 1)\, x$



$$\alpha = \alpha_o + \beta$$

$$\alpha_o = \text{constant} \tag{B-4}$$

$$\beta = \tan^{-1}\left(\frac{V_v}{V_{ac}}\right) \cong \frac{V_v}{V_{ac}} \quad . \tag{B-5}$$

Assume that V_v is defined as the actual velocity perpendicular to the encountering aircraft's flight path, taking into consideration the rolling motion of the encountering aircraft, as a result of the tangential component of the vortex velocity; i.e.,

$$V_v = V_{vv} - R_R x \tag{B-6}$$

x varies from wing tip to wing tip of the encountering aircraft, b/2 to -b/2.

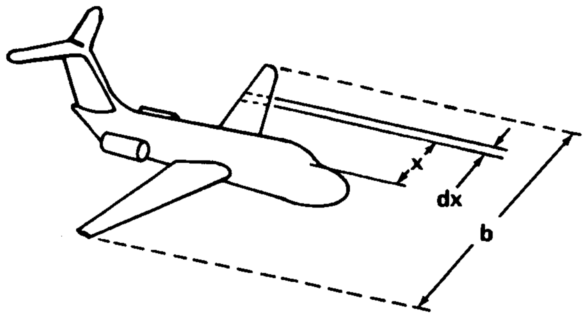
b = the wingspan of the encountering aircraft

R_R = aircraft roll rate.

Then, the lift-and-roll moments [34, 35] are given by

$$L_o = \int_{-b/2}^{b/2} l\, dx = \int_{-b/2}^{b/2} 1/2\, \rho (V_{ac})^2\, c \left[a_o(\alpha_o + \beta) + C_{L_o}\right] dx \tag{B-7}$$

$$R_o = \int_{-b/2}^{b/2} lx\, dx = \int_{-b/2}^{b/2} 1/2\, \rho (V_{ac})^2\, c \left[a_o(\alpha_o + \beta) + C_{L_o}\right] x\, dx \tag{B-8}$$



The roll moment coefficient, C_R , is defined as

$$C_R = \frac{R_o}{Sbq} \tag{B-9}$$

S = wing surface area (encountering aircraft).

Upon substitution, the following is obtained:

$$C_R = \frac{1}{Sb} \int_{-b/2}^{b/2} \frac{q(x)}{q(x)} C(x) \left[a_o(\alpha + \beta) + C_{L_o} \right] x \, dx \tag{B-10}$$

$$C_R = \frac{1}{Sb} \int_{-b/2}^{b/2} C(x) \left[a_o(\alpha_o + \beta) + C_{L_o} \right] x \, dx \quad . \tag{B-11}$$

Assume no roll acceleration and no lateral control inputs of the encountering aircraft; then, the rolling moment coefficient is equal to zero.

Setting $C_R = 0$ and solving for R_R, then the assumed constant gives:

$$R_R = \frac{\int_{-b/2}^{b/2} c(x)\, a_o \alpha_o\, x\, dx + \int_{-b/2}^{b/2} c(x)\, a_o \frac{V_{vv}}{V_{ac}}\, x\, dx + \int_{-b/2}^{b/2} c(x)\, C_{L_o}\, x\, dx}{\int_{-b/2}^{b/2} \frac{c(x)\, a_o\, x^2}{V_{ac}}\, dx} \quad . \tag{B-12}$$

The wing span of the aircraft is divided into 100 equal increments, Δx, and the vertical velocity component of the vortex velocity flowfield (thus, the incremental angle of attack) is calculated for each increment.

This equation is numerically evaluated by the Aircraft/Vortex Interaction Module to determine theoretically the maximum roll rate induced on various aircraft by vortices from other aircraft as a function of relative position of the aircraft and vortex system, types of aircraft (both generating and encountering), and age of the vortices.

The Aircraft/Vortex Interaction Module uses the calculation of the average angle of attack across the wing to serve as an indication of when the aircraft experiences a stall caused by the vortex velocity flowfield.

The average angle of attack across the wing is given by $\bar{\alpha}$ where $\bar{\alpha}$ is defined as follows:

$$\bar{\alpha} = \alpha_o + \bar{\beta} \tag{B-13}$$

where

$$\bar{\beta} = \frac{1}{101\,\bar{c}} \sum_{i=1}^{101} \beta_i c_i \quad . \tag{B-14}$$

From equations (B-4) and (B-5), it can be seen $\overline{\alpha}$ can approach a stall angle only if the vortex velocity flowfield acts on the wing span with a significant vertical component.

In this module, two situations are considered to represent a stall: stall type 1 and stall type 2.

Stall type 1 is defined as the situation where the average angle of attack across the wing is greater than 18 deg (for landing or departing aircraft).

Stall type 2 is defined as the situation where the average angle of attack is less than 50 percent of the magnitude of the flight path angle for landing aircraft and less than 50 percent of 5 deg + flight path angle for departing aircraft. The stall type 2 situation is to be used as a check on possible forced settling or lack of climbout due to vortex velocity flowfield.

The following is a computer listing of the Aircraft/Vortex Interaction Subroutine used in the Total System Simulation Model to calculate peak roll rates and stall or no-stall conditions. Following the subroutine are two sample outputs of the total systems simulation: one for a DC-9 aircraft following a 747 aircraft on takeoff with simulated scan plane at 7000 ft down the runway, and the other sample output is for a DC-9 aircraft following a 747 aircraft in the landing corridor. Here, the scan plane analysis is listed for 13 000 ft from the landing end of the runway.

Note in the sample output scan plane analysis that roll rate induced on the encountering aircraft as a function of relative location between aircraft and vortex system, time separation between the two aircraft, and atmospheric conditions is an output.

```
      SUBROUTINE NTRACT(VEL1,VEL2,VISC,SPAN2,XLAM,CBAR,A0,CL0,RR,GAMMA,
     1 VPK, RCORE, THETAD, ISTALL,COSINB,IFLITE,BETBAR)
      COMMON/COORD/TG, TE, XG,YG,ZG, XE, YE, ZE, XVGT1, YVGT1, ZVGT1,
     1           XVGT2, YVGT2,  ZVGT2
      COMMON/CNST/PI, PI2,PI3,  PI4, TERM1, TERM2
      DIMENSION XN(101),XD(101),XN1(101)
      PI=3.1415873
      DX=.01*SPAN2
      X=-SPAN2/2.
      RRN=0.
      RRN1=0.
      RRD=0.
      Z=VEL1*(TE-TG)
      TERMX=SQRT(VEL1/(Z*VISC))                                        2
      VPK=.639*GAMMA*TERMX*PI4
      RCORE=2.24/TERMX
      SUM=0.0
      DO 10 I=1,101
      R1=SQRT(((XE+X)-XVGT1)**2+(YE-YVGT1)**2)                         6
      R2=SQRT(((XE+X)-XVGT2)**2+(YE-YVGT2)**2)                         7
      RLV1=R1/2.*SQRT(VEL1/(Z*VISC))                                   8
      RLV2=R2/2.*SQRT(VEL1/(Z*VISC))                                   9
      VLV1=(1.0-EXP(-RLV1**2))/RLV1                                    10
      VLV2=(1.0-EXP(-RLV2**2))/RLV2                                    11
      VTANL1=VLV1*GAMMA*SQRT(VEL1/(Z*VISC))/(4.0*PI)                   12
      VTANL2=VLV2*GAMMA*SQRT(VEL2/(Z*VISC))/(4.0*PI)                   13
      VVERT1=VTANL1*((XE+X)-XVGT1)/R1
      VVERT2=-VTANL2*((XE+X)-XVGT2)/R2
      VVERT=VVERT1+VVERT2
      BETA=ATAND(VVERT/VEL2)                                           14
      ALPHA= BETA + 5.0
      A0=.07775 + ALPHA*(.0104 + ALPHA*(-.0015 + .00004*ALPHA))
      C=4.*CBAR/(SPAN2*(XLAM+1.))*(SPAN2/2.+(XLAM-1.)*ABS(X))
      SUM= SUM + BETA*C
      XN(I)=C*A0*VVERT*X/VEL2
      XN1(I)=C*CL0*X
      XD(I)=C*A0*X*X/VEL2
      X=X+DX
   10 CONTINUE
      DO 14 J=2,101
      RXN=(XN(J)+XN(J-1))*DX/2.
      RXN1=(XN1(J)+XN1(J-1))*DX/2.
      RXD=(XD(J) + XD(J-1))*DX/2.
      RRN=RRN+RXN
      RRN1=RRN1+RXN1
   14 RRD=RRD+RXD
      RR=(RRN+RRN1)/RRD*57.29577951
      ISTALL=0
      BETBAR=SUM/(101.*CBAR)
      IF(BETBAR.GE.13.) ISTALL=3
      GO TO (20,30),IFLITE
C     TAKE-OFF OPERATION
   20 CONTINUE
      IF(BETBAR.LE.(-.50*THETAD)) ISTALL=4
      RETURN
C    LANDING OPERATION
   30 CONTINUE
      THETAD=(90.-ARCOSD(COSINB))                                      41
      IF(BETBAR.LE.(-.50*ABS(THETAD))) ISTALL=4
      RETURN
      END
```

```
                         FLIGHT PATTERN ANALYSIS FOR CASE NUMBER **    1    **

                         TAKE-OFF OPERATION

     THERE ARE **  2  ** AIRCRAFT IN THE FLIGHT PATTERN, AND SCANS FOR HAZARDOUS CONDITIONS WILL BE MADE AT ** 10 ** LOCATIONS
ALONG THE FLIGHT PATH. THE MINIMUM TIME OF SEPARATION BETWEEN AIRCRAFT IS **   30. ** SECONDS, AND THE STEP-SIZE FOR INCREASING
SEPARATION IS **     5. SECONDS.

                                        ATMOSPHERIC CONDITIONS

          EDDY VISCOSITY IS **  0.3140 ** (FT**2/SEC)                       CROSS WIND IS **    -2.60 ** (FT/SEC)
          DENSITY IS **  0.002380 ** (SLUGS/FT**3)                         HEAD WIND IS  **     3.00 ** (FT/SEC)

*** AIRCRAFT DATA ***

AIRCRAFT TYPE            747        DC-9
WEIGHT (LB.)             710000.     77700.
VELOCITY (FT/SEC)        287.       243.
WING SPAN (FT.)          196.        89.
MEAN AERO. CHORD (FT)    27.3       11.8
SURFACE AREA (FT**2)     5500.      9340.
ASPECT RATIO             6.95       7.40
TAPER RATIO              0.38       0.38
ROLL RATE LIMIT (DEG)    11.75      21.90
LIFT COEFFICIENT         1.00       1.00
FLT. PATH ANGLE (DEG)     8.        12.
LIFT-OFF POINT (FT)      6400.      6500.
TAKE-OFF TIME FOR PLANE NO. 1   10.

                              *-*-*-*  SCAN PLANE ANALYSIS  *-*-*-*

SCAN PLANE NO.   4 AT   7000. FEET
-----------------------------------

VORTEX GENERATING AIRCRAFT IS A * 747     *
ENCOUNTERING AIRCRAFT IS A       * DC-9   *
TIME (SEC) OF GENERATION OF VORTEX BEING SCANNED IS .... .......,          12.45
AIRCRAFT LOCATION (FT) DURING GENERATION OF VORTEX BEING SCANNED     X=     0.00     Y=    96.84     Z=  7089.04
TIME (SEC) OF ENCOUNTER IS....................................             42.13
ENCOUNTERING AIRCRAFT LOCATION (FT) AT TIME OF ENCOUNTER .......     X=     0.00     Y=   106.28     Z=  7000.00
VORTEX 1 LOCATION (FT) AT TIME OF ENCOUNTER .....................    X=   108.68     Y=    49.00     Z=  7000.00
VORTEX 2 LOCATION (FT) AT TIME OF ENCOUNTER .....................    X=  -263.01     Y=    49.00     Z=  7000.00
TIME SEPARATION (SEC) BETWEEN AIRCRAFT IS .......................          30.00
ROLL RATE (DEG/SEC) INDUCED IN ENCOUNTERING AIRCRAFT IS .........          -1.28
ROLL RATE CAPABILITY (DEG/SEC) OF ENCOUNTERING AIRCRAFT IS .....           21.90
PEAK TANGENTIAL VELOCITY OF VORTEX AT TIME OF ENCOUNTER IS .....          112.14
RADIUS OF CORE OF VORTEX AT TIME OF ENCOUNTER IS ................           6.84
THEORETICAL ROLL RATE WHICH WOULD RESULT FROM A CENTER COLLISION           93.79
FLIGHT PATH ANGLE OF ENCOUNTERING AIRCRAFT IS ...................          12.00
AVERAGE VORTEX CONTRIBUTION TO ANGLE OF ATTACK IS ..............           -2.66
```

```
                         FLIGHT PATTERN ANALYSIS FOR CASE NUMBER **     1   **

                         LANDING OPERATION

     THERE ARE **  2  ** AIRCRAFT IN THE FLIGHT PATTERN, AND SCANS FOR HAZARDOUS CONDITIONS WILL BE MADE AT ** 10 ** LOCATIONS
 ALONG THE FLIGHT PATH. THE MINIMUM TIME OF SEPARATION BETWEEN AIRCRAFT IS **   30. ** SECONDS, AND THE STEP-SIZE FOR INCREASING
 SEPARATION IS **     5. SECONDS.

                                        ATMOSPHERIC CONDITIONS

           EDDY VISCOSITY IS **  0.3140 ** (FT**2/SEC)                       CROSS WIND IS **   -2.60 ** (FT/SEC)
           DENSITY IS **  0.002330 ** (SLUGS/FT**3)                          HEAD WIND IS  **    3.00 ** (FT/SEC)

*** AIRCRAFT DATA ***

 AIRCRAFT TYPE             747        DC 9
 WEIGHT (LB.)              710000.     77700.
 VELOCITY (FT/SEC)         297.       243.
 WING SPAN (FT.)           196.        89.
 MEAN AERO. CHORD (FT)     27.3       11.8
 SURFACE AREA (FT**2)      5500.      9340.
 ASPECT RATIO              6.95        7.40
 TAPER RATIO               0.38        0.38
 ROLL RATE LIMIT (DEG)     11.75       21.90
 LIFT COEFFICIENT          1.00        1.00
 X DESCENT COORD. (FT)      200.          0.
 Y DESCENT COORD. (FT)      850.        650.
 Z DESCENT COORD. (FT)   -15000.     -15000.
 X TOUCHDOWN COORD(FT)        0.          0.
 Z TOUCHDOWN COORD(FT)     2000.       1000.
 DESCENT INITIATION TIME FOR PLANE NO. 1   10.

                               *-*-*-*  SCAN PLANE ANALYSIS  *-*-*-*

SCAN PLANE NO.   2 AT  -13000. FEET
-------------------------------------

VORTEX GENERATING AIRCRAFT IS A * 747    *
ENCOUNTERING AIRCRAFT IS A      * DC 9   *
TIME (SEC) OF GENERATION OF VORTEX BEING SCANNED IS .... .......          17.38
AIRCRAFT LOCATION (FT) DURING GENERATION OF VORTEX BEING SCANNED    X=   175.38     Y=   745.36     Z=-12907.12
TIME (SEC) OF ENCOUNTER IS......................................          48.34
ENCOUNTERING AIRCRAFT LOCATION (FT) AT TIME OF ENCOUNTER .......    X=    -0.00     Y=   568.75     Z=-13000.00
VORTEX 1 LOCATION (FT) AT TIME OF ENCOUNTER ....................    X=   166.42     Y=   528.13     Z=-13000.00
VORTEX 2 LOCATION (FT) AT TIME OF ENCOUNTER ....................    X=    23.34     Y=   528.13     Z=-13000.00
TIME SEPARATION (SEC) BETWEEN AIRCRAFT IS ......................          30.00
ROLL RATE (DEG/SEC) INDUCED IN ENCOUNTERING AIRCRAFT IS ........         -16.13
ROLL RATE CAPABILITY (DEG/SEC) OF ENCOUNTERING AIRCRAFT IS .....          21.90
PEAK TANGENTIAL VELOCITY OF VORTEX AT TIME OF ENCOUNTER IS .....         109.80
RADIUS OF CORE OF VORTEX AT TIME OF ENCOUNTER IS ...............           6.98
THEORETICAL ROLL RATE WHICH WOULD RESULT FROM A CENTER COLLISION          34.84
FLIGHT PATH ANGLE OF ENCOUNTERING AIRCRAFT IS ..................          -2.33
AVERAGE VORTEX CONTRIBUTION TO ANGLE OF ATTACK IS ..............           0.17
```

APPENDIX C. THE LASER DOPPLER VELOCIMETER VOLUME SCAN SIMULATION PROGRAM

The computer program is designed to simulate the output of a single LDV system as it scans a vortex system at various angles and ranges from its various positions along the corridors of interest. The output of the program is air velocity as a function of observation angles, range, and system design.

The capabilities of the sensor simulation include the following:

1. The LDV system can be located at any position in the airport layout coordinate system.

2. The LDV system can scan any desired volume in any direction.

3. The tangential velocity profile of the vortex system along the line of sight of the LDV system can be calculated as a function of LDV systems design, range-angle of observation, sensor system location, and vortex velocities.

4. The size of the LDV system's transmitting/receiving apertures for a focused system design can be selected. (For pulse LDV systems the pulse length can be selected.)

The limitations and assumptions of this module include the following:

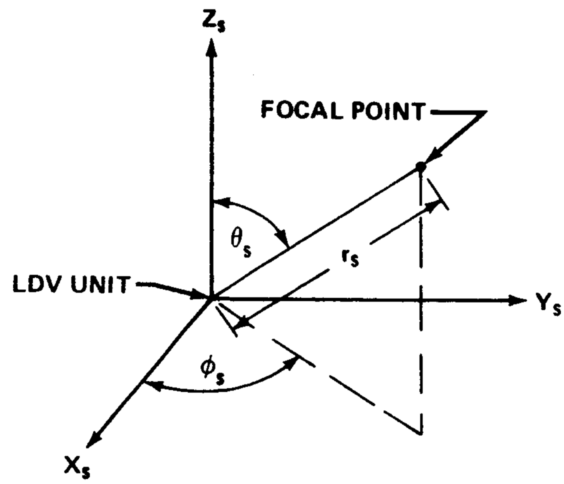
1. The module assumes that no wind signal is mixed with the vortex flowfield signal.

2. The backscattering aerosols are assumed to be constant throughout the vortex.

3. The vortex is assumed to be approximated by a straight line.

4. A limitation of the simulation is the assumption of an infinite signal-to-noise (S/N) ratio.

The LDV system is located in the airport layout coordinate system and uses a spherical coordinate system, origin at the sensor unit, to locate the LDV's focal point and scan plane orientation.



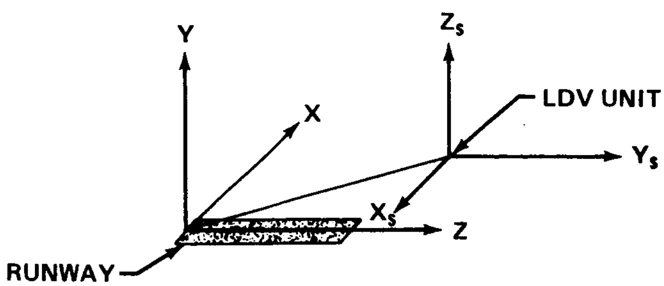
The transformation from the spherical coordinate to the LDV's rectangular coordinate system is

$$X_s = r_s \sin \theta_s \cos \phi_s$$

$$Y_s = r_s \sin \theta_s \sin \phi_s$$

$$Z_s = r_s \cos \theta_s \quad .$$

The transformation and translation of the LDV's rectangular coordinate system to the airport layout reference coordinate system is



$$X = X_o - X_s$$

$$Y = Y_o + Z_s$$

$$Z = Z_o + Y_s$$

where X_o, Y_o, Z_o is the position coordinates of the LDV unit in the airport layout reference coordinate system.

The scanning bounds of the LDV system are input:

ϕ_{s1} lower bound of ϕ_s

ϕ_{s2} upper bound of ϕ_s

θ_{s1} lower bound of θ_s

θ_{s2} upper bound of θ_s

r_{s1} lower bound of r_s

r_{s2} upper bound of r_s .

Also, the number of divisions within each of the above intervals is input.

The scanning starts at the lower bound of each parameter. The simulated sensor first scans across ϕ_s then moves along θ_s while it continues to scan ϕ_s and, finally, it moves along r_s while still scanning ϕ_s and θ_s. For each point of the scan, the simulation computes the velocity component of the vortex in the direction of the LDV unit.

The computation of the tangential velocity component of the vortex system in the direction of the LDV system is accomplished first by transforming the focal point of the LDV system from the spherical system to the airport coordinate system.

$$X = X_o - r_s \sin\theta_s \cos\phi_s$$

$$Y = Y_o + r_s \cos\theta_s$$

$Z = Z_o + r_s \sin\theta_s \sin\phi_s$ (Fig. C-1)

$S = \left(X_{s_s}, Y_{s_s}, Z_{s_s}\right)$ focal point in LDV's rectangular system

$P_1 = (X_{sp_1}, Y_{sp_1}, Z_{sp_1})$ point on vortex$_1$ axis in a particular scan plane in the airport layout coordinate system

$P_2 = (X_{sp_2}, Y_{sp_2}, Z_{sp_2})$ point on vortex$_2$ axis in a particular scan plane in the airport layout coordinate system

$\overrightarrow{P_1Q_1}$ = direction of vortex$_1$ axis

$\overrightarrow{P_2Q_2}$ = direction of vortex$_2$ axis.

In this simulation, the vectors $\overrightarrow{P_1Q_1}$ and $\overrightarrow{P_2Q_2}$ are taken to be unit vectors, but they can be chosen to be of arbitrary length since

$$\frac{|\overrightarrow{SP_1} \times \overrightarrow{P_1Q_1}|}{|\overrightarrow{P_1Q_1}|} = \frac{|SP_1| \cdot |P_1Q_1| \sin(\text{angle between them})}{|P_1Q_1|}$$

and the P_1Q_1 magnitudes cancel.

The distance, D_1, from S to the vortex$_1$ axis is given by

$$D_1 = \frac{|\overrightarrow{SP_1} \times \overrightarrow{P_1Q_1}|}{|\overrightarrow{P_1Q_1}|}$$

$$\overrightarrow{SP_1} = \overrightarrow{OP_1} - \overrightarrow{OS} \quad .$$

The distance, D_2, from S to the vortex$_2$ axis is given by

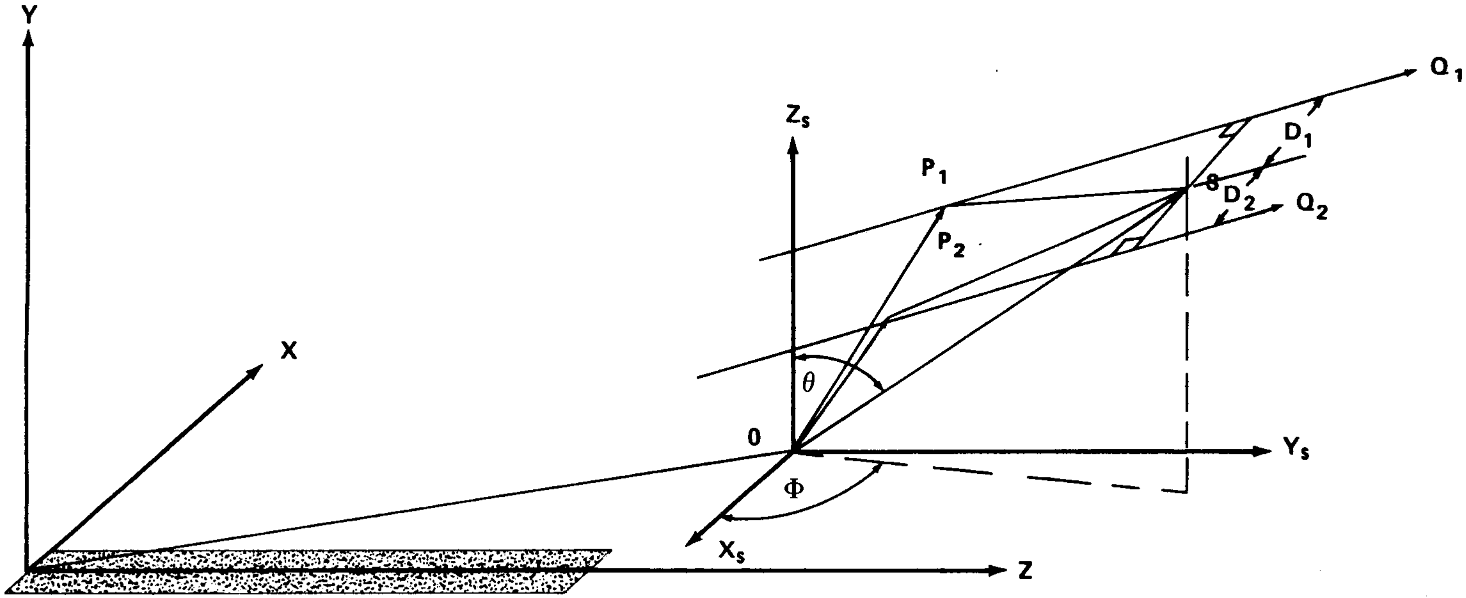


Figure C-1. Sensor coordinate system with respect to the Airport Layout Modules' reference coordinate system.

$$D_2 = \frac{|\overrightarrow{SP_2} \times \overrightarrow{P_2Q_2}|}{|\overrightarrow{P_2Q_2}|}$$

$$\overrightarrow{SP_2} = \overrightarrow{OP_2} - \overrightarrow{OS} \quad .$$

The total tangential velocity magnitude of the vortex system at point S is given by the aircraft vortex module as $V_T = V_{T_1} + V_{T_2}$

V_{T_1} = vortex$_1$ tangential velocity

V_{T_2} = vortex$_2$ tangential velocity

$$\overrightarrow{VT1} = V_{T_1} \frac{(\overrightarrow{SP_1} \times \overrightarrow{P_1Q_1})}{|\overrightarrow{SP_1} \times \overrightarrow{P_1Q_1}|}$$

$$\overrightarrow{VT2} = V_{T_2} \frac{(\overrightarrow{SP_2} \times \overrightarrow{P_2Q_2})}{|\overrightarrow{SP_2} \times \overrightarrow{P_2Q_2}|}$$

$$\overrightarrow{V_T} = \overrightarrow{VT1} + \overrightarrow{VT2} \quad .$$

The magnitude of the component of $\overrightarrow{V_T}$ in the direction of $\overrightarrow{SO}$ is

$$V = \frac{(-\overrightarrow{V_T} \cdot \overrightarrow{OS})}{|\overrightarrow{OS}|} \quad .$$

The detection of the vortex velocity flowfield using the coaxial-focused LDV system is simulated as follows: The LDV does not make a point measurement. It effectively samples a finite volume of space along a line and consequently observes a variety of velocities depending on the inhomogenuity of the flowfield. The signal is the strongest from the focal point and decreases as the elemental volume considered increases its distance from the focus.

The power contained in the radiation detector due to N particles per cubic centimeter is proportional to the current squared, as shown in the following equation [36]:

$$i^2 = 1/4\ \pi\eta^2\ \alpha^2\sigma\ R^4 A^4 N \int_0^\infty \frac{dL}{L^2 + \dfrac{\pi^2 R^2}{\lambda^2 f^2}(f - L)^2}$$

where

η = quantum efficiency of the detector (electrons/photon)

σ = backscattering coefficient of the particles

α = power level of the local oscillator

R = radius of the transmitter lens

A^2 = transmitted light flux in photon/s

N = effective number of identical particles/cm^3

f = nominal range of focusing

λ = optical wavelength of transmitter

L = range from transmitter lens.

The current squared due to an infinitesimal unit of length dL is

$$\frac{di^2}{dL} = (1/4\ \pi\eta^2\ \alpha^2\ \sigma\ A^4 N) \frac{R^4}{\left[L^2 + \dfrac{\pi^2 R^4}{\lambda^2 f^2}(f - L)^2\right]} .$$

Let $\Delta L = \lambda Lf/\pi R^2$; then,

$$\frac{di^2}{dL} = (1/4\pi\eta^2 \alpha^2 \sigma A^4 N) \frac{R^4}{\left[\left(1 + \frac{(f - L)^2}{\Delta L^2}\right) L^2\right]} .$$

Assume η, α, σ, A, N are constants and let

$$K_1 = 1/4 \pi\eta^2 \alpha^2 \sigma A^4 N,$$

then

$$\frac{di^2}{dL} = K_1 \frac{R^4}{\left[\left(1 + \frac{(f - L)^2}{\Delta L^2}\right) L^2\right]} .$$

This is proportional to the equation used in this module:

$$\frac{di^2}{dL} \approx I_N = \left(\frac{\psi_o \pi}{\lambda}\right)^2 \frac{R^4}{\left(1 + \frac{(f - L)^2}{\Delta L^2}\right) L^2}$$

where $(\psi_o \pi/\lambda)^2$ is a constant.

Therefore, the equation in this module is proportional to the power in the detector because of a volume of space having an infinitesimal unit of length in the line of sight direction from the detector.

In this module, only the interval $(f - \Delta f, f + \Delta f)$ along the laser line of sight is considered:

$$\Delta f = 2\lambda f^2/\pi R^2 .$$

It is assumed that this interval provides 50 percent of the signal. Within this interval, a finite number of equally spaced points are sampled. The velocity at each point $\left(V_{T_n}\right)$ is multiplied by the weighting function (I_N) at that point; these points are then summed and divided by the sum of the weighting factors:

$$V_L = \sum_{n=1}^{N} V_{T_n} I_n \bigg/ \sum_{n=1}^{N} I_n \quad .$$

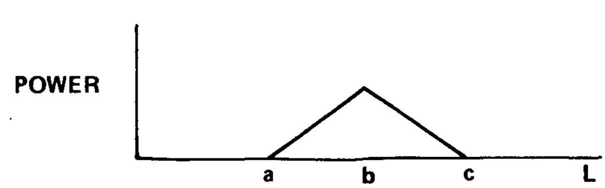
This V_L function thus provides an LDV system output which is weighted according to a calculated system spatial resolution. This type weighting function provides a data output similar to that which was recorded by using the spectrum analyzer in the one-dimensional LDV field test at MSFC.

The bistatic LDV system is simulated increasing the diameter of the transmitter/receiving optics of the system and employing one portion of the optics for transmitting and another for receiving. The distance between the two portions is known as the base leg distance of the bistatic system.

The detection of the vortex velocity flowfield employing the coaxial pulsed LDV system is simulated in the following way: The transmitted pulse is assumed to be a square wave. The detector is open for the same length of time as that of the transmitter; therefore, a pulse of length equal to the transmitted pulse enters the detector.

The pulse length is also assumed to be short relative to the distance to the volume being observed. Therefore, the detected power of an elemental volume is directly proportional to the length of time its illumination passes into the detector.

The power going into the detector relative to time is a square wave. But as a function of location of elemental volume, the curve looks like



"b" is the center of the pulse where the incoming pulse and the outgoing pulse coincide exactly. At this instant of time, "a" is the back of the outgoing pulse and "c" is the front of the outgoing pulse. Figure C-2 shows the outgoing and incoming pulses passing each other. Then, the power curve shown above is the weighting function (I_n) as in the focused cases and V_L is computed the same.

The flow chart for the Sensor Simulation Module of the Total System Simulation Model is given in Figure C-3.

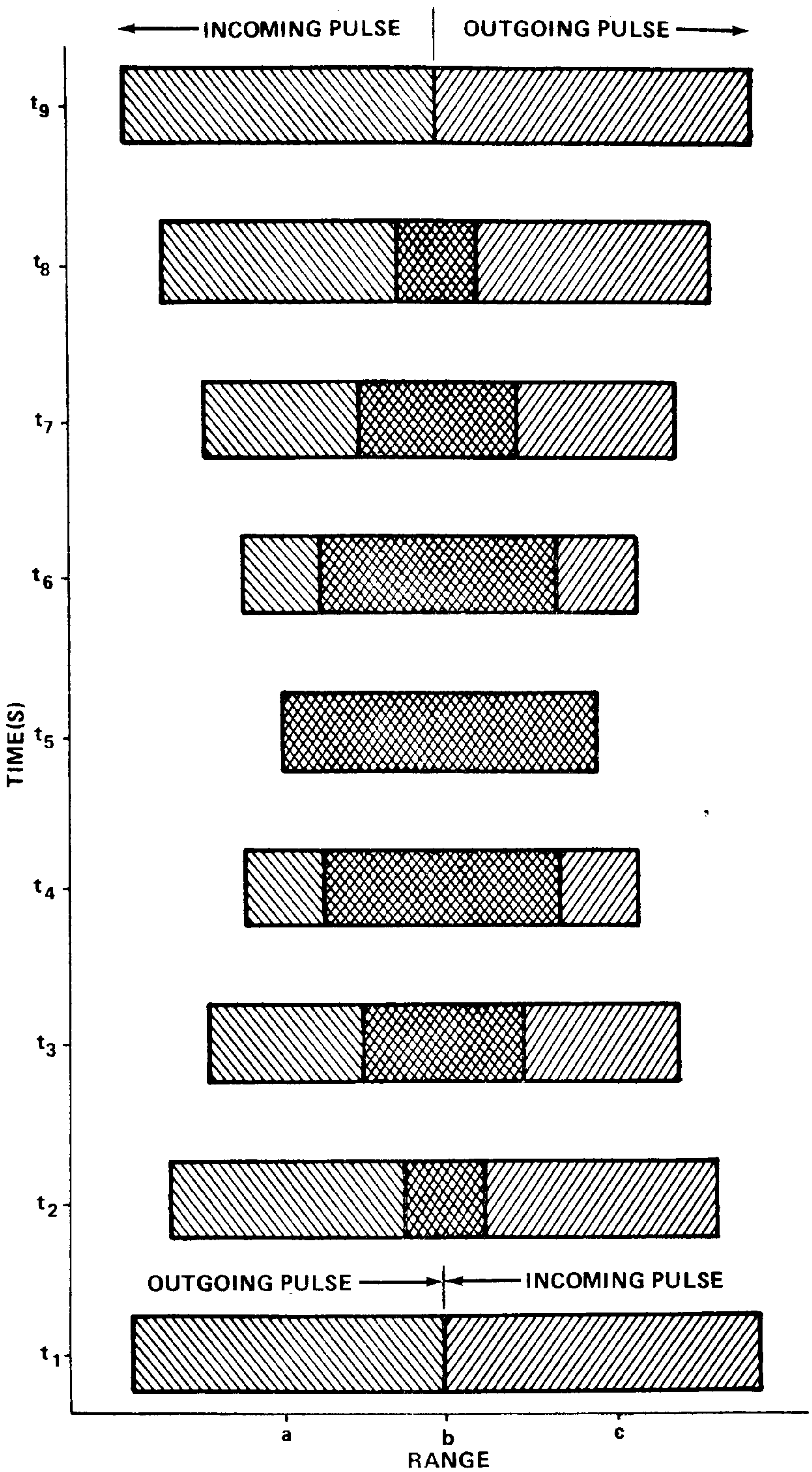


Figure C-2. Schematic of outgoing and incoming pulses.

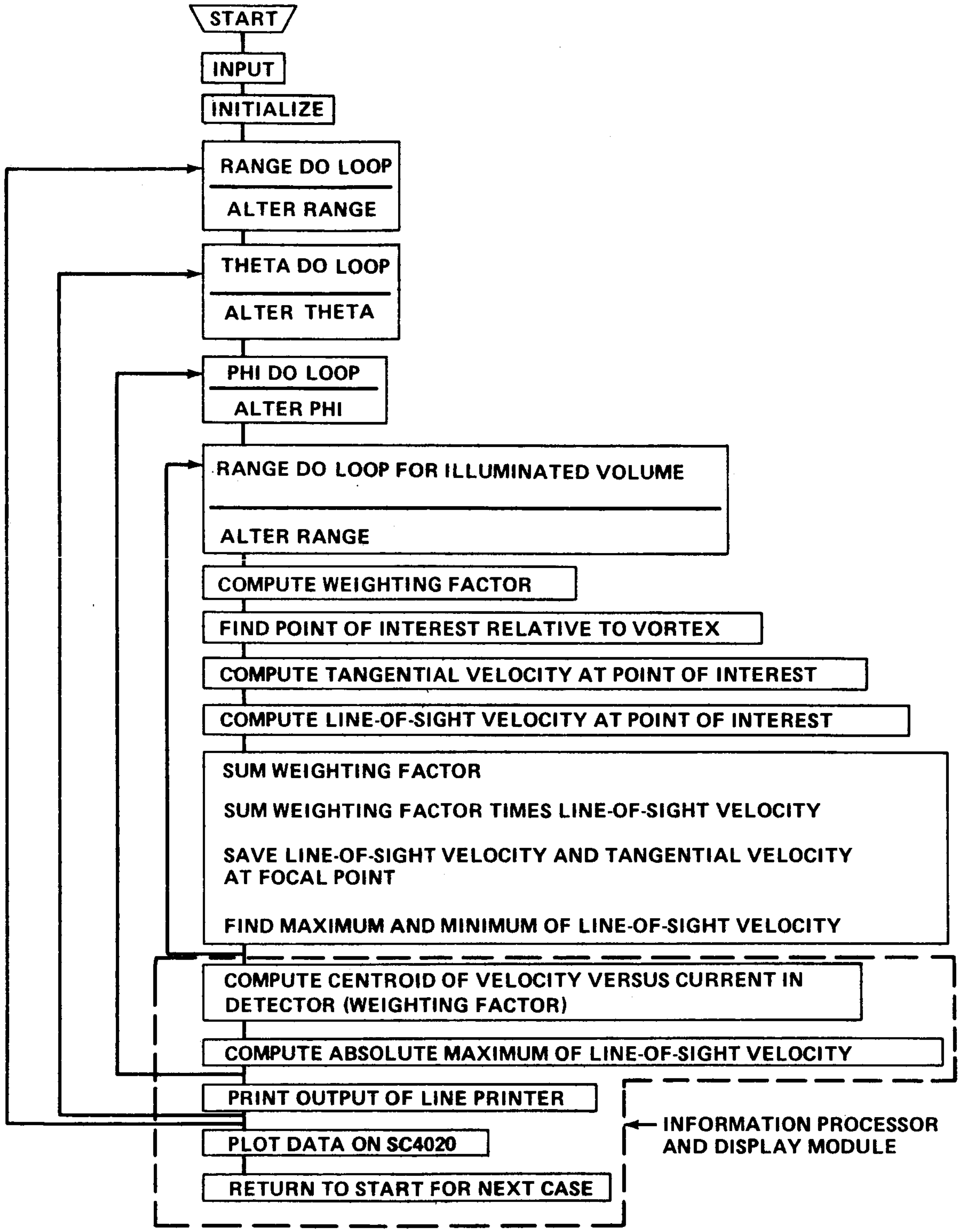


Figure C-3. Flow chart of LDV scan simulation program.

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APPROVAL

SYSTEMS SIMULATION FOR AN AIRPORT TRAILING VORTEX WARNING SYSTEM

By Harold B. Jeffreys

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Office. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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