CHROMOSPHERIC ACTIVITY AND STELLAR EVOLUTION

Rudolf Kippenhahn
Göttingen University Observatory

STELLAR EVOLUTION AND MECHANICAL FLUX

Stellar evolution carries a star through the Hertzsprung-Russell-Diagram. For a given mass, M. one obtains both luminosity, L, and radius, R, as functions of time, t. From these parameters we determine the surface gravity and the effective temperature as functions of t:

$$g(t), T_{eff}(t).$$

It is these two parameters which determine the properties of the outermost layers of a star, the atmosphere and the top of the hydrogen convective zone.

From the equations of mixing lengths theory (Böhm-Vitense, 1958) one can derive for the Mach number, M, that:

$$M^2 < \frac{1}{8} \frac{\ell_2}{H_p^2} \left(\nabla - \nabla_{ad}\right)$$

where ℓ = mixing length, H_p = pressure scale height and $\nabla = d \ln T/d \ln P$, $\nabla_{ad} = (d \ln T/d\ln P)_{ad}$. We thus see that the Mach number can approach 1 only in regions where $\nabla - \nabla_{ad}$ is large, that is in those regions where the stratification is highly superadiabatic – as it is at the top of the convective zone. Sound waves can be formed in these layers only; thus the mechanical flux also depends only on g(t) and $T_{eff}(t)$. Therefore, for the determination of the mechanical flux a grid of models of stellar outer layers as functions of the two parameters g, T_{eff} is necessary.

Recently de Loore (1970) has computed the flux for a set of model atmospheres. The mechanical flux F_{mech} which he derived is given in the log T_{eff} - log g - plane of Figure IV-1. As has been said yesterday, de Loore's models exaggerate the mechanical flux for those models which have convective zones thinner than the mixing lengths. In the next three figures evolutionary tracks are plotted in the log T_{eff} - log g - plane together with de Loore's mechanical flux areas. In Figure IV-2 the pre-main sequence evolution as well as the post main sequence evolution up to helium flash are plotted for a star of one solar mass. One can see that the star is always in the region of strong mechanical flux. This holds also

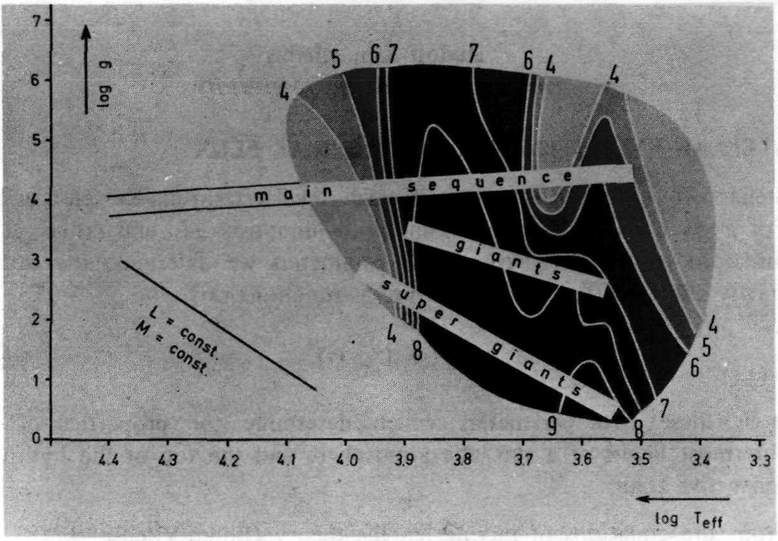

Figure IV-1 The mechanical flux F_{mech} *as a function of g and* T_{eff} *computed by de* Loore (1970) with the Lighthill-Proudman theory. The numbers at the white lines give log F_{mech} where F_{mech} is in c-g-s units. The straight line in the lower left corner gives the slope of an evolutionary track which is horizontal in the HRD.

for the post main sequence evolution of a 1, 3 solar mass star (Figure IV-2). Stars of 1, 3 solar masses settle down on the main sequence near F5. This is the region where on the main sequence one observes the transition from stars with Ca emission to those without. One therefore is surprised that according to de Loore's computations such a star is right in the middle of the region of strong mechanical flux. One would expect the star to be on the left border of the area of strong mechanical flux instead. This is probably due to the enhanced mechanical flux in the thin convective zones in de Loore's computations. Figures IV-3 and IV-4 show that stars of higher masses start on the Hayashi track in the region of strong mechanical flux, move into the low flux region and then come back into the high flux region during central helium burning and further later evolution. While the more massive stars make loops they go several times from high flux to low flux regions and vice versa.

It has been indicated during this conference that the mechanical flux computed according to the Lighthill-Proudman theory is not very reliable due to uncertainties in the theory of convection. We were confronted yesterday with at least two new and different possible mechanisms of heating. Certainly these mechanisms have to be worked out more thoroughly before one can decide whether we really have the correct

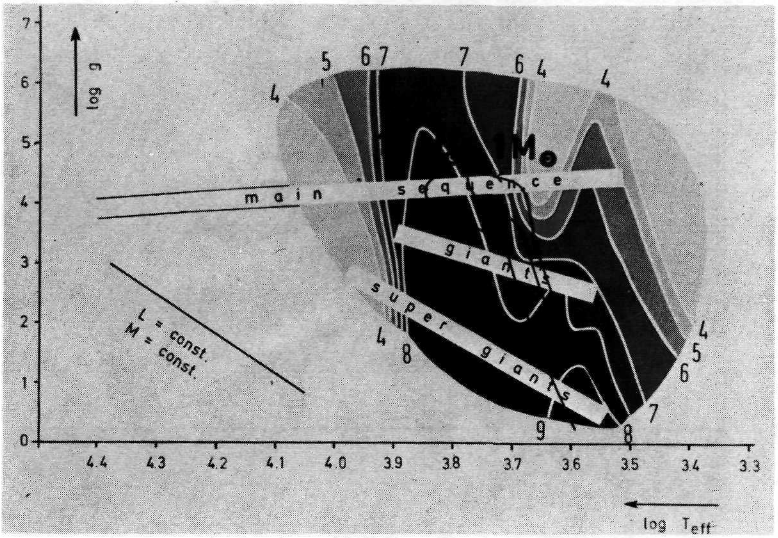

Figure IV-2 Evolutionary tracks for 1 $M_\odot$ and for 1.3 $M_\odot$ in the log g-log T_{eff} plane. The 1 $M_\odot$ star starts in the lower right corner, moves into its pre-main sequence evolution towards the main sequence and goes back into the lower right corner in the post ms evolution. For the 1.3 $M_\odot$ star only the post ms evolution is plotted.

theory of mechanical heating. It is, for instance, not sufficient to show that a certain type of motion is unstable by making only a linear analysis.

What one has to show is that such an instability, if it is fully developed, has sufficient energy to produce the mechanical flux necessary for chromospheric heating. In the case of convection we know that in many stars all the energy of the star is transported through such motion and it is therefore easy to get the required energy from convection. It should be kept in mind that in the HRD the observed transition from stars with observed calcium emission to those where calcium emission is not, or is only seldom, observed seems to agree fairly well with a line of constant mechanical flux generated by convection.

In particular on the main sequence there is a sharp transition between calcium emission and no calcium emission (as it is observed by O. C. Wilson, 1964) which coincides with the well known transition from convection to no convection. Since the flux depends on the eighth power of the turbulent velocity one would expect a sharp cut-off in the mechanical flux at this transition. That this cut-off is not so pronounced in Figures IV-1 to IV-4 may be due to de Loore's treatment of thin convective zones.

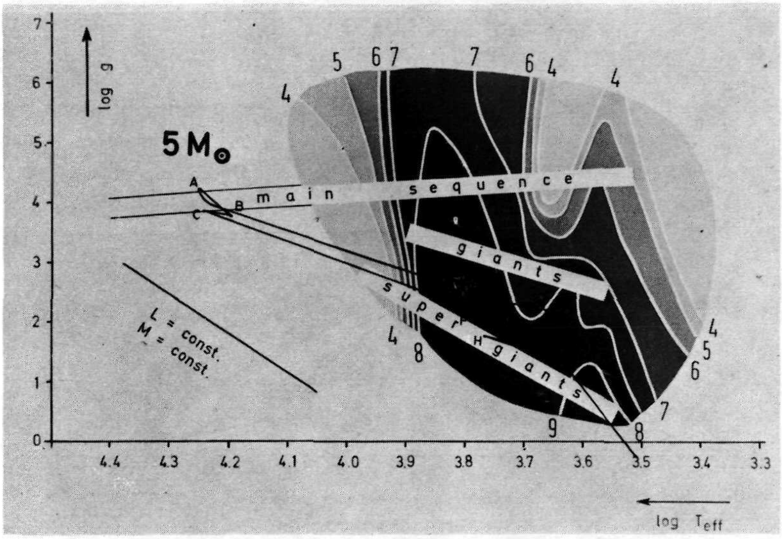

Figure IV-3 The evolutionary track for 5 $M_\odot$ from the pre-ms evolution to the ms. Central hydrogen burning starts at point A and is terminated at point B. Further evolutionary stages go from C to H.

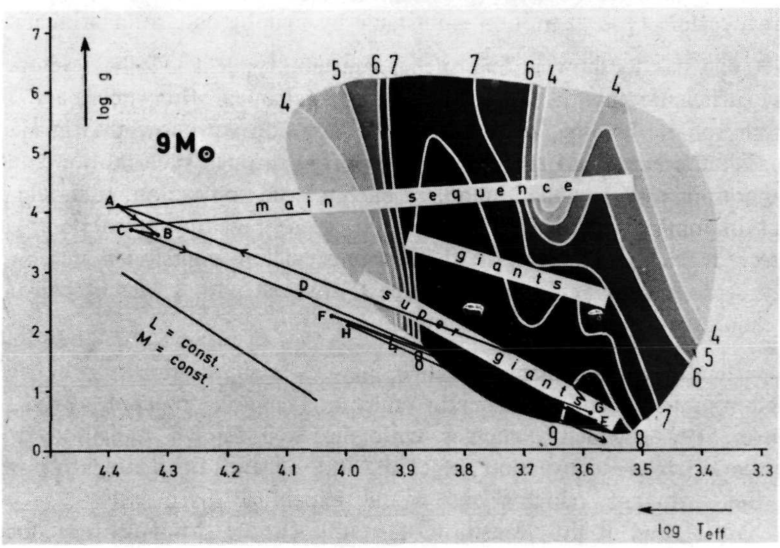

Figure IV-4 The evolutionary track for 9 $M_\odot$ from the pre-ms evolution to the ms. Central hydrogen burning: A-B/further evolutionary stages: C-H.

INFLUENCE ON STELLAR EVOLUTION
OUTER BOUNDARY CONDITIONS

I do not think that stellar models would be drastically different if the normal grey or nongrey atmospheric boundary conditions were replaced by a fit to an outer layer with a more complicated temperature profile. Only cool stars are sensitive to their outer boundary conditions – but only in the sense that their radii and therefore their position in the HRD is dependent on boundary conditions.

But the evolution itself is steered by the very deep interior and the interior of an evolved star does not know about the envelope.

MASS LOSS BY STELLAR WIND

The mass per year blown into space by the solar wind is small. It is less than the decrease in mass of the Sun due to the mass equivalent of its radiated energy. From the point of view of stellar evolution this mass loss can therefore be neglected. According to Weyman (1962) a Ori has a mass loss of

$$\frac{dM}{dt} = 4 \times 10^{-6}\ M_{\odot}/yr.$$

a Ori is a star of about 20 solar masses in its post main sequence evolution. In the most favorable case this mass loss might add up during central helium and carbon burning to a mass loss of a few percent for that star.

The luminosity of a main sequence star is reduced by mass loss according to the mass-luminosity relationship. But a star with shell burning remains at the same luminosity even if 90% of its hydrogen rich envelope is removed. This is well known from computations of mass exchange in close binary systems. Therefore it is very difficult to decide from observations whether an evolved star has undergone mass loss.

This is the reason why for years an argument has been going on between the non-linear cepheid pulsation theory people on the one side and the evolutionary and linear pulsation theory people on the other side. Christy (1968) claims that he can get agreement with observed light curves only if he assumes that cepheids have but half of the mass given by the normal evolution theory. On the other hand Lauterborn, et al., (1971), give an evolutionary track for a 5 $M_{\odot}$ star which has loops in the red giant region with several slow crossings of the cepheid strip. They found that if more than 5% of the mass of the star were taken off the envelope, the loops disappear. Therefore, they argue, if mass loss takes place there would be no slow crossings of the cepheid strip, there would then be no cepheids and Christy would then have no observed light curves to compare his theoretical curves with.

Since the mass of the cepheids is still undetermined (Fricke, Strittmatter, Stobie, 1972, Cox, King, Stellingwarf, 1972) if we wish to understand whether mass loss from coronas influences the evolution of stars we certainly have to look for the masses of the cepheids since this offers a chance to obtain information.

LATE PRE-MAIN SEQUENCE AND MAIN-SEQUENCE-EVOLUTION AND CHROMOSPHERIC ACTIVITY

When O. C. Wilson (1963) found that field stars have less chromospheric activity than the same type of stars in galactic clusters a completely new point of view came into play. Imagine: stars at the same place in the HRD and (since they are, therefore, also on the main sequence) stars of the same mass, differ in their Ca + emission! These stars should have the same atmospheres since g and T_{eff} are the same. They certainly have the same mechanical flux if it is computed in the same way as de Loore, but they differ in their chromospheric activity. The puzzle would remain even if one of the two new mechanisms mentioned yesterday were to replace the mechanical flux due to sound waves coming out from the convective zone. All those mechanisms would produce the same mechanical flux for the same values of g and T_{eff}.

Kraft (1967) found the correlation between chromospheric activity and rotational velocity. Now we know from the work of Skumanich (1972) that roughly

$$Ca^{+}\text{-emission} \sim \Omega \sim t^{-1/2}$$

where Ω is the angular velocity of the surface. From the Sun we know the Ca^+ emission is correlated to the magnetic field. Beckers and Sheeley during this conference told me that for fields between 0 and 100Γ there is a positive correlation between Ca^+ emission and the magnetic field strength $|B|$ although there is a large scatter around this relationship. Finally, we know that the solar magnetic field is related to the rotation of the Sun. We therefore come up with the following logical scheme, as shown in Figure IV-5.

The outer five boxes, forming a pentagon, give the logical structure as it follows from the first two sections of this article. Stellar evolution changes effective temperature and surface gravity of the stars, and these two parameters determine the top of the convective layers in which the

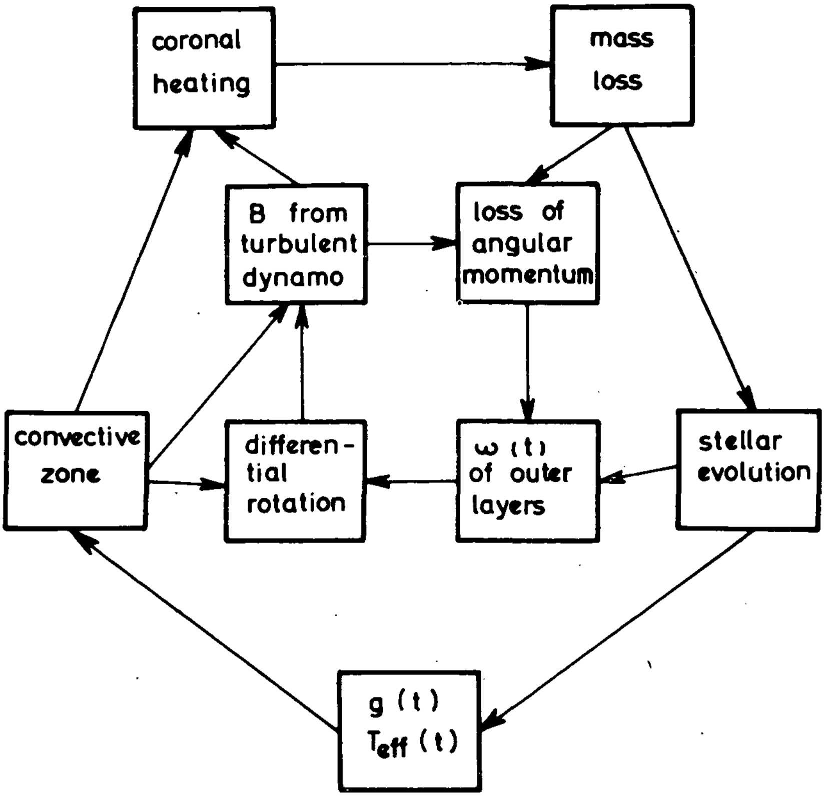

Figure IV-5 The logical structure which connects stellar evolution with nonthermal heating of chromospheres coronas.

mechanical flux is generated which heats the outer layers. Heated outer layers may produce a stellar wind which may influence the stellar evolution.

Due to the effects mentioned in this section one must also take into account the inner boxes. We know much less about these boxes inside the pentagon. What seems to go on inside the pentagon is more secret to us. As you see in the figure, almost all the arrows, that is all the information, goes into the interior of the pentagon and almost nothing comes out. But there is one leak.

If rotation is taken into account we must keep in mind that during stellar evolution when the star is contracting or expanding the angular velocity distribution will change. The angular velocity Ω, near the surface might therefore also be influenced by stellar evolution. For the Sun there is an indication that convective zones show differential rotation. Differential rotation together with convection can produce magnetic fields which on the one hand can enhance the outcoming magnetic flux and especially can determine the region where the dissipation takes place. It therefore influences the heating of the outer layers. On the other hand the stellar wind together with magnetic fields can produce a strong loss of angular momentum which, together with stellar evolution, influences the angular velocity distribution of the star. In the following we will discuss in more detail the interior of the pentagon.

EVOLUTION AND STELLAR ROTATION

Even if we assume that the star does not lose angular momentum the problem is difficult. We do not know how effective mechanisms, such as large or small scale motions or magnetic fields, are at redistributing angular momentum in the stellar interior. We do know that only very restricted angular momentum distributions are stable, but we do not know what the time scales of some of the instabilities are and whether they are really important during the life time of a star.

If we knew the true theory of the flow of angular momentum inside the star during evolution, the surface angular velocity would be known as a function of time: $\Omega = \Omega\,(t)$. From numerical calculations with different assumptions about the redistribution of angular momentum during stellar evolution Kippenhahn, Meyer-Hofmeister, Thomas 1970) one can derive, as a very crude thumb rule, that

$$\Omega(t) \sim \frac{1}{R^2\,(t)}$$

This relationship is valid in the case of local conservation of angular momentum. It turns out that this is a fairly good approximation in the physically more realistic case when one assumes that the hydrogen convective zone rotates as a solid body and that in the radiative regions angular momentum is locally conserved.

For our purpose in this review it is not so important to know the numerical details but rather to understand the logical structure, that is to find out what determines what. For this purpose it is sufficient to know that, when stars from the main sequence evolve into the red giant region, the surface angular velocity goes down roughly as indicated by the above formula. Observed rotational velocities for red giants (Oke, Greenstein, 1954) support the above formula.

DIFFERENTIAL ROTATION

Winding and unwinding of magnetic fields seems to be important for the solar dynamo. Therefore differential rotation is essential. The turbulent viscosity of the hydrogen convective zone gives a time scale of only 100 years for adjustment. The differential rotation therefore is certainly not a fossil relic from earlier phases of evolution. It must be maintained by some unknown mechanism.

Many attempts have been made to explain the solar rotation law. I think everybody now agrees that it is a pure hydrodynamic phenomenon; that the magnetic fields there have to follow the gas in the hydrodynamic flow and do not influence the rotation. This is indicated by the fact that the differential rotation does not vary with the solar cycle during which the magnetic field changes sign.

Among the hydrodynamic approaches there is that via *non-isotropic* viscosity proposed by L. Biermann (1951), Kippenhahn (1963) and Köhler (1970). This approach did not encounter much enthusiasm from the professional hydrodynamicists. On the other hand there are the attempts by Busse (1970) and recently by Gilman (1972).

In the present we do not know if any of these approaches will really turn out to be true. But, for the moment, we can just assume that convective regions like to rotate differentially – whatever the reason is.

TURBULENT DYNAMOS

During the last years, theories for the solar cycle have been developed by Babcock (1960) and Leighton (1969) and also by Steenbeck and Krause (1966). Both approaches have in common that turbulence and rotation are considered in a statistical theory which yields equations for the mean velocities and for the mean magnetic field. These equations contain terms in addition to those of ordinary magnetohydrodynamics due to correlations in the turbulent quantities. In normal magnetichydrodynamics one has

$$\frac{\partial B}{\partial t} = \frac{c^2}{4\pi\sigma} \Delta \underline{B} + \text{cure } [\underline{V}\,\underline{B}]$$

where $\underline{B}$, $\underline{V}$, are the magnetic field, the velocity field and σ the electric conductivity. The first term on the right hand describes the dissipation of magnetic energy due to ohmic losses while the second term alone would give the frozen-in condition. From Cowling's theorem there follows that in the axisymmetric case a given velocity field $\underline{V}$ can not maintain a magnetic field against the dissipation. But in the case of turbulent motion one obtains an equation similar to that above for the mean field but this equation contains an additional term as indicated in the lower part of Figure IV-6. This additional term has been derived by Steenbeck and Krause, and it contains the fact that rising and falling turbulent elements

LEIGHTON'S NONLINEAR MODEL:

$$\frac{\partial B_\varphi}{\partial t} = \underbrace{\sin \ell \left(B_\vartheta \frac{\partial \Omega}{\partial \vartheta} + \tau B_\tau \frac{\partial \Omega}{\partial \tau} \right)}_{\text{Winding of Frozen-in Field}} - \underbrace{\delta \cdot \text{const.} \, |B_\varphi| B_\varphi}_{\text{Depletion Due to Eruption}}$$

$$\frac{\partial B_\tau}{\partial t} = \underbrace{\frac{1}{T_D} \frac{\partial}{\partial \cos \ell} \left[m.^2 \, \ell \, \frac{\partial B_\tau}{\partial \cos \ell} \right]}_{\text{Diffusion}} - \underbrace{\delta \cdot \text{const.} \frac{\partial (B_\varphi \cos \ell)}{\partial \cos \ell}}_{\text{Creation of } B_\tau \text{ by Tilt''}}$$

$$\delta = \begin{cases} 0 & |B_\varphi| < |B_o| \\ 1 & \text{otherwise} \end{cases}$$

B_ϑ from $\nabla \cdot \underline{B} = 0$

KRAUSE-STEENBECK'S LINEAR THEORY:

$$\frac{\partial \underline{B}}{\partial t} = \underbrace{\text{const.} \, \Delta \underline{B}}_{\text{Diffusion}} + \underbrace{\text{curl.} \, (\underline{V} \times \underline{B})}_{\text{Winding of Frozen-in Field}} + \underbrace{\text{curl} \, (\alpha \underline{B})}_{\alpha\text{-effect}}$$

$\nabla \cdot \underline{B} = 0$

Figure IV-6 Formulae for the two types of models for turbulent dynamos.

are forced to a helical motion by Coriolis forces. The magnetic field is tilted by these elements in such a way that the mean field behaves as if there is a mean electric current parallel to the mean magnetic field (α-effect). This effect was already indicated by Parker (1955). The papers by Krause, Rädler and Steenbeck on the turbulent dynamo have recently been translated by Roberts and Stix (1971) (See also Deinzer, 1971).

Similar additional terms have been introduced into the magnetohydrodynamic equations by Leighton as one can see in Figure IV-6. In this theory similar to the α-effect of Steenbeck and Krause a "tilt" is assumed when a pair of sunspots appear when a magnetic "rope" comes to the surface.

The Babcock-Leighton theory is non-linear and one therefore obtains for any given angular velocity distribution, and for a given differential rotation, a magnetic field configuration. Recently Durney and Stenflo (1971) have investigated the strength of the magnetic fields in Leighton's dynamo in dependence on the angular velocity assuming the differential rotation to be the same. They found that the magnetic field is approximately proportional to the angular velocity.

$$|\underline{B}| \sim \Omega$$

New solutions of the Steenbeck-Krause equations have recently been found by Köhler (1972) who used a solar model with a realistic convective zone, and derived from the properties of the convective layer the factor in front of the $\Delta\underline{B}$ term of the Steenbeck-Krause equation as well as their α as functions of depth. He indeed obtained periodic solutions. Certainly the linear theory can not give amplitudes. But Stix (1972) investigated the case of non-linear limiting which would set in if the amplitudes become sufficiently high. Then, as already suggested by Steenbeck and Krause, the magnetic fields would be so strong that they would react on turbulence and inhibit the helical motion. With such a cut-off he found that the amplitudes roughly go like

$$|\underline{B}| \sim \Omega^{3/2}$$

The theory of the solar cycle is incomplete but one might already dare to make some predictions for other stars. If stars have a convective zone and are rotating, one would expect that they also have differential rotation. In this case the turbulent dynamo may work and one would expect the magnetic field to increase with rotational velocity if everything else including the degree of differential rotation is kept constant.

LOSS OF ANGULAR MOMENTUM

It had first been pointed out by Schatzman (1954) that mass loss from a rotating star with a magnetic field gives a high loss of angular momentum. This is due to the fact that the outstreaming material gains angular momentum from the magnetic field until it has reached a point where it is released into space. Following Weber and Davis (1967) one can write

$$\frac{d}{dt}\,(k M.\, R^2\, \Omega) = \frac{2}{3}\, r_A{}^2\, \Omega\, \frac{dM}{dt} \qquad (1)$$

$k M\ R^2$ is the inertial momentum of the star. The factor k can be computed for any given stellar model. The radius r_A is the distance from the star at which the Cowling number

$$C^2 \equiv \frac{v_r^2}{B_r^2 / 4 \pi \rho}$$

is one. Here v_r and B_r are the radial components of velocity and magnetic field. If we follow recent work by Durney (1972) in a more generalized way one can show that

$$r_A \sim B_0 v_A^{-½} \left(\frac{dM}{dt} \right)^{-½}$$

where v_A is the value of v_r at the point where $C = 1$ and B_o the field at the surface of the star. If we assume from the dynamo theory it follows that $B_o \sim \Omega^\gamma$ we can then write

$$r_A \sim \Omega^\gamma v_A^{-½} \left(\frac{dM}{dt} \right)^{-½}$$

From equation (1) it then follows that – as long as the radius of the star is not varying with time, as it is in the case for the main sequence stage to a high degree of approximation, one can write

$$\frac{1}{\Omega} \frac{d\Omega}{dt} = \text{const.} \frac{\Omega^{2\gamma}}{v_A}$$

Therefore for any given rate of mass loss and for any assumption as to how the radial velocity, v_A, varies with time, one can determine the angular velocity as a function of time. Probably v_A as well as dM/dt will vary with the angular velocity since the angular velocity will enhance the turbulent dynamo and therefor enhance the heating and therefor the mass loss. Generally one can assume

$$v_A \sim \Omega^\zeta$$

with a free exponent ζ. Then equation (2) can be integrated and gives (as long as $\zeta \neq 2\gamma$)

$$\Omega = \text{const.} \, (t - t_0)^{\frac{1}{\zeta - 2\gamma}}$$

Durney has used this formula for the special case $\zeta = 0$, $\gamma = 1$ in order to obtain Skumanich's law $\Omega \sim t^{-½}$. Certainly one must know more about the mechanisms inside the pentagon of Figure IV-5. The main purpose here is to show that, in principle, the time dependence of the angular velocity distribution is determined.

We have now discussed the boxes inside the pentagon and I must say I have the feeling that the whole logical structure indicates quite a closed picture although many details still have to be worked out.

TURBULENT VELOCITIES IN THE ATMOSPHERES OF ROTATING STARS

I would like to add a comment on the question of hot main sequence stars where convective theory gives practically no turbulent velocities. It has been shown by Baker and Kippenhahn (1959) that near the surfaces of rotating stars meridional circulation can reach fairly high velocities. I will give a different approach here. We consider very rapidly rotating stars where, near the equator, the centrifugal force almost balances gravity. Then it follows from von Zeipel's theorem that the effective temperature at each latitude is connected with the effective gravity:

$$T_{eff} \sim g^{1/4}.$$

It follows that pressure and temperature are constant on equipotential surfaces for hydrostatic equilibrium. But when we try to construct atmospheres in each latitude it turns out that the mean optical depth τ is not constant on equipotential surfaces ϕ = const:

$$d\tau = -\kappa\, d\tau = -\kappa\, d\phi/g,\ \kappa = \kappa(P,T) = \kappa(\phi),$$

$$\tau = -\int \kappa(\phi)\, d\phi /\, g$$

Therefore τ varies on equipotential surfaces like g^{-1}. Solution of the transfer equations yields the temperature which is not constant on equipotential surfaces. This can be most easily seen in the case of a grey atmosphere where radiative equilibrium in the simplest approximation is given by

$$T^4 = \text{const. x } T_{eff}^{\ 4} \left(\tau + \frac{2}{3}\right).$$

$T_{eff}^{\ 4}$ varies on equipotential surfaces like g and τ like g^{-1}. Therefore T is not constant on equipotential surfaces. This is in contradiction to the condition of hydrostatic equilibrium. The equilibrum condition with the longer time scale will not be fulfilled. This is the equation of hydrostatic equilibrium. We therefore must assume that there are strong horizontal motions with velocities high enough that the inertia terms are of the same order as the pressure gradient. This means the velocities are near the velocity of sound.

The theory of atmospheres of rotating stars has recently been worked out by C. Smith (1970) and indeed he found that there are velocities which come near the velocity of sound. Therefore if chromospheric activity is found in rapidly rotating hot stars where convection cannot account for it, turbulent atmospheric motions in the atmospheres caused by rotation may be responsible.

ACKNOWLEDGEMENT

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