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STATISTICAL COMPLEX FATIGUE DATA FOR SAE 4340 STEEL AND ITS USE IN DESIGN BY RELIABILITY

by Dr. Dimitri Kececioglu and John L. Smith

Prepared under Grant No. NGL 03-002-044 by The University of Arizona

Tucson, Arizona

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION - WASHINGTON, D. C.

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November 15, 1970

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ABSTRACT

A brief description of the complex fatigue machines used in the test program is presented. The data generated from these machines are given and discussed. Two methods of obtaining strength distributions from the data are also discussed. Then follows a discussion of the construction of statistical fatigue diagrams and their use in designing by reliability. Finally, some of the problems encountered in the test equipment and a corrective modification are presented.

CHAPTER I

INTRODUCTION

1.1 Background on Testing Program

The purpose of the test program is to generate cycles-to-failure data by testing rotating specimens subjected to a bending moment and a constant torque. The test machines were designed and built at the University of Arizona.

The following is a brief discussion of the test machines. For a more complete discussion of the design and development of the test machines see Ref. 1. Presently there are three machines at the University of Arizona.

The test specimen is subjected to a bending moment by weights hung at the end of a lever arm. Fig. 1.1 shows a schematic diagram of the machine illustrating the loading geometry. The torque is applied by turning the torque coupling, which rotates shaft A with respect to shaft B and holds the relative position of the shafts. After the specimen is tightened in the holding collets the adjusting nut on the torque coupling can be turned while holding shaft A from rotating. This will rotate shaft B with respect to shaft A. It requires a full 360 degree turn of the adjusting nut to rotate shaft B one degree with respect to shaft A. Once the play in the gear boxes is taken up

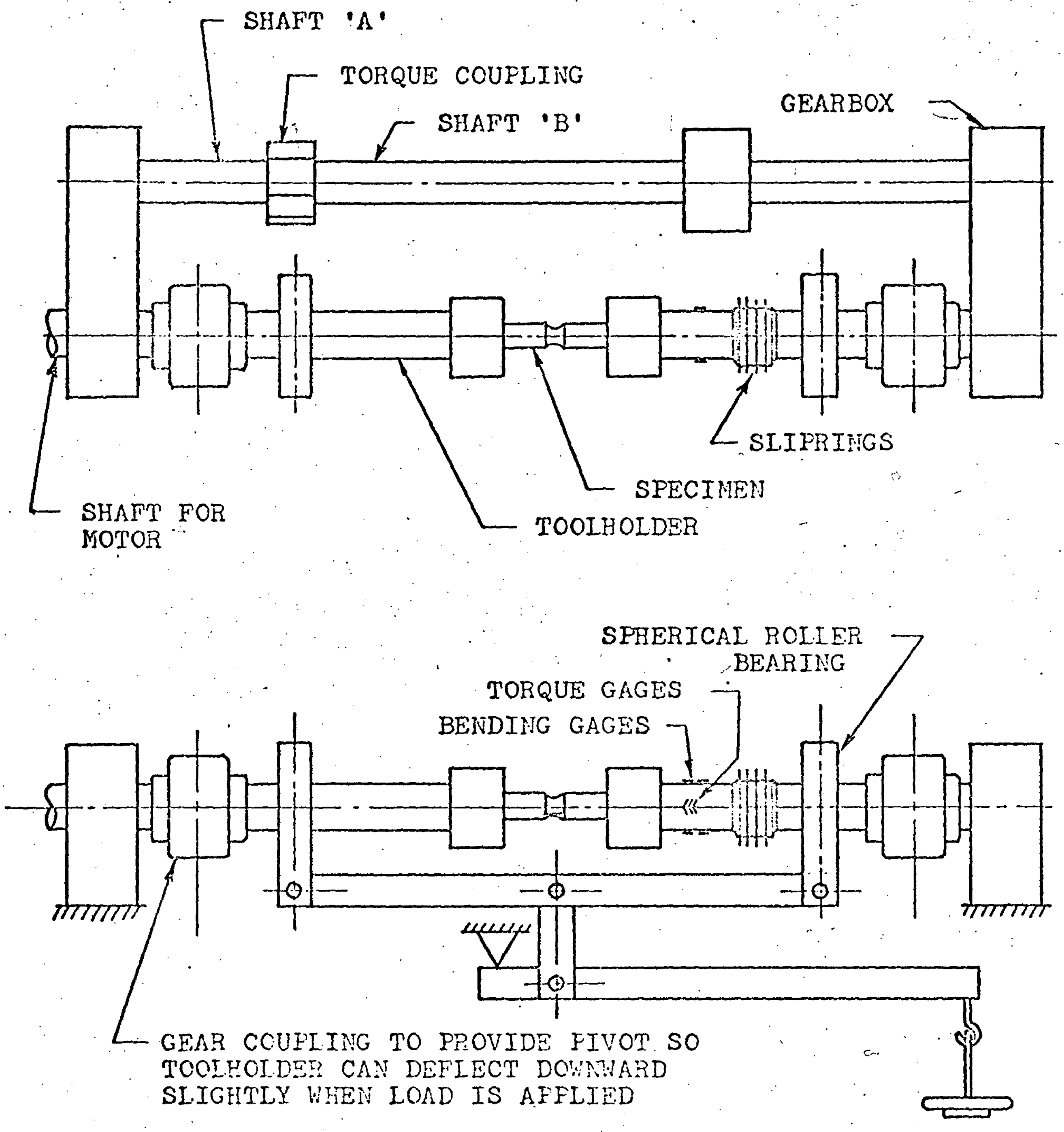

Fig. 1.1 Schematic Diagram of Test Machines Showing the Loading Configuration

then torque is exerted on the specimen.

The magnitudes of the shear stress and the normal stress that the specimen experiences are related to the stresses which the toolholder experiences. Two strain gage bridge curcuits are mounted on the toolholder in the position shown in Fig. 1.1. Acting through an amplifier the output of one bridge is a measure of the normal stress and the output of the other bridge is a measure of the shear stress that the toolholder experiences. The outputs are in the form of a Visicorder® record. The strain gage outputs and how they are used to determine the stresses in the specimen will be further discussed in Sect. 2.1.

Since the machines are capable of subjecting a specimen to combined stresses, tests can be conducted for any combination of alternating and mean stresses. The alternating stress is in the form of a normal stress caused by a constant bending moment on the rotating specimen. Any element of volume located at the surface of the specimen will experience an alternating tension and compression of equal magnitude. If torque is applied the element will also experience a constant shear stress perpendicular to the normal stress. Figure 1.2 show the stress element.

Early in the research program a study was made of the various failure theories. On the basis of this study it was concluded that the von Mises-Hencky theory most closely predicts fatigue failures in steel (1, p. 41).

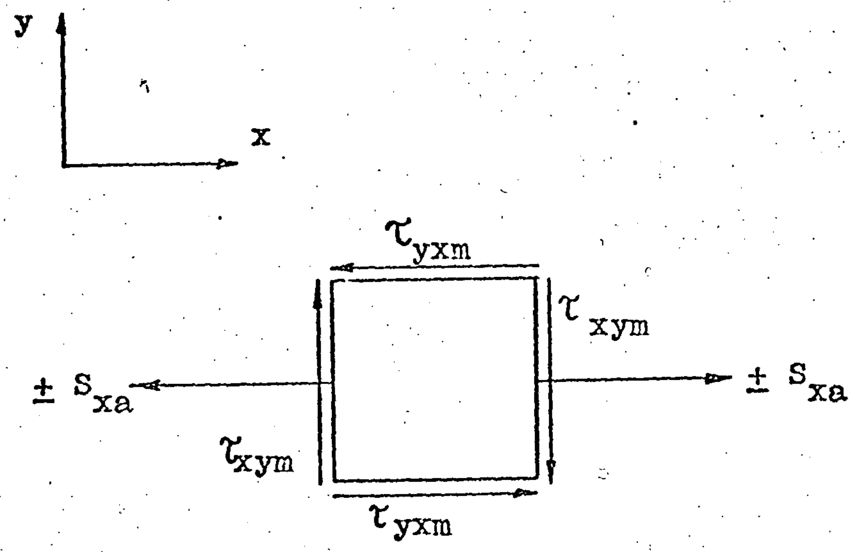

Fig. 1.2 Stress Element on Surface of Test Specimen

Shigley (2, P. 185) makes the point that the von Mises-Hencky theory (also called the distortion-energy theory) was developed to predict yield under static loads. However, because of the good agreement between fatigue data and the theory it was accepted as the failure governing criterion for the research program.

In order to have some measure of the relative magnitudes of the alternating and mean stresses a stress ratio was defined in terms of the von Mises stresses (2, P. 188). For an ordinary element subjected to bi-axial stresses the von Mises stresses are given by:

$$S_a = [S_{xa}^2 - S_{xa}\, S_{ya} + S_{ya}^2 + 3\tau_{xya}^2]^{\frac{1}{2}}$$

$$S_m = [S_{xm}^2 - S_{xm}\, S_{ym} + S_{ym}^2 + 3\tau_{xym}^2]^{\frac{1}{2}}$$

For the case of an alternating normal stress along the x axis and a constant shear stress perpendicular to the x axis, the above equations reduce to

$$S_a = S_{xa} \tag{1}$$

$$S_m = \sqrt{3}\,\tau_{xym} \tag{2}$$

The stress ratio is defined as

$$R = \frac{S_a}{S_m} = \frac{S_a}{\sqrt{3}\,\tau_{xym}} \tag{3}$$

where

R = stress ratio

S_a = alternating normal stress

S_m = mean normal stress

τ_{xym} = mean shear stress.

The specimens being used for the test program are made of SAE 4340 steel, condition C-4, heat treated to 35 - 40 R_c . The specimens were all manufactured from the same heat. Fig. 1.3 shows the specimen geometry.

The ultimate purpose of the test program is to develop statistical fatigue diagrams which can be used to design a specified reliability into a rotating shaft subjected to a bending moment and a constant torque for a specified life.

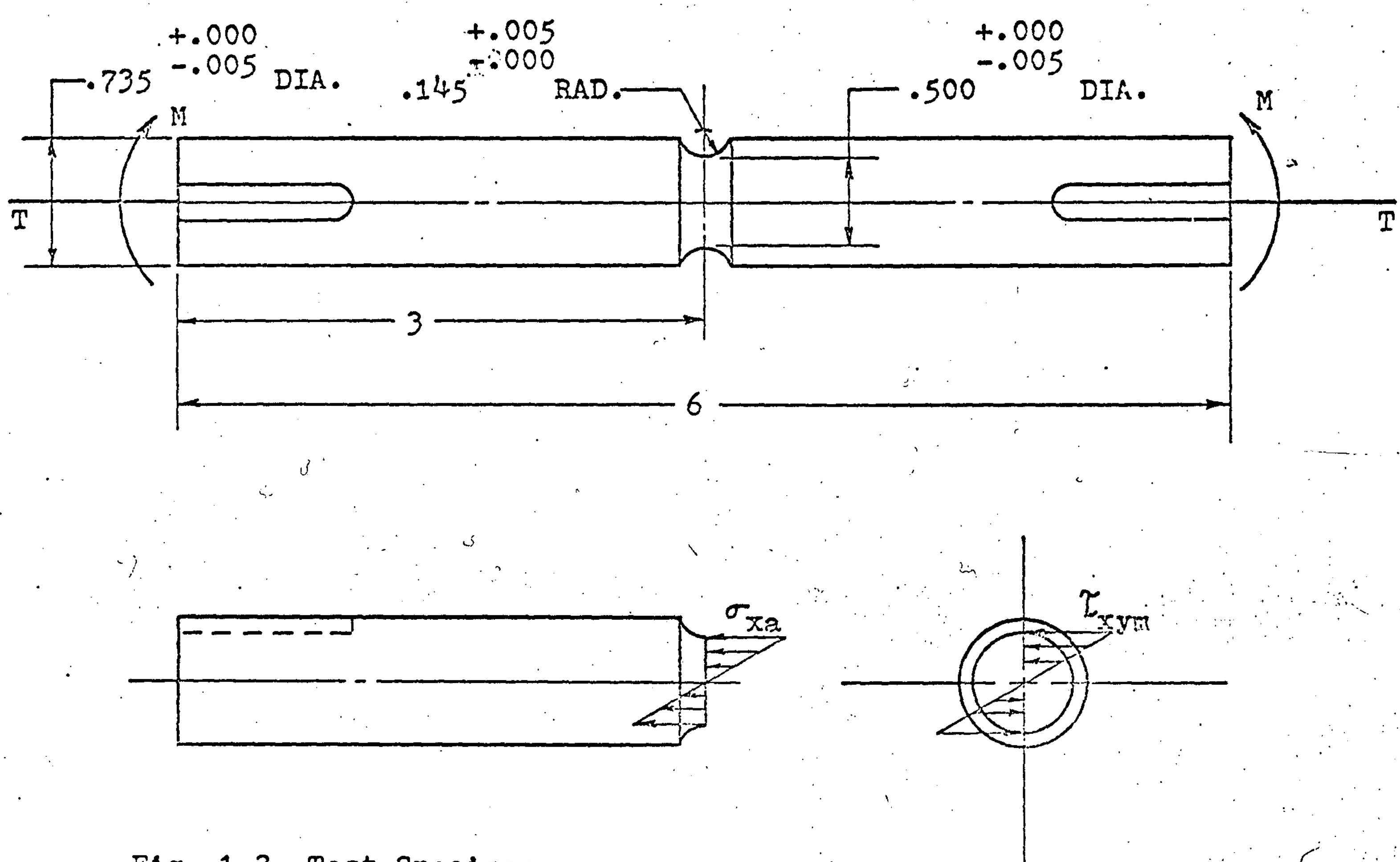

Fig. 1.3 Test Specimen

In order to secure data to construct the statistical fatigue surfaces the following test plan was proposed (1, pp. 132-138):

1. Tests are to be conducted at various alternating stress levels, holding the stress ratio constant, to determine the cycles-to-failure distributions at the various levels. Tests, of 18 specimens at each level, are to be conducted at 4 to 6 different stress levels for each stress ratio. This data can then be plotted on log-log paper with stress level on the ordinate and cycles-to- failure on the abscissa. The resulting diagram will look like fig. 1.4 . Such a diagram can be obtained for each stress ratio except R = 0.
2. Once the cycles-to-failure diagrams are obtained then the strength distributions for specified lives would be obtained and these distributions used in constructing the fatigue diagrams to be used to design by reliability.

1.2 Data Reduction

The data as taken from the testing machines is in the form of a Visicorder record which contains two traces for each specimen. One trace records the amplitude of the alternating normal stress as seen by the strain gages on the

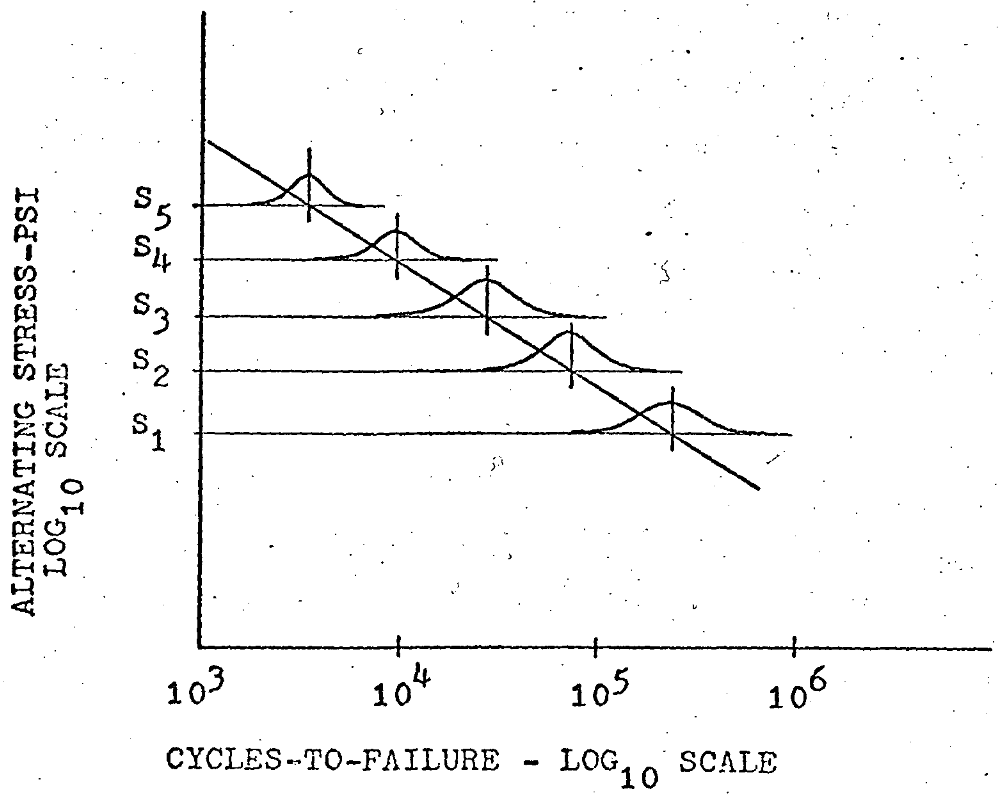

Fig. 1.4 S-N Diagram Showing Cycles-to-Failure Distributions for a Specific Non-Zero Stress Ratio.

toolholder and the other trace records the magnitude of the constant shear stress. From these records the nominal normal stress and average nominal shear stress can be obtained for all the specimens which were tested at a given level. The determination of these stress levels from the Visicorder records is discussed in Section 2.1.

The terms "nominal normal stress" and "nominal shear stress" refer to the stresses the specimen would experience at the outermost fiber of the cross section if it did not contain a stress riser; ie, if the test section were of a constant diameter and that diameter were equal to the diameter across the base of the groove in the present specimen.

Section 2.2 discusses the methods employed in determining the endurance level at the various stress ratios.

Sections 2.3, 2.4 and 2.5 deal with the calculation of the mean and standard deviation, and the coefficients of skewness and kurtosis of both the cycles-to-failure data and the natural logarithms of the cycles-to-failure data.

Two computer programs were developed to aid in the reduction of the test data:

1. A program to reduce the Visicorder records to stresses and stress ratios.

2. A program to calculate the mean and standard deviation of the cycles-to-failure data and test how well the data fits the normal distribution. The program also makes these calculations for the log-normal distribution.

The programs are discussed in Appendices A and B. Included in the discussion is a complete description of the input data card formats for each deck and a flow chart for each program.

The first program mentioned above was developed by this author whereas the second was previously written but was modified to be used with the data of this research program.

1.3 Goodness-of-Fit Tests

Section 2.6.1 discusses the Chi-square goodness-of-fit test and its applicability to the test data. Section 2.6.2 discusses the Kolmogorov-Smirnov goodness-of-fit test and its applicability.

1.4 Generation of Statistical Fatigue Diagrams From Test Data

The generation of the statistical fatigue diagrams requires that the strength distributions at various cycles of life be known. The test plan that was proposed at the beginning of the research effort was set up with the objective of obtaining test data from which estimates could be made of these strength distributions. Section 2.8 discusses the proposed method and its limitations. In the same section another test plan and method is proposed for obtaining the required distribution based on actual data.

Section 2.8.1 discusses how to use statistical fatigue diagrams to design a shaft to a specified reliability. The assumptions and limitations of the method are also discussed.

1.5 Modification of One Test Machine

With the approval of Dr. Kececioglu and Mr. Vincent R. Lalli of NASA-Lewis this author made a modification to one of the three test machines in December, 1969. It is believed that this modification will eliminate some of the problems encountered with the machines. None of the data presented in this report was obtained from the modified machine. Details of the modification are presented in Section 2.9.

CHAPTER II

DATA REDUCTION

2.1 Determination of Stress Levels and Stress Ratios

As stated in the introduction, the raw data is in the form of a Visicorder record containing two traces. Figure 2.1 is an illustration of what such a record looks like. The traces are a measure of the alternating normal stress and constant shear stress as seen by the strain gages which are mounted on the machine toolholder. The values of these traces, in terms of divisions, can be converted into values of nominal normal stress and nominal shear stress in the groove of the specimen. However, to be able to make this conversion several constants must be known which relate the output of the strain gages on the toolholder to the nominal stresses in the groove of the specimen. The calibrations required to obtain these constants are quite extensive and since the original calibrations were not conducted by this author only a brief discussion will be undertaken here. For a complete discussion of the calibration procedure see Ref. 3.

There are five constants required to make the conversion from divisions on the Visicorder record to nominal

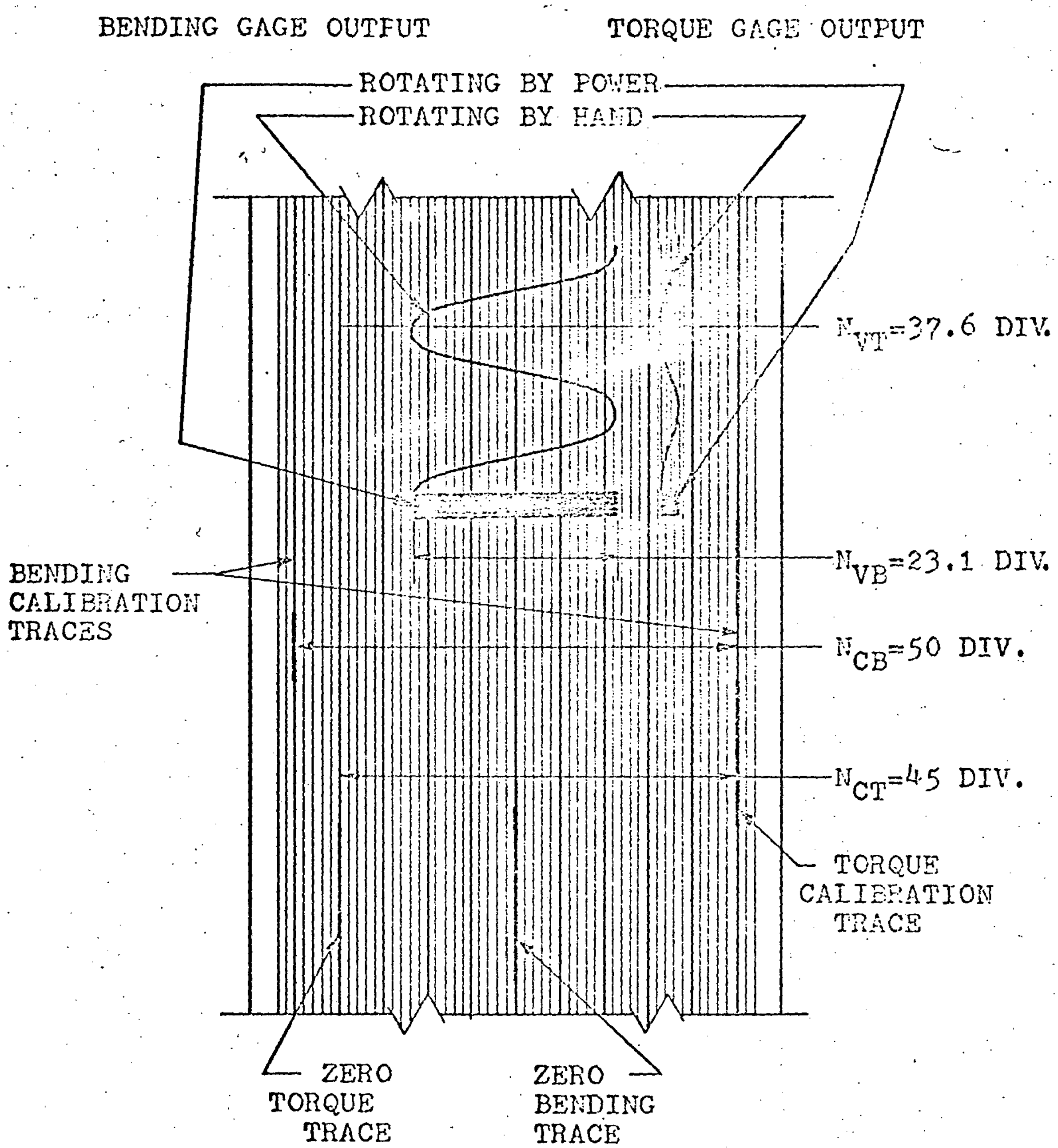

Fig. 2.1 Illustration of a Visicorder Record Showing the Bending, Torque and Calibration Traces

stresses in the test specimen. They are:

$K_{bgr} = \frac{\sigma_{os}}{\sigma_{as}}$ = constant to relate the output of the strain gages on the specimen to the actual stress in the specimen.

$K_{gr-th} = \frac{\sigma_{ot}}{\sigma_{os}}$ = constant to relate the output of the bending strain gages on the toolholder to the output of the strain gages on the specimen.

$K_t = \frac{\tau_{as}}{\tau_{ot}}$ = constant to relate the actual shear stress in the specimen to the output of the torque strain gages on the toolholder.

$K_{t/b} = \frac{\sigma_{ot}}{\tau_{ot}}$ = constant to relate the output of the bending gages on the toolholder to the output of the torque gages on the toolholder when the toolholder is subjected to a changing torque while the bending moment is held constant.

$K_{b/t} = \frac{\tau_{ot}}{\sigma_{ot}}$ = constant to relate the output of the torque gages on the toolholder to the output of the bending gages on the toolholder when the toolholder is subjected to a changing bending moment while the torque is held constant.

where

σ_{os} = normal stress in the specimen as indicated by the output of strain gages on the test specimen.

σ_{as} = actual normal stress in the specimen.

σ_{ot} = normal stress in the toolholder as indicated by the strain gages on the toolholder.

τ_{as} = actual shear stress in the specimen.

τ_{ot} = shear stress as indicated by the strain gages on the toolholder.

The last two constants listed are interaction constants. Due to misalignment, a bending gage may record an output when the shaft is subjected to pure torsion. Likewise, a misaligned torque gage may react to a pure bending on the shaft. These two constants take this interaction into account.

Whenever a strain gage on the toolholder is replaced or when a change is made to the machine that alters its loading characteristics some of the calibration constants may change. Therefore, whenever such changes are made, calibration tests must be performed to obtain corrected values for the constants that are affected by the change. Changes that affected the calibration constants were made at three different times throughout the test program. In order to know what constants were in effect during given time periods each time period was designated as a mode of operation. The values of these constants are given in Appendix C.

Figure 2.2 shows the sequence of calculations required to convert the strain gage outputs, as recorded by the Visicorder, to nominal stresses in the groove of the test specimen. The meanings of the symbols used in Fig. 2.2 which have not been defined earlier in this section are:

N_{cb} = number of Visicorder record divisions used when adjusting the gain in the bending channel of the amplifier.

N_{ct} = number of Visicorder record divisions used when adjusting the gain in the torque channel of the amplifier.

R_{cb} = value in ohms of the calibrating resistance used when adjusting the gain in the bending channel amplifier.

R_{ct} = value in ohms of calibrating resistance used when adjusting the gain in the torque channel of the amplifier.

N_{vb} = amount of deflection, in Visicorder divisions, caused by the toolholder bending strain gage bridge output when load is applied to the specimen.

N_{vt} = amount of deflection, in Visicorder divisions, caused by the toolholder torque strain gage bridge output when load is applied to specimen.

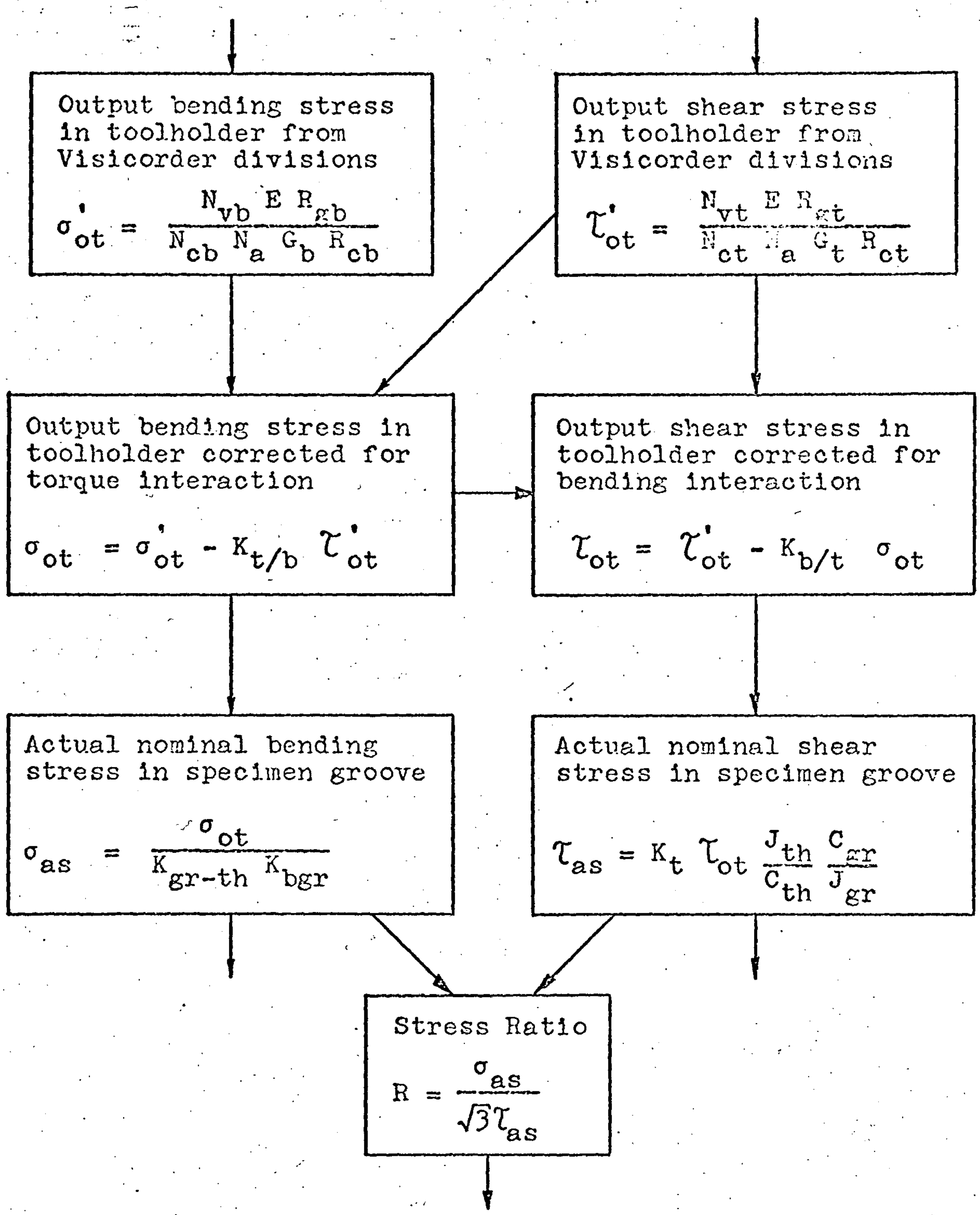

Fig. 2.2 Flow Chart of Calculations to Obtain Nominal Stresses in the Specimen Groove From the Strain Gage Outputs.

E = modulus of elasticity for steel
= 30×10^6 psi.

R_{gb} = resistance of each bending gage
= 190 ohms.

R_{gt} = resistance of each torque gage
= 120 ohms.

G_b = bending gage factor
= 3.23

G_t = torque gage factor
= 2.05

N_a = number of active gages in each bridge
= 4

J_{th} = polar moment of inertia of the toolholder cross section where the gages are mounted.

C_{th} = radius of the toolholder where the gages are mounted.
= 1.000 inches.

J_{gr} = polar moment of inertia of the specimen cross section at the base of the groove.

C_{gr} = radius of the specimen at the base of the groove.
= .249 inches.

In Fig. 2.2 the equation for σ_{ot} , in the second box on the left, is an approximation. The full equation is:

$$\sigma_{ot} = \frac{\sigma'_{ot} - K_{t/b}\,\tau'_{ot}}{1 + K_{t/b}\,K_{b/t}} \qquad (4)$$

Upon examination of the values of $K_{t/b}$ and $K_{b/t}$ in Appendix C it is seen that they are small and the product is negliglible compared to 1. Thus, Eq. (4) reduces to the one given in Fig. 2.2 .

A program for the CDC 6400 computer at The University of Arizona using FORTRAN IV was developed to determine σ_{as}, τ_{as} and R, as per Fig. 2.2, from the data. The program, along with user instructions and flow chart, is presented in Appendix A.

Table 2.1 lists the average stress levels and ratios at which test data were collected. The computer outputs listing the stresses in the individual specimens are given in Appendix D.

2.2 Stress-to-Failure Data (Staircase Method)

The method used for testing specimens to determine the distribution of the endurance strength was the staircase method (4, p. 48; 5) sometimes called the "up and down method". Briefly, the method consists of testing a specimen subjected to a stress equal to the estimated endurance strength. If the specimen fails, the next specimen is subjected to a stress one increment lower than the failed specimen. If the specimen does not fail by a

TABLE 2.1

TABLE SHOWING THE AVERAGE STRESS LEVELS AND RATIOS AT WHICH SPECIMENS WERE TESTED

Average Stress Ratio	Average Normal Stress*-psi	Standard Dev. Normal Stress**	Number of Specimens Tested	Average Shear Stress***	Standard Dev. of Shear Stress**
∞	144,000	1500	12	0	0
	114,000	900	18	0	0
	98,000	2700	18	0	0
	81,000	900	18	0	0
	73,000	1800	18	0	0
3.5	151,000	3850	12	25,000	800
	115,000	1820	18	19,500	700
	83,000	1200	18	13,500	750
	74,000	750	18	12,500	600
0.825	111.000	1250	12	73,500	2050
	92,000	6700	18	66,500	3400
	76,000	3150	18	53,500	3450
	65,000	3850	18	47,000	1300
0.44	69,000	1400	18	90,000	1600
	60,000	700	18	78,500	1400

* Rounded off to nearest 1,000 psi.

** Rounded off to nearest 50 psi.

*** Rounded off to nearest 500 psi.

predetermined time the test is terminated and the next specimen is subjected to a stress one increment higher. This procedure continues until the desired sample size is obtained. It should be noted that only approximately 50% of the tested specimens are used in the calculations of mean and standard deviation. The calculations are based on either the successes or failures, whichever has occurred the least number of times. The specimen just preceding the first change of mode is considered as the beginning of the test. A change of mode is a success followed by a failure or a failure followed by a success.

The equations for calculating the estimates of the mean and standard deviation of the endurance strength are given by (5, p. 114).

$$m = y + d\left(\frac{A}{N} \pm \frac{1}{2}\right) \tag{5}$$

and

$$s = 1.620\, d\left(\frac{NB-A^2}{N^2} + 0.029\right) \tag{6}$$

where

m = mean

s = standard deviation

y = the lowest stress level at which a success or failure (whichever the analysis is based on) occurred.

d = stress increment

N = effective sample size; ie, the number of specimens used in the calculations.

$$A = \sum_{i=o}^{n} i\, n_i$$

$$B = \sum_{i=o}^{n} i^2\, n_i$$

n = number of success (or failures) which occurred at the i^{th} level.

The lowest level is considered the zeroth level, the next the 1^{st} level, etc. In Eq. (5) above the (+) is used if the calculations are based on successes and the (-) if based on failures.

The above analysis requires that the variate being tested is assumed to be normally distributed or can be transformed to a normal distribution (5, p. 111). Also, the stress increment should be in the range of $.5\sigma$ to 2σ, where σ is the standard deviation of the distribution. Therefore, some prior knowledge of the variance is helpful for good results.

The staircase method is very good for the determination of the mean of the variate being tested. However, the estimate of the variance can be poor if the sample size is not large. Mood and Dixon state that (5, p. 112), "Measures of reliability may well be very misleading if the sample size is less than forty or fifty." It is not clear, however, whether they are referring to effective or actual sample size, the effective sample size being approximately 50% of the actual sample size. Because of research program limitations the effective sample sizes of the endurance tests range from 16 to 18.

Endurance tests were conducted at R = ∞, 3.5, 1.0 and 0.44. As stated before, the tests of specimens which do not fail are terminated at a predetermined time. For stress ratio of ∞ the tests were terminated after 90 hours which is equivalent to more than 9.5 million cycles. In conducting the endurance tests at stress ratio of ∞, of the eighteen specimens which failed, sixteen failed before 48 hours of running time.

For stress ratio of 1.0 the tests were terminated at 48 hours which is equivalent to over 5 million cycles. Of the eighteen specimens which failed during the endurance tests at R = 1.0 all failed before 24 hours of running time. The endurance tests at R = 3.5 were terminated at 24 hours which represents more than 2.5 million cycles.

The endurance tests at R = 0.44 have not been completed as of the date of this report. They are also being terminated at 24 hours of running time.

The calculation of the means and standard deviations from the staircase data will be discussed separately in the next two sections. The results of those calculations are given in Table 2.2

TABLE 2.2

ESTIMATES OF THE MEAN AND STANDARD DEVIATION OF THE ENDURANCE STRENGTH FOR STRESS RATIOS OF ∞, 3.5 AND 1.0

Stress Ratio	Mean* (psi)	Standard Deviation**
∞	57,000	3,800
3.5	55,000	3,700
1.0	57,000	3,300

Estimates are based on the calculations discussed in Sections 2.2.1 and 2.2.2 .

* Rounded off to nearest 1,000 psi.

** Rounded off to nearest 100 psi.

2.2.1 Endurance Tests for R = ∞ and R = 1.0

The tests were based on the amount of weight placed on the loading arm. An increment of one pound in the pan was used to obtain the staircase. The calculations for mean and standard deviation were also based on weight and these values were then converted to psi.

Figures 2.3 and 2.4 show the staircase plots for the tests. They were taken from an earlier report (6, pp. 48-49). Tables 2.3 and 2.4 list the specimens that were used in the endurance level calculations for R = ∞ and R = 1.0 respectively.

To obtain a better estimate of the endurance strength distribution the average of the stresses in all the specimens for a given pan weight was found and then the increment between these averages for the different pan weights was calculated. In order to obtain a uniform increment an average was found for the increments.

For stress ratio of ∞ the calculations are based on successes. The average stress at each pan weight and the number of specimens which succeeded at each level is:

Pan Weight pounds	Stress psi	Number of Successes
27	61,857	1
26	59,999	2
25	54,430	10
24	51,915	2
23	59,421	1

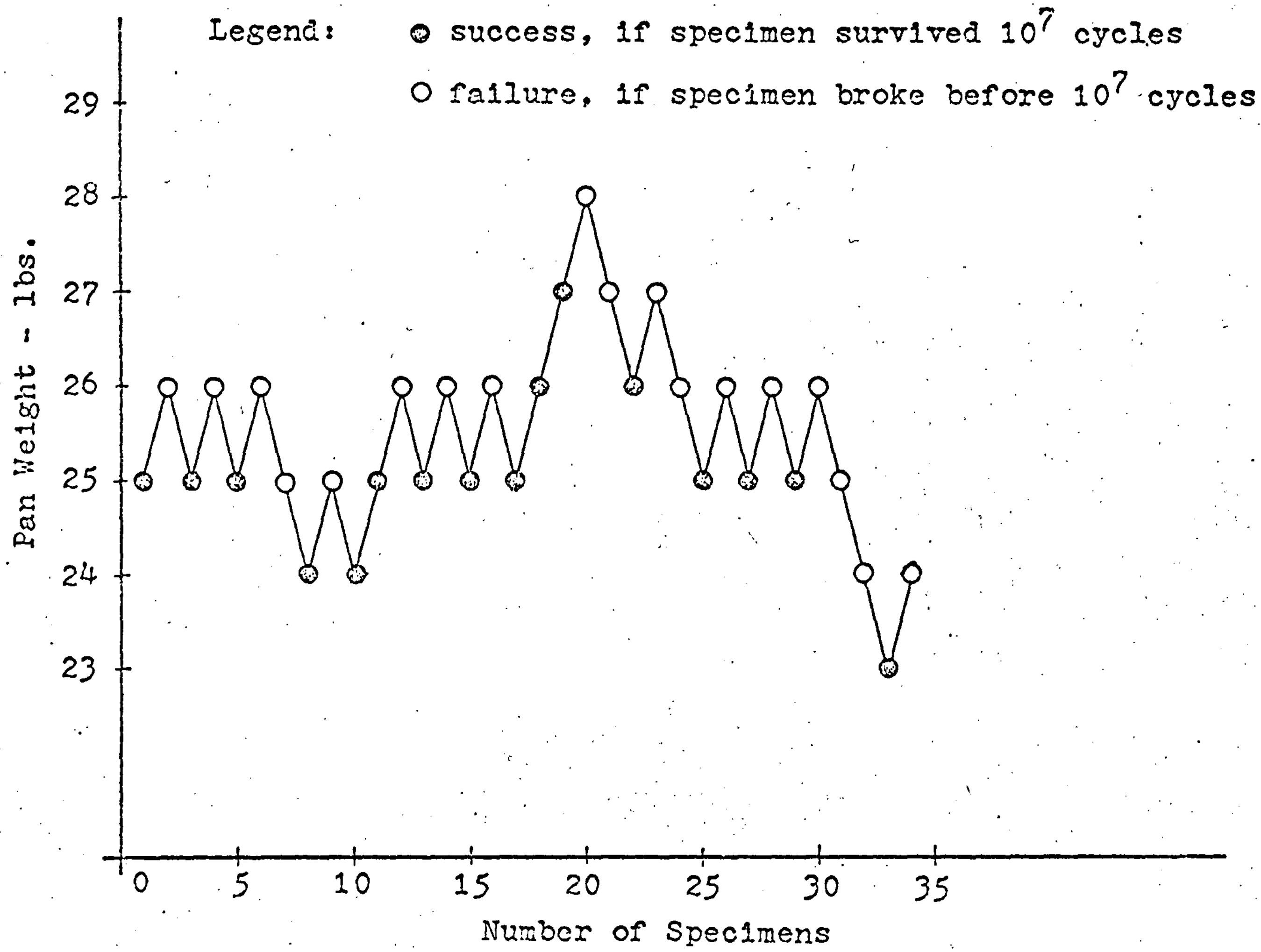

Fig. 2.3 Endurance Strength Data Obtained by the Staircase Method for Stress Ratio of ∞ for SAE 4340 Steel, MIL-S-5000B, Condition C4, Rockwell C 35/40.

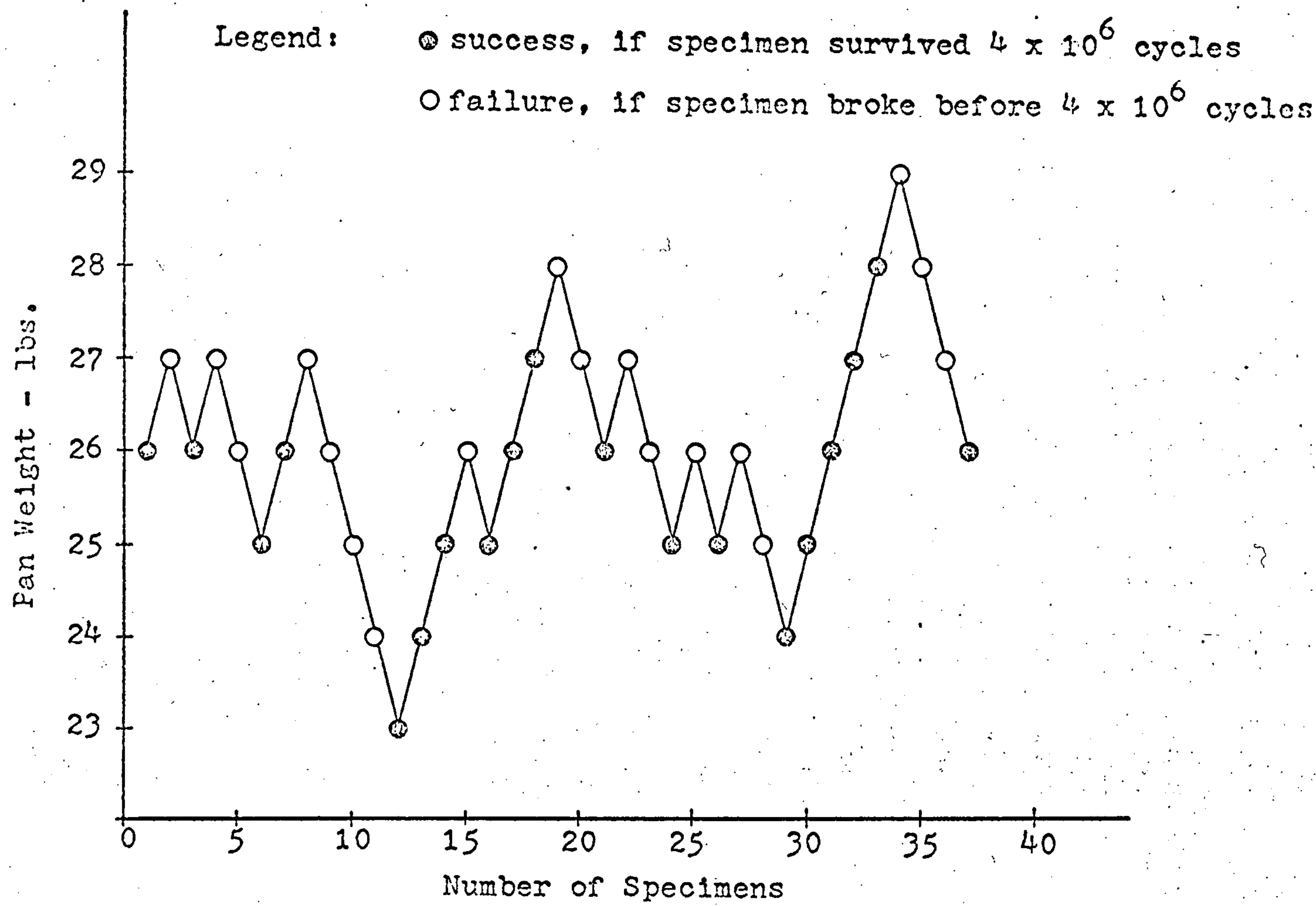

Fig. 2.4 Endurance Strength Data Obtained by the Staircase Method for Stress Ratio of 1.0 for SAE 4340 Steel, MIL-S-5000B, Condition C4, Rockwell C 35/40.

TABLE 2.3

LIST OF SPECIMENS AND THE CORRESPONDING STRESSES AND PAN WEIGHTS FOR ENDURANCE TESTS AT STRESS RATIO = ∞

Specimen Number	Pan Weight pounds	Nominal Normal Stress in Specimen psi.
87	25.0	60,519
17	25.0	37,476
74	25.0	46,827
202	25.0	51,125
182	25.0	57,353
124	25.0	60,411
216	26.0	58,017
205	24.0	49,580
199	25.0	54,036
209	25.0	53,903
177	24.0	54,249
168	25.0	57,132
190	26.0	61,981
207	27.0	61,857
142	25.0	65,528
208	23.0	59,422

TABLE 2.4

LIST OF SPECIMENS AND THE CORRESPONDING STRESSES AND PAN WEIGHTS FOR ENDURANCE TESTS AT STRESS RATIO = 1.0

Specimen Number	Pan Weight pounds	Nominal Normal Stress in Specimen psi.	Stress Ratio
248	27.0	48,979	.721
288	27.0	61,723	1.072
267	26.0	58,350	1.045
300	27.0	55,537	1.005
275	26.0	59,810	1.169
409	25.0	57,067	1.066
364	24.0	52,863	1.040
372	26.0	58,885	1.042
362	28.0	61,561	1.031
429	27.0	58,918	1.019
359	27.0	58,727	1.055
417	26.0	55,581	.996
336	26.0	54,156	.909
435	26.0	56,298	1.114
401	25.0	57,266	1.081
400	29.0	60,166	1.064
386	28.0	58,619	1.063
402	27.0	58,288	1.087

Note that the average stress for the one specimen at a pan weight of 23 pounds is greater than the average stress for both 25 and 24 pounds. To eliminate this inconsistency, the one data point taken at 23 pounds was eliminated from the calculations. Then, taking the average of the increments between the other four levels the value of 3,314 psi. for the stress increment was obtained.

In order to get values for the number of specimens which succeeded at each level, it was assumed that each specimen tested at a given pan weight was subjected to a stress equal to the average stress calculated for that pan weight. Thus, 2 specimens were run at 24 pounds, 10 specimens at 25 pounds, 2 specimens at 26 pounds and 1 specimen at 27 pounds. Using Eqs. (5) and (6) where

$$y = 51{,}915 \text{ psi.}$$

$$d = 3{,}314 \text{ psi.}$$

$$N = 15$$

$$A = \sum_{i=o}^{3} i\, n_i = 0(2) + 1(10) + 2(2) + 3(1) = 17$$

$$B = \sum_{i=o}^{3} i^2\, n_i = 1^2(10) + 2^2(2) + 3^2(1) = 27$$

then

$$m = 51{,}915 + 3{,}314 \left(\frac{17}{15} + \frac{1}{2} \right)$$

$$m = 57,317 \text{ psi.}$$

and

$$s = 1.620\ (3,314)\ \left[\frac{15(27)-(17)^2}{(15)^2} + 0.029\right]$$

$$s = 3,818 \text{ psi.}$$

Thus the estimates for the mean and standard deviation of the endurance strength distribution for stress ratio of ∞ are 57,317 psi. and 3,818 psi. respectively.

Similarly, the estimates for the mean and standard deviation of the endurance strength distribution for stress ratio of 1.0 were found to be 56,785 psi. and 3,290 psi., respectively.

2.2.2 Endurance Level for R = 3.5

The staircase method was used to conduct the tests at this ratio also, but instead of basing the tests on pan weight they were based on stress in the specimen. The stress increment was chosen to be 3,000 psi. Figure 2.5 shows the staircase plot for these tests. Table 2.5 lists the specimens which were used in the calculation of the distribution parameters. Also listed are the stress as recorded by the strain gages, the target stress and actual stress ratio in each specimen.

Table 2.5 indicates that the stresses were not held as close as targeted when the tests were conducted. The scatter may be due to carelessness on the part of the operators and test machine operating inconsistencies.

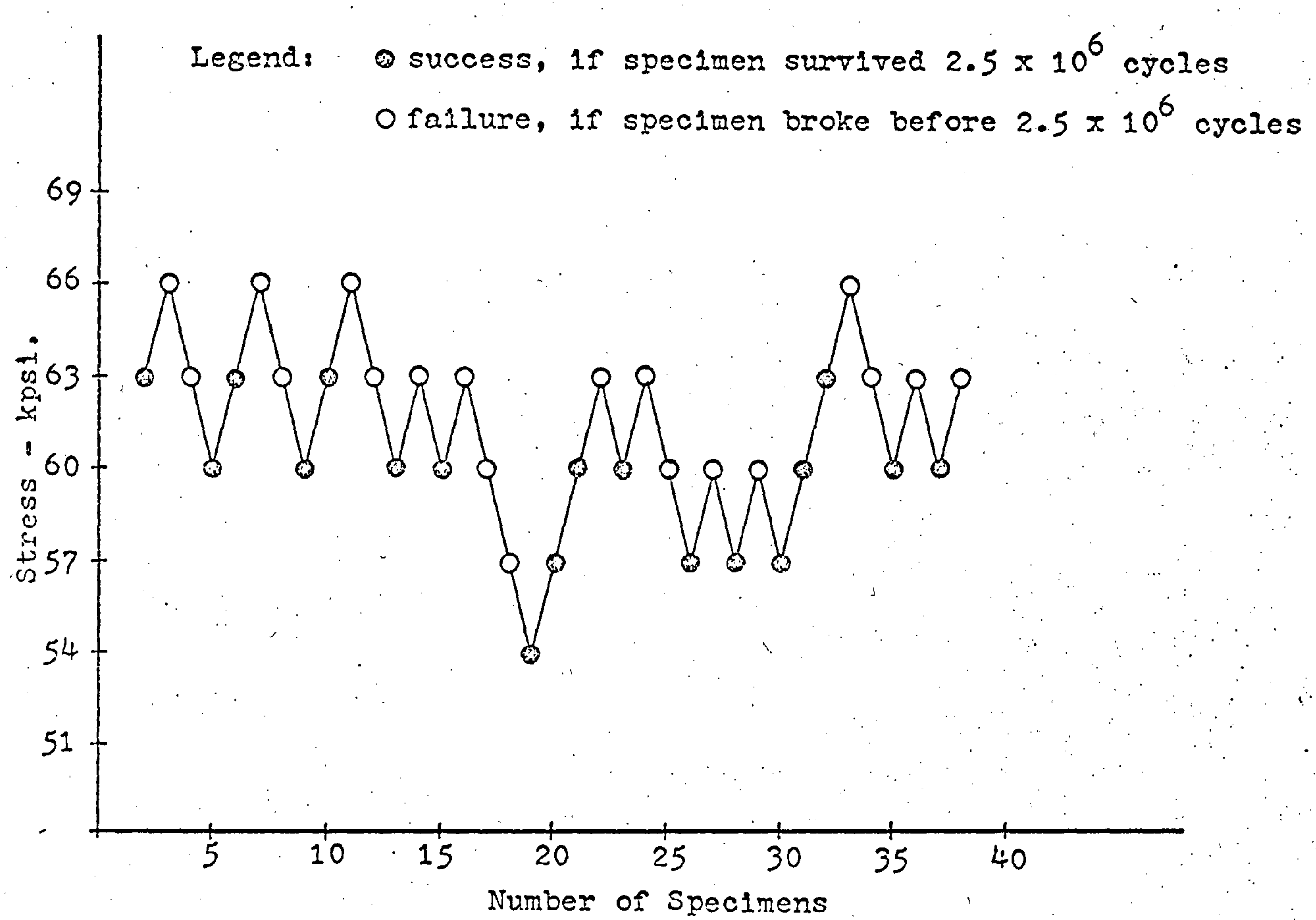

Fig. 2.5 Endurance Strength Data Obtained by the Staircase Method for Stress Ratio of 3.5 for SAE 4340 Steel, MIL-S-5000B, Condition C4, Rockwell C 35/40.

TABLE 2.5

LIST OF SPECIMENS AND CORRESPONDING ACTUAL STRESSES AND INTENDED STRESSES FOR ENDURANCE TESTS AT STRESS RATIO = 3.5

Specimen Number	Actual Normal Stress in Specimen (psi)	Intended Normal Stress in Specimen (psi)	Actual Stress Ratio
477	55,917	60,000	3.54
465	55,825	57,000	4.26
478	58,291	60,000	3.64
506	51,256	57,000	3.22
528	57,376	60,000	3.78
444	53,410	57,000	3.26
466	54,236	57,000	3.60
524	47,071	51,000	3.03
546	50,292	54,000	3.93
499	60,903	57,000	5.72
532	54,533	57,000	4.43
483	50,828	54,000	3.10
529	51,381	54,000	3.56
587	52,067	54,000	3.50
581	54,320	57,000	3.67
611	56,826	60,000	3.42
589	54,118	57,000	3.47
603	53,813	57,000	3.41

These are discussed in Section 2.9. Because of this the average actual stress at each target level was determined as follows:

Average Actual Stress (psi)	Target Stress (psi)	Number of Successes
57,103	60,000	4
54,823	57,000	9
51,142	54,000	4
47,071	51,000	1

The average stress increment between these levels is 3,377 psi. Using Eqs. (5) and (6) where

$$y = 47{,}071 \text{ psi.}$$

$$d = 3{,}377 \text{ psi.}$$

$$N = 18$$

$$A = \sum_{i=1}^{3} i\, n_i = 1(4) + 2(9) + 3(4) = 34$$

$$B = \sum_{i=1}^{3} i^2\, n_i = 1(4) + 4(9) + 9(4) = 76$$

then

$$m = 47{,}071 + 3{,}377 \left(\frac{34}{18} + \frac{1}{2} \right) = 55{,}139 \text{ psi.}$$

and

$$s = 1.620\ (3{,}377) \left[\frac{(18)(76)-(34)^2}{(18)^2} + 0.029\right] = 3{,}738 \text{ psi.}$$

Thus, the estimated mean and standard deviation of the endurance strength distribution for stress ratio of 3.5 are 55,139 psi. and 3,738 psi., respectively.

2.3 Cycles-to-Failure Data at Specific Stress Ratios and Levels

It was desired to determine how the cycles-to-failure data at each stress level and ratio was distributed.

Based on a study made by Broome (7) in an earlier report on this research program it was decided to fit the normal and log-normal distributions to the data and conduct goodness-of-fit tests.

The parameters for these two distributions were estimated using the unbiased estimates for mean and standard deviation. For the normal case the cycles-to-failure is the variate and for the log-normal case the natural logarithms of the cycles-to-failure is the variate.

The moment coefficients of skewness and kurtosis were also estimated from the data. For the normal distribution the values of these two parameters are 0 and 3, respectively.

The Kolmogorov-Smirnov goodness-of-fit test was used as a measure of how well the distributions fit the data.

A computer program was used to make the calculations. The program had been developed previously but it was modified by this author to include the Kolmogorov-Smirnov test. The original program, which was written by Patel (8), used only the Chi-square goodness-of-fit test. The program and user instructions are included in Appendix B.

Table 2.6 lists the results obtained by estimating the normal distribution parameters from the data and conducting the Kolmogorov-Smirnov goodness-of-fit test. Table 2.7 lists the results obtained by using the natural logarithms of the cycles-to-failure data.

2.4 Normal Distribution Fit to the Data

The probability density function for the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty \qquad (7)$$

where μ = mean

σ = standard deviation

If μ and σ are not known from prior knowledge they must be estimated from the data as discussed in the next section.

2.4.1 Determination of Mean, Standard Deviation, Skewness and Kurtosis

By definition the moment generating function, denoted by m(t), is the expected value of e^{tx} which is denoted by $E[e^{tx}]$. For the normal distribution

TABLE 2.6

NORMAL DISTRIBUTION PARAMETER ESTIMATES AND THE MAX D VALUES

Stress Ratio	Average Alternating Stress Level (psi)	Sample Size	Normal Dist. Parameters				Max. D Value*
			Mean	Standard Deviation	α_3 Skew-ness	α_4 Kurtosis	
∞	144,000	12	2,773	.532	-1.344	3.561	.175
	114,000	18	9,029	1,024	-.247	1.995	.099
	98,000	18	22,171	3,815	-.042	1.855	.082
	81,000	18	77,977	12,549	.873	3.060	.184
	73,000	18	161,984	34,286	-.616	2.201	.174
3.5	151,000	12	1,451	275	-.222	1.725	.113
	115,000	18	6,203	1,010	.797	3.397	.111
	83,000	18	39,608	12,990	1.876	7.241	.239
	74,000	18	74,212	19,531	.147	2.057	.132
0.825	111,000	12	6,554	1,005	.893	2.493	.296
	92,000	18	20,467	5,595	.239	2.257	.119
	76,000	18	61,028	11,132	-.428	2.182	.109
	65,000	18	127,577	21,227	.171	1.942	.141
0.44	69,000	18	53,573	15,470	1.667	4.833	.272
	60,000	18	142,550	41,585	.393	3.057	.148

* Maximum D-Value From K-S Test

TABLE 2.7

LOG-NORMAL DISTRIBUTION PARAMETER ESTIMATES AND MAX D-VALUES

Stress Ratio	Average Alternating Stress Level (psi)	Sample Size	Log-Normal Dist. Parameters: Mean	Standard Deviation	α_3 Skew-ness	α_4 Kurtosis	Max. D Value*
∞	144,000	12	7.906547	.227914	-1.586	4.258	.207
	114,000	18	9.101961	.116130	-.407	2.123	.092
	98,000	18	9.992123	.176382	-.265	1.945	.094
	81,000	18	11.252667	.153954	.575	2.697	.159
	73,000	18	11.970997	.234941	-.906	2.707	.198
3.5	151,000	12	7.262676	.197789	-.435	2.006	.128
	115,000	18	8.720894	.157247	.403	2.869	.086
	83,000	18	10.545160	.286664	.686	4.407	.184
	74,000	18	11.180338	.274219	-.342	2.501	.084
0.825	111,000	12	8.777704	.145966	.742	2.295	.280
	92,000	18	9.890129	.280831	-.171	1.916	.120
	76,000	18	11.001940	.195062	-.703	2.457	.111
	65,000	18	11.743320	.167235	-.030	1.769	.147
0.44	69,000	18	10.856736	.248421	1.210	3.899	.214
	60,000	18	11.825351	.305282	-.457	3.182	.118

* Maximum D-Value From K-S Test

$$m(t) = E\,[e^{tx}] = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Carrying out the integration yields

$$m(t) = e^{t\mu + \sigma^2 t^2/2}$$

which is the moment generating function for the normal distribution.

The procedure for obtaining the r^{th} moment is to take the r^{th} derivative of the moment generating function with respect to t and evaluate it at t = 0.

By definition the r^{th} sample moment is given by

$$\mu_r' = \frac{\sum_{i=1}^{n} x_i^{\,r}}{n} \,.$$

Taking the first derivative of the moment generating function with respect to t and evaluating at t = 0 yields

$$m_1 = \mu$$

Then equating the first distribution moment and the first sample moment yields the estimate for the mean

$$\hat{\mu} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (8)$$

It can be shown that this is an unbiased estimator.

Taking the second derivative of the moment generating function with respect to t and evaluating at t = 0 yields

$$m_2 = \sigma^2 + \mu^2$$

Equating the second moment yields

$$\hat{\sigma}^2 + \hat{\mu}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

but

$$\hat{\mu}^2 = \left[\frac{\sum_{i=1}^{n} x_i}{n}\right]^2$$

thus

$$\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \frac{\left[\sum_{i=1}^{n} x_i\right]^2}{n^2} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \text{sample mean.}$$

It can be shown that this is a biased estimator for σ^2. For unbiasedness, the expected value of the parameter estimator must equal the parameter (9, p. 72) thus in this case for unbiasedness

$$E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}\right] = \sigma^2 .$$

Going through the computations yields

$$E\left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}\right] = \frac{n-1}{n} \sigma^2 .$$

Therefore the estimator

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$

is a biased estimate for σ^2. However, the unbiased estimator for σ^2 is

$$s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} \quad . \qquad (9)$$

The moment coefficients of skewness and kurtosis are given by

$$\alpha_3 = \frac{m_3}{(m_2)^{3/2}} \quad \text{and} \quad \alpha_4 = \frac{m_4}{(m_2)^2}$$

respectively, where

$$m_2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$

$$m_3 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^3}{n}$$

and

$$m_4 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^4}{n}$$ (See Ref. 10, p.48).

Evaluating the third and fourth derivatives of the moment generating function at $t = 0$ and substituting into the equations for α_3 and α_4 will lead to values of $\alpha_3 = 0$ and $\alpha_4 = 3$ for the normal distribution.

The estimates of the mean, standard deviation and the moment coefficients of skewness and kurtosis have been calculated for each group of cycles-to-failure data. The estimates of the moment coefficients of skewness and kurtosis were calculated in order to give a further indication of how well the data fits the normal distribution. However, it is important to note that even though α_3 of a symmetrical distribution is zero, obtaining a value of zero for the estimate of α_3 from the data does not necessarily mean that the distribution is symmetrical. Mood and Graybill make this point (9, p. 109) and state that, "knowledge of the third moment gives almost no clue as to the shape of the distribution." Therefore, the moment coefficient of skewness is not a good measure of whether or not a distribution is symmetrical.

The moment coefficient of kurtosis is a measure of the peakedness of the distribution.

2.5 Log-Normal Distribution Fit to the Data

A variate is distributed log-normal if the logarithm of the variate is distributed normal. That is, by letting $y = \log_e x$ then

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$

where the unbiased estimates of μ and σ are

$$\hat{\mu} = \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$$

2.5.1 Determination of Mean, Standard Deviation, Skewness and Kurtosis

The determination of these parameters for the log-normal distribution is the same as for the normal distribution as covered in Section 2.4.1 except the log of the variate is used in the calculations.

2.6 Goodness-of-Fit Tests

In order to determine how well the normal and log-normal distributions fit the data, two goodness-of-fit tests were proposed. They are the Chi-square and Kolmogorov-Smirnov goodness-of-fit tests. However, both tests were not used in determining whether or not to reject a particular distribution. In all cases the Chi-square goodness-of-fit test was not valid because of the small sample sizes. Further reasons for it not being valid are given in the next section.

Since the computer program is written using both tests it should be pointed out that when analyzing data where the Chi-square test is valid, if both tests are used to determine rejection, the level of significance changes. In other words, if the critical values for rejection are based on a significance level of α for both tests and the criteria for rejection of the null hypothesis is if either one or the other or both tests rejects the distribution then the level of significance in rejecting is not α. It is not clear what the confidence level would be since the extent to which the two tests are correlated is not known. Therefore, only one test should be used in determining goodness-of-fit. For the data in this report only the Kolmogorov-Smirnov test was used.

2.6.1 Chi-Square Goodness-of-Fit Test

The Chi-square goodness-of-fit test can be used only with grouped data, that is, data divided into cells. The total Chi-square value is the sum of the Chi-square values of each cell. This can be written as

$$V_{k-1} = \sum_{i=1}^{k} \frac{(E_i - O_i)^2}{E_i}$$

where

E_i = the expected frequency of the i^{th} cell

O_i = the observed frequency of the i^{th} cell

k = total number of cells

V_{k-1} = total Chi-square value

V_{k-1} can be shown to be distributed X^2_{k-1} hence the name Chi-square test, where k-1 is the number of degrees of freedom. However, if the parameters of the hypothesized distribution must be estimated from the data the degrees of freedom must be decreased by the number of parameters estimated. In the case where the fit of the normal distribution to the data is being tested, if the mean and standard deviation are estimated from the data the number of degrees of freedom is k-1-2 or k-3.

The derivation of the Chi-square test is based on the law of large numbers in such a way that in order for the test to be valid the number of observations in each cell must be large. In actual practice it is generally accepted that the observed value of each cell must be greater than five.

Since the two parameters of the normal distribution were estimated from the data there must be at least four cells in order for the degrees of freedom to be greater than zero. Therefore, there would have to be at least 24 observations to have greater than five in each cell. In each case in this test program there were either twelve or eighteen specimens tested. Hence, the Chi-square test would be invalid for the sample sizes tested.

2.6.2 Kolmogorov-Smirnov Goodness-of-Fit Test

Briefly, the Kolmogorov-Smirnov test is a comparison of a hypothesized cumulative frequency distribution $F_n(x)$ and the observed cumulative distribution $S_n(x)$. Rejection or non-rejection is based on the absolute value of the maximum difference between the two functions. In mathematical terms

$$D_n = |F_n(x) - S_n(x)| \qquad (10)$$

where

D_n = Kolmogornov statistic

$$F_n(x) = \int_{-\infty}^{x_1} f(x)\,dx$$

f(x) = probability density function of the hypothesized distribution

x_1 = any specific value of the variate X

$$S_n(x) = \frac{X}{N} \qquad (11)$$

X = number of observations less than or equal to x_1

N = sample size

If D_n is greater than some critical value (D_c) the hypothesized distribution is rejected at some level of significance (α). The probability statement is $P(D_n > D_c) = \alpha$. Table 2.8 lists the critical values for sample sizes of 12 and 18 at various levels of significance.

TABLE 2.8

TABLES OF CRITICAL VALUES OF D TO USE IN THE KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST

Sample Size	Level of Significance (α)		
	0.10	0.05	0.01
12	0.338	0.375	0.450
18	0.278	0.309	0.371

In using the Kolmogorov-Smirnov test the D-value is found at each data point. If at any point the calculated D-value is greater than the critical value at the desired significance level, the hypothesis is rejected. If not, the hypothesis is not rejected.

2.7 Discussion of Results

In 8 of the 15 stress levels tested the D-value (Kolmogorov Statistic) was less for the log-normal distribution than for the normal distribution. The moment coefficient of kurtosis was closer to 3.0 for the log-normal distribution in 9 of the 15 cases. The moment coefficient of skewness was closer to zero for the log-normal distribution in 7 of the 15 cases. However, as was mentioned in Section 2.4.1, obtaining a value close to zero for the moment coefficient of skewness does not necessarily mean that the distribution is symmetrical or normal.

Based on the Kolmogorov-Smirnov test, in no case can either the normal or the log-normal distribution be rejected with 90% confidence. For stress ratio of 0.44 at stress level 69,000 psi. the normal distribution can ? be rejected with 85% confidence.

From the above results it appears that the log-normal distribution is favored.

The cycles-to-failure data for stress ratios of ∞

and 0.825 were previously analyzed and reported by Broome (7). He also concluded that the log-normal distribution was favored. He also points out that from a phenomenological viewpoint the log-normal distribution is justified.

Figures 2.6 through 2.9 show plots on log scales at the average stress levels of the cycles-to-failure distributions for stress ratios of ∞, 3.5 , 0.825 and 0.44 . The distributions shown are log-normal. The mean line shown on each figure was obtained by fitting a straight line, by the method of least squares, to the estimates of the means of the distributions on each figure. The 3 standard deviation limits for each distribution are also shown. To obtain a smooth envelope the dashed lines were drawn in by sight.

Specimens were tested at only two stress levels for the stress ratio of 0.44. They were 69,000 psi. and 60,000 psi. An attempt was made to test at a level of 75,000 psi., but at a stress ratio of 0.44 this requires a shear stress of 98,500 psi. The specimen began yielding and finally broke as the torque was applied. Thus, for stress ratio of 0.44 the cycles-to-failure tests were restricted to two stress levels. The endurance tests for stress ratio of 0.44 have not yet been completed but it appears as though the mean endurance limit is about 51,000 psi.

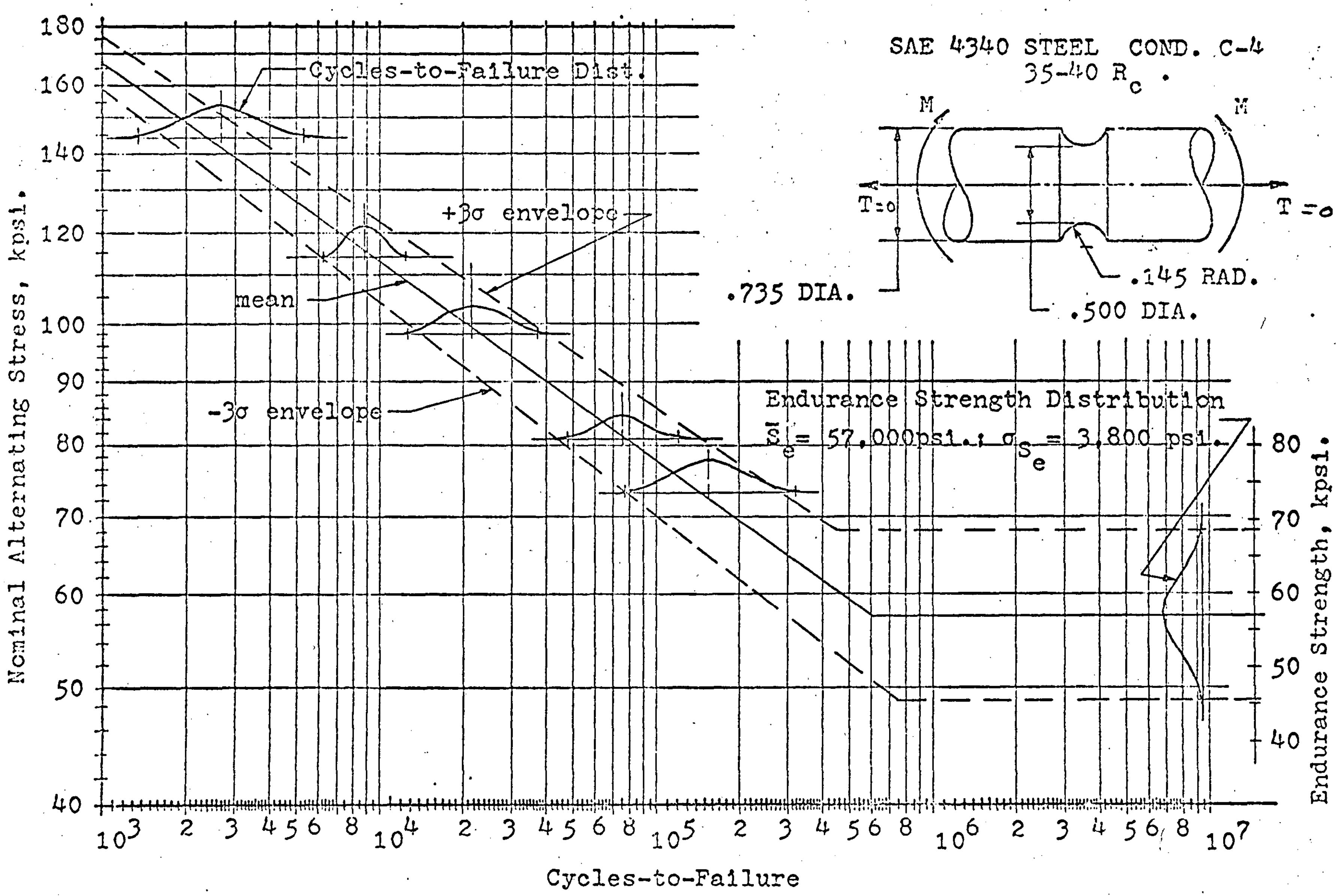

Fig. 2.6 Cycles-to- Failure Distributions and Endurance Strength Distribution for Stress Ratio of ∞

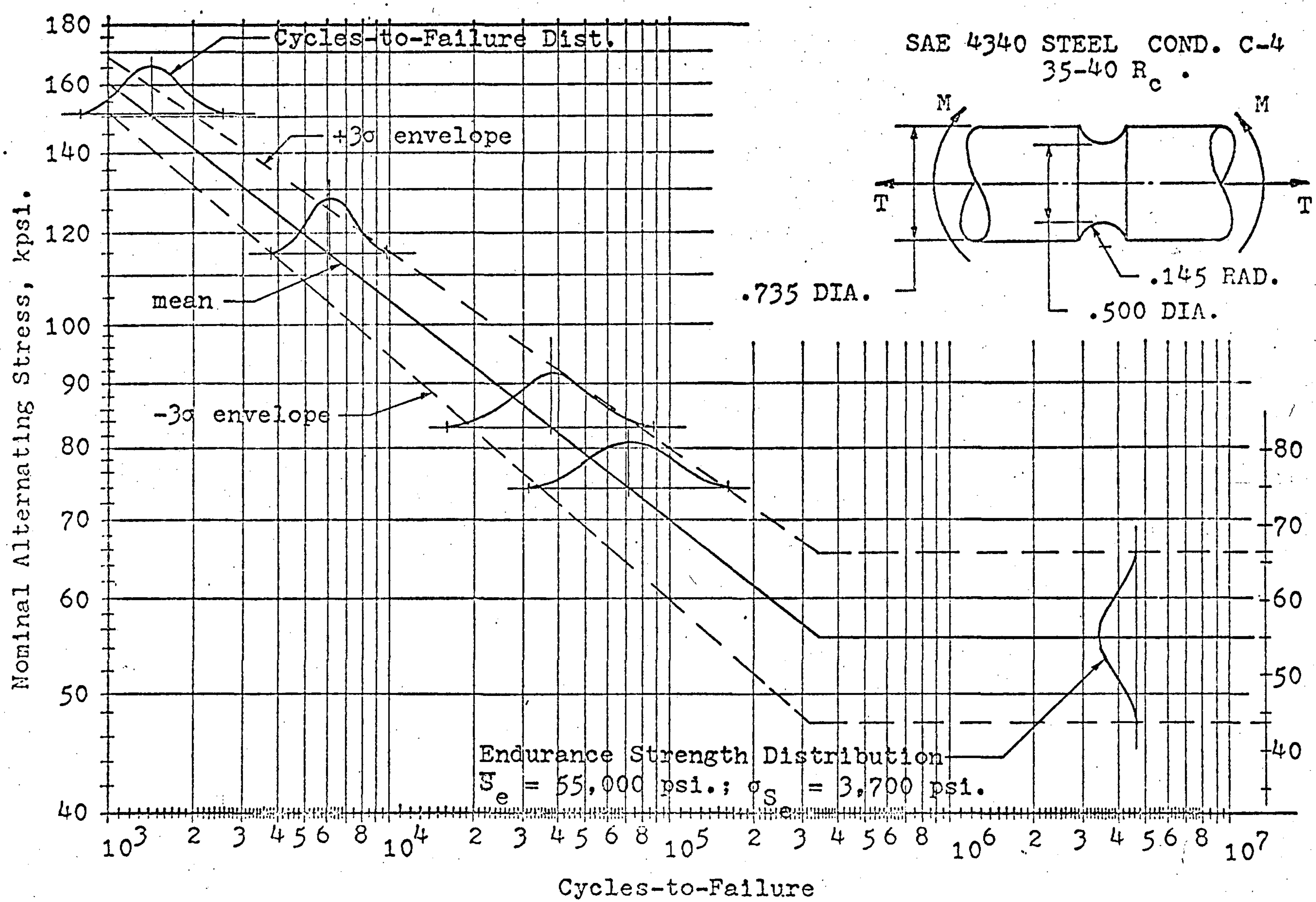

Fig. 2.7 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 3.5.

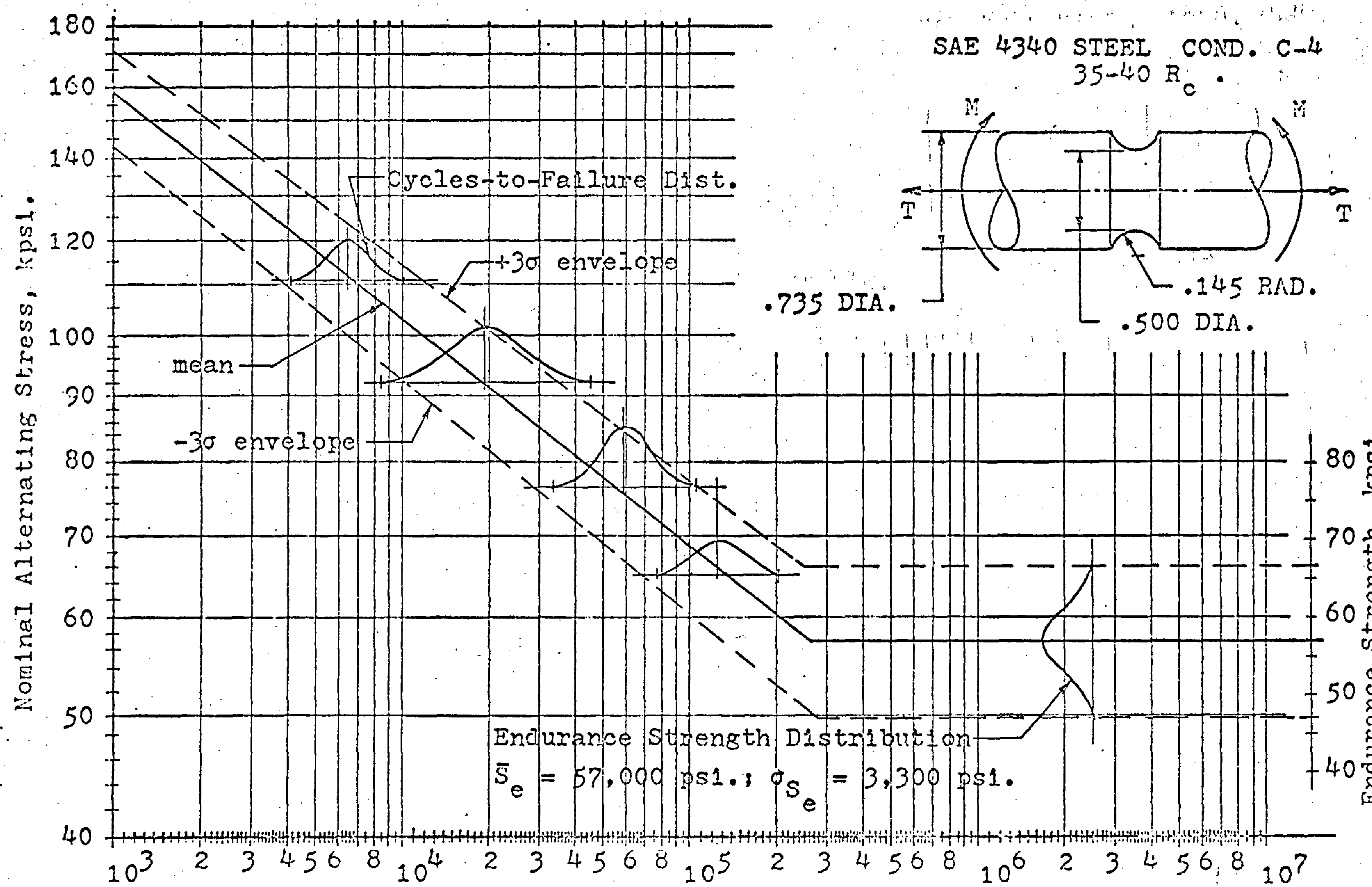

Fig. 2.8 Cycles-to-Failure Distributions and Endurance Strength Distribution for Stress Ratio of 0.825.

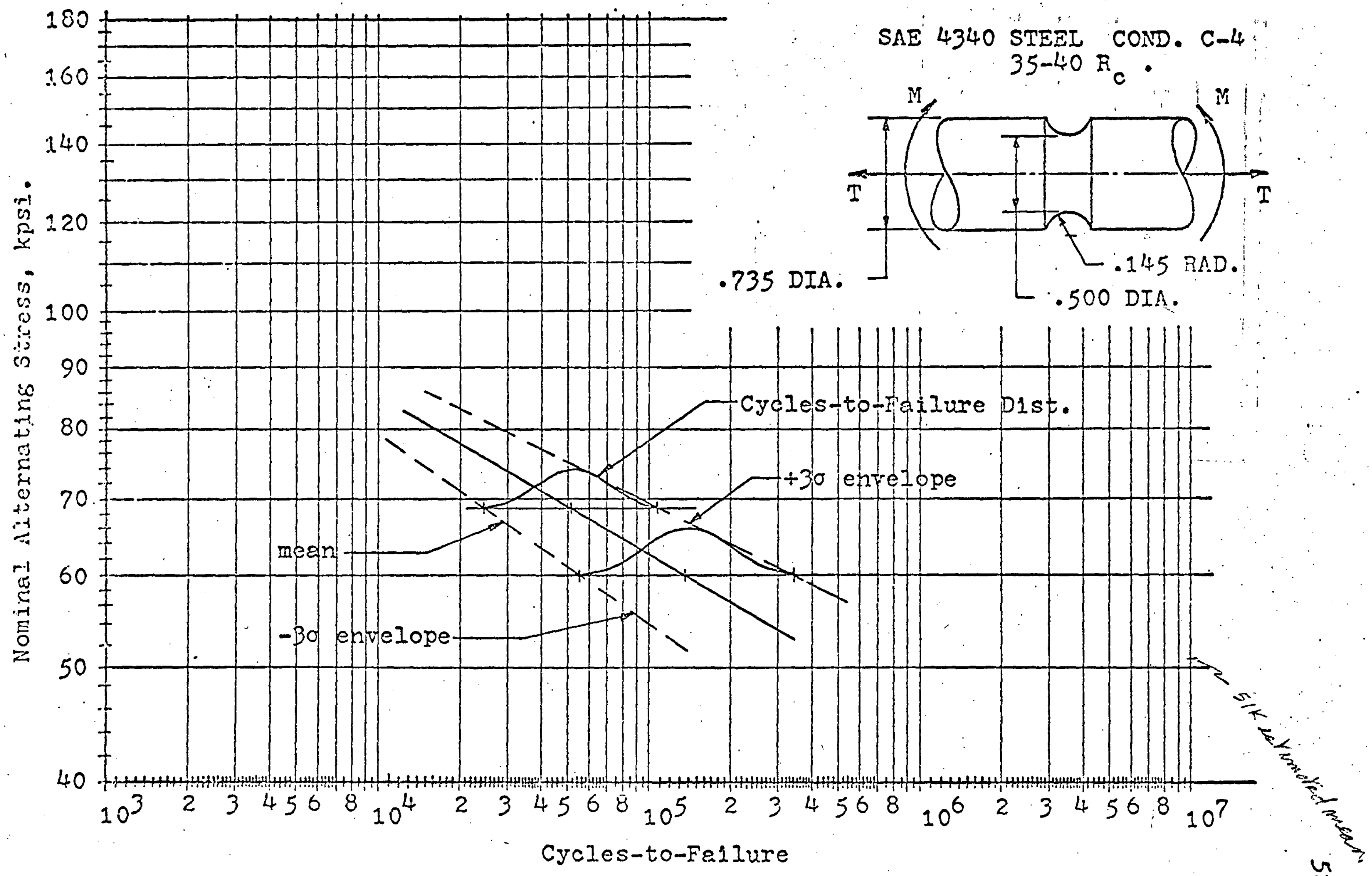

Fig. 2.9 Cycles-to-Failure Distributions at Stress Ratio of 0.44.

2.8 Construction of Statistical Fatigue Diagrams at Various Numbers of Cycles of Life

The ultimate objective of the research program is to construct statistical fatigue diagrams from which shafts can be designed for a specified cycle life and reliability.

One kind of conventional fatigue diagram is one using the modified Goodman line as shown in Fig. 2.10 .

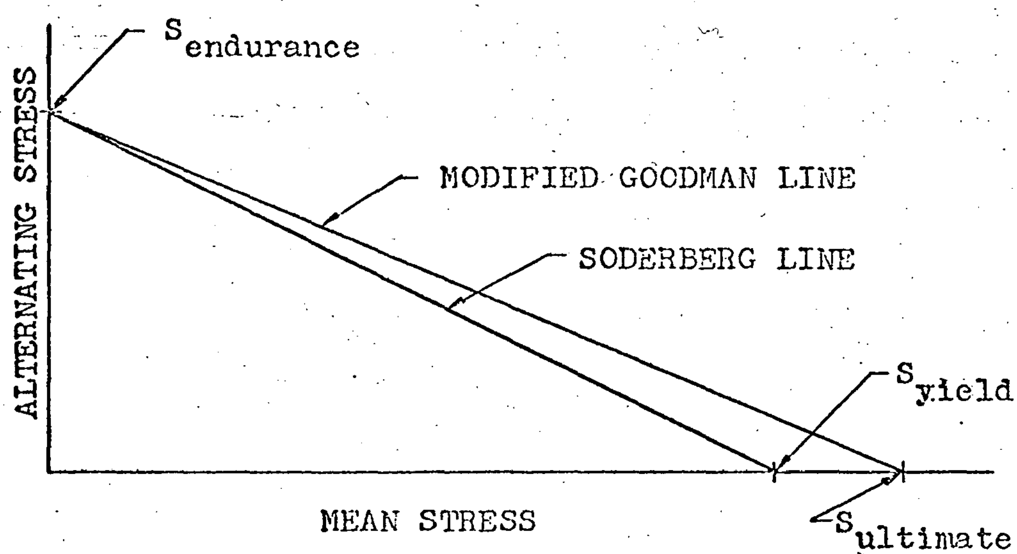

Fig. 2.10 Fatigue Diagram Showing the Modified Goodman Line and the Soderberg Line.

The diagram is a plot of alternating stress on the ordinate versus mean stress on the abscissa. The modified Goodman line is a line connecting the endurance strength and ultimate tensile strength. Another line, which connects the endurance strength and yield strength, is called the Soderberg line. Both the modified Goodman line and Soderberg line are conservative (2, p. 178).

The modified Goodman line is used to design for an infinite life. Other lines can be constructed connecting the ultimate strength to some alternating stress corresponding to a specified life. This line would be used to design for that finite life. However, these types of lines are deterministic and do not account for variability in the strength of a material. If the strength distributions can be determined for various stress ratios and cycles of life then fatigue diagrams can be defined in terms of a mean line and standard deviation about the line.

The method of determining the strength distributions which has been proposed for this research program has been discussed in earlier reports (1, pp. 134-135; 11, pp. 24-26). Briefly, once the mean line and 3σ envelopes (σ = standard deviation) have been established on the S-N diagrams as discussed in Section 2.7 then distribution parameters can be interpolated at evenly spaced stress levels. Suppose the strength distribution is desired at N cycles. A histogram can be constructed along the N cycle line such that the midpoints of each histogram cell is one of the interpolated stress levels, as shown in Fig. 2.11 . The ordinate of each cell is the area, to the left of N_1 , under the distribution curve corresponding to that cell. For example, denoting the bottom distribution function on Fig. 2.11 as $f(N|S_1)$ then the ordinate of the bottom histogram cell is

$$F(N|S_1) = \int_{-\infty}^{N_1} f(N|S_1)\, dN. \qquad (12)$$

Likewise, the ordinate of the next cell will be

$$F(N|S_2) = \int_{-\infty}^{N_1} f(N|S_2)\, dN. \qquad (13)$$

In general the ordinate of the i^{th} cell will be

$$F(N|S_i) = \int_{-\infty}^{N_1} f(N|S_i)\, dN \qquad (14)$$

and the total histogram will look like that shown in Fig. 2.11 . This total histogram, as a first approximation, is taken to be cumulative strength histogram of specimens failing by N_1 cycles. The probability density histogram can be obtained from this cumulative histogram. Denoting the strength random variable along the N_1 axis as S then the value of the i^{th} cell of the probability density histogram is $f(S_i) = F(N|S_i) - F(N|S_{i-1})$. After the probability density histogram is found then the normal distribution parameters can be estimated using statistical methods.

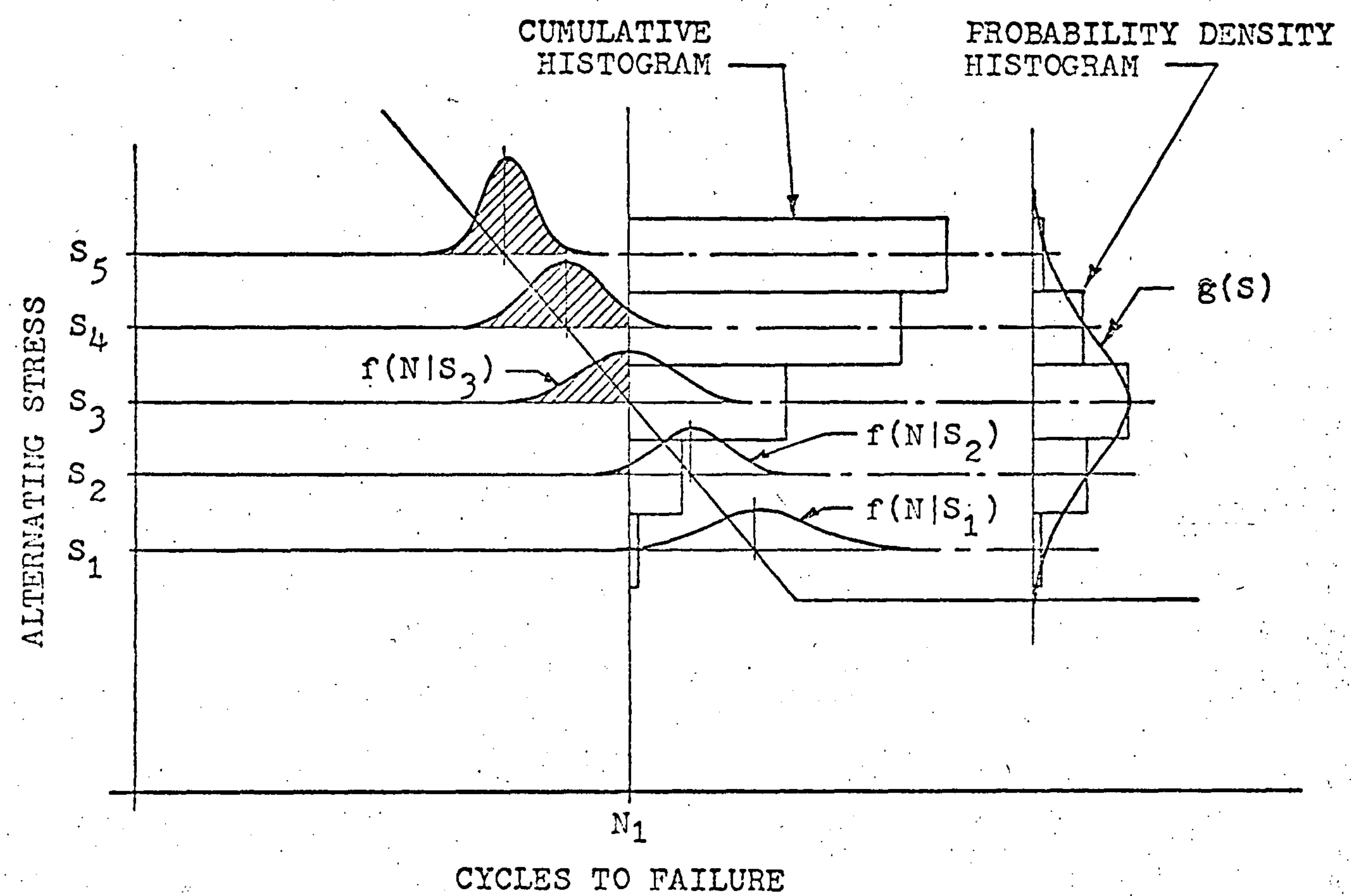

Fig. 2.11 Histogram Obtained From Cycles-to-Failure Distributions

A computer program was developed by R. E. Smith (11) using this method of estimating the strength distribution parameters. A printout of the program is given in Appendix F. The estimates of the strength distribution parameters at cycles of 10,000, 50,000 and 100,000 for a stress ratio of 3.5 are listed in Table 2.9 . These distributions are plotted on Fig. 2.12 . Note that the $\pm 3\sigma$ limits are inside the $\pm 3\sigma$ envelope for the cycles-to-failure distributions; hence, for this case the method appears to yield unconservative estimates of the strength distribution parameters. The reason for this will be investigated.

TABLE 2.9

ESTIMATED NORMAL PARAMETERS FOR STRENGTH DISTRIBUTIONS AT VARIOUS CYCLES-OF-LIFE

Cycles of Life N	Parameter Estimates of Normal Distribution	
	Mean (psi)	Standard Dev. (psi)
10,000	106,639	3,256
50,000	79,021	3,253
100,000	70,972	2,313

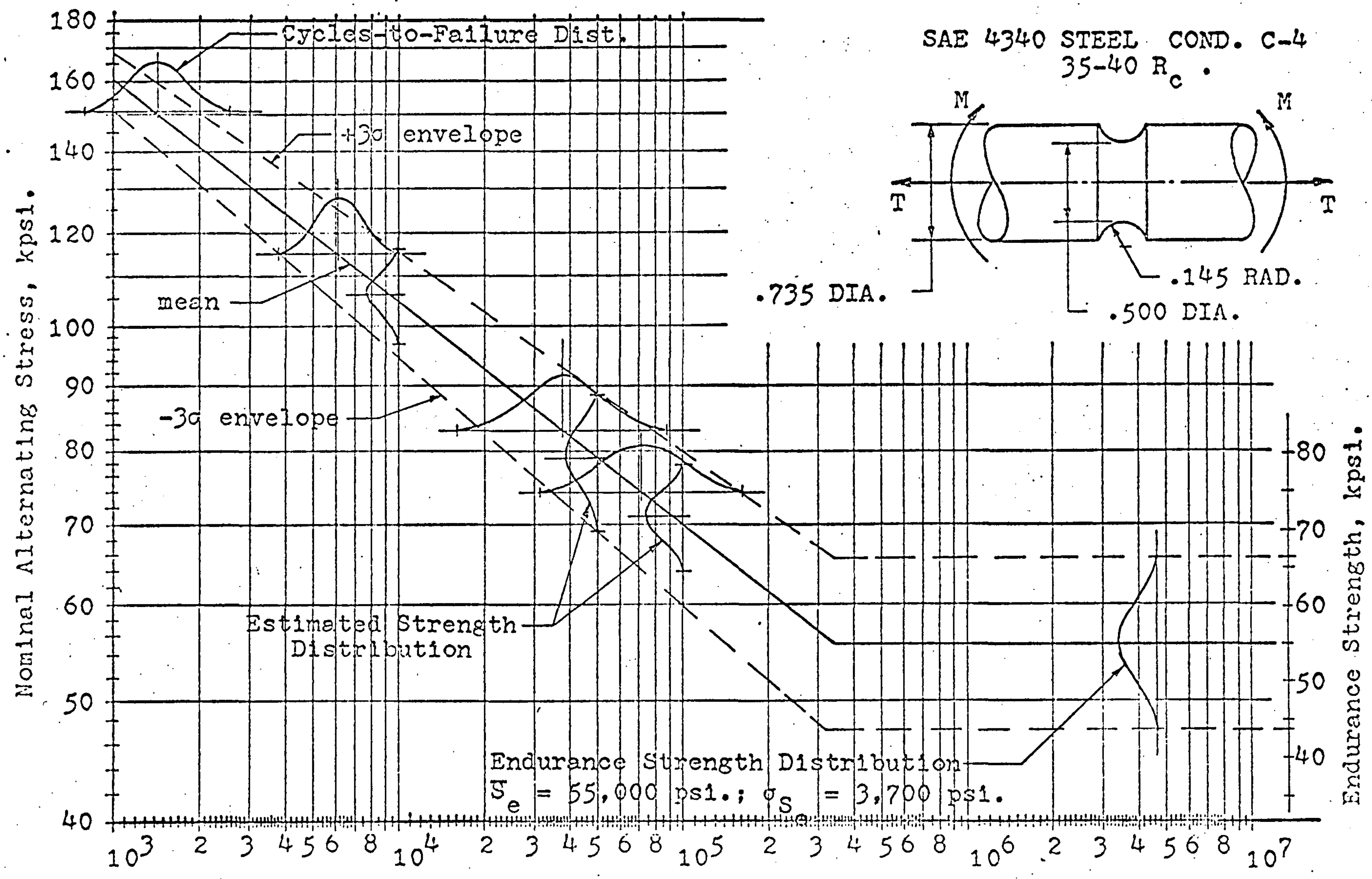

Fig. 2.12 Plot of Estimated Strength Distributions at Various Cycles of Life and Estimated Cycles-to-Failure Distributions at Various Stress Levels for Stress Ratio of 3.5

Another approach might be to base the strength distribution estimates on actual data so that some measure of accuracy can be obtained through the use of an appropriate goodness-of-fit test. The approximate distributions of strength at a specified number of cycles is wanted, then the distribution of the stresses to which the specimens failing at N cycles are subjected might be obtained as follows. N is a random variable and the probability of even one specimen failing at exactly N cycles is zero. Therefore, the strength distribution at a discrete N is unattainable, but a distribution of specimens failing within a band of N is possible to obtain.

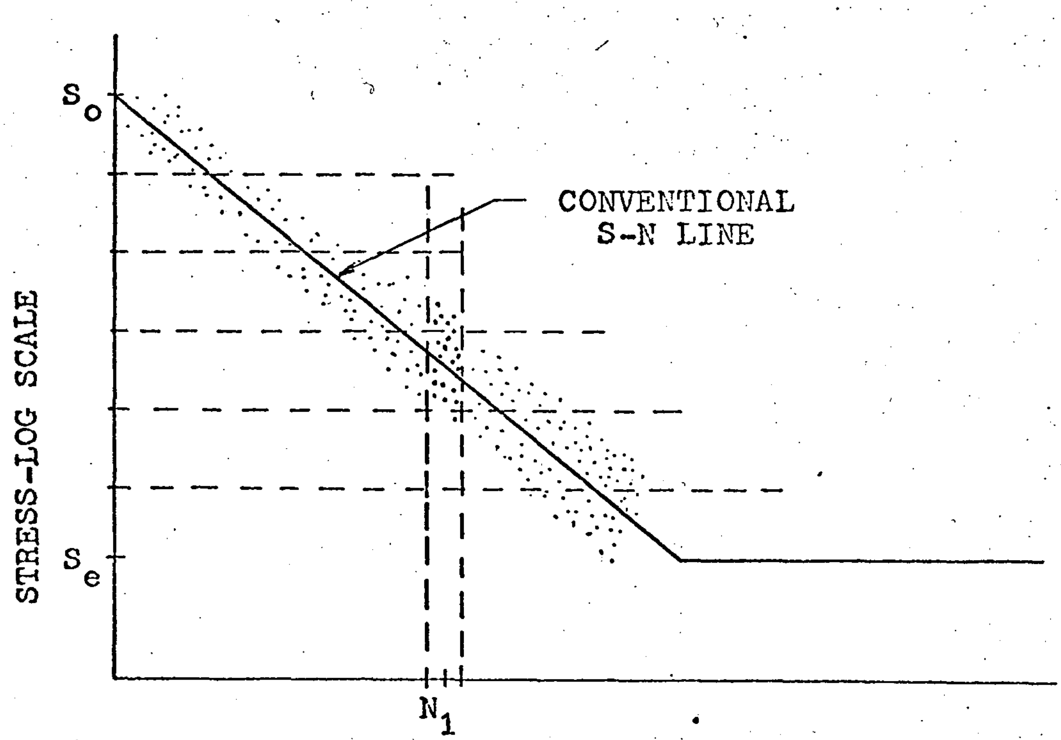

Fig. 2.13 S-N Diagram Showing Random Data Points and Cell Structure for Testing and Obtaining Distributions.

Suppose the strength distribution is desired at N_1 cycles. From existing data an estimate of the conventional S-N curve can be obtained and a range of stress corresponding to 10^3 cycles and the endurance limit can be obtained; S_o and S_e on Fig. 2.13 . Then specimens could be tested at random levels within this range. The stress range could be divided into cells, as shown in Fig. 2.13 and specimens tested at random levels within each cell. This would insure a more even distribution of data points.

Once the random data is obtained then the parameter estimates can be calculated and goodness-of-fit tests conducted. A cell with N_1 as its midpoint can be constructed as shown in Fig. 2.13 . For a conservative estimate the cell may be constructed so that N_1 is the lower boundary. The estimate of the mean strength at N_1 cycles would be the average of the stresses at which the specimens were tested that failed within the N_1 cell (the darkened data points on Fig. 2.13). The width of the cell would depend on how good an estimate was desired. Using these data points the estimates of any distribution parameters could be obtained and goodness-of-fit tests conducted to determine which distributions to reject. As an example, Fig. 2.14 is the S-N diagram for stress ratio of ∞ showing the estimated distributions of the cycles-to-failure data. Suppose it is desired to find the strength distribution for 70,000 cycles. If a cell of width 30,000 cycles is constructed on the conservative side of 70,000 cycles it passes through two distributions. The actual data points which fall within the cell can be obtained from Table 2.10 which lists the cycles to failure data at these two stress levels for stress ratio of ∞ . For stress level 73,000 psi. one point falls in the cell and for stress level 81,000 psi. 13 points fall in the cell for a total of 14 points. Calculating the mean and standard deviation of the strength for these points yields

$$\bar{S} = 80{,}516 \text{ psi. and } \sigma_S = 2{,}632 \text{ psi.}$$

Then, using the Kolmogorov-Smirnov goodness-of-fit test it can be determined whether or not the normal distribution is a good estimate for the strength distribution at 70,000 cycles.

The D statistic, as given by Eq. (10), is

$$D = |F_n(x) - S_n(x)|$$

where

$$F_n(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{z^2}{2}} dz$$

and

$$z_1 = \frac{S-\bar{S}}{\sigma_S}$$

Euy Engleman

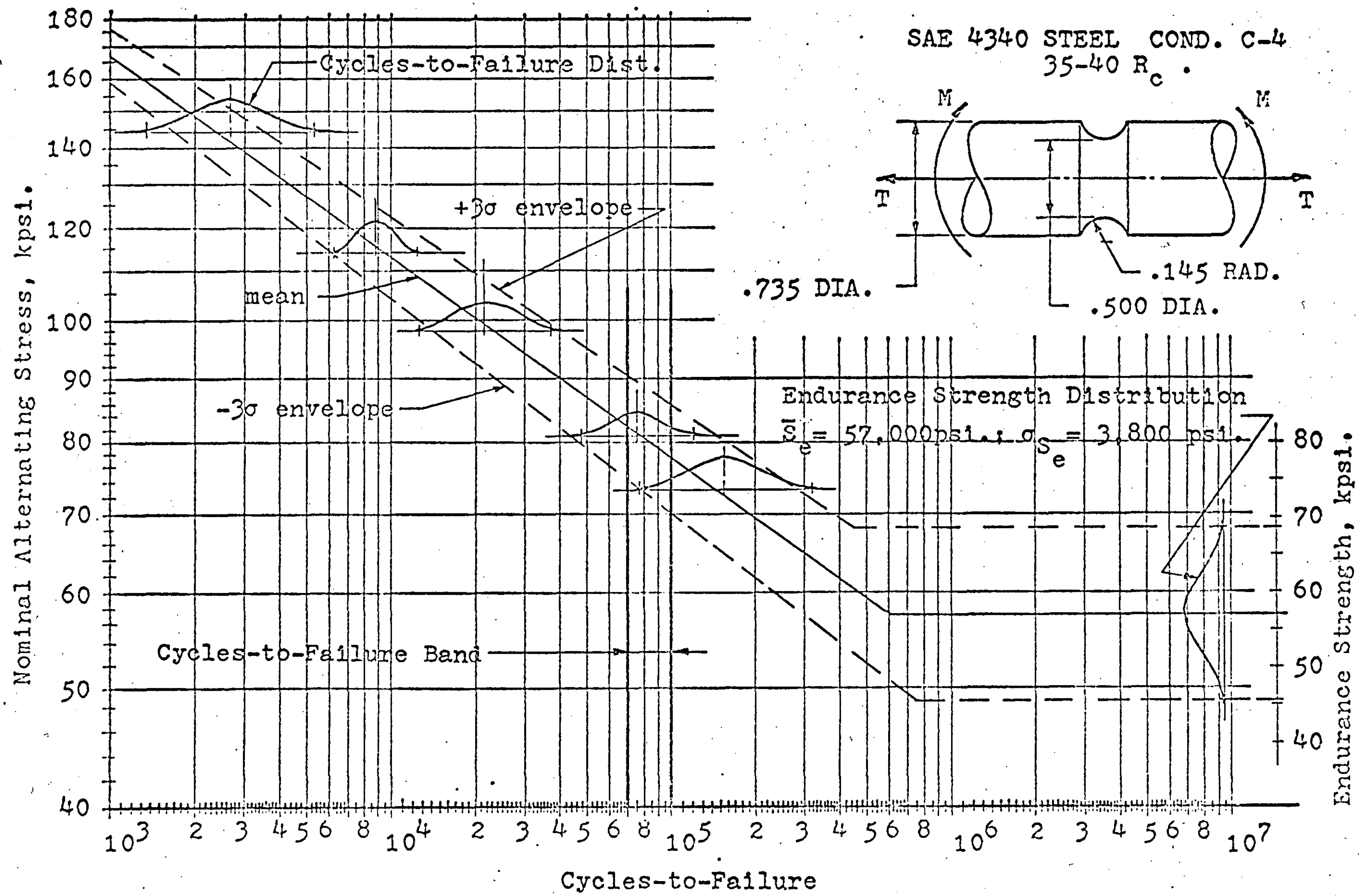

Fig. 2.14 S-N Diagram for Stress Ratio of ∞ Showing the Estimated Cycles-to-Failure Distributions and Test Band.

TABLE 2.10

CYCLES-TO-FAILURE AND STRESS DATA FOR STRESS RATIO OF ∞ AT STRESS LEVELS OF 73,000 PSI. AND 81,000 PSI.

Average Stress Level of 73,000 psi.		Average Stress Level of 81,000 psi.	
Actual Stress-psi.	Cycles-to-Failure	Actual Stress-psi.	Cycles-to-Failure
73,796	125,950	81,820	64,997
73,886	103,413	81,282	67,759
70,969	145,783	81,321	83,812*
72,029	195,824	80,421	78,852*
71,973	93,303*	82,641	60,002
74,342	178,192	81,027	64,857
73,404	196,894	80,667	71,968*
72,572	183,306	79,201	86,349*
74,931	177,508	81,832	82,248*
72,374	203,019	80,032	74,690*
72,925	155,178	80,871	75,662*
74,819	205,041	83,112	71,166*
75,982	172,661	82,001	71,556*
73,647	124,137	81,224	73,089*
72,961	127,794	81,423	96,004*
74,629	172,572	81,363	107,415
68,029	172,275	81,363	98,468*
72,164	182,860	81,784	74,698*

* Indicates Specimens Whose Cycles-to-Failure Fall Within the Range of 70,000 to 100,000 Cycles.

S is the value of the stress of the data point being examined and

$$S_n(x) = \frac{S'}{N}$$

where

S' = the number of points which have a stress less than or equal to S.

and

N = total number of points.

For example, using the last data point under the 81,000 psi. stress level column in Table 2.10;

S = 81,784 psi.

N = 14

Since 81,784 psi. is the second highest stress value of the data points there are 13 points less than or equal to it, so

$$S_n(x) = \frac{13}{14} = 0.929$$

The value of z is

$$z_1 = \frac{81,784-80,516}{2,632} = 0.482$$

From standard normal tables this value of z_1 corresponds to

$$F_n(x) = 0.620$$

Therefore

$$D = 0.309$$

From a table of critical D-values (which can be found in most statistics books which discuss the Kolmogorov-Smirnov test) it is seen that for a sample size of 14

$$P(D > 0.292) = 0.15$$

Thus, since the D obtained here is greater than 0.292 this normal distribution can be rejected with 85% confidence that the distribution does not fit the data, but can not be rejected with 90% confidence.

Past studies by others indicate that for a given cycle life the fatigue strength distribution is normal (12, p. 351). One reason the normal distribution can be rejected here with 85% confidence may be that the sample used to estimate the mean and standard deviation is not random and therefore the estimates may be biased. Definite stress levels were aimed for and although there is some scatter about those levels, the sample cannot be considered a random one. The properties of the estimators for the mean and standard deviation are known to be good if the sample is random. If the sample is not random the estimates may be biased or insufficient. It is for this reason, if this method is used, that the data can give little more than gross approximations of the strength distribution parameters. The present data might be useful if more tests were conducted randomly throughout the stress range to give an even distribution of points.

? This seems wrong.

It must be pointed out that a very large number of specimens may have to be tested to obtain enough specimens, so that preferably more than 35 fail within the narrow cycle life range desired for sufficient accuracy, particularly for strength distributions at lower cycles of life.

The staircase method should also be tried for finite life to see how the results from the three methods compare.

Once the random data is obtained and the strength distribution parameters have been calculated at the desired cycles of life, construction of the fatigue diagrams is relatively simple.

Assuming the stress ratio is held constant for the data points on each S-N diagram then the alternating strength distribution can be transformed to the mean strength distribution through the constant R. When either distribution and the stress ratio, R, is known then the other distribution is completely defined, so only the alternating strength distributions will be worked with.

The means and variances are related by

$$\bar{S}_a = R\,\bar{S}_m \quad \text{and} \quad \sigma_{S_a}{}^2 = R^2\,\sigma_{S_m}{}^2$$

where

$\bar{S}_a$ = mean of the alternating strength distribution

$\bar{S}_m$ = mean of the mean strength distribution

σ_{S_a} = standard deviation of the alternating strength distribution

σ_{S_m} = standard deviation of the mean strength distribution

As an example, a fatigue diagram using the endurance strength distributions calculated in Sections 2.2.1 and 2.2.2 will be constructed. The endurance strength distribution parameters are listed in Table 2.11 .

The distribution for stress ratio of 0 is the ultimate strength distribution for unnotched specimens as obtained from tensile tests. The details and results of these tests were reported in an earlier report (6) and therefore will not be discussed here. The results are presented in Tables 2.12 and 2.13 . Twenty specimens were tested. Ten were identical to those used in the fatigue tests (Fig. 1.3) and ten were unnotched specimens. The unnotched specimens exhibited a yield point, which was recorded, and the notched specimens did not have a noticeable yield point.

The reasons for using the ultimate strength distribution obtained from the unnotched specimens rather than that for the notched specimens is as follows. The mean or constant stress that the specimens are subjected to in the test program is a shear stress. However, a mean normal stress is desired for the mean stress axis of the fatigue diagram. If the specimens would have been tested to fracture in static torsion the shear stress distribution would have been converted to normal stress by the relationship $S_m = \sqrt{3}\,\tau$.

TABLE 2.11

ALTERNATING STRENGTH DISTRIBUTION PARAMETERS AND THE CORRESPONDING DISTRIBUTION PARAMETERS ALONG THE VARIOUS STRESS RATIO LINES FOR INFINITE LIFE

Stress Ratio R	Endurance Strength Distribution Parameters		Corresponding Dist. Parameters Along R Axis	
	Mean psi** $\bar{S}_a$	Standard Dev. psi*** σ_{S_a}	Mean psi** $\bar{S}_r$	Standard Dev. psi*** σ_{S_r}
∞	57,000	3,800	57,000	3,800
3.5	55,000	3,700	57,000	3,800
1.0	57,000	3,300	81,000	4,700
* 0	178,000	2,500	178,000	2,500

* These values are the distribution parameters of the ultimate strength of unnotched specimens obtained from tensile tests.

** Rounded off to nearest 1,000 psi.

*** Rounded off to nearest 100 psi.

TABLE 2.12

DATA AND RESULTS FROM STATIC TESTS ON NOTCHED SPECIMENS*

(Stress Ratio = 0)

Test No.	Ultimate Load 1,000 lbs.	Breaking Load 1,000 lbs.	Ultimate Strength psi. **	True Breaking Strength psi. **
1	49.3	47.0	253,500	305,000
2	49.6	47.0	255,000	305,000
3	49.4	46.3	254,000	299,500
4	50.3	47.4	259,000	299,500
5	48.8	46.0	251,000	306,500
6	49.2	46.0	253,000	302,500
7	49.6	46.8	255,000	304,500
8	49.8	47.1	256,000	305,500
9	50.5	47.7	260,000	309,500
10	49.9	47.5	256,500	302,000
		Mean	255,500	304,000
		Standard Deviation	2,500	3,000

* Average specimen diameter at the base of the notch is 0.4975 inches.

** All strengths rounded to nearest 500 psi.

TABLE 2.13

DATA AND RESULTS FROM STATIC TESTS ON UNNOTCHED SPECIMENS

(Stress Ratio = 0)

Test No.	Yield Load 1000 lb	Ultimate Load 1000 lb	Breaking Load 1000 lb	Diameter Average In.	Area Aver. In^2	Yield Strength psi*	Ultimate Strength psi*	True Breaking Strength psi*
1	31.5	32.5	24.3	0.4753	0.1774	177,500	183,000	264,000
2	30.6	31.5	23.5	0.4764	0.1784	171,500	176,500	254,000
3	30.6	31.6	23.3	0.4754	0.1775	172,500	178,000	249,000
4	20.8	31.0	22.8	0.4755	0.1776	168,000	174,500	251,500
5	29.9	31.1	22.9	0.4758	0.1778	168,000	175,000	254,500
6	30.1	31.3	23.2	0.4722	0.1752	172,000	178,500	256,500
7	30.4	32.5	25.0	0.4787	0.1800	169,000	181,000	256,000
8	30.2	31.4	24.2	0.4757	0.1777	170,000	178,000	253,000
9	30.6	31.6	23.6	0.4763	0.1782	172,000	177,500	259,000
10	30.3	31.2	24.2	0.4755	0.1766	171,000	176,500	250,500
				Mean		171,000	178,000	255,000
				Standard Deviation		3,000	2,500	4,500

* All Strengths Rounded to the Nearest 500 psi.

It is generally accepted that the static strength of ductile steel is not affected by a stress concentration. Therefore, theoretically, the same shear strength distribution should be obtained whether the specimens are notched or not.

However, the mean normal strength distribution was obtained directly from the tensile tests. In the case of tensile tests a notch does have an affect on the strength. The notched specimen has a higher ultimate strength due to a radial stress being introduced into the specimen at the root of the groove (6, p. 19). A grooved specimen subjected to a static torque load would not experience this radial stress. Therefore, the strength distribution used at stress ratio of 0 is the ultimate strength distribution for the unnotched specimens.

In order to construct the fatigue diagram the distributions along the various stress ratio lines must be calculated. The required relationships can be derived from Fig. 2.15 . They are:

$$\text{Tan}\ \theta = \frac{\bar{S}_a}{\bar{S}_m} = \bar{R} = \text{stress ratio}$$

$$\bar{S}_R = \frac{\bar{S}_a}{\text{Sin}\theta} \qquad (15)$$

but

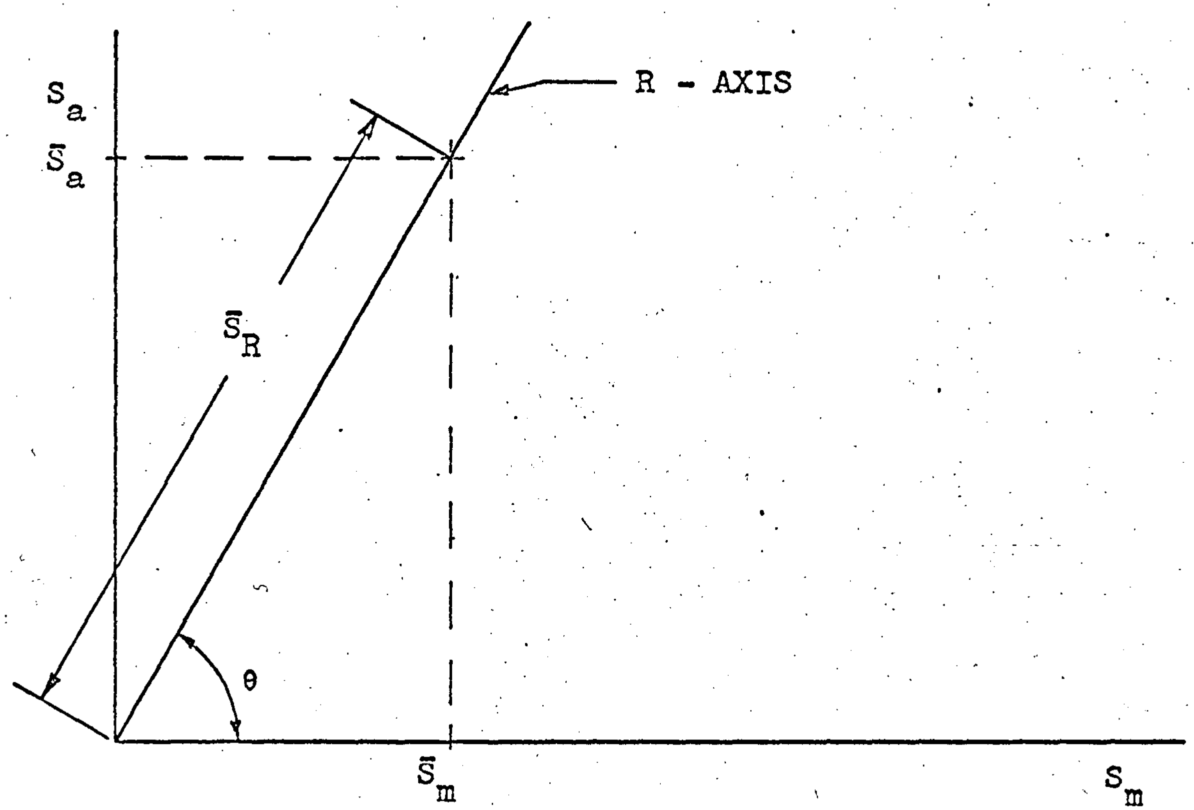

Fig. 2.15 Relationship Between the Stress Ratio Lines

$$\text{Sin } \theta = \frac{\text{Tan } \theta}{[1+(\text{Tan } \theta)^2]^{\frac{1}{2}}} = \frac{\bar{R}}{[1+\bar{R}^2]^{\frac{1}{2}}}$$

thus

$$\bar{S}_R = \frac{\bar{S}_a(1+\bar{R}^2)^{\frac{1}{2}}}{\bar{R}} \tag{16}$$

Using the same derivation the expressions for the standard deviation along the R axis are

$$\sigma_{S_R} = \frac{\sigma_{S_a}}{\text{Sin}\theta} \tag{17}$$

and

$$\sigma_{S_R} = \frac{\sigma_{S_a}(1+R^2)^{\frac{1}{2}}}{R} \tag{18}$$

Referring to Fig. 2.16 the distribution along the alternating stress axis ($R = \infty$) can be plotted directly. For this axis $\theta = 90^\circ$ and Eq. (15) yields $\bar{S}_{R,\infty} = \bar{S}_{a,\infty}$ and Eq. (17) yields $\sigma_{S_R,\infty} = \sigma_{S_a,\infty}$.

For the distribution along the stress ratio line 3.5 Eqs. (16) and (18) give

$$\bar{S}_{R,3.5} = \frac{\bar{S}_{a,3.5}(1+R^2)^{\frac{1}{2}}}{R}$$

$$= \frac{55,000[1+(3.5)^2]^{\frac{1}{2}}}{3.5} = 57,201 \text{ psi.}$$

for the mean and

$$\sigma_{S_R,3.5} = \frac{\sigma_{S_a,3.5}(1+R^2)^{\frac{1}{2}}}{R}$$

$$= \frac{3,700\ [1+(3.5)^2]^{\frac{1}{2}}}{R} = 3,848 \text{ psi.}$$

for the standard deviation.

The same procedure would be used along any other stress ratio axis except the appropriate values of S_a and σ_{s_a} would be used. The results of the calculations for the parameters for endurance are listed in Table 2.11 and the resulting fatigue diagram is Fig. 2.16 . The endurance tests for R = 0.44 have not yet been completed but from the tests that have been run it appears as though the endurance level will be in the range of 49,000 to 53,000 psi. and the scatter will be wide. The actual distribution will probably be very close to what the distribution shown on Fig. 2.16, in dashed lines, looks like.

It probably will not be possible to conduct tests at ratios much lower than 0.44, especially at high stress levels. The stress ratio is $R = S_a / S_m$ and the lower limit on S_a is the endurance level. The upper limit on S_m is the yield strength which is 171,000 psi. Thus the lower limit on stress ratio, assuming the endurance does not fall much below 51,000 psi. is

$$R = \frac{51,000}{171,000} = 0.28$$

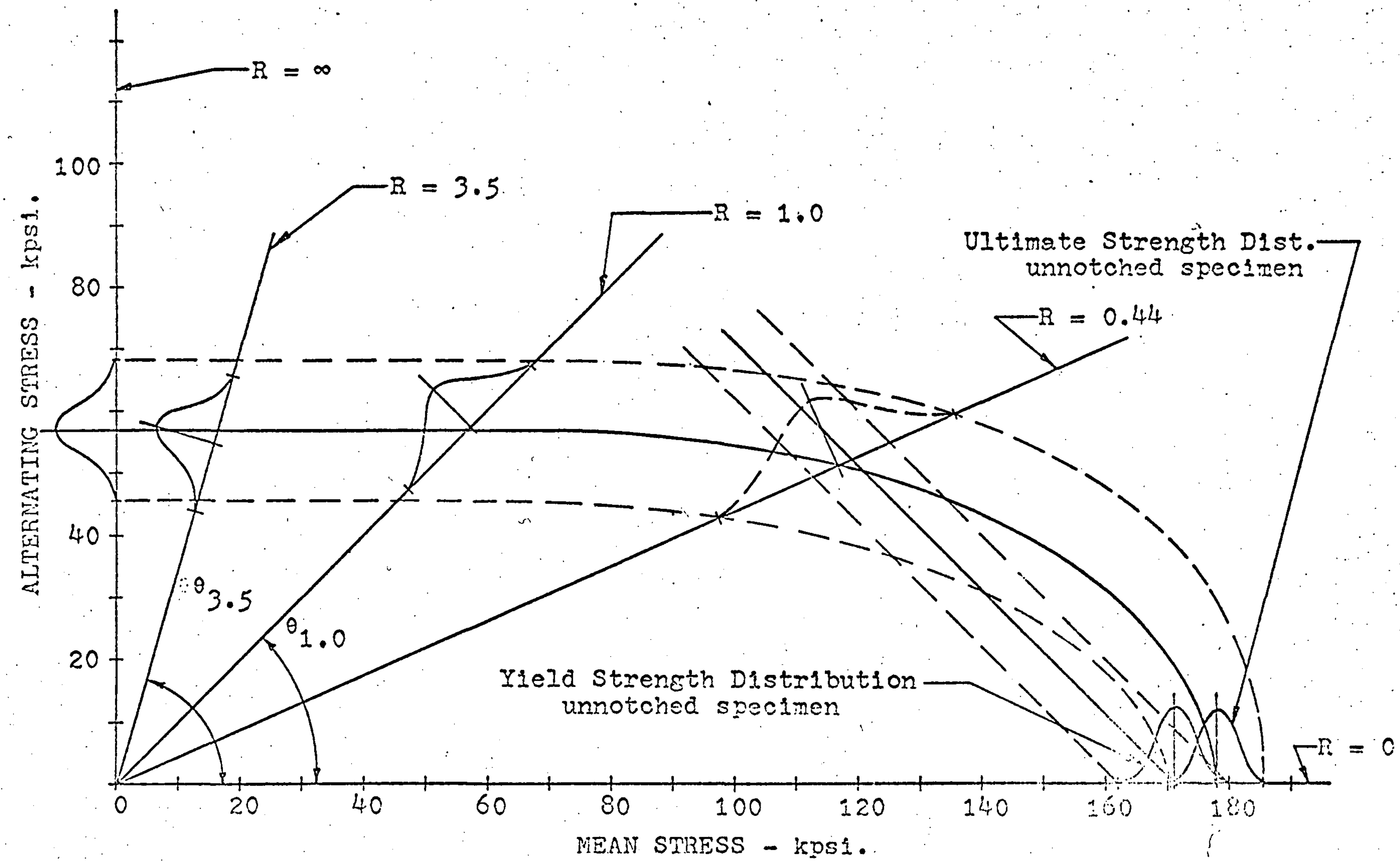

Fig. 2.16 Statistical Fatigue Diagram For Infinite Life.

This may not be a very good assumption since the fatigue diagram shows that it does fall off rapidly at low stress ratios. At these low ratios yielding will be the predominant failure mode. Unless the capability exists to monitor the torque on the specimen continuously, it will not be known whether or not the initial torque load was constant throughout the test.

2.8.1 Use of Statistical Fatigue Diagrams in Design

The conventional use of fatigue diagrams is to determine what combinations of mean and alternating stress are safe for a given number of cycles of operation. For example, in Fig. 2.17 the stress combination of S_{a_1} and S_{m_1} would be considered safe for a design life of 10^5 cycles but not for infinite life. The combination of S_{a_1} and S_{m_2} is not safe for 10^5 cycles of life. The question arises, "With what confidence can one say that a combination is safe?" To answer this question the distributions of the alternating and mean stresses to which the component in question is subjected and the distribution of the limiting fatigue boundary must be considered. A method of obtaining the distribution of the fatigue boundary at various cycles of life has been previously discussed.

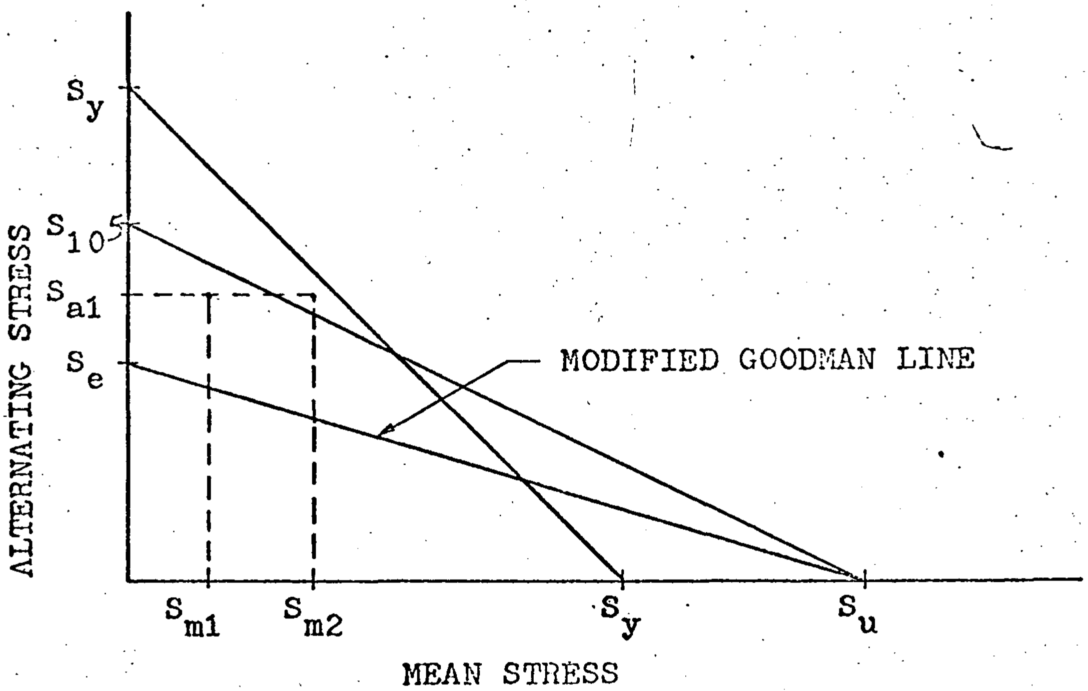

Fig. 2.17 Conventional Fatigue Diagram Showing the Modified Goodman Line

If the distributions of the alternating and mean stresses to which a shaft is subjected are known or can be estimated then they can be plotted on the fatigue diagram as shown in Fig. 2.18 . If the shaft stress distributions are normal and the stress ratio can be assumed to be constant then the distribution along the stress ratio axis (R_2 axis in Fig. 2.18) can be obtained from the relationships

$$\bar{s}_{R_2} = \frac{s_a}{\text{Sin } \theta_2} \quad \text{and} \quad \sigma_{s_{R_2}} = \frac{\sigma_{s_a}}{\text{Sin } \theta_2}$$

where

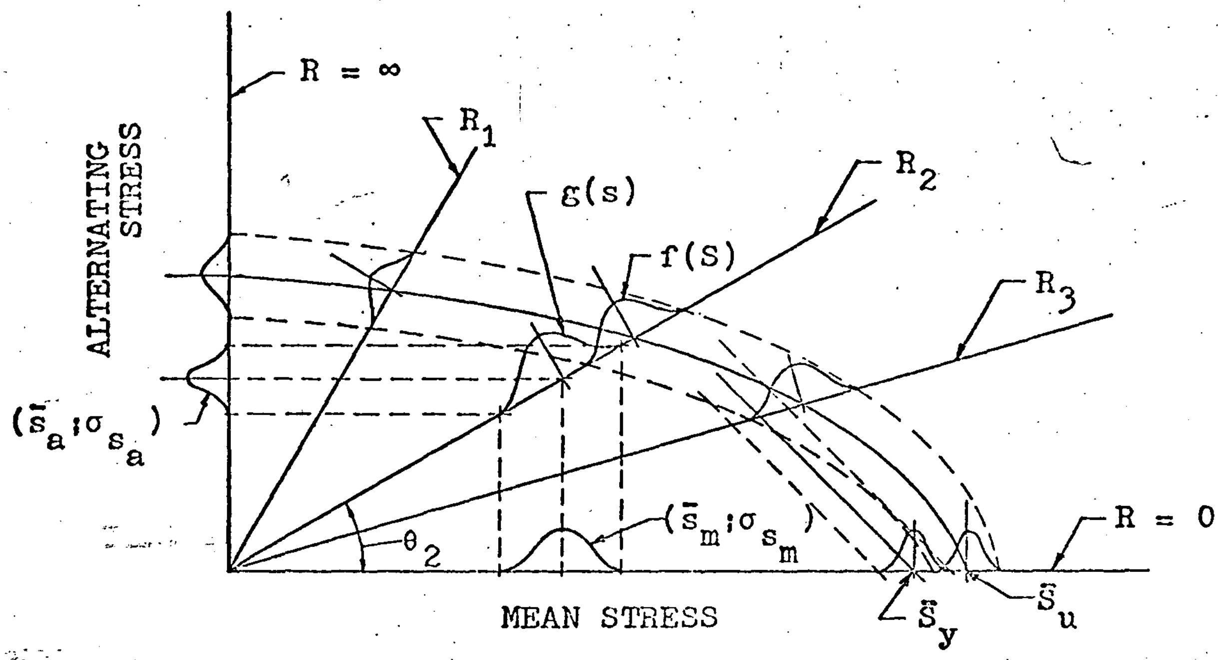

Fig. 2.18 Statistical Fatigue Diagram

$\bar{s}_{R_2}$ = mean of the stress distribution along the R_2 axis.

$\sigma_{s_{R_2}}$ = standard deviation of the stress distribution along the R_2 axis.

$\bar{s}_a$ = mean of the alternating stress of the shaft.

σ_{s_a} = standard deviation of the alternating stress of the shaft.

Figure 2.19 shows the stress and strength distributions along the R_2 axis. Once the parameters of the distributions are known the probability of failure can be calculated. Defining the random variable Z as

$$Z = S - s$$

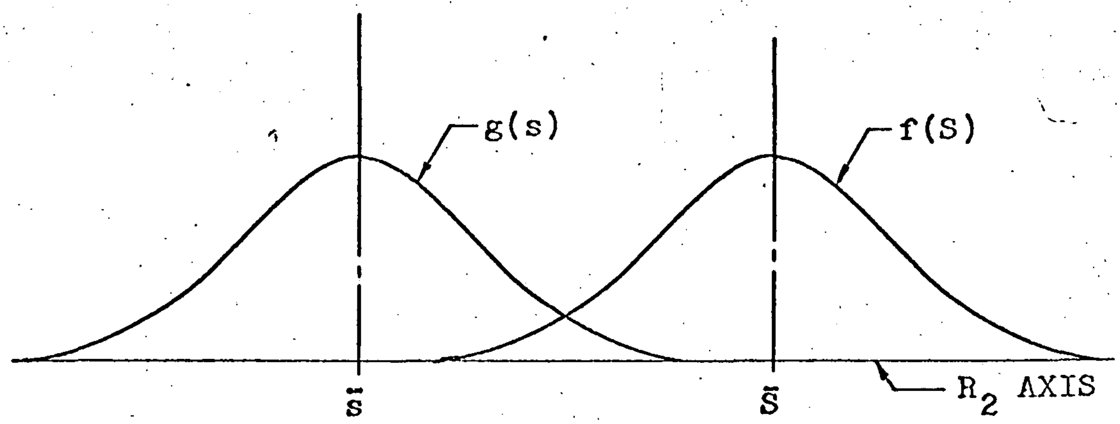

Fig. 2.19 Enlargement of Stress and Strength Distributions on R_2 Axis of Fig. 2.18 .

where

S = the random variable, strength.

s = the random variable, stress.

then the mean and standard deviation of Z can be shown to be normally distributed with parameters (13, p. 113)

$$\bar{Z} = \bar{S} - \bar{s} \tag{19}$$

$$\sigma_z = \sqrt{\sigma_S^2 + \sigma_s^2} \tag{20}$$

Failure occurs when $s > S$ or when $S - s < 0$. In Fig. 2.20 the shaded area represents the probability of failure. Stated in mathematical terms

$$P(Z < 0) = \int_{-\infty}^{0} \frac{1}{\sigma_z\sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma_z}\right)^2} dz$$

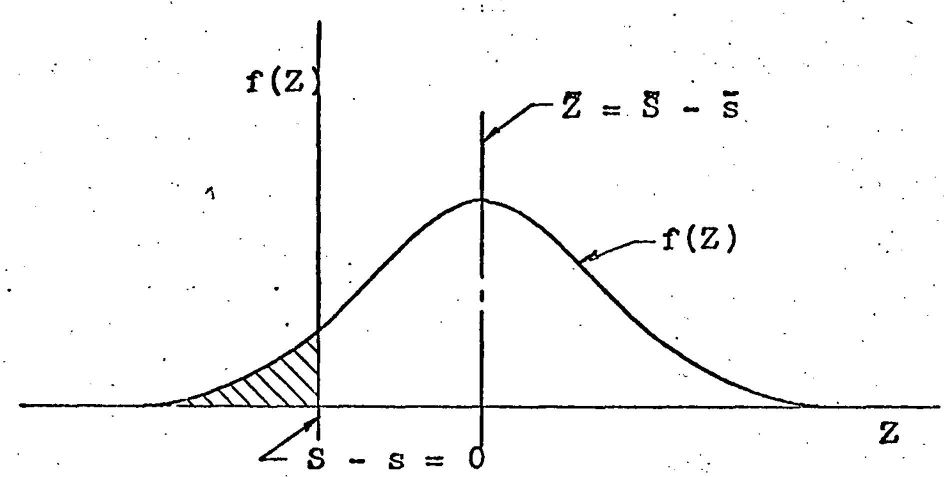

Fig. 2.20 The Distribution of Z = S - s.

Putting into the standard normal from by letting

$$x_1 = \frac{0-\bar{Z}}{\sigma_Z} = - \frac{\bar{S} - \bar{s}}{\sqrt{\sigma_S^2 + \sigma_s^2}} \qquad (21)$$

then

$$P(X < x_1) = \int_{-\infty}^{x_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2} dx$$

For a known x_1 the value of $P(X < x_1)$, which is the probability of failure, can be obtained from standard normal distribution area tables. The reliability R_e is defined as

$$R_e = 1 - P(\text{failure}) = 1 - P(X < x_1)$$

which can be expressed as

$$R_e = \int_{x_1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} x^2} dx \qquad (22)$$

Thus the reliability of the shaft can be determined.

In most cases the designer must start with a level of reliability and determine the shaft diameter such that for the imposed loads the specified reliability will be met. The procedure in this case is to work backwards through the preceding derivation. The alternating stress in the shaft can be expressed in terms of the diameter by

$$s = \frac{MD}{2I} = \frac{32M}{\pi D^3} \quad (23)$$

where

M = the bending moment on the section in question.

D = diameter of the shaft.

$I = \frac{\pi D^4}{64}$ = moment of inertia of the shaft cross section.

The standard deviation of the stress can be expressed in terms of the mean and the standard deviation of the diameter of the shaft. Using the approximate partial derivative method (13, p. 90) for the standard deviation of the alternating stress yields

$$\sigma_s \approx \left\{ \left[\frac{32}{\pi D^3} \sigma_M\right]^2 + \left[- \frac{96}{\pi} \frac{M}{D^4} \sigma_D \right]^2 \right\}^{\frac{1}{2}} \quad (24)$$

The expression obtained using the approximate partial derivative method may not be valid in certain cases.

For example, in the case of a product Z = XY the standard deviation of Z is given by (13, p. 123)

$$\sigma_Z = \sqrt{(\bar{X}\ \sigma_Y)^2 + (\bar{Y}\ \sigma_X)^2 + (\sigma_X \sigma_Y)^2} \qquad (25)$$

whereas the expression given by the partial derivative method is

$$\sigma_Z = \sqrt{(X\ \sigma_y)^2 + (Y\ \sigma_x)^2} \qquad (26)$$

When the term $(\sigma_X \sigma_Y)^2$ is small compared to other terms on the right side of Eq. (25) then the approximation will yield good answers. If the values of σ_X and σ_Y differ from each other by 3 or 4 orders of magnitude then the term is no longer negligible. Then exact expressions for the standard deviations must be used. They can be found in Chapter 3 of Ref. 13.

The standard deviation of the shaft diameter is a function of the tolerance on the diameter. Assuming the distribution of the shaft diameter is normal then 99.73% of the shaft diameters will fall within $\pm\ 3\ \sigma_D$. Denoting the shaft diameter as $D\ ^{\pm t}$, where in most cases the tolerance 2t is known, then $3\sigma_D = t$.

The mean and standard deviation of the moment can be determined from the loading distribution. The only unknown in equations 21, 23, and 24 is D the diameter of the shaft and it can be solved for.

This, then, is a method of designing a rotating shaft, subjected to a bending moment and a torque, for a specified number of cycles of life with a predetermined reliability.

However it is important to recall the limitations and assumptions.

1. The amount of data used in estimating the distribution parameters can have a substantial effect on the shape of the distribution. The most pronounced effect is in the tails. Since the calculation of reliability is based on the overlap of the tails of two distributions it is necesary to have good estimates of the distributions in order to obtain accurate values of reliability.

2. The tests were conducted on specimens of given geometry, hardness, and material at room temperature. The fatigue surfaces generated from these tests are valid only for this type of specimen, environment and loading. The loading was a bending moment and a torque causing an alternating normal stress acting parallel to the centerline of the shaft and a constant shear stress acting perpendicular to the centerline.

3. It was assumed that along any given stress ratio axis the strength distribution is normal.
4. It was also assumed that the ratio of the alternating and mean stresses, to which the shaft being designed is subjected, is constant.

Obviously, it would be an almost impossible task to generate statistical fatigue surfaces to cover every type of material and configuration. However, it is feasible that enough tests could be conducted to allow empirical equations to be developed relating different stress concentration factors and conventional physical properties for the more common steels. This would make it possible to design from a diagram which was not generated for that particular configuration and material.

2.9 Modification of One Test Machine

This section will discuss some of the problems encountered in working with the test machines and the measures taken to overcome some of the problems. Two of the most troublesome areas are the instrumentation and the gear couplings. The gear couplings are the means by which the toolholder arms are allowed to swing downward slightly when the bending load is applied. Referring to Fig. 1.1 it is seen that the bending load configuration also applies a shear load to the couplings. However, the couplings were not designed to withstand radial shear loading; their purpose being to transmit torsional loads only. The couplings consist of three major parts. Two of the parts are pressed onto the shafts that are being coupled and have an external gear. The third part is an internal gear sleeve which slides over with the other two parts with the gears meshing as shown in Fig. 2.21 .

A radial shear load causes the sleeve to cock allowing the centerline of one shaft to fall below the centerline of the other. This misalignment, along with the angular deflection downward caused by the bending load, causes varying degrees of vibration in the toolholders and specimen. When the gear couplings are new the misalignment is small and there is very little vibration, but as the teeth in the couplings wear the misalignment and vibration become greater.

Note, also that there is no definite pivot point in the couplings, thus the length of the moment arm between the loading bearing and the pivot point can change while the machine is running causing a change in stress applied to the specimen. However, from observing tests in progress it appears as though the magnitude of the alternating stress remains quite constant for runs of up to about two hours. This investigator has never monitored a run longer than about two hours. The reason for this is that it is necessary

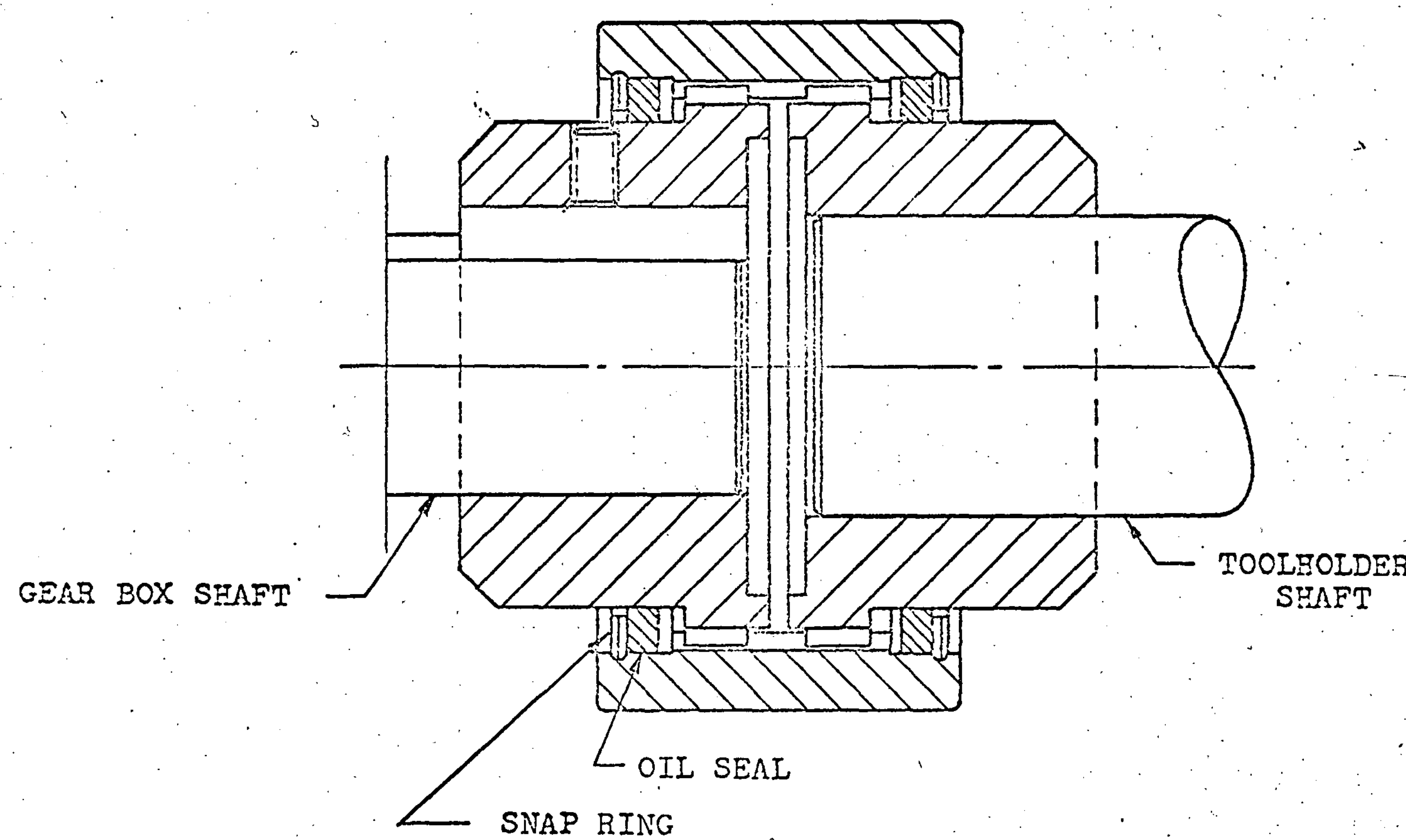

Fig. 2.21 Gear Coupling Used on Fatigue Testing Machine.

to have the contact points off the slip rings when the specimen breaks. Just prior to the specimen breaking, it and the toolholder arms vibrate quite a bit. If the points are down on the sliprings when this happens they can become damaged or broken. Usually the endurance tests are monitored for about 20 minutes.

Another problem is that there is a difference in the number of divisions, as recorded by the Visicorder, between the static and dynamic outputs. In other words, after the specimen is loaded, the machine is rotated by hand and the Visicorder trace is observed. Then the loads are adjusted until the desired output is attained. However, when the machine is rotated under power the output is a different amount, sometimes as much as four divisions. This can represent as high as 8,000 psi. depending upon the gain setting of the amplifier. The change can be easily corrected for the bending stress simply by changing the weight on the loading arm. This can be done while the machine is running. To make the correction for torque load the machine must be shut down and the torque coupling adjusted. However, the amount of change between static and dynamic outputs is not constant. After adjusting the torque to allow for the change, when the machine is restarted the change may be a different amount so the machine must again be shut down and readjusted.

There is another complication entering the problem. After about 30 seconds of running time the torque trace begins to drift quite a bit. The amount of drift depends upon which machine is being used and the torque load applied to the specimen. The higher the torque load the less the drift. Tests were made to determine if the torque load was changing or if the drift was due to amplifier drift. Several factors led to the conclusion that the drift is caused by drift in the amplifier; one being the fact that the torque output drifts upward in two machines. It is hard to believe that if the torque were changing, it would become greater. The tests seemed to indicate that the zero datum point was drifting. However, it drifted only when the machine was running. The drift, when the machine was not running would amount to one division, at the most, in about 2 hours whereas the drift amounted to as high as 3 divisions in 2 minutes with the machine running. One result of the drift is that if, when adjusting the torque load, the proper number of divisions is not obtained in one or two tries the torque will start to drift and no accurate measure of the amount of torque on the specimen can be obtained. When this happens the loads must be removed from the specimen, one holding collet must be loosened and the amplifier recalibrated. This is very time consuming, especially if this procedure must

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be gone through two or three times.

The drift problem could probably be eliminated by obtaining solid state amplification equipment. The present equipment has been in use since the start of the research program and is not solid state. Obtaining solid state equipment is planned for the near future.

It is not quite clear what is causing the difference between the static and dynamic outputs. For the bending stress it seems feasible that such a difference may be caused by the difference in the rate of elastic deformation of the toolholder. When the machine is rotated by hand the rate of deformation is slow but under power the machine rotates at about 1,750 rpm and the deformation of an element on the surface of the toolholder may lag behind the applied load. Using this line of reasoning it would seem that as the peak load is reached, if the corresponding deformation is lagging, then as the load begins to decrease the deformation will continue to increase only to that point where the deformation corresponds to the decreasing load. In other words, the toolholder never reaches the amount of deformation corresponding to the peak load. This reasoning would mean that the divisions of Visicorder output should be greater for the static case than for the dynamic case. However, the opposite is true. In all cases, where a difference in the bending output has been experienced, the bending gage output has been greater with the machine running under power than when turned by hand.

The above reasoning cannot explain the difference in outputs in the torque gage bridge either, because the torque load is not alternating but is constant.

The torque gage bridge output for the static case has been greater than for the dynamic case in most tests. This can be easily explained. Torque is exerted on the specimen through the torque coupling on the backshaft (Fig. 1.1). The torque is transmitted from the backshaft to the toolholder shafts through the gear boxes. When torque is applied through the torque coupling the rate at which it is applied is very slow. When torque is applied the play in the gearboxes is taken up along with any other play in the system. Then when the machine is started the initial impact causes more play to be taken up and the torque decreases. At very high torque levels all the play is taken up in the initial application of the torque. The initial starting impact causes very little decrease in torque gage bridge output, but the lower the torque level the greater is the amount of decrease. For the lower levels it is more difficult to take the decrease into account by setting the torque higher because at any given torque level the amount of decrease from specimen to specimen is not the same. Several trials enable one to determine the change and set the divisions accordingly.

Late in November, 1969 one machine was partially dismantled for the purpose of modifying the gear couplings. The modification consisted of placing a spherical bearing inside of each of the two couplings. The bearings do several things to eliminate some of the problems. They absorb the radial shear load and create a definite point about which the toolholder arms can pivot. The bearings have not been in use long enough to determine whether or not the rate of gear tooth wear will decrease in the couplings, but it is believed that it will. Since there is now a definite pivot point, even if there is excessive tooth wear the outer ring will not be able to misalign and cause vibration. In Fig. 2.22 note that the inner diameter of the spherical bearing has a sliding fit with the shaft through it. This is true of both bearings. This eliminates the possibility of subjecting the specimen to an axial load when tightening the specimen in the collets.

Since the spherical bearings have been installed, about 34 endurance tests at a stress ratio of 0.44 have been run on the modified machine. No vibrational problems have been encountered and it appears as though the difference between the static and dynamic outputs is minimal.

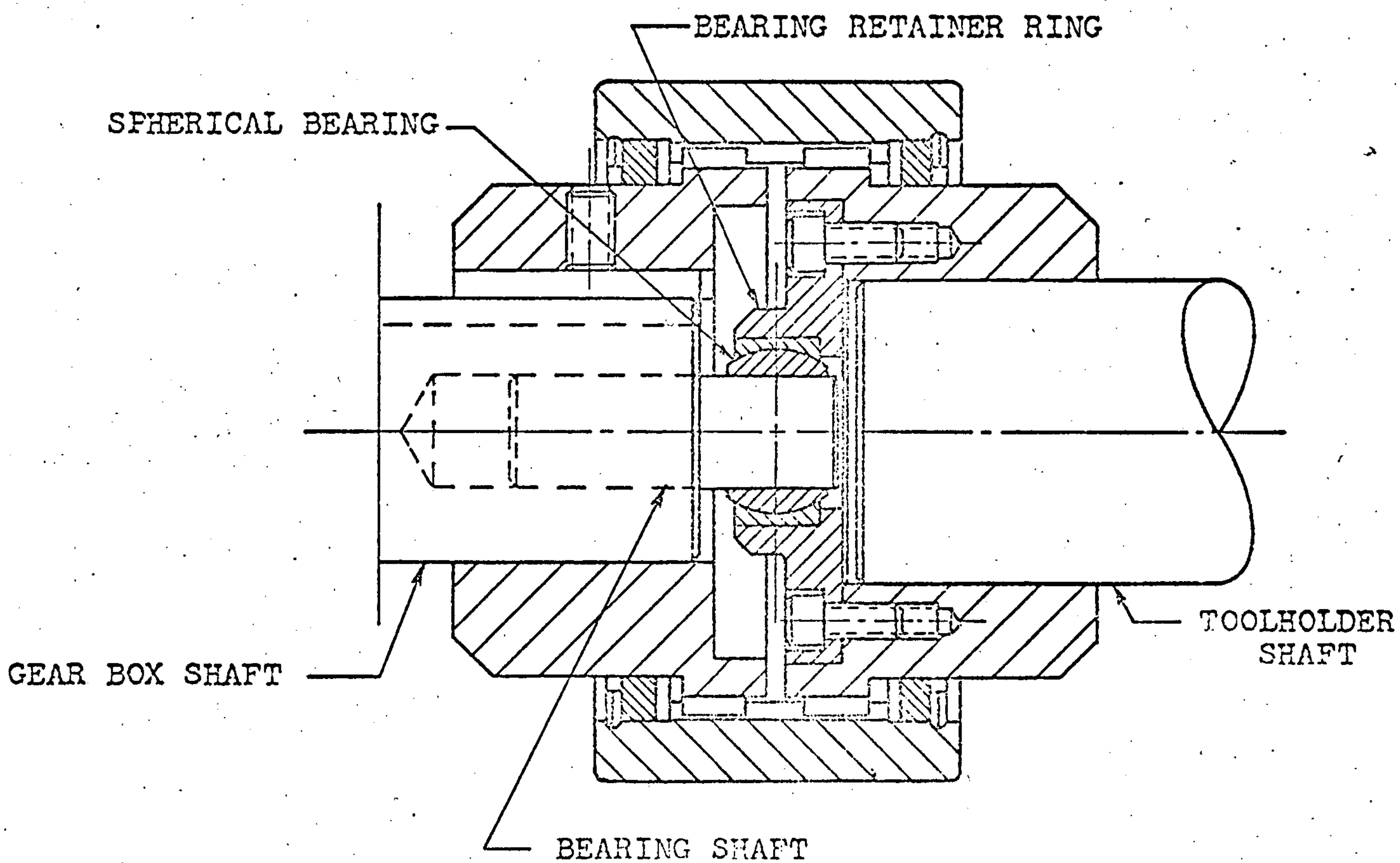

Fig. 2.22 Modified Gear Coupling.

CHAPTER III

OVERALL CONCLUSIONS

1. Estimates for the normal distribution parameters were obtained for the endurance level at stress ratios of ∞; 3.5 and 1.0 . The tests at stress ratio of 0.44 are in progress.

2. Estimates for the normal and log-normal distribution parameters were obtained from the cycles-to-failure data. By comparing the D-values and the coefficients of skewness and kurtosis it was concluded that the data tends towards the log-normal distribution. It was also concluded that in order to make a more conclusive decision a larger sample size at each stress level would be required.

3. The method previously proposed to obtain strength distribution parameter estimates yields optimistic values and should be studied further.

4. If random data is obtained by the method proposed in this report estimates which might have favorable properties may be obtained.

5. Fatigue surface was generated using the endurance data which is among the first few attempts to obtain such design data. Efforts to generate such data based on larger samples, with solid state instrumentation, and with

greater operator care, should be continued.

6. On the basis of the 34 specimens tested on the machine with the modified couplings it is concluded that the spherical bearings improved the running characteristics of the machine. It is hoped that the long range performance would also be satisfactory.

CHAPTER IV

RECOMMENDATIONS

1. New solid-state instrumentation should be purchased. The new instrumentation should be drift free and should have the capability of constantly monitoring the strain gage outputs.

2. The gear couplings should be modified on the remaining two machines.

3. The endurance tests for stress ratios of ∞ and 1.0 should be checked.

4. Before the next phase of the test program is embarked upon the machines should be recalibrated to see if the calibration constants are the same.

5. Larger sample sizes should be used in future tests, preferably 30 or more at each stress level to obtain the much needed cycles-to-failure design data.

6. The feasibility of incorporating random stress levels to produce data from which better strength distribution parameters might be estimated should be investigated.

APPENDIX A:

COMPUTER PROGRAM TO CALCULATE STRESS LEVELS AND RATIOS

The purpose of this program is to convert the visicorder records into normal stresses, shear stresses and stress ratios. The program also calculates the means and standard deviations of the stresses and ratio of each group of data. It also calculates the cycles-to-failure from times to failure data, but the program will also accept cycles-to- failure data. The program distinguishes between the two through the use of a code number. The code also tells the program whether or not a group of data are endurance test data. This discrimination is necessary because of the calculation of mean and standard deviation of the stresses. The program will not calculate the endurance level distribution parameters. The discrimination code is fed in as data and is as follows:

0 if the failure data is in times to failure
1 if the data is an endurance test
2 if the failure data is in cycles-to-failure.

The input format for the code will be discussed shortly.

The program will accept as many sets of data as desired and the groups may be mixed; ie, endurance test, group with cycles-to-failure data and group with times to failure data. A group is all the data for one stress level. A group of data consists of the following. The first card contains in this order, the number of specimens in the level, the mode of the run and the code. The mode is dependent upon the date the run was made. For a further discussion on mode see Section 2.1.

The fields on the data card are as follows:

spaces 1 to 5 - number of specimens
spaces 6 to 10 - mode
spaces 11 to 15 - code

The number of specimens, mode and code are fixed point numbers and have no decimals but the numbers must be placed to the right in each field.

The next sequence of cards reads in the cycles-to-failure or times to failure in hours, minutes and seconds. If the data is in times to failure there are ten groups on a card, so the number of cards required will depend upon how many specimens are in the level. The format across the card is:

spaces 1 and 2 - blank
spaces 3 and 4 - hours
spaces 5 and 6 - minutes
spaces 7 and 8 - seconds
spaces 9 and 10 - blank

and the sequence continues in this manner. If the failure data is in cycles-to-failure the format is 8 fields of 10 spaces each and the decimals appear in the last space of each field; ie, spaces 10, 20, 30, etc. If the group of data is for an endurance test there is no failure data and these cards are left out. The program will automatically handle it if the proper code number is put on the first card.

Following the cycles-to-failure cards are the cards containing the information for each specimen in the stress level. The information must be placed on each card as follows:

spaces 1 to 5 - test number
spaces 6 to 10 - specimen number
spaces 11 to 15 - machine number
spaces 16 to 20 with a decimal in space 20 - pan weight

spaces 21 to 30 with a decimal in space 28 - bending calibration resistance

spaces 31 to 40 with a decimal in space 38 - number of bending calibration divisions

spaces 41 to 50 with a decimal in space 48 - number of divisions of bending

spaces 51 to 60 with a decimal in space 58 - torque calibration resistance

spaces 61 to 70 with a decimal in space 68 - number of torque calibration divisions

spaces 71 to 80 with a decimal in space 78 - number of divisions of torque

The test number, specimen number and machine number are fixed point numbers and must be placed to the right in each field. There is one card for each test specimen and the cards must be placed in the same order as the failure data is placed on the cards preceding these cards. For data at stress ratio of ∞ there will be no torque stress data. In this case these fields can be left blank. The computer reads blanks on data cards as zeros.

This makes up one group of data at a given stress level and ratio. As many groups may be run as desired by simply placing the groups one behind the other.

Following is a listing of important variables in the program and Fig. A - 1 is a flow chart of the program. Figure A - 2 is a program listing. The outputs from this program are given in Appendixes D and E.

List of Definitions for Program to Find Stress Levels and Ratios (PROGRAM STRESS)

NCARDS	=	number of specimens tested at given level.
MODE	=	number of mode depending on date of test.
NCODE	=	0 if failure data is in times to failure.
	=	1 if data is from an endurance level.
	=	2 if failure data is in cycles-to-failure.
XHOURS(I) XMIN(I) SECS(I)	=	times to failure in hours, minutes and seconds.
TOTCY(I)	=	cycles-to-failure.
NOTEST	=	test number.
NOSPEC	=	speciman number.
MACHNO	=	machine number.
PANWT	=	amount of weight on loading arm.
RCALB	=	calibration resistance used in bending channel.
ENCALB	=	number of visicorder divisions used when calibrating bending channel.
ENVISB	=	number of divisions during actual test.
RCALT	=	calibration resistance used in torque channel.
ENCALT	=	number of visicorder divisions used when calibrating torque channel.
ENVIST	=	number of divisions during actual test.
ENA	=	number of active arms in strain gage bridge.
RGAGEB	=	resistance of bending strain gages.
RGAGET	=	resistance of torque strain gages.
GB	=	bending gage factor.

GT	=	torque gage factor.
CBGR	=	calibration constant K_{BGR} .
CGRTH	=	calibration constant K_{GR-TH} .
CT	=	calibration constant K_T .
CTB	=	calibration constant $K_{T/B}$.
CBT	=	calibration constant $K_{B/T}$.
RPM	=	revolutions per minute of machine.
SOUTH	=	output normal stress corrected for interaction.
TAUTH	=	output shear stress corrected for interaction.
STRGR(I)	=	normal stress in specimen groove.
TAUGR(I)	=	shear stress in specimen groove.
SOUTHP	=	output stress not corrected for interaction.
TAUTHP	=	output stress not corrected for interaction.

Program to Calculate Stress Levels and Ratios

MAIN PROGRAM (STRESS)

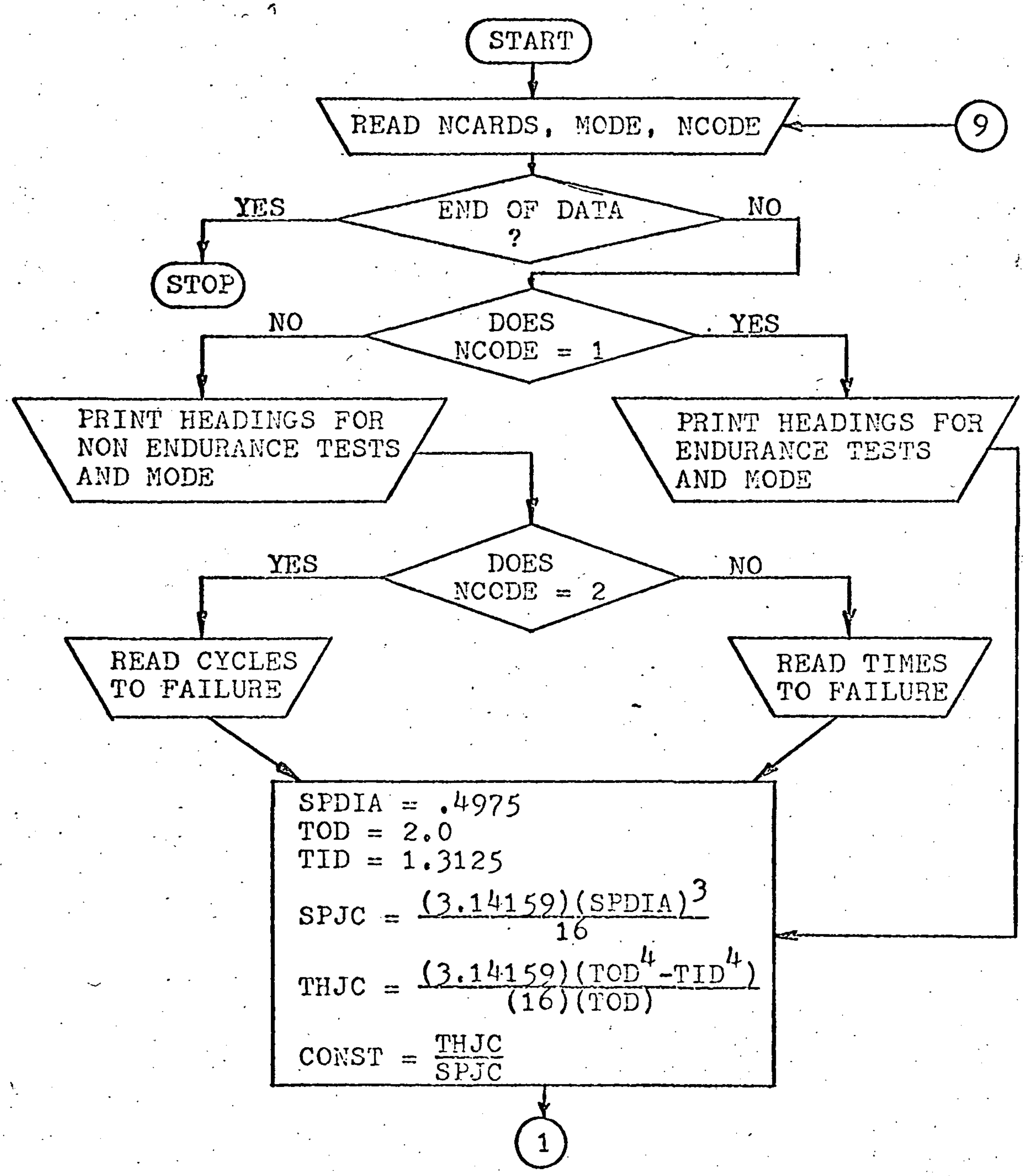

Fig. A - 1

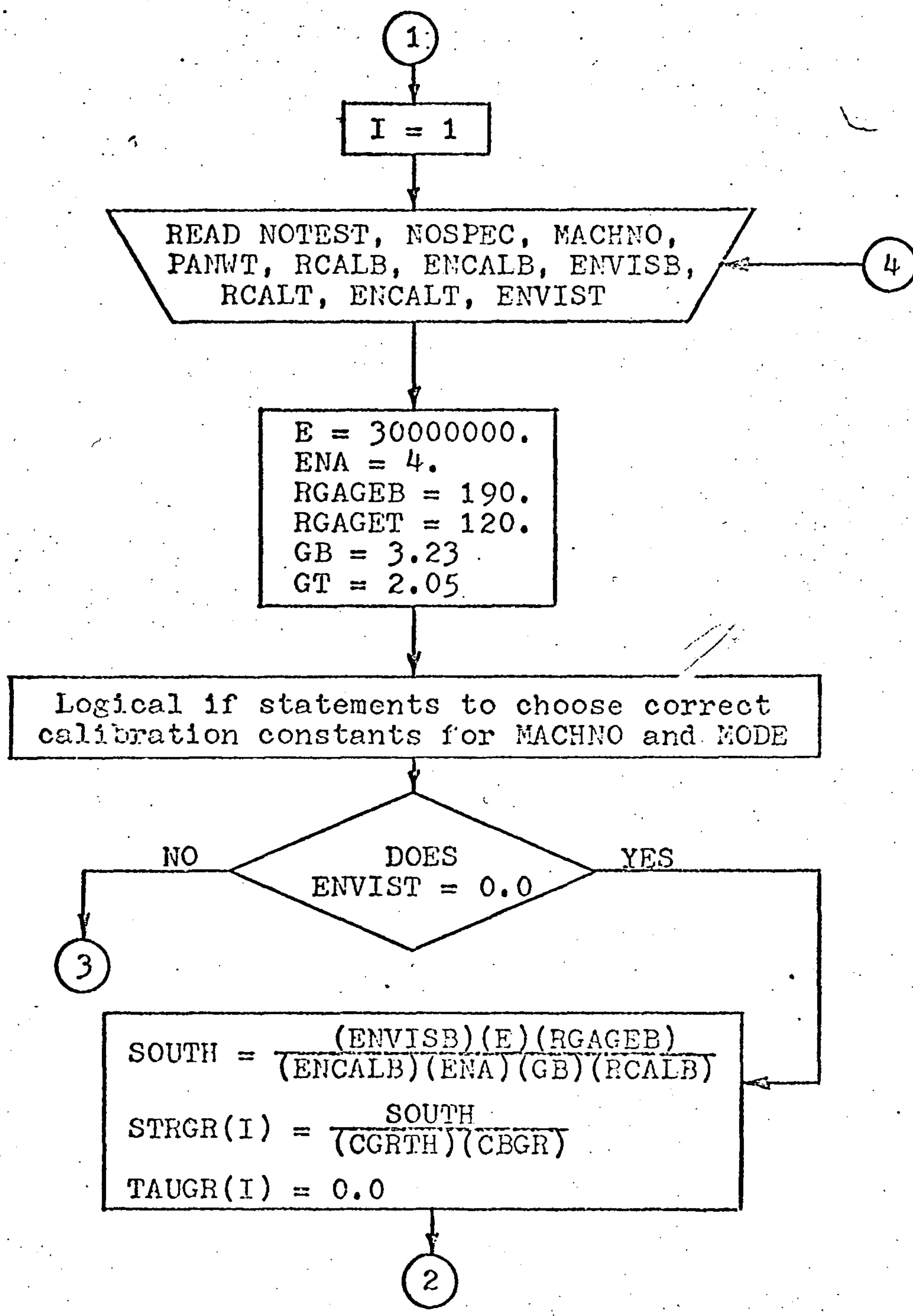

Fig. A - 1 (continued)

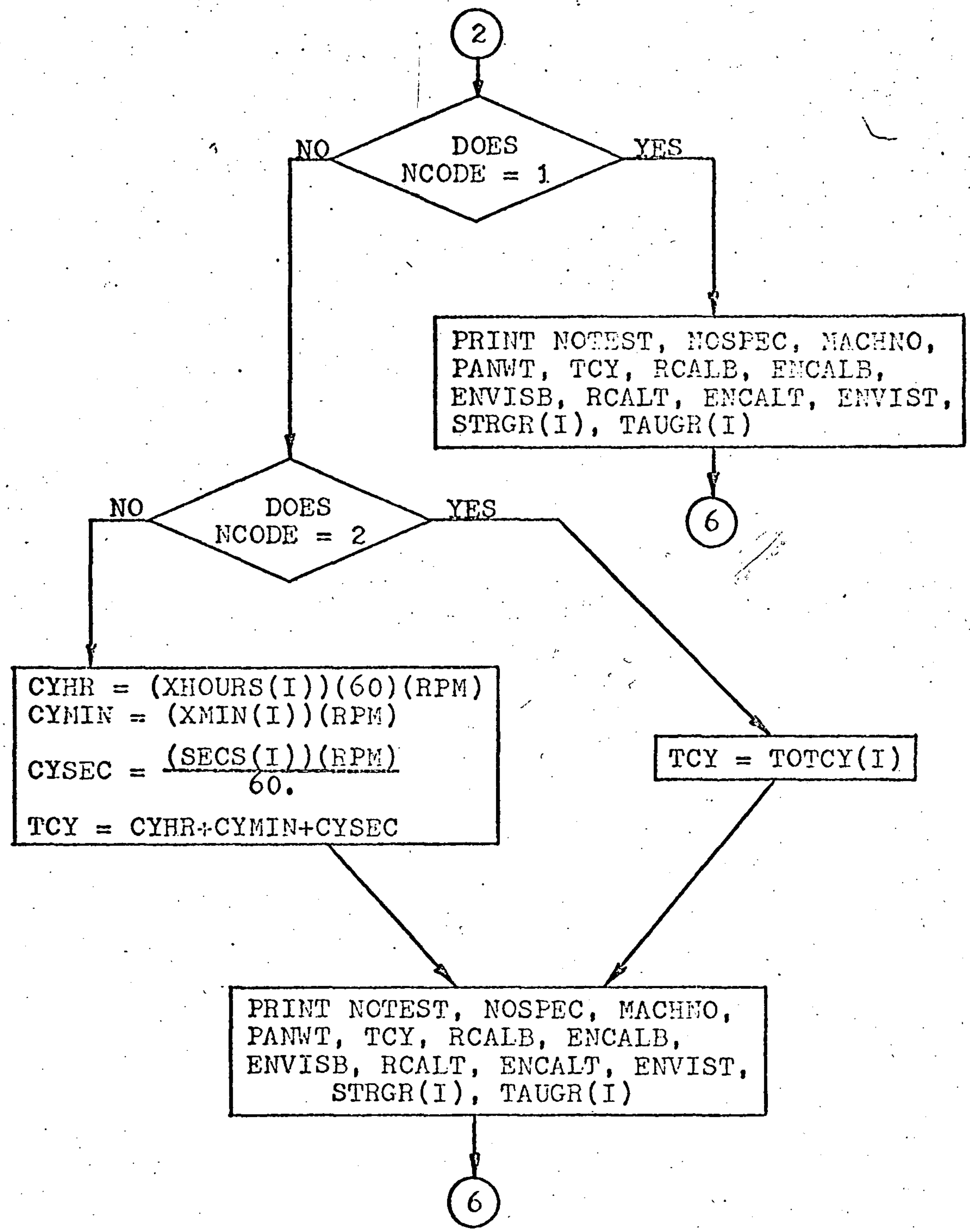

Fig. A - 1 (continued)

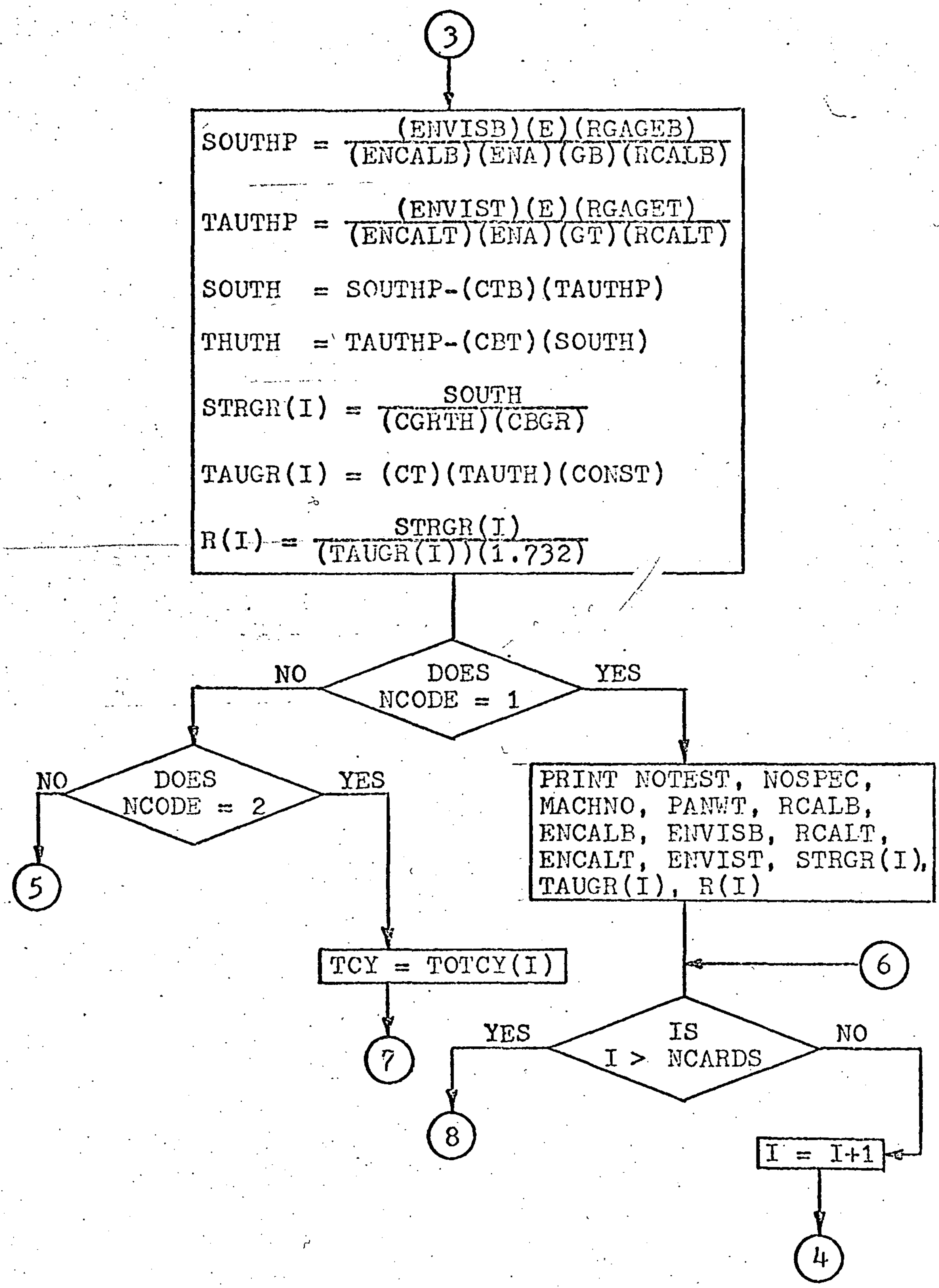

Fig. A-1 (continued)

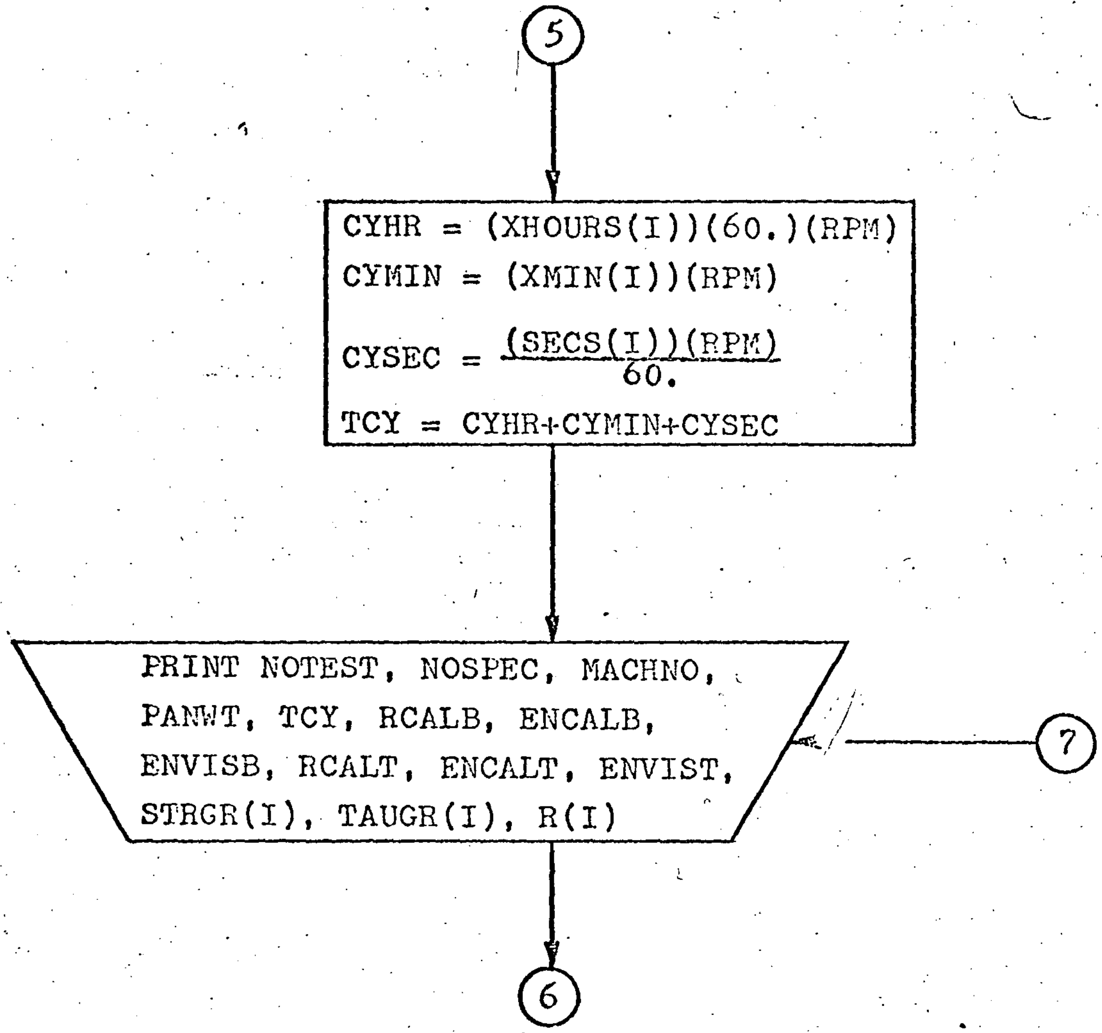

Fig. A - 1 (continued)

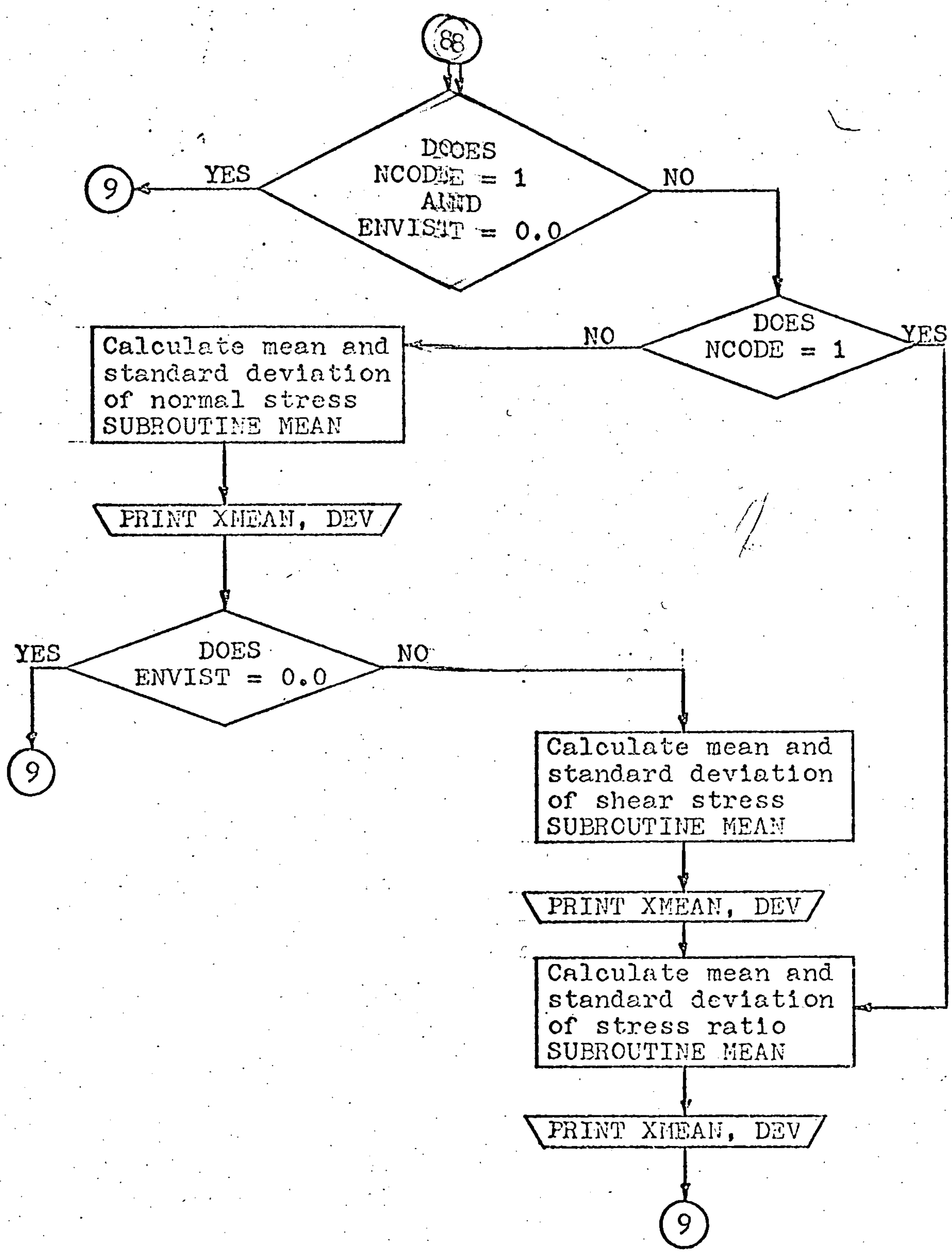

Fig. A-1 (continued)

Program to Find Mean and Standard Deviation

SUBROUTINE MEAN

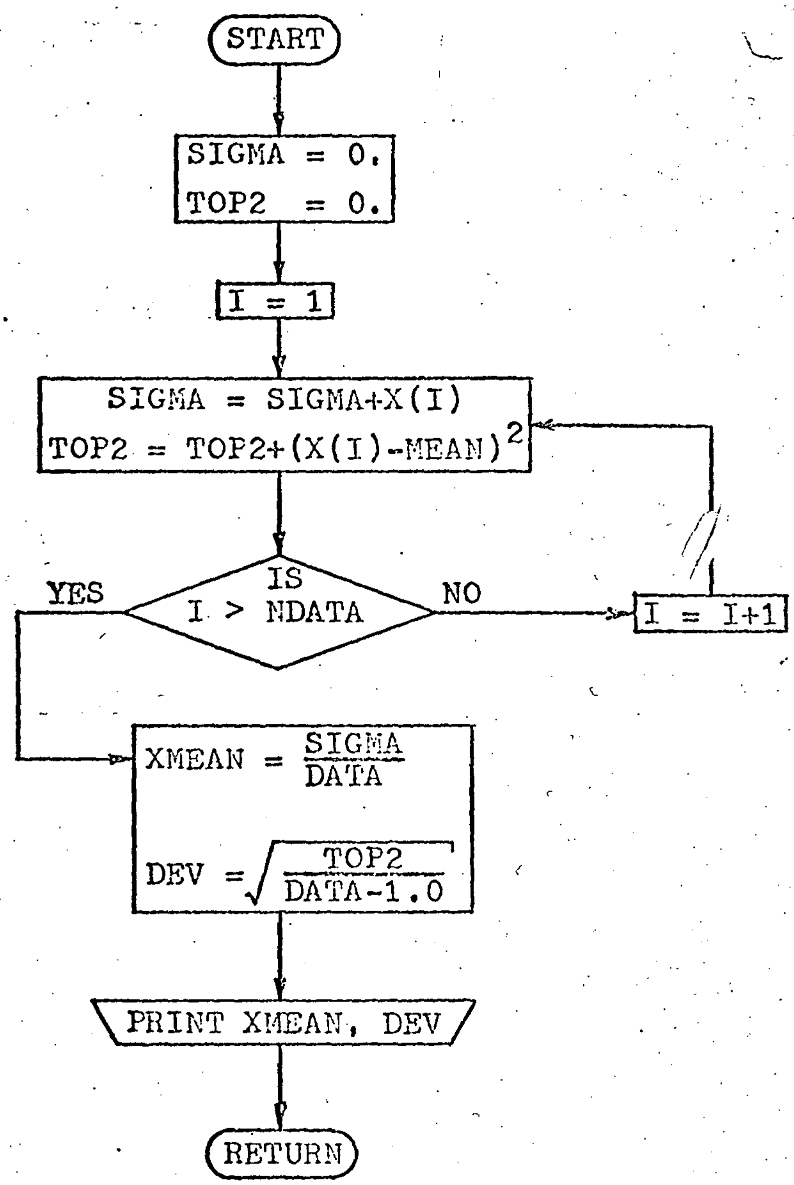

Fig. A-1 (continued)

APPENDIX A (continued)

Fig. A-2 Computer Printout of PROGRAM (STRESS)

```
            PROGRAM STRESS (INPUT,OUTPUT,TAPE1 =INPUT)
000003      DIMENSION STRGR(50), TAUGR(50), R(50),XHOURS(50),XMIN(50),
           1SECS(50),TOTCY(50)
      C     READ IN THE NUMBER OF CARDS IN THE STRESS LEVEL AND THE MODE
      C     OF OPERATION OF THE MACHINE
      C-----NCODE = 1 FOR ENDURANCE LEVEL, NCODE = 0 IF FAILURES IN TIMES TO FAILURE.
      C--------NCODE = 2 IF FAILURES IN CYCLES TO FAILURE.
000003 35   READ 100, NCARDS, MODE, NCODE
000015 100  FORMAT (3I5)
000015      IF (EOF,1) 201, 171
000020 171  PRINT 91
000024 91   FORMAT(1H1//)
000024 170  IF (NCODE.EQ.1) GO TO 175
      C-----FORMAT FOR OUTPUT HEADINGS INCLUDING TIMES TO FAILURE.
000026      PRINT 310, MODE
000034 310  FORMAT(62X,11HTEST MODE =,I2/)
000034      PRINT 61
000040 61   FORMAT (2X,34HTEST  SPEC.  MACH.  PAN     CYCLES,7X,4HRCAL,7X
           1,4HNCAL,7X,4HNVIS,6X,4HRCAL,6X,4HNCAL,6X,4HNVIS,5X,7HBENDING,4X, 1
           26HSHEAR     STRESS/3X,35HNO.   NO.    NO.   WT.   TO FAILURE,3X,
           37HBENDING,4X,7HBENDING,4X,7HBENDING,4X,6HTORQUE,4X,6HTORQUE,4X,
           46HTORQUE,5X,6HSTRESS,4X,6HSTRESS,4X,5HRATIO//)
000040      IF (NCODE.EQ.2) GO TO 320
      C------ROUTINE TO READ IN TIMES TO FAILURE.
000042      READ 401, (XHOURS(I),XMIN(I),SECS(I), I = 1, NCARDS)
000061 401  FORMAT (8(2X,3F2.0,2X))
000061      GO TO 300
      C-----READ IN CYCLES TO FAILURE.
000062 320  READ 402, (TOTCY(I), I=1,NCARDS)
000075 402  FORMAT ( 8F10.0)
000075      GO TO 300
      C-----FORMAT FOR OUTPUT HEADINGS WITHOUT TIMES TO FAILURE.
000076 175  PRINT 90, MODE
000104 90   FORMAT (50X,14HENDURANCE TEST,10X,11HTEST MODE =,I2/)
000104 172  PRINT 60
000110   60 FORMAT (3X,4HTEST,3X,8HSPECIMEN,3X,7HMACHINE,5X,3HPAN,7X,4HRCAL,7X
           1,4HNCAL,7X,4HNVIS,6X,4HRCAL,6X,4HNCAL,6X,4HNVIS,5X,7HBENDING,4X,
           25HSHEAR,4X,6HSTRESS/4X,3HNO.,6X,3HNO.,7X,3HNO.,5X,6HWEIGHT,4X,
           37HBENDING,4X,7HBENDING,4X,7HBENDING,4X,6HTORQUE,4X,6HTORQUE,4X,
           46HTORQUE,5X,6HSTRESS,4X,6HSTRESS,4X,5HRATIO/ )
      C-----ROUTINE TO CALCULATE POLAR MOMENTS OF INERTIA.
      C-----SPDIA = SPECIMEN DIA.     TOD = TOOLHOLDER O. D.   TID = TOOLHOLDER I. D.
000110 300  SPDIA=.4975
000111      TOD=2.0
000113      TID=1.3125
000114      SPJC=(3.14159*SPDIA**3)/16.0
000117      THJC = (3.14159*(TOD**4-TID**4))/(16.*TOD)
000126      CONST=THJC/SPJC
000130      DO 120 I = 1, NCARDS
      C     READ IN THE TEST NO., SPECIMEN NO., MACHINE NO., PANWEIGHT, AND THE
      C     CALIBRATION RESISTANCE, VISICORDER CALIBRATION DISTANCE AND VISICORDER
      C     OUTPUT DIVISIONS FOR BENDING AND TORQUE
000132   80 READ 10,NOTEST,NOSPEC,MACHNO, PANWT,RCALB,ENCALB,ENVISB,RCALT,
           1ENCALT,ENVIST
000162 10   FORMAT (3I5,F5.0,6F10.2)
000162      CARDS = NCARDS
      C     DEFINE THE ELASTIC MODULUS, NO. OF ACTIVE ARMS OF THE BRIDGES, THE
```

```
              C     RESISTANCES OF THE BENDING AND TORQUE GAUGES, AND THE BENDING AND
              C     TORQUE GAUGE FACTORS
000163              E=30000000.
000165              ENA=4.
000166              RGAGEB=190.
000170              RGAGET=120.
000171              GB=3.23
000173              GT = 2.05
              C     SELECTION OF MACHINE AND MODE
000174              IF(MACHNO.EQ.1.AND.MODE.EQ.1) GO TO 11
000204              IF(MACHNO.EQ.2.AND.MODE.EQ.1) GO TO 24
000213              IF(MACHNO.EQ.3.AND.MODE.EQ.1) GO TO 31
000222              IF(MACHNO.EQ.1.AND.MODE.EQ.2) GO TO 11
000230              IF(MACHNO.EQ.2.AND.MODE.EQ.2) GO TO 24
000236              IF(MACHNO.EQ.3.AND.MODE.EQ.2) GO TO 31
000245              IF(MACHNO.EQ.1.AND.MODE.EQ.3) GO TO 11
000254              IF(MACHNO.EQ.2.AND.MODE.EQ.3) GO TO 24
000263              IF(MACHNO.EQ.3.AND.MODE.EQ.3) GO TO 33
000271              IF(MACHNO.EQ.1.AND.MODE.EQ.4) GO TO 11
000300              IF(MACHNO.EQ.2.AND.MODE.EQ.4) GO TO 24
000307              IF(MACHNO.EQ.3.AND.MODE.EQ.4) GO TO 34
              C     CALIBRATION PARAMETERS FOR GIVEN NODE AND MACHINE
000316        11    CBGR = 1.0123
000317              CGRTH = .0208
000321              CT = .8752
000322              CTB = -.0459
000324              CBT = .029
000325              RPM=1786.
000327              GO TO 50
000330        31    CBGR = 1.0946
000331              CGRTH = .0211
000333              CT = .933
000334              CTB = .0
000335              CBT = -.0149
000337              RPM=1780.
000341              GO TO 50
000341        33    CBGR = 1.0946
000342              CGRTH = .0211
000344              CT = .7721
000345              CTB = .0
000346              CBT = -.0127
000350              RPM=1780.
000352              GO TO 50
000352        24    CBGR = 1.0123
000353              CGRTH = .0188
000355              CT = .8201
000356              CTB = 0.0344
000360              CBT = .0422
000361              RPM=1784.
000363              GO TO 50
000364        34    CBGR = 1.0123
000365              CGRTH = .0197
000367              CT = .7721
000370              CTB = .0
000371              CBT = -.0127
000373              RPM=1780.
000375           50 IF(ENVIST.EQ.0.0) GO TO 160
000376              GO TO 51
```

```
          C       CALCULATION OF BENDING STRESS LEVEL FOR INFINITY RATIO
000377      160 SOUTH=(ENVISB*E*RGAGEB)/(ENCALB*ENA*GB*RCALB)
000405            STRGR(I) = SOUTH / (CGRTH * CBGR)
000411            TAUGR(I) = 0.0
000412            IF (NCODE.EQ.1) GO TO 301
000414            IF (NCODE.EQ.2) GO TO 330
          C-----CALCULATE CYCLES TO FAILURE FROM TIMES TO FAILURE.
000415            CYHR = XHOURS(I)*60.*RPM
000417            CYMIN = XMIN(I)*RPM
000421            CYSEC = (SECS(I)*RPM)/60.0
000423            TCY = CYHR+CYMIN+CYSEC
000427            GO TO 335
000427    330     TCY = TOTCY(I)
000431    335     PRINT 72,NOTEST,NOSPEC,MACHNO,PANWT,TCY,RCALB,ENCALB,ENVISB,RCALT,
                 1ENCALT,ENVIST,STRGR(I),TAUGR(I)
000467    72      FORMAT(3I6,F8.1,2F11.0,F10.2,F11.2,F11.0,F10.2,
                 1F10.2,F11.0,F10.0,4X,6HINFIN./)
000467            GO TO 120
000470    301     PRINT 71,NOTEST,NOSPEC,MACHNO, PANWT,RCALB,ENCALB,ENVISB,RCALT,
                 1ENCALT,ENVIST,STRGR(I),TAUGR(I)
000524    71      FORMAT(4X,I3,6X,I3,8X,I1,6X,F5.1,F12.0,F10.2,F11.2,F11.0,F10.2,
                 1F10.2,F11.0,F10.0,3X,6HINFIN./)
000524            GO TO 120
          C       CALCULATION OF BENDING STRESS, SHEAR STRESS AND STRESS RATIO FOR
          C       ALL FINITE RATIOS
000525       51 SOUTHP=(ENVISB*E*RGAGEB)/(ENCALB*ENA*GB*RCALB)
000533            TAUTHP=(ENVIST*E*RGAGET)/(ENCALT*ENA*GT*RCALT)
000541            SOUTH=SOUTHP-CTB*TAUTHP
000544            TAUTH=TAUTHP-CBT*SOUTH
000547            STRGR(I) = SOUTH / (CGRTH * CBGR)
000553            TAUGR(I) = CT*TAUTH*CONST
000555            R(I) = STRGR(I) / (TAUGR(I) * 1.732)
000561            IF (NCODE.EQ.1) GO TO 302
000563            IF (NCODE.EQ.2) GO TO 340
          C-----CALCULATE CYCLES TO FAILURE FROM TIMES TO FAILURE.
000564            CYHR = XHOURS(I)*60.*RPM
000566            CYMIN = XMIN(I)*RPM
000570            CYSEC = (SECS(I)*RPM)/60.0
000572            TCY = CYHR+CYMIN+CYSEC
000576            GO TO 345
000576    340     TCY = TOTCY(I)
000600    345     PRINT 73,NOTEST,NOSPEC,MACHNO,PANWT,TCY,RCALB,ENCALB,ENVISB,RCALT,
                 1ENCALT,ENVIST,STRGR(I),TAUGR(I),R(I)
000640    73      FORMAT(3I6,F8.1,2F11.0,F10.2,F11.2,F11.0,F10.2,
                 1F10.2,F11.0,F10.0,F9.3/)
000640            GO TO 120
000641    302     PRINT 70,NOTEST,NOSPEC,MACHNO, PANWT,RCALB,ENCALB,ENVISB,RCALT,
                 1ENCALT,ENVIST,STRGR(I),TAUGR(I),R(I)
000677    70      FORMAT(4X,I3,6X,I3,8X,I1,6X,F5.1,F12.0,F10.2,F11.2,F11.0,F10.2,
                 1F10.2,F11.0,F10.0,F9.3/)
000677      120 CONTINUE
000702            IF (NCODE.EQ.1.AND.ENVIST.EQ.0.0) GO TO 35
000711            IF (NCODE.EQ.1) GO TO 200
          C       CALCULATION OF MEAN AND STANDARD DEVIATION OF BENDING STRESS, SHEAR
          C       STRESS, AND STRESS RATIO
000713            CALL MEAN (STRGR,  CARDS,NCARDS, XMEAN, DEV)
000716            PRINT 3
000722    3       FORMAT (1H )
```

```
000722          PRINT 130, XMEAN, DEV
000732   130    FORMAT (19X,31HMEAN BENDING STRESS IN GROOVE =,F10.1,5H PSI.//11X,
               139HSTD. DEV. OF BENDING STRESS IN GROOVE =,F10.2,5H PSI./)
000732   132    IF(ENVIST.EQ.0.0) GO TO 35
000733          IF (NCODE.EQ.1) GO TO 200
000735          CALL MEAN (TAUGR, CARDS, NCARDS, XMEAN, DEV)
000741          PRINT 3
000745          PRINT 140, XMEAN, DEV
000755   140    FORMAT (20X,30HMEAN TORQUE STRESS IN GROOVE =,F10.1,5H PSI./ 12X,
               138HSTD. DEV. OF TORGUE STRESS IN GROOVE =,F10.2,5H PSI. )
000755   200    CALL MEAN (     R,  CARDS,NCARDS, XMEAN, DEV)
000761          PRINT 3
000765          PRINT 150, XMEAN, DEV
000775   150    FORMAT (31X,19HMEAN STRESS RATIO =,F10.5/ 23X,27HSTD. DEV. OF STRE
               1SS RATIO =,F10.5)
000775   37     GO TO 35
000776   201    STOP
001000          END
```

```
                SUBROUTINE MEAN (X, DATA, NDATA, XMEAN, DEV)
          C-----SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.
000010          DIMENSION X(NDATA)
000010          SIGMA= 0.0
000011          DO 8 I=1, NDATA
000012    8     SIGMA=SIGMA+ X(I)
000016          XMEAN = SIGMA/DATA
000017          TOP2 = 0.0
000020          DO 9 I=1,NDATA
000021    9     TOP2 = TOP2 + (X(I) - XMEAN)**2
000026          DEV =SQRT(TOP2/(DATA - 1.0))
000036          RETURN
000036          END
```

APPENDIX B

Program to Calculate the Parameters of the Normal and Log-Normal Distributions and Conducts Goodness-of-Fit Tests

This program calculates the mean and standard deviation of the cycles-to-failure data for both the normal and log-normal distributions and calculates the moment coefficients of skewness and kurtosis. It also performs the Chi-square and Kolmorgorov-Smirnov goodness-of-fit tests. The input consists of:

1. a card containing the number of data points at that stress level, the accuracy of the data, the stress level and the stress ratio.
2. a card or series of cards containing the cycles-to-failure data listed in descending order.
3. a card or series of cards containing the cumulative frequency of the cycles-to-failure up to that point. These must be listed in the same order as the cycles-to-failure data.

The input format for the first card is:

spaces 1 to 3 - number of data points in fixed point format.

spaces 4 to 8 with a decimal in space 7 - number of data points in floating point format spaces 9 to 17 with a decimal in space 13 - accuracy of the cycles-to-failure data.

spaces 18 to 27 with a decimal in space 26 - stress level at which the data was taken.

spaces 28 to 35 with a decimal in space 30 - stress ratio at which the data was taken.

The cards containing the cycles-to-failure data have eight fields of ten spaces each with the decimal at the right of each field. In other words the first data point is in spaces 1-10 with the decimal in space 10, the second data point in spaces 11-20 with the decimal in space 20, etc.

The format of the cards containing the cumulative frequency data is 26 fields of three spaces each with the decimal to the right of each field. The first value is in spaces 1-3 with the decimal in space 3, the second value is in spaces 4-6 with the decimal in space 6, etc.

The program is set up to accommodate as many sets of data as desires.

Following is a list of important variables used in the program. Figure B-1 is a flow diagram of the program and Fig. B-2 is a printout of the program. Figure B-3 is an example of the output for one set of data. The output shown is for data tested at a stress level 114,000 psi. and stress ratio of ∞.

List of Definitions for Program to Fit Normal and Log-Normal Distributions to Cycles-to-Failure Data (PROGRAM CYTOFR)

Main Program:

NDATA = DATA = number of observations.

STRLV = stress level in psi.

AKURCY = accuracy to which cycles-to-failure data are known.

RATIO = stress ratio

X(I) = cycles-to-failure data

CUMFRQ(I) = cumulative frequency of each X(I); ie, number of X's less than or equal to X(I).

PCAREA(I) = CUMFRQ(I)/NDATA

Subroutine to calculate the mean and standard deviation of the cycles-to-failure data (SUBROUTINE MEAN)

SIGMA = sum of the X(I)'s

XMEAN = average of the X(I)'s

TOP2 = $\sum_{i=1}^{n} (X(I)-XMEAN)^2$

DEV = standard deviation of the X(I)'s

Function subroutine to find the area under the normal curve (FUNCTION PROB(X)).

X = abscissa value for which corresponding area is desired.

PROB = desired area.

Subroutine for Chi-square goodness-of-fit test (SUBROUTINE CHISQA).

K	=	number cells.
XMAX	=	largest value of cycles-to-failure.
XMIN	=	smallest value of cycles-to-failure.
CSV	=	cell starting value.
CEV	=	cell end value.
CLB	=	cell lower bound.
CUB	=	cell upper bound.
FREQ(J)	=	number of observations in J^{th} cell.
REQAREA(J)	=	expected value of J^{th} cell.
CHISQR	=	total Chi-square value.
U(I)	=	Chi-square value of I^{th} cell.

Subroutine for Kolmogorov-Smirnov test (SUBROUTINE DTEST).

Z(I)	=	abscissa value on standard normal curve for a given X(I).
ARUNCN	=	area under standard normal curve from - to Z(I).
DSTAT(I)	=	absolute difference between the data cumulative frequency and the hypothesized cumulative frequency.
XMEAN	=	average of the X(I)'s.
DEV	=	standard deviation of the X(I)'s
PROB(T)	=	area under the standard normal curve from -T to +T.

Subroutine to calculate the moment coefficients of skewness and kurtosis (SUBROUTINE ALPHA).

ALPHA3 = moment coefficient of skewness.

ALPHA4 = moment coefficient of skewness.

VAR $= \sum_{i=1}^{n} (X(I)-\underline{X})^2$

TOP3 $= \sum_{i=1}^{n} (X(I)-\underline{X})^3$

SKEW = third moment of the data.

STDEV = biased estimator for standard deviation.

TOP4 $= \sum_{i=1}^{n} (X(I)-\underline{X})^4$

TKURT = fourth moment of the data.

Program to Calculate Parameter Estimates for the Normal and Log-Normal Distributions and Conduct Goodness-of-Fit Tests

MAIN PROGRAM (CYTOFR)

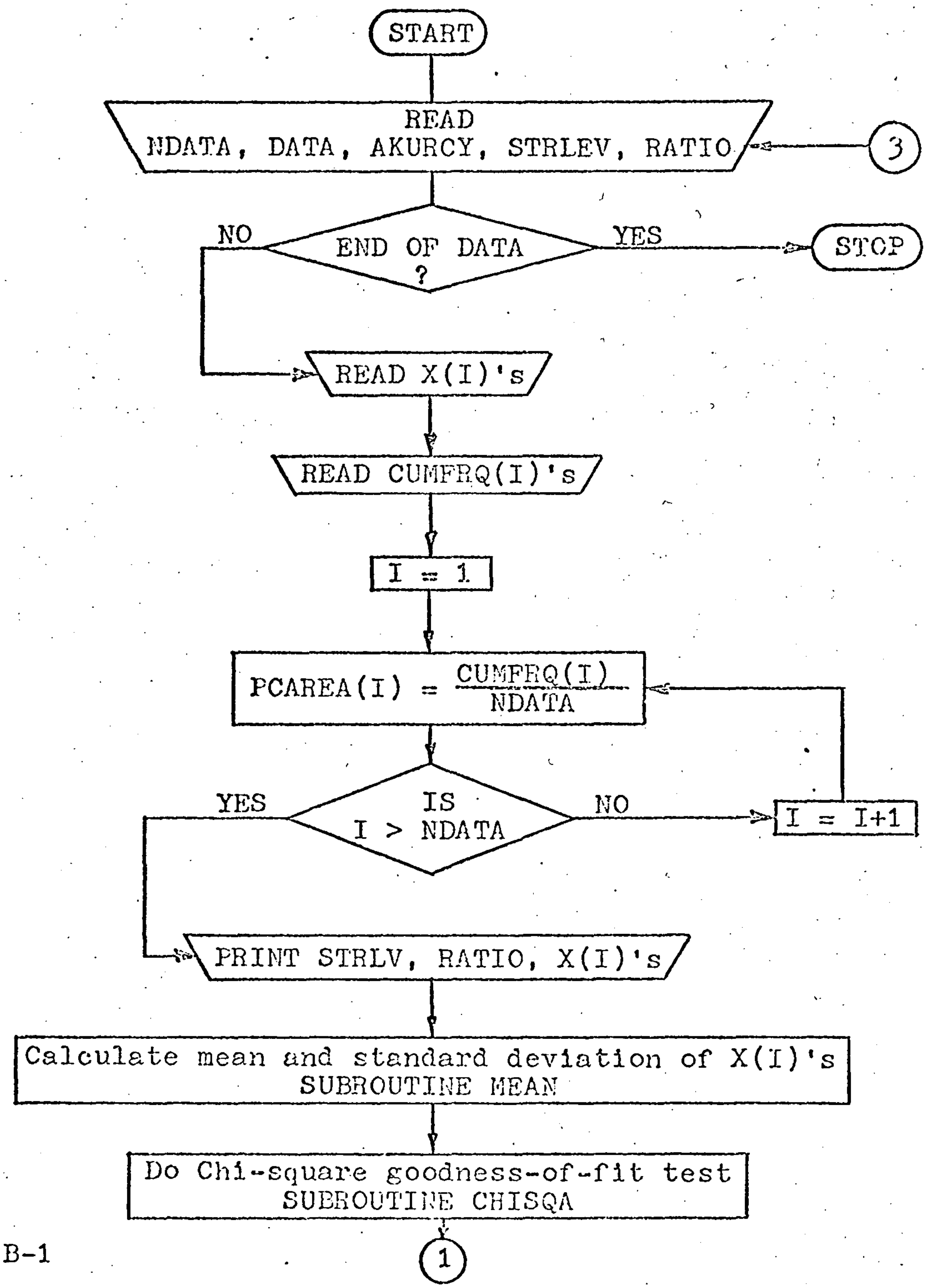

Fig. B-1

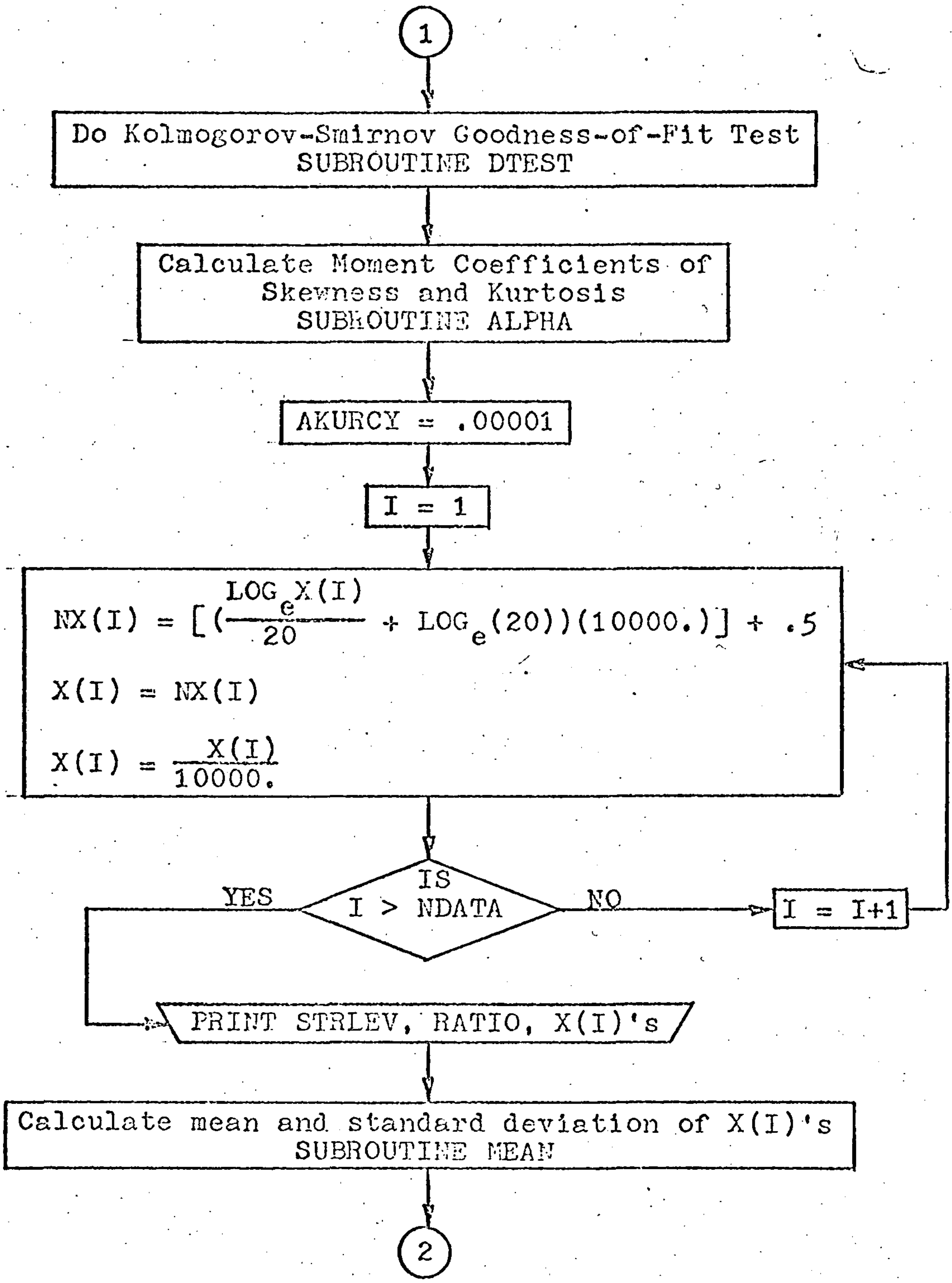

Fig. B-1 (continued)

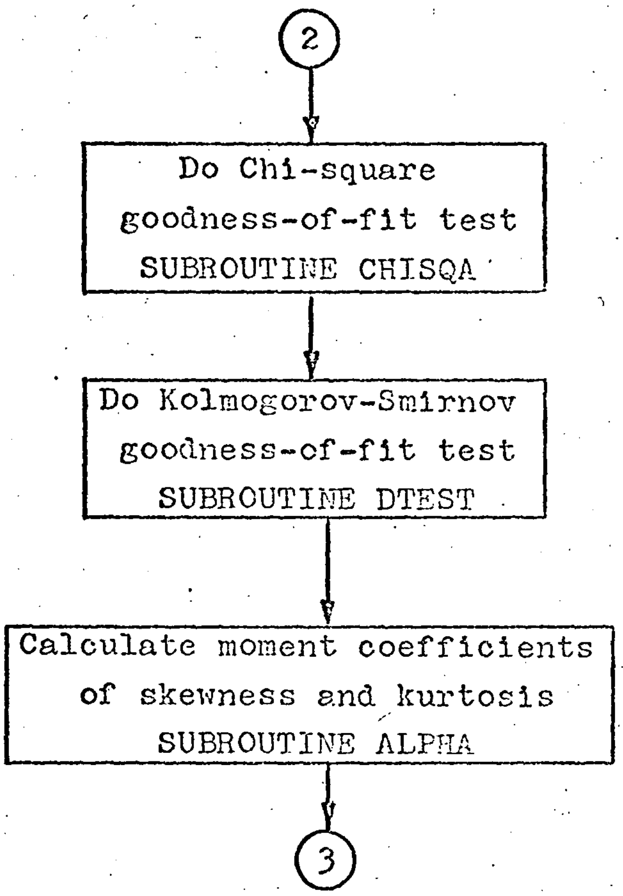

Fig. B-1 (continued)

Subroutine to Find Mean and Standard Deviation

SUBROUTINE MEAN

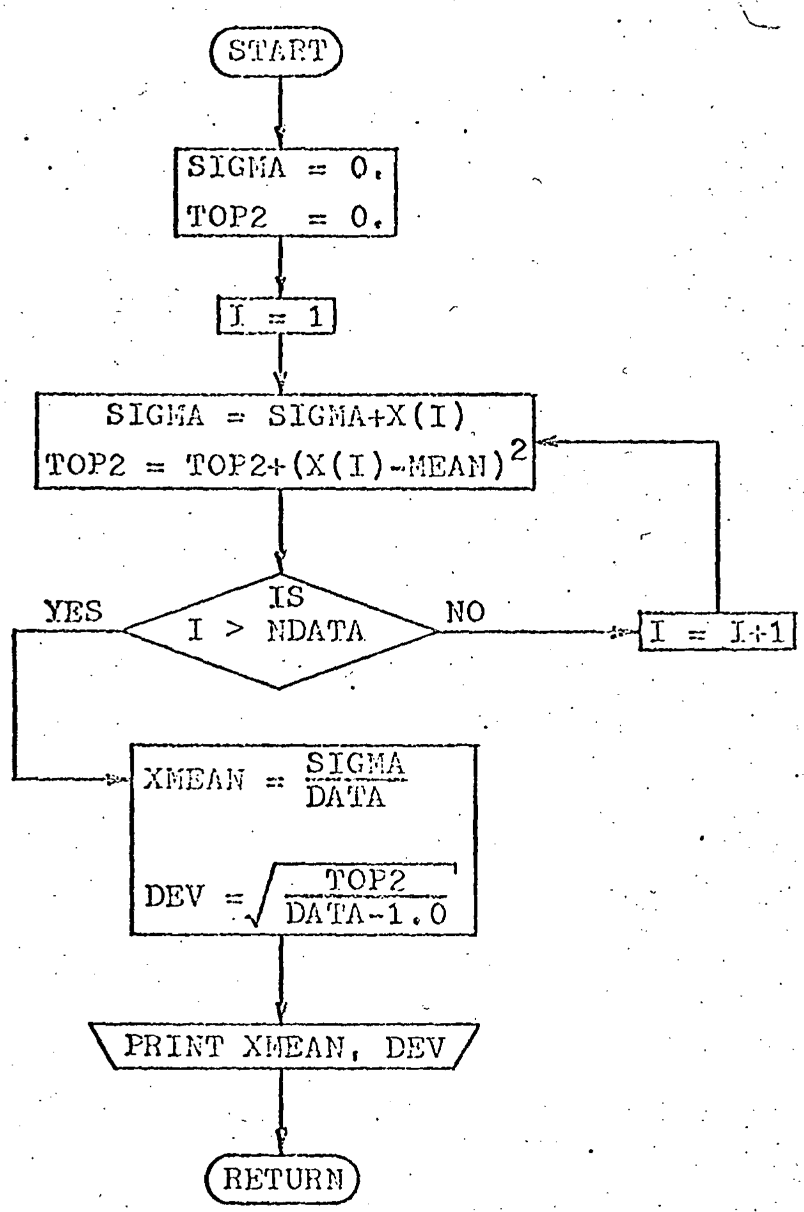

Fig. B-1 (continued)

Subroutine to Find Area Under Standard Normal Curve

FUNCTION PROB(X)

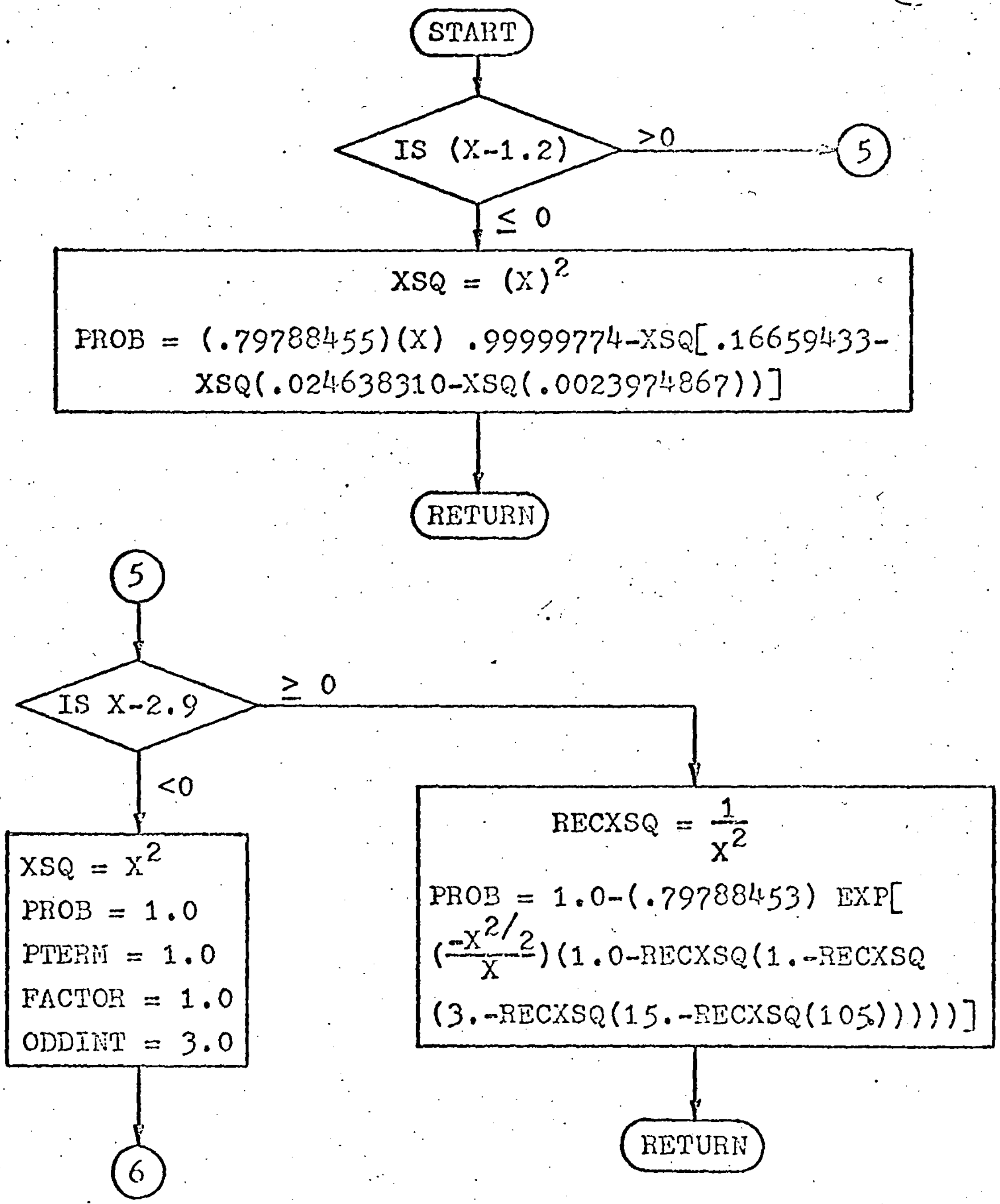

Fig. B-1 (continued)

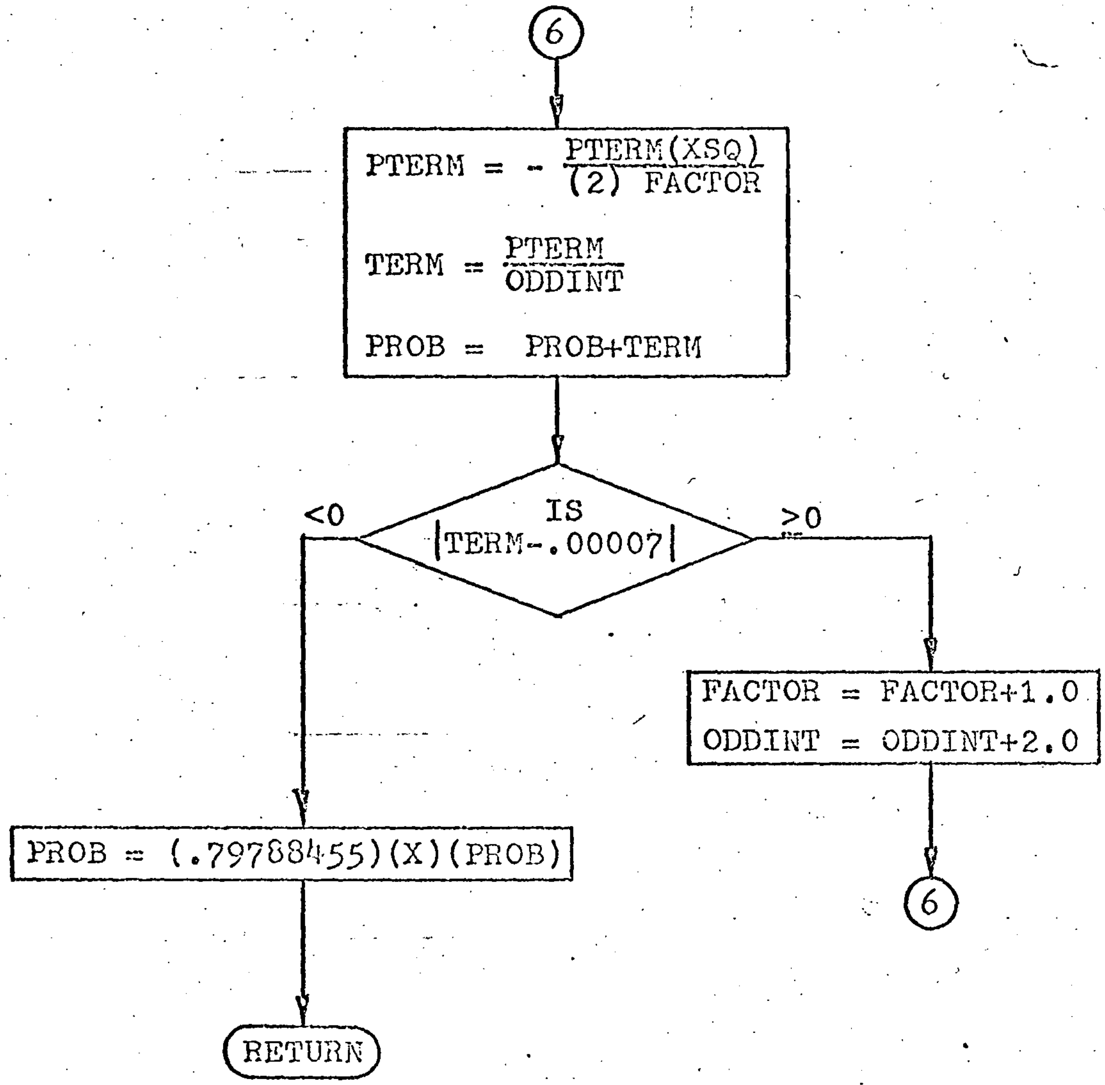

Fig. B-1 (continued)

Subroutine to Conduct Chi-Square Goodness-of-Fit Test

SUBROUTINE CHISQA

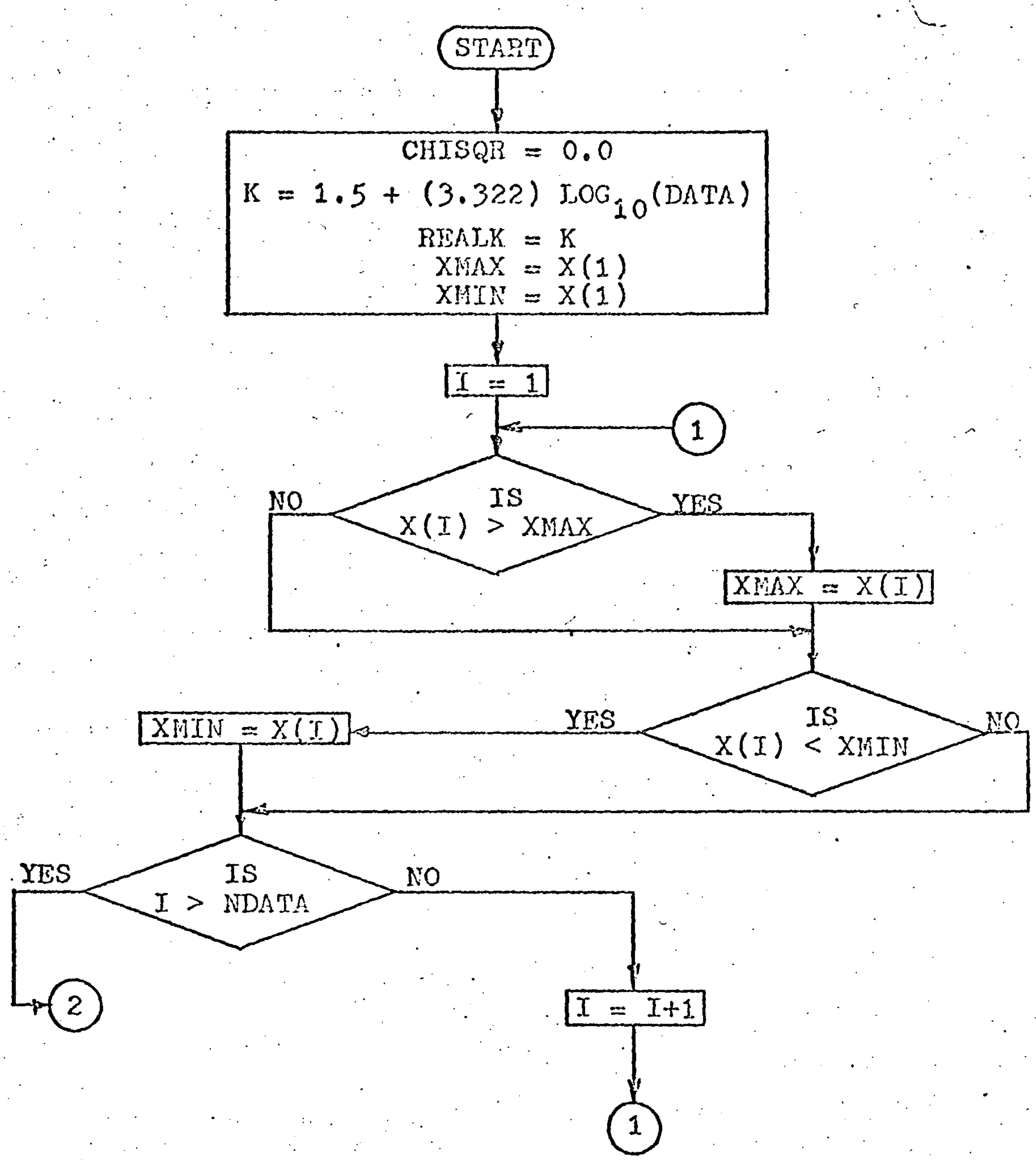

Fig. B-1 (continued)

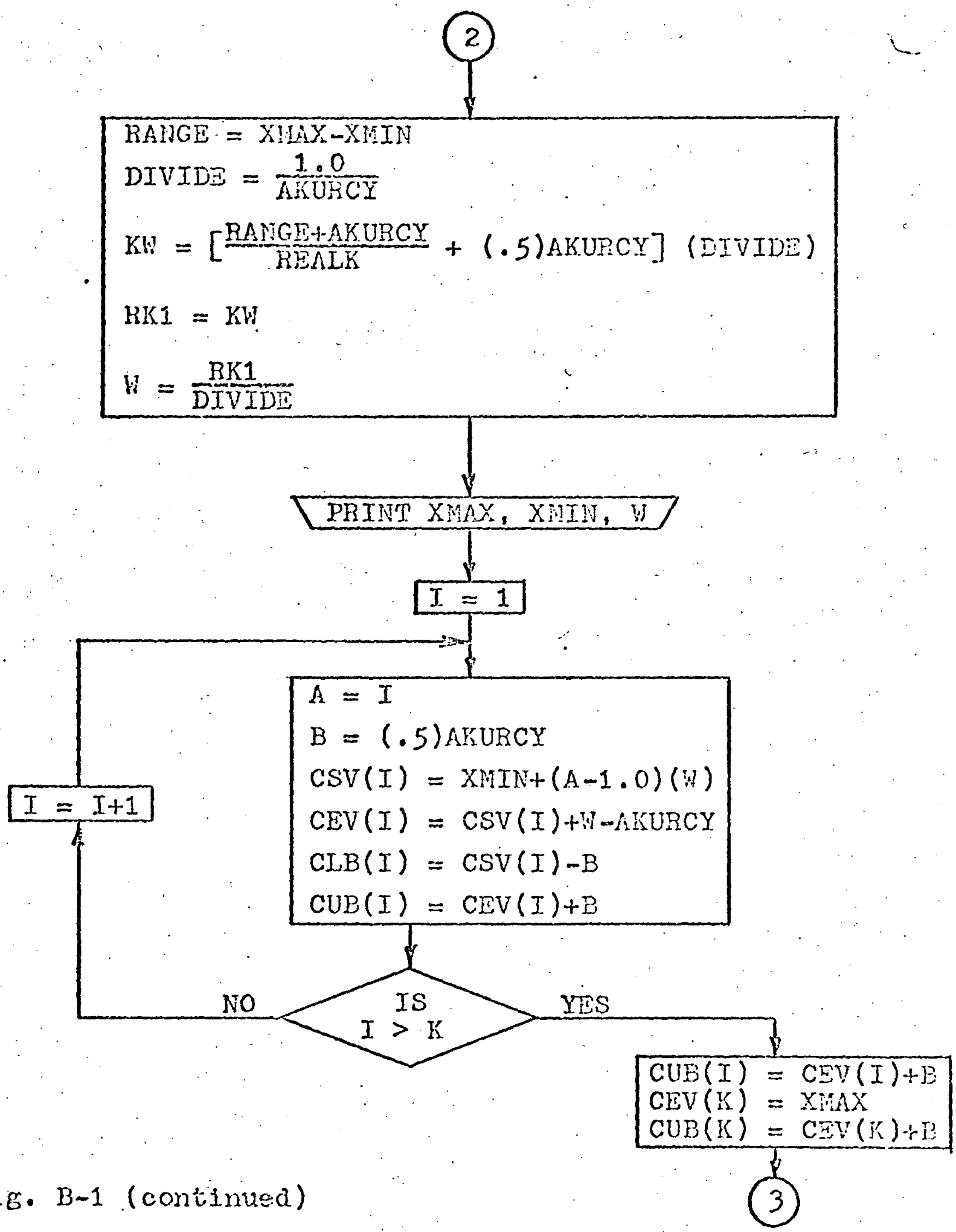

Fig. B-1 (continued)

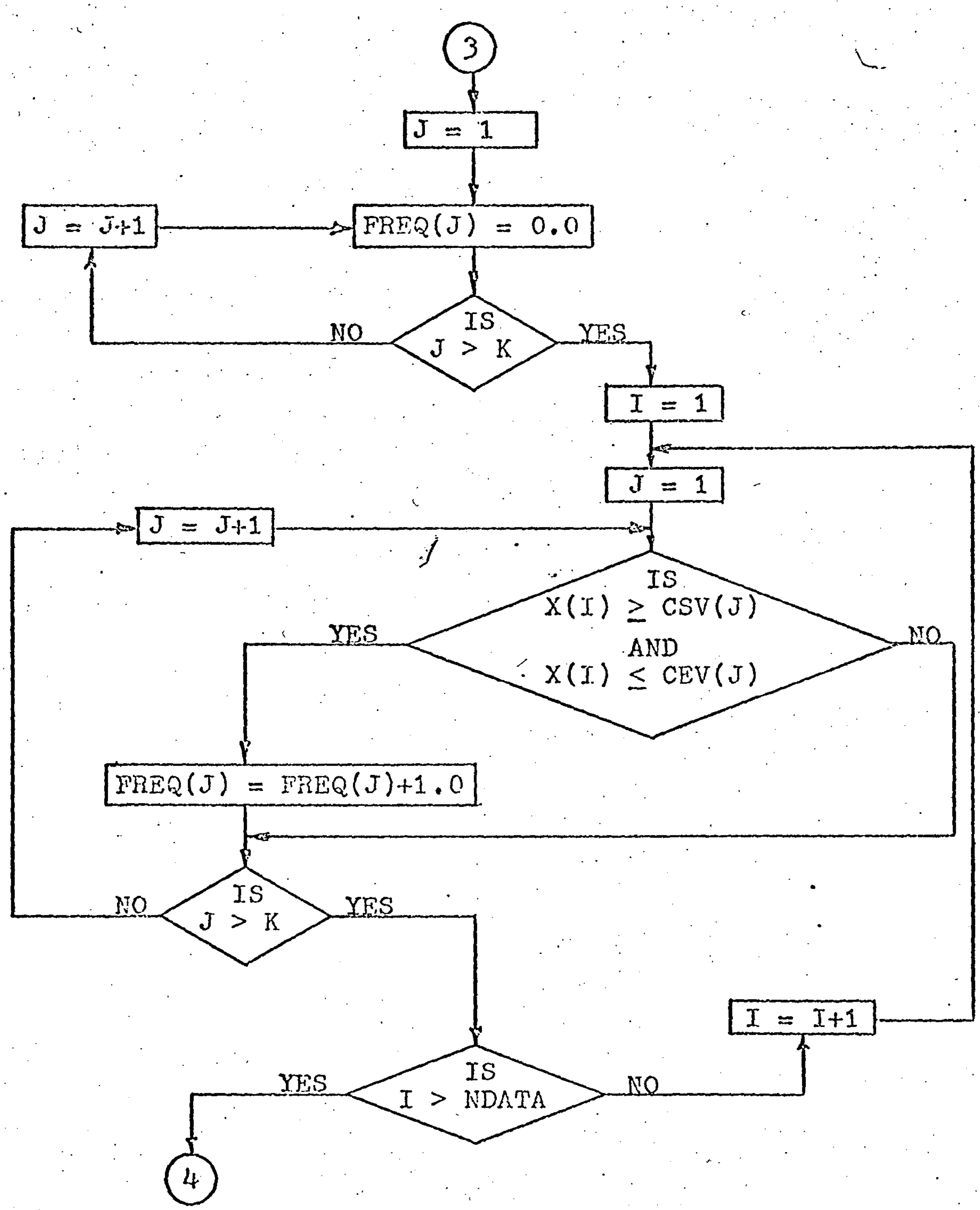

Fig. B-1 (continued)

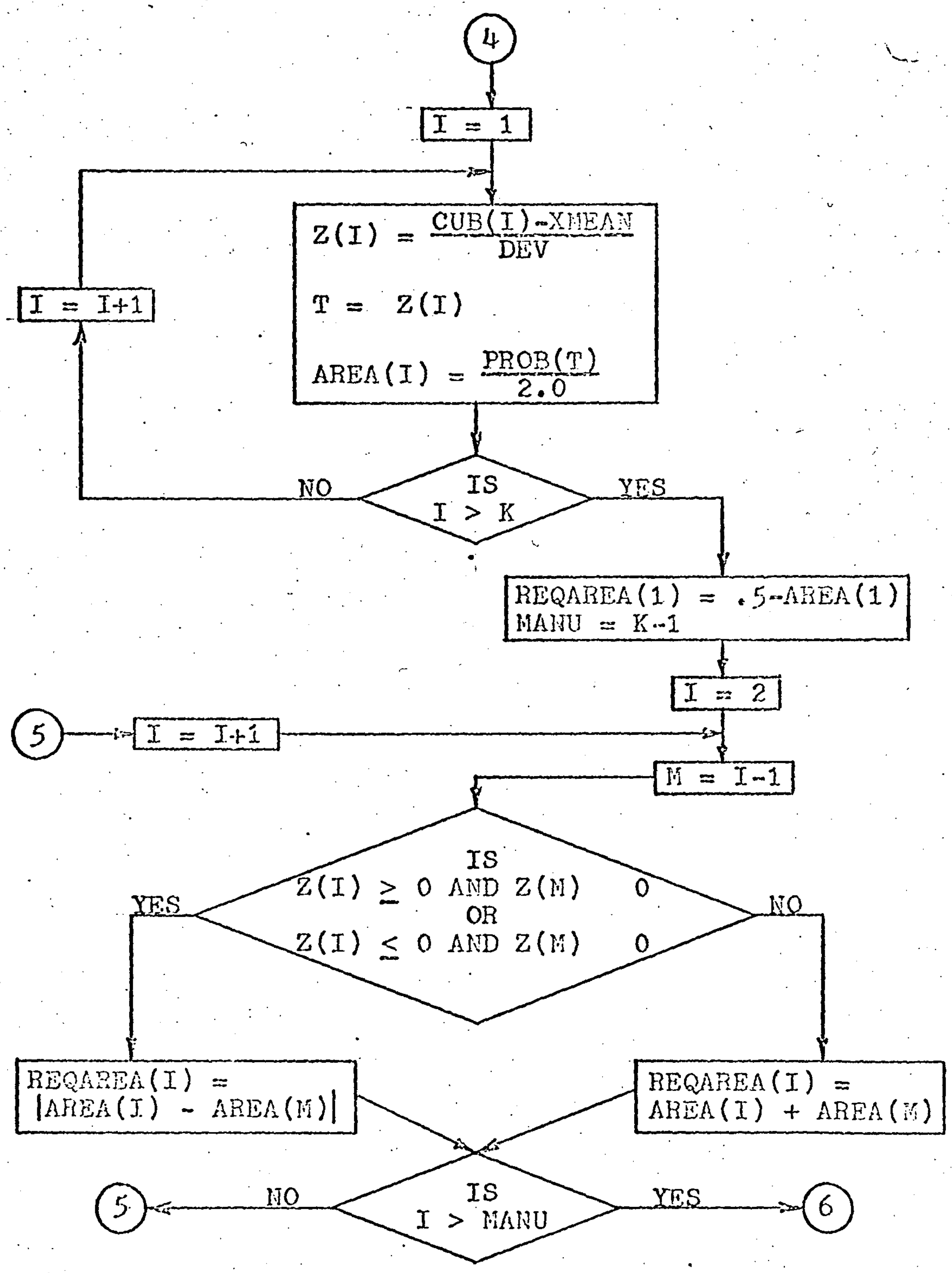

Fig. B-1 (continued)

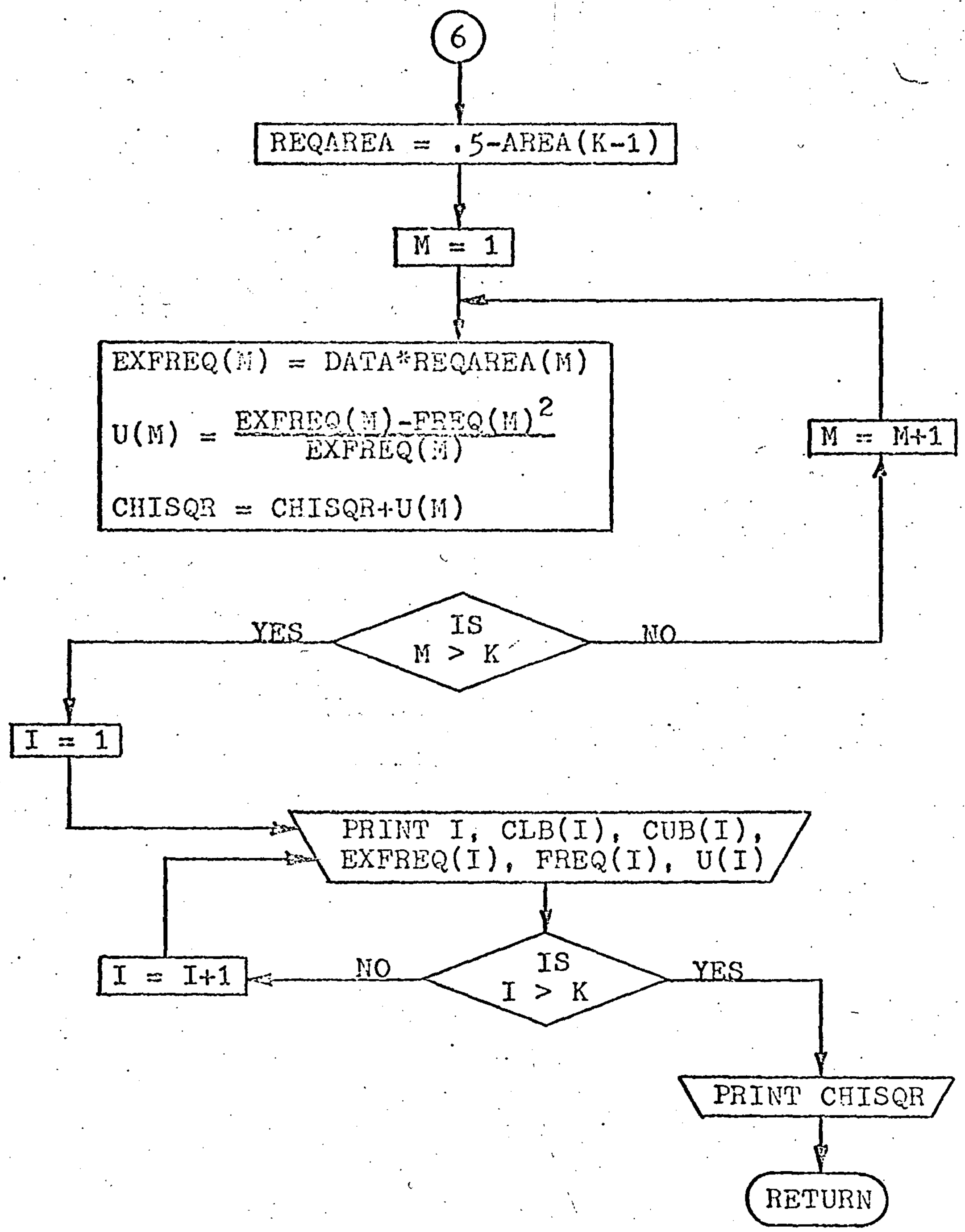

Fig. B-1 (continued)

Subroutine to Conduct Kolmogorov-Smirnov Goodness-of-Fit Test

SUBROUTINE DTEST

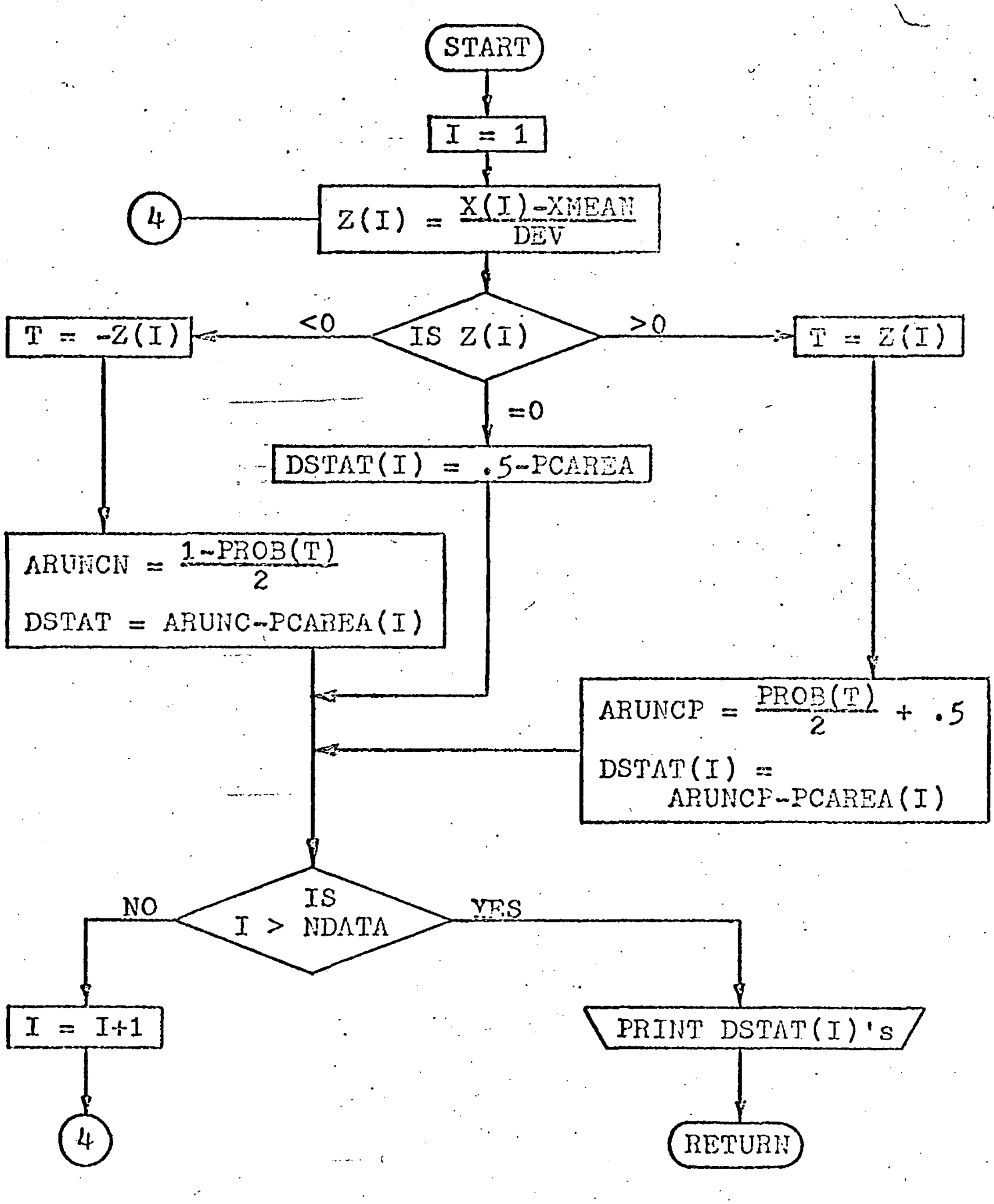

Fig. B-1 (continued)

Subroutine to Find the Moment Coefficients of Skewness and Kurtosis

SUBROUTINE ALPHA

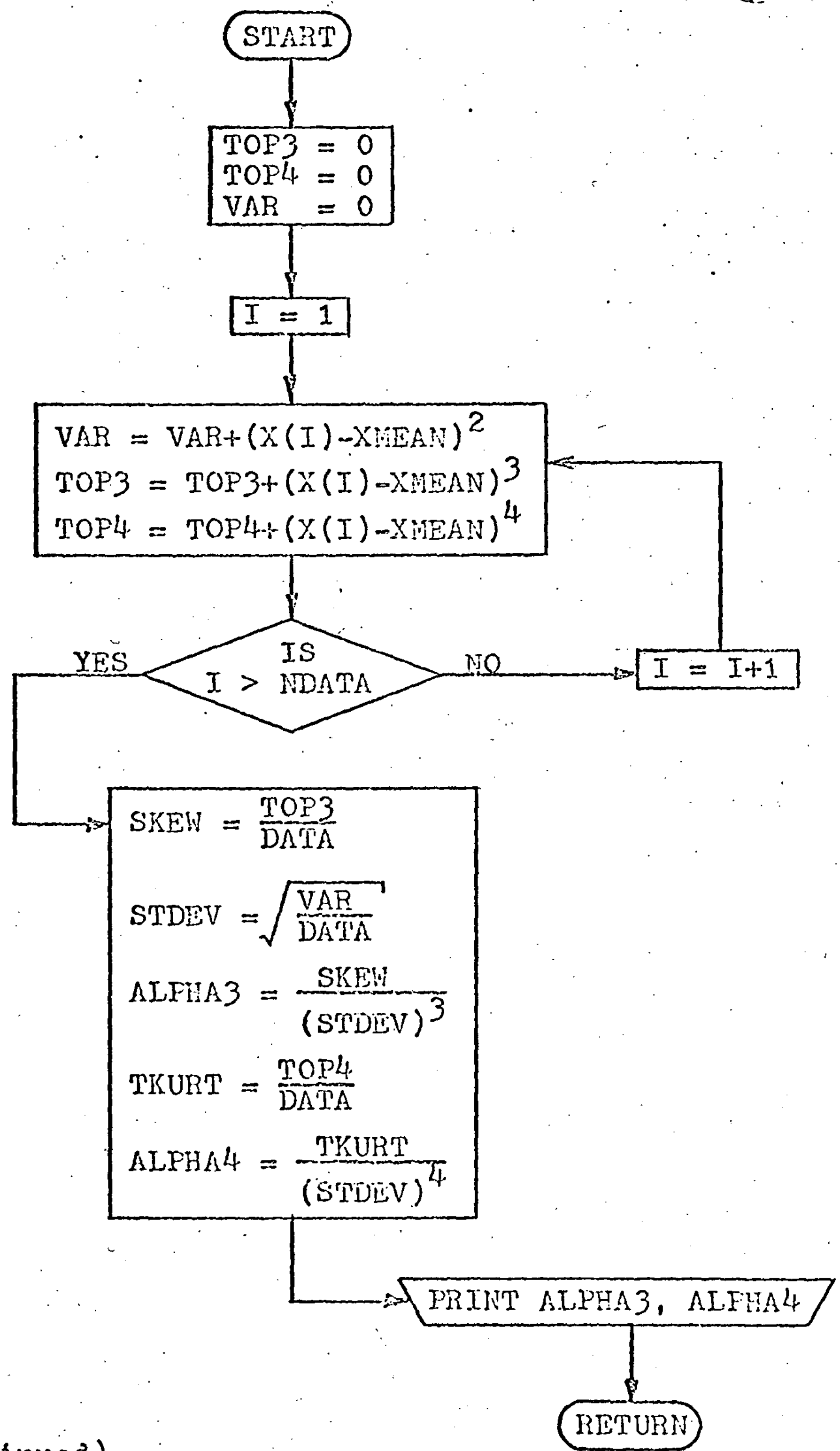

Fig. B-1 (continued)

APPENDIX B (continued)

Fig. B-2 Computer Printout of PROGRAM (CYTOFR)

```
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

                  PROGRAM CYTOFR (INPUT,OUTPUT,TAPE1=INPUT)
            C-----PROGRAM TO FIT NORMAL AND LOG-NORMAL CURVE TO DATA AND CHECK
            C-----GOODNESS OF FIT.
000003            DIMENSION X(100),CSV(9),CEV(9),CLB(9),CUB(9),CUMFRQ(100),
                 1PCAREA(100),DSTAT(100),FREQ(9),AREA(9),REQAREA(9),EXFREQ(9),U(9),
                 2Z(100),NX(100),RANK(100)
000003            EXTERNAL PROB
000003      710   PRINT 1
            C-----NDATA=DATA=NUMBER OF OBSERVATIONS
            C-----STRLV = STRESS LEVEL IN PSI.
000007            READ 6,NDATA,DATA,AKURCY,STRLEV, RATIO
            C-----X= NUMBER OF CYCLES TO FAILURE
000025      6     FORMAT(I3,F5.1,F9.4,F10.1,F8.5)
000025            IF (EOF,1) 56,55
000030      55    READ 7,( X(I), I=1,NDATA)
000043      7     FORMAT(8F10.0)
            C-----ROUTINE TO CALCULATE CUMULATIVE DISTRIBUTION FOR DATA.
            C-----READ CUMULATIVE VALUE FOR EACH POINT.
000043            READ 701, (CUMFRQ(I), I=1, NDATA)
000056      701   FORMAT (26F3.0)
            C-----PCAREA = F(N) OF OBSERVATIONS
000056            DO 759 I=1, NDATA
000060      759   PCAREA(I) = CUMFRQ(I)/DATA
000064            PRINT 405
000070      405   FORMAT (40X,53HNORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DAT
                 1A.///)
000070            IF (RATIO.EQ.0.0) GO TO 414
000071            PRINT 402, STRLEV, RATIO
000101      402   FORMAT(29X,14H STRESS LEVEL=,F10.1,5H PSI.,16X
                 1,14HSTRESS RATIO =,F6.3//)
000101            GO TO 415
000102      414   PRINT 416, STRLEV
000110      416   FORMAT(29X,14HSTRESS LEVEL=,F10.1,5H PSI.,16X
                 1,23HSTRESS RATIO = INFINITY//)
000110      415   PRINT 404
000114      404   FORMAT(55X,22HCYCLES TO FAILURE DATA/)
000114            PRINT 403, (X(I),I=1,NDATA)
000127      403   FORMAT (6(10X,F10.1))
000127            PRINT 3
000133      3     FORMAT (1H0)
000133            CALL MEAN(X, DATA, NDATA, XMEAN, DEV)
000137            CALL CHISQA(X, DATA, NDATA, PROB, AKURCY, XMEAN, DEV, Z)
000147            CALL DIEST (PCAREA,NDATA,X,DEV,DSTAT,PROB,XMEAN, Z)
000157            CALL ALPHA(X, NDATA, DATA, XMEAN, DEV, ALPHA3, ALPHA4)
000166            AKURCY =  .00001
000170            DO 54 I=1,NDATA
000171            NX(I) =(ALOG(X(I)/20.)+ALOG(20.))*100000.  + .5
000204            X(I) = NX(I)
000206      54    X(I) = X(I)/100000.
000212            PRINT 1
000216      1     FORMAT (1H1,3HASD/// )
000216            PRINT 401
000222      401   FORMAT (38X,57HLOG-NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE
                 1 DATA.///)
000222            IF (RATIO.EQ.0.0) GO TO 417
000223            PRINT 402, STRLEV, RATIO
000233            GO TO 418
```

```
000234    417    PRINT 416, STRLEV
000242    418    PRINT 2
000246    2      FORMAT (49X,34HLOGS OF THE CYCLES TO FAILURE DATA/)
000246           PRINT 413, (X(I),I=1,NDATA)
000261    413    FORMAT (6(8X,F12.5))
000261           PRINT 3
000265           CALL MEAN(X, DATA, NDATA, XMEAN, DEV)
000271           CALL CHISQA(X, DATA, NDATA, PROB, AKURCY, XMEAN, DEV, 7)
000301           CALL DTEST (PCAREA,NDATA,X,DEV,DSTAT,PROB,XMEAN, 4)
000311           CALL ALPHA(X, NDATA, DATA, XMEAN, DEV, ALPHA3, ALPHA4)
000320           GO TO 718
000321    56     STOP
000323           END
```

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```
              SUBPOUTINE MEAN (X, DATA, NDATA, XMEAN, DEV)
        C-----SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.
000010        DIMENSION X(NDATA)
000010        SIGMA= 0.0
000011        DO 8 I=1, NDATA
000012  8     SIGMA=SIGMA+ X(I)
000016        XMEAN = SIGMA/DATA
000017        TOP2 = 0.0
000020        DO 9 I=1,NDATA
000021  9     TOP2 = TOP2 + (X(I) - XMEAN)**2
000026        DEV =SQRT(TOP2/(DATA - 1.0))
000036        PRINT 14, XMEAN
000043        PRINT 15, DEV
000054  14    FORMAT( 10X,  12HSAMPLE MEAN=, F17.6)
000054  15    FORMAT( 10X,  15HSTD. DEVIATION=, F14.6)
000054        RETURN
000055        END
```

```
                 FUNCTION PROB(X)
          C-----THIS SUBROUTINE GIVES AREA UNDER NORMAL CURVE FROM -Z TO +Z
          C      WITH AN ACCURACY OF 0.00005
          C-----Z VALUE GIVEN BY CALLING PROGRAM MUST BE A POSITIVE NUMBER.
000003           IF (X-1.2)11,11,12
000005    11     XSQ=X*X
000006           PROB= 0.79788455*X*(0.99999774-XSQ*(0.16659433-XSQ*(0.024638310-XS
                1Q*0.0023974867)))
000016           RETURN
000017    12     IF(X-2.9) 13,14,14
000022    13     XSQ=X*X
000023           PROB=1.0
000024           PTERM=1.0
000025           FACTOR=1.0
000026           ODDINT=3.0
000030    970    PTERM=-PTERM*XSQ/(2.0*FACTOR)
000034           TERM=PTERM/ODDINT
000035           PROB=PROB+TERM
000037           IF( ABS (TERM) - 0.00007 ) 80,90,90
000042    90     FACTOR =FACTOR+1.0
000044           ODDINT=ODDINT+2.0
000046           GO TO 970
000046    80     PROB=0.79788455*X*PROB
000051           RETURN
000051    14     RECXSQ= 1.0 / (X*X)
000053           PROB= 1.0 - 0.79788453*EXP(-X*X/2.0)/X*(1.0-RECXSQ*(1. -RECXSQ*(3.
                1 - RECXSQ*(15. - RECXSQ*105. ))))
000075           RETURN
000076           END
```

```
              SUBROUTINE CHISQA (X, DATA, NDATA, PROB, AKURCY, XMEAN, DEV,Z)
        C-----SUBROUTINE TO FIT A HISTOGRAM TO THE DATA AND PERFORM THE CHI-SQUARE
        C-----TEST FOR THE NORMAL OR LOG-NORMAL DISTRIBUTIONS.
000013        DIMENSION X(NDATA),Z(NDATA),CSV(9),CEV(9),CLB(9),CUB(9),
             1REQAREA(9),AREA(9),EXFREQ(9),    FREQ(9),    U(9)
000013        CHISQR= .0
        C-----TO DETERMINE THE NUMBER OF CLASS INTERVALS,K
000013        K= 1.5 +3.322*ALOG10(DATA)
000024        REALK=K
        C-----IN ORDER TO DETERMINE THE RANGE,FIND X(MAX) AND X(MIN)
000025        XMAX=X(1)
000027        XMIN= X(1)
000030        DO 17 I=1,NDATA
000032        IF( X(I).GT.XMAX ) XMAX = X(I)
000037  17    IF(X(I).LT. XMIN) XMIN=X(I)
000046         RANGE=  XMAX- XMIN
        C------TO DETERMINE THE CLASS INTERVAL WIDTH,W
        C-----ROUTINE TO ROUND OFF CLASS WIDTH TO SAME NUMBER OF PLACES AS THE ACCURACY
000050        DIVIDE = 1.0/AKURCY
000051        KW =(((RANGE+AKURCY)/REALK)+.5*AKURCY)*DIVIDE
000057        RK1 = KW
000060        W = RK1/DIVIDE
000062        PRINT 62,XMAX
000067        PRINT 63,XMIN
000075        PRINT 65,W
000103        DO 22 I=1,K
000110        A=I
000111        B = 0.5*AKURCY
000112        CSV(I)= XMIN+(A-1.0)*W
000117        CEV(I)= CSV(I)+W-AKURCY
000123        CLB(I)= CSV(I)-B
000127  22    CUB( I ) = CEV(I)+B
000135        CEV(K) = XMAX
000137        CUB(K) = CEV(K) +B
000143        DO 23 J=1,K
000144  23    FREQ(J)=0.0
000150        DO 24 I=1,NDATA
000152        DO 24 J=1,K
000153        IF( X(I).GE.CSV(J).AND. X(I).LE.CEV(J)   ) FREQ(J)=FREQ(J)+ 1.0
000172  24    CONTINUE
        C-----CHI-SQUARE TEST
000177        PRINT 41
000202        PRINT 406
000206        DO 30 I=1,K
000213        Z(I)=( CUB(I)- XMEAN) / DEV
000220        T= ABS( Z(I) )
000224  30    AREA(I)= PROB(T)/2.0
000243        REQAREA(1) = 0.5 - AREA(1)
000244         MANU=K-1
000246        DO 32 I=2,MANU
000247        M=I-1
000250        IF(  (Z(I).GE.0.0.AND.Z(M).GE.0.0).OR.( Z(I).LE.0.0.AND.Z(M).LE.0.
             1 )  ) GO TO 31
000272        REQAREA(I)= AREA(I)+AREA(M)
000300        GO TO 32
000300  31    REQAREA(I) = ABS( AREA(I)-AREA(M) )
000310  32    CONTINUE
```

```
000313          REQAREA(K)= 0.5-AREA(K-1)
000317          DO 80 M=1,K
000320          EXFREQ(M)=DATA*REQAREA(M)
000323          U(M)=(( EXFREQ(M)-FREQ(M))**2)/EXFREQ(M)
000331    80    CHISQR=CHISQR+U(M)
          C-----TO PRINT THE TABLE FOR CHI-SQUARE TEST
000337          DO 33 I=1,K
000341    33    PRINT 34,I,CLB(I),CUB(I), EXFREQ(I),FREQ(I),U(I)
000412          PRINT 35, CHISQR
000420    62    FORMAT( 10X, 14HMAXIMUM VALUE=,F15.6)
000420    63    FORMAT( 10X, 14HMINIMUM VALUE=, F15.6)
000420    65    FORMAT( 10X, 12HCLASS WIDTH=, F17.6)
000420    41    FORMAT(1H0)
000420    406   FORMAT (5X,5H CELL,10X,10HLOWER CELL,11X,10HUPPER CELL,13X,8HEXPEC
               1TED,13X,8HOBSERVED,13X,11HCHI-SQUARED/8X,6HNUMBER,10X,8HBOUNDARY,
               213X, 8HBOUNDARY,13X,9HFREQUENCY,12X,9HFREQUENCY,12X,13HVALUE OF CE
               3LL/)
000420    34    FORMAT (10X,I2,5F21.6)
000420    35    FORMAT (1H0,81X,25HTOTAL CHI-SQUARED VALUE =,F10.6)
000420          RETURN
000421          END
```

```
                  SUBROUTINE DTEST (PCAREA,NDATA,X,DEV,DSTAT,PROB,XMEAN,Z)
            C-----SUBROUTINE TO CALCULATE THE KOLMOGOROV-SMIRNOV D-VALUES.
000013            DIMENSION PCAREA(NDATA),X(NDATA),Z(NDATA),DSTAT(NDATA)
000013            DO 706 I=1, NDATA
000014            Z(I) = (X(I) - XMEAN)/DEV
000021            IF (Z(I)) 703, 704, 705
000024      703   T =ABS(Z(I))
            C-----ARNCN=AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR NEGATIVE Z.
000027            ARUNCN = (1.0-PROB(T))/2.0
000043            DSTAT(I) = ARUNCN - PCAREA(I)
000050            GO TO 706
000050      704   DSTAT(I) = .5 - PCAREA(I)
000054            GO TO 706
000054      705   T = Z(I)
            C-----ARUNCP=AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR POSITIVE Z.
000057            ARUNCP = PROB(T)/2.0 +  .500
000072            DSTAT(I) =     (ARUNCP - PCAREA(I))
000077      706   CONTINUE
000102            PRINT 708
000105            PRINT 707, (DSTAT(I),I=1,NDATA)
000130      707   FORMAT (6(10X,F10.5))
000130      708   FORMAT (//40X,53H D VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT
                 1TEST/41X,52H(LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA)/)
000130            RETURN
000131            END
```

```
                SUBROUTINE ALPHA (X, NDATA, DATA, XMEAN, DEV, ALPHA3, ALPHA4)
000012          DIMENSION X(NDATA)
          C------SUBROUTINE TO CALCULATE THE COEFFICIENTS OF SKEWNESS AND KURTOSIS
          C------CALCULATE THE THIRD MOMENT OF THE DATA (SKEWNESS)
000012          TOP3 = 0.0
000012          VAR = 0.0
000014          DO 710 I =1, NDATA
000015          VAR = VAR + (X(I) - XMEAN)**2
000020    710   TOP3 = TOP3 +(X(I) - XMEAN)**3
000027          SKEW = TOP3 / DATA
000030          STDEV = SQRT(VAR/DATA)
          C------ALPHA3 = MOMENT COEFFICIENT OF SKEWNESS.
000037          ALPHA3 = SKEW/(STDEV**3)
          C------CALCULATE THE FOURTH MOMENT OF THE DATA (KURTOSIS).
000041          TOP4 = 0.0
000042          DO 711 I = 1, NDATA
000044    711   TOP4 = TOP4 + (X(I) - XMEAN)**4
000052          TKURT = TOP4 / DATA
          C------ALPHA4 = MOMENT COEFFICIENT OF KURTOSIS.
000053          ALPHA4 = TKURT/(STDEV**4)
000056          PRINT 712
000062          PRINT 713
000066          PRINT 714, ALPHA3, ALPHA4
000102    712   FORMAT (///19X,39HMOMENT COEFFICIENT OF SKEWNESS (ALPHA3),18X,39HM
               1OMENT COEFFICIENT OF KURTOSIS (ALPHA4)/)
000102    713   FORMAT     (21X,34HFOR NORMAL DISTRIBUTION ALPHA3 = 0,23X,36HFOR NO
               1RMAL DISTRIBUTION ALPHA4 = 3.0/)
000102    714   FORMAT (28X,25HFOR ABOVE DATA---ALPHA3 =F6.3,26X,25HFOR ABOVE DATA
               1---ALPHA4 =,F6.3)
000102          RETURN
000103          END
```

Fig. B-3 Output of PROGRAM (CYTOFR)

ASD

NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DATA.

STRESS LEVEL=, 114000.0 PSI. STRESS RATIO = INFINITY

CYCLES TO FAILURE DATA

10540.0	10353.0	10347.0	9990.0	9818.0	9747.0
9362.0	9302.0	9261.0	9092.0	8925.0	8860.0
8376.0	8088.0	8015.0	7717.0	7622.0	7112.0

SAMPLE MEAN= 9029.277778
STD. DEVIATION= 1024.216820
MAXIMUM VALUE= 10540.000000
MINIMUM VALUE= 7112.000000
CLASS WIDTH= 686.000000

CELL NUMBER	LOWER CELL BOUNDARY	UPPER CELL BOUNDARY	EXPECTED FREQUENCY	OBSERVED FREQUENCY	CHI-SQUARED VALUE OF CELL
1	7111.500000	7797.500000	2.261984	3.000000	.426713
2	7797.500000	8483.500000	3.245106	3.000000	.024744
3	8483.500000	9169.500000	4.632969	3.000000	.575568
4	9169.500000	9855.500000	4.241336	5.000000	.135708
5	9855.500000	10540.500000	3.774612	4.000000	.012971

TOTAL CHI-SQUARED VALUE = 1.175703

D VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
(LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA) .

-.07611	-.04255	.01199	-.00745	.00159	.03605
-.03931	-.00613	.03394	.02442	.01501	.04547
-.07154	-.09874	-.06122	-.06662	-.02639	-.02495

MOMENT COEFFICIENT OF SKEWNESS (ALPHA3)

FOR NORMAL DISTRIBUTION ALPHA3 = 0

FOR ABOVE DATA---ALPHA3 = -.247

MOMENT COEFFICIENT OF KURTOSIS (ALPHA4)

FOR NORMAL DISTRIBUTION ALPHA4 = 3.0

FOR ABOVE DATA---ALPHA4 = 1.995

ASD

LOG-NORMAL DISTRIBUTION FITTED TO CYCLES-TO-FAILURE DATA.

STRESS LEVEL=, 114000.0 PSI.   STRESS RATIO = INFINITY

LOGS OF THE CYCLES TO FAILURE DATA

9.26293	9.24503	9.24445	9.20934	9.19197	9.18471
9.14441	9.13798	9.13357	9.11515	9.09661	9.08930
9.03313	8.99814	8.98907	8.95118	8.93879	8.86954

SAMPLE MEAN=	9.101961
STD. DEVIATION=	.116130
MAXIMUM VALUE=	9.262930
MINIMUM VALUE=	8.869540
CLASS WIDTH=	.078680

CELL NUMBER	LOWER CELL BOUNDARY	UPPER CELL BOUNDARY	EXPECTED FREQUENCY	OBSERVED FREQUENCY	CHI-SQUARED VALUE OF CELL
1	8.869535	8.948215	1.669727	2.000000	.065328
2	8.948215	9.026895	2.992467	3.000000	.000019
3	9.026895	9.105575	4.561237	3.000000	.534386
4	9.105575	9.184255	4.469620	4.000000	.049343
5	9.184255	9.262935	4.306949	6.000000	.665534

TOTAL CHI-SQUARED VALUE = 1.314610

D VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
(LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA)

-.08286	-.05342	.00120	-.01090	.00307	.03972
-.02402	.01067	.05171	.04521	.03718	.06770
-.05665	-.09212	-.05673	-.06959	-.03111	-.03288

MOMENT COEFFICIENT OF SKEWNESS (ALPHA3)

FOR NORMAL DISTRIBUTION ALPHA3 = 0

FOR ABOVE DATA---ALPHA3 = -.407

MOMENT COEFFICIENT OF KURTOSIS (ALPHA4)

FOR NORMAL DISTRIBUTION ALPHA4 = 3.0

FOR ABOVE DATA---ALPHA4 = 2.123

APPENDIX C

CALIBRATION CONSTANTS OF EACH MACHINE FOR THE DIFFERENT MODES

Mode 1: 26 July, 1966 to 10 August, 1967

Mode 2: 11 August, 1967 to 1 June, 1969

Machine	K_{bgr}	K_{gr-th}	K_t	$K_{t/b}$	$K_{b/t}$
1	1.0123	0.0208	0.8752	-0.0459	0.0290
2	1.0123	0.0188	0.8201	0.0344	0.0422
3	1.0946	0.0211	0.9330	0.0	-0.0149

Mode 3: 2 June, 1969 to 1 February, 1970

Machine	K_{bgr}	K_{gr-th}	K_t	$K_{t/b}$	$K_{b/t}$
1	1.0123	0.0208	0.8752	-0.0459	0.0290
2	1.0123	0.0188	0.8201	0.0344	0.0422
3	1.0946	0.0211	0.7721	0.0	-0.0127

APPENDIX C (continued)

CALIBRATION CONSTANTS OF EACH MACHINE FOR THE DIFFERENT MODES

Mode 4: 2 February, 1970 to next change

Machine	K_{bgr}	K_{gr-th}	K_t	$K_{t/b}$	$K_{b/t}$
1	1.0123	0.0208	0.8752	-0.0459	-0.0290
2	1.0123	0.0188	0.8201	0.0344	0.0422
3	1.0123	0.0197	0.7721	0.0	-0.0127

APPENDIX D

COMPUTER OUTPUTS LISTING STRESS
LEVELS AND RATIOS OF INDIVIDUAL
SPECIMENS FOR THE VARIOUS STRESS LEVELS

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
125	166	2	29.0	2973	190000	50.20	59.30	-0	-0.00	-0.00	144126	0	INFIN.
126	204	2	29.0	2973	90000	50.10	28.20	-0	-0.00	-0.00	144982	0	INFIN.
130	225	2	29.0	3063	90000	49.70	27.40	-0	-0.00	-0.00	142003	0	INFIN.
131	133	2	29.0	3033	90000	50.00	27.90	-0	-0.00	-0.00	143727	0	INFIN.
132	196	2	29.0	2884	90000	49.40	28.00	-0	-0.00	-0.00	145994	0	INFIN.
133	220	2	29.0	3181	90000	49.90	28.20	-0	-0.00	-0.00	145563	0	INFIN.
134	193	2	29.0	2765	90000	49.60	28.00	-0	-0.00	-0.00	145405	0	INFIN.
135	191	2	29.0	3181	90000	50.40	28.40	-0	-0.00	-0.00	145141	0	INFIN.
136	163	2	29.0	3271	90000	50.20	28.40	-0	-0.00	-0.00	145719	0	INFIN.
425	342	1	30.0	2471	150000	50.20	51.60	-0	-0.00	-0.00	143580	0	INFIN.
426	341	1	30.0	1548	150000	50.10	50.70	-0	-0.00	-0.00	141357	0	INFIN.
428	365	1	30.0	1935	150000	50.30	51.55	-0	-0.00	-0.00	143156	0	INFIN.

MEAN BENDING STRESS IN GROOVE = 144229.4 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 1505.57 PSI.

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
29	63	1	20.0	10540	290000	32.40	51.00	-0	-0.00	-0.00	113728	0	INFIN.
30	22	1	20.0	8088	290000	33.20	52.10	-0	-0.00	-0.00	113381	0	INFIN.
31	109	1	20.0	9990	290000	32.90	51.40	-0	-0.00	-0.00	112878	0	INFIN.
32	82	1	20.0	10347	190000	50.20	51.40	-0	-0.00	-0.00	112913	0	INFIN.
33	42	1	20.0	10353	190000	50.50	52.40	-0	-0.00	-0.00	114426	0	INFIN.
34	59	1	20.0	9818	190000	50.50	52.70	-0	-0.00	-0.00	115081	0	INFIN.
35	52	1	20.0	9747	190000	49.30	50.80	-0	-0.00	-0.00	113633	0	INFIN.
36	76	1	20.0	8860	190000	50.70	51.50	-0	-0.00	-0.00	112017	0	INFIN.
37	64	1	20.0	9302	190000	50.30	51.60	-0	-0.00	-0.00	113127	0	INFIN.
38	111	1	20.0	8925	190000	50.40	52.00	-0	-0.00	-0.00	113778	0	INFIN.
39	20	1	20.0	8376	190000	50.40	52.00	-0	-0.00	-0.00	113778	0	INFIN.
40	73	1	20.0	9261	190000	49.90	51.30	-0	-0.00	-0.00	113371	0	INFIN.
41	69	1	20.0	7717	190000	49.30	50.90	-0	-0.00	-0.00	113856	0	INFIN.
43	86	1	20.0	7622	190000	49.30	51.40	-0	-0.00	-0.00	114975	0	INFIN.
44	96	1	20.0	7112	190000	48.90	50.60	-0	-0.00	-0.00	114111	0	INFIN.
45	80	1	20.0	9362	190000	49.20	51.50	-0	-0.00	-0.00	115433	0	INFIN.
46	101	1	20.0	9092	190000	49.40	51.20	-0	-0.00	-0.00	114296	0	INFIN.
47	89	1	20.0	8015	190000	49.00	51.10	-0	-0.00	-0.00	115003	0	INFIN.

MEAN BENDING STRESS IN GROOVE = 113877.1 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 888.70 PSI.

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
10	26	1	15.0	22544	290000	39.40	51.00	-0	-0.00	-0.00	93522	0	INFIN.
11	62	2	16.0	27464	290000	41.10	49.80	-0	-0.00	-0.00	96858	0	INFIN.
12	88	1	15.0	27250	290000	38.20	51.00	-0	-0.00	-0.00	96460	0	INFIN.
13	46	2	16.0	21080	290000	40.00	51.00	-0	-0.00	-0.00	101920	0	INFIN.
14	58	1	15.0	27024	290000	38.20	51.00	-0	-0.00	-0.00	96460	0	INFIN.
15	99	2	16.0	16920	290000	40.00	50.50	-0	-0.00	-0.00	100920	0	INFIN.
16	33	1	15.0	18136	290000	38.40	50.50	-0	-0.00	-0.00	95017	0	INFIN.
17	85	2	16.0	21192	290000	40.00	50.00	-0	-0.00	-0.00	99921	0	INFIN.
18	55	1	15.0	16500	290000	38.40	50.50	-0	-0.00	-0.00	95017	0	INFIN.
19	49	2	16.0	22886	290000	39.40	49.10	-0	-0.00	-0.00	99617	0	INFIN.
20	110	1	15.0	23640	290000	39.70	52.00	-0	-0.00	-0.00	94636	0	INFIN.
21	104	2	16.0	27558	290000	40.60	50.40	-0	-0.00	-0.00	99232	0	INFIN.
22	113	1	15.0	21204	290000	37.30	49.70	-0	-0.00	-0.00	96270	0	INFIN.
23	65	2	16.0	20576	290000	40.00	50.30	-0	-0.00	-0.00	100521	0	INFIN.
24	39	1	15.0	16258	290000	38.00	51.70	-0	-0.00	-0.00	98299	0	INFIN.
25	13	2	16.0	25196	290000	39.40	49.50	-0	-0.00	-0.00	100428	0	INFIN.
26	71	1	15.0	19352	290000	38.90	51.10	-0	-0.00	-0.00	94910	0	INFIN.
27	84	2	16.0	24304	290000	39.90	50.60	-0	-0.00	-0.00	101374	0	INFIN.

MEAN BENDING STRESS IN GROOVE = 97854.5 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 2687.70 PSI.

150

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
49	102	1	10.0	64997	290000	45.30	51.30	-0	-0.00	-0.00	81820	0	INFIN.
50	118	1	10.0	67757	290000	45.60	51.30	-0	-0.00	-0.00	81282	0	INFIN.
51	75	1	10.0	83812	290000	46.20	52.00	-0	-0.00	-0.00	81321	0	INFIN.
52	34	1	10.0	78852	290000	45.10	50.20	-0	-0.00	-0.00	80421	0	INFIN.
53	45	1	10.0	60002	290000	45.20	51.70	-0	-0.00	-0.00	82641	0	INFIN.
54	94	1	10.0	64857	290000	46.10	51.70	-0	-0.00	-0.00	81027	0	INFIN.
55	35	1	10.0	71968	290000	45.50	50.80	-0	-0.00	-0.00	80667	0	INFIN.
56	66	1	10.0	86349	290000	44.70	49.00	-0	-0.00	-0.00	79201	0	INFIN.
57	70	1	10.0	82248	290000	46.00	52.10	-0	-0.00	-0.00	81832	0	INFIN.
58	25	1	10.0	74690	290000	45.50	50.40	-0	-0.00	-0.00	80031	0	INFIN.
60	31	1	10.0	75662	290000	46.10	51.60	-0	-0.00	-0.00	80871	0	INFIN.
61	114	1	10.0	71166	290000	45.90	52.80	-0	-0.00	-0.00	83112	0	INFIN.
62	29	1	10.0	71556	290000	45.20	51.30	-0	-0.00	-0.00	82001	0	INFIN.
63	18	1	10.0	73089	290000	46.70	52.50	-0	-0.00	-0.00	81224	0	INFIN.
65	48	1	10.0	96004	290000	44.90	50.60	-0	-0.00	-0.00	81423	0	INFIN.
67	81	1	10.0	107415	290000	44.40	50.00	-0	-0.00	-0.00	81363	0	INFIN.
68	19	1	10.0	98468	290000	44.40	50.00	-0	-0.00	-0.00	81363	0	INFIN.
69	30	1	10.0	74698	290000	43.20	48.90	-0	-0.00	-0.00	81784	0	INFIN.

MEAN BENDING STRESS IN GROOVE = 81299.1 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 902.19 PSI.

152

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
139	135	2	6.0	125950	190000	49.60	30.00	-0	-0.00	-0.00	73796	0	INFIN.
140	167	2	6.0	103413	190000	50.20	30.40	-0	-0.00	-0.00	73886	0	INFIN.
142	198	2	6.0	145783	190000	50.20	29.20	-0	-0.00	-0.00	70969	0	INFIN.
143	214	2	6.0	195824	190000	49.80	29.40	-0	-0.00	-0.00	72029	0	INFIN.
145	215	2	6.0	93303	190000	49.50	29.20	-0	-0.00	-0.00	71973	0	INFIN.
146	134	2	6.0	178192	190000	49.40	30.10	-0	-0.00	-0.00	74341	0	INFIN.
147	125	2	6.0	196894	190000	49.20	29.60	-0	-0.00	-0.00	73404	0	INFIN.
148	186	2	6.0	183306	190000	50.10	29.80	-0	-0.00	-0.00	72572	0	INFIN.
150	159	2	6.0	177508	190000	49.50	30.40	-0	-0.00	-0.00	74931	0	INFIN.
151	148	2	6.0	203019	190000	49.90	29.60	-0	-0.00	-0.00	72374	0	INFIN.
152	128	2	6.0	155178	190000	52.20	31.20	-0	-0.00	-0.00	72925	0	INFIN.
153	219	2	6.0	205041	190000	49.90	30.60	-0	-0.00	-0.00	74819	0	INFIN.
154	123	2	6.0	172661	190000	50.10	31.20	-0	-0.00	-0.00	75982	0	INFIN.
155	121	2	6.0	124137	190000	49.70	30.00	-0	-0.00	-0.00	73647	0	INFIN.
156	145	2	6.0	127794	190000	50.00	29.90	-0	-0.00	-0.00	72961	0	INFIN.
157	171	2	6.0	172572	190000	49.70	30.40	-0	-0.00	-0.00	74629	0	INFIN.
160	156	2	6.0	172275	190000	49.50	27.60	-0	-0.00	-0.00	68029	0	INFIN.
163	183	2	6.0	182860	190000	49.20	29.10	-0	-0.00	-0.00	72164	0	INFIN.

MEAN BENDING STRESS IN GROOVE = 73079.6 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 1788.48 PSI.

TEST MODE = 2

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
326	361	1	34.0	1518	125000	49.60	46.00	304000	45.00	19.70	156834	24846	3.644
327	418	1	34.0	1310	125000	49.50	45.10	304000	44.80	19.40	154085	24607	3.615
329	433	1	34.0	1280	125000	49.90	45.00	304000	45.10	19.40	152516	24458	3.600
331	349	1	31.0	1697	125000	49.90	45.10	304000	45.00	20.00	152897	25403	3.475
332	381	1	32.0	1607	125000	50.30	44.10	304000	45.00	20.30	148381	25977	3.298
333	369	1	32.0	1399	125000	49.90	42.60	304000	45.00	19.00	144429	24156	3.452
334	348	1	33.0	1756	125000	50.00	44.30	304000	45.00	19.10	149849	24152	3.582
338	368	1	31.0	1756	125000	49.90	43.50	304000	45.00	19.80	147508	25258	3.372
584	632	1	35.0	982	125000	49.95	45.65	304000	45.20	20.15	154595	25446	3.508
586	647	1	35.0	1131	125000	50.00	45.65	304000	45.10	19.65	154410	24776	3.598
627	612	1	32.0	1191	125000	24.90	22.11	304000	22.48	9.21	150130	23157	3.743
628	658	1	32.0	1786	125000	25.00	21.58	304000	22.52	10.06	146097	25747	3.276

MEAN BENDING STRESS IN GROOVE = 150977.5 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 3841.69 PSI.

MEAN TORQUE STRESS IN GROOVE = 24832.0 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 794.91 PSI.

MEAN STRESS RATIO = 3.51372
STD. DEV. OF STRESS RATIO = .14394

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
532	473	1	25.0	8722	125000	49.70	33.35	304000	45.20	14.50	113488	18247	3.591
605	584	1	20.0	6489	125000	49.85	33.15	304000	44.95	15.35	112543	19658	3.305
607	656	1	21.0	7531	125000	49.90	33.50	304000	45.30	15.95	113640	20337	3.226
608	570	1	20.0	5120	125000	50.00	33.55	304000	45.15	15.60	113562	19899	3.295
612	588	1	20.0	6638	125000	50.20	33.55	304000	45.10	14.10	113010	17715	3.683
613	563	1	20.0	5060	125000	49.85	33.95	304000	45.10	14.75	115187	18618	3.572
614	597	1	20.0	6846	125000	49.85	34.40	304000	45.20	15.05	116719	18970	3.552
618	624	1	22.0	5924	125000	49.77	33.50	304000	44.95	15.24	113893	19456	3.380
619	630	1	22.0	4644	125000	49.88	34.16	304000	45.10	15.50	115876	19711	3.394
620	569	1	22.0	5715	125000	49.90	34.85	304000	44.90	15.60	118160	19897	3.429
621	581	1	22.0	5507	125000	49.83	33.70	304000	44.96	15.55	114451	19697	3.321
622	637	1	22.0	6191	125000	49.80	33.75	304000	45.08	15.22	114662	19340	3.423
623	582	1	21.0	7561	125000	49.92	34.40	304000	44.80	14.75	116545	18726	3.593
624	578	1	22.0	5418	125000	50.00	33.28	304000	44.90	15.55	112659	19778	3.256
625	622	1	22.0	5864	125000	50.10	33.40	304000	44.92	15.63	112843	20082	3.244
626	583	1	21.0	5745	125000	49.85	33.55	304000	45.00	15.20	113876	19372	3.394
629	554	1	22.0	6400	125000	49.96	34.94	304000	45.06	15.12	118284	19099	3.576
630	579	1	22.0	6281	125000	49.76	33.30	304000	44.95	14.57	113195	18479	3.537

MEAN BENDING STRESS IN GROOVE = 114588.5 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 1820.89 PSI.

MEAN TORQUE STRESS IN GROOVE = 19304.5 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 718.79 PSI.

MEAN STRESS RATIO = 3.43179
STD. DEV. OF STRESS RATIO = .14174

TEST MODE = 3

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
366	434	1	12.0	32624	190000	50.10	38.00	304000	45.30	10.25	84356	12749	3.820
368	420	1	12.0	37089	190000	50.20	37.55	304000	45.25	11.00	83254	13905	3.457
369	350	1	12.0	46198	190000	50.30	38.50	304000	45.30	12.00	85241	15308	3.215
374	410	1	11.0	42001	190000	50.00	38.20	304000	45.15	11.50	85054	14631	3.356
376	428	1	12.0	36494	190000	50.00	37.60	304000	45.10	11.20	83710	14243	3.393
435	520	1	11.0	37298	190000	49.85	37.20	304000	45.10	10.30	83012	12928	3.707
438	479	1	11.0	54116	190000	50.00	37.60	304000	45.10	10.10	83634	12614	3.828
439	542	1	11.0	27981	190000	49.95	37.70	304000	45.00	10.75	83984	13603	3.565
440	521	1	11.0	34797	190000	50.15	37.75	304000	45.10	11.15	83789	14167	3.415
445	539	1	10.0	38072	190000	50.05	37.30	304000	45.15	10.90	82945	13802	3.470
448	501	1	10.0	48668	190000	50.10	37.50	304000	45.15	10.15	83251	12682	3.790
453	523	1	11.0	39054	190000	50.15	37.90	304000	45.05	10.60	84081	13360	3.634
459	454	1	10.0	81590	190000	50.25	36.95	304000	45.30	10.80	81840	13632	3.466
461	547	1	10.0	34291	190000	50.30	36.75	304000	45.05	10.70	81318	13587	3.456
463	453	1	10.0	36673	190000	50.15	36.80	304000	45.10	10.45	81651	13189	3.574
590	562	1	13.0	37982	190000	49.90	37.00	304000	45.10	10.70	82516	13535	3.520
592	643	1	13.0	22176	190000	50.05	36.50	304000	44.95	10.70	81171	13626	3.439
593	599	1	13.0	25837	190000	49.90	37.25	304000	45.10	11.55	83127	14779	3.248

MEAN BENDING STRESS IN GROOVE = 83218.5 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 1178.33 PSI.

MEAN TORQUE STRESS IN GROOVE = 13685.5 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 740.78 PSI.

MEAN STRESS RATIO = 3.51963
STD. DEV. OF STRESS RATIO = .18021

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TEST MODE = 3

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
467	505	1	8.0	95819	190000	50.17	33.49	304000	45.00	9.23	74259	11619	3.690
469	507	1	8.0	103112	190000	49.40	33.05	304000	45.10	10.13	74486	12917	3.329
470	544	1	8.0	82870	190000	49.95	33.32	304000	44.95	9.35	74217	11814	3.627
471	449	1	8.0	47716	190000	50.20	33.60	304000	45.08	10.30	74530	13175	3.266
472	447	1	8.0	72244	190000	50.00	33.50	304000	45.08	9.75	74567	12358	3.484
476	495	1	8.0	65219	190000	50.05	33.25	304000	45.10	10.35	73984	13257	3.222
477	508	1	7.0	105701	190000	50.10	32.88	304000	45.00	9.75	73056	12426	3.394
482	543	1	8.0	94985	190000	49.90	33.65	304000	45.00	9.30	75016	11702	3.701
486	460	1	8.0	101058	190000	50.10	33.20	304000	45.10	9.85	73766	12522	3.401
487	531	1	8.0	75905	190000	50.00	34.05	304000	45.00	10.25	75816	13091	3.344
489	502	1	8.0	55991	190000	49.95	33.30	304000	45.05	9.35	74172	11784	3.634
490	474	1	8.0	65635	190000	50.10	33.15	304000	44.95	10.30	73689	13243	3.213
491	510	1	8.0	55068	190000	50.00	33.90	304000	45.00	9.80	75454	12433	3.504
594	568	1	10.0	39679	190000	50.00	32.60	304000	44.90	9.55	72570	12174	3.442
596	625	1	10.0	62123	190000	49.85	33.15	304000	45.00	9.50	73998	12028	3.552
597	593	1	10.0	68076	190000	49.90	33.50	304000	45.00	9.85	74723	12528	3.444
598	628	1	8.0	75369	190000	49.85	33.10	304000	45.10	10.40	73949	13333	3.202
599	602	1	10.0	69237	190000	50.00	33.15	304000	45.00	9.20	73757	11589	3.675

MEAN BENDING STRESS IN GROOVE = 74222.7 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 774.73 PSI.

MEAN TORQUE STRESS IN GROOVE = 12443.9 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 610.09 PSI.

MEAN STRESS RATIO = 3.45139
STD. DEV. OF STRESS RATIO = .16903

TEST MODE = 3

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
411	354	1	19.0	6906	190000	50.10	47.75	200000	45.20	32.50	108545	70032	.895
412	395	1	21.0	5626	190000	50.35	49.65	200000	45.20	34.30	112375	73972	.877
413	345	1	21.0	8603	190000	50.45	49.90	200000	45.10	33.55	112635	72445	.898
415	335	1	22.0	6072	190000	50.10	49.30	200000	45.15	33.30	112046	71815	.901
416	358	1	21.0	6102	190000	49.95	48.45	200000	45.10	32.90	110456	71042	.898
417	406	1	20.0	7799	190000	50.50	48.60	200000	45.10	34.30	109768	74217	.854
418	387	1	21.0	6043	190000	50.10	49.25	200000	45.10	33.60	111971	72577	.891
419	356	1	21.0	6162	190000	50.45	48.75	200000	45.30	35.50	110311	76554	.832
420	397	1	22.0	5388	190000	50.25	49.60	200000	45.30	33.90	112432	72903	.890
421	432	1	21.0	5894	190000	50.35	49.15	200000	45.15	34.55	111311	74651	.861
422	392	1	21.0	6221	190000	50.30	48.65	200000	45.10	35.50	110426	76904	.829
423	390	1	21.0	7829	190000	49.80	48.75	200000	45.15	33.75	111529	72843	.884

MEAN BENDING STRESS IN GROOVE = 111150.5 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 1257.68 PSI.

MEAN TORQUE STRESS IN GROOVE = 73329.6 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 2051.25 PSI.

MEAN STRESS RATIO = .87577
STD. DEV. OF STRESS RATIO = .02569

TEST MODE = 2

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
223	295	3	15.0	32129	190000	50.00	41.60	304000	45.00	42.60	83646	68922	.701
226	274	3	15.0	24356	190000	50.00	41.64	304000	45.00	42.14	83726	68195	.709
228	262	3	15.0	24653	190000	49.80	43.70	304000	45.00	43.10	88221	69792	.730
230	261	3	15.0	21004	190000	50.06	46.00	304000	45.30	43.44	92382	69946	.763
232	312	3	15.0	25009	190000	49.50	51.20	304000	45.10	42.30	103989	68644	.875
233	256	3	15.0	23110	190000	49.90	43.50	304000	45.10	43.40	87641	70105	.722
235	258	3	15.0	25157	190000	50.00	43.40	304000	45.08	41.60	87265	67282	.749
236	254	3	15.0	12964	190000	49.60	44.10	304000	44.88	42.10	89388	68406	.754
237	237	2	15.0	20189	190000	50.10	40.70	304000	45.00	46.50	96420	61404	.907
238	292	2	15.0	20457	190000	49.70	39.60	304000	44.84	42.96	94713	56747	.964
239	307	3	15.0	20203	190000	49.80	41.90	304000	45.00	38.80	84587	62917	.776
240	314	3	15.0	19105	190000	49.80	40.90	304000	44.80	41.20	82569	66977	.712
242	227	3	15.0	28243	190000	49.90	41.30	304000	44.80	40.30	83209	65556	.733
403	426	1	16.0	13038	190000	50.10	43.75	250000	44.90	36.20	99386	62764	.914
405	408	1	16.0	13365	190000	50.00	42.90	250000	44.60	37.50	97837	65619	.861
406	367	1	16.0	14883	190000	50.00	42.70	250000	45.00	37.90	97401	65747	.855
407	370	1	16.0	16699	190000	50.55	43.25	250000	44.80	39.00	97685	68042	.829
408	379	1	16.0	13841	190000	50.15	42.70	250000	44.90	37.80	97118	65726	.853

MEAN BENDING STRESS IN GROOVE = 91510.2 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 6706.89 PSI.

MEAN TORQUE STRESS IN GROOVE = 66266.2 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 3478.00 PSI.

MEAN STRESS RATIO = .80029
STD. DEV. OF STRESS RATIO = .08238

TEST MODE = 2

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
207	172	1	10.0	77840	190000	49.80	32.20	304000	44.30	32.50	73613	46988	.905
209	200	1	10.0	65844	190000	50.10	34.60	304000	45.10	36.90	78735	52499	.866
210	185	1	10.0	69148	190000	50.20	33.90	304000	44.80	35.10	76937	50229	.884
211	176	1	10.0	72006	190000	49.80	33.00	304000	45.00	36.00	75594	51371	.850
212	141	1	10.0	59980	190000	49.80	29.70	304000	44.60	37.00	68379	53555	.737
213	154	1	10.0	67600	190000	50.00	34.00	304000	44.80	35.30	77469	50512	.885
215	218	1	10.0	70190	190000	49.90	33.70	304000	44.90	36.00	77000	51451	.864
217	277	3	10.0	67492	190000	49.90	34.70	304000	44.00	36.60	69912	60500	.667
218	326	3	10.0	75798	190000	50.00	35.30	304000	44.50	37.50	70978	61294	.669
222	285	3	10.0	61796	190000	49.90	36.50	304000	45.00	35.60	73538	57659	.736
384	363	1	10.0	58372	190000	50.30	34.30	304000	45.10	37.50	77817	53415	.841
386	427	1	10.0	59980	190000	50.10	33.75	304000	45.20	37.65	76911	53539	.829
387	404	1	10.0	62242	190000	50.20	34.90	304000	45.25	37.80	79297	53632	.854
390	441	1	10.0	43936	190000	50.10	34.10	304000	44.95	37.80	77706	54050	.830
391	416	1	10.0	46466	190000	49.70	33.45	304000	45.00	37.80	76865	54011	.822
392	405	1	10.0	47567	190000	50.15	34.20	304000	44.90	37.40	77826	53513	.840
393	382	1	10.0	39560	190000	50.15	33.60	304000	45.00	37.60	76515	53724	.822
433	471	1	10.0	52687	190000	49.80	33.95	304000	45.00	38.50	77872	55023	.817

MEAN BENDING STRESS IN GROOVE = 75720.3 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 3145.07 PSI.

MEAN TORQUE STRESS IN GROOVE = 53720.2 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 3437.55 PSI.

MEAN STRESS RATIO = .81769
STD. DEV. OF STRESS RATIO = .06975

TEST MODE = 2

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
245	287	2	7.0	149945	190000	49.80	29.10	304000	44.60	32.30	69404	42972	.933
252	228	3	7.0	137327	190000	49.60	30.70	304000	44.80	27.80	62227	45304	.793
253	230	3	7.0	131957	190000	49.80	30.50	304000	44.60	28.70	61573	46930	.758
254	327	3	7.0	104367	190000	49.50	30.20	304000	44.80	28.00	61337	45607	.777
256	304	3	7.0	129228	190000	49.60	31.00	304000	44.50	29.20	62835	47856	.758
257	321	3	7.0	101193	190000	49.40	30.70	304000	44.80	28.20	62479	45945	.785
258	232	3	7.0	167735	190000	49.20	30.20	304000	44.80	30.10	61711	48956	.728
259	255	3	7.0	159280	190000	49.80	30.30	304000	44.60	28.60	61169	46763	.755
260	272	3	7.0	99383	190000	49.50	31.60	304000	44.70	29.20	64180	47669	.777
261	247	3	7.0	143201	190000	49.70	30.80	304000	45.00	28.70	62304	46535	.773
262	231	3	7.0	135962	190000	49.60	30.90	304000	44.80	29.10	62632	47380	.763
263	229	3	7.0	120565	190000	49.30	31.70	304000	44.60	29.10	64645	47622	.784
264	268	3	7.0	117895	190000	49.50	32.00	304000	44.80	28.90	64993	47102	.797
394	385	1	7.0	100879	190000	49.90	30.55	304000	44.80	32.70	69812	46847	.860
396	337	1	7.0	140350	190000	50.10	30.00	304000	44.85	33.10	68358	47430	.832
398	442	1	7.0	110851	190000	50.15	31.05	304000	44.80	33.35	70621	47794	.853
400	413	1	7.0	142404	190000	50.15	31.15	304000	45.00	33.00	70806	47048	.869
402	360	1	7.0	103856	190000	49.95	31.70	304000	45.00	32.80	72280	46709	.893

MEAN BENDING STRESS IN GROOVE = 65187.0 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 3869.69 PSI.

MEAN TORQUE STRESS IN GROOVE = 46803.8 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 1288.33 PSI.

MEAN STRESS RATIO = .80488
STD. DEV. OF STRESS RATIO = .05562

TEST MODE = 1

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
497	445	1	7.0	121656	300000	50.00	39.00	200000	45.00	35.80	58284	79234	.425
499	548	1	7.0	103290	300000	49.80	39.50	200000	44.50	35.90	59257	80344	.426
501	470	1	6.0	206372	300000	49.80	39.75	200000	45.00	36.00	59576	79649	.432
502	462	1	6.0	84656	300000	49.70	39.45	200000	44.90	35.80	59254	79387	.431
503	545	1	7.0	144517	300000	49.90	40.15	200000	45.00	33.70	59779	74447	.464
504	489	1	7.0	162675	300000	49.85	39.60	200000	44.90	35.50	59265	78707	.435
505	512	1	7.0	67392	300000	49.95	40.25	200000	45.00	35.50	60054	78506	.442
506	484	1	6.0	115137	300000	49.95	39.65	200000	44.90	34.95	59165	77465	.441
507	496	1	6.0	188215	300000	50.05	39.90	200000	44.95	35.45	59452	78499	.437
508	488	1	7.0	116923	300000	49.85	39.90	200000	44.95	36.30	59766	80413	.429
509	500	1	7.0	136034	300000	49.85	40.95	200000	45.30	35.50	61123	77745	.453
510	538	1	6.0	153626	300000	50.00	39.20	200000	45.15	34.90	58455	76934	.439
512	451	1	6.0	116685	300000	49.90	40.85	200000	45.30	35.40	60915	77726	.452
513	539	1	6.0	161395	300000	50.00	40.05	200000	45.15	35.80	59738	78924	.437
514	550	1	7.0	161693	300000	50.10	39.80	200000	45.10	35.70	59271	78801	.434
515	481	1	6.0	148684	300000	49.95	40.30	200000	44.95	35.50	60128	78593	.442
516	515	1	6.0	140350	300000	49.70	39.95	200000	44.90	34.80	59850	77106	.448
633	651	1	7.0	165324	300000	50.22	40.28	200000	45.00	35.82	59827	79236	.436

MEAN BENDING STRESS IN GROOVE = 59620.0 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 704.25 PSI.

MEAN TORQUE STRESS IN GROOVE = 78439.7 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 1399.67 PSI.

MEAN STRESS RATIO = .43899
STD. DEV. OF STRESS RATIO = .01008

TEST MODE = 4

TEST NO.	SPEC. NO.	MACH. NO.	PAN WT.	CYCLES TO FAILURE	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
634	613	1	9.0	93259	300000	24.96	22.43	200000	22.57	20.33	67073	89681	.432
635	601	1	10.0	43757	300000	24.95	23.38	200000	22.53	20.41	69782	90128	.447
637	571	1	9.0	90044	300000	25.04	22.68	200000	22.52	20.19	67550	89238	.437
639	619	1	10.0	53878	300000	25.02	22.88	200000	22.50	20.42	68211	90340	.436
641	558	2	11.0	40675	300000	25.03	23.45	200000	22.39	21.94	68507	90965	.435
642	638	2	11.0	44511	300000	24.96	23.36	200000	22.33	21.48	68502	89253	.443
643	641	3	12.0	55951	300000	24.88	23.12	200000	22.48	22.20	68526	89283	.443
644	551	2	11.0	36631	300000	24.93	23.65	200000	22.55	21.69	69488	89212	.450
645	594	2	11.0	40973	300000	24.92	23.42	200000	22.62	21.93	68774	89964	.441
646	621	3	12.0	67432	300000	24.91	24.08	200000	22.60	22.55	71285	90230	.456
647	646	3	12.0	49425	300000	24.90	24.58	200000	22.56	22.30	72794	89411	.470
648	623	3	12.0	46517	300000	24.94	22.87	200000	22.47	21.94	67622	88275	.442
650	685	3	12.0	50522	300000	24.91	22.89	200000	22.51	22.70	67762	91149	.429
651	651	3	12.0	50404	300000	24.98	23.49	200000	22.48	22.20	69344	89292	.448
662	659	2	11.0	48019	300000	25.02	23.50	200000	22.53	21.98	68707	90547	.438
663	689	2	11.0	50963	300000	25.00	23.25	200000	22.59	21.72	68048	89226	.440
664	660	2	10.0	54769	300000	24.93	23.07	200000	22.50	21.59	67700	89054	.439
666	675	2	10.0	46533	300000	24.91	23.80	200000	22.46	21.94	69953	90623	.446

MEAN BENDING STRESS IN GROOVE = 68868.3 PSI.

STD. DEV. OF BENDING STRESS IN GROOVE = 1424.63 PSI.

MEAN TORQUE STRESS IN GROOVE = 89770.6 PSI.
STD. DEV. OF TORQUE STRESS IN GROOVE = 761.94 PSI.

MEAN STRESS RATIO = .44295
STD. DEV. OF STRESS RATIO = .00946

APPENDIX E

COMPUTER OUTPUTS LISTING STRESS LEVELS AND RATIOS OF INDIVIDUAL SPECIMENS FOR THE ENDURANCE TESTS

ENDURANCE TEST TEST MODE = 1

TEST NO.	SPECIMEN NO.	MACHINE NO.	PAN WEIGHT	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
64	87	2	25.0	290000	49.40	37.40	-0	-0.00	-0.00	60519	0	INFIN.
101	17	1	25.0	490000	53.40	46.80	-0	-0.00	-0.00	37476	0	INFIN.
102	74	2	25.0	490000	49.00	48.50	-0	-0.00	-0.00	46827	0	INFIN.
110	202	1	25.0	390000	49.60	47.20	-0	-0.00	-0.00	51125	0	INFIN.
117	182	1	25.0	190000	49.80	25.90	-0	-0.00	-0.00	57353	0	INFIN.
122	124	1	25.0	190000	50.20	27.50	-0	-0.00	-0.00	60411	0	INFIN.
141	216	1	26.0	190000	49.80	26.20	-0	-0.00	-0.00	58017	0	INFIN.
149	205	1	24.0	190000	49.60	22.30	-0	-0.00	-0.00	49580	0	INFIN.
158	199	1	25.0	190000	50.00	24.50	-0	-0.00	-0.00	54036	0	INFIN.
161	209	1	25.0	190000	49.10	24.00	-0	-0.00	-0.00	53903	0	INFIN.
169	177	1	24.0	190000	49.60	24.40	-0	-0.00	-0.00	54249	0	INFIN.
175	168	1	25.0	190000	49.80	25.80	-0	-0.00	-0.00	57132	0	INFIN.
185	190	2	26.0	190000	50.00	25.40	-0	-0.00	-0.00	61981	0	INFIN.
186	207	2	27.0	190000	50.10	25.40	-0	-0.00	-0.00	61857	0	INFIN.
190	142	2	25.0	190000	49.90	26.80	-0	-0.00	-0.00	65528	0	INFIN.
195	208	2	23.0	190000	50.10	24.40	-0	-0.00	-0.00	59422	0	INFIN.

ENDURANCE TEST TEST MODE = 4

TEST NO.	SPECIMEN NO.	MACHINE NO.	PAN WEIGHT	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
445	477	2	22.0	250000	25.00	15.20	304000	22.47	3.97	55917	9125	3.538
455	465	2	21.0	250000	25.07	15.20	304000	22.55	3.42	55825	7560	4.263
457	478	2	23.0	250000	25.17	15.95	304000	22.52	4.05	58291	9240	3.642
465	506	2	21.0	250000	24.96	13.92	304000	22.50	3.94	51256	9189	3.221
484	528	2	24.0	250000	25.00	15.59	304000	22.42	3.85	57376	8763	3.780
520	444	2	27.0	250000	25.04	14.55	304000	22.50	4.06	53410	9448	3.264
522	466	2	25.0	250000	24.98	14.73	304000	22.50	3.81	54236	8723	3.590
527	524	2	21.0	250000	25.00	12.81	304000	22.54	3.82	47071	8981	3.026
528	546	2	23.0	250000	25.07	13.70	304000	22.50	3.28	50292	7384	3.933
529	499	2	26.0	250000	24.95	16.48	304000	22.50	2.97	60903	6150	5.717
533	532	2	26.0	250000	24.96	14.78	304000	22.48	3.23	54533	7105	4.432
536	483	2	22.0	250000	25.00	13.83	304000	22.48	4.03	50828	9464	3.101
539	529	2	22.0	250000	25.06	14.00	304000	22.53	3.64	51381	8335	3.559
587	587	2	22.0	250000	25.08	14.20	304000	22.58	3.75	52067	8594	3.498
591	581	2	25.0	250000	25.03	14.78	304000	22.53	3.75	54320	8539	3.673
600	611	2	26.0	250000	24.95	15.42	304000	22.52	4.16	56826	9597	3.419
603	589	2	25.0	250000	24.91	14.66	304000	22.45	3.90	54118	9002	3.471
606	603	2	25.0	250000	24.98	14.62	304000	22.50	3.94	53813	9100	3.414

MEAN STRESS RATIO = 3.69675
STD. DEV. OF STRESS RATIO = .61987

ENDURANCE TEST   TEST MODE = 2

TEST NO.	SPECIMEN NO.	MACHINE NO.	PAN WEIGHT	RCAL BENDING	NCAL BENDING	NVIS BENDING	RCAL TORQUE	NCAL TORQUE	NVIS TORQUE	BENDING STRESS	SHEAR STRESS	STRESS RATIO
280	248	3	27.0	390000	50.15	50.15	304000	45.15	24.30	48979	39208	.721
317	288	2	27.0	190000	50.10	25.95	304000	44.55	25.15	61723	33231	1.072
318	267	2	26.0	190000	49.70	24.35	304000	44.45	24.30	58350	32230	1.045
321	300	2	27.0	190000	49.60	23.15	304000	44.65	24.10	55537	31894	1.005
323	275	2	26.0	190000	49.60	24.85	304000	44.10	22.25	59810	29537	1.169
330	409	2	25.0	190000	49.90	23.90	304000	44.95	23.60	57067	30917	1.066
339	364	2	24.0	190000	53.05	23.55	304000	45.00	22.40	52863	29356	1.040
348	372	2	26.0	190000	49.65	24.55	304000	45.20	25.00	58885	32613	1.042
377	362	2	28.0	190000	49.80	25.75	304000	45.00	26.30	61561	34485	1.031
378	429	2	27.0	190000	50.00	24.75	304000	45.00	25.45	58918	33393	1.019
381	359	2	27.0	190000	50.00	24.65	304000	45.10	24.60	58727	32140	1.055
383	417	2	26.0	190000	49.55	23.15	304000	44.60	24.30	55581	32211	.996
388	336	2	26.0	190000	50.30	22.95	304000	45.15	26.15	54156	34413	.909
395	435	2	26.0	190000	50.10	23.65	304000	47.10	23.40	56298	29176	1.114
397	401	2	25.0	190000	49.95	24.00	304000	45.00	23.40	57266	30595	1.081
430	400	2	29.0	190000	50.10	25.30	304000	44.65	24.75	60166	32645	1.064
431	386	2	28.0	190000	50.10	24.65	304000	47.15	25.50	58619	31854	1.063
432	402	2	27.0	190000	50.00	24.45	304000	47.10	24.80	58288	30970	1.087

MEAN STRESS RATIO = 1.03214
STD. DEV. OF STRESS RATIO = .09408

APPENDIX F

PRINTOUT OF PROGRAM TO CALCULATE THE STRENGTH DISTRIBUTION ESTIMATES FOR MEAN AND STANDARD DEVIATION

```
              PROGRAM STRENG (INPUT,OUTPUT,TAPE1 =INPUT)
        C-----PROGRAM FINDS NORMAL STRENGTH DISTRIBUTION FROM LOGNORMAL
        C-----CYCLES-TO-FAILURE DISTRIBUTIONS.
000003        DIMENSION ASTR(10), AMLOG(10), ASIGL(10), SLAM(9),SIG3M(23),
             1   XMLOG(301), XSIGL(301), AREA(301,23), FREQ(301,23),SIG3P(23),
             2   CYC(23), SMLOG(23), SSIGL(23), SK3(23), SK4(23),CYTOFR(10,25),
             3 NUM(10),INDEX(10),OFREQ(10,23),GFREQ(10,23),DMAX(10,23),AN2(10)
             4  ,CHISQ(10,23),ST(10),SLSIG(9), EFREQ(10,23),INDEX2(10),ANUM(10)
             5,CYCLES(301),ACYCLE(25), STRLOG(10), STMEAN(23), XCYCLE(23)
000003        EQUIVALENCE (AREA,FREQ),
             1   (XSIGL,SSIGL), (XMLOG,OFREQ), (AREA(500),EFREQ),
             2   (AREA,CHISQ), (AREA(250),GFREQ)
000003        INT = 1
000004      5 READ10,XJMIN,XJMAX,SINT,M,N,RATIO
000024     10 FORMAT (2F10.6,F10.0,2I5,F10.5)
        C     FAILURE DISTRIBUTION VALUES ARE READ IN FROM
        C        LOWEST TO HIGHEST STRESS AND STRESSES ARE
        C        INTEGER VALUES OF SINT
000024        IF (EOF,1) 510, 20
000027     20 READ 30, (ASTR(J),AMLOG(J),ASIGL(J),NUM(J), J=1,N)
000050     30 FORMAT (F10.0,2F10.6,I5)
000050        READ 36, (XCYCLE(J), J=1,M)
000063  36    FORMAT (8F10.0)
000063        DO 34 I=1,M
000065  34    ACYCLE(I)=EXP(AMLOG(I))
000074        NM1 = N-1
000075        L = 1
000076        INDEX(1) = 1
000077        DO 85 J=1,NM1
000101        DSTR = ASTR(J+1)-ASTR(J)
000103        DMEAN = AMLOG(J+1)-AMLOG(J)
000105        DSIG = ASIGL(J+1)-ASIGL(J)
000107        SLAM(J) = DSTR/DMEAN
000111        IF(DSIG) 40,50,40
000112     40 SLSIG(J) = DSTR/DSIG
000115     50 CONTINUE
000115        JP1 = J+1
000117        MX = DSTR/SINT
000122        INDEX(JP1) = L+MX
000124        SMIN = ASTR(1)
000126        XMLOG(1) = AMLOG(1)
000127        XSIGL(1) = ASIGL(1)
000131        DO 80 K=1,MX
000132        LX = L + K
000134        XMLOG(LX) = XMLOG(LX-1)+SINT/SLAM(J)
000140        IF(DSIG) 70,60,70
000141     60 XSIGL(LX) = XSIGL(LX-1)
000143        GO TO 80
000144     70 XSIGL(LX) = XSIGL(LX-1)+SINT/SLSIG(J)
000151     80 CONTINUE
000154        L = LX
000155     85 CONTINUE
000160        PRINT 87
000163  87    FORMAT(1H1///44X,    44HNORMAL STRENGTH DISTRIBUTIONS FROM LOGNORMA
             1L/49X,32HCYCLES TO FAILURE DISTRIBUTIONS.///)
000163        IF(RATIO.EQ.0.0) GO TO 91
000164        PRINT 88, RATIO
```

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000172   88     FORMAT (44X,17HEXPERIMENTAL DATA,8X,14HSTRESS RATIO =,F6.3//)
000172          GO TO 93
000173   91     PRINT 92
000177   92     FORMAT(44X,17HEXPERIMENTAL DATA,8X,23HSTRESS RATIO = INFINITY//)
000177   93     PRINT 94
000203   94     FORMAT(8X,13HSTRENGTH PSI.,9X,32HMEAN-CYCLES      LOG MEAN-CYCLES,
               18X,11HLOG STD DEV/)
000203          NP = N-1
000205          PRINT 95, (ASTR(I), ACYCLE(I), AMLOG(I),ASIGL(I), I=2,NP)
000226   95     FORMAT(2F20.2,2F20.8)
000226          PRINT 96
000232   96     FORMAT(///54X,26HINTERPOLATED STRESS LEVELS//)
000232          PRINT 90
000236   90     FORMAT(8X,13HSTRENGTH PSI.,9X,32HMEAN-CYCLES      LOG MEAN-CYCLES,
               18X,11HLOG STD DEV,14X,10HINTEGER(I)/)
000236          XINT = INT
000240          STR = SMIN
000241          DO110 I=1,LX,INT
000243          CYCLES(I)=EXP(XMLOG(I))
000247          PRINT 100,STR,CYCLES(I),XMLOG(I), XSIGL(I),I
000265   100    FORMAT (2F20.2,2F20.8,I20)
000265          STR = STR+SINT*XINT
000270     110 CONTINUE
000273          NOLD = N
000274          N = LX
         C      CONVERTING LOGNORMAL FAILURE DIST. PARAMETERS AT N
         C         STRESS LEVELS TO CUMMULATIVE LOGNORMAL FAILURE
         C         DISTRIBUTION
000275          DO 180 I=1,N
000276          DO 180 J=1,M
000277          CYC(J) = ALOG(XCYCLE(J))
000303          Z= (CYC(J)-XMLOG(I))/XSIGL(I)
000307          IF(Z) 120,140,140
000310     120 IF(Z+4.5)160,130,130
000313     130 Z = -Z
000314          CALL NORMAL (Z,PROB)
000316          AREA(I,J) = (1.0-PROB)/2.0
000324          GO TO 180
000324     140 IF(Z-3.5) 150,150,170
000327   150    CALL NORMAL (Z,PROB)
000331          AREA(I,J) = PROB/2.0+0.5
000336          GO TO 180
000337     160 AREA(I,J) = 0.0
000343          GO TO 180
000343     170 AREA(I,J) = 1.0
000347   180    CONTINUE
000354          PRINT 225
000360          INT2 = INT*4
000362          STR = SMIN
000363          PRINT 185
000367   185    FORMAT(///46X,42HCUMULATIVE LOGNORMAL FAILURE DISTRIBUTIONS)
000367          DO 220 I=1,N,INT2
000371          PRINT 190,STR,XMLOG(I),XSIGL(I)
000402     190 FORMAT(1H0,5X,11HSTRENGTH = ,F10.0,5X, 9HLOG MEAN ,
               1    9HCYCLES = ,F9.6,5X,20HLOG STD DEVIATION = ,F9.6)
000402          PRINT 200
000406     200  FORMAT (1X, 33HDATA BELOW IS J, AND CUMMULATIVE ,
               1    13HDIST UP TO J.)
```

```
000406          PRINT 210, (J, AREA(I,J),J=1,M)
000424      210 FORMAT    ((1X,I3,F9.6,I3,F9.6,I3,F9.6,I3,F9.6,
               1  I3,F9.6,I3,F9.6,I3,F9.6,I3,F9.6,I3,F9.6))
000424          STR = STR+SINT*XINT*4.
000430      220 CONTINUE
000433          DO 222 J=1,NOLD
000434          I = INDEX(J)
000436          XNUM = NUM(J)
000437          DO 224 K=1,M
000441          OFREQ(J,K) = XNUM*AREA(I,K)
000451      224 CONTINUE
000454      222 CONTINUE
          C     FREQUENCY DIST AND NORMAL DISTRIBUTION PARAMETERS
000456          XJMIN = XJMIN-XJINT
000460          PRINT 225
000464     225  FORMAT (1H1)
000464          PRINT 230
000470     230  FORMAT(///51X,33HCUMULATIVE STRENGTH DISTRIBUTIONS)
000470          DO 260 J=1,M
000472          XJ = J
000473          PRINT 240, J, CYC(J), XCYCLE(J)
000505      240 FORMAT (1H0,4HJ = ,I3 ,5X,13HLOG CYCLES = ,F9.6,
              15X,8HCYCLES =,F10.1)
000505          PRINT 250, (I,AREA(I,J),I=1,N,INT)
000523      250 FORMAT (2X,3H(I),1X,9HFREQUENCY/(1X,I4,F10.6,I5,F10.6
               1  ,I5,F10.6,I5,F10.6,I5,F10.6,I5,F10.6,I5,F10.6,I5,F10.6))
000523      260 CONTINUE
000526          NM1 = N-1
000530          DO 270 J=1,M
000531          DO 270 I=1,NM1
000532          IP1 = I+1
000534          FREQ(I,J) = AREA(IP1,J) -AREA(I,J)
000542      270 CONTINUE
000547          N = NM1
          C     MEAN AND NORMAL DISTRIBUTION PARAMETERS OF HISTOGRAM
000551          DO 300 J=1,M
000552          FI = 0.
000553          FIUI = 0.
000554          UI = SMIN-SINT*0.5
000557          DO 280 I=1,N
000560          UI = UI+SINT
000562          FI = FI+FREQ(I,J)
000566          FIUI = FIUI+FREQ(I,J)*UI
000573      280 CONTINUE
000575          SMLOG(J) = FIUI/FI
000577          STMEAN(J) = ALOG(SMLOG(J))
000603          SM2 = 0.
000604          SM3 = 0.
000605          SM4 = 0.0
000606          UI = SMIN-SINT*0.5
000611          DO 290 I=1,N
000612          UI = UI+SINT
000614          SQ = (UI-SMLOG(J))*(UI-SMLOG(J))
000617          SM2 = SM2 + SQ*FREQ(I,J)
000624          SM3 = SM3 + FREQ(I,J)*(UI-SMLOG(J))*SQ
000632          SM4 = SM4+SQ*SQ*FREQ(I,J)
000637      290 CONTINUE
000642          SM2 = SM2/FI
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000643          SM3 = SM3/FI
000644          SM4 = SM4/FI
000645          SSIGL(J) = SORTF(SM2)
000651          SIG3P(J) = SMLOG(J) + 3.0*SSIGL(J)
000655          SIG3M(J) = SMLOG(J) - 3.0*SSIGL(J)
000660          SK3(J) = SM3/(SSIGL(J)*SM2)
000663          SK4(J) = SM4/(SM2*SM2)
000665      300 CONTINUE
000670          PRINT 225
000673          PRINT 305
000677     305  FORMAT(///30X,73HPARAMETERS OF NORMAL STRESS DISTRIBUTIONS AT SPEC
               1IFIED CYCLES TO FAILURE.//)
000677          PRINT 310
000703     310  FORMAT (83X,20H-3 SIGMA     +3 SIGMA  /3X,20HNUMBER     LOG CYCLES ,
               1 6X,6HCYCLES,10X,13HMEAN STRENGTH,10X,9HSTD. DEV.,7X,5HLIMIT   ,
               2 7X,34HLIMIT        SKEWNESS       KURTOSIS  /)
000703          PRINT 320, (J,CYC(J),XCYCLE(J),SMLOG(J), SSIGL(J),
               1 SIG3M(J),SIG3P(J),SK3(J),SK4(J), J =1,M)
000735     320  FORMAT (4X,I3,F16.6,F13.0,2F20.0,F15.0,F12.0,2F12.4)
000735          PRINT 225
000741          PRINT 322
000745     322  FORMAT (1H0)
           C-----ROUTINE FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST
000745          PRINT 325
000751     325  FORMAT (//41X,52HD-VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT T
               1EST  /)
000751          DO 600 I=2,NP
000753          NDATA = NUM(I)
000755          READ 605,(CYTOFR(I,J),J=1,NDATA)
000770     605  FORMAT (8F10.0)
000770     600  CONTINUE
000773          DO 500 J=1,M
000774          ANUM(1) = 0.0
000775          TOTAL = 0.0
000776          DO 665 I=2,NP
000777          NDATA = NUM(I)
001001          AN1 = 0.0
001002          DO 660 K = 1,NDATA
001003          IF (CYTOFR(I,K).LE.XCYCLE(J))  AN1 = AN1 + 1.0
001012     660  CONTINUE
001015          AN2(I) = AN1
001017          ANUM(I) = ANUM(I-1) + AN1
001021          TOTAL = TOTAL + AN1
001022     665  CONTINUE
001024          DO 480 I=2,NP
001026          Z =(ASTR(I) -SMLOG(J))/SSIGL(J)
001032          IF(Z) 330,350,350
001033      330 Z = -Z
001034          IF(Z-4.5) 340,360,360
001037      340 CALL NORMAL (Z,PROB)
001041          PROBA = 0.5-PROB*0.5
001044          GO TO 370
001044     350  IF(Z-4.5)355,365,365
001047      355 CALL NORMAL (Z,PROB)
001051          PROBA = PROB*0.5+0.5
001054          GO TO 370
001054      360 PROBA = 0.0
001055          GO TO 370
```

```
001056     365 PROBA = 1.0
001060    370    GFREQ(I,J)=PROBA
001064           IF (TOTAL.EQ.0.0) GO TO 375
001065           DAREA = ANUM(I)/TOTAL
001067           GO TO 380
001067    375    DAREA = 0.0
001070    380    DMAX(I,J) = ABS(DAREA-GFREQ(I,J))
001101    480    CONTINUE
001103           PRINT 490, XCYCLE(J),  TOTAL,(AN2(I),DMAX(I,J), I=2,NP)
001125    490    FORMAT (6X,F8.0,21H CYCLES      TOTAL N =,F3.0/(9(F9.0,F6.3)))
001125           PRINT 322
001131    500    CONTINUE
001134           GO TO 5
001134    510    STOP
001136           END
```

```
              SUBROUTINE NORMAL(Z,PROB)
        C     PROB = THE AREA UNDER NORMAL DISTRIBUTION BETWEEN
        C        PLUS AND MINUS Z STANDARD DEVIATIONS
000005        IF(Z-1.2) 1000,1000,1010
000007   1000 ZSQ = Z*Z
000010        PROB = 0.79788455*Z*(0.99999774-ZSQ*(0.16659433
             1  -ZSQ*(0.024638310 - ZSQ*0.0023974867)))
000020        GO TO 1070
000021   1010 IF(Z-2.9) 1020,1060,1060
000024   1020 ZSQ = Z*Z
000025        PROB = 1.0
000026        PTERM = 1.0
000027        FACT = 1.0
000027        ODDIN = 3.0
000031   1030 PTERM = -PTERM*ZSQ/(2.0*FACT)
000035        TERM = PTERM/ODDIN
000036        PROB = PROB + TERM
000037        IF ( ABS(TERM) - 0.00007) 1050,1040,1040
000043   1040 FACT = FACT + 1.0
000045        ODDIN = ODDIN + 2.0
000047        GO TO 1030
000047   1050 PROB = 0.79788455*Z*PROB
000051        GO TO 1070
000052   1060 REC = 1./(Z*Z)
000054        PROB = 1.-0.79788453*  EXP(-Z*Z/2.0)/Z*
             1    (1. - REC*(1. - REC*(3. - REC*(15.-REC*105.))))
000076   1070 CONTINUE
000076        RETURN
000077        END
```

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