TEMPERATURE DISTRIBUTION IN A STELLAR ATMOSPHERE-DIAGNOSTIC BASIS

John T. Jefferies
Nancy D. Morrison
Institute for Astronomy
University of Hawaii

Presented by John T. Jefferies

INTRODUCTION

As is well known, the word "chromosphere" was coined to denote the bright, thin, colored ring seen as the solar limb was obscured by the Moon at the time of a total eclipse. This region of the Sun's atmosphere was found to be the source of many strong emission lines — the flash spectrum — some persisting to such heights as to leave no doubt that their cores originated quite high in the chromosphere. The presence of such an emission line region is not unexpected; however, what gives the solar chromosphere special interest is the fact that its observed spectroscopic properties cannot be explained on the basis that it is a simple extension of a "classical" atmosphere for which radiative, hydrostatic, and local thermodynamic equilibrium all apply. Thus, the height above the limb to which most eclipse lines persist is inconsistent with the predicted density scale height. The observation of neutral and ionized helium lines in the flash spectrum demands temperatures far in excess of those predicted for a radiative equilibrium model. Further difficulty is encountered in attempting to explain in classical terms the shapes and strengths of certain chromospheric lines in the disk spectrum, notably the self reversals in the cores of H and K. Such observations, coupled with the recognition that the coronal temperature is in the range of millions of degrees and the discovery of the peculiar inhomogeneities in the chromospheric gas, e.g., the spicules and the supergranular flow pattern and such transitory phenomena as surges, flares and prominences, all contributed to the recognition that the properties of the chromosphere are controlled by factors that lie outside the scope of a classical atmosphere. Thus, the partitioning of the Sun into photosphere, chromosphere, and corona is seen to be far more fundamental than the simple geometrical division based on eclipse observation. It appears that there are different mechanisms at work in these layers, especially in the way energy is transferred.

We recognize now that some, at least, of the spectroscopic features of the solar chromosphere are consistent with the hypothesis that the temperature increases outward above some minimum value found a few hundred

kilometers above the limb. The temperature rise is thought to be a result of the dissipation of mechanical energy generated in the photosphere, and if this is so we will naturally expect this process to take place in other stars, leading to the formation of stellar chromospheres. A direct approach to the study of these layers might be to concentrate on the kinematic motion of the line-forming layers as deduced from the shapes, strengths, and wavelength shifts of spectral lines. It is also fruitful, however, to consider the *symptom* of the dissipation of energy, namely the temperature rise, as a basis for comparison between solar and stellar chromospheres and this is the approach we shall adopt here. Thus, we shall consider a stellar chromosphere as a region where the temperature increases outward, and we shall examine spectroscopic methods for inferring the existence and properties of a temperature rise.

The following section sets out the physical basis for the discussion with some general considerations on how (or whether) the temperature structure of a gas controls the shapes of spectral lines. In particular, we shall discuss why some lines are very sensitive temperature indicators while others are much less so. Following that, we shall consider emission lines and what they can tell us about the atmosphere of the star, and we shall discuss methods for determining the temperature structure of the atmosphere from the analysis of line profiles. The final section contains a brief discussion of the information in the stellar continuum, together with some miscellaneous indicators.

THE INFLUENCE OF TEMPERATURE STRUCTURE ON LINE PROFILES

The monochromatic flux F_ν emerging from a plane parallel semi-infinite gas is given by

$$F_\nu = 2 \int_0^\infty S_\nu (\tau_\nu) E_2 (\tau_\nu) d\tau_\nu , \qquad (1)$$

where τ_ν is the monochromatic optical depth, E_2 is the second exponential integral, and S_ν is the source function, defined as

$$S_\nu = \frac{\epsilon_\nu}{\kappa_\nu} , \qquad (2)$$

where ϵ_ν and κ_ν represent respectively the monochromatic volume emissivity and the absorption coefficient per unit length in the gas. In

general, both ϵ_ν and κ_ν will contain components from continuum and line processes; however we are here primarily interested in the cores of strong lines formed in the outer atmospheric layers, and we shall neglect the continuum contribution.

Clearly, the emergent flux will reflect the temperature distribution only to the extent that S_λ (or ϵ_λ and κ_λ) depends on the temperature. For a spectral line it is well known — see, e.g., Jefferies (1968) — that

$$S_\nu = \frac{2h\nu^3}{c^2} \left[\frac{g_2}{g_1} \frac{n_1}{n_2} - 1 \right]^{-1} \psi(\nu) \quad , \tag{3}$$

where n_1, n_2 are the concentrations of atoms in the lower and upper levels of the line g_1, g_2 are the statistical weights of the levels and $\psi(\nu)$ is a function which we shall set equal to unity, following Jefferies (1968) and Hummer (1969). This latter approximation implies that the line source function is independent of frequency over the core of the line, and we shall therefore drop the subscript ν. The physical basis of our arguments remains unchanged if we neglect stimulated emission, in which case equation (3) reduces to

$$S_\ell = \frac{2h\nu^3}{c^2} \frac{g_1}{g_2} \frac{n_2}{n_1} \quad . \tag{4}$$

Thus, the dependence of the emergent flux on the temperature structure of the gas is fixed by the temperature dependence of the population ratio. Now this ratio can be expressed, formally, as

$$\frac{n_2}{n_1} = \frac{R_{12}}{R_{21}} \quad , \tag{5}$$

where R_{ij} is the rate of all transition paths, direct and indirect, which carry the atom from level i to level j. Recognizing that there are, in general, two mechanisms (collisional and radiative) by which transitions can take place, we can write, equivalently,

$$\frac{n_2}{n_1} = \frac{\int_0^\infty J_\nu \kappa_\nu \, d\nu/h\nu + C_{12} + I_{12}}{A_{21} + C_{21} + I_{21}} \quad , \tag{6}$$

where the C's are direct collisional rates and the first terms in numerator and denominator are respectively the direct radiative absorption and spontaneous transition rates, while the terms I_{ij} represent the rates of *indirect* transitions taking the atom from level i to level j. In this formulation, the "source" terms C_{12} and I_{12} represent the creation of fresh photons into the radiation field, while the sink terms C_{21} and I_{21} represent the destruction of absorbed photons by de-excitation of the atom. The source terms thus represent the ultimate source of the radiation in the gas.

Thomas (1957) distinguished two classes of lines according to whether direct collisional transitions or indirect processes are chiefly responsible for creation and destruction of photons. If $C_{12} \gg I_{12}$ and $C_{21} \gg I_{21}$, equation (3) reduces to

$$S_{\ell} = \frac{\int_0^\infty J_\nu \phi_\nu \, d\nu + \epsilon \, B_\nu(T)}{1 + \epsilon} , \qquad (7)$$

where $B_\nu(T)$ is the Planck function at the local kinetic temperature T, ϕ_ν is a normalized profile of the absorption coefficient and the important parameter ϵ is defined as

$$\epsilon \simeq \frac{C_{21}}{A_{21}} \qquad (8)$$

Thus, ϵ measures the importance of direct collisional relative to radiative de-excitations of an atom in the upper level of the line. In this case, therefore, the gas temperature enters directly into the line source function; the physical reason is that the collisions then control the production of new photons in the line, and the rate of these collision transitions depends on the kinetic temperature, through the Boltzmann distribution. Thus, for such a "collisionally controlled" line, the atmospheric temperature structure should be reflected in the line profile. The essential questions of interest to us here are, can we know *a priori* whether a line is "collisionally controlled," and, if so, exactly how is the temperature structure reflected in the profile of the observed line?

Thomas (1957) gave a partial analysis of the first question. In particular, he showed that, for stars of solar type and later, one would expect strong resonance lines of non-metals, and of ionized metals, to be collisionally controlled. The dichotomy depends on the atomic level structure and on the color temperature of the stellar continuum; as particularly important

cases in this category, we identify the resonance lines of Ca^+, Mg^+, H, C, N, and O when formed in stars of solar type and later. Thomas also showed that the ratio of the populations of the levels of the resonance lines of neutral metals should be controlled less by collisions than by indirect processes, which should, in turn, be controlled by the strength of the continuum radiation field streaming through the gas. As a consequence, the source functions of such lines should *not* reflect the local temperature distribution in the region where the lines are formed, but rather the temperature in the region where the *continua* originate. Thomas' corresponding partitioning of lines into "collisional" and "photoelectric" control is important to keep in mind when designing observational programs, but it must be applied with an intelligent understanding of its basis. Thus, whether a given line falls into one or the other of the classifications depends on the gas temperature, the stellar continuum flux, and the local density; the classification is not an immutable property of the line. For example, the cooler the star, the closer a given line will be to collisional control.

Considerable insight into the question of just how sensitively the temperature structure is reflected in the line profile has been obtained over the past ten to fifteen years. For a collisionally controlled line, for which S_ϱ is given by equation (7), we can compute the emergent radiation for a given temperature model by solving (with appropriate boundary conditions) the transfer equation

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\varrho = I_\nu - (1-\lambda)\int_0^\infty J_\nu \phi_\nu \, d\nu - \lambda B_\nu(T) , \quad (9)$$

where $\lambda = C_{21}/(A_{21} + C_{21})$ is the probability of a collisional deexcitation of an atom in the upper state of the line. We consider solutions of equation(9) for two general cases, an isothermal semi-infinite layer of gas, and secondly, a model in which the temperature increases outward.

ISOTHERMAL LAYER

Schematic results for an isothermal layer are illustrated in Figures I-1 and I-2 for a set of values of the scattering parameter λ. Two aspects of these figures should be particularly noted. Firstly, the line source function saturates to the Planck function at an optical depth of λ^{-1} as measured in the line center. This characteristic distance is known as the "thermalization length," corresponding physically to the average optical distance which a photon will travel from its point of creation as a new photon,
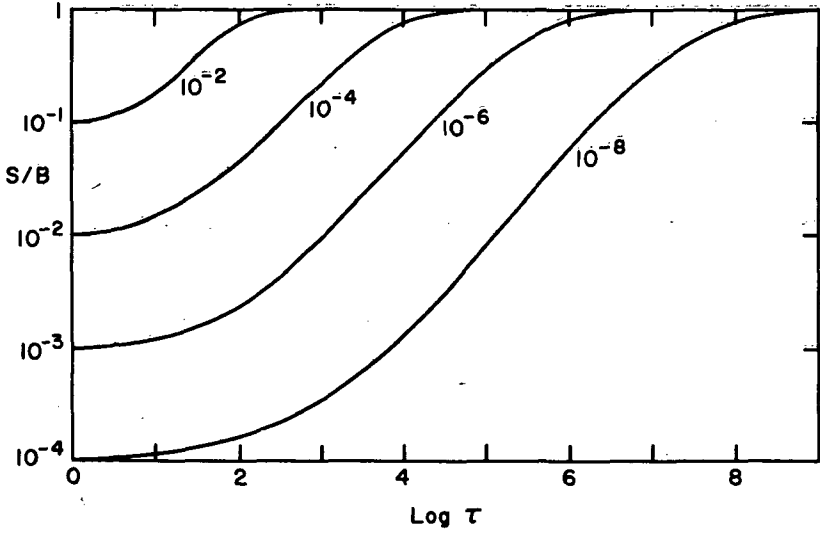
Figure I-1   The ratio of the line source function to the Planck function, for an iso-
thermal gas, as a function of optical depth at the line center. The differ-
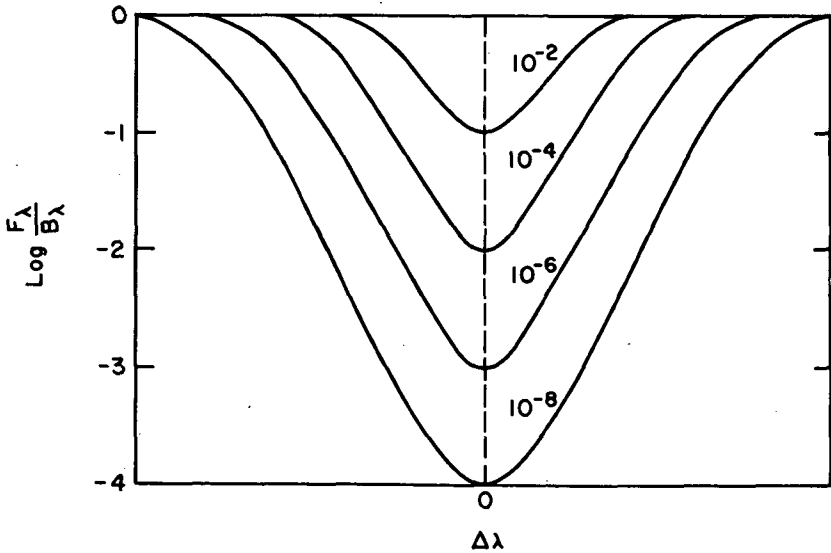ent curves refer to different values of the scattering parameter λ.

Figure I-2   The logarithm of the ratio of the emergent flux in the line to the Planck
function, for an isothermal gas, as a function of wavelength. The profiles
refer to different values of the scattering parameter λ.

following collisional excitation, to its point of destruction by collisional de-excitation; this occurs, on the average, after λ^{-1} successive absorptions and reemissions. Detailed discussions of the thermalization length are given, e.g., by Finn and Jefferies (1968a, b) and by Hummer and Rybicki (1971a). It is important in the present context to point out that an equivalent interpretation of the thermalization length is the distance to which a change in atmospheric conditions will be reflected in the radiation field. Thus, for example, a discontinuous jump in temperature at some point in the gas would be reflected in the radiation field up to an optical distance (measured in the line center) of λ^{-1} away. Clearly, therefore, the degree of line scattering in the gas has a profound effect on the depth distribution of intensity in the line and so, via equation (7), on the line source function and on the profile of the emergent flux.

This fact is reflected in the second point we wish to emphasize, which is illustrated in Figure I-2. The line profiles shown there are obviously different, yet they are computed for atmospheres with *identical* temperature structure; in fact, the gas is isothermal in the kinetic temperature (although the profiles are in absorption). The differences among the profiles arise from the differences in λ (or ϵ), not from any differences in temperature structure. Only in the case of LTE (for which $\lambda = 1$) is the temperature uniquely reflected in the profile; the line is then completely filled in. From the definition (8) it can be shown that $\epsilon \sim 10^{-16} n$ for a strong line in the visible, where n is the electron density in cm^{-3}. In the solar chromosphere where Ca II H and K are formed, $n \sim 10^{11}$ cm^{-3}, consequently, $\lambda \sim \epsilon \sim 10^{-5}$, a value assuring a major departure from local thermodynamic equilibrium. Since λ is proportional to the density the profiles of collisionally controlled lines certainly reflect the gas density. In a more general case of a non-isothermal gas, we shall show that both the temperature and density distributions determine the profile; evidently, the problem of separating these two effects from the line profile will not be straightforward.

THE CHROMOSPHERIC CASE

Collisionally Controlled Lines

Jefferies and Thomas (1959) studied the formation of a collisionally controlled line in a gas whose temperature increases towards the top of the atmosphere. Their temperature model is shown in Figure I-3 as the full line; the source functions derived by solving equation (9) are shown schematically as broken curves; the corresponding emergent line profiles are shown in Figure I-4. It is immediately clear that the temperature structure is certainly reflected in the profiles but to a degree which is controlled by the
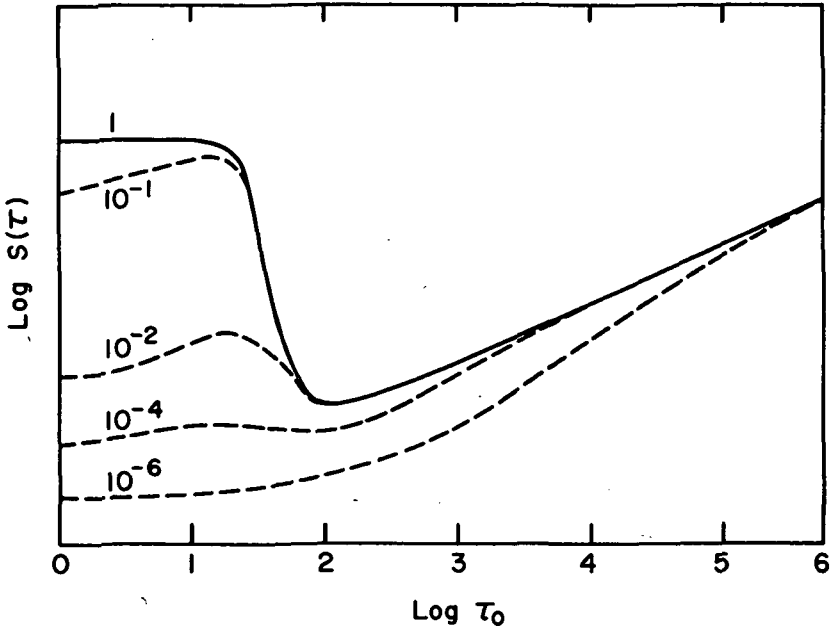
Figure I-3   The line source function as a function of optical depth in the line center. The solid curve represents the Planck function according to the model of Jefferies and Thomas (1958). The dashed curves are solutions of the equation of radiative transfer for this model, for different values of λ.
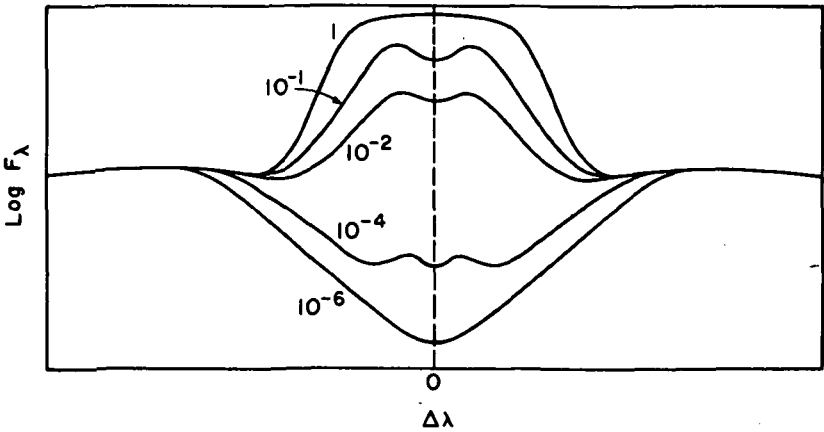

Figure I-4   Emergent line profiles predicted by the model of Jefferies and Thomas (1958) for different values of λ.

parameter λ, as would be expected from the arguments given above. There is a striking qualitative agreement between the computed line profiles and those observed in late type stars, particularly for Ca II H and K, and the Fe II (3100 A) lines (cf. Weymann 1962, Boesgaard 1972), as well as in solar Lyα and Mg II H and K. The general consistency of the predictions is evidence that these self-reversed lines do arise in a gas with a positive temperature gradient.

The shape of the profile depends not only on the amplitude of the temperature rise, but also on the relative values of the optical depth τ_m where this rise begins and the thermalization depth $\tau^*(=\lambda^{-1})$. The K-line reversals should be or absent if $\tau_m \ll \tau^*$ and strong if $\tau_m \gg \tau^*$. Thus, emission features should go with high densities and deep chromospheres, and weak or no emissions should accompany low densities and thin chromospheres.

In summary, the observation of self-reversed emission cores in H and K give direct evidence of the existence of an outward temperature rise in the stellar atmosphere. Their absence in these lines is not, however, necessarily an indication of the absence of such a temperature rise since the density and temperature characteristics of the gas may be such that a temperature rise could not be reflected in the K line profile. It may be relevant in this regard that observations of some F stars show deep normal K line profiles and others show reversals (e.g., Warner 1968).

Photoelectrically Controlled Lines

Thomas' arguments also give us some insight into the reason why lines like Hα, which are as "strong" as K, do not normally show an emission reversal. We shall not go into detail here, but merely note, with Stromgren (1935), that certain lines, of which solar Hα is in fact a good example, derive their excitation mainly through indirect transitions which transfer atoms from the lower to the upper state via an intermediate state, commonly the continuum. Such processes are governed by absorption of radiation generated lower in the atmosphere, which is essentially present as a background illumination. As a result, the local temperature in a chromospheric region where the line is formed plays little or no role in determining the emergent line shape (although it may control the Doppler width and so set a scale to the profile). In that case, the line source function takes the form

$$S_\ell = (1 - n) \int_0^\infty J_\nu \phi_\nu \, d\nu + \eta \, B^* \, , \qquad (10)$$

both η and B* being controlled by the strength and "color" of the continuous and weak line radiation streaming through the chromosphere, and so being constant with depth in line forming regions. Thus, independently of the *kinetic temperature* structure in the chromosphere, emergent profiles of photoelectrically controlled lines will have the same form as those shown in Figure I-2 since the source and sink terms for the cases illustrated there are also constant with depth. Such lines will then appear in absorption even when H and K show strong emission cores. This is not to say that Hα must always be in absorption; e.g., at high densities direct collisions can become more important than indirect photoelectric processes and such a shift to collisional control is probably the reason that Hα goes into emission in solar flares and in flare stars.

INFLUENCE OF INCLUDING MORE LEVELS

The simple physical arguments presented above give great insight into the response of line profiles to the temperature and density structure of a gas. For a quantitative discussion, however, greater detail is needed in the specification of the atomic level structure, particularly the incorporation of more than the two levels (plus continuum) to which earlier treatments were confined. Many calculations of multi-line problems have been carried out – cf. e.g., Avrett (1966), Finn and Jefferies (1968b, 1969), Cuny (1968), Athay *et al.* (1968) – but they change the above physical picture little if at all. One significant general conclusion from such calculations, however, is that the source functions of multiple lines (such as H and K) share an essentially common depth dependence over much of the gas. Waddell (1962) showed that this equality is required by a comparison of solar center-to-limb observations of D_1 and D_2. If generally correct, the conclusion is of great importance for the analysis of stellar spectra.

SUMMARY

We have seen that profiles of certain spectral lines should be sensitive to the temperature distribution in a gas and so can presumably be considered indicators of the presence of chromospheres. We have seen that these lines are, in fact, observed to have profiles which indicate the presence of an outward temperature rise, and have seen why others, comparably strong, should not, and do not, show the same features. For the temperature-sensitive lines, we have seen that the profiles reflect both temperature and density structure, but it is not clear that we can disentangle these dependences in a unique way.

The basic physical ideas seem clear and give self-consistent (if qualitative) results. Their application to stellar problems will demand more

sophisticated computations, particularly taking into account many atomic levels in order to allow predictions on a number of lines formed in the same atom or ion.

The theoretician also faces the fact that the inhomogeneous structure seen in the solar chromosphere may be expected to be present in other stars, and he must seek to compute its influence on the space averaged profiles observed from a star. The averaged spectrum from a multidimensional medium does not necessarily reflect average temperature or density conditions, but the extent of this failure is not yet clear. The techniques for studying such questions are available in Monte Carlo programs or in the more conventional solution of three-dimensional transfer problems, and it seems that only by model calculations can we obtain some idea of the sensitivity of different lines in stellar spectra to inhomogeneities. Such data are essential if we are ever to develop sound methods of analysis, or even to design meaningful observation programs.

DO EMISSION LINES IMPLY A CHROMOSPHERE?

The problem of what the presence of an emission line implies about the structure of a stellar atmosphere is still unsolved. Following Gebbie and Thomas (1968), we can characterize the problem in the following specific terms: The observed flux is given as

$$F_\nu = \int I_\nu \mu \, d\omega \quad , \tag{11}$$

and the central question is whether the emission line is intrinsic, over all or some of the star's surface (i.e., $I_\nu > I_c$), or whether it has a geometrical origin because the area of integration is much greater for the line than for the continuum. It is of basic importance to try to develop a diagnostic tool to discriminate between these two possible sources of emission lines. Although we have no concrete ideas to suggest here, a reasonable first step would be to study some model problems to clarify the consequences of postulating a geometrical origin for emission lines. The theoretical tools for handling such problems are available, especially since the development (cf. Hummer and Rybicki, 1971 b) of simpler methods for handling transfer problems in spherical atmospheres. A model problem based on a pure hydrogen atmosphere could greatly clarify this question.

A line will be *intrinsically* in emission if the line and continuum source functions are related according to the inequality,

$$S_\ell\,(\tau_0 = 1) > S_c\,(\tau_c = 1) \quad , \qquad\qquad (12)$$

where the optical depths are measured along the direction of observation, and τ_0 refers to the line center, τ_c to the continuum at a neighboring wavelength. We may obviously satisfy this inequality either by reducing S_c or by increasing S_ℓ. The latter possibility occurs most naturally, at least for a collisionally controlled line, if the temperature increases outwards. This mechanism explains the fact that, in the solar atmosphere, emission lines are abundant below $\sim$1600 A (and present up to $\sim$2000 A). To some extent, their appearance is favored by the increasing continuum opacity below about 1800 A which places the region of formation of the continuum near the temperature minimum, while that of the lines lies in the chromosphere.

The alternative notion that S_c is depressed below S_ℓ was originally explored by Schuster (1905), later by Underhill (1949) and more recently by Gebbie and Thomas (1968). In its simplest form, and the one most favorable for emission line formation in a "classical" atmosphere, Schuster's mechanism supposes that the line is formed in LTE so that $S_\ell = B_\nu$, while the continuum is formed partly by thermal and partly by scattering processes so that S_c is given as

$$S_c = (1 - \lambda_\nu)\,J_\nu + \lambda_\nu\,B_\nu\,(T) \quad , \qquad\qquad (13)$$

with $\lambda_\nu < 1$.

For an *isothermal* gas, $S_c/B_\nu(T)$ will be less than unity near the front of the atmosphere because the escape of photons from the surface reduces J_ν below B_ν. Consequently, inequality (12) is satisfied and the line appears in emission. For a normal radiative equilibrium gradient, however, the continuum intensity J_ν increases substantially and it becomes much more difficult to satisfy the inequality (12). Gebbie and Thomas (1968) concluded that, except perhaps in the infrared, the Schuster mechanism would be ineffective in a classical atmosphere. Their conclusion, supported by the work of Harrington (1970), is only strengthened if the line source function is represented by the more physically correct expression (7). In this case, the emergent central intensity drops below its LTE value, making it still more difficult for the line to appear in emission. The applicability of the Schuster mechanism is further reduced by the fact that it requires that the continuum not be formed in LTE; for most stars, however, LTE is believed to hold in the continuum. Still, exceptions exist, especially for hot stars, where electron scattering is significant

while, even for the Sun, LTE fails below the Lyman limit. Hence, the possibility that S_c is reduced by some departure from LTE in the continuous spectrum needs to be kept in mind in connection with the appearance of emission lines in a stellar spectrum.

We believe that a rich field of investigation of great potential to stellar spectroscopic diagnostics is to be found in a concentrated attack on the appearance of emission lines in stars. So far, the confusion between processes forming intrinsic emission lines and those arising from extended envelopes (stationary or expanding) has limited our ability to use these lines for diagnostic purposes. The sophistication of present-day computational methods is sufficient, and the rewards sufficiently attractive, to merit a full-scale attack on the problem of differentiating between these two entirely different origins for emission lines.

ANALYSIS OF SPECTRAL LINES

OPTICALLY THICK GASES

We saw above that certain lines are expected to contain information in their profiles on, among other things, the distribution of temperature with depth in the gas. We now wish to discuss briefly the problem of using the information in an observed line profile to infer the temperature distribution in the gas; in a sense, this is the inverse of the problem, discussed above, of computing the line profile given the atmospheric structure.

The best starting point currently available is the expression

$$F_\nu = 2 \int_0^\infty S_\nu(t_\nu) E_2(t_\nu) dt_\nu \quad , \tag{14}$$

which already restricts the scope of our analysis to a homogeneous semi-infinite plane — parallel layer. A more complicated expression suitable for spherically symmetric geometry could no doubt be obtained; an extension to more general expressions incorporating stochastic spatial variations is beyond the present development of the subject.

From equation (14), the first part of the analysis consists in determining from the observed profile F_ν the run of $S_\nu(t_\nu)$ for each point on the line profile. As it stands, however, infinitely many possible distributions $S_\nu(t_\nu)$ will satisfy the integral equation (14). Some limitation of these solutions can be obtained if we restrict attention to those parts of the profile where continuum processes are negligible compared to those in the line, so that S_ν

and t_ν refer only to the spectral line. However, even in that case, the depth distribution of S_ν is not uniquely determined. In order to invert equation (14) uniquely, Jefferies and White (1967) have shown that it is necessary to have observational profiles of two or more lines whose source functions at all depths are related in some known way. Since, as mentioned above, studies of multiline transfer problems have indicated that the source functions of close-lying multiplet lines are essentially equal at all depths, except perhaps close to the surface, such lines should provide the necessary data for an analysis. This principle has been applied by Curtis and Jefferies (1967) to the solar D lines (for which the availability of center-to-limb data greatly simplifies the problem, and allows us to retrieve information on the continuum parameters also). Wilson and Worrall (1969) have also attempted an analysis of the solar D lines, using data at one point on the disk; their procedure is essentially equivalent, therefore, to that which would in practice be applied to stellar spectra, where no geometrical resolution is obtainable. It is not our purpose here to discuss in detail such analytical methods, or the closely related method of deJager and Neven (1967), but rather to draw attention to their existence, since they offer an alternative interpretive method to that based purely on model calculations. The theory of such analytical processes also allows a more incisive study of such important questions as the uniqueness of a particular derived model, a subject quite beyond the scope of this paper but one nevertheless deserving closer study than it has received.

While the analytical method has promise, at least, of determining the depth variation of S_ϱ and the frequency and depth variation of the line absorption coefficient, its application so far (to solar data) has not been wholly satisfying. The difficulty may lie in inadequate data, in uncertainties in the inversion of the integral equation, in limitations in the basic formulation (14), or in the degree to which S_ϱ is independent of wavelength within the line and the same from one line to another.

If such problems can be resolved, the depth variation of S_ϱ would still require interpretation in terms of the density and temperature structure of the gas. We can see no way of approaching this other than through a model calculation. At least, the depth and wavelength dependence of the line absorption coefficient that is yielded by the analysis would be helpful by setting constraints on the model.

OPTICALLY THIN GASES

The specific intensity emitted from a gas in an optically thin line reflects the integral of the volume emissivity along the line of sight; in general, the line profile does not reflect the way in which emitting material is

distributed and an infinite number of geometrical rearrangements of the emitting material will yield the same emission in all optically thin lines. A satisfying technique for spectroscopic diagnosis of an optically thin line would therefore seek some way of specifying the physical state of the gas which is unique and so preserved under such geometrical rearrangement. This general problem has been studied by Jefferies, Orrall, and Zirker (1972). While their particular interest lay in its use for the analysis of coronal forbidden lines, the method is of general application, in particular to the optically thin lines of the solar chromosphere.

For a transparent gas, the specific intensity I, integrated over the line profile, can be written

$$I = \frac{h\nu}{4\pi} A \int_0^\infty n_u(x) \, dx \; , \tag{15}$$

where n_u is the population of the upper state of the line and x is the geometrical coordinate in the line of sight. The emissivity is controlled only by the local electron density and temperature, and the intensity of interacting radiation fields of known strength, provided that n_u is determined by electron collisions or by the absorption of radiation in spectral regions which are themselves thin. In the usual way, we expand the population n_u in the form

$$n_u = \frac{n_u}{n_i} \frac{n_i}{n_A} \frac{n_A}{n_H} \frac{n_H}{n} n \; , \tag{16}$$

with n_i the concentration of the ionization stage to which the line belongs, n_A and n_H the concentrations of the element and of hydrogen, and n the electron density. The ratios n_i/n_A, n_u/n_i, and n_H/n are all functions of n and T only. If we define

$$a_{el} = \frac{n_A}{n_H} \tag{17}$$

as the abundance of the element with respect to hydrogen and define an ionization-excitation function

$$\chi(n,T) \equiv \frac{n_u}{n_i} \frac{n_i}{n_A} \frac{n_H}{n} \; , \tag{18}$$

then equation (15) takes the form

$$I = \frac{h\nu}{4\pi} Aa_{el} \int_0^\infty \chi(n, T) \, n \, dx. \qquad (19)$$

Specific distributions $n(x)$ and $T(x)$ would, of course, characterize the gas uniquely and would yield straightforwardly a value of I. However, as stated above, we could never determine such distributions uniquely from the observed intensities. We therefore abandon the geometrical distribution as a characterization of the gas and instead transfer the analytical problem to an n, T space by introducing a distribution function $\mu(n,T)$ through the definition

$$dN(n,T) \equiv N \mu(n,T) \, dn \, dT, \qquad (20)$$

with $dN(n,T)$ the number of electrons in the sampled column that are in neighborhoods where the electron temperature lies between T and T + dT and, simultaneously, the electron density lies between n and n + dn. The distribution dN is normalized to the total electron content N in the column so that $\mu(n,T)$ is normalized to unity. In these terms we can write

$$I = C a_{el} N \int_0^\infty \int_0^\infty \chi(n,T) \, \mu(n,T) \, dn \, dT, \qquad (21)$$

where $C \equiv (h\nu/4\pi) A$. Equation (21) is a double integral equation with kernel $\chi(n,T)$ which may, in principle, be solved for the distribution function $\mu(n,T)$, given data on the number of lines for which the functions χ are sufficiently different.

While, in principle, we might hope to infer the bivariate function $\mu(n,T)$, in practice we shall probably have to accept the more limited description of the gas implicit in the assumption that n and T are uniquely related everywhere along the line of sight.

In that more restrictive case, equation (21) becomes

$$I = C a_{el} N \int_0^\infty \chi[n(T), \; T] \; \phi(T) \, dT, \qquad (22)$$

where the distribution function $\phi(T)$ is given by

$$\phi(T) = \int_0^\infty \mu(n,T)\ dn \qquad (23)$$

and n(T) is the single valued function relating the electron density to the temperature. Clearly ϕ (T) dT is the fraction of all the electrons in the column whose temperatures lie between T and T + dT.

The finesse of an analysis based on the above formulation will depend on the degree to which the excitation-ionization functions $X(n,T)$ differ from one line to another. Since we can calculate χ in advance once we know the cross sections for radiative and collisional transitions, we can decide in advance which set of lines of a particular ion will best suit our needs for analysis.

ZETA AURIGAE-TYPE ECLIPSING BINARIES

Because of their special geometry, a class of eclipsing binaries present favorable cases for the study of a stellar chromosphere. Of the bright stars of this type, the prototype ζ Aur is the best observed but the observational results are similar for 31 and 32 Cygni — cf. Wilson (1961), Wright (1970). These binary systems are composed of a K-type supergiant and an early B-type dwarf or subgiant which undergoes total eclipse. As it passes behind the extended atmosphere of the supergiant absorption lines appear in the spectrum. Since the radiation field of the B star may affect the temperature structure of the chromosphere of the K giant it is not clear to what extent results from these systems apply to single stars. In the absence of any other way of obtaining direct detailed information about the temperature structure of a star other than the sun, the method nevertheless has great value.

In spite of this fact, few observers have attempted to draw conclusions about the variation of temperature with height in the chromosphere. Those who have done so have used a curve-of-growth analysis for the line spectrum to derive values for the excitation and ionization temperatures at one or several heights in the chromosphere. In their study of ζ Aurigae, Wilson and Abt (1954) were able to reproduce their observations only by supposing the envelope to be slumpy. Otherwise, the B-type star ought to ionize the envelope of the supergiant to a greater degree than that observed. Wright (1959) concurred that the chromospheric spectrum ought to be produced mainly in small condensations where the density is much

greater than in the rest of the gas. Further evidence for the existence of condensations is given by the observation that the chromospheric K line usually contains several components of different radial velocities. As in the Sun, then, a correct analysis must take into account the inhomogeneity of the medium.

MISCELLANEOUS INDICATORS

SYMBIOTIC FEATURES

Unambiguous indications of the presence of a temperature rise are given by what we will call symbiotic spectral features: features whose behavior in a stellar spectrum yields, through elementary analysis, values of temperature or abundance that are anomalous or disagree with the values derived from other spectral features in the same star. For example, calculation of the population of the lower level for conditions expected in stars yields an estimate for the strength of the line at a given temperature. If the line has a large excitation potential and is stronger than expected, it must arise in a hot layer above the photosphere. For example, the Balmer lines in some M-type giants are anomalously strong, indicating overpopulation of the second level by a large factor (Deutsch 1970). For a Orionis, Spitzer (1939) showed that the great strength of Ha implies a radiation density of Lya that corresponds to a temperature of 17000°K. This value contrasts sharply with the effective temperature of the star, which is near 3300°K. Another example of this type of indicator is a group of lines near 1 micron wavelength due to Si I and Mg I (Spinrad and Wing 1969). Since they have excitation potentials of about 6 eV, their presence is favored by temperatures of 5000° or 6000°K. Nevertheless, they are as strong in a Ori as they are in the Sun. Finally, the most important symbiotic features are the lines arising from excited states of He I. Though λ10830 is the most prominent of these lines, others, such as λ5876, may be observable in cool stars also. Vaughn and Zirin (1968) calculated the population of the lower level of the line at 10830 Å and found that, for all densities considered, it is negligible for T < 20000 °K and large for all higher temperatures. This line must therefore be regarded as a clear indicator of a large rise in temperature in the upper atmosphere of any star whose effective temperature is substantially less than this value.

CONTINUOUS SPECTRA

For a region of a stellar continuous spectrum where the opacity is known as a function of wavelength, the wavelength variation of the emergent

flux will contain information about the temperature gradient. In some regions of the spectrum, the opacity may be so high that even the continuous radiation arises effectively in the chromosphere; in the sun this happens for millimetre radiowaves, and again in the near UV at about 700 Å. If the color temperature, or the brightness temperature, of the radiation increases as the opacity increases, an outward temperature rise is indicated.

An example of such a case is found in the ultraviolet below about 0.3 μm, where a high opacity is provided by the bound-free continua of hydrogen and the metals. The opacity generally increases toward shorter wavelengths, and, at some wavelength, the continuous radiation originates effectively at the height in the atmosphere where the temperature has a minimum. Naturally, this wavelength is shorter than the wavelength at which the chromosphere begins to influence the cores of the lines and where, consequently, emission lines begin to appear. Since the transition of the line spectrum from absorption to emission occurs at longer and longer wavelengths for later and later spectral types, it is reasonable that the influence of the chromosphere on the continuum should also extend to longer wavelengths for later spectral types. Doherty (1970) has studied the ultraviolet continua of K and M stars near 3000 Å as observed by OAO-2. In particular, he considered the wavelength dependence of the color temperature of the continuum, which should reflect the variation of the electron temperature with height. For stars of spectral type earlier than about K5, the flux below 0.28 μm decreases rapidly toward shorter wavelengths. For α Tauri and α Orionis, however the flux decreases much more slowly in this region, and both stars show a minimum in the color temperature at about 0.30 μm. Whether this minimum indicates a temperature minimum in the stellar atmosphere or only a maximum in the opacity is not clear.

Another source of continuous opacity that may be important to this discussion is the H-ion. Beyond 1.6 μm, the free-free opacity of this ion increases monotonically, in a known manner (Geltman, 1965). If, in the chromosphere of a cool star, the temperature is low enough and the electron density is high enough, the H-ions might produce enough opacity so that the continuous radiation in the observable infrared would arise in the chromosphere. Thus, limb brightening or even an infrared excess might be observable at wavelengths shorter than 20 μm. Noyes, Gingerich, and Goldberg (1966) searched unsuccessfully for limb brightening at 24 μm in the Sun. From a model of the solar chromosphere, they predicted that the Sun should show an infrared excess at 50 μm. They suggested further that other stars, in which there is an additional opacity source in the infrared or in which the temperature minimum lies at greater optical

depth than in the Sun, might show an infrared excess at shorter wavelengths.

Another source of excess radiation at long wavelengths might be free-free emission from hydrogen. If the characteristics of the long-wave radiation were to require that the source have an electron density lower than that expected in the photosphere, the radiation would have to arise in an extended envelope surrounding the star. If, in addition, the temperature required for the source is substantially higher than the effective temperature of the star, a temperature rise above the photosphere is indicated. For example, Wallerstein (1971) has considered whether the excess at 10 μm of the KO supergiant W Cephei could be produced by free-free emission. He found that the size (but not the wavelength dependence) of the infrared excess could be produced by free-free emission in a sphere with radius 15 A.U. if the electron density is 2 - 4 x 10^9 cm^{-3} and the electron temperature is 5000-6000°K. Since the effective temperature of a KO supergiant is only about 4000°K (Allen 1963, p. 201), the star presumably has a chromosphere, but the free-free emission, which comes from a very extended region, apparently does not originate there.

In the Sun, free-free emission at radio frequencies arises in the corona. From a simple model of a stellar corona, Weymann and Chapman (1965) have predicted that free-free emission should be detectable in the microwave region, and this emission has been detected in cool stars (Kellermann and Pauliny-Toth 1966; Seaquist 1967).

CONCLUSION

In this review, we have tried to indicate areas where further theoretical work would improve the present understanding of stellar chromospheres. In several places, we have emphasized that calculations of line profiles need to take into account inhomogeneities in the gas. This necessity arises from the fact that all stars that can be observed with spatial resolution over the disk — the Sun and the ζ Aur variables — show inhomogeneities in the chromosphere. Past work on the ζ Aur variables has shown that the effect of inhomogeneities can be striking.

We have also considered the problem of obtaining the distribution of temperature with height from an observed line profile. Although we pointed out that such methods exist and should be applied to stellar spectra, we also noted that the methods cannot yet be applied with complete success. Not only are better line profiles needed than are usually obtainable from stars, but further improvements in the theory are also required, from improvements in the basic formulation to refinements in numerical techniques.

The most general area of research that we have suggested is, however, the question of what emission lines mean. We mentioned three situations that are thought capable of producing emission lines in a stellar atmosphere: a temperature rise, the Schuster mechanism, and the case where the effective emitting area is larger in the line than in the continuum. Of these suggestions, only the first is thought to exist generally; still, the others cannot be entirely ruled out. It would be desirable to know in detail what conditions would permit the Schuster mechanism or the geometrical mechanism to operate. This area of research promises to be one of the most fruitful in the area of stellar chromospheres.

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DISCUSSION FOLLOWING THE INTRODUCTORY TALK BY JEFFERIES

Skumanich — There is a hidden parameter in Jefferies' curves which I think should be brought out, namely, the thickness of the chromosphere or, conversely, the scale height. If the optical thickness of the chromosphere is held constant and there are changes in the density, then the physical thickness must change. Such a change is governed by the momentum equation (e.g. hydrostatic equilibrium).

Jefferies — You are quite right. I should have mentioned that for illustration I took a constant optical thickness for the chromosphere and varied λ independently.

Skumanich — This makes three parameters not two, and we should worry about the variations of all three.

Jefferies — Yes, that's correct, and the relationship of the thickness of the chromosphere to the density is of fundamental importance. In point of fact, if the optical depth where the chromospheric temperature rise begins

is greater than the thermalization length then you'll have strong chromospheric emission, and if it's less the emission will be weak.

Athay — There is another aspect of the uncertainty coming into the profile, and that is the opacity of the atmosphere. Even the photoelectrically controlled lines depend on temperature due to the temperature dependence of the line opacities.

Skumanich — One more parameter is that which describes the kinematic situation, and this brings the number of basic parameters to four.

Poland — In reference to the statement that an increasing source function is the result of a chromosphere, it should be mentioned that Auer and Mihalas have obtained results that show it is possible to get emission lines due to optical pumping without a chromosphere. For example HeII lines can be pumped by hydrogen lines if the overlap is sufficient.