2

X-621-72-442
PREPRINT

NASA TM X- 66119

THE PHASE DISCREPANCY BETWEEN TEMPERATURE AND DENSITY IN THE THERMOSPHERE

PETER W. BLUM

(NASA-TM-X-66119) THE PHASE DISCREPANCY BETWEEN TEMPERATURE AND DENSITY IN THE THERMOSPHERE P.W. Blum (NASA) Nov. 1972 31 p CSCL 04A
N73-12395
Unclas
G3/13 49147

NOVEMBER 1972

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

30

X-621-72-442

THE PHASE DISCREPANCY BETWEEN TEMPERATURE AND DENSITY IN THE THERMOSPHERE

by

Peter W. Blum

November 1972

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

THE PHASE DISCREPANCY BETWEEN TEMPERATURE AND DENSITY IN THE THERMOSPHERE

Peter W. Blum

ABSTRACT

The observations of thermospheric densities by satellite drag analysis and the thermospheric temperatures by back scatter radar measurements show a phase difference. The dependence of this phase difference on various factors is briefly reviewed. It is shown that by chosing certain boundary conditions at 120 km the observations of both temperature and densities become consistent. The possible range of boundary conditions is analysed. Furthermore it is shown that the phase discrepancy is largely due to the higher harmonic components of both the temperature and the density variation in the thermosphere.

CONTENTS

TABLES

ILLUSTRATIONS

THE PHASE DISCREPANCY BETWEEN TEMPERATURE AND DENSITY IN THE THERMOSPHERE

INTRODUCTION

Much effort has been spent in the last decade on the theoretical explanation of the observed behaviour of the thermosphere. Especially the time of the maximum of the diurnal density variation was a subject to a great many investigations on thermospheric dynamics. The atmospheric densities in the height region between 250 to more than 500 km peak between 14 and 14.5 hours L. T., while the earlier theoretical models of the thermosphere (Harris and Priester, 1962; Mahoney) predicted the maximum of density about three hours later. In these early computations it was implied that, at least in the upper thermosphere, the temperature has its maximum at the same time as the density. For this reason the problem was apparently further complicated when incoherent back-scatter-radar observations made it possible to deduce thermospheric temperatures directly. These temperatures show a maximum between 15 to 17 hours and have therefore a phase difference relative to the densities of about two hours. This phase difference depends on season, but at least to a first approximation is height-independent from 250 to 500 km.

A satisfactory solution of the problem requires the understanding of the dynamics of the thermosphere and involves therefore the solution of the three-dimensional Navier-Stokes equations, possibly with the inclusion of the non-linearities, and coupled to the energy-balance equation. No complete solution has been given up to the present; Bailey and Moffet (1972) have recently stated that claims to the contrary appear premature.

In this paper the energy budget of the thermosphere will not be investigated. Its understanding is essentially identical with the complete solution of the phase problem. We shall limit ourselves to a partial aspect, namely a possible reconciliation of the phase difference between temperature and density based on the observational evidence of satellite drag analysis and incoherent scatter radar. We shall treat these observations of temperature and density as empirical data and show them to be consistent, or rather determine the conditions which make such a consistency possible.

OBSERVATIONAL EVIDENCE

a. The Time of the Density Maximum

Satellite drag analysis has provided us with a great amount of data on the distribution of thermospheric densities as a function of height, local time, latitude, season, solar activity and geomagnetic activity. The diurnal variation of density becomes only observable by rockets and satellite drag analysis above 150 to 170 km. In the lower height region between 150 to 250 km the data are sparse. King-Hele et al. (1966) have analysed both rockets and satellites having low perigees and have concluded that above 170 km there exists a definite diurnal variation with a higher day-time than night-time density. This contrasts with Harris and Priester's (CIRA 65) prediction of lower day-time densities than night-time densities below 150 to 180 km, and a corresponding very small diurnal variation in the height region from 150 to 180 km. Until now the hour of the day-time maximum below 250 km has not been ascertained, so that 250 km is the lowest height for which a definite observation derived from satellite drag on the phase of the diurnal density variation can be made. Above 250 km the time of the maximum is remarkable constant and falls between 14.2 and 14.5 L.T. for a great many of the satellites analysed (Jacchia and Slowey, 1968; Jacchia, 1970). It shows some scatter, but no discernable dependence on height and little, if any, on the other parameters like solar activity, latitude and season. The density minimum is less pronounced, has more scatter, and may also show a height-dependence, varying from 3.4 hours at 350 km to 2.15 hours L.T. above 500 km (Jacchia, 1970). From the fact that the time of the maximum and the minimum do not differ by exactly twelve hours it must be deduced that in addition to the diurnal component of the density variation, also a semi-diurnal and possibly higher components must be present. This is included in Jacchia's expression for the exospheric model temperature (Jacchia, 1965 and 1972) which represent in Jacchia's model the observed densities. On the other hand, it must be remembered that the process by which density data are obtained from drag analysis has inherently a low time resolution. While this may be advantageous by excluding sporadic deviations from the steady state densities, it unfortunately reduces the accuracy of the method regarding the determination of the amplitudes and phases of the semi-diurnal and higher components of the diurnal density variation. For this reason it must be concluded that the semi-diurnal component of the density variation which is derived from Jacchia's formula, which in turn is based on a tremendous amount of satellite drag data, is not the total semi-diurnal density amplitude, but only a part of it. With regard to the hour of the maximum density, it is also possible that a height-dependent trend may exist, but not have been discovered due to the inherent low time resolution and the relatively large scatter of the observations. But should this be the case, then this trend must be less than 40 minutes, otherwise it would have become apparent due to the very large number of individual observations.

Within this accuracy we may therefore postulate that satellite drag observations have established that the time of the maximum densities of the diurnal variation in the height region between 250 to 500 km is height-independent, that to the maximum densities contribute besides the fundamental component also the semi-diurnal and possibly higher components, but that no accurate statement regarding the amplitude and phases of the higher components is at present available from an analysis of the drag data.

Voiskovsky et al. (1971) have tested the possibility that drag analysis gives an erroneous result for the time of the maximum densities when the density variation has a substantial content of higher harmonics similar to the radar-deduced temperature variation. They have found that at least for satellites having eccentricities of the order 0.1 or larger, the error in the time of the maximum that could result from such a density variation is less than 0.5 hours.

A very valuable addition to the satellite drag data are the densities derived from mass-spectrometer measurements. By this method it has been ascertained from the OGO-6 observations (Hedin et al., 1972) that the time of the maximum of the density of the various atmospheric constituents is not the same for all constituents. Especially the maximum of the helium density takes place much earlier than the maximum of the total density. At 450 km helium seems to peak at about 10 hours L. T.

b. The Diurnal Variation of Atomic Oxygen at the Lower Boundary

Few reliable data on the variation of atomic oxygen at 120 km are available. On theoretical grounds Shimazaki (1971) has calculated an amplitude for this variation of less than 0.1 with a peak density in the late afternoon. We shall define the amplitude and phase of a variation as used in this paper as the parameters a_n and φ_n in the expression

$$\rho(z,t) = \rho_0(z)\left(1 + \Sigma\, a_n \cos n\,\omega\,(t - \varphi_n)\right) \quad (1)$$

Alcayde et al. (1972) have extrapolated the behaviour of atomic oxygen down to 120 km from their observations. These show considerable scatter, but may be interpreted as having a diurnal as well as higher order components. The maximum to minimum density ratio can be estimated to be between 2 and 3, this corresponds to an amplitude of 0.3 to 0.5. The time of the maximum is approximately at 7 hours. As no data between 19 and 3.5 hours are available, possible estimates regarding the amplitudes and amplitude ratios of the various harmonic components are not possible, or at least not accurate.

King-Hele (1966) has derived a larger amplitude of the diurnal variation of density in the lower thermosphere than shown by most other results. He has determined fróm Cosmos rockets shot during periods of low solar activity that at a height of 170 km there already exists a ratio of 1.7 between day and night densities. This corresponds to an amplitude of 0.3. It is hard to explain such a large variation, without the assumption of the existence of a diurnal variation already at 120 km. As nitrogen will probably vary little at 120 km, the observations of King-Hele strengthen the assumption that the diurnal variation of atomic oxygen at 120 km is not negligible.

From regressive analysis of incoherent back scatter data (Champion, 1971) a maximum to minimum ratio of the atomic oxygen densities of 3 to 1, or even 4 to 1 at a height of 225 km is computed. This would correspond to an amplitude greater than 0.1 at 120 km. As the maximum densities at 225 km seem to be in the late morning hours, a phase maximum in the early morning hours at 120 km is indicated. It should be remarked that it is uncertain whether this particular observation refers to the steady state or a sporadic deviation from it.

The O/O_2 ratio at 120 km has been measured by many investigators and has been the subject of a number of discussions and papers (v. Zahn, 1970, Hickman and Nier). The changes that the estimates of this parameter have undergone is reflected in the development of the Jacchia models. The Jacchia 65 models (Jacchia, 1965) assume a ratio of unity, the 1972 models (Jacchia, 1972) a ratio of 2.6 and a still later version of the 1972 models a ratio of 1.9 depending slightly on solar activity. In the Jacchia models there is a definite relationship between the model exospheric temperature that corresponds to a certain density profile and the O/O_2 ratio at the lower boundary. The lower the exospheric model temperature, the higher this ratio must be. Because of this sensitivity of the Jacchia model temperatures to variations of the O/O_2 ratio at 120 km, there seems little point in comparing the magnitude of the Jacchia model temperatures with the temperatures deduced from incoherent scatter measurements and to deduce from a near-equality of both temperatures that there exists a consistency between both sets of data, i.e., the density drag data and the incoherent scatter observations, as has sometimes been argued (Waldteufel, 1971). It is rather the phase behaviour, i.e., the diurnal variation of temperature from incoherent scatter observations that must be shown to be consistent with the drag determined variations of thermospheric densities.

From the above it may be concluded that at present insufficient definite information on the behaviour of atomic oxygen at 120 km is available. In the model computations represented in this paper the Jacchia 1972 average densities were used, but contrary to Jacchia's model a diurnal variation of the atomic oxygen densities at the lower boundary was introduced.

c. Thermospheric Temperatures

Thermospheric temperatures have been observed by incoherent scatter methods for several years by various groups (Nisbet, 1967; Carru and Waldteufel, 1969; Evans, 1969; McLure, 1971). A great amount of measurements have been obtained and many of the details of the dependence of the temperature profile on the various physical parameters like solar activity, geomagnetic activity and season have been discovered (Waldteufel and Cogger, 1971). All groups have obtained results that are consistent in the essentials. A generally accepted model that describes thermospheric temperatures as a function of all the parameters does not exist at present, though partial models have been constructed (Waldteufel, 1971). We shall describe briefly the observations of Salah and Evans (1972) as they are one of the latest published results. These observations extend over a continuous period and were analysed by Salah and Evans in terms of harmonic components. It should be remembered that the uncertainties of the temperature measurements are higher at night than during day-time. This introduces in turn uncertainties in the higher harmonic components when a Fourier-analysis is made. In Table 1 the first three harmonic components of the exospheric temperature for the winter, equinox and summer season according to Salah and Evans results are given. These measurements, especially for the winter season, yield times of the temperature maxima that are earlier by about half an hour compared to Waldteufel and Cogger's data (1971). This cannot be entirely explained as a latitudinal effect according to Waldteufel's empirical expression for the exospheric temperature dependence. If harmonic components of higher order than three are also taken into account, somewhat later maxima for the total temperature are obtained, but even if these are included then the diurnal temperature maximum in winter as measured by Salah and Evans, still seems to take place earlier than corresponding values found by other groups.

As seen from Table 1 the phase of the maximum temperature shows a considerable dependence on season, while the phase of the maximum density does not. For this reason the phase discrepancy is smallest in winter and most pronounced in summer, at equinoxes it has an intermediate value. It may even be considered doubtful whether the winter phase discrepancy is at all above the possible errors of observations. On the other hand the summer phase discrepancy of 1.5 to 2 hours is well established. In our model calculations we refer only to the summer data, as there the problem is most critical.

The temperature variation in the 120 km height region was also measured by Salah and Evans. It was interpreted by them to consist of a semi-diurnal tidal oscillation that is rapidly damped below 135 km and shifts in phase by a considerable local time interval over the vertical distance of less than 30 km. From the data published so far it seems that it is difficult to deduce with some degree of certainty the amplitude or phase of the temperature variation and its various harmonic components in this height region. The possibility that the measurements, or some of them, represent sporadic deviations from the steady state values of temperature, possibly caused by gravity waves, cannot be rejected entirely. Earlier Wand (1969) also observed temperature variations in the 120 km region. These had phase shifts of 1 to 4 hours within a 15 km vertical distance and periods that varied from 7.4 to 13 hours on various days. In this paper we are only concerned with the steady state behaviour of the thermosphere. As the data obtained by the various groups and the data of one group on various days, show considerable scatter, it seems premature to deduce from the observations so far published results which determine the steady state behaviour of the temperature at 120 km.

PREVIOUS EXPLANATIONS OF THE PHASE DISCREPANCY

Chandra and Stübbe (1970) suggested that variable boundary conditions at 120 km can explain the phase discrepancy between temperature and density. They have assumed instantaneous diffusive equilibrium above 120 km. Thus the density of the atmosphere is uniquely determined by the boundary conditions at 120 km and the temperature profile above 120 km. This concept, also discussed earlier in a somewhat different context by Harris and Priester (1965), is called dynamic diffusion by Chandra and Stubbe. Based on these assumptions a one-dimensional simplified model of the dynamics and the heat budget of the thermosphere was solved. In this model the density variation at 120 km of all constituents was assumed to have the same phase and amplitude and the temperature varied in phase with the density. The equality between the phases of temperature and density was deduced from simple one-dimensional considerations of the region below the turbopause. This constraint is probably not true for a more realistic atmospheric model. In their model calculations the phase of the density maximum increases from 14 hours at 200 km to about 16 hours at 500 km. This increase of the time of the maximum is not in accordance with satellite observations.

Cummack and Butler (1972) objected for other reasons to the explanation given by Chandra and Stubbe. They argued that in Chandra and Stubbe's model considerable vertical motions result from the equality of the phases of the temperature and density variation at 120 km. This breathing of the atmosphere is additional to the EUV-induced breathing above 120 km. It would in effect raise the atmosphere periodically by one scale height which, Cummack and Butler conclude cannot be reconciled with ionospheric observations.

Mayr and Völland (1972) have suggested that wind induced diffusion causes a time delay of one to two hours between temperature and density and that this furnishes the required explanation of the phase discrepancy. Their theoretical dynamical model of the thermosphere (Volland and Mayr, 1970) is used as a first approximation for the thermospheric temperature density and wind distribution. It is postulated that below 250 km the atmosphere is not in diffusive equilibrium, mainly due to the non-vanishing divergence of the horizontal wind field. A perturbation treatment for the density distribution of atomic oxygen as deduced from the continuity and diffusion equation is applied. The arguments, why Mayr and Volland's treatment cannot be the full explanation of the problem seem mainly to be the following:

1. No observational evidence for deviations from diffusive equilibrium above 120 km exist. If indeed there are deviations, then they are probably small.

2. Their results show a phase discrepancy between temperature and density of 5 hours at 160 km, 4 hours at 200 km and about one hour at 360 km. They limit their model to heights below 360 km. The theoretical foundations of their approach are valid to 500 km, only in the exosphere the hydrodynamic equations become questionable. Their limitation to heights below 360 km is therefore unwarranted. If their computations are extended to 500 km, as can easily be done, the phase discrepancy is reduced to less than half an hour. For this reason their results are contradicted by satellite drag data and do not explain the observations.

From these difficulties of Mayr and Volland's approach it should not be concluded that wind - induced deviations from diffusive equilibrium in the lower thermosphere do not have a pronounced effect on the structure of the thermosphere as suggested by Mayr and Volland.

Swartz and Nisbet (1971) have suggested that the densities derived by satellite drag analysis and the temperatures from radar observations are self-consistent, not due to a diurnal variation of density or temperature at the lower boundary, but rather due to a diurnal variation of the temperature gradient at 120 km. They did not verify that this reconciles the drag and back-scatter data over the whole height range in which results are available, but have limited themselves to one height, namely 300 km. They have derived from their observations temperature gradients at 120 km that have between 9 to 14 hours L. T. a maximum that is almost double the day-time average. As only the measurements for a single day were published it is not certain if the data represent the average values over a longer period. Wand (1972) has taken the average of 5 days in September 1970 and has found the highest temperature gradient to occur at 9 hours L. T. with a gradual decrease up to 18 hours L. T. Figure 1 shows the smoothed data of both sets of observations. From Salah and Evans (1972) observations it is difficult to deduce a consistent average behaviour of the temperature gradient at 120 km. Therefore it cannot be considered as completely established that the variation of the temperature gradient at 120 km determined by Swartz and Nisbet corresponds to the steady state behaviour of this quantity. Even if such a variation of the steady state values of the temperature gradient would be assumed, we have not been able to verify Swartz and Nisbet's ascertion that this behaviour explains the discrepancy between the phase of temperature and density completely and no further variation of the density at 120 km is required. For one height, i.e., 300 km this is probably true, but a corresponding fit for the whole height region from 250 to 500 km seems not to be possible. The variation of the temperature gradient at the lower boundary may play an important part in the reconciliation of the phase discrepancy, but it alone cannot explain it.

THERMOSPHERIC MODEL

Our approach to the reconciliation of the phase discrepancy between thermospheric densities and temperatures is essentially based on variable boundary conditions at the 120 km level similar to the solution suggested by Chandra and Stubbe.

Our deduction from observations and assumptions are:

1. The diurnal variations of exospheric temperature includes several harmonics as detailed in Table 1. The amplitude of the higher harmonics is relatively high, the third has about 10% of the amplitude of the fundamental.

2. The phase of the exospheric temperature depends on season. It is latest in summer and earliest in winter.

3. The temperature profile has a height-dependence according to the Bates formula, i.e., is similar to Jacchia's temperature profile.

4. There may exist a steady state temperature and temperature gradient variation at the lower boundary.

5. The atmosphere is in diffusive equilibrium.

6. The time of the maximum densities is determined by satellite drag data and is independent of height.

7. Atomic oxygen and helium have a diurnal variation at 120 km. There is no constraint regarding the phase of the temperature and density variation.

By including the temperature distribution as given by observations we have avoided a dynamical treatment of the thermosphere, as we consider the observed temperatures the result of the dynamical behaviour of the thermosphere. Thereby we have included dynamic effects of horizontal winds and other energy sources without a need to specify them. Our approach therefore limits the problem to the derivation of the observed density profile, and especially the constancy of the density maxima as a function of height, from the given temperature profile and the assumed boundary conditions. In contrast, the semi-empiric Jacchia model describes the density observations by the introduction of an empirical temperature profile, which is not identical with the true kinetic temperatures. In our treatment the temperatures are identical with the true kinetic temperatures, at least as far as they can reliably be derived from back scatter measurements.

The consistency of the temperature and density observations depends on the assumed boundary conditions. They are not uniquely determined from the condition of the consistency. Nevertheless, the variety of boundary conditions that fulfills the demand of consistency is limited. We may therefore draw at least preliminary conclusions on the boundary conditions at 120 km that exist in reality.

DIFFUSIVE EQUILIBRIUM

Diffusive equilibrium above 120 km is one of the assumptions of our model. We shall briefly discuss this. The assumption that atomic oxygen and helium and not the other constituents have a diurnal variation at 120 km is not strictly consistent with diffusive equilibrium, because it implies that atomic oxygen and helium have a velocity differing from the velocity of the major constituents which is zero as the density of N_2 at 120 km is assumed to be time-independent. This in turn causes a deviation of the atomic oxygen and helium density profile, from the diffusive equilibrium densities. We have disregarded this slight inconsistency because the deviations from diffusive equilibrium that result from the assumed variations are small and therefore, the density profiles that results from this assumption are a close approximation to the density profiles that would result from an inclusion of the diffusion velocities.

The existence of barometric equilibrium for the total mass density of the thermospheric height region is assured by the vertical equation of motion for the total mass (Rishbeth, 1969). The terms of this equation that could cause deviations from barometric equilibrium are the inertial term, the Coriolis term and the ion-drag and viscious drag terms. The horizontal flow appears in this equation only through the non-linearities of the convective derivative of the vertical motion and the Coriolis force. All the terms that could cause deviations from the barometric law for the total density are at least three orders of magnitude less than the term due to the gravitational attraction. Their combined effect is possibly a slight modification of the effective gravitational force, but no deviations of the total mass distribution from the barometric law results. Horizontal flow may effectively modify the vertical temperature profile, but will not cause deviations from the barometric law. From somewhat different considerations Yanowitch (1966) has derived similar results. On the other hand, effects like eddy diffusion, dissociation and recombination of atomic oxygen and horizontal having a non-zero divergence may cause deviations from diffusive equilibrium of the individual constituents. These deviations may change the average molecular weight and average scale height and thereby cause a change in the distribution of the total mass density, but this distribution would still be in accordance with the barometric law.

Deviations from diffusive equilibrium in the lower thermosphere above 120 km have not been observed, because they are below the threshold of the accuracy of our present methods of observations. That such deviations do exist has been demonstrated on purely theoretical ground by Shimazaki (1967, 1968, 1971) and Mayr and Volland (1971). Especially atomic oxygen and helium are the two constituents that may have deviations from diffusive equilibrium. In our model calculations we have not taken into account these deviations, as they are probably small and reliable data on their amplitudes and phases are at present not available. By neglecting these deviations we have not limited the accuracy of our model calculations as we have treated the boundary conditions at 120 km as free parameters. Any deviation from diffusive equilibrium can be absorbed in our revised boundary values. In other words, our boundary values for atomic oxygen and helium would probably be somewhat modified if deviations from diffusive equilibrium would have been taken into account, but no appreciable change in atmospheric behaviour above 250 km would result from this. Once a correct description of the diurnal variation at 250 km is obtained and the temperature profile above this height is given, the atmospheric behaviour above 250 km is determined, irrespective of the exact mechanism by which the diurnal variation at 250 km has arisen.

According to the above we assume that the atmospheric densities above 120 km are given by the well-known expression for diffusive equilibrium

$$n_i\,(z,t) = n_i\,(z_0,t) \left(\frac{T(z_0,t)}{T(z,t)}\right)^{1+\alpha_i} \exp\left(-\int_{z_0}^{z} \frac{dz}{H_i(z,t)}\right) \qquad (2)$$

where α_i is zero for all constituents except helium, which has a thermal diffusion coefficient $\alpha_{he} = -0.4$.

The boundary conditions represented by $n_i(z_0, t)$ in expression (2) are to be understood as the effective boundary conditions that would result in an identical behaviour above 250 km when no deviations from diffusive equilibrium above 120 km would exist.

EFFECTS OF HIGHER HARMONICS

The observations and analysis of Salah and Evans shows that the diurnal component of exospheric temperature peaks at 14 hours and is therefore in phase with the maximum of the diurnal component of the density variation. For this reason it becomes apparent that the phase discrepancy is an effect of the higher harmonics of density and temperature. The density according to expression (1) is a product of several time-dependent factors, especially the exponential function. Its variation shows therefore effects equivalent to non-linearities. With a temperature profile that has only a diurnal component a harmonic analysis of the relevant density profile shows an induced semi-diurnal component. The amplitude of this semi-diurnal component of the density variation is seen in Figure 2 for two cases: (a) no temperature or density variation at the lower boundary, (b) no density variation at the lower boundary, but a temperature variation with an amplitude of 0.1 and a phase equal to the phase of the exospheric temperature. About 10-15% semi-diurnal variation is induced by the fundamental of temperature. A comparison of the curves also shows that the amplitude of the density variation in the lower thermosphere depends strongly on the temperature variation at the lower boundary. With no temperature variation the density variation below 170 km is negligible. In this context it may be recalled that King-Hele (1966) has found in the lower thermosphere an amplitude of the density variation that is considerable in excess of the CIRA 65 or the Jacchia model densities. His results give additional support to the assumption that the steady state values of the lower boundary conditions have a non-negligible variation.

The interaction of the various harmonics of the temperature profile to give rise to higher harmonics of the density variation is further exemplified in Figures 3 and 4. Here the exospheric temperature is in accordance with Salah and Evans analysis and includes harmonics up to the third order. The resulting densities are compared for three cases: Only diurnal component of the density variation taken into account, the first three harmonics of the density variation are summed, and the first five harmonics of the density variation are summed. From Figure 3 it is evident that the neglect of the 4th and 5th harmonic of the density variation results in a time of maximum that deviates from the correct values. At 500 km the error is about 0.3 hours. The phase difference between the maximum density of the fundamental component and the total density is about 2 hours.

NUMERICAL RESULTS

The total density distribution in our model is given by

$$\rho\,(z,t) = \sum_{i=1}^{4} m_i\, n_i\,(z_0,t)\, \frac{T(z_0,t)}{T(z,t)} \exp\left(- \int_{z_0}^{z} \frac{dz}{H_i(z,t)} \right) \tag{3}$$

with the four atmospheric constituents N_2, O_2, O and He. The first two constituents are assumed to show no variations at 120 km. The average densities for all constituents is taken from the Jacchia 1972 models. The temperatures are deduced from Salah and Evans observations and the temperature profile follows the Bates (1958) formula. The consistency between the observations of density and temperature is obtained by chosing suitable boundary conditions. As the accuracy of the determination of the time of the maximum of the density variation from satellite drag is about 0.5 hours, boundary conditions that resulted in a density profile having a maximum in the height region from 250 to 500 km that did not show a height dependent trend of more than 0.7 hours and had an average between 14.3 and 14.8 hours were considered a solution that satisfied the demands of consistency of the temperature and density data.

In order to obtain this the parameters of the boundary conditions were varied. These parameters are the amplitudes and phases of the variation of the density of atomic oxygen, of temperature and of the temperature gradient. The variations of the temperature gradient were additional to the slight temperature gradient variation of the Jacchia model that is proportional to exospheric temperature. This additional variation was obtained in the computation by including a time-dependence of the shape parameters of the Jacchia temperature profile. Not all parameters were varied simultaneously. Three different sets of boundary conditions were tested:

1. Variation of amplitudes and phases of the atomic oxygen density and the temperature T.

2. Variation of the amplitudes and phases of the atomic oxygen density and the temperature gradient T'.

3. Variation of the amplitude and phase of the atomic oxygen density and a temperature variation at the lower boundary that was in phase with the exospheric temperature.

Special cases of (1) and (2) were variations of only one of the three parameters T, T' and atomic oxygen density. The third model corresponds to a time variation of temperature that is essentially independent of height. Except for the temperature variation in the model (3) all variations included only a diurnal component with no higher harmonic content. A semi-diurnal or higher harmonic component at the lower boundary results generally in a variation at exospheric heights that has a shape not in accordance with observations, especially it may have more than two extrema. Various theoretical models treat the thermospheric variations in terms of harmonic components (Volland, 1970; Mayr and Volland, 1970), for the very simple computations required in our model this was not necessary, but in order to facilitate a comparison with other models such a harmonic analysis was also made.

The diurnal variation of the helium density at the lower boundary was assumed to have an amplitude of 0.1 with a maximum between 8 and 9 hours. This assumption accounted approximately for the observations of the helium variation at higher altitudes as determined by OGO-6 and the San Marco satellite (Newton et al, 1971). The helium variation has little if any effect on the total density in the height region investigated in this paper.

The optimum boundary conditions were determined by a non-linear search program that selected a set of boundary conditions that minimized a parameter related to our demand of consistency of temperature and density data. As large amplitudes of variation at 120 km are not likely, these amplitudes were limited in the various cases to 0.1, 0.2 and 0.3 etc. For limits that were set too low no good solutions could be obtained. Some of the results for the optimum boundary conditions are shown in Figures 5, 6, and 7. Table 2 summarizes these results. Figure 8 shows the best possible result for the case where only the density is varied at the lower boundary with both temperature and temperature gradient constant (except for the small variation of the temperature gradient also included in the Jacchia model). In this case the time of the maximum of the total density has a height dependent shift of more than one hour, it is therefore somewhat above the limits set by the observations. An even larger height-dependent phase shift results if at the lower boundary the density is kept constant and only either temperature or temperature gradient are varied.

RESULTS

On the premise that thermospheric observations establish the following:

a. The total density between 250 and 500 km has a maximum between 14 to 14.8 hours L.T. The height dependent shift of this time of maximum is less than 0.7 hours for the steady state. No seasonal shifts exists.

b. The diurnal variation of thermospheric temperatures is in accordance with radar back scatter observations, i.e., its time of maximum depends on season and is in summer about two hours after the peak of the density. It can be deduced:

1. The boundary conditions at 120 km must be strongly dependent on season as the large change in the phase discrepancy between summer and winter can only be due to a change in boundary conditions.

2. For summer, where the phase discrepancy is most pronounced, the boundary conditions must comply with certain conditions:

   a. Without a variation of the density of atomic oxygen at the lower boundary the observed temperature profile cannot give rise to a height-independent time of maximum density that is in accordance with drag data. The optimum solution for constant lower boundary densities results in a height-average value for the phase discrepancy that is correct, but its shift in the height region from 240 to 500 km is about 1.6 hours. Such a height trend would have been discovered by drag analysis.

   b. Boundary conditions with a constant temperature and a constant temperature gradient may explain the observations, but the required amplitude of the density variations is between 0.4 to 0.5. This large amplitude is hard to explain theoretically and makes it therefore likely that in addition to the density the temperature or the temperature gradient must vary at 120 km.

   c. Consistent solutions may be obtained with variations of the density of atomic oxygen with an amplitude between 0.1 and 0.2 and a time of maximum in the morning hours. The corresponding required variation of the temperature would have an amplitude of 0.1 and a maximum in the early afternoon. If the temperature is kept constant than the temperature gradient must be varied with an amplitude of 0.5. The model calculations do not allow to distinguish between the likelihood of a variation of the temperature or the temperature gradient. Also, a simultaneous variation of both parameters seems possible, but was not tested.

   d. The phase discrepancy is an effect strongly influenced by the presence of higher harmonic components of the diurnal density and temperature variation. For this reason it must be concluded that no dynamical theory that neglects the higher order components can give an adequate account of the observed phase discrepancy, as it scarcely exists for the fundamental components.

e. The boundary conditions determined by the model computations are effective boundary conditions that assume the existence of diffusive equilibrium above 120 km. It is possible that the actual boundary conditions that take account of possible deviations from diffusive equilibrium above 120 km have smaller amplitudes of variations than the effective boundary conditions we have determined. Until more details on the thermospheric variations between 120 to 250 km are known, it is difficult to determine the influence of deviations from diffusive equilibrium.

3. It seems possible to construct empirical models of the thermosphere that represent consistently the observations of both temperature and density in the height range where they have been observed.

ACKNOWLEDGEMENTS

The author is indebted to the National Academy of Sciences for a Research Associateship and to Dr. I. Harris, Goddard Space Flight Center, Greenbelt, Md. for numerous discussions concerning this paper.

REFERENCES

1. Alcayde, D., P. Bauer, C. Jaeck, and J. C. Falin. J. Geophys. Res. 77, 13, 2368, 1972.

2. Bailey, G. J., and R. J. Moffett. Planet Space Sci. 20, 1085, 1972.

3. Bates, D. M. Proc. Roy. Soc. London A, 253, 451, 1958.

4. Carru, H., and P. Waldteufel. Ann. Geophys. t25, 485, 1969.

5. CIRA 1965. COSPAR International Reference Atmosphere, North-Holland Publ. Co., Amsterdam, 1965.

6. Champion, K. S. W. COSPAR Conference, Seattle, The properties of the neutral atmosphere, 1971.

7. Chandra, S., and P. Stubbe. Planet. Space Sci. 18, 1021, 1970.

8. Cummack, C. H., and P. H. Butler. Planet. Space Sci. 20, 289, 1972.

9. Harris, I., and W. Priester. J. Atm. Sci. 19, 286, 1962.

10. Harris, I., and W. Priester. J. Atmos. Sci. 22, 3, 1965.

11. Hedin, A. E., H. G. Mayr, C. A. Reber, G. R. Carignan, and N. W. Spencer. COSPAR Conference, Madrid, 1972.

12. Hickman, D. R., and A. O. Nier. J. Geophys. Res. 77, 2880, 1972.

13. Jacchia, L. Smithsonian Contrib. Astrophys. 8, 9, 1965.

14. Jacchia, L. Space Research X, 367, 1970.

15. Jacchia, L. S.A.O. Special Report 332, 1972.

16. Jacchia, L., and J. W. Slowey. Planet. Space Sci. 16, 509, 1968.

17. King-Hele, D. G., and E. Quinn. Planet. Space Sci. 14, 1023, 1966.

18. Mahoney, J. R., Meter. Momographs 9, 90, 1968.

19. Mayr, H. G., and H. Volland. J. Geophys. Res. 77, 13, 2359, 1972.

20. McLure, J. P. J. Geophys. Res. 76, 3106, 1971.

21. Newton, G. P., D. T. Pelz, and W. T. Kasprzak. COSPAR Conference, Madrid, 1972.

22. Nisbet, J. S. J. Atm. Sci. 24, 586, 1967.

23. Rishbeth, H. Ann. Geophys. t25, 495, 1969.

24. Salah, J. E., and J. G. Evans. COSPAR Conference, Madrid, 1972.

25. Shimazaki, T. J. Atm. Terr. Phys. 29, 723, 1967.

26. Shimazaki, T. J. Atm. Terr. Phys. 30, 1279, 1968.

27. Shimazaki, T. J. Atm. Terr. Phys. 33, 1383, 1971.

28. Swartz, W. E., and J. S. Nisbet. J. Geophys. Res. 76, 185, 1971.

29. Voiskovsky, M. I., B. V. Kugaenko, V. M. Synitsyn, and P. E. Elyashberg. Space Research XI, 953, 1971.

30. Volland, H. J. Geophys. Res. 75, 28, 5618, 1970.

31. Volland, H., and H. G. Mayr. Ann. Geophys. t26, 907, 1970.

32. Wand, R. H. J. Geophys. Res. 74, 5688, 1969.

33. Wand, R. H. USNC/URSI Spring Meeting, 1972.

34. Waldteufel, P., and L. Cogger. J. Geophys. Res. 76, 5322, 1971.

35. Waldteufel, P. J. Geophys. Res. 76, 6990, 1971.

36. Yanowitch, M. Pure and Applied Geophysics, 64, 169, 1966.

37. v. Zahn, U. J. Geophys. Res. 75, 5517, 1970.

Table 1

Harmonic Analysis of Exospheric Temperature

(ACC. TO SALAH AND EVANS)

	EQUINOX		SUMMER		WINTER	
COMPONENTS	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE
DIURNAL COMP.	14 L.T.	0.1	14 L.T.	0.1	14 L.T.	0.1
SEMI-DIURN. COMP.	15.58 L.T.	0.019	17 L.T.	0.029	13 L.T.	0.011
3RD HARMONIC	16.8 L.T.	0.011	18 L.T.	0.011	16 L.T.	0.011
TOTAL FIRST THREE HARMONICS	15.35 L.T.		16. 22 L.T.		14. 48 L.T.	
TOTAL	16 L.T.		17 L.T.		15 L.T.	

Table 2

Boundary Conditions at 120 km Required for Explanation of Phase Discrepancy from Drag Data and Radar Temperature

AMP. OF VAR. ATOM. OXYGEN	AMP. OF TEMP. VARIATION	AMP. OF TEMP. GRAD. VAR. (ADDITION TO JACCHIA MODEL)	PHASE SHIFT TOT. DENSITY (HOURS) 240–500 km	
0.01 OR LESS	UP TO 0.1	ZERO	1.6	NO DENSITY VAR.
0.01 OR LESS	ZERO	UP TO 0.6	1.4	NO DENSITY VAR.
0.2 – 0.5	ZERO	ZERO	1–1. 2	ONLY DENSITY VAR.
0.3–0.6	ZERO	0.35	0.7	
0.2	0.1	ZERO	0.7	
0.1–0.15	ZERO	0.5	0.8	
0.2–0.3	0.1	ZERO	0.6	
0.2–0.3	ZERO	0.5	LESS THAN 0.6	
0.3	PROP. TO T_∞	ZERO	LESS THAN 0.4	

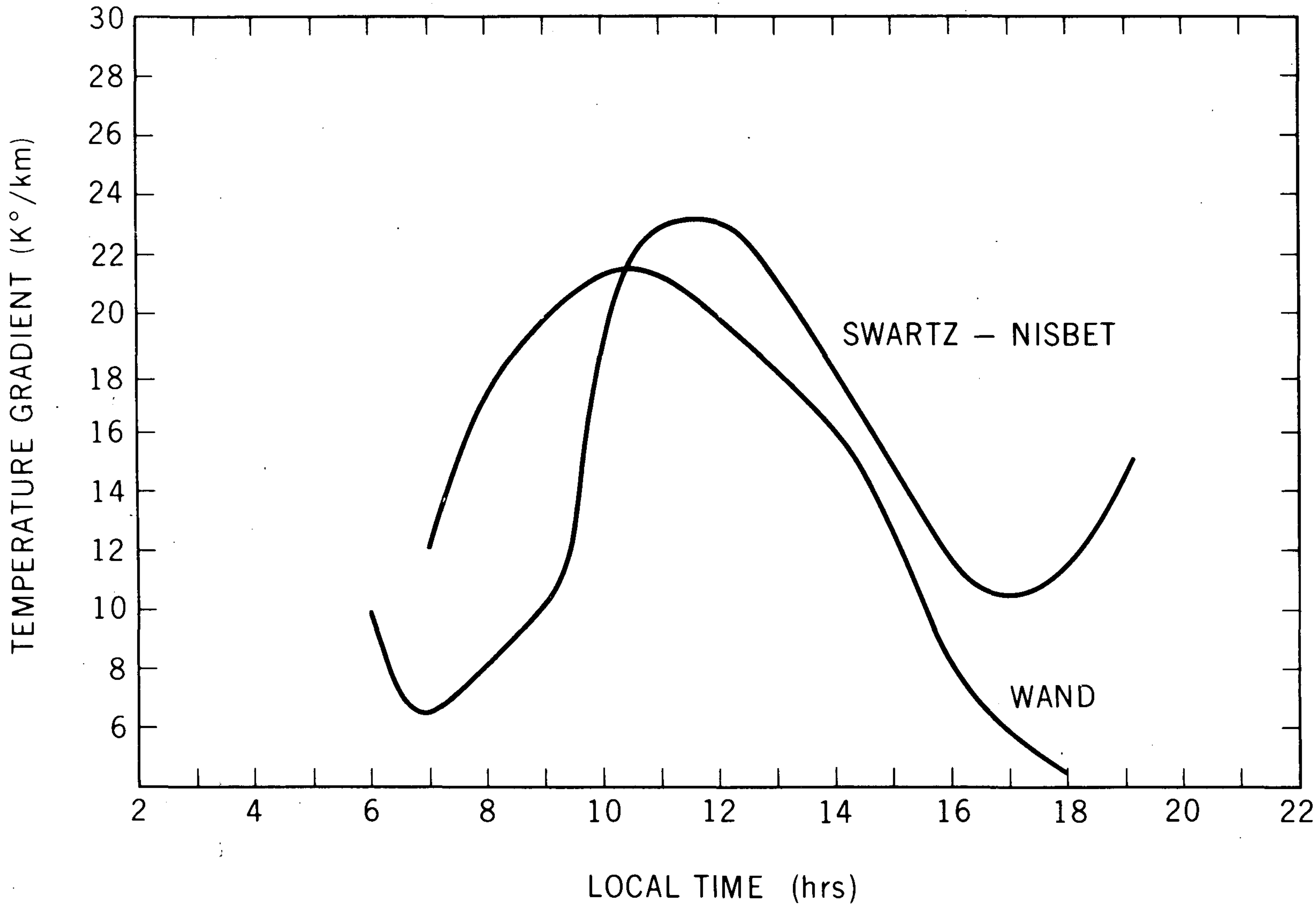

Figure 1. Induced Semi-diurnal Component of Density Variation by Purely Diurnal Temperature Variation.

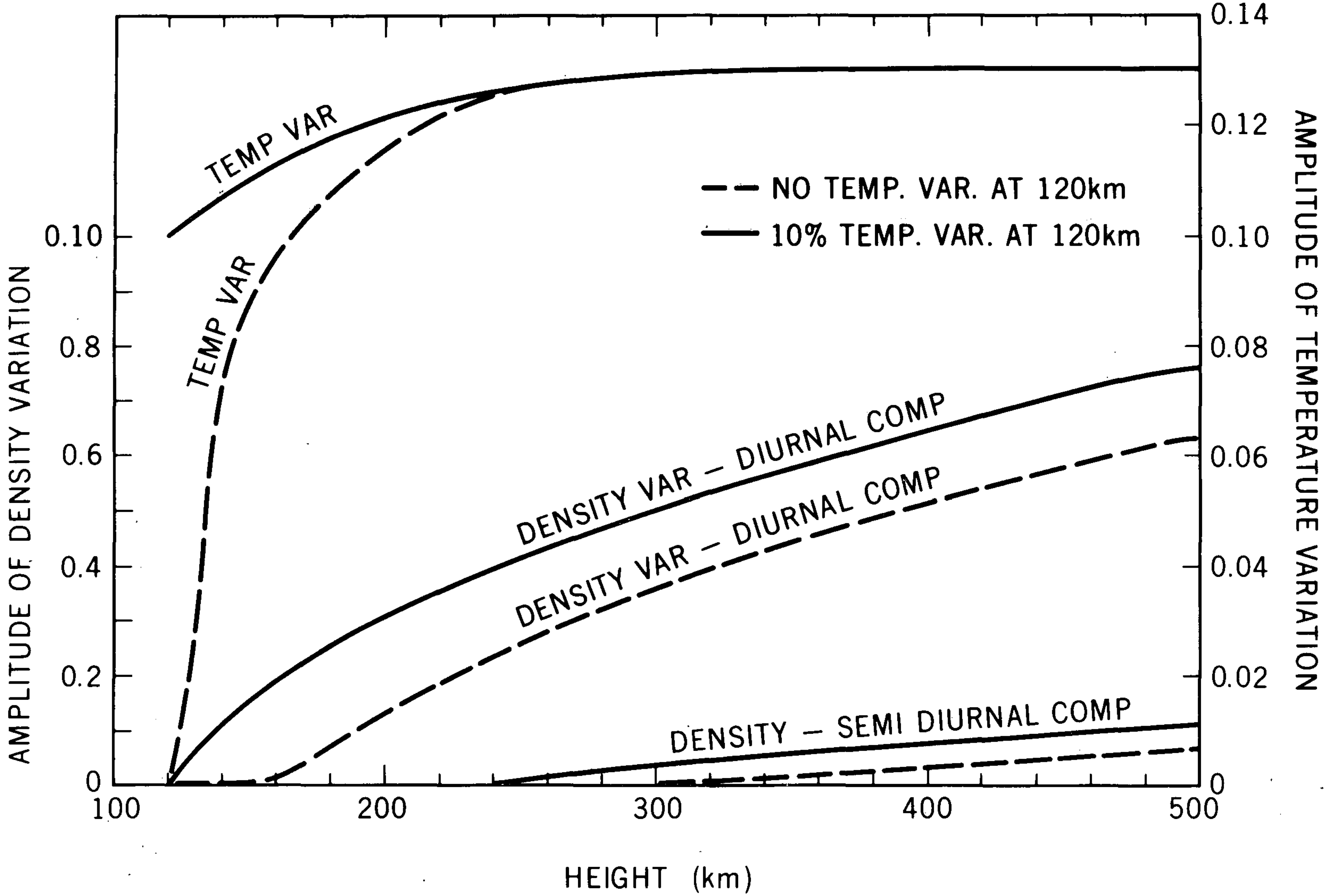

Figure 2. Smoothed Temperature Gradient at 120 km Deduced from the Observations of Wand and Nisbet-Swartz.

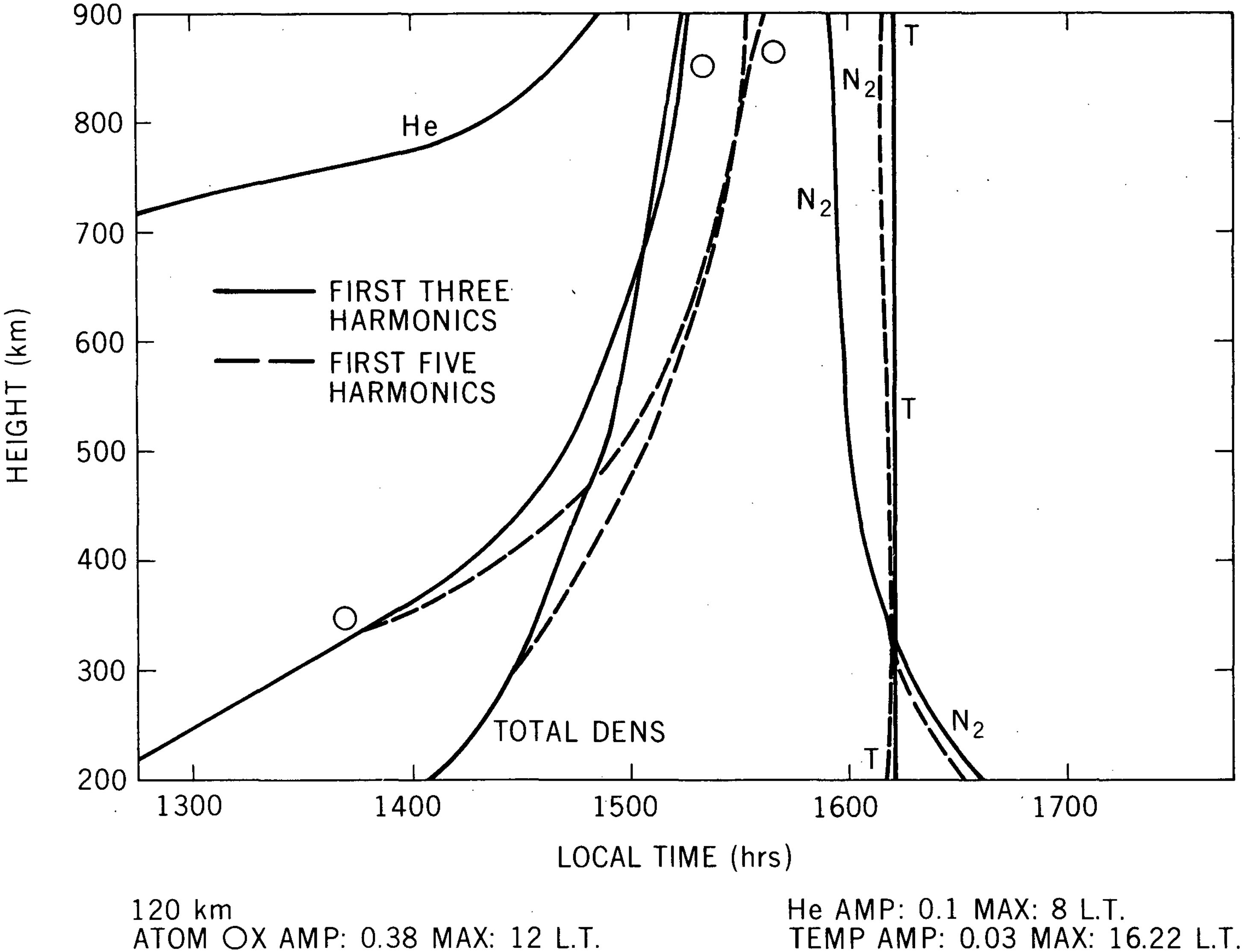

Figure 3. Effect of Higher Harmonics on the Time of Maximum Densities.

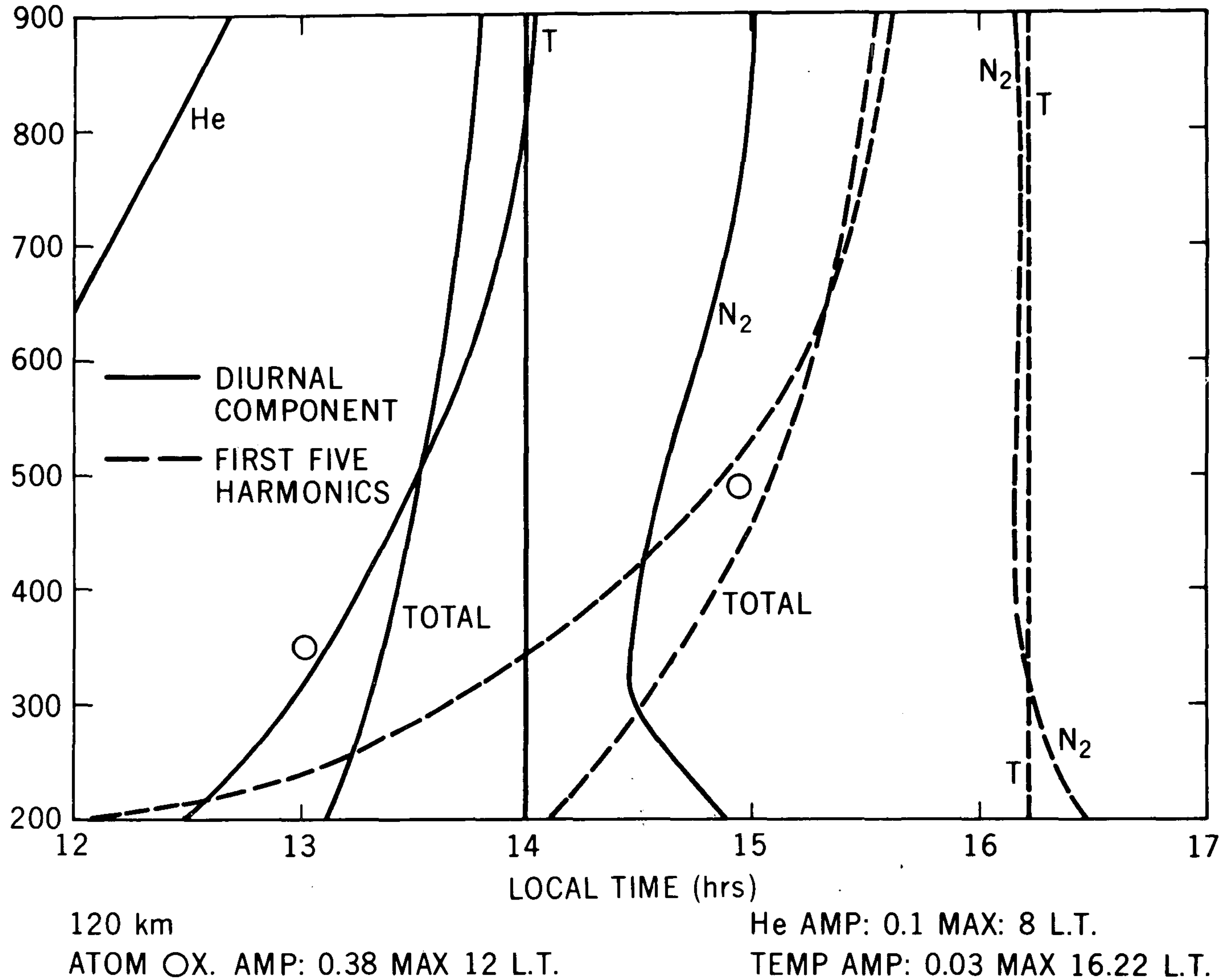

Figure 4. Comparison of the Time of Maximum Densities Between Fundamental Component and Total Densities.

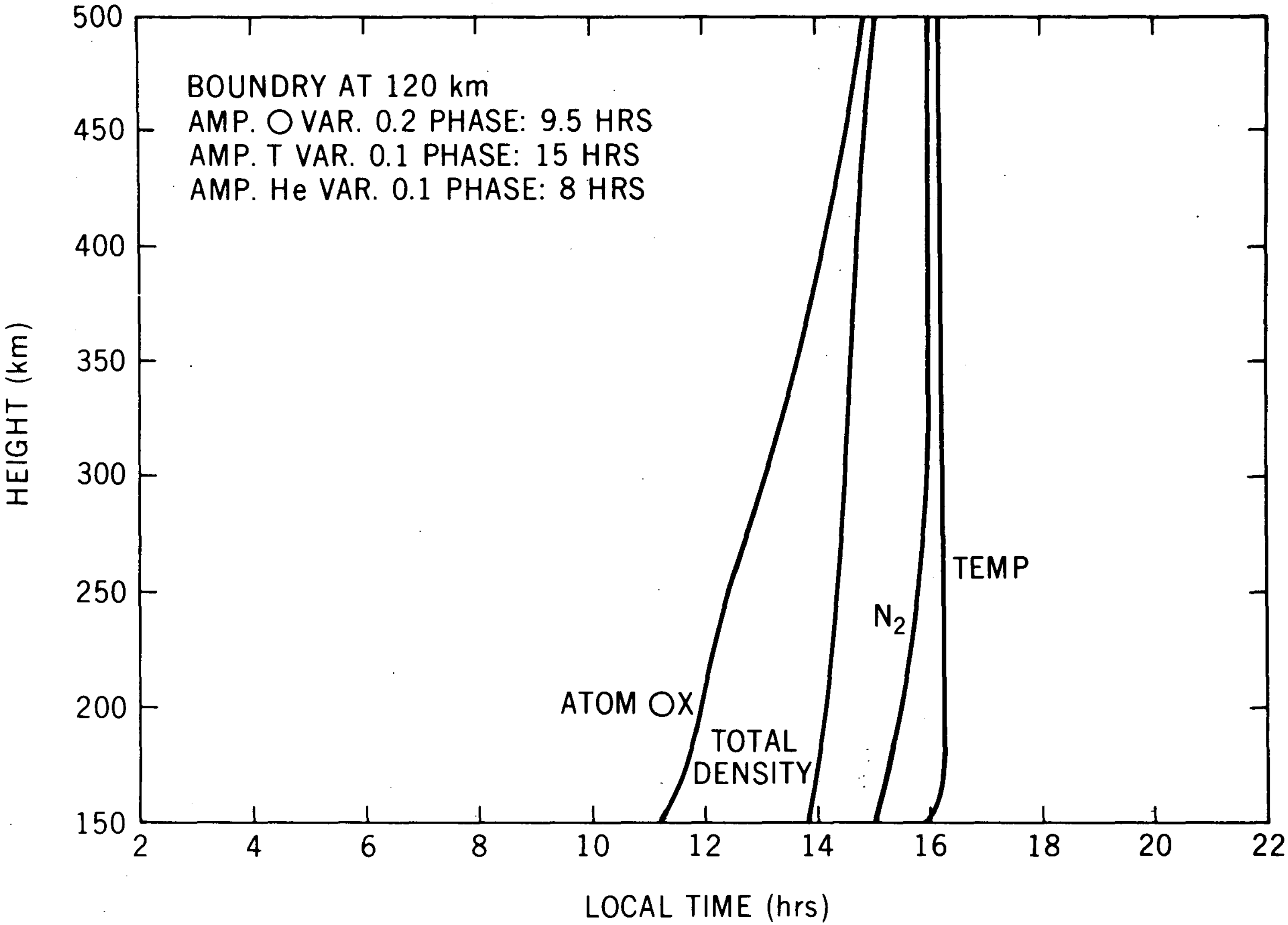

Figure 5. Time of Maximum Densities and Temperatures for Boundary Conditions at 120 km that have a Variation of the Atomic Oxygen Density and Temperature.

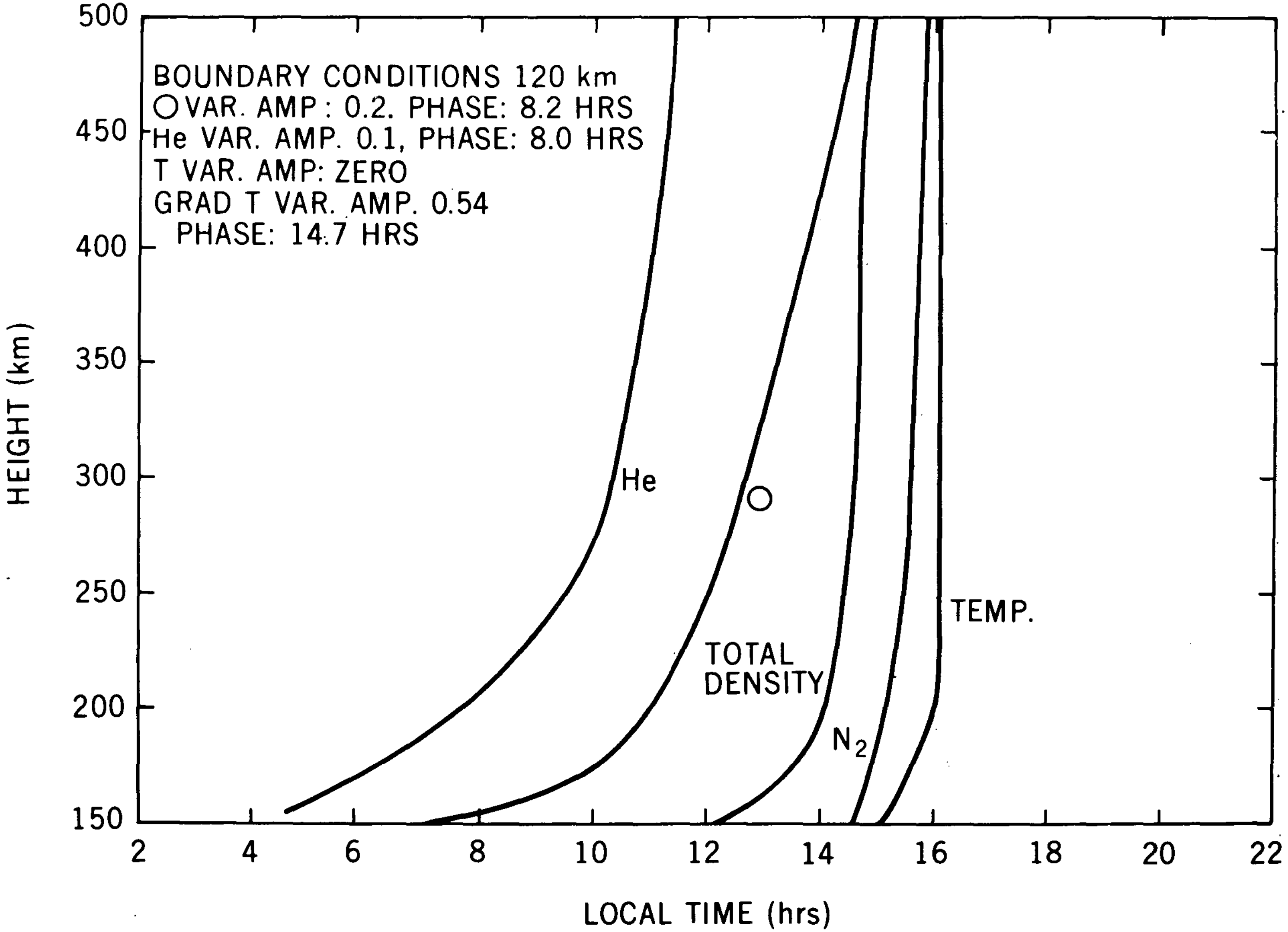

Figure 6. Time of Maximum Densities and Temperatures for Boundary Conditions at 120 km that have a Variation of the Atomic Oxygen Density and the Temperature Gradient.

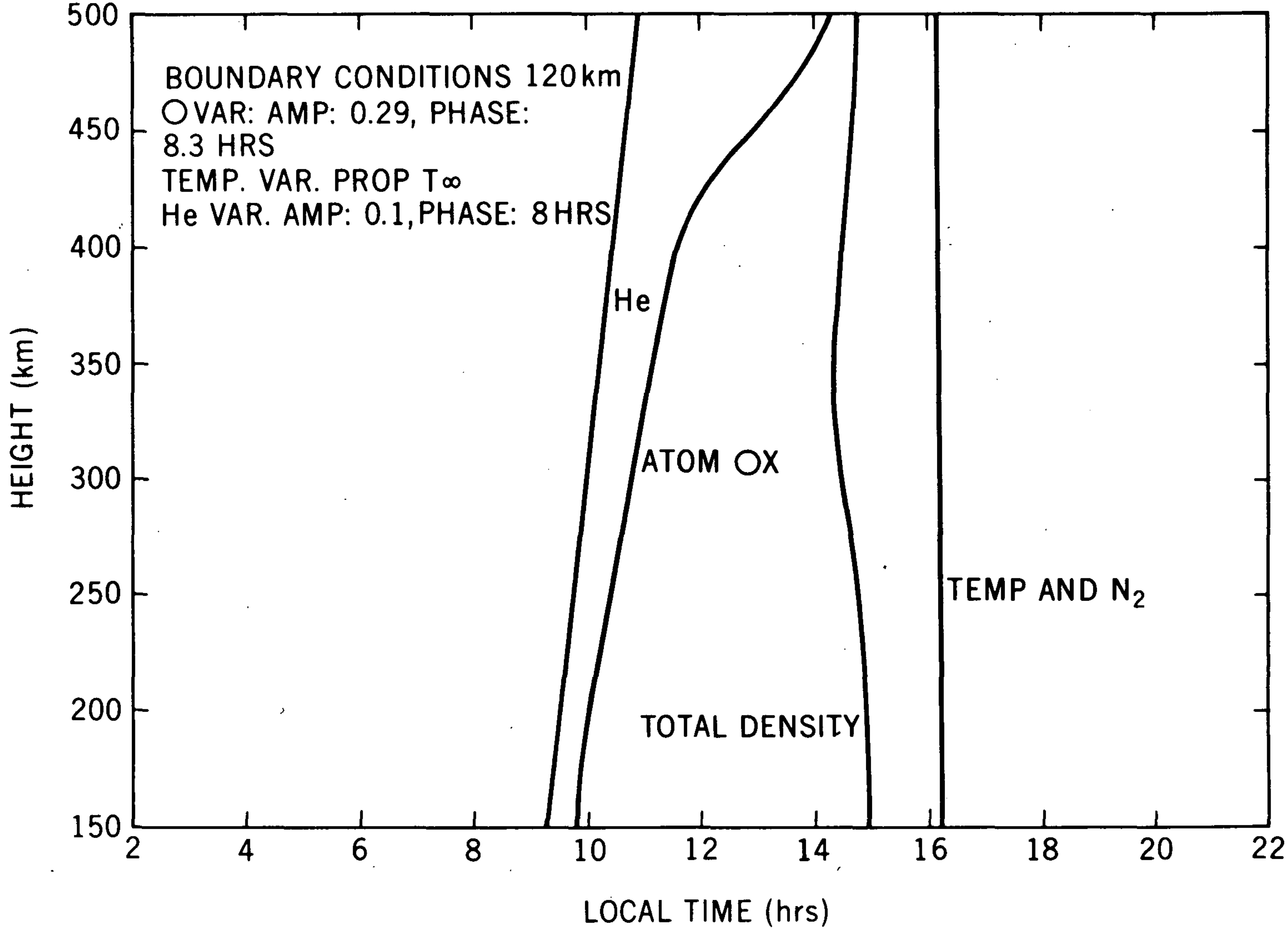

Figure 7. Time of Maximum Densities and Temperatures for Boundary Conditions at 120 km that have a Variation of Atomic Oxygen Densities and a Temperature Variation of Equal Amplitude and Phase as the Exospheric Temperature.

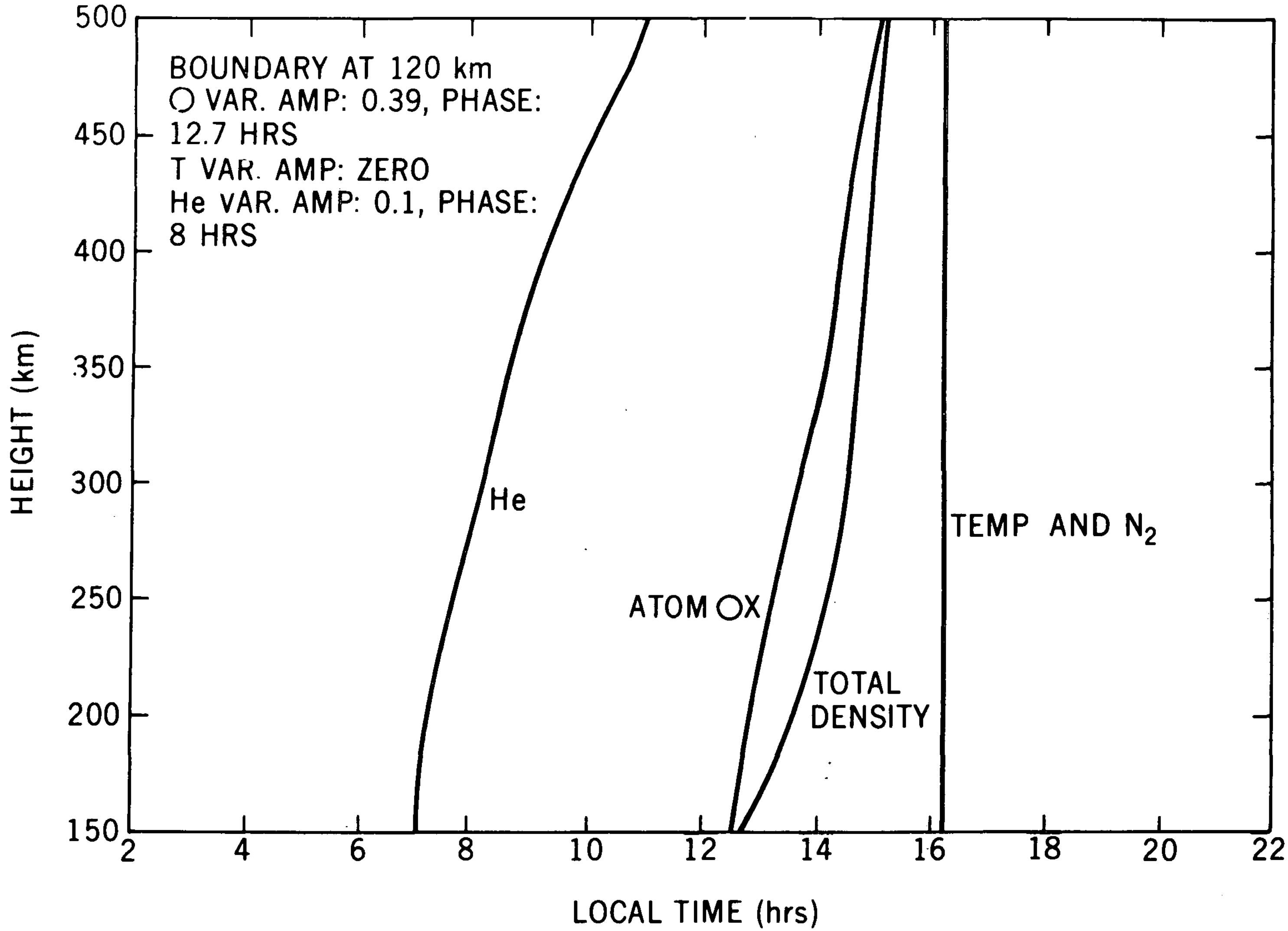

Figure 8. Time of Maximum of Densities and Temperature for Boundary Conditions that have a Variation of Atomic Oxygen Densities and Constant Temperatures and Temperature Gradients.