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GEOS-1 LASER PULSE RETURN SHAPE ANALYSIS

T. L. FELSENTREGER

(NASA-TM-X-66123) GEOS-1 LASER PULSE RETURN SHAPE ANALYSIS T.L. Felsentreger (NASA) Sep. 1972 43 p CSCL 20E

N73-13490

Unclas
G3/16 50358

SEPTEMBER 1972

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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T. L. Felsentreger
Geodynamics Branch
Trajectory Analysis & Geodynamics Division

September 1972

GODDARD SPACE FLIGHT CENTER
Greenbelt, Maryland

GEOS-1 LASER PULSE RETURN SHAPE ANALYSIS

ABSTRACT

An attempt has been made to predict the shape of the laser return pulse from the corner cube retroreflectors on the GEOS-1 spacecraft. The study is geometrical only, and neglects factors such as optical interference, atmospheric perturbations, etc. A function giving the "intensity" of the return signal at any given time has been derived. In addition, figures are given which show the predicted return pulse shape as a function of time, the angle between the beam and the spin axis, and an "In-plane" angle (designating the orientation of the intersection of the planar waves with the plane of the corner cubes).

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GEOS-1 LASER PULSE RETURN SHAPE ANALYSIS

T. L. Felsentreger
Geodynamics Branch

INTRODUCTION

Several earth physics projects have been planned (e.g., the SAFE Experiment) which will make it necessary for laser tracking of artificial Earth satellites to have accuracies on the order of 10 cm. or better. Present methods for interpreting the return pulses to locate the center of mass of a satellite give accuracies considerably worse than this, one of the major sources of error being the large numbers of corner cube reflectors arranged in an asymmetrical pattern on the satellite. Additional planned modifications of the GSFC lasers make it possible for us to have more detailed knowledge of the shape of the return pulse.

The GEOS-1 spacecraft was chosen for initial study because (i) it is one of the satellites to be used in the SAFE Experiment, (ii) the corner cube reflectors all lie in the plane of the base of the spacecraft, and (iii) the corner cube reflectors are arranged symmetrically with respect to the geometric center of the satellite's base. There are four "panels" of reflectors, two of which have 94 cubes each and the other two having 73 cubes each for a total of 334 corner cube reflectors (see Figure 6).

It should be mentioned here that the "corners" of the cubes all lie in essentially the same plane, but the tolerances exceed the wavelength of the laser light. Thus, the phases of the return signals from the corner cubes will be randomly distributed, resulting in a large deviation of the "actual" return intensity from the "theoretical" return intensity. Therefore, the theoretical return will be a "mean" from which the actual return deviates. The actual return will be analyzed statistically and a mean obtained to be compared with the theoretical mean.

However, before GEOS-1 was tackled, a preliminary study was made of one asymmetrical rectangular array of corner cubes. The following section is devoted to a discussion of this study.

STUDY OF A RECTANGULAR CORNER CUBE ARRAY

The following assumptions are made:

1. Each corner cube reflector is hexagonally-shaped, as viewed head-on.
2. The array is rectangular with mxn cubes as shown in Figure 1, and the "centers" of the hexagons all lie in the same plane.
3. The return signals emanate from the same point in each cube, e.g., the "center" of the hexagon.

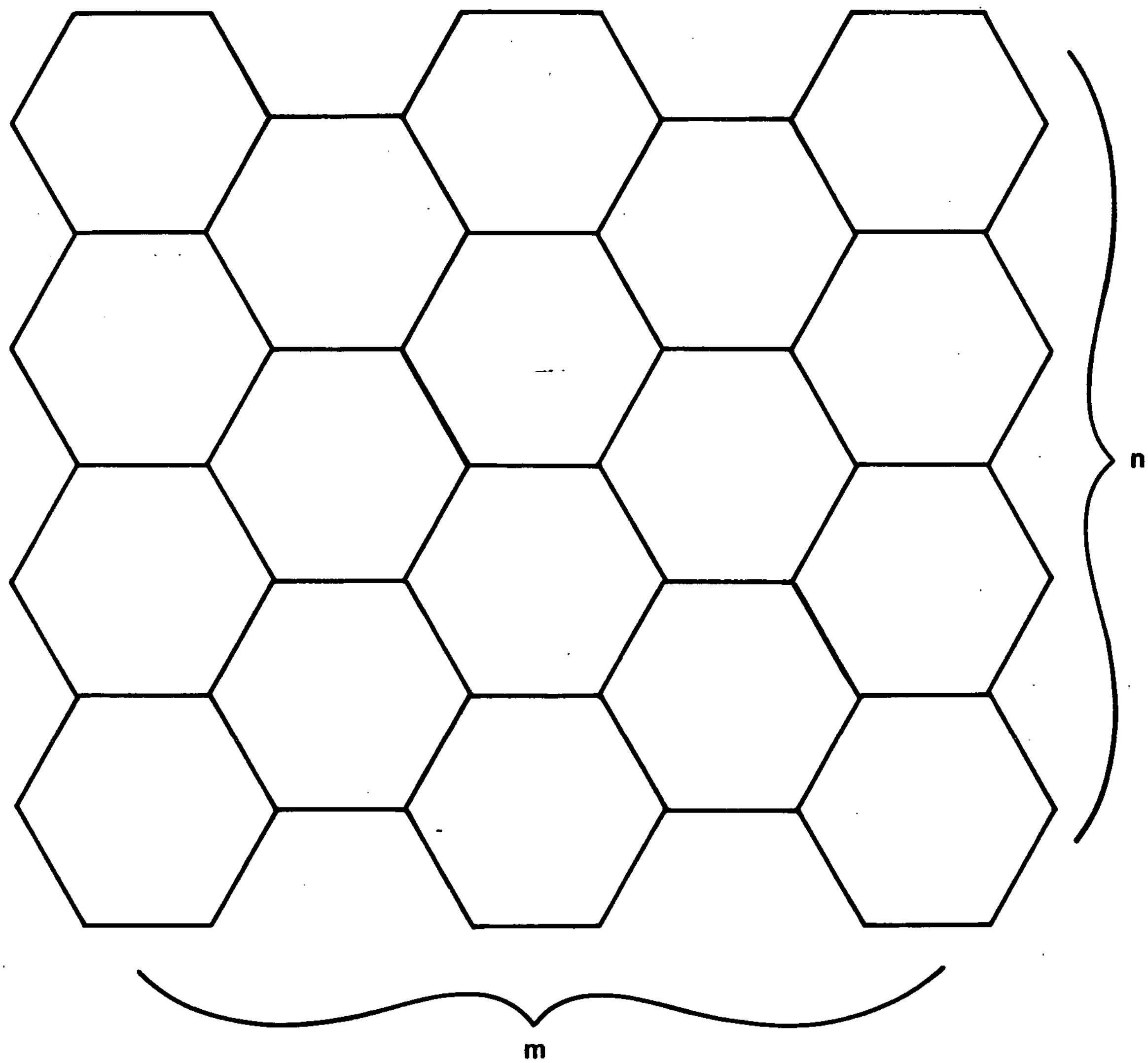


Figure 1. Rectangular Corner Cube Array

4. Each planar wave in the laser pulse intersects the plane of the array with the angle $\frac{\Pi}{2} - \alpha$, as shown in Figure 2.

5. The lines of intersection between the planar waves and the plane of the array intersect the 2n-1 "rows" of the array with the angle β, as shown in Figure 3.

Let d designate the distance between the centers of any two adjacent corner cubes in any of the 2m-1 columns (see Figures 1 and 3). Since the reflectors

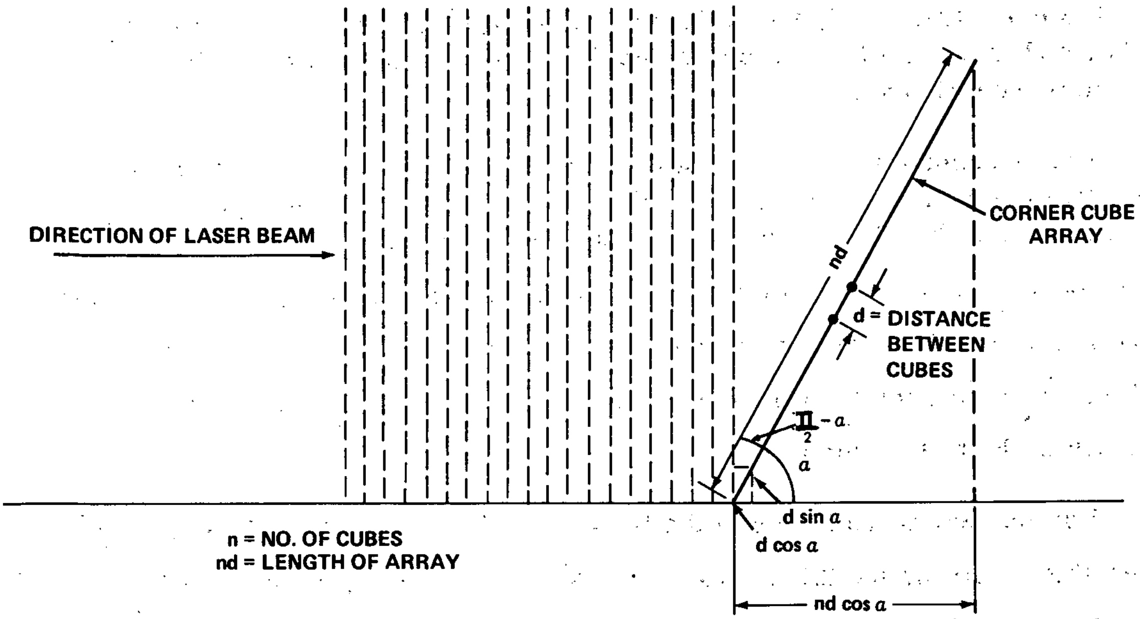


Figure 2. Intersection of Planar Laser Waves with Array ($\beta = 0$)

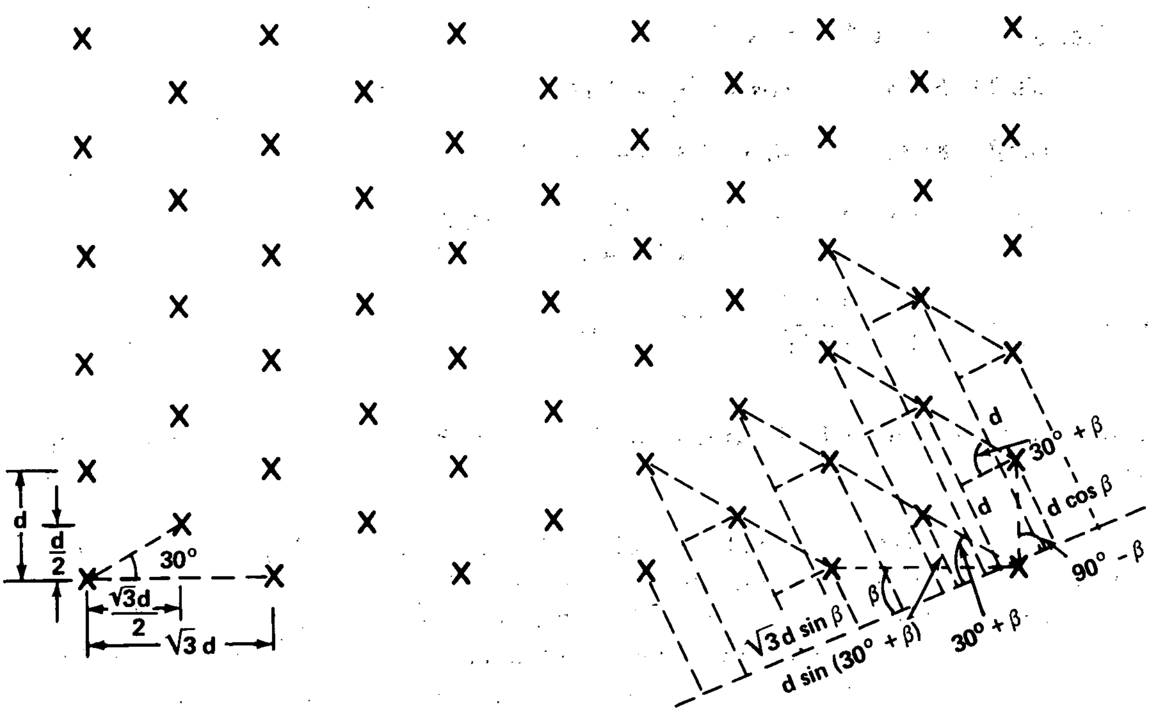


Figure 3. Rectangular Corner Cube Array

are hexagonally-shaped, the distance between two centers in the same row is $\sqrt{3}d$, and there is a 30° angle between the "first" centers in two adjacent columns (see Figure 3).

Let us assume that the first wave of the pulse strikes the first reflector in the extreme right-hand column, as shown in Figure 3. From Figures 2 and 3, it can be seen that this wave will need to travel a distance of $d \cos\beta \cos\alpha$ to strike the next reflector in this column, an elapsed time of $d \cos\beta \frac{\cos\alpha}{c}$ (c = speed of light) from incidence with the first reflector. Similarly, an elapsed time of $2d \cos\beta \frac{\cos\alpha}{c}$ for the third reflector, up to $(n-1)\, d \cos\beta \frac{\cos\alpha}{c}$ for the last reflector in the first column.

For the second reflector in the first row, the elapsed time is $\sqrt{3} \sin\beta \frac{\cos\alpha}{c}$ (see Figure 3); for the third reflector, $2\sqrt{3}d \sin\beta \frac{\cos\alpha}{c}$, up to $(m-1)\sqrt{3}d \sin\beta \frac{\cos\alpha}{c}$ for the last reflector. Let us designate a reflector in the i^{th} column, j^{th} row by (i, j). We can now specify the "elapsed times" for the reflectors along any "diagonal" of centers. Figure 3 indicates the geometry involved and the method of computation. Table 1 gives these elapsed times for a 3 X 3 array, starting in the lower right-hand corner.

Let $\beta = 35°$, $\alpha = 45°$. Then,

$$\sin\beta = .5736 \qquad \cos\alpha = \frac{\sqrt{2}}{2} = .7071$$

$$\cos\beta = .8192 \qquad \sin(30° + \beta) = \sin 65° = .9063$$

Let $d = 3$ cm., $c = 3 \times 10^{10}\ \text{cm/sec} \cdot \dfrac{1\ \text{sec}}{10^9\ \text{nsec}} = 3 \times 10 \dfrac{\text{cm}}{\text{nsec}}$

Then, $\dfrac{d \cos\alpha}{c} = \dfrac{(3\ \text{cm.})(.7071)}{3 \times 10\ \text{cm/nsec}} = .07071\ \text{nsec}$

Table 2 gives the numerical values for Table 1 in this example, in nanoseconds (nsec.).

The return pulse from each reflector will have a time delay equal to twice the elapsed time of incidence from the (1, 1) reflector. Thus, if $I_{11}(t)$ is the function representing the intensity of the return pulse from the (1, 1) reflector at time t, and if t_{ij} is the elapsed time of incidence of any planar wave (for the (i, j) reflector) from the (1, 1) reflector, then $I_{ij}(t-2t_{ij})$ represents the intensity of the return pulse from the (i, j) reflector at time t. A simple addition of all the I_{ij}'s at time t should give the total intensity of the return.

Table 1. Elapsed Times of Incidence From (1, 1) Reflector

$j \downarrow$ \ $i \rightarrow$	5	4	3	2	1
5	$4d \sin(30° + \beta) \frac{\cos\alpha}{c}$		$d[2 \sin(30° + \beta) + \cos\beta] \frac{\cos\alpha}{c}$		$2d \cos\beta \frac{\cos\alpha}{c}$
4		$3d \sin(30° + \beta) \frac{\cos\alpha}{c}$		$d[\sin(30° + \beta) + \cos\beta] \frac{\cos\alpha}{c}$	
3	$d[2 \sin(30° + \beta) + \sqrt{3} \sin\beta] \frac{\cos\alpha}{c}$		$2d \sin(30° + \beta) \frac{\cos\alpha}{c}$		$d \cos\beta \frac{\cos\alpha}{c}$
2		$d[\sin(30° + \beta) + \sqrt{3} \sin\beta] \frac{\cos\alpha}{c}$		$d \sin(30° + \beta) \frac{\cos\alpha}{c}$	
1	$2\sqrt{3} d \sin\beta \frac{\cos\alpha}{c}$		$\sqrt{3}d \sin\beta \frac{\cos\alpha}{c}$		0

Table 2. Elapsed Times of Incidence From (1, 1) Reflector (nsec)

$j \downarrow$ \ $i \rightarrow$	5	4	3	2	1
5	0.25634		0.18610		0.11585
4		0.19225		0.12201	
3	0.19842		0.12817		0.05793
2		0.13433		0.06408	
1	0.14050		0.07025		0

Table 3. Intensities at nsec Intervals

t(nsec.)	Intensity	z
0	.5000	1.17740
1	.5704	1.05966
2	.6417	.94192
3	.7120	.82418
4	.7792	.70644
5	.8409	.58870
6	.8950	.47096
7	.9395	.35322
8	.9727	.23548
9	.9931	.11774
10	1.0000	0
11	.9931	-.11774
12	.9727	-.23548
13	.9395	-.35322
14	.8950	-.47096
15	.8409	-.58870
16	.7792	-.70644
17	.7120	-.82418
18	.6417	-.94192
19	.5704	-1.05966
20	.5000	-1.17740

Let us assume that each transmitted pulse is Gaussian-shaped, beginning at a "threshold" intensity of 1/2 the peak intensity. If we normalize the peak intensity to the value 1, then the intensity can be represented by

$$I(z) = e^{-z^2/2}, \qquad -1.17740 \leqslant z \leqslant 1.17740,$$

as shown in Figure 4.

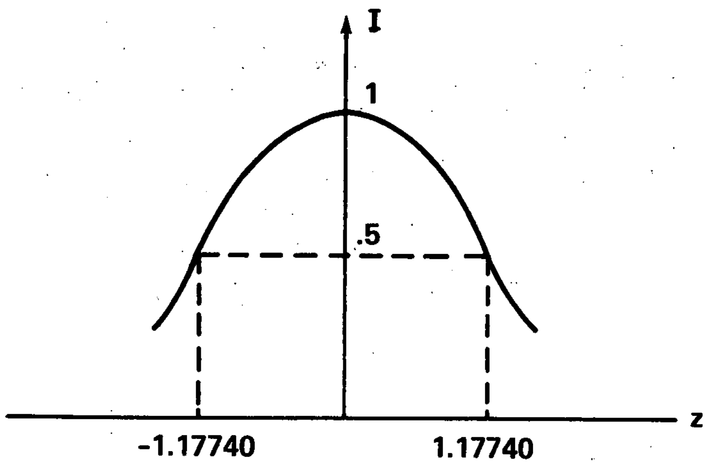


Figure 4. Intensity (Gaussian Distribution)

This pulse will be returned by each reflector unaltered in shape (neglecting interference, atmospheric perturbations, etc.). If we assume that the transmitted pulse is 20 nsec long, then

$$z = .11774\,t - 1.1774 \qquad (0 \leq t \leq 20)$$

We also assume that the return pulse is read out at one nsec intervals; hence, the total intensity I(t) at time t is

$$I(t) = \sum_{i,j} I_{ij}\,(t - 2t_{ij}) \qquad (t = 0, 1, 2, \ldots, 20 \text{ nsec.})$$

Table 3 gives the values of z, $I(z)$, and t for $t = 0, 1, 2, \ldots, 20$ nsec. A simple computation shows that

$$z(t - 2t_{ij}) = .11774\,(t - 2t_{ij}) - 1.7774 = z(t) - .11774\,(2t_{ij}).$$

Table 4 gives values of $z(t - 2t_{ij})$, for $t = 0, 1, 2, \ldots, 20$ nsec. Table 5 gives values of $I_{ij}\,(t - 2t_{ij})$ and the total intensity I(t) for $t = 0, 1, 2, \ldots, 20$ nsec. Figure 5 shows the intensity of the return pulse as a function of time.

GEOS-1 ANALYSIS

The GEOS-1 spacecraft has four panels of corner cube reflectors, all lying in the plane of the bottom of the spacecraft as shown in Figure 6. The "dots" represent the "centers" of the hexagonally shaped corner cubes. Panels 1 and 3 each contain 94 cubes, while panels 2 and 4 each contain 73 corner cubes. The distance d across the "flats" of each corner cube is 2.489 cm. There is a 59.06 cm. broad band spiral antenna and a conical range and range rate antenna which cause eclipsing of the reflectors, but this factor was ignored.

Table 4. The Argument z for Rectangular Corner Cube Array

	Column 1			Column 2		Column 3			Column 4		Column 5		
t (nsec)	z(t)	$z(t-2t_{13})$	$z(t-2t_{15})$	$z(t-2t_{22})$	$z(t-2t_{24})$	$z(t-2t_{31})$	$z(t-2t_{33})$	$z(t-2t_{35})$	$z(t-2t_{42})$	$z(t-2t_{44})$	$z(t-2t_{51})$	$z(t-2t_{53})$	$z(t-2t_{55})$
0	-1.17740												
1	-1.05966	-1.07330	-1.08694	-1.07475	-1.08839	-1.07620	-1.08984	-1.10348	-1.09129	-1.10493	-1.09274	-1.10638	-1.12002
2	-0.94192	-0.95556	-0.96920	-0.95701	-0.97065	-0.95846	-0.97210	-0.98574	-0.97355	-0.98719	-0.97500	-0.98864	-1.00228
3	-0.82418	-0.83782	-0.85146	-0.83927	-0.85291	-0.84072	-0.85436	-0.86800	-0.85581	-0.86945	-0.85726	-0.87090	-0.88454
4	-0.70644	-0.72008	-0.73372	-0.72153	-0.73517	-0.72298	-0.73662	-0.75026	-0.73807	-0.75171	-0.73952	-0.75316	-0.76680
5	-0.58870	-0.60234	-0.61598	-0.60379	-0.61743	-0.60524	-0.61888	-0.63252	-0.62033	-0.63397	-0.62178	-0.63542	-0.64906
6	-0.47096	-0.48460	-0.49824	-0.48605	-0.49969	-0.48750	-0.50114	-0.51478	-0.50259	-0.51623	-0.50404	-0.51768	-0.53132
7	-0.35322	-0.36686	-0.38050	-0.36831	-0.38195	-0.36976	-0.38340	-0.39704	-0.38485	-0.39849	-0.38630	-0.39994	-0.41358
8	-0.23548	-0.24912	-0.26276	-0.25057	-0.26421	-0.25202	-0.26566	-0.27930	-0.26711	-0.28075	-0.26856	-0.28220	-0.29584
9	-0.11774	-0.13138	-0.14502	-0.13283	-0.14647	-0.13428	-0.14792	-0.16156	-0.14937	-0.16301	-0.15082	-0.16446	-0.17810
10	0	-0.01364	-0.02728	-0.01509	-0.02873	-0.01654	-0.03018	-0.04382	-0.03163	-0.04527	-0.03308	-0.04672	-0.06036
11	0.11774	0.10410	0.09046	0.10265	0.08901	0.10120	0.08756	0.07392	0.08611	0.07247	0.08466	0.07102	0.05738
12	0.23548	0.22184	0.20820	0.22039	0.20675	0.21894	0.20530	0.19166	0.20385	0.19021	0.20240	0.18876	0.17512
13	0.35322	0.33958	0.32594	0.33813	0.32449	0.33668	0.32304	0.30940	0.32159	0.30795	0.32014	0.30650	0.29286
14	0.47096	0.45732	0.44368	0.45587	0.44223	0.45442	0.44078	0.42714	0.43933	0.42569	0.43788	0.42424	0.41060
15	0.58870	0.57506	0.56142	0.57361	0.55997	0.57216	0.55852	0.54488	0.55707	0.54343	0.55562	0.54198	0.52834
16	0.70644	0.69280	0.67916	0.69135	0.67771	0.68990	0.67626	0.66262	0.67481	0.66117	0.67336	0.65972	0.64608
17	0.82418	0.81054	0.79690	0.80909	0.79545	0.80764	0.79400	0.78036	0.79255	0.77891	0.79110	0.77746	0.76382
18	0.94192	0.92828	0.91464	0.92683	0.91319	0.92538	0.91174	0.89810	0.91029	0.89665	0.90884	0.89520	0.88156
19	1.05966	1.04602	1.03238	1.04457	1.03093	1.04312	1.02948	1.01584	1.02803	1.01439	1.02658	1.01294	0.99930
20	1.17740	1.16376	1.15012	1.16231	1.14867	1.16086	1.14722	1.13358	1.14577	1.13213	1.14432	1.13068	1.11704

Table 5. Intensities for Rectangular Corner Cube Array

	Column 1			Column 2		Column 3			Column 4		Column 5			
t(nsec)	I(t)	$I(t-2t_{13})$	$I(t-2t_{15})$	$I(t-2t_{22})$	$I(t-2t_{24})$	$I(t-2t_{31})$	$I(t-2t_{33})$	$I(t-2t_{35})$	$I(t-2t_{42})$	$I(t-2t_{44})$	$I(t-2t_{51})$	$I(t-2t_{53})$	$I(t-2t_{55})$	I(t)
0	.5000													.5000
1	.5704	.5622	.5539	.5613	.5531	.5604	.5522	.5440	.5513	.5431	.5504	.5422	.5341	7.1786
2	.6417	.6335	.6252	.6326	.6243	.6317	.6234	.6152	.6226	.6143	.6217	.6134	.6051	8.1047
3	.7120	.7040	.6959	.7031	.6951	.7023	.6942	.6861	.6934	.6852	.6925	.6844	.6762	9.0244
4	.7792	.7716	.7640	.7708	.7632	.7700	.7624	.7547	.7616	.7539	.7608	.7530	.7453	9.9105
5	.8409	.8341	.8272	.8334	.8265	.8326	.8257	.8187	.8250	.8179	.8242	.8171	.8101	10.7334
6	.8950	.8892	.8833	.8886	.8826	.8880	.8820	.8759	.8814	.8752	.8807	.8746	.8684	11.4649
7	.9395	.9349	.9302	.9344	.9297	.9339	.9291	.9242	.9286	.9237	.9281	.9231	.9180	12.0774
8	.9727	.9695	.9661	.9691	.9657	.9687	.9653	.9617	.9650	.9614	.9646	.9610	.9572	12.5480
9	.9931	.9914	.9895	.9912	.9893	.9910	.9891	.9870	.9889	.9868	.9887	.9866	.9843	12.8569
10	1.0000	.9999	.9996	.9999	.9996	.9999	.9995	.9990	.9995	.9990	.9995	.9989	.9982	12.9925
11	.9931	.9946	.9959	.9947	.9960	.9949	.9962	.9973	.9963	.9974	.9964	.9975	.9984	12.9487
12	.9727	.9757	.9786	.9760	.9789	.9763	.9791	.9818	.9794	.9821	.9797	.9823	.9848	12.7274
13	.9395	.9440	.9483	.9444	.9487	.9449	.9492	.9533	.9496	.9537	.9500	.9541	.9580	12.3377
14	.8950	.9007	.9063	.9013	.9068	.9019	.9074	.9128	.9080	.9134	.9086	.9139	.9192	11.7953
15	.8409	.8476	.8542	.8483	.8549	.8490	.8556	.8620	.8563	.8627	.8570	.8634	.8697	11.1216
16	.7792	.7866	.7940	.7874	.7948	.7882	.7956	.8029	.7964	.8037	.7972	.8044	.8116	10.3420
17	.7120	.7200	.7279	.7209	.7288	.7217	.7296	.7375	.7305	.7383	.7313	.7392	.7444	9.4821
18	.6417	.6500	.6582	.6508	.6590	.6517	.6599	.6681	.6608	.6690	.6617	.6699	.6780	8.5788
19	.5704	.5786	.5869	.5795	.5878	.5804	.5887	.5969	.5895	.5978	.5904	.5987	.6070	7.6526
20	.5000	.5081	.5161	.5089	.5170	.5098	.5179	.5260	.5187	.5268	.5196	.5277	.5359	6.7325

3

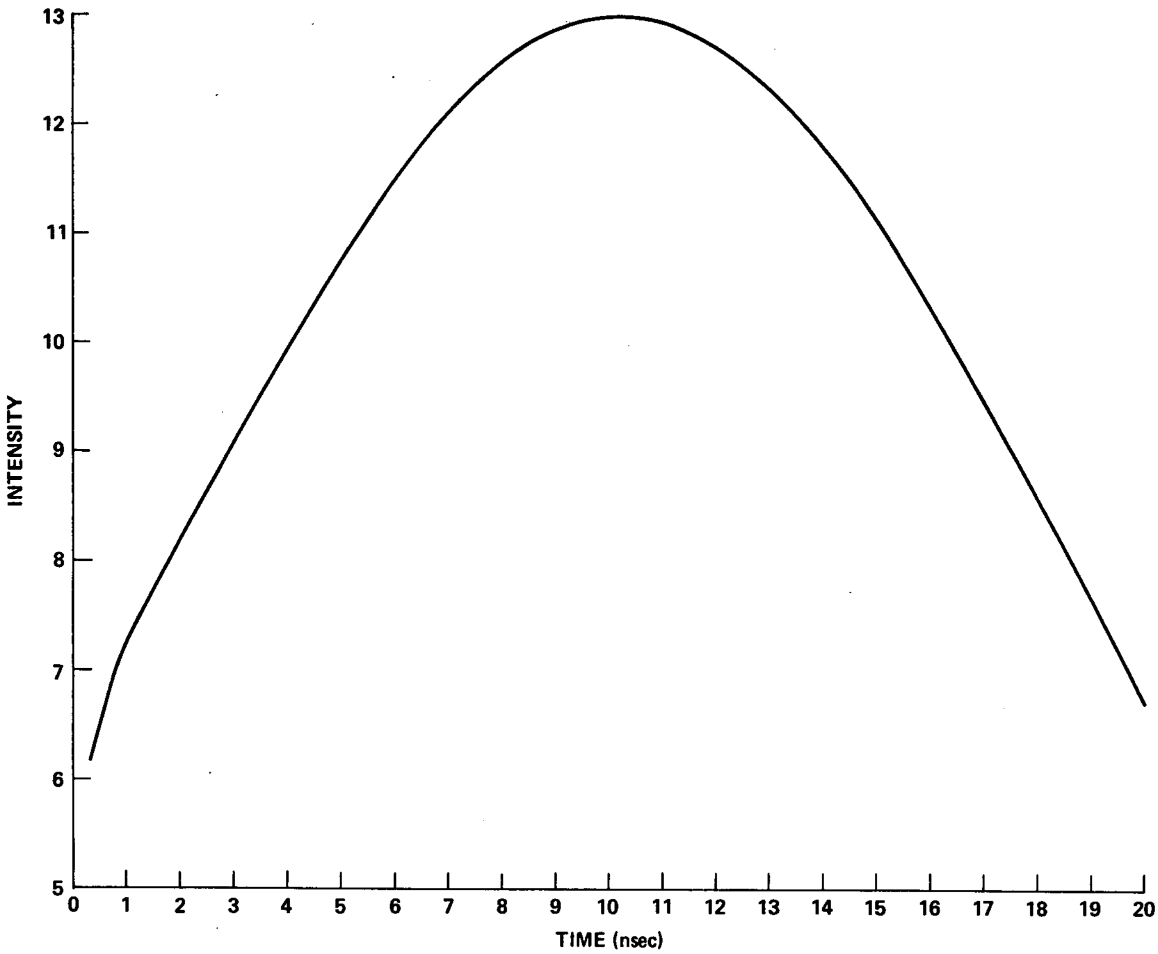


Figure 5. Intensity of Return Pulse for Rectangular Array

We consider the line of intersection of a planar wave with the plane of the spacecraft's base as it sweeps across the base from left to right on Figure 6. It will intersect the indicated "rows" of reflectors in panels 1, 2, 3, and 4 with the angles $\beta_1, \beta_2, \beta_3$, and β_4, respectively, as shown in Figure 6. (For convenience, a "fictitious" row of reflectors has been created for panel 2--to be explained later). To indicate the approach taken for analysis, the case $0 \leqslant \beta_1 \leqslant 22^\circ.5$ will be described in detail. The various cases correspond to the different sequences in which the panels are "struck" by the line of intersection as it sweeps across the spacecraft's base.

From Figure 6, we have

$$\beta_3 = \beta_1, \qquad \beta_2 = \beta_4 = 45^\circ - \beta_1.$$

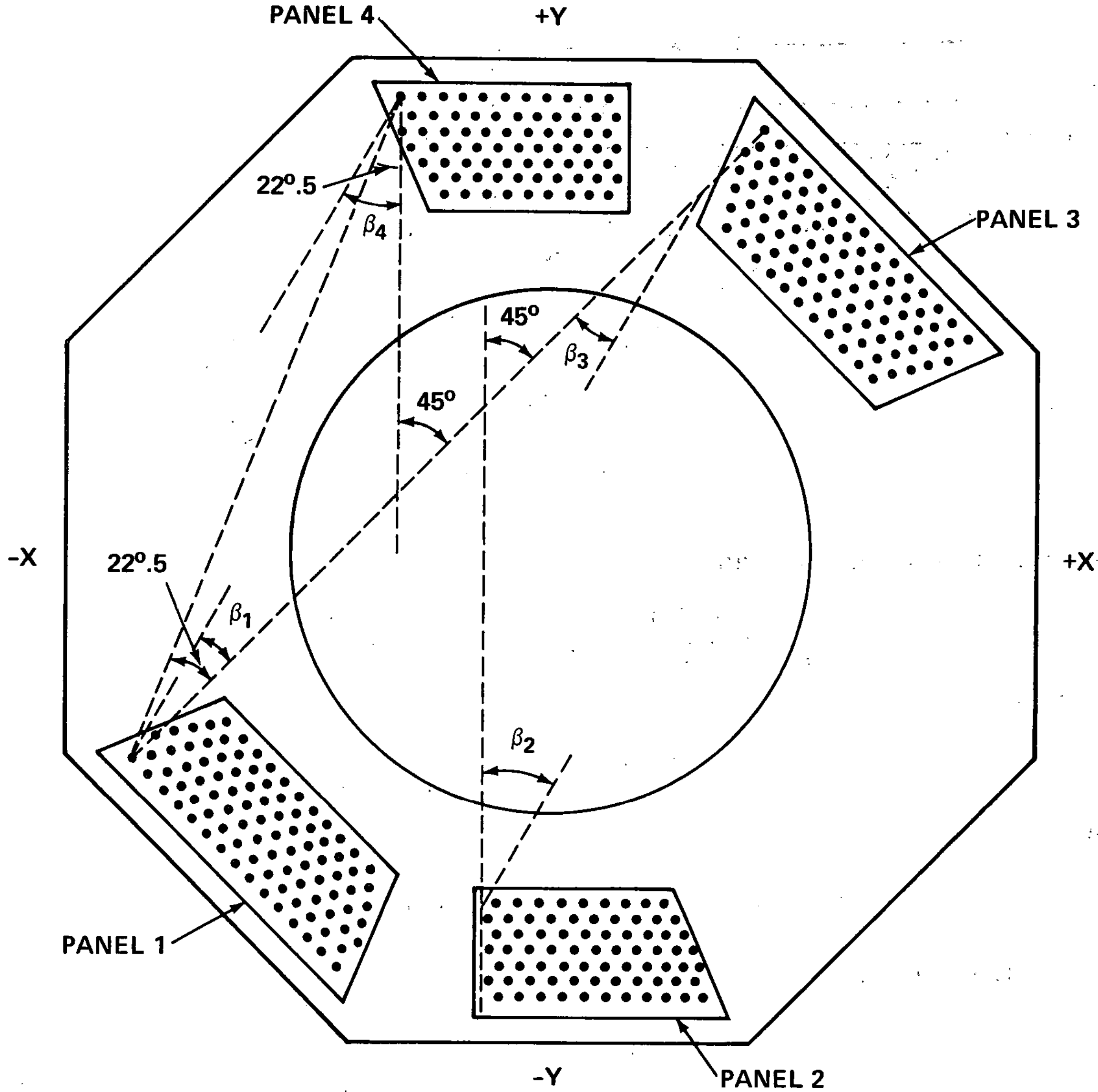
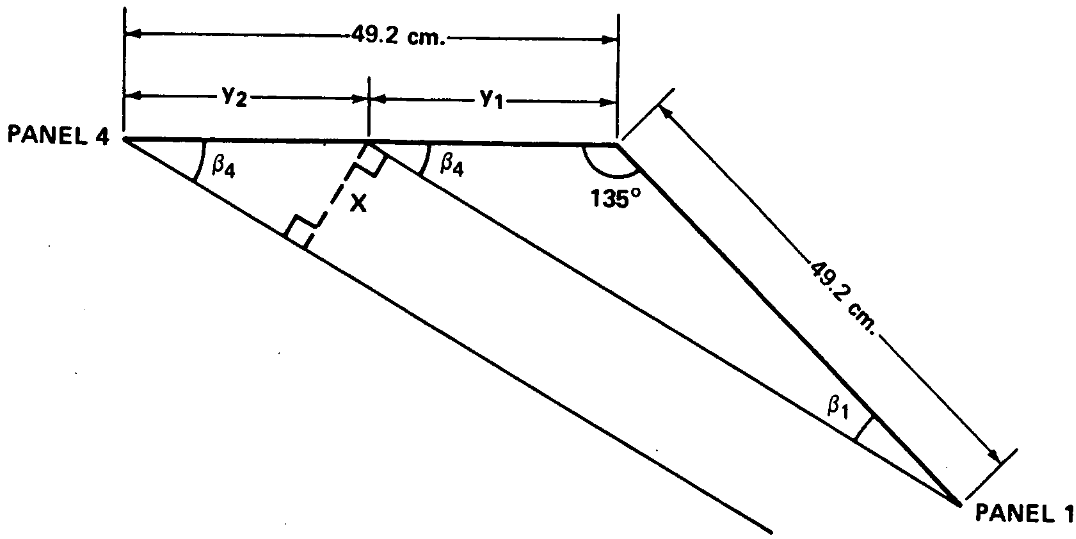


Figure 6. GEOS-1 Panel Configuration

Thus, for $0 \leqslant \beta_1 \leqslant 22^\circ.5$, panel 4 will be encountered first by the line of intersection. The first problem is to determine the time differences between the encounter with panel 4 and the encounters with the other panels (or, equivalently, the relative projected distances between panel 4 and the other panels as determined by the line of intersection). For panels 1 and 4, we have the following configuration:



We wish to determine the distance x. We have

$$\frac{y_1}{\sin \beta_1} = \frac{49.2 \text{ cm.}}{\sin \beta_4} \Rightarrow y_1 = (49.2 \text{ cm.}) \frac{\sin \beta_1}{\sin \beta_4},$$

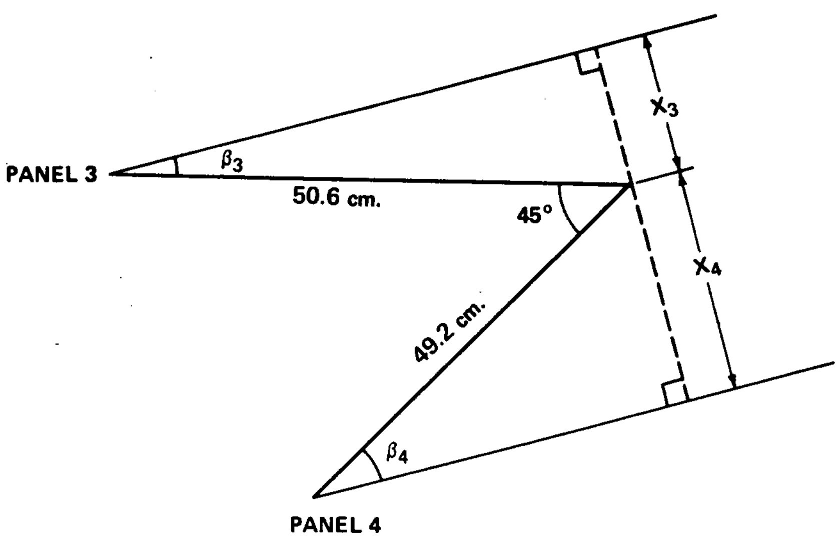
Then,

$$y_2 = 49.2 \text{ cm.} - y_1 = (49.2 \text{ cm.}) \left(1 - \frac{\sin \beta_1}{\sin \beta_4}\right),$$

and

$$x = y_2 \sin \beta_4 \ (49.2 \text{ cm.}) (\sin \beta_4 - \sin \beta_1).$$

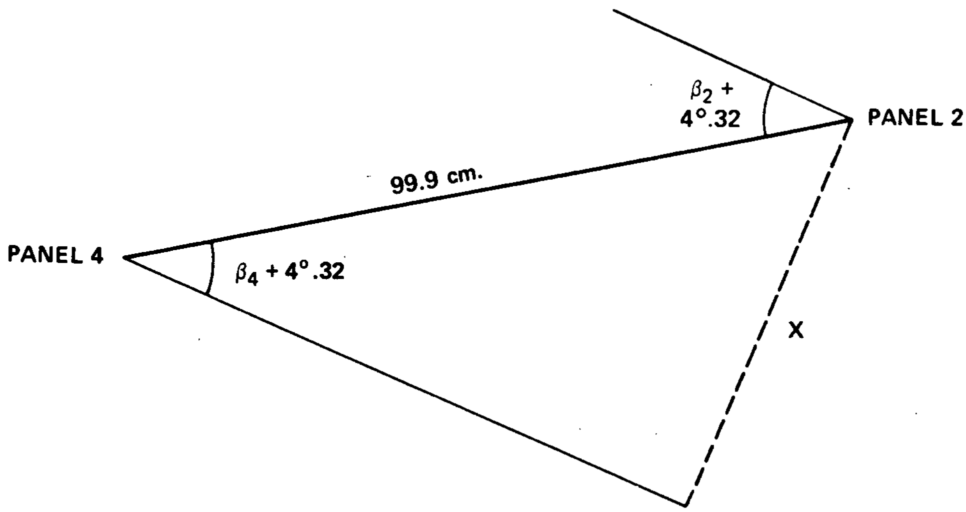
For panels 3 and 4:



We wish to determine $x_3 + x_4$. We have

$$x_3 + x_4 = (50.6 \text{ cm.}) \sin \beta_3 + (49.2 \text{ cm}) \sin \beta_4 .$$

<u>For panels 2 and 4:</u>



$$x = (99.9 \text{ cm.}) \sin (\beta_4 + 4^\circ.32)$$

The reflectors in panel 4 have the following configuration:

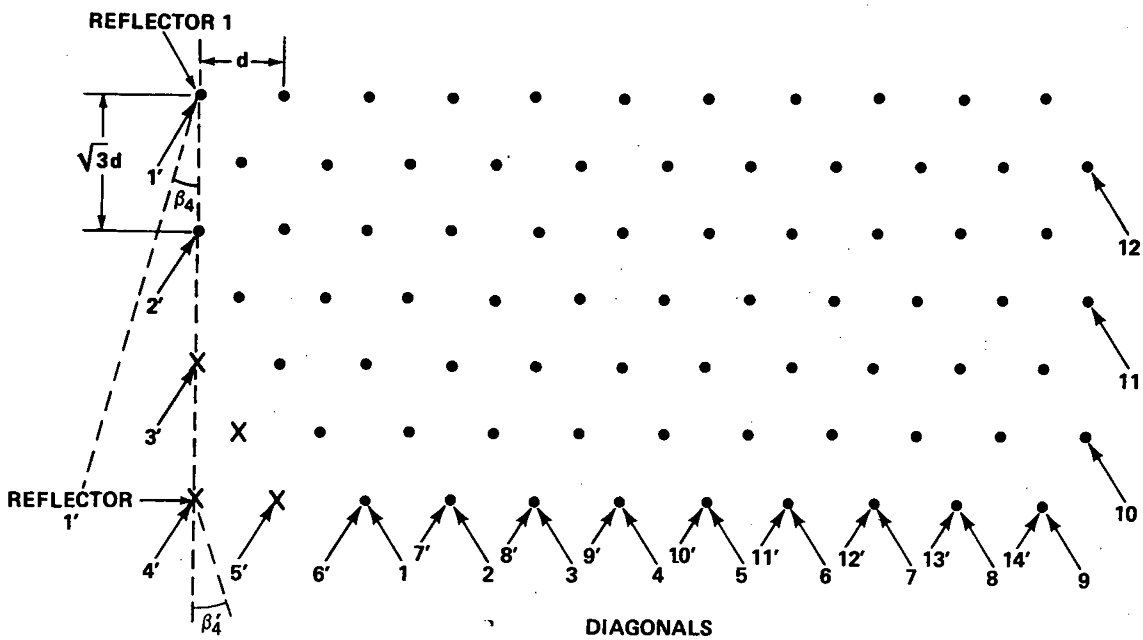


Figure 7. Panel 4.

Using the results previously obtained for a rectangular array, we have the following elapsed times of incidence (from the indicated reflector 1) for the reflectors in the various diagonals:

Diagonal 1: $d\,[i \sin(30^\circ + \beta_4) + \sqrt{3} \sin \beta_4]\,\frac{\cos \alpha}{c}$, $i = 0, 1, 2, 3, 4$

Diagonal 2: $id \sin(30^\circ + \beta_4)\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 3: $d\,[i \sin(30^\circ + \beta_4) + \cos \beta_4]\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 4: $[i \sin(30^\circ + \beta_4) + 2 \cos \beta_4]\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

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Diagonal 9: $d\,[i \sin(30^\circ + \beta_4) + 7 \cos \beta_4]\,\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 10: $d\,[i \sin(30^\circ + \beta_4) + 8 \cos \beta_4]\,\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 5$

Diagonal 11: $d\,[i \sin(30^\circ + \beta_4) + 9 \cos \beta_4]\,\frac{\cos \alpha}{c}$, $i = 0, 1, 2, 3$

Diagonal 12: $d\,[i \sin(30^\circ + \beta_4) + 10 \cos \beta_4]\,\frac{\cos \alpha}{c}$, $i = 0, 1$

For panel 1, the reflectors have the following configuration:

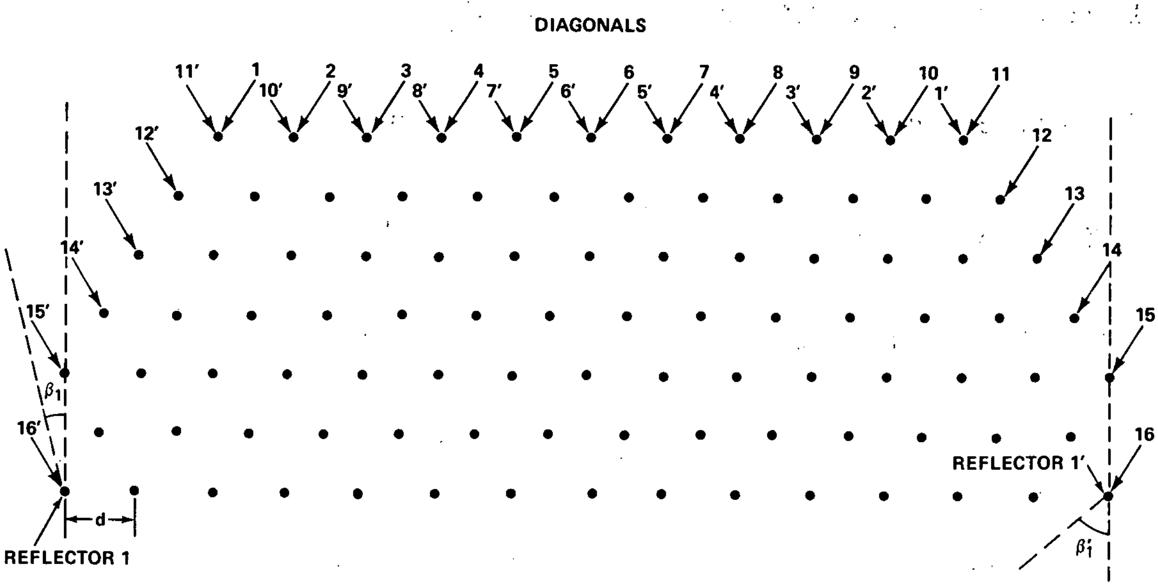

Figure 8. Panel 1.

We have the following elapsed times of incidence (from the indicated reflector 1) for the reflectors in the various diagonals:

Diagonal 1: $d\,[i \sin(30° + \beta_1) + \sqrt{3} \sin \beta_1]\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 4$

Diagonal 2: $id \sin(30° + \beta_1)\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 3: $d\,[i \sin(30° + \beta_1) + \cos \beta_1]\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 4: $d\,[i \sin(30° + \beta_1) + 2 \cos \beta_1]\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

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Diagonal 11: $d\,[i \sin(30° + \beta_1) + 9 \cos \beta_1]\,\dfrac{\cos a}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 12: $d\,[i \sin(30^\circ + \beta_1) + 10 \cos\beta_1]\,\dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 5$

Diagonal 13: $d\,[i \sin(30^\circ + \beta_1) + 11 \cos\beta_1]\,\dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 4$

Diagonal 14: $d\,[i \sin(30^\circ + \beta_1) + 12 \cos\beta_1]\,\dfrac{\cos\alpha}{c}$, $i = 0, 1, 2, 3$

Diagonal 15: $d\,[i \sin(30^\circ + \beta_1) + 13 \cos\beta_1]\,\dfrac{\cos\alpha}{c}$, $i = 0, 1, 2$

Diagonal 16: $d\,[i \sin(30^\circ + \beta_1) + 14 \cos\beta_1]\,\dfrac{\cos\alpha}{c}$, $i = 0$

To refer back to reflector 1, panel 4, we add the time

$$(49.2 \text{ cm.})\,(\sin\beta_4 - \sin\beta_1)\,\frac{\cos\alpha}{c}$$

to each of the above times.

For panel 2, we have

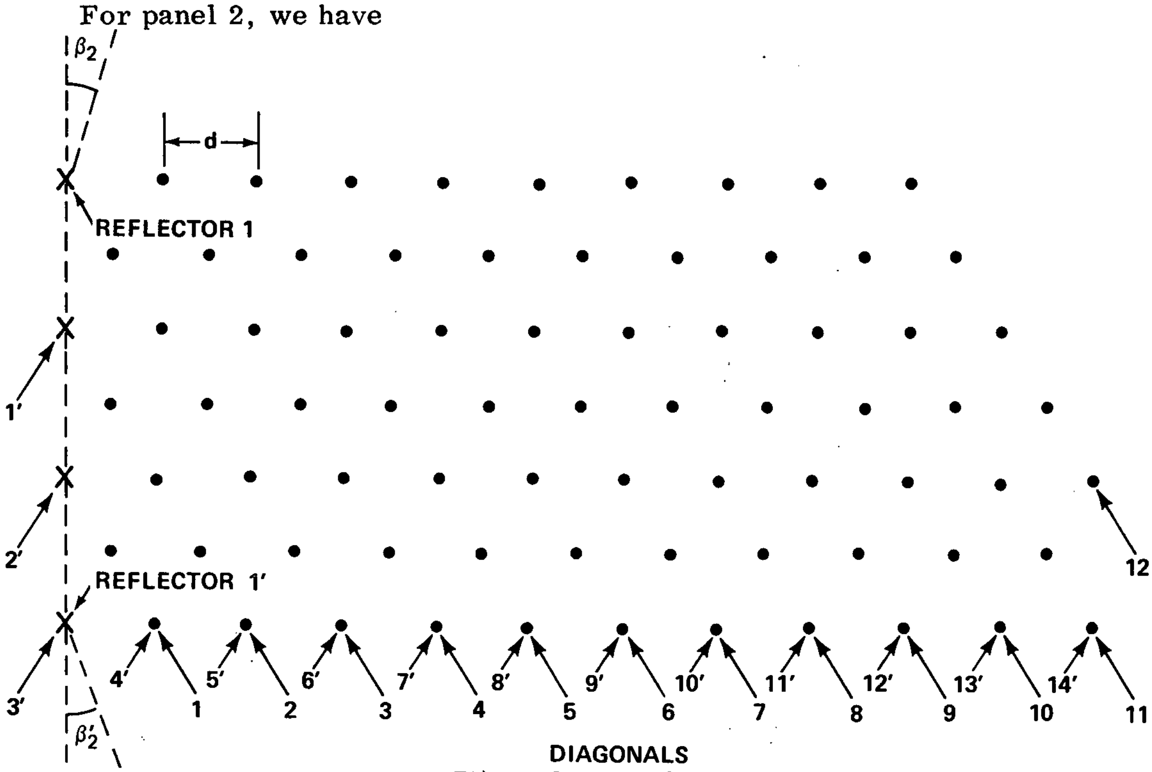

Figure 9. Panel 2.

A "fictitious" row of reflectors, indicated by the x's, has been introduced here to simplify computations. The elapsed times of incidence (from the indicated reflector 1) for the reflectors in the various diagonals are as follows:

Diagonal 1: $d\,[i \sin(30^\circ + \beta_2) + 2\sqrt{3} \sin \beta_2]\,\dfrac{\cos \alpha}{c}$, $i = 1, 2$

Diagonal 2: $d\,[i \sin(30^\circ + \beta_2) + \sqrt{3} \sin \beta_2]\,\dfrac{\cos \alpha}{c}$, $i = 1, 2, 3, 4$

Diagonal 3: $id \sin(30^\circ + \beta_2)\dfrac{\cos \alpha}{c}$, $i = 1, 2, \ldots, 6$

Diagonal 4: $d\,[i \sin(30^\circ + \beta_2) + \cos \beta_2]\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

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Diagonal 11: $d\,[i \sin(30^\circ + \beta_2) + 8 \cos \beta_2]\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 12: $d\,[i \sin(30^\circ + \beta_2) + 9 \cos \beta_2]\,\dfrac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 4$

We need to add the time

$$(99.9 \text{ cm.}) \sin(\beta_4 + 4^\circ.32)\frac{\cos \alpha}{c}$$

to each of the above times to refer back to reflector 1, panel 4.

Finally, we have the following configuration for panel 3, with the introduction of fictitious reflectors indicated by x's:

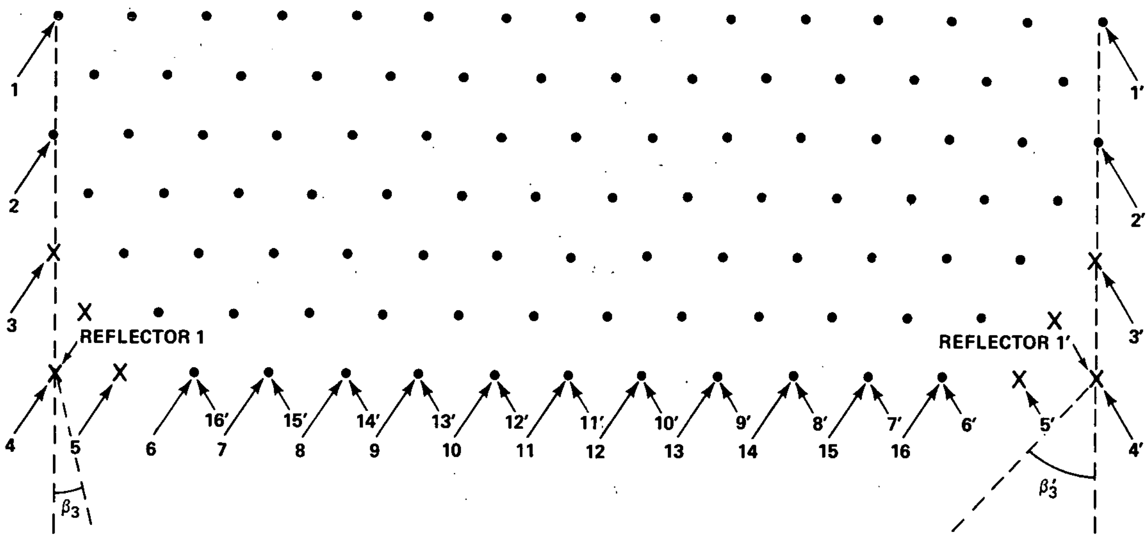

Figure 10. Panel 3.

The elapsed times of incidence from reflector 1 are as follows:

Diagonal 1: $d\,[i \sin(30^\circ + \beta_3) + 3\sqrt{3}\sin\beta_3]\,\dfrac{\cos\alpha}{c}$, $i = 0$

Diagonal 2: $d\,[i \sin(30^\circ + \beta_3) + 2\sqrt{3}\sin\beta_3]\,\dfrac{\cos\alpha}{c}$, $i = 0, 1, 2$

Diagonal 3: $d\,[i \sin(30^\circ + \beta_3) + \sqrt{3}\sin\beta_3]\,\dfrac{\cos\alpha}{c}$, $i = 1, 2, 3, 4$

Diagonal 4: $id \sin(30^\circ + \beta_3)\dfrac{\cos\alpha}{c}$, $i = 2, 3, \ldots, 6$

Diagonal 5: $d\,[i \sin(30^\circ + \beta_3) + \cos\beta_3]\,\dfrac{\cos\alpha}{c}$, $i = 1, 2, \ldots, 6$

Diagonal 6: $d\,[i \sin(30^\circ + \beta_3) + 2\cos\beta_3]\,\dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 6$

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Diagonal 15: $d\,[i \sin(30^\circ + \beta_3) + 11 \cos \beta_3]\,\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 16: $d\,[i \sin(30^\circ + \beta_3) + 12 \cos \beta_3]\,\frac{\cos \alpha}{c}$, $i = 0, 1, \ldots, 4$

To each of the above times, we add

$$[(50.6 \text{ cm.}) \sin \beta_3 + (49.2 \text{ cm.}) \sin \beta_4]\,\frac{\cos \alpha}{c}$$

to refer back to reflector 1, panel4.

This completes the analysis for $0 \leqslant \beta_1 \leqslant 22^\circ.5$. Panel 4, reflector 1 is encountered first; panel 1, reflector 1 second; panel 3, reflector 1 third; and panel 2, reflector 1 fourth (for $\beta_1 = 0^\circ$, panels 1 and 3, reflectors 1 are encountered simultaneously; and for $\beta_1 = 22^\circ.5$, panels 1 and 4, reflectors 1, are encountered simultaneously).

Case (ii) $22^\circ.5 < \beta_1 \leqslant 45^\circ$

In this case, the same panel configurations as for case (i) are used. However, panel 1 (reflector 1) is encountered first, so all times are referred back to this reflector. The following times need to be added to the elapsed times for the various panels:

panel 4: $(49.2 \text{ cm.})(\sin \beta_1 - \sin \beta_4)\frac{\cos \alpha}{c}$

panel 3: $(99.7 \text{ cm.}) \sin \beta_1 \frac{\cos \alpha}{c}$

panel 2: $[(59.8 \text{ cm.}) \sin \beta_1 + (57.8 \text{ cm.}) \sin \beta_2]\,\frac{\cos \alpha}{c}$

As in case (i),

$$\beta_3 = \beta_1, \beta_2 = \beta_4 = 45^\circ - \beta_1.$$

When $\beta_1 = 26^\circ.85$, panels 2 and 3 are encountered simultaneously.

Case (iii) $45° < \beta_1 \leqslant 90°$.

Panel 1 is still encountered first. However, for panels 2 and 4 different (fictitious) reflectors are chosen as reference reflectors 1 to ease the computation. For panel 4, refer back to Figure 7--the reference fictitious reflector is reflector 1', and the diagonals are indicated by primes also. The elapsed times are:

Diagonal 1': $d\,[i \sin(30° + \beta_4') + 3\sqrt{3} \sin \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 0$

Diagonal 2': $d\,[i \sin(30° + \beta_4') + 2\sqrt{3} \sin \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 0, 1, 2$

Diagonal 3': $d\,[i \sin(30° + \beta_4') + \sqrt{3} \sin \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 1, 2, 3, 4$

Diagonal 4': $id \sin(30° + \beta_4') \dfrac{\cos\alpha}{c}, \; i = 2, 3, \ldots, 6$

Diagonal 5': $d\,[i \sin(30° + \beta_4') + \cos \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 1, 2, \ldots, 6$

Diagonal 6': $d\,[i \sin(30° + \beta_4') + 2\cos \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 0, 1, \ldots, 6$

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Diagonal 11': $d\,[i \sin(30° + \beta_4') + 7\cos \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 0, 1, \ldots, 6$

Diagonal 12': $d\,[i \sin(30° + \beta_4') + 8\cos \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 0, 1, \ldots, 5$

Diagonal 13': $d\,[i \sin(30° + \beta_4') + 9\cos \beta_4'] \dfrac{\cos\alpha}{c}, \; i = 0, 1, 2, 3$

Diagonal 14': $d\,[i \sin(30° + \beta_4') + 10\cos \beta_4'] \dfrac{\cos a}{c}, \; i = 0, 1$

To refer back to panel 1, reflector 1, add the time

$$[(49.2 \text{ cm.}) \sin \beta_1 + (35.9 \text{ cm.}) \sin \beta_4'] \frac{\cos \alpha}{c}$$

to each of the above elapsed times.

For panel 2, refer back to Figure 9--the reference fictitious reflector is reflector 1'. The elapsed times are

Diagonal 1': $d\,[i \sin (30° + \beta_2') + 2\sqrt{3} \sin \beta_2'] \frac{\cos \alpha}{c}, i = 1, 2$

Diagonal 2': $d\,[i \sin (30° + \beta_2'] + \sqrt{3} \sin \beta_2'] \frac{\cos \alpha}{c}, i = 1, 2, 3, 4$

Diagonal 3': $id \sin (30° + \beta_2') \frac{\cos \alpha}{c}, i = 1, 2, \ldots, 6$

Diagonal 4': $d\,[i \sin (30° + \beta_2') + \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0, 1, \ldots, 6$

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Diagonal 9': $d\,[i \sin (30° + \beta_2') + 6 \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0, 1, \ldots, 6$

Diagonal 10': $d\,[i \sin (30° + \beta_2') + 7 \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0, 1, \ldots, 5$

Diagonal 11': $d\,[i \sin (30° + \beta_2') + 8 \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0, 1, \ldots, 4$

Diagonal 12': $d\,[i \sin (30° + \beta_2') + 9 \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0, 1, 2, 3$

Diagonal 13': $d\,[i \sin (30° + \beta_2') + 10 \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0, 1, 2$

Diagonal 14': $d\,[i \sin (30° + \beta_2') + 11 \cos \beta_2'] \frac{\cos \alpha}{c}, i = 0.$

To refer back to panel 1, reflector 1, add

$$[(59.8 \text{ cm.}) \sin \beta_1 - (71.2 \text{ cm.}) \sin \beta_2'] \frac{\cos \alpha}{c}$$

to these elapsed times. The relations between panels 1 and 3 remain the same as in case (ii). When $\beta_1 = 49°.32$, panels 2 and 4 are encountered simultaneously. In addition, we have

$$\beta_3 = \beta_1, \beta_2' = \beta_4' = \beta_1 - 45°.$$

Case (iv) $57°.73 \leqslant 90°$.

Refer back to Figures 8 and 10 for the new configurations for panels 1 and 3. The elapsed times will be the same as in case (i) for panels 1 and 3, with $\beta_1 = \beta_1', \beta_3 = \beta_3'$, and the diagonals numbered with primes. The elapsed times for panels 2 and 4 will be the same as in case (iii). Also,

$$\beta_3' = \beta_1', \beta_2' = \beta_4' = 135° - \beta_1'.$$

Panel 1, reflector 1' will be encountered first—at $\beta_1' = 57°.73$, panels 1 and 2 will be encountered simultaneously. The following times need to be added to the elapsed times for the various panels:

panel 2: $[(24.4 \text{ cm.}) \sin \beta_1' - (21.2 \text{ cm.}) \sin \beta_2'] \frac{\cos \alpha}{c}$

panel 4: $[(13.5 \text{ cm.}) \sin \beta_1' + (86.2 \text{ cm.}) \sin \beta_4'] \frac{\cos \alpha}{c}$

panel 3: $(99.7 \text{ cm.}) \sin \beta_1' \frac{\cos \alpha}{c}$

Case (v) $45° \leqslant \beta_1' < 57°.73$

Same as case (iv), except that panel 2 is encountered first. Add the following times to the elapsed times for the various panels:

panel 1: $[(21.2 \text{ cm.}) \sin \beta_2' - (24.1 \text{ cm.}) \sin \beta_1'] \frac{\cos \alpha}{c}$

panel 3: $[(21.2 \text{ cm.}) \sin \beta_2' + (75.6 \text{ cm.}) \sin \beta_3'] \frac{\cos \alpha}{c}$

panel 4: $(99.9 \text{ cm.}) \sin(\beta_2' + 4°.32) \dfrac{\cos\alpha}{c}$

Case (vi) $0 \leqslant \beta_1' < 45°$

Panel 2 is encountered first, and the configurations and elapsed times for panels 1 and 3 are the same as in cases (iv) and (v). However, panels 2 and 4 are changed as follows--for panel 2:

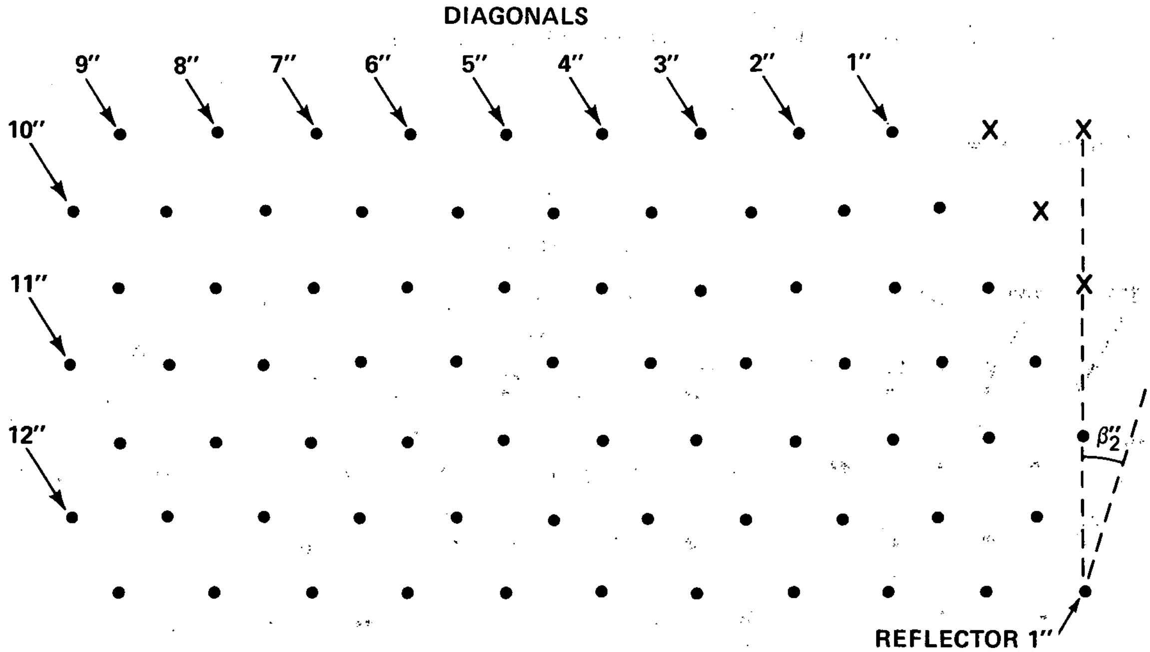

Figure 11. Panel 2.

The elapsed times of incidence are:

Diagonal 1'': $d\,[i \sin(30° + \beta_2'') + \sqrt{3} \sin\beta_2''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 4$

Diagonal 2'': $id \sin(30° + \beta_2'') \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 3'': $d\,[i \sin(30° + \beta_2'') + \cos\beta_2''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 6$

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Diagonal 9'': $d\,[i \sin(30° + \beta_2'') + 7 \cos\beta_2''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 10'': $d\,[i \sin(30° + \beta_2'') + 8 \cos\beta_2''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 5$

Diagonal 11'': $d\,[i \sin(30° + \beta_2'') + 9 \cos\beta_2''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, 2, 3$

Diagonal 12'': $d\,[i \sin(30° + \beta_2'') + 10 \cos\beta_2''] \dfrac{\cos\alpha}{c}$, $i = 0, 1$

For panel 4:

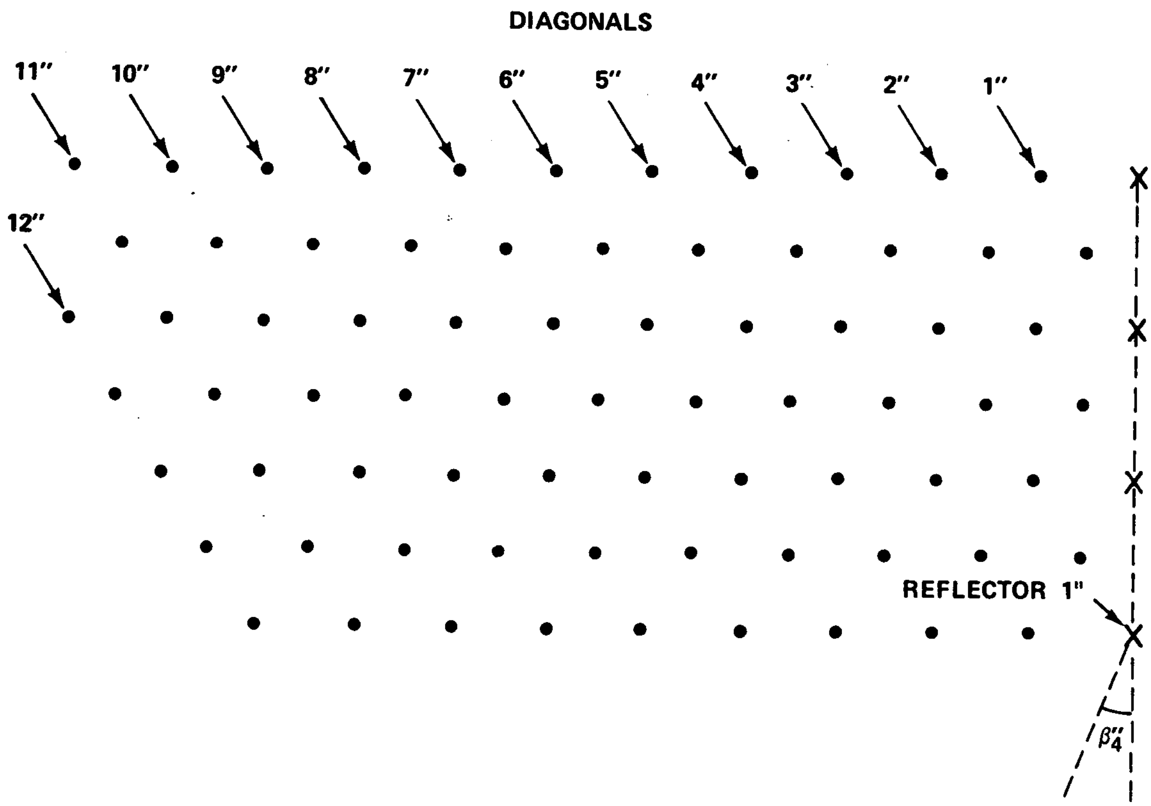

Figure 12. Panel 4.

The elapsed times of incidence are:

Diagonal 1'': $d\,[i \sin(30^\circ + \beta_4'') + 2\sqrt{3} \sin \beta_4''] \dfrac{\cos\alpha}{c}$, $i = 1, 2$

Diagonal 2'': $d\,[i \sin(30^\circ + \beta_4'') + \sqrt{3} \sin \beta_4''] \dfrac{\cos\alpha}{c}$, $i = 1, 2, 3, 4$

Diagonal 3'': $id \sin(30^\circ + \beta_4'') \dfrac{\cos\alpha}{c}$, $i = 1, 2, \ldots, 6$

Diagonal 4'': $d\,[i \sin(30^\circ + \beta_4'') + \cos \beta_4''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 6$

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Diagonal 11'': $d\,[i \sin(30^\circ + \beta_4'') + 8 \cos \beta_4''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 6$

Diagonal 12'': $d\,[i \sin(30^\circ + \beta_4'') + 9 \cos \beta_4''] \dfrac{\cos\alpha}{c}$, $i = 0, 1, \ldots, 4$

For this case,

$$\beta_3' = \beta_1', \beta_2'' = \beta_4'' = 45^\circ + \beta_1'.$$

The following times are to be added to the elapsed times for the various panels:

panel 1: $[(49.4 \text{ cm.}) \sin \beta_2'' - (64.0 \text{ cm.}) \sin \beta_1'] \dfrac{\cos\alpha}{c}$

panel 3: $[(49.4 \text{ cm.}) \sin \beta_2'' + (36.0 \text{ cm.}) \sin \beta_3'] \dfrac{\cos\alpha}{c}$

panel 4: $(99.9 \text{ cm.}) \sin(c4 + 4^\circ.32) \dfrac{\cos\alpha}{c}$

CONCLUSIONS

Figures 13 through 24 show the predicted return intensities for GEOS-1 for the various cases discussed. In these figures, θ is the angle between the laser beam and the spin axis of the satellite. For $\theta = 0^\circ$, each of the 334 corner cube reflectors is encountered simultaneously by the laser beam--thus, the normalized peak return intensity is 334 at the 10 nsec. mark. For any $\theta \neq 0^\circ$, the peak intensity will occur at some time later than 10 nsec. and will have a value less than 334; the difference between this time and 10 nsec. is the time between the first encounter of a planar beam wave with a reflector and the last encounter (or, alternatively, twice the time between the wave's first encounter with a reflector and its encounter with the geometric center of the base of the satellite). The maximum time differential is about 3-4 nsec., or about 1 meter in distance.

Variation in the in-plane angle β_1 causes time differences of a fraction of a nanosecond, and are therefore negligible. Thus, for a given θ (which is determinable from attitude and elevation information), one can determine the time of peak return intensity, which corresponds to the geometric center of the base of the spacecraft.

ACKNOWLEDGEMENT

The author is grateful to Mr. J. P. Murphy of the Geodynamics Branch for helpful discussions and suggestions.

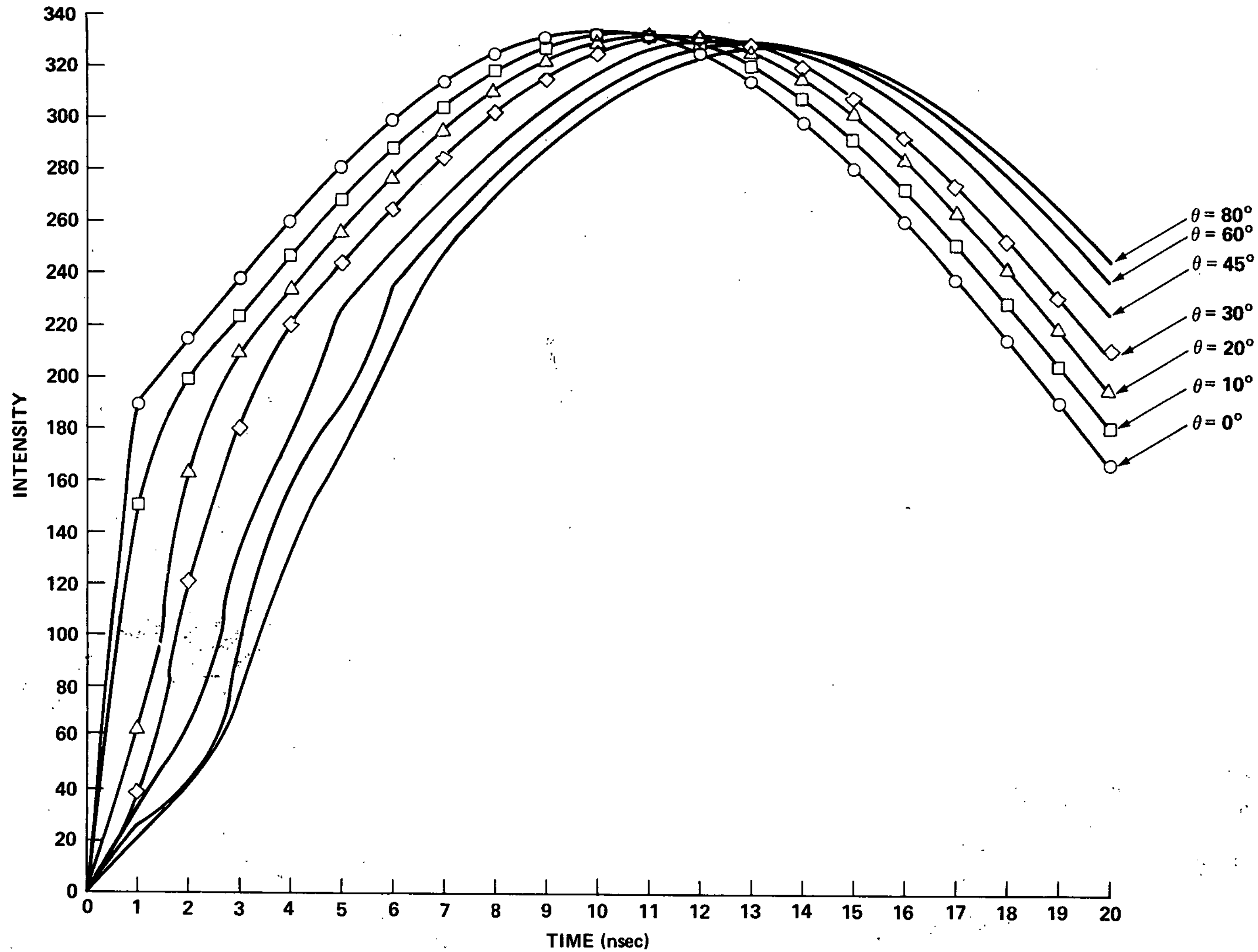

Figure 13. Return Intensity for GEOS-1 ($\beta_1 = 0°$), Case (i)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

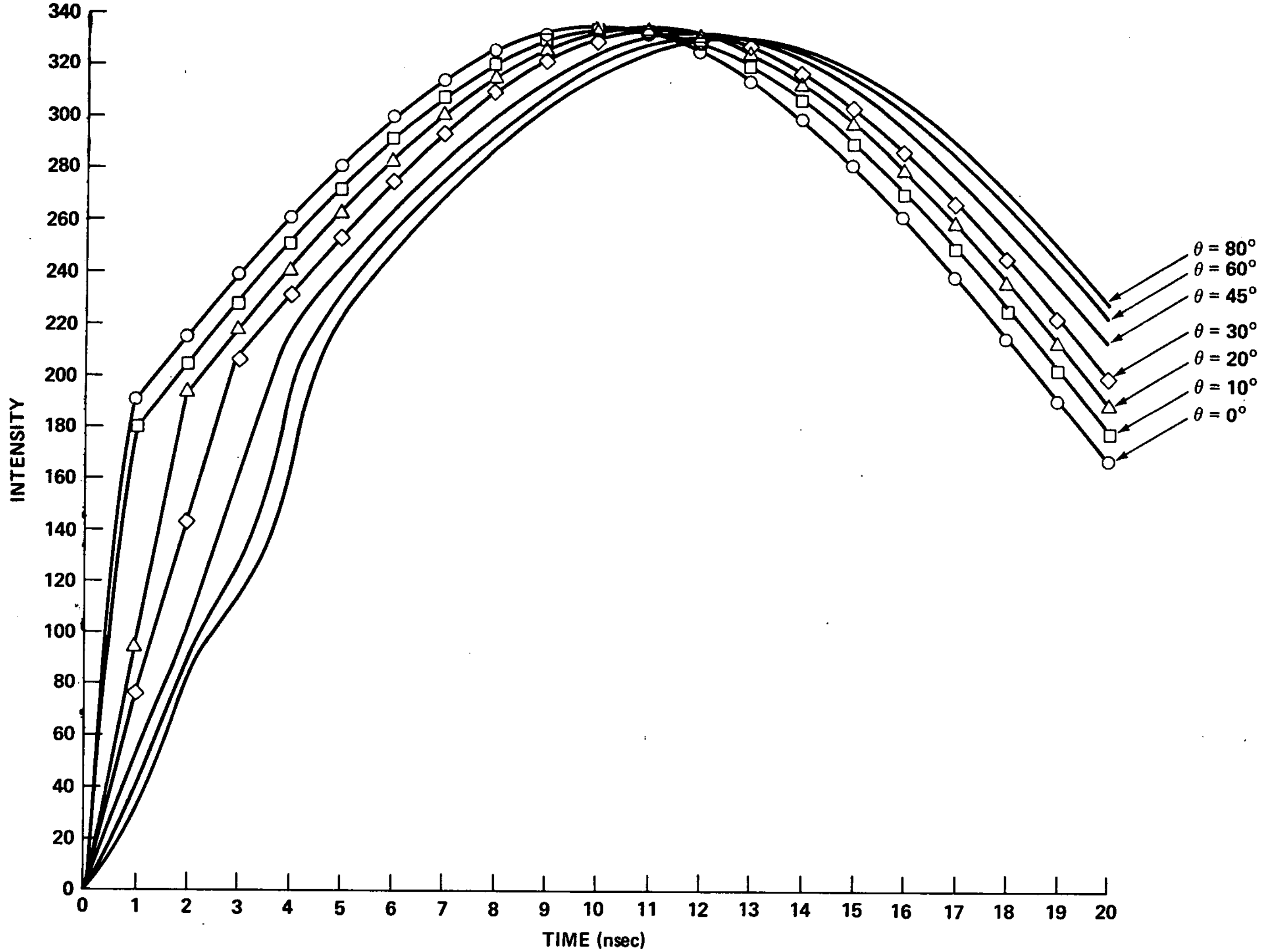

Figure 14. Return Intensity for GEOS-1 ($\beta_1 = 20°$), Case (i)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

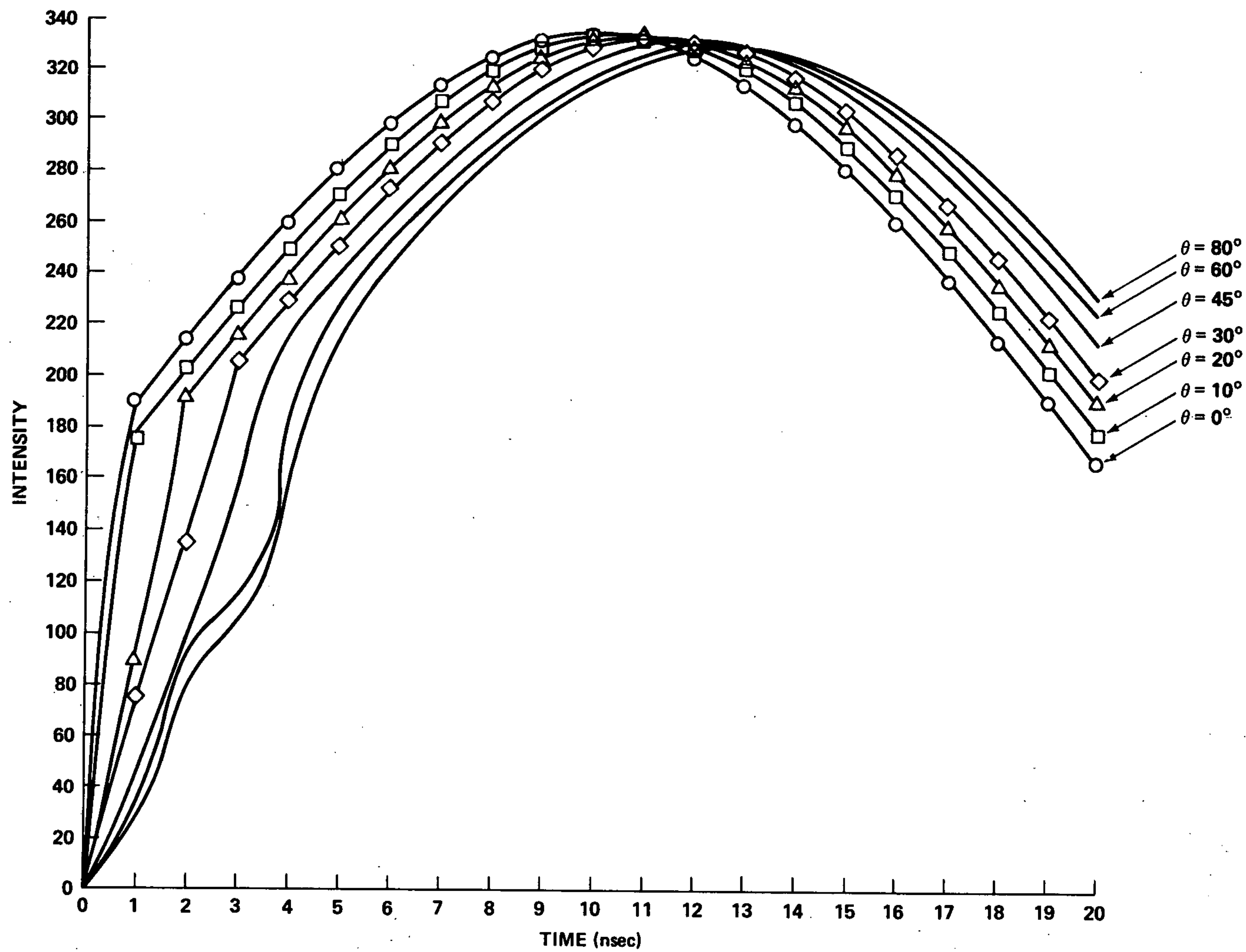

Figure 15. Return Intensity for GEOS-1 ($\beta_1 = 27°.5$), Case (ii)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

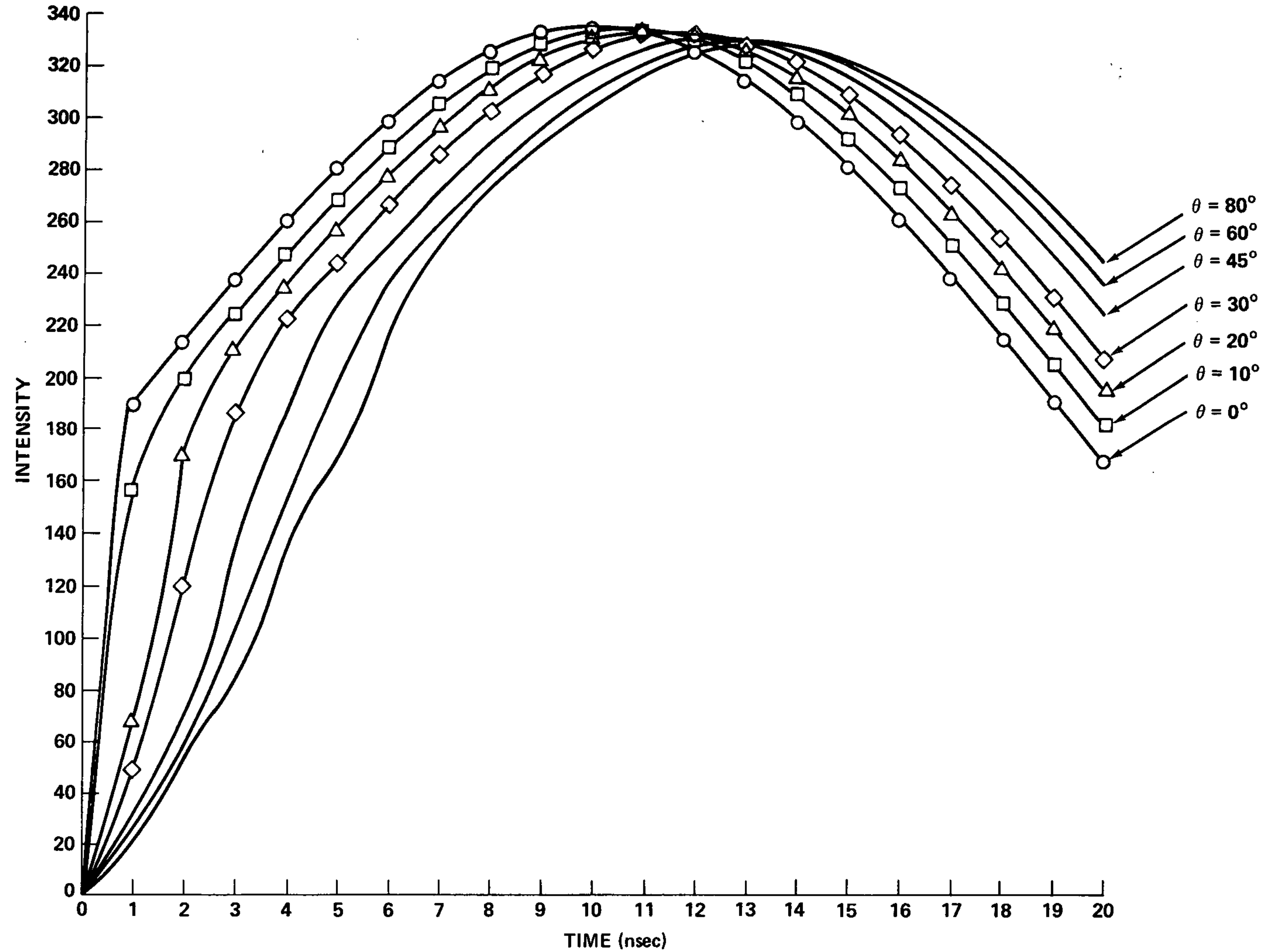

Figure 16. Return Intensity for GEOS-1 (β_1 = 42°.5), Case (ii)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

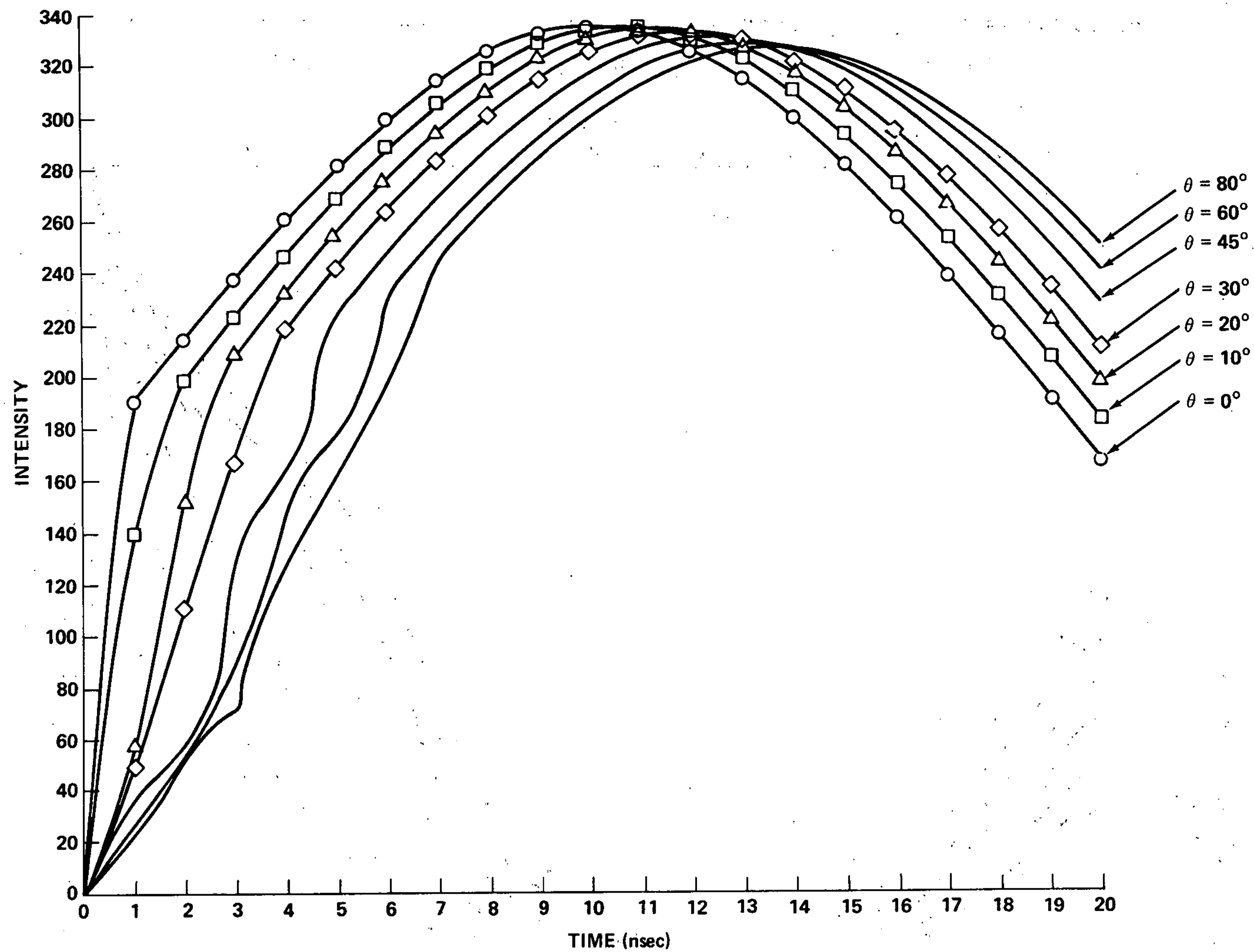

Figure 17. Return Intensity for GEOS-1 ($\beta_1 = 50°$), Case (iii)
(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

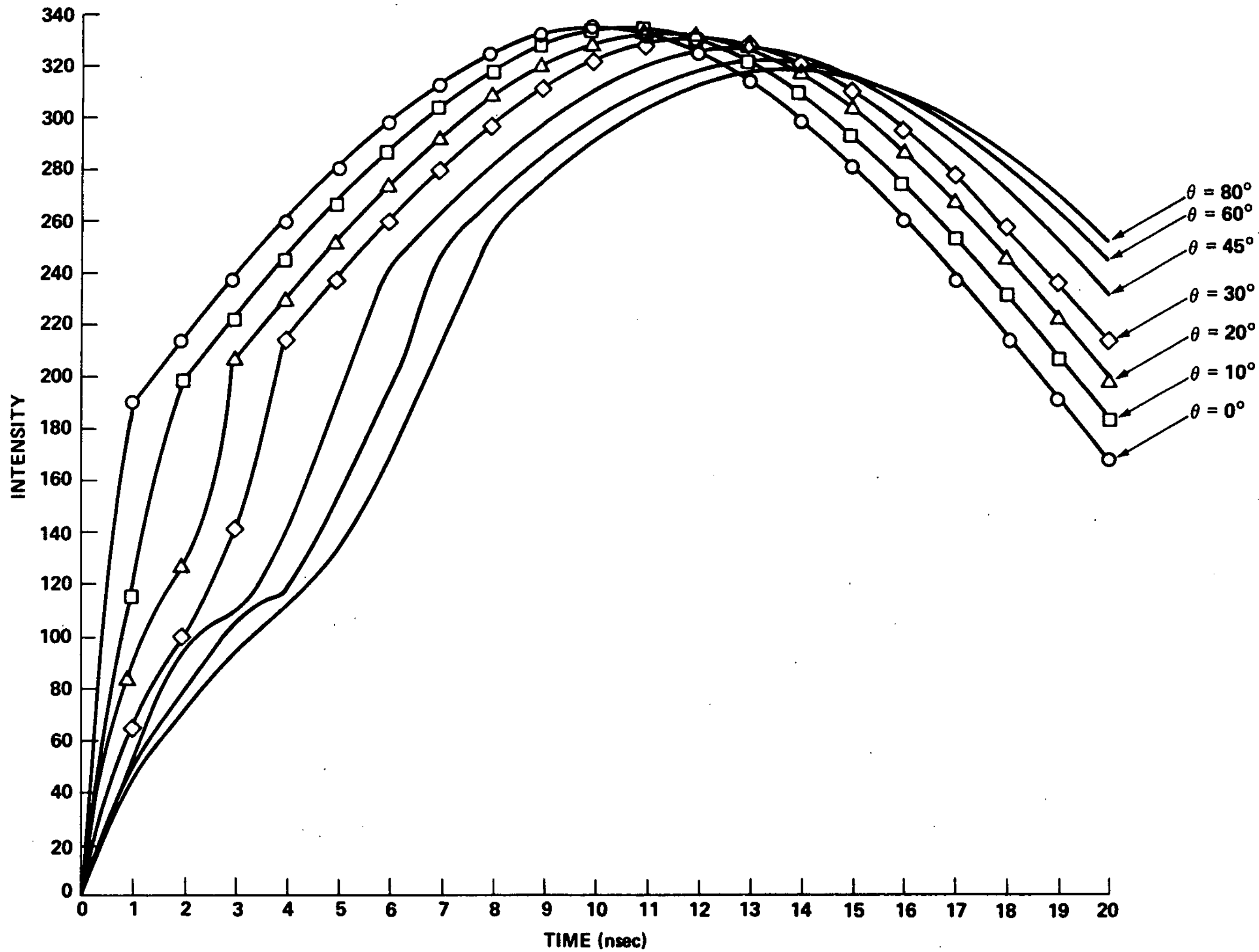

Figure 18. Return Intensity for GEOS-1 (β_1 = 80°), Case (iii)
(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

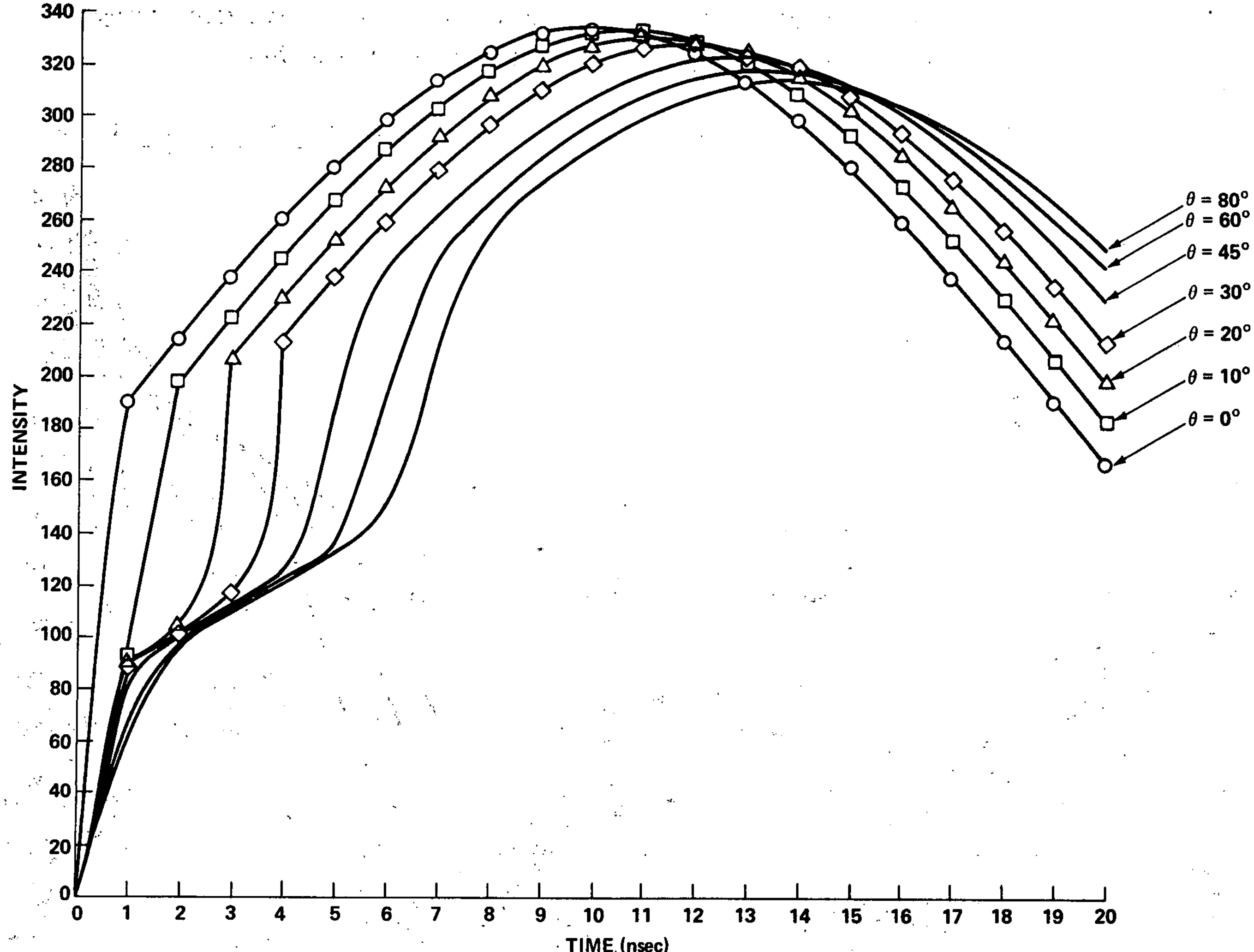

Figure 19. Return Intensity for GEOS-1 ($\beta_1 = 60°$), Case (iv)
(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

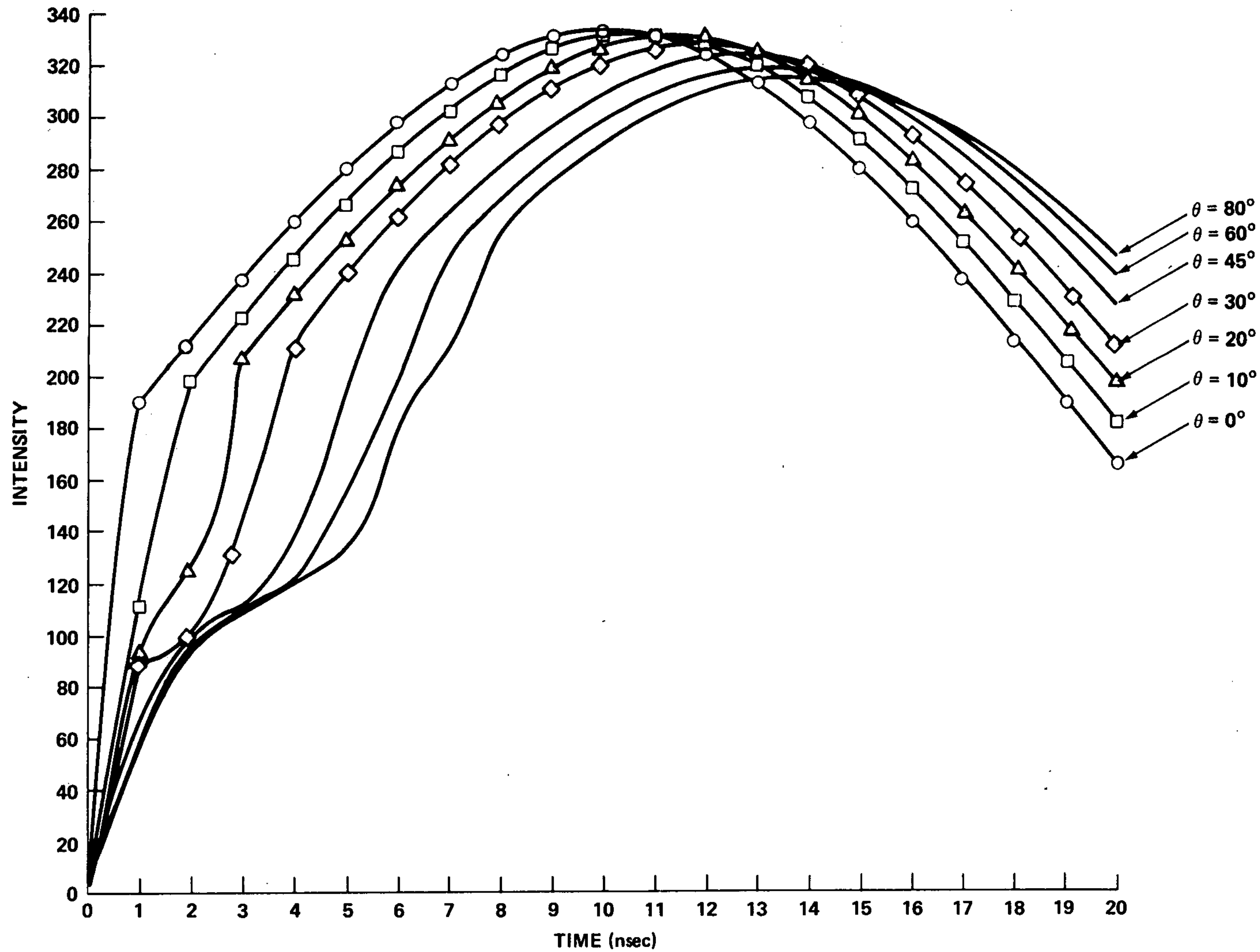

Figure 20. Return Intensity for GEOS-1 ($\beta_1 = 85°$), Case (iv)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

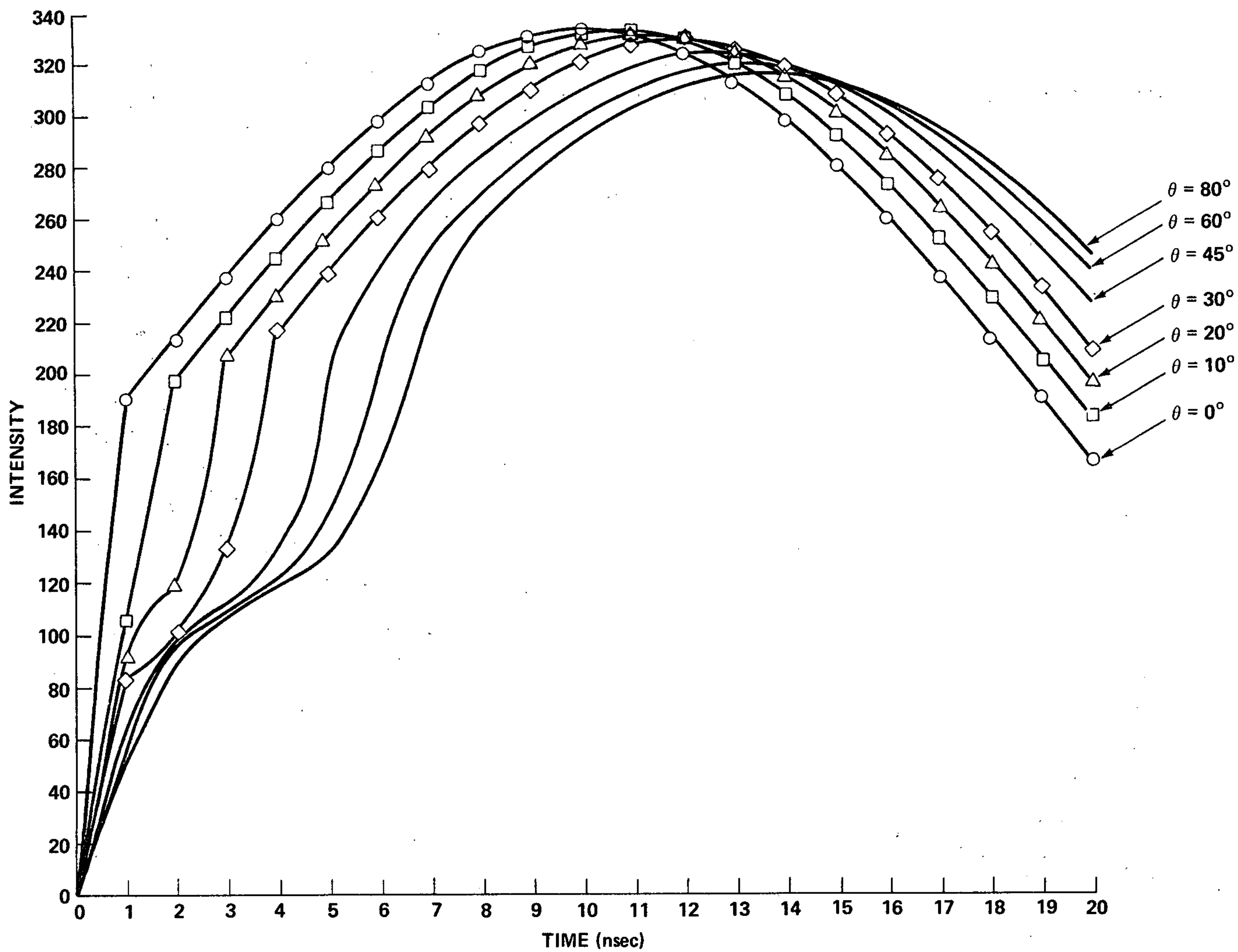

Figure 21. Return Intensity for GEOS-1 ($\beta_1 = 45°$), Case (v)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

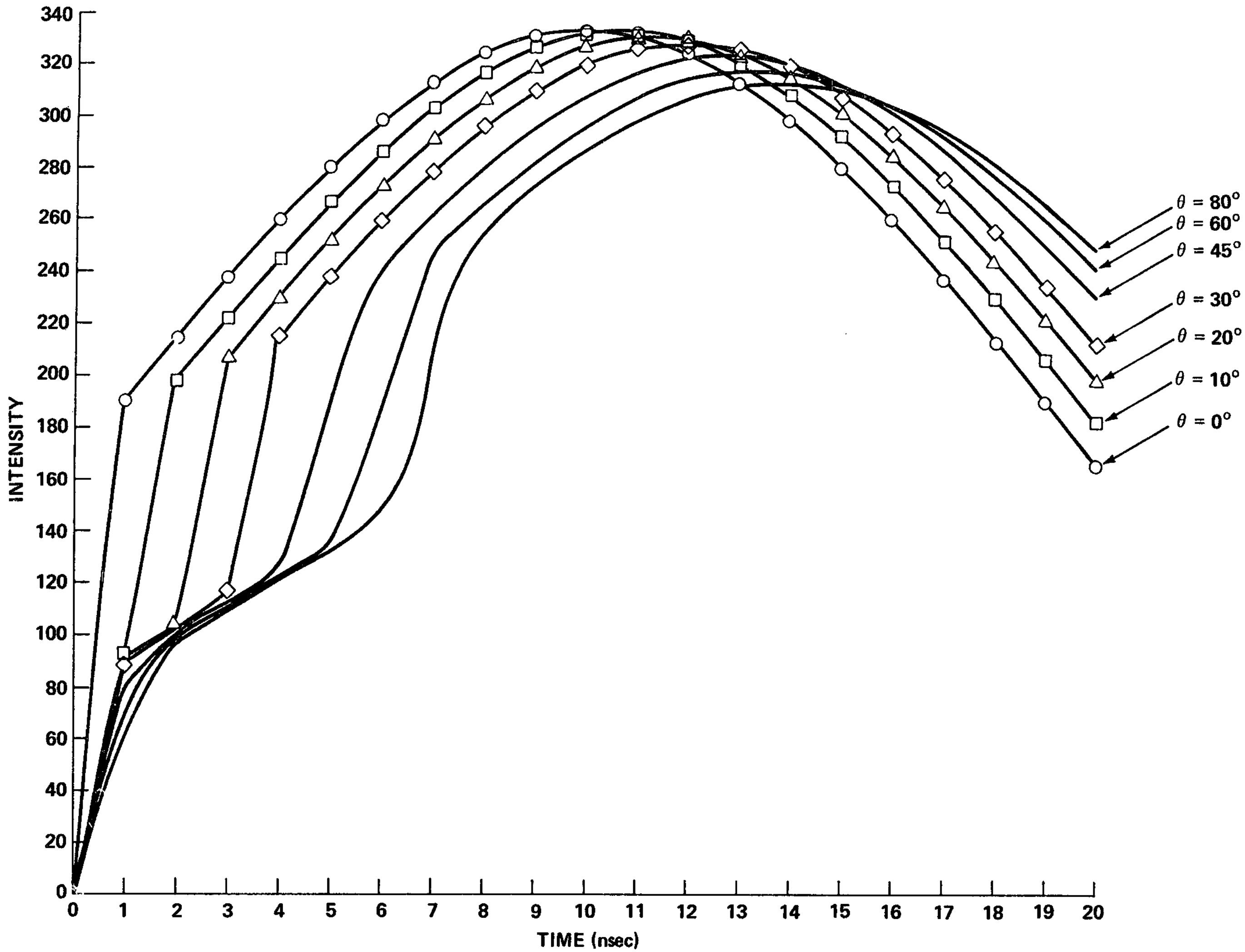

Figure 22. Return Intensity for GEOS-1 (β_1 = 57°.732628), Case (v)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

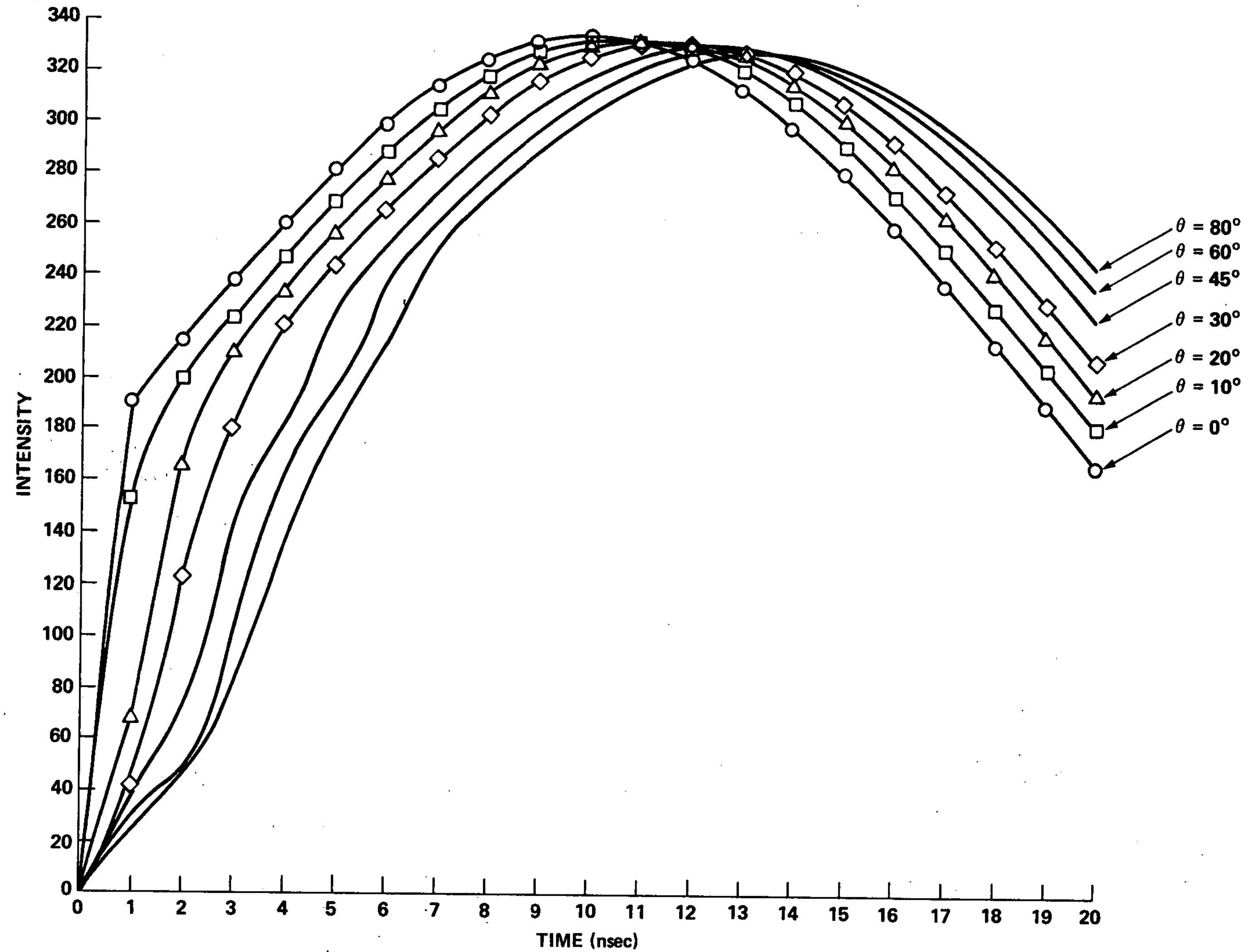

Figure 23. Return Intensity for GEOS-1 ($\beta_1 = 0°$), Case (vi)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)

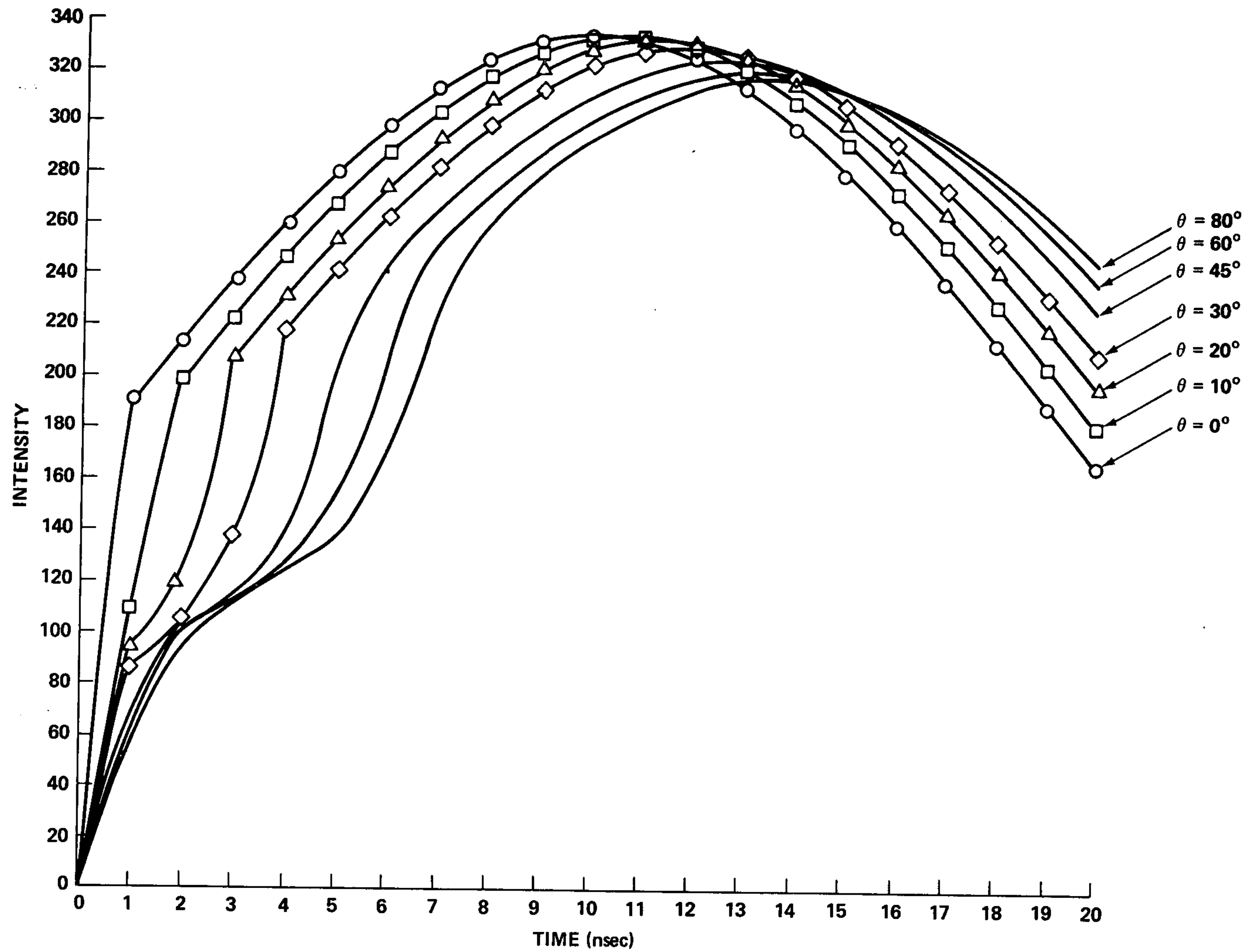

Figure 24. Return Intensity for GEOS-1 (β_1 = 45°), Case (vi)

(Note: Intensity is a measure of the equivalent number of corner cubes reflecting simultaneously the peak of the pulse.)