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SMOOTHING OF FUNCTIONS OF RANGE AND RANGE RATE MEASUREMENTS FROM EARTH ORBITING SATELLITES

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GODDARD SPACE FLIGHT CENTER

GREENBELT, MARYLAND

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ABSTRACT

In this analysis it is shown that for satellites in circular Earth orbits with altitudes of 500 kilometers to 1500 kilometers, and for satellites in elliptical orbits with an approximate 4000 kilometer height of perigee, a high degree least squares polynomial (e.g., a 9th or 10th degree in some cases) is required to smooth both range and range rate data for purposes of input to orbit determination programs. This is especially true for overhead passes. In order to circumvent this problem, functions of range and range rate are smoothed with lower degree least squares polynomials (e.g., 3rd and 4th degree) and it is shown that under the above geometric constraints the standard deviation of fit (i.e., the standard deviation of the errors between the approximating function for range or range rate and the true function of range or range rate) can be reduced to levels commensurate with typical S-band tracking system resolution which is 1 to 2 meters in range and 0.005 meters/second in range rate for a 1 per second data rate. The computational time required for fitting functions of range and range rate for low degree polynomials is shorter than the computational time required to fit the range or range rate directly for higher degree polynomials.

Also shown in the analysis are the effects of Gaussian random noise, biases, and periodic noise. This analysis includes numerous examples applied to the 44 point data smoothing interval currently used in much of the operational preprocessing at the Goddard Space Flight Center.

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SUMMARY

In orbit determination, range, range rate, and other types of tracking measurements are made at various tracking sites, then sent to a central control for preprocessing and input to an orbit determination program after some smoothing technique has been used to reduce the effects of random and periodic noise on the data. For the Goddard Range and Range Rate System, both data types (range and range rate) are smoothed by a 3rd degree least squares polynomial (in blocks of 44 data points) and the midpoint of the smoothed curve is selected as the best estimate of the measurement over the data arc for input to the orbit determination program.

In this analysis it is shown that for satellites in circular Earth orbits with altitudes of 500 kilometers to 1500 kilometers and for satellites in elliptical orbits with an approximate 4000 kilometer height of perigee, a high degree least square polynomial (e.g., 9th or 10th in some cases) is required to smooth both range and range rate data for purposes of input to orbit determination programs. This is especially true for overhead circular passes and for satellites in the perigee portion of elliptical orbits directly overhead the tracking station. In order to circumvent this problem, functions of the range and range rate measurements are smoothed by lower degree least squares polynomials (e.g., 3rd and 4th degree), and it is shown that the standard deviation of the errors between the true range and range rate and the approximating functions of range and range rate can be reduced to levels commensurate with typical S-band tracking system resolution: 1 to 2 meters in range and 0.005 meters/second in range rate (sample error for a sample rate of one per second).

Also shown in the analysis are the effects of Gaussian random noise, biases, and periodic noise.

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SMOOTHING OF FUNCTIONS OF RANGE AND RANGE RATE MEASUREMENTS FROM EARTH ORBITING SATELLITES

1.0 STATEMENT OF PROBLEM

In scientific satellite orbit determination, practical experience has shown the advantages of editing, filtering, and compacting tracking data prior to usage in an orbit determination program. Since computer running time for an orbit determination program is generally proportional to the number of input observations, it has been proven economical to fit, in the least squares sense, blocks of consecutive raw data points to low order polynomial functions in a separate preprocessor computer program. Each polynomial function evaluated at the midpoint can be selected as one input data value to the orbit determination program. At Goddard for example, 44 raw data points are used as one block and a 3rd degree least squares polynomial calculated for both range and range rate data. The value of the polynomial at the midpoint is then selected as input to the orbit determination program. By this procedure a data compaction of 44:1 of input to the Goddard orbit determination programs is effected. In addition, obvious "wild" data points can be rejected in the polynomial function calculation by comparison of residuals between the function evaluated at a data point and that data point. Editing criteria, such as 3.5 times the root-mean-square error calculation from all residuals, can be used to reject individual data points that clearly are not noise-like errors. After rejection of "wild" data points, a "closer" polynomial function can be fit to the remaining unedited data points in the data block.

Along with the advantages of preprocessing raw tracking data, experience has shown that pitfalls are also present. It is the intent of this document to illustrate a primary shortcoming of preprocessor programs, and to propose an improvement. This demonstration of difficulty in fitting data points in the least squares sense to low degree polynomials is contained within Appendix A in Figures A-1 to A-120. In generating each of the figures, the following procedure was applied:

1. Generate observations from a hypothetical ground station located on a spherical Earth and rotating within the orbit plane of a satellite in a two-body system. Fix the observation sample rate at one observation per second.

2. Generate the data such that zero time occurs as the satellite crosses station zenith.

3. Calculate the coefficients of low order polynomials fitted to the observation data in the least squares sense.

4. Plot the difference between the generated observation value and the calculated polynomial at each data point. (These are Figures A-1 to A-120.)

These differences between the polynomial function and the unperturbed observation or "true" values are errors caused by the insufficiency of low order polynomials to accurately fit the equations which define the measurements of range, range rate and angles.

Four different situations were considered in Figures A-1 through A-120:

1. Figures A-1 to A-30 reflect the polynomial fitting error for range, range rate, and elevation data for a circular orbit, altitude of 1000 kilometers, where the satellite passes over the station at time equals zero seconds, and data points are taken at time = 1, 2, 3. . . , 44 seconds. Each data type is fitted to a one through tenth degree polynomial. In each figure 'a' is the nominal Earth radius plus altitude and eccentricity 'e' is zero.

2. Figures A-31 to A-60 show the polynomial fitting error for range, range rate, and elevation data for a circular orbit, altitude of 1000 kilometers where the satellite passes over the station at time equals zero seconds, and data points are taken at time = 1, 2, . . . , 200 seconds. Figures A-31 to A-60 differ from Figures A-1 to A-30 only in the length of time (200 seconds compared to 44 seconds) over which data is taken.

3. Figures A-61 to A-90 depict the polynomial error for range, range rate, and elevation data for an elliptic orbit, semi-major axis of 95,000 kilometers, eccentricity of 0.89 (nominal values for IMP-G), where the satellite passes over the station at perigee and at time equals zero seconds, and data points are taken at time = 1, 2, . . . , 44 seconds. Each data type is fitted to a one through tenth degree polynomial.

4. Figures A-91 to A-120 differ from Figures A-61 to A-90 only in the span of time over which data is taken. Figures A-91 to A-120 cover 200 seconds of time for the assumed elliptic orbit conditions.

The figures in Appendix A show clearly the error in polynomial smoothing at the time of each data point on the polynomial and also shows that the number of zero crossings is equal to the degree of the polynomial plus one. Kruger in Reference 1 also considered this topic but the pictorial illustration of error is lacking. The report shows efficient methods of reducing this kind of error, and these methods achieve a significant reduction in computer operation time when compared to the standard method of polynomial smoothing.

2.0 SMOOTHING OF FUNCTIONS OF RANGE AND RANGE RATE BY LEAST SQUARES POLYNOMIALS

It has been shown previously that high degree least squares polynomials (e.g., 9th and 10th degree in some cases) must be fit to range and range rate measurements in order to reduce the rms (root mean square) of fit to levels commensurate with the resolution of a typical S-band system (1 to 2 meters in range and approximately 0.005 meters/second in range rate). This is particularly true for satellites in a near-Earth circular orbit at the overhead portion of the orbit and for satellites in an elliptical orbit around the earth at the perigee portion of the orbit. During these portions of the orbit, accelerations and higher derivatives are significant.

The purpose of this analysis is to show that lower degree least squares polynomials (e.g., 3rd and 4th degree) can be fit to functions of range and range rate with the result that the standard deviation of fit (the standard deviation of fit is defined as the standard deviation of the errors between the approximating function and the true function of range or range rate) is within the resolution of a typical S-band system. We will first treat analytically the case of a satellite in a circular near-Earth orbit at the overhead portion of the orbit, and secondly, a satellite in an elliptical orbit about the Earth simulating the orbit of IMP-G at the perigee portion of the orbit. In addition, we will simulate errors in range and range rate (random, bias, and periodic errors) and note the effects of these errors upon the standard deviation of fit.

Briefly, the technique consists in squaring the range measurements and fitting least squares polynomials to this function, then taking the square root of the fitted function at various points along the polynomial. It will be demonstrated that the standard deviation of fit between the square root of a 4th degree polynomial fit to range squared and the true range function is within the resolution of a typical S-band system. For the case of range rate each of the values of range as calculated by taking the square root of the 4th degree polynomial fit to the square of the range measurements is multiplied by the range rate value at the proper time. It should be noted that the range measurements and the range rate measurements do not occur at the same instant of time, that is, the time at the satellite for the range measurement is different from the time at the satellite for the range rate measurement. The appropriate time for the range, however, can be determined from the range polynomial fit. Polynomials are then fit to the range rate times the range or $\dot{r}R$ ($\dot{r}$ refers to the range rate measurement and R refers to the range measurement). Values from the desired points along the fitted $\dot{r}R$ polynomial are then divided by the range value (again properly time tagged to coincide with the time at the satellite for range rate) to obtain a new function. It will be seen that for a 3rd degree least squares polynomial fit to $\dot{r}R$ the resulting function is close to the true generated range rate function in the sense that the standard deviation of fit between the approximating function and the true range rate function is within the resolution of a typical S-band system.

3.0 ANALYSIS FOR RANGE

In this section we will consider the satellite to be in a circular orbit around the Earth, in the equatorial plane and rotating in the same direction as Earth rotation. The geometry is indicated in Figure 1 with an equatorial tracking station and observations referenced to the point of closest approach:

$$R_s = R_e + h_s$$

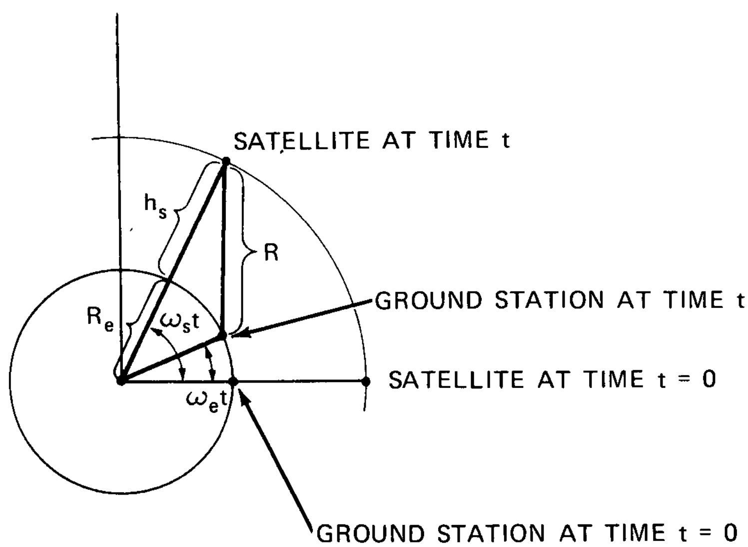

Figure 1. Geometry of Circular Orbit

In Figure 1, ω_e is the angular velocity of the Earth, ω_s is the angular velocity of the satellite, R_e is the radius of the Earth, R_s is the radius to the satellite, h_s is the height of the satellite above the Earth, and R is the range of the satellite from the ground tracking station.

From the geometry in Figure 1 we can write an expression for R and expand it in an infinite series as follows:

$$R = \{R_s^2 + R_e^2 - 2R_s R_e \cos(\omega_s - \omega_e)t\}^{1/2} \tag{1}$$

Letting $k_1 = \left(R_s^2 + R_e^2\right)^{1/2}$, $k_2 = 2R_s R_e/\left(R_s^2 + R_e^2\right)$, and $k_3 = (\omega_s - \omega_e)$, we have

$$R = k_1 \left(1 - k_2 \cos k_3 t\right)^{1/2} \tag{2}$$

which can be expanded in an infinite series

$$R = k_1 \{1 - \tfrac{1}{2} k_2 \cos k_3 t - (1 \cdot 1)/(2 \cdot 4)\, k_2^2 \cos^2 k_3 t$$

$$- (1 \cdot 1 \cdot 3)/(2 \cdot 4 \cdot 6)\, k_2^3 \cos^3 k_3 t - \ldots\}$$

or

$$R = k_1 \left\{ 1 - \sum_{m=0}^{\infty} \frac{\Gamma(m+\frac{1}{2})\, k_2^m}{(2m-1) \quad \pi \quad \Gamma(m+1)} \left(\sum_{n=0}^{\infty} (-1)^n \frac{(k_3 t)^n}{(2n)!} \right)^m \right\} \tag{3}$$

Writing the square of R we have

$$R^2 = R_s^2 + R_e^2 - 2 R_s R_e \cos(\omega_s - \omega_e)\, t$$

or

$$R^2 = k_1^2 \left(1 - k_2 \cos k_3 t\right) \tag{4}$$

Expanding R^2 in an infinite series we have

$$R^2 = k_1^2 \left(1 - k_2 \sum_{m=0}^{\infty} (-1)^m \frac{(k_3 t)^{2m}}{(2m)!}\right) \tag{5}$$

The series representation in equation (5) requires fewer terms than (3) to accurately approximate the function.

In order to show this, we can choose a value of t = 200 seconds and compute the error in range for a 4th degree polynomial for both R and R^2. Letting h_s = 1000 kilometers (i.e., R_s = 7378 kilometers), R_e = 6378 kilometers, the period of the satellite is

$$P = 2\pi \sqrt{\frac{a^3}{\mu_e}} = 2\pi \sqrt{\frac{(7378)^3 \text{ km}^3}{398603.2 \text{ km}^3/\text{sec}^2}}$$

or

$$P = 6307 \text{ seconds} \tag{6}$$

where 'a' is the semi-major axis of the orbit and μ_e is the gravitational constant for the Earth. Therefore

$$\omega_s = \frac{2\pi}{P} = 0.000996 \text{ radians/second}$$

$$\omega_e = 2\pi/24(60)^2 = 0.0000727 \text{ radians/second}$$

$$k_3 = (\omega_s - \omega_e) = 0.000923 \text{ radians/second}$$

The true range at t = 200 seconds is from equation (1)

$$R = 1612.698 \text{ kilometers}$$

Using equation (3) we have for m = 4

$$R_1 = 2954.467 \text{ kilometers}$$

and the error is

$$\Delta R_1 = R_1 - R = 1341.769 \text{ kilometers} \tag{7}$$

Using equation (5) for t = 200 seconds and m = 4 results in

$$R_2 = 1612.696 \text{ kilometers}$$

and the error is

$$\Delta R_2 = R_2 - R = -0.002 \text{ kilometers} \tag{8}$$

It can be seen from the above example that for the same time t = 200 seconds and the same degree polynomial approximation of R^2 in (5) (here R^2 is being approximated as a 4th degree polynomial in t) and R in (3) (here R is being approximated as a 4th degree polynomial in $\cos(K_3 t)$) taking the square root of the polynomial in R^2 is more accurate than using the polynomial in R. The reason for this is that when using series expansions the R^2 function converges much more rapidly than does the R function. That is, a lower degree polynomial can be fit to R^2 than to R to attain a given computation accuracy.

4.0 ANALYSIS FOR RANGE RATE

Recalling equation (4) of the previous section:

$$R^2 = k_1^2 (1 - k_2 \cos k_3 t) \tag{4}$$

we can differentiate to obtain

$$2R\dot{R} = k_1^2 k_2 k_3 \sin k_3 t \tag{9}$$

and combining with (2)

$$\dot{R} = \frac{k_1^2\ k_2\ k_3\ \sin k_3 t}{2k_1\ (1-k_2\ \cos k_3 t)^{1/2}} \tag{10}$$

Expanding (10) in an infinite series we have

$$\begin{aligned}
\dot{R} &= \tfrac{1}{2}k_1\ k_2\ k_3\ \sin k_3 t\ \{1 + \tfrac{1}{2}k_2 \cos k_3 t + (1\cdot 3)/(2\cdot 4)\ k_2^2 \cos^2 k_3 t \\
&\qquad + (1\cdot 3\cdot 5)/(2\cdot 4\cdot 6)\ k_2^3 \cos^3 k_3 t + \ldots\} \\
&= \tfrac{1}{2}k_1\ k_2\ k_3 \sum_{m=1}^{\infty} (-1)^{m-1} \frac{(k_3 t)^{2m-1}}{(2m-1)!} \\
&\qquad \sum_{n=0}^{\infty} \frac{\Gamma(n+\frac{1}{2})\,k_2^n}{\sqrt{\pi}\ \Gamma(n+1)} \left(\sum_{r=0}^{\infty} (-1)^r \frac{(k_3 t)^{2r}}{(2r)!} \right)^n
\end{aligned} \tag{11}$$

The function $2R\dot{R}$ in equation (9) can be expanded in an infinite series to obtain

$$R\dot{R} = k_1^2 k_2'\ k_3 \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(k_3 t)^{2n-1}}{(2n-1)!} \tag{12}$$

where the factor of two has been absorbed into k_2'.

The series representation in equation (12) is much easier to work with than equation (11) since it converges more quickly.

In order to show this we can choose a value of $t = 200$ seconds and compute the error in range rate for a 3rd degree polynomial for both $\dot{R}$ and $R\dot{R}$. Letting $h_s = 1000$ kilometers (i.e., $R_s = 7378$ kilometers), $R_e = 6378$ kilometers, we have for the true range rate $\dot{R}$ from equation (10)

$$\dot{R} = 4948.9772 \text{ meters/second}$$

Using equation (11) for $t = 200$ seconds and $m = 3$ we have for the range rate

$$\dot{R}_1 = 1742.0195 \text{ meters/second}$$

and the error in range rate is

$$\Delta\dot{R}_1 = \dot{R}_1 - \dot{R} = -3206.9577 \text{ meters/second} \tag{13}$$

Using equation (12) for t = 200 seconds, n = 3, and an approximate value for range of R_2 = 1612.696 kilometers obtained from equation (5) for m = 4, we have for the range rate

$$\dot{R}_2 = 4948.9336 \text{ meters/second}$$

and the error in range rate is

$$\Delta\dot{R}_2 = \dot{R}_2 - \dot{R} = -0.0436 \text{ meters/second} \tag{14}$$

In this case it is seen that for a given degree of polynomial it is much better to fit to $R\dot{R}$ and extract the range rate by dividing by R (R being determined from a fit to R^2) than to fit $\dot{R}$ directly. Again the reason for this is the $R\dot{R}$ function converges much more rapidly when expanded in series than does the $\dot{R}$ function.

5.0 RESULTS FROM GENERATING RANGE DATA FOR A SATELLITE IN A CIRCULAR EARTH ORBIT AND FOR A SATELLITE IN AN ELLIPTICAL EARTH ORBIT

In all cases to be described it is assumed that the satellite and the Earth tracking station are in the same plane (equatorial) and both rotating in the same direction. The circular orbit is illustrated in Figure 1 and the eccentric orbit in Figure 2. In the case of the circular orbit the satellite is rotating with an angular velocity of ω_s and the Earth is rotating with angular velocity of ω_e . The data are taken at one/second. It should be noted that for a sample rate of one/second the resolution in range is on the order of 1 to 2 meters for a typical S-band tracking system. The standard deviation of fit is the standard deviation of the errors between the approximating function of range or range rate and the true function of range or range rate.

Table 1 is a comparison between fitting to R^2 and fitting to R for a circular overhead pass. The satellite height is 1000km and the number of points is 44. No random noise, bias, or periodic noise has been added to the range values. For a fourth degree fit to R^2 the standard deviation of fit is 0.11×10^{-5} meters while for a fourth degree fit to R the standard deviation is 0.09 meters. It can be seen that fitting to R^2 for a specific polynomial degree is much better than fitting to R. Effects of biases caused by the squaring process were noted to be insignificant compared to the standard deviation shown. This will be shown in succeeding tables.

Table 2 is again a comparison between fitting to R^2 and R for a circular 1000km altitude overhead pass but with the number of points increased to 200.

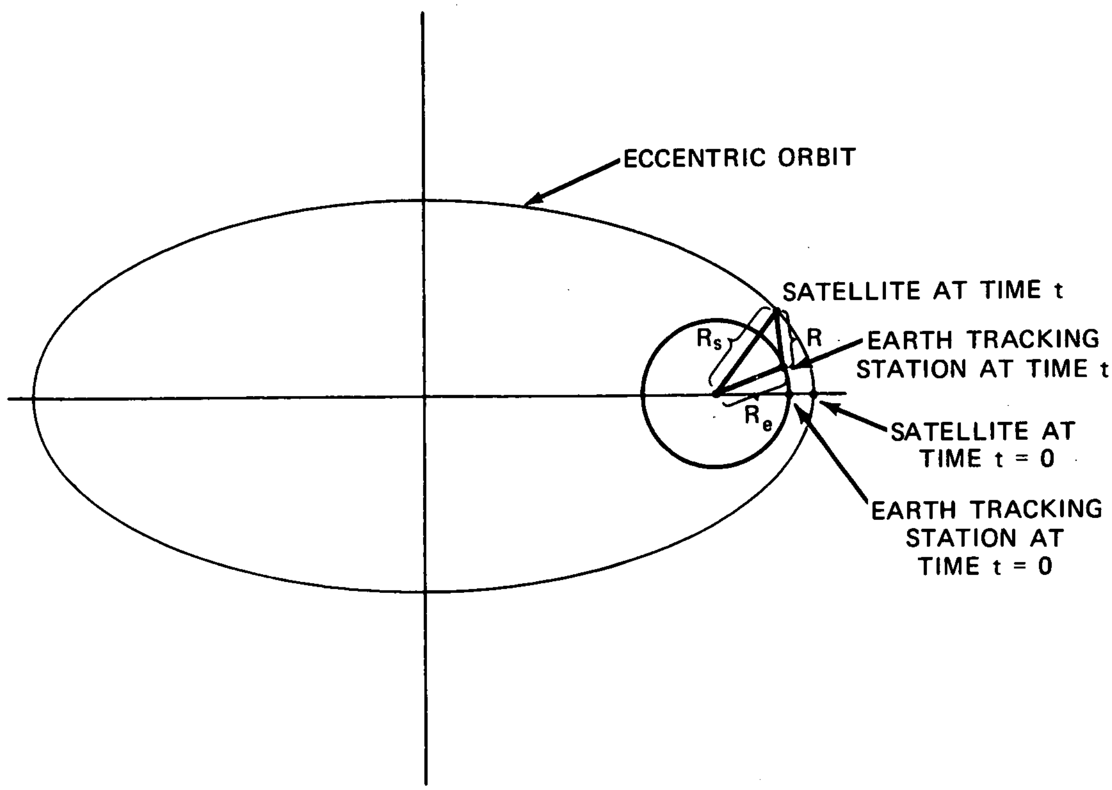

Figure 2. Geometry for Eccentric Orbit at Perigee

No bias, or random, or periodic noise has been added to the range. For a fourth degree fit to R^2 the standard deviation of fit is 0.008 meters while for a fourth degree fit to R the standard deviation is 107.5 meters. It should be noted that for a third degree fit to R^2 the standard deviation of fit is 9.09 meters which is much higher than the resolution of a typical S-band tracking system.

Table 3 is a comparison between fitting to R^2 and fitting to R for an eccentric orbit simulating the orbit of the IMP-G satellite at the perigee portion of the orbit. The semi-major axis is 95000km and the eccentricity is 0.89. The number of points is 44. No random noise, bias or periodic noise has been added to the range values. For a fourth degree fit to R^2 the standard deviation of fit is 0.16×10^{-5} meters while for a fourth degree fit to R the standard deviation of fit is 0.19×10^{-3} meters.

Table 4 is a comparison between fitting to R^2 and fitting to R for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit. The semi-major axis is 95000km and the eccentricity is 0.89. In this case the number of points is 200. Again, no random noise, bias, or periodic noise has been added to the range values. For a fourth degree fit to R^2 the standard deviation of fit is 0.01 meters, while for a fourth degree fit to R the standard deviation is 1.40 meters.

Table 1
Range Analysis

Comparison between fitting to R^2 and fitting to R for a circular overhead pass. $h_s = 1000$km, n = 44 points. No random noise or bias, no periodic noise.

Degree of Polynomial Fit	σ_{R^2} (meters)	σ_R (meters)
1	0.28518453×10^4	0.27952192×10^4
2	0.20395929×10^0	0.27590778×10^2
3	$0.24890964 \times 10^{-1}$	0.30444532×10
4	$0.10513290 \times 10^{-5}$	$0.92926701 \times 10^{-1}$
5	$0.84968737 \times 10^{-7}$	$0.58031350 \times 10^{-2}$
6	$0.33051808 \times 10^{-9}$	$0.33328270 \times 10^{-3}$
7	$0.32909997 \times 10^{-9}$	$0.11612154 \times 10^{-4}$
8	$0.33795794 \times 10^{-9}$	$0.11953913 \times 10^{-5}$
9	$0.34314998 \times 10^{-9}$	$0.17982671 \times 10^{-7}$
10	$0.30374976 \times 10^{-9}$	$0.41618490 \times 10^{-8}$

Table 2
Range Analysis

Comparison between fitting to R^2 and fitting to R for a circular overhead pass. $h_s = 1000$km, n = 200 points. No random noise or bias, no periodic noise.

Degree of Polynomial Fit	σ_{R^2} (meters)	σ_R (meters)
1	0.51020112×10^5	0.36713759×10^5
2	0.71120520×10^2	0.48267764×10^4
3	0.90921999×10^1	0.17821334×10^3
4	$0.77245760 \times 10^{-2}$	0.10753328×10^3
5	$0.65134129 \times 10^{-3}$	0.24781483×10^2
6	$0.39555368 \times 10^{-6}$	0.12653316×10^1
7	$0.24905413 \times 10^{-7}$	0.87747226×10^0
8	$0.13722842 \times 10^{-9}$	0.21976958×10^0
9	$0.26623817 \times 10^{-9}$	$0.11882227 \times 10^{-1}$
10	$0.41170183 \times 10^{-6}$	$0.93721581 \times 10^{-2}$

Table 3
Range Analysis

Comparison between fitting to R^2 and fitting to R for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000 km, eccentricity = 0.89, n = 44 points. No random noise or bias, no periodic noise.

Degree of Polynomial Fit	σ_{R^2} (meters)	σ_R (meters)
1	0.87843840×10^{3}	0.87710562×10^{3}
2	$0.54749199 \times 10^{-1}$	0.71638679×10^{0}
3	$0.66625852 \times 10^{-2}$	$0.86594629 \times 10^{-1}$
4	$0.15986661 \times 10^{-5}$	$0.19494156 \times 10^{-3}$
5	$0.12877727 \times 10^{-6}$	$0.15462630 \times 10^{-4}$
6	$0.10486462 \times 10^{-8}$	$0.58462630 \times 10^{-7}$
7	$0.98764667 \times 10^{-9}$	$0.34437905 \times 10^{-8}$
8	$0.10706711 \times 10^{-8}$	$0.10317410 \times 10^{-8}$
9	$0.56929905 \times 10^{-8}$	$0.98555667 \times 10^{-8}$
10	$0.33281828 \times 10^{-6}$	$0.43096297 \times 10^{-6}$

Table 4
Range Analysis

Comparison between fitting to R^2 and fitting to R for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000 km, eccentricity = 0.89, n = 200 points. No random noise or bias, no periodic noise.

Degree of Polynomial Fit	σ_{R^2} (meters)	σ_R (meters)
1	0.17756301×10^{5}	0.17236634×10^{5}
2	0.22282125×10^{2}	0.27444208×10^{3}
3	0.27383164×10^{1}	0.29418330×10^{2}
4	$0.13417197 \times 10^{-1}$	0.13969013×10^{1}
5	$0.10714135 \times 10^{-2}$	$0.76762590 \times 10^{-1}$
6	$0.93704336 \times 10^{-5}$	$0.75774230 \times 10^{-2}$
7	$0.54130339 \times 10^{-6}$	$0.17361715 \times 10^{-3}$
8	$0.69497875 \times 10^{-8}$	$0.41028219 \times 10^{-4}$
9	$0.13151758 \times 10^{-8}$	$0.26686856 \times 10^{-6}$
10	$0.23331270 \times 10^{-5}$	$0.26377387 \times 10^{-6}$

For the next three tables (Tables 5, 6, and 7) the satellite is in a circular overhead pass at a height of 1000 km. No random noise, bias or periodic noise has been added to the range values. Table 5 is for 50 points, Table 6 for 100 points, and Table 7 for 200 points. The standard deviations of fit are respectively, 0.23×10^{-5} meters, 0.14×10^{-3} meters, and 0.008 meters for the 4th degree fit for a 1/second sample rate. It can be seen that for longer data arcs the standard deviation of fit is larger for a fixed degree polynomial. The mean value of the differences is given to show that no significant biases are introduced by the squaring process.

Tables 8 and 9 are also for a circular overhead pass at a height of 500 kilometers and 1500 kilometers respectively. The number of points is 200, and no random noise, bias, or periodic noise have been added to the range values. The fit is to R^2. Comparing the two tables it can be seen that for satellites in lower orbits the standard deviation of fit is greater. This is true since the orbital dynamics are greater at lower heights.

Table 10 is for a satellite in a circular overhead pass, the number of points is 200 and the satellite height is 1000 km. In this case Gaussian noise has been added to the range values, but no bias or periodic noise. The standard deviation of the random noise is 2 meters and the fit is to R^2.

For a sample error of 2 meters standard deviation and a 200 uncorrelated point sample, it can be expected that the resulting standard deviation between the calculated range and the true range would be no better than $\frac{1}{\sqrt{200-k-1}}$ times the 2 meter sample error, where k is the polynomial degree. For example the minimum possible 4th degree error would be 0.14 meters, but note that in Table 10, the minimum error is 0.16 meters. This should be expected since the polynomial error is also included in the 0.16 meters. Also note that increasing the polynomial degree increases the standard deviation. No significant bias effects are caused by the squaring process as indicated by the mean value of the difference between the calculated value and the true value in Table 10.

Table 11 is for a satellite in a circular overhead pass of 1000 km. The number of points is 200. Here a bias of 5 meters has been added to each of the range values, but no random or periodic noise. The standard deviation of fit for a fourth degree fit to R^2 is 0.008 meters. It should be noted that although the standard deviation of fit is low, the bias of 5 meters has not been removed from the data. This is shown in Table 11 by the calculation of the mean value of the errors between the calculated ranges and the true values.

Table 12 is for a satellite in a circular overhead pass of 1000 km. The number of points is 200. In this case a bias of 10 meters has been added to

Table 5
Range Analysis

Circular orbit, overhead pass, n = 50 points, h = 1000 km, no random noise or bias, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	0.56232410×10^{1}	0.36697705×10^{4}
2	$0.43846117 \times 10^{-4}$	0.33792588×10^{0}
3	$0.52249932 \times 10^{-6}$	$0.41412746 \times 10^{-1}$
4	$-0.64028427 \times 10^{-11}$	$0.22551938 \times 10^{-5}$
5	$-0.15133992 \times 10^{-10}$	$0.18329509 \times 10^{-6}$
6	$-0.22409949 \times 10^{-10}$	$0.89572210 \times 10^{-10}$
7	$-0.15425030 \times 10^{-10}$	$0.88103885 \times 10^{-10}$
8	$-0.22118911 \times 10^{-10}$	$0.82792086 \times 10^{-10}$
9	$-0.15716068 \times 10^{-10}$	$0.12349933 \times 10^{-8}$
10	$-0.29103830 \times 10^{-11}$	$0.22028116 \times 10^{-7}$

Table 6
Range Analysis

Circular orbit, overhead pass, n = 100 points, h_s = 1000 km, no random noise or bias, no periodic noise. Fit to R^2.

Degree of Polynomial of Fit R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	0.43412137×10^{2}	0.14070889×10^{5}
2	$0.76245816 \times 10^{-2}$	0.50892545×10^{1}
3	$0.31838125 \times 10^{-4}$	0.63581281×10^{0}
4	$0.41197927 \times 10^{-8}$	$0.13705393 \times 10^{-3}$
5	$-0.48894435 \times 10^{-10}$	$0.11360777 \times 10^{-4}$
6	$-0.20518200 \times 10^{-10}$	$0.17534497 \times 10^{-8}$
7	$-0.39435690 \times 10^{-10}$	$0.17001838 \times 10^{-9}$
8	$-0.40017767 \times 10^{-10}$	$0.14958597 \times 10^{-9}$
9	$-0.42928150 \times 10^{-10}$	$0.43774422 \times 10^{-8}$
10	$-0.54569682 \times 10^{-10}$	$0.12456672 \times 10^{-6}$

Table 7
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000km, no random noise or bias, no periodic noise. Fit to R^2.

Degree of Polynomial of Fit R^2	Mean Value at Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57311709 \times 10^{3}$	0.51020112×10^{5}
2	0.65392906×10^{0}	0.71120520×10^{2}
3	$-0.13274929 \times 10^{-1}$	0.90921999×10^{1}
4	$0.12320237 \times 10^{-5}$	$0.77245760 \times 10^{-2}$
5	$-0.78485391 \times 10^{-7}$	$0.65134129 \times 10^{-3}$
6	$-0.65556378 \times 10^{-10}$	$0.39555368 \times 10^{-6}$
7	$-0.33687684 \times 10^{-10}$	$0.24905413 \times 10^{-7}$
8	$-0.65629138 \times 10^{-10}$	$0.13722842 \times 10^{-9}$
9	$-0.32378011 \times 10^{-10}$	$0.26623817 \times 10^{-9}$
10	$-0.56388672 \times 10^{-10}$	$0.41170183 \times 10^{-6}$

Table 8
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 500km, no random noise or bias, no periodic noise. Fit to R^2.

Degree of Polynomial of Fit R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.14017085 \times 10^{5}$	0.11479317×10^{6}
2	0.22011470×10^{1}	0.16554739×10^{3}
3	$-0.28085236 \times 10^{0}$	0.21908235×10^{2}
4	$-0.11024715 \times 10^{-3}$	$0.22829275 \times 10^{-1}$
5	$0.13522350 \times 10^{-5}$	$0.19670138 \times 10^{-2}$
6	$-0.48545189 \times 10^{-9}$	$0.14739109 \times 10^{-5}$
7	$0.31286618 \times 10^{-11}$	$0.94197462 \times 10^{-7}$
8	$-0.23937901 \times 10^{-10}$	$0.14093511 \times 10^{-9}$
9	$-0.36161509 \times 10^{-10}$	$0.45868546 \times 10^{-9}$
10	$-0.14697434 \times 10^{-10}$	$0.83170198 \times 10^{-6}$

Table 9
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1500 km, no random noise or bias, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	0.73014488×10^{2}	0.31550632×10^{5}
2	0.10515184×10^{0}	0.36535050×10^{2}
3	$-0.87464878 \times 10^{-4}$	0.46033781×10^{1}
4	$0.22908323 \times 10^{-6}$	$0.31902773 \times 10^{-2}$
5	$-0.23760367 \times 10^{-8}$	$0.26649971 \times 10^{-3}$
6	$-0.43073669 \times 10^{-10}$	$0.13167802 \times 10^{-6}$
7	$-0.61700121 \times 10^{-10}$	$0.82462316 \times 10^{-8}$
8	$-0.53551048 \times 10^{-10}$	$0.16415799 \times 10^{-9}$
9	$-0.48894435 \times 10^{-10}$	$0.25140319 \times 10^{-9}$
10	$-0.44237822 \times 10^{-10}$	$0.74615831 \times 10^{-6}$

Table 10
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, Gaussian noise with standard deviation of 2 meters added to range, no bias, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57326496 \times 10^{3}$	0.51020087×10^{5}
2	0.50526911×10^{0}	0.71133472×10^{2}
3	$-0.16213166 \times 10^{0}$	0.90966450×10^{1}
4	$-0.14866744 \times 10^{0}$	0.15549463×10^{0}
5	$-0.14870137 \times 10^{0}$	0.27298950×10^{0}
6	$-0.14869614 \times 10^{0}$	0.26911890×10^{0}
7	$-0.14869552 \times 10^{0}$	0.27292880×10^{0}
8	$-0.14869539 \times 10^{0}$	0.27497664×10^{0}
9	$-0.14869537 \times 10^{0}$	0.27571468×10^{0}
10	$-0.14869535 \times 10^{0}$	0.30533768×10^{0}

Table 11
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000km, no random noise, bias of 5 meters, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.56810792×10^3	0.51020242×10^5
2	0.56541091×10^1	0.71140992×10^2
3	0.49867262×10^1	0.90917396×10^1
4	0.50000013×10^1	$0.81804169 \times 10^{-2}$
5	0.49999993×10^1	$0.54635935 \times 10^{-3}$
6	0.50000000×10^1	$0.45810348 \times 10^{-5}$
7	0.50000000×10^1	$0.38591418 \times 10^{-5}$
8	0.50000000×10^1	$0.88206819 \times 10^{-6}$
9	0.50000000×10^1	$0.59604645 \times 10^{-6}$
10	0.50000000×10^1	$0.93676052 \times 10^{-6}$

Table 12
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000km, no random noise, bias of 10 meters, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.56309875×10^3	0.51020171×10^5
2	0.10654289×10^2	0.71160893×10^2
3	0.99867274×10^1	0.90912056×10^1
4	0.10000001×10^2	$0.86382979 \times 10^{-2}$
5	0.99999999×10^1	$0.44143008 \times 10^{-3}$
6	0.10000000×10^2	$0.94133833 \times 10^{-5}$
7	0.10000000×10^2	$0.77458523 \times 10^{-5}$
8	0.10000000×10^2	$0.17711708 \times 10^{-5}$
9	0.10000000×10^2	$0.59604645 \times 10^{-7}$
10	0.10000000×10^2	$0.41723251 \times 10^{-6}$

each of the range values, but no random or periodic noise. The standard deviation of fit for a fourth degree fit to R^2 is 0.008 meters. The mean value of the errors is also shown in Table 12 and is equal to the 10 meters bias added to the range values.

Table 13 is for a satellite in a circular overhead pass of 1000km height. The number of points is 200. In this case Gaussian random noise with a standard deviation of 2 meters plus a bias of 10 meters have been added to the range values. There is no periodic noise. The standard deviation of fit for a fourth degree polynomial fit to R^2 is 0.16 meters. Compare this table with Table 10.

Table 14 is for a satellite in a circular overhead pass of 1000km height. The number of points is 200. Here Gaussian random noise with a standard deviation of 2 meters plus a bias of 100 meters have been added to the range values. There is no periodic noise. The standard deviation of fit for a fourth degree polynomial fit to R^2 is 0.15 meters. Compare this table with Table 13.

Table 15 is for a satellite in a circular overhead pass of 1000km height. The number of points is 200. Gaussian random noise with a standard deviation of 2 meters plus a bias of -100 meters have been added to the range values. There is no periodic noise. The standard deviation of fit for a fourth degree polynomial fit to R^2 is 0.16 meters. Compare this table with Tables 10, and 14.

From the above comparison it can be seen that the squaring operation and taking the square root of values from the calculated polynomial has no significant effect on the standard deviation of fit. This is true because the biases and noise are very small compared to the range values.

Table 16 is for a satellite in a circular overhead pass of 1000km height. The number of points is 200. No random noise or bias has been added. However, periodic noise (i.e., a sinusoidal variation) with one period over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 7.02 meters, or approximately the root mean square level of the sinusoidal variation.

Table 17 is for a satellite in a circular overhead pass of 1000km height. The number of points is 200. No random noise or bias has been added. However, periodic noise with one period over the data arc and an amplitude of 100 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 70.19 meters.

Comparing Tables 16 and 17 it can be seen that if the amplitude of the periodic noise is increased by an order of magnitude, the standard deviation of fit is also increased by an order of magnitude.

Table 13
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, Gaussian random noise with standard deviation of 2 meters, bias of 10 meters, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.56324662×10^3	0.51019945×10^5
2	0.10505629×10^2	0.71173274×10^2
3	0.98378706×10^1	0.90955771×10^1
4	0.98513327×10^1	0.15537387×10^0
5	0.98512987×10^1	0.27295453×10^0
6	0.98513039×10^1	0.26911919×10^0
7	0.98513045×10^1	0.27292849×10^0
8	0.98513046×10^1	0.27497671×10^0
9	0.98513046×10^1	0.27571469×10^0
10	0.98513047×10^1	0.30533773×10^0

Table 14
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, Gaussian random noise with standard deviation of 2 meters, bias of 100 meters, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.47308153×10^3	0.51018667×10^5
2	0.10050887×10^3	0.71531482×10^2
3	0.99837891×10^2	0.90859705×10^1
4	0.99851334×10^2	0.15454660×10^0
5	0.99851299×10^2	0.27264695×10^0
6	0.99851304×10^2	0.26912184×10^0
7	0.99851304×10^2	0.27292569×10^0
8	0.99851305×10^2	0.27497738×10^0
9	0.99851305×10^2	0.27571477×10^0
10	0.99851305×10^2	0.30533770×10^0

Table 15
Range Analysis

Circular orbit, overhead pass, $n = 200$ points, $h_s = 1000$ km, Gaussian random noise with standard deviation of 2 meters, bias of -100 meters, no periodic noise. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.67344845×10^3	0.51021508×10^5
2	-0.99498332×10^2	0.70735443×10^2
3	-0.10016215×10^3	0.91073307×10^1
4	-0.10014867×10^3	0.15701463×10^0
5	-0.10014870×10^3	0.27334791×10^0
6	-0.10014870×10^3	0.26911599×10^0
7	-0.10014870×10^3	0.27293193×10^0
8	-0.10014870×10^3	0.27497589×10^0
9	-0.10014870×10^3	0.27571460×10^0
10	-0.10014870×10^3	0.30533752×10^0

Table 16
Range Analysis

Circular orbit, overhead pass, $n = 200$ points, $h_s = 1000$ km, no random noise or bias, periodic noise - one period over the data arc with amplitude of 10 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.57306064×10^3	0.51018713×10^5
2	0.61348907×10^0	0.71388901×10^2
3	$-0.12593838 \times 10^{-1}$	0.10576680×10^2
4	$-0.84866522 \times 10^{-4}$	0.70193983×10^1
5	$0.12394707 \times 10^{-4}$	0.70555806×10^1
6	$0.57059493 \times 10^{-6}$	0.70532513×10^1
7	$0.22727618 \times 10^{-7}$	0.70533856×10^1
8	$0.97767042 \times 10^{-9}$	0.70533680×10^1
9	$-0.25480404 \times 10^{-9}$	0.70533680×10^1
10	$-0.15388650 \times 10^{-9}$	0.70533680×10^1

Table 17
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000km, no random noise or bias, periodic noise - one period over the data arc with amplitude of 100 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57255206 \times 10^{3}$	0.51004346×10^{5}
2	0.25027554×10^{0}	0.90946409×10^{2}
3	$-0.65065861 \times 10^{-2}$	0.69946409×10^{2}
4	$-0.84345918 \times 10^{-3}$	0.70192585×10^{2}
5	$0.12241143 \times 10^{-3}$	0.70555939×10^{2}
6	$0.57291405 \times 10^{-5}$	0.70532511×10^{2}
7	$0.21139887 \times 10^{-6}$	0.70533855×10^{2}
8	$0.10894728 \times 10^{-7}$	0.70533680×10^{2}
9	$-0.12016972 \times 10^{-8}$	0.70533680×10^{2}
10	$-0.15126716 \times 10^{-9}$	0.70533680×10^{2}

Table 18 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. However, periodic noise with only one half period over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 3.10 meters, but a bias (mean value) of 6.33 meters appears in the calculation because the periodic noise is only one half of a period. In the symmetrical cases, (Tables 16 and 17), the mean value is very small. In Table 18 the comparable fit to range alone (no squaring) is also shown. It can be seen that the squaring process has not changed the calculated bias that results from the periodic noise.

Table 19 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. However, periodic noise with one half period over the data arc and an amplitude of 100 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 31.04 meters, and a mean value of 63.34 meters. This table should be compared with Table 18.

Table 20 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. However, periodic noise with two periods over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 3.73 meters.

Table 21 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. Periodic noise with 4 periods over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 2.30 meters.

Table 22 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. Periodic noise with 10 periods over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 0.98 meters.

Table 23 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. Periodic noise with 20 periods over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 0.48 meters.

Table 24 is for a satellite in a circular overhead pass of 1000 km height. The number of points is 200. No random noise or bias has been added. Periodic

Table 18
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000km, no random noise or bias, periodic noise - one half period over the data arc with amplitude of 10 meters.

Fit to R^2		
Degree of Polynomial Fit	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.56679742×10^3	0.51019905×10^5
2	0.69920845×10^1	0.72002611×10^2
3	0.63211163×10^1	0.97093607×10^1
4	0.63342420×10^1	0.31036987×10^1
5	0.63342351×10^1	0.31028176×10^1
6	0.63342352×10^1	0.31028168×10^1
7	0.63342352×10^1	0.31028156×10^1
8	0.63342352×10^1	0.31028156×10^1
9	0.63342352×10^1	0.31028156×10^1
10	0.63342352×10^1	0.31028156×10^1

Fit to R		
Degree of Polynomial Fit	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	0.63342352×10^1	0.36713658×10^5
2	0.63342352×10^1	0.48267502×10^4
3	0.63342352×10^1	0.17823768×10^3
4	0.63342352×10^1	0.10757746×10^3
5	0.63342352×10^1	0.24974704×10^2
6	0.63342352×10^1	0.33508881×10^1
7	0.63342352×10^1	0.32245014×10^1
8	0.63342352×10^1	0.31105887×10^1
9	0.63342352×10^1	0.31028384×10^1
10	0.63342352×10^1	0.31028298×10^1

Table 19
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, no random noise or bias, periodic noise - 1/2 period over the data arc with amplitude of 100 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.50992007×10^3	0.51018042×10^5
2	0.64035451×10^2	0.85021258×10^2
3	0.63330638×10^2	0.32596197×10^2
4	0.63342409×10^2	0.31035110×10^2
5	0.63342351×10^2	0.31028170×10^2
6	0.63342352×10^2	0.31028168×10^2
7	0.63342352×10^2	0.31028156×10^2
8	0.63342352×10^2	0.31028156×10^2
9	0.63342352×10^2	0.31028156×10^2
10	0.63342352×10^2	0.31028156×10^2

Table 20
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, no random noise or bias, periodic noise - 2 periods over the data arc with amplitude of 10 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	-0.57304880×10^3	0.51019275×10^5
2	0.67885475×10^0	0.71363720×10^2
3	$-0.11782631 \times 10^{-1}$	0.10294925×10^2
4	$0.87714913 \times 10^{-3}$	0.37264259×10^1
5	$-0.94999505 \times 10^{-4}$	0.67701815×10^1
6	$-0.38696852 \times 10^{-4}$	0.66901152×10^1
7	$-0.13922318 \times 10^{-5}$	0.70589196×10^1
8	$-0.54686156 \times 10^{-6}$	0.70419848×10^1
9	$0.54413977 \times 10^{-8}$	0.70539728×10^1
10	$-0.10403892 \times 10^{-8}$	0.70532474×10^1

Table 21
Range Analysis

Circular orbit, overhead pass, $n = 200$ points, $h_s = 1000$ km, no random noise or bias, periodic noise - 4 periods over the data arc with amplitude of 10 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57307828 \times 10^{3}$	0.51019689×10^{5}
2	0.67153820×10^{0}	0.71245103×10^{2}
3	$-0.13908271 \times 10^{-1}$	0.98094827×10^{1}
4	$-0.10454206 \times 10^{-3}$	0.22957449×10^{1}
5	$0.37628817 \times 10^{-4}$	0.24174527×10^{1}
6	$0.61536298 \times 10^{-4}$	0.24396834×10^{1}
7	$0.11472527 \times 10^{-4}$	0.34990706×10^{1}
8	$0.97367085 \times 10^{-5}$	0.34483242×10^{1}
9	$0.11239810 \times 10^{-4}$	0.47282819×10^{1}
10	$0.11098580 \times 10^{-4}$	0.47616309×10^{1}

Table 22
Range Analysis

Circular orbit, overhead pass, $n = 200$ points, $h_s = 1000$ km, no random noise or bias, periodic noise - 10 periods over the data arc with amplitude of 10 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57310116 \times 10^{3}$	0.51019944×10^{5}
2	0.66149021×10^{0}	0.71168460×10^{2}
3	$-0.13717537 \times 10^{-1}$	0.93396096×10^{1}
4	$-0.12423038 \times 10^{-3}$	0.97543444×10^{0}
5	$0.30591526 \times 10^{-4}$	0.13130648×10^{1}
6	$0.71848698 \times 10^{-5}$	0.12966610×10^{1}
7	$0.19724603 \times 10^{-4}$	0.14410911×10^{1}
8	$0.19955211 \times 10^{-4}$	0.14350532×10^{1}
9	$0.19999883 \times 10^{-4}$	0.14357364×10^{1}
10	$0.19961966 \times 10^{-4}$	0.14401765×10^{1}

Table 23
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, no random noise or bias, periodic noise - 20 periods over the data arc with amplitude of 10 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57310928 \times 10^{3}$	0.51020030×10^{5}
2	0.65766279×10^{0}	0.71143456×10^{2}
3	$-0.13494873 \times 10^{-1}$	0.92017682×10^{1}
4	$-0.56432316 \times 10^{-4}$	0.47985581×10^{0}
5	$0.25893803 \times 10^{-4}$	0.67310787×10^{0}
6	$0.11611734 \times 10^{-4}$	0.66390283×10^{0}
7	$0.20885959 \times 10^{-4}$	0.81419273×10^{0}
8	$0.21186552 \times 10^{-4}$	0.80656632×10^{0}
9	$0.20405610 \times 10^{-4}$	0.89263636×10^{0}
10	$0.20430135 \times 10^{-4}$	0.88827017×10^{0}

Table 24
Range Analysis

Circular orbit, overhead pass, n = 200 points, h_s = 1000 km, no random noise or bias, periodic noise - 100 periods over the data arc with amplitude of 10 meters. Fit to R^2.

Degree of Polynomial Fit to R^2	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters
1	$-0.57311709 \times 10^{3}$	0.51020113×10^{5}
2	0.65394037×10^{0}	0.71120465×10^{2}
3	$-0.13253378 \times 10^{-1}$	0.90919487×10^{1}
4	$0.22352436 \times 10^{-4}$	$0.78996626 \times 10^{-2}$
5	$0.20819964 \times 10^{-4}$	$0.19801803 \times 10^{-2}$
6	$0.20937789 \times 10^{-4}$	$0.17386356 \times 10^{-2}$
7	$0.20910955 \times 10^{-4}$	$0.22044689 \times 10^{-2}$
8	$0.20910024 \times 10^{-4}$	$0.21811915 \times 10^{-2}$
9	$0.20912562 \times 10^{-4}$	$0.25762194 \times 10^{-2}$
10	$0.20912495 \times 10^{-4}$	$0.25558498 \times 10^{-2}$

noise with 100 periods over the data arc and an amplitude of 10 meters has been added to the range values. For a fourth degree polynomial fit to R^2 the standard deviation of fit is 0.008 meters.

Summing up the above noise analysis, it is seen that purely Gaussian random noise added to the range values is "smoothed out" in the data processing, whereas biases are not. In the case of periodic noise, for a fixed data rate the higher the amplitude (for a fixed number of periods in the data arc), the higher the standard deviation of fit. Also, for a fixed data rate the greater the number of whole periods in the data arc (for a fixed noise amplitude), the less the standard deviation of fit and, in this latter case the statistics of the periodic noise approaches that of purely random noise. (Tables 20 through 24.)

S-band system resolution per sample is on the order of 1 to 2 meters in range. For a typical circular orbit of 1000 km where the satellite is passing directly overhead and the data arc is 200 seconds at a 1/second data rate, S-band system resolution is obtained with a 4th degree polynomial fit to range squared; whereas the same resolution is obtained with a 9th degree polynomial fit to the range (see Table 2). For a satellite in a circular orbit of 500 km passing directly overhead the tracking station and for a data arc of 200 seconds at a 1/second data rate, S-band resolution is obtained with a fourth degree fit to range squared (see Table 8).

6.0 RESULTS FROM GENERATING RANGE RATE DATA FOR A SATELLITE IN CIRCULAR EARTH ORBIT AND FOR A SATELLITE IN AN ELLIPTICAL EARTH ORBIT

In all cases to be described it is assumed that the satellite and the Earth tracking station are in the same plane (equatorial) and both are rotating counterclockwise. The circular orbit is illustrated in Figure 1 and the eccentric orbit in Figure 2. In the case of the circular orbit the satellite is rotating with an angular velocity of ω_s and the Earth is rotating with an angular velocity of ω_e. The data are taken at 1/second. It should be noted that the per sample resolution of range rate measurements is on the order of 0.005 meters/second for a typical S-band system.

Table 25 is a comparison between fitting to $\dot{r}R$ where R is calculated from a 4th degree fit to R^2, and fitting to $\dot{r}$ for a circular orbit of 1000 kilometers, an overhead pass with the number of points equal to 44. No random noise, bias, or periodic noise has been included. S-band system resolution is achieved with a third degree polynomial fit to $\dot{r}R$ (0.5×10^{-6} meters/second) whereas to achieve the same resolution or better it takes an eighth degree polynomial fit to $\dot{r}$.

Table 26 is a comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for a circular orbit of 1000 kilometers, an overhead pass with the number of points equal to

Table 25
Range Rate Analysis

Comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for a circular orbit of 1000 kilometers, overhead pass, number of points = 44, no random noise, no bias, and no periodic noise. Fourth order fit to R^2.

Degree of Polynomial Fit	$\sigma_{\dot{r}R}$(meters/sec)	$\sigma_{\dot{r}}$ (meters/sec)
1	$0.55251670 \times 10^{-1}$	0.74415775×10^{1}
2	$0.90169777 \times 10^{-2}$	0.11006757×10^{1}
3	$0.47860247 \times 10^{-6}$	$0.42157274 \times 10^{-1}$
4	$0.45710565 \times 10^{-7}$	$0.31784487 \times 10^{-2}$
5	$0.50297534 \times 10^{-9}$	$0.21341051 \times 10^{-3}$
6	$0.22455044 \times 10^{-10}$	$0.85684969 \times 10^{-5}$
7	$0.27358737 \times 10^{-11}$	$0.99355434 \times 10^{-6}$
8	$0.11666926 \times 10^{-12}$	$0.16837249 \times 10^{-7}$
9	$0.22492701 \times 10^{-11}$	$0.42796710 \times 10^{-8}$
10	$0.15643117 \times 10^{-9}$	$0.13646136 \times 10^{-9}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 26
Range Rate Analysis

Comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for a circular orbit of 1000 kilometers, overhead pass, number of points = 200. No random noise, no bias, and no periodic noise. Fourth order fit to R^2.

Degree of Polynomial Fit	$\sigma_{\dot{r}R}$ (meters/sec)	$\sigma_{\dot{r}}$ (meters/sec)
1	0.41853490×10^{1}	0.27792616×10^{3}
2	0.72166192×10^{0}	0.14122315×10^{2}
3	$0.76412732 \times 10^{-3}$	0.10873037×10^{2}
4	$0.62547192 \times 10^{-4}$	0.28487932×10^{1}
5	$0.80294421 \times 10^{-5}$	0.17028630×10^{0}
6	$0.21490185 \times 10^{-5}$	0.14174102×10^{0}
7	$0.14087773 \times 10^{-6}$	$0.37740126 \times 10^{-1}$
8	$0.14233777 \times 10^{-8}$	$0.22903342 \times 10^{-2}$
9	$0.36281798 \times 10^{-7}$	$0.20753311 \times 10^{-2}$
10	$0.36957427 \times 10^{-8}$	$0.54031244 \times 10^{-3}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

200. No random noise, bias, or periodic noise has been included. S-band system resolution is achieved again with a third degree polynomial fit to $\dot{r}R$ whereas to achieve the same resolution or better it takes a tenth order polynomial fit to $\dot{r}$.

Table 27 is a comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis of 95000 kilometers, eccentricity of 0.89, number of points equal to 44 with a fourth degree polynomial fit to R^2. S-band system resolution is achieved with a second degree polynomial fit to $\dot{r}R$ (0.0024 meters/second) whereas the same or better resolution is achieved with a third degree polynomial fit to $\dot{r}$. No random noise, bias, or periodic noise are present.

Table 28 is a comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis of 95000 kilometers, eccentricity of 0.89, number of points equal to 200 with a fourth degree polynomial fit to R^2. Here S-band system resolution is achieved with a third degree fit to $\dot{r}R$ (0.0013 meters/second) whereas the same or better resolution is achieved with a fifth degree fit to $\dot{r}$ (0.0011 meters/second). No random noise, bias, or periodic noise are present.

Table 29 is for a satellite in an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis of 95000 kilometers, eccentricity of 0.89, number of points equal to 44 with a third degree fit to R^2. S-band system resolution is achieved with a second degree polynomial fit to $\dot{r}R$ (0.0024 meters/second). No random noise, bias, or periodic noise are present. Compare this with Table 27 (4th degree fit to R^2). The mean value of the differences is shown to illustrate that a bias is not introduced by the method of fitting.

Table 30 is for a satellite in an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis of 95000 kilometers, eccentricity of 0.89, number of points equal to 200 with a third degree fit to R^2. No random noise, bias, or periodic noise is present. The standard deviation of fit for a third degree polynomial fit to $\dot{r}R$ is 0.0021 meters/second, within the resolution of a typical S-band system. Compare this with Table 28 (4th degree fit to R^2).

Table 31 is for a satellite in an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis of 95000 kilometers, eccentricity of 0.89, number of points equal to 200. Guassian random noise with a standard deviation of 0.005 meters/second has been added to the range rate values. No bias or periodic noise has been added and a fourth degree polynomial has been fit to R^2. For a third degree fit to $\dot{r}R$ the standard deviation of fit is

Table 27
Range Rate Analysis

Comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000km, eccentricity = 0.89, number of points = 44. Fourth order fit to R^2.

Degree of Polynomial Fit	$\sigma_{\dot{r}R}$ (meters/sec)	$\sigma_{\dot{r}}$ (meters/sec)
1	$0.14843337 \times 10^{-1}$	0.19416260×10^{0}
2	$0.24136823 \times 10^{-2}$	$0.31365064 \times 10^{-1}$
3	$0.72864384 \times 10^{-6}$	$0.88834831 \times 10^{-4}$
4	$0.70570925 \times 10^{-7}$	$0.84818951 \times 10^{-5}$
5	$0.95270103 \times 10^{-10}$	$0.37503272 \times 10^{-7}$
6	$0.64165907 \times 10^{-11}$	$0.25111475 \times 10^{-8}$
7	$0.21837252 \times 10^{-12}$	$0.22702691 \times 10^{-10}$
8	$0.20768250 \times 10^{-12}$	$0.83556286 \times 10^{-11}$
9	$0.37381279 \times 10^{-12}$	$0.70736022 \times 10^{-11}$
10	$0.33854018 \times 10^{-10}$	$0.43032299 \times 10^{-10}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 28
Range Rate Analysis

Comparison between fitting to $\dot{r}R$ and fitting to $\dot{r}$ for an eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000km, eccentricity = 0.89, number of points = 200. Fourth order fit to R^2.

Degree of Polynomial Fit	$\sigma_{\dot{r}R}$ (meters/sec)	$\sigma_{\dot{r}}$ (meters/sec)
1	0.13235152×10^{1}	0.16209897×10^{2}
2	0.21728503×10^{0}	0.23296691×10^{1}
3	$0.13335383 \times 10^{-2}$	0.13825187×10^{0}
4	$0.12431866 \times 10^{-3}$	$0.91762163 \times 10^{-2}$
5	$0.34060858 \times 10^{-5}$	$0.10524417 \times 10^{-2}$
6	$0.14739791 \times 10^{-6}$	$0.27904480 \times 10^{-4}$
7	$0.20301123 \times 10^{-7}$	$0.73389407 \times 10^{-5}$
8	$0.14609493 \times 10^{-9}$	$0.55924325 \times 10^{-7}$
9	$0.14172266 \times 10^{-9}$	$0.46887330 \times 10^{-8}$
10	$0.19497726 \times 10^{-8}$	$0.21698210 \times 10^{-8}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 29
Range Rate Analysis

Eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000 km, eccentricity = 0.89, number of points = 44. No random noise, bias or periodic noise. Third order fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	$-0.31415660 \times 10^{-5}$	$0.14843303 \times 10^{-1}$
2	$0.11973341 \times 10^{-8}$	$0.24134640 \times 10^{-2}$
3	$-0.94828186 \times 10^{-13}$	$0.12075541 \times 10^{-5}$
4	$-0.40391932 \times 10^{-13}$	$0.28866104 \times 10^{-6}$
5	$-0.31939098 \times 10^{-13}$	$0.60276757 \times 10^{-9}$
6	$-0.27361951 \times 10^{-13}$	$0.86537384 \times 10^{-10}$
7	$-0.21573652 \times 10^{-13}$	$0.36686199 \times 10^{-12}$
8	$-0.25873243 \times 10^{-13}$	$0.20022048 \times 10^{-12}$
9	$-0.24621719 \times 10^{-13}$	$0.40106217 \times 10^{-12}$
10	$-0.48557118 \times 10^{-13}$	$0.46081116 \times 10^{-10}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 30
Range Rate Analysis

Eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000 km, eccentricity = 0.89, number of points = 200. No random noise, bias or periodic noise. Third order fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	$-0.50857727 \times 10^{-2}$	0.13234910×10^{1}
2	$0.39501185 \times 10^{-4}$	0.21694633×10^{0}
3	$-0.34286505 \times 10^{-7}$	$0.21404468 \times 10^{-2}$
4	$0.74316297 \times 10^{-9}$	$0.49350656 \times 10^{-3}$
5	$-0.13638624 \times 10^{-11}$	$0.20279421 \times 10^{-4}$
6	$-0.14587997 \times 10^{-12}$	$0.24368930 \times 10^{-5}$
7	$-0.22972513 \times 10^{-12}$	$0.21041812 \times 10^{-6}$
8	$-0.19679591 \times 10^{-12}$	$0.11313188 \times 10^{-7}$
9	$-0.16329160 \times 10^{-12}$	$0.18525224 \times 10^{-8}$
10	$-0.24185765 \times 10^{-12}$	$0.21254859 \times 10^{-8}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 31
Range Rate Analysis

Eccentric orbit simulating the orbit of IMP-G at the perigee portion of the orbit, semi-major axis = 95000km, eccentricity = 0.89, number of points = 200. Gaussian noise with standard deviation of 0.005 meters/second added to range rate measurements. No bias or periodic noise. Fourth degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Second
1	$-0.52521228 \times 10^{-2}$	0.13235272×10^{1}
2	$-0.12710253 \times 10^{-3}$	0.21728637×10^{0}
3	$-0.16666557 \times 10^{-3}$	$0.14042741 \times 10^{-2}$
4	$-0.16663907 \times 10^{-3}$	$0.48046468 \times 10^{-3}$
5	$-0.16663930 \times 10^{-3}$	$0.46270123 \times 10^{-3}$
6	$-0.16663933 \times 10^{-3}$	$0.60050733 \times 10^{-3}$
7	$-0.16663933 \times 10^{-3}$	$0.74288198 \times 10^{-3}$
8	$-0.16663933 \times 10^{-3}$	$0.76385666 \times 10^{-3}$
9	$-0.16663933 \times 10^{-3}$	$0.78017102 \times 10^{-3}$
10	$-0.16663933 \times 10^{-3}$	$0.79971799 \times 10^{-3}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

0.0014 meters/second. Compare Table 31 with Table 28 to see the effects of the Gaussian random noise. In this case the $\dot{r}$ noise is small compared to the value of $\dot{r}$.

Table 32 is for a satellite in a circular orbit of 500 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree fit to R^2. No random noise, bias, or periodic noise has been added. S-band system resolution is achieved with a third degree polynomial fit to $\dot{r}R$ where the standard deviation of fit is 0.0023 meters/second.

Table 33 is for a satellite in a circular orbit of 1500 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree polynomial fit to R^2. No random noise, bias, or periodic noise has been added. The standard deviation of fit for a third degree polynomial fit is 0.32×10^{-3} meters/second.

Table 34 is for a satellite in a circular orbit of 1000 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree polynomial fit to R^2. No random noise or bias has been added. Periodic noise has been added with a range amplitude of 2 meters and a range rate amplitude of 1 meter/second. There are two periods over the data arc. The standard deviation of fit for a 3rd degree polynomial fit to $\dot{r}R$ is 0.36 meters/second which is caused by the $\dot{r}$ cyclic noise.

Table 32
Range Rate Analysis

Circular orbit of 500 km, overhead pass, number of points = 200. No random noise or bias, no periodic noise. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	0.47953194×10^{0}	0.96246538×10^{1}
2	$0.24847058 \times 10^{-1}$	0.17378154×10^{1}
3	$0.29492329 \times 10^{-4}$	$0.22904419 \times 10^{-2}$
4	$-0.59403423 \times 10^{-6}$	$0.14613763 \times 10^{-3}$
5	$-0.52965080 \times 10^{-8}$	$0.46734438 \times 10^{-4}$
6	$0.10620666 \times 10^{-7}$	$0.32032858 \times 10^{-4}$
7	$0.29504444 \times 10^{-8}$	$0.11446048 \times 10^{-4}$
8	$0.17628773 \times 10^{-9}$	$0.19062020 \times 10^{-5}$
9	$-0.32519232 \times 10^{-11}$	$0.13304742 \times 10^{-5}$
10	$0.57038285 \times 10^{-11}$	$0.89235640 \times 10^{-6}$

Note: $\dot{r}$ refers to range rate measurement and R refers to the range measurement.

Table 33
Range Rate Analysis

Circular orbit of 1500 km, overhead pass. Number of Points = 200. No random noise or bias, no periodic noise. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	$-0.25329401 \times 10^{-1}$	0.21625184×10^{1}
2	$0.10486964 \times 10^{-2}$	0.36537740×10^{0}
3	$0.76490996 \times 10^{-8}$	$0.31597987 \times 10^{-3}$
4	$0.20410022 \times 10^{-8}$	$0.28381520 \times 10^{-4}$
5	$0.15667219 \times 10^{-10}$	$0.18873398 \times 10^{-5}$
6	$-0.57786664 \times 10^{-12}$	$0.18084428 \times 10^{-6}$
7	$-0.41014303 \times 10^{-12}$	$0.39209944 \times 10^{-7}$
8	$-0.42284398 \times 10^{-12}$	$0.10342901 \times 10^{-7}$
9	$-0.27872815 \times 10^{-12}$	$0.32922406 \times 10^{-9}$
10	$-0.49690030 \times 10^{-12}$	$0.38911794 \times 10^{-8}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 35 is for a satellite in a circular orbit of 1000 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree polynomial fit to R^2. No random noise or bias has been added. However, periodic noise has been added to the data with a range amplitude of 2 meters and a range rate amplitude of 1 meter/second. There is one period over the data arc. The standard deviation of fit for a 3rd degree polynomial fit to $\dot{r}R$ is 0.71 meters/second.

Table 36 is for a satellite in a circular orbit of 1000 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree polynomial fit to R^2. No bias or periodic noise has been added. However, Gaussian random noise has been added with a range standard deviation of 2 meters and a range rate standard deviation of 0.005 meters/second. For a 3rd degree polynomial fit to $\dot{r}R$ the standard deviation of fit is 0.71×10^{-3} meters/second.

Table 37 is for a satellite in a circular orbit of 1000 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree polynomial fit to R^2. Gaussian random noise has been added with a range standard deviation of 2 meters and a range rate standard deviation of 0.005 meters/second. A bias of 5 meters has been added to the range measurements and a bias of 5 meters/second added to the range rate measurements. Periodic noise has also been added to the data with a range amplitude of 2 meters and a range rate amplitude of 1 meter/second. There are two periods over the data arc. For a 3rd degree polynomial fit to $\dot{r}R$ the standard deviation of fit is 0.36 meters/second. Compare this with Table 34, the noise free case, and note that the mean value converges to the bias of 5 meters/second.

Table 38 is for a satellite in a circular orbit of 1000 kilometers, an overhead pass, the number of points equal to 200, and a fourth degree polynomial fit to R^2. Gaussian random noise with a range standard deviation of 2 meters and a range rate standard deviation of 0.005 meters/second has been added to the data. No biases have been added. However, periodic noise with a range amplitude of 2 meters and a range rate amplitude of 1 meter/second has been added. There are two periods over the data arc. For a 3rd degree polynomial fit to $\dot{r}R$ the standard deviation of fit is 0.36 meters/second. Compare with Tables 34 and 37.

Summing up the previous range rate analysis it is seen that for satellites directly overhead the tracking station in circular orbits of 500 kilometers and greater and for satellites that are in elliptical orbits at the perigee portion of the orbit (where the perigee height is only a few thousand kilometers), in the noise free case where the data arcs are 200 seconds or less (at a sample rate of one/second) typical S-band resolution (on the order of 0.005 meters/second) can be achieved by fitting a 3rd degree least squares polynomial to $\dot{r}R$ (and a 4th degree least squares polynomial to R^2) (see Tables 25, 26, 27, 28, 32, and 33).

Table 34
Range Rate Analysis

Circular orbit of 1000 km, overhead pass. Number of points = 200. No random noise or bias. Periodic noise-range amplitude of 2 meters, range rate amplitude of 1 meter/second: two periods over the data arc. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	$-0.40882026 \times 10^{-1}$	0.42674498×10^{1}
2	$0.91430595 \times 10^{-2}$	0.79177834×10^{0}
3	$0.14924478 \times 10^{-3}$	0.36361170×10^{0}
4	$0.86128632 \times 10^{-4}$	0.37262966×10^{0}
5	$-0.96187081 \times 10^{-5}$	0.67699432×10^{0}
6	$-0.40732068 \times 10^{-5}$	0.66901158×10^{0}
7	$-0.12848551 \times 10^{-6}$	0.70589165×10^{0}
8	$-0.60574703 \times 10^{-7}$	0.70419849×10^{0}
9	$0.12498209 \times 10^{-8}$	0.70539724×10^{0}
10	$-0.13428403 \times 10^{-9}$	0.70532474×10^{0}

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 35
Range Rate Analysis

Circular orbit of 1000 km, overhead pass, number of points = 200. No random noise or bias. Periodic noise - range amplitude of 2 meters, range rate amplitude of 1 meter/second: one period over the data arc. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	$-0.46776306 \times 10^{-1}$	0.43560247×10^{1}
2	$0.25762243 \times 10^{-2}$	0.90776251×10^{0}
3	$0.69169003 \times 10^{-4}$	0.70877599×10^{0}
4	$-0.86556339 \times 10^{-5}$	0.70192154×10^{0}
5	$0.12504086 \times 10^{-5}$	0.70556025×10^{0}
6	$0.57350466 \times 10^{-7}$	0.70532509×10^{0}
7	$0.22965466 \times 10^{-8}$	0.70533855×10^{0}
8	$0.11151414 \times 10^{-9}$	0.70533680×10^{0}
9	$-0.10936816 \times 10^{-10}$	0.70533680×10^{0}
10	$-0.65533357 \times 10^{-12}$	0.70533680×10^{0}

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 36
Range Rate Analysis

Circular orbit of 1000 km, overhead pass, number of points = 200. Gaussian random noise-range standard deviation of 2 meters and range rate standard deviation of 0.005 meters/second. No bias or periodic noise. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Second
1	$-0.42710922 \times 10^{-1}$	$0.41855463 \times 10^{-1}$
2	$0.64746148 \times 10^{-2}$	0.72173969×10^{0}
3	$-0.16561654 \times 10^{-3}$	$0.70583749 \times 10^{-3}$
4	$-0.16668431 \times 10^{-3}$	$0.46814769 \times 10^{-3}$
5	$-0.16669457 \times 10^{-3}$	$0.46984210 \times 10^{-3}$
6	$-0.16664764 \times 10^{-3}$	$0.57422594 \times 10^{-3}$
7	$-0.16663969 \times 10^{-3}$	$0.76158320 \times 10^{-3}$
8	$-0.16663899 \times 10^{-3}$	$0.77240557 \times 10^{-3}$
9	$-0.16663938 \times 10^{-3}$	$0.78505025 \times 10^{-3}$
10	$-0.16663933 \times 10^{-3}$	$0.80063179 \times 10^{-3}$

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 37
Range Rate Analysis

Circular orbit of 1000 km, overhead pass, number of points = 200. Gaussian random noise: range standard deviation of 2 meters; range rate standard deviation of 0.005 meters/second. Range bias of 5 meters: range rate bias of 5 meters/second. Periodic noise: range amplitude of 2 meters; range rate amplitude of 1 meter/second. Two periods over the data arc. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Second
1	0.49608727×10^{1}	0.41174103×10^{1}
2	0.50091630×10^{1}	0.81110932×10^{0}
3	0.49999835×10^{1}	0.36369805×10^{0}
4	0.49999195×10^{1}	0.37283249×10^{0}
5	0.49998237×10^{1}	0.67711807×10^{0}
6	0.49998293×10^{1}	0.66909882×10^{0}
7	0.49998332×10^{1}	0.70613452×10^{0}
8	0.49998333×10^{1}	0.70442591×10^{0}
9	0.49998334×10^{1}	0.70561445×10^{0}
10	0.49998334×10^{1}	0.70554436×10^{0}

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

Table 38
Range Rate Analysis

Circular orbit of 1000 km, overhead pass, number of points = 200. Gaussian random noise: range amplitude of 2 meters; range rate amplitude of 0.005 meters/second. No biases. Periodic noise: range amplitude of 2 meters; range rate amplitude of 1 meter/second. Two periods over the data arc. 4th degree fit to R^2.

Degree of Polynomial Fit to $\dot{r}R$	Mean Value of Differences Between Calculated Values and True Values	Standard Deviation of Fit in Meters/Sec
1	$-0.41050371 \times 10^{-1}$	0.42676703×10^{1}
2	$0.89757266 \times 10^{-2}$	0.79197794×10^{0}
3	$-0.17495436 \times 10^{-4}$	0.36379782×10^{0}
4	$-0.80544685 \times 10^{-4}$	0.37282381×10^{0}
5	$-0.17631361 \times 10^{-3}$	0.67713959×10^{0}
6	$-0.17072026 \times 10^{-3}$	0.66909879×10^{0}
7	$-0.16676820 \times 10^{-3}$	0.70613482×10^{0}
8	$-0.16669956 \times 10^{-3}$	0.70442590×10^{0}
9	$-0.16663813 \times 10^{-3}$	0.70561445×10^{0}
10	$-0.16663946 \times 10^{-3}$	0.70554436×10^{0}

Note: $\dot{r}$ refers to the range rate measurement and R refers to the range measurement.

In the case of a satellite in a 1000 kilometer circular orbit at the overhead portion of the orbit for a 200 second data arc where the data rate is 1/second and there is no random noise, bias, or periodic noise on the range rate data, the standard deviation of fit (between the approximating function for range rate and the true range rate function) is 0.76×10^{-3} meters/second for a third degree least squares polynomial fit to the range times the range rate. For a third degree polynomial fit to the range rate the standard deviation of fit (between the fitted range rate polynomial and the true range rate function) is 10.9 meters/second. In order to achieve a standard deviation of 0.76×10^{-3} meters/second for a least squares polynomial fit to range rate directly, a 10th degree polynomial is required (see Table 26). The advantage of a lower degree polynomial fit to the range rate times the range occurs because the computation time for fitting a 3rd degree polynomial to this mixed data type is much less than the computation time required to fit a 10th degree polynomial to range rate directly. This will be shown in Section 8.

When purely random Gaussian noise with a standard deviation of 0.005 meters/second is added to the true range rate, the noise is "smoothed out" in the fitting process (compare Table 28 with 31 and Table 26 with 36).

In the case of periodic noise, the greater the number of whole periods in the data arc (for the same noise amplitude), the smaller the standard deviation of fit (compare Tables 34 and 35).

The effect of adding a bias to the true range rate values is not removed in the smoothing process, but shows up as the mean value of the differences between the approximating function for range rate and the true function of range rate (see Table 37).

7.0 COMPARISON OF RANGE RATE POLYNOMIAL ERROR FOR VARIOUS METHODS OF SMOOTHING — 200 SECOND DATA ARC, CIRCULAR ORBIT OF 1000 KILOMETER HEIGHT

In each of the preceeding tables the effects of polynomial degree on the polynomial smoothing error for one specific case were shown. In this section a specific orbit is assumed: 200 second length of data with 200 data points for a 1000 kilometer height, circular orbit, passing directly over the station at time equal to zero seconds; and comparisons are made for different methods of smoothing. The data has been simulated for a 2 dimensional situation as before.

In Table 39, no data noise has been assumed so that the data are "perfect". In column 1, a polynomial is fitted to data points consisting of the true range (R_T) and the true range rate ($\dot{R}_T$) multiplied together ($R_T \dot{R}_T$). The values of the calculated range rate (R_{CALC}) are obtained by dividing the polynomial value at $t = 1, 2, 3 \ldots$, by the true range value. The error is formed by subtracting the true range rate from the calculated range rate (error = $\dot{R}_{CALC} - \dot{R}_T$). Biases and standard deviations were computed and summed in a root-sum-square manner. The standard deviation of these calculated errors are shown in column 1. Note that the computer truncation error introduces a minimum error point at an 8th degree polynomial. For higher degree polynomials, an infinite digit computer would show smaller error. Here the error increased for a 9th degree polynomial. The specific computer used here works with 64 bits or approximately 16 ~~significant digits.~~

Table 39
No Noise on Data

Range rate polynomial smoothing error for various methods of fitting a 200 second arc, 1000 kilometer height, circular orbit, no noise on data.

Polynomial Degree	$\overline{R_T \dot{R}_T}$	6th $R_T^2 \Rightarrow R_{CALC}$ / $R_{CALC}\ \dot{R}_T$	4th $R_T^2 \Rightarrow R_{CALC}$ / $R_{CALC}\ \dot{R}_T$	2nd $R_T^2 \Rightarrow R_T$ / $R_{CALC}\ \dot{R}_T$	$\dot{R}_T$
	M/Sec	M/Sec	M/Sec	M/Sec	M/Sec
1	$0.42(10^{1})$	$0.42(10^{1})$	$0.42(10^{1})$	$0.42(10^{1})$	$0.28(10^{3})$
2	$0.72(10^{0})$	$0.72(10^{0})$	$0.72(10^{0})$	$0.58(10^{0})$	$0.14(10^{2})$
3	$0.77(10^{-3})$	$0.77(10^{-3})$	$0.76(10^{-3})$	$0.80(10^{-1})$	$0.11(10^{2})$
4	$0.78(10^{-4})$	$0.78(10^{-4})$	$0.63(10^{-4})$	$0.18(10^{-1})$	$0.28(10^{1})$
5	$0.55(10^{-7})$	$0.55(10^{-7})$	$0.80(10^{-5})$	$0.16(10^{-2})$	$0.17(10^{0})$
6	$0.40(10^{-8})$	$0.32(10^{-8})$	$0.21(10^{-5})$	$0.14(10^{-2})$	$0.14(10^{0})$
7	$0.23(10^{-11})$	$0.40(10^{-9})$	$0.14(10^{-6})$	$0.29(10^{-3})$	$0.38(10^{-1})$
8	$\boxed{0.11(10^{-11})}$	$0.12(10^{-9})$	$0.14(10^{-6})$	$0.31(10^{-4})$	$0.23(10^{-2})$
9	$0.17(10^{-11})$	$\boxed{0.80(10^{-11})}$	$0.36(10^{-7})$	$0.25(10^{-4})$	$0.21(10^{-2})$
10	$0.37(10^{-8})$	$0.24(10^{-9})$	$\boxed{0.37(10^{-8})}$	$\boxed{0.49(10^{-5})}$	$\boxed{0.54(10^{-3})}$

In column 2, the calculated range, R_{CALC}, is found by squaring the true range and fitting the squared data points to a 6th degree polynomial in range squared. Polynomials are then fitted to values of the true range rate ($\dot{R}_T$) and the calculated range (R_{CALC}) multiplied together. The error is defined as above. Note that the minimum error is larger than for column 1. Minimum errors are shown enclosed in a rectangle.

Columns 3 and 4 are similar to column 2 except that the range values calculated (R_{CALC}) are formed from a 4th and 2nd degree polynomial to R_T squared.

Column 5 shows the error from a fit to the true range rate and is the poorest method of fitting as stated previously, since low degree polynomials are poor approximations to the true range rate function.

Table 39 is an artificial case since the true data are always perturbed by noise. The noise-free conditions of Table 39 were presented only for illustration of polynomial smoothing error. Table 40 shows the more realistic comparisons for data with typical noise-like errors. Column 1 is also artificial in the sense that it includes range noise only, that is, range rate data is assumed perfect. The noisy range data as squared, fit to a 4th degree polynomial, square root values are taken from the calculated polynomial (R_{CALC}), and the calculated range and the true range rate (multiplied together) are fit to various degree polynomials. The calculated range rate, $\dot{R}_{CALC}$, results from dividing values from the polynomial by R_{CALC}. A minimum occurs at a 10th degree polynomial.

Column 2 uses a value of R_{CALC} from squaring R_T and fitting to a 4th degree polynomial. Column 2 is also unrealistic since there is no noise on the range data. Noise with standard deviation of 0.005 meters per second is imposed upon $\dot{R}_T$, and R_{CALC} and the noisy range rate data are multiplied together, R_{CALC} $(\dot{R}_T + N)$, and fit to a polynomial of degree 1 through 10. A minimum value of 0.0005 meters/second occurs for a 5th degree polynomial fit. Here the minimum value of error should be $\frac{1}{\sqrt{200-k-1}}$ times the value of the sample standard deviation (0.005 meters/second) or 0.00036 meters/second for a 5th degree polynomial fit ($k = 5$).

Columns 3, 4, and 5 have noise on both the range and the range rate data; the differences are in the method of calculating the range (R_{CALC}). Column 3 calculates range from a 6th degree polynomial to range squared, column 4 from a 4th degree polynomial, and column 5 from a 2nd degree polynomial. Although column 3 fits to a higher degree polynomial in $(R_T + N)^2$, computer truncation error seems to cause a larger minimum error than for the 4th degree fit to

Table 40

Noise on One or Both Data Types $\sigma_R = 2\,M$, $\sigma_{\dot{R}} = 0.005\,M/sec$

Range rate polynomial smoothing error for various methods of fitting a 200 second arc, 1000 kilometer height, circular orbit, noise on one or both data types.

Poly-nomial Degree	4th $(R_T+N)^2 \Rightarrow R_{CALC}$ $\overline{\dot{R}_{CALC}\ \dot{R}_T}$	4th $R_T^2 \Rightarrow R_{CALC}$ $\overline{\dot{R}_{CALC}\ (\dot{R}_T+N)}$	6th $(R_T+N)^2 \Rightarrow R_{CALC}$ $\overline{\dot{R}_{CALC}\ (\dot{R}_T+N)}$	4th $(R_T+N)^2 \Rightarrow R_{CALC}$ $\overline{\dot{R}_{CALC}\ (\dot{R}_T+N)}$	2nd $(R_T+N)^2 \Rightarrow R_{CALC}$ $\overline{\dot{R}_{CALC}\ (\dot{R}_T+N)}$	4th $R_T+N \Rightarrow R_{CALC}$ $\overline{\dot{R}_{CALC}\ (\dot{R}_T+N)}$	$\overline{(\dot{R}_T+N)}$
	M/Sec	M/Sec	M/Sec	M/Sec	M/Sec	M/Sec	M/Sec
1	$0.42(10^{1})$	$0.42(10^{1})$	$0.42(10^{1})$	$0.42(10^{1})$	$0.42(10^{1})$	$0.42(10^{1})$	$0.28(10^{3})$
2	$0.72(10^{0})$	$0.72(10^{0})$	$0.72(10^{0})$	$0.72(10^{0})$	$0.58(10^{0})$	$0.71(10^{0})$	$0.14(10^{2})$
3	$0.50(10^{-3})$	$0.94(10^{-3})$	$0.88(10^{-3})$	$0.73(10^{-3})$	$0.80(10^{-1})$	$0.23(10^{0})$	$0.11(10^{2})$
4	$0.52(10^{-4})$	$0.51(10^{-3})$	$0.72(10^{-3})$	$\boxed{0.50(10^{-3})}$	$0.17(10^{-1})$	$0.23(10^{0})$	$0.28(10^{1})$
5	$0.36(10^{-4})$	$\boxed{0.50(10^{-3})}$	$\boxed{0.58(10^{-3})}$	$0.50(10^{-3})$	$0.17(10^{-2})$	$0.73(10^{-1})$	$0.17(10^{0})$
6	$0.28(10^{-5})$	$0.60(10^{-3})$	$0.59(10^{-3})$	$0.60(10^{-3})$	$0.16(10^{-2})$	$0.45(10^{-1})$	$0.14(10^{0})$
7	$0.20(10^{-5})$	$0.78(10^{-3})$	$0.78(10^{-3})$	$0.78(10^{-3})$	$0.87(10^{-3})$	$0.82(10^{-2})$	$0.38(10^{-1})$
8	$0.61(10^{-6})$	$0.80(10^{-3})$	$0.79(10^{-3})$	$0.79(10^{-3})$	$\boxed{0.79(10^{-3})}$	$0.15(10^{-2})$	$0.24(10^{-2})$
9	$0.43(10^{-7})$	$0.80(10^{-3})$	$0.80(10^{-3})$	$0.80(10^{-3})$	$0.80(10^{-3})$	$0.11(10^{-2})$	$0.22(10^{-2})$
10	$\boxed{0.36(10^{-7})}$	$0.82(10^{-3})$	$0.82(10^{-3})$	$0.82(10^{-3})$	$0.82(10^{-3})$	$\boxed{0.84(10^{-3})}$	$\boxed{0.98(10^{-3})}$

$(R_T + N)^2$ in column 4. In column 5, the 2nd degree fit to $(R_T + N)^2$ is not sufficient to calculate any useful value of range (R_{CALC}). In column 5, the minimum error occurs for an 8th degree polynomial.

Column 6 extracts a calculated range from a 4th degree fit to $R_T + N$. Note that this is not a fit to the square of the data. Here again the calculated range, R_{CALC}, is poor and minimum error occurs for a 10th degree polynomial.

Column 7 are errors for a polynomial fit directly to noisy range rate. Minimum error occurs for a 10th degree polynomial fit and is approximately twice the value for column 4 fit to 4th degree.

8.0 TIMING OF METHODS

The contention has been made that performing operations on the data points (such as squaring the range) prior to polynomial fitting in the least squares senses, reduces the degree of the polynomial required, and thus conserves computer operation time. The purpose of this section is to describe some timing experiments made which demonstrate the validity of this claim.

Several programs were run on the IBM 360-75 computer in order to time various operations. These runs were made with no other jobs in the computer so that complications in timing would not be encountered. In addition the machine timing function, TIME, was called before and after each operation to be timed. TIME returns the number of hundredths of seconds at the time of the call which have occurred since midnight on that day. Since computer operations take relatively small time intervals compared to hundredths of a second, the identical operation was repeated many times between calls to TIME, in order to calculate an accurate estimate for a single operation. For example, the square root operation takes only $73.5\,\mu$seconds on the 360/75 computer but timing was performed on 20,000 of these operations. In timing the polynomial least squares fitting, the program used was D00107 from the GSFC computer library. In program D00107 the normal equations are formed, and the pivotal element method is used for the matrix inversion process. Table 41 illustrates the results of this timing.

Table 41
Timing of Methods

METHOD 1:	
Square 200 Range Measurements	0.0039 sec
Generate 4th Degree Polynomial fit to 200 Squared Measurements	0.4140 sec
Square Root 200 Values from Squared Polynomial	0.0160 sec
TOTAL	0.4339 sec
METHOD 2:	
Generate 9th Degree Polynomial fit to 200 Range Measurements	1.0796 sec
RATIO: $\frac{\text{METHOD 1}}{\text{METHOD 2}} = 2.5$	

9.0 CONCLUSIONS

In this analysis it has been shown that:

1. High degree least squares polynomials (e.g., 9th and 10th degree in some cases) are required to fit range and range rate measurements in order to reduce the root mean square error of fit to levels that are commensurate with the resolution of a typical S-band tracking system: 1 to 2 meters in range and 0.005 meters/second in range rate (sample error for sample rate of one per second). This is especially true for satellites that are in a near-Earth circular orbit or satellites that are in an elliptical orbit at the perigee portion of the orbit, where perigee height is only a few thousand kilometers.

2. A method to circumvent the problem above is to least squares polynomial fit functions of range and range rate, and extract the range and range rate from these functions. For a given accuracy the degree of the least squares polynomial required to fit these functions is much

lower (3rd and 4th degree) than the degree of the least squares polynomial required to fit the range and range rate directly (9th and 10th degree in some cases).

3. By fitting to the square of range, and extracting the range by taking the square root of the fitted range-squared polynomial, the accuracy of the range is greatly increased as compared to fitting the range directly. For example, in the case of a satellite in a 1000 kilometer circular orbit at the overhead portion of the orbit, for a 200 second data arc where the data rate is 1/second and there is no random noise, bias, or periodic noise on the range data, the standard deviation of fit (between the square root of the fitted range-squared-polynomial and the true range function) is 0.8 centimeters for a 4th degree fit to the square of range. For a 3rd degree polynomial fit to the square of range the standard deviation of fit is 9.09 meters. For a 4th degree polynomial fit to the range, the standard deviation of fit (between the fitted range polynomial and the true range function) is 108 meters. In order to achieve a standard deviation of 0.8 centimeters for a least squares polynomial fit to range directly, a 10th degree polynomial is required. For a typical S-band system, range resolution is on the order of 1 to 2 meters. The advantage of a lower degree polynomial fit to range squared occurs because the computation time for fitting a 4th degree polynomial to range squared is much less than the computation time required to fit a 10th degree polynomial to range directly.

4. By multiplying the range rate by the range and fitting to this mixed data type, and extracting the range rate by dividing the fitted polynomial by the range (where the range has been previously determined by fitting a 4th degree least squares polynomial to range squared as discussed in #3 on page 42), the accuracy of the range rate is greatly increased as compared to fitting to the range rate directly for the same degree polynomial. For example, in the case of a satellite in a 1000 kilometer circular orbit at the overhead portion of the orbit for a 200 second data arc where the data rate is 1/second and there is no random noise, bias, or periodic noise on the range rate data, the standard deviation of fit (between the approximating function for range rate just discussed above and the true range rate function) is 0.76×10^{-3} meters/second for a 3rd degree least squares polynomial fit to the range times the range rate. For a 3rd degree polynomial fit to the range rate the standard deviation of fit (between the fitted range rate polynomial and the true range rate function) is 10.9 meters/second. In order to achieve a standard deviation of 0.76×10^{-3} meters/second for a least squares polynomial fit to range rate directly, a 10th degree polynomial is required. For a typical S-band system the range rate resolution is on the order of 0.5 x

10^{-2} meters/second for a sample rate of one per second. The advantage of a lower degree polynomial fit to the range rate times the range occurs because the computation time for fitting a 3rd degree polynomial to this mixed data type is much less than the computation time required to fit a 10th degree polynomial to range rate directly.

5. For satellites in near-Earth circular orbits ranging from 500 kilometers to 1500 kilometers and for satellites in elliptical orbits around the Earth (such as IMP-G), a 4th degree least squares polynomial fit to range squared and a 3rd degree least squares polynomial fit to the range rate times the range are sufficient for achieving a typical S-band range resolution of 1 to 2 meters and a typical S-band range rate resolution of 0.005 meters/second.

6. Gaussian random noise on the range and range rate data is unaffected by the above process of fitting and reduction of uncertainties by the square root of the number of points follows just as linear least squares polynomial smoothing. Biases are not smoothed out but remain unchanged in the data. Periodic noise on the data is not removed by this processing technique and such noise has a pronounced effect on the results. For a fixed amplitude the greater the number of periods of periodic noise in the data span, the less the standard deviation of fit and the more this type of noise appears to behave like random noise. In addition, for a fixed number of periods of this noise in the data span, the less the amplitude of the periodic noise the less the effect in the smoothing process and the less the standard deviation of fit. It must be emphasized that the effects of periodic noise are not increased by the recommended process.

10.0 RECOMMENDATIONS

For satellites in circular Earth orbits at altitudes ranging from 500 kilometers to 1500 kilometers and for satellites in highly elliptical Earth orbits with low perigee, it is recommended that:

1. A 4th degree least squares polynomial be fit to the square of the range measurements and the value for range obtained by taking the square root of the range-squared polynomial.

2. A 3rd degree least squares polynomial be fit to the range times the range rate measurements (where the values of range are obtained by the method outlined in #1 and properly time tagged to coincide with the time of the range rate measurements). The value for range rate

is obtained by dividing the fitted polynomial by the range (where again the values of range are obtained by the method outlined in #1 and properly time tagged to coincide with the time extracted from the fitted $\dot{r}R$ polynomial).

These methods will be tested in the near future by the authors on actual tracking data, and comparisons will be made of computation time and estimates of errors between the present smoothing methods and the methods proposed above. Estimating error in smoothing actual tracking data will be more difficult than for this documented case of simulated data, but actual orbital uncertainties can be used for this judgement.

REFERENCE

1. Kruger, B., "Filter Properties of Least Squares Fitted Polynomials," GSFC X-551-68-47, January 1968.

APPENDIX A

ILLUSTRATION OF SMOOTHING ERRORS FROM FITTING TRACKING DATA IN THE LEAST SQUARES SENSE TO POLYNOMIALS OF 1st THROUGH 10th DEGREE

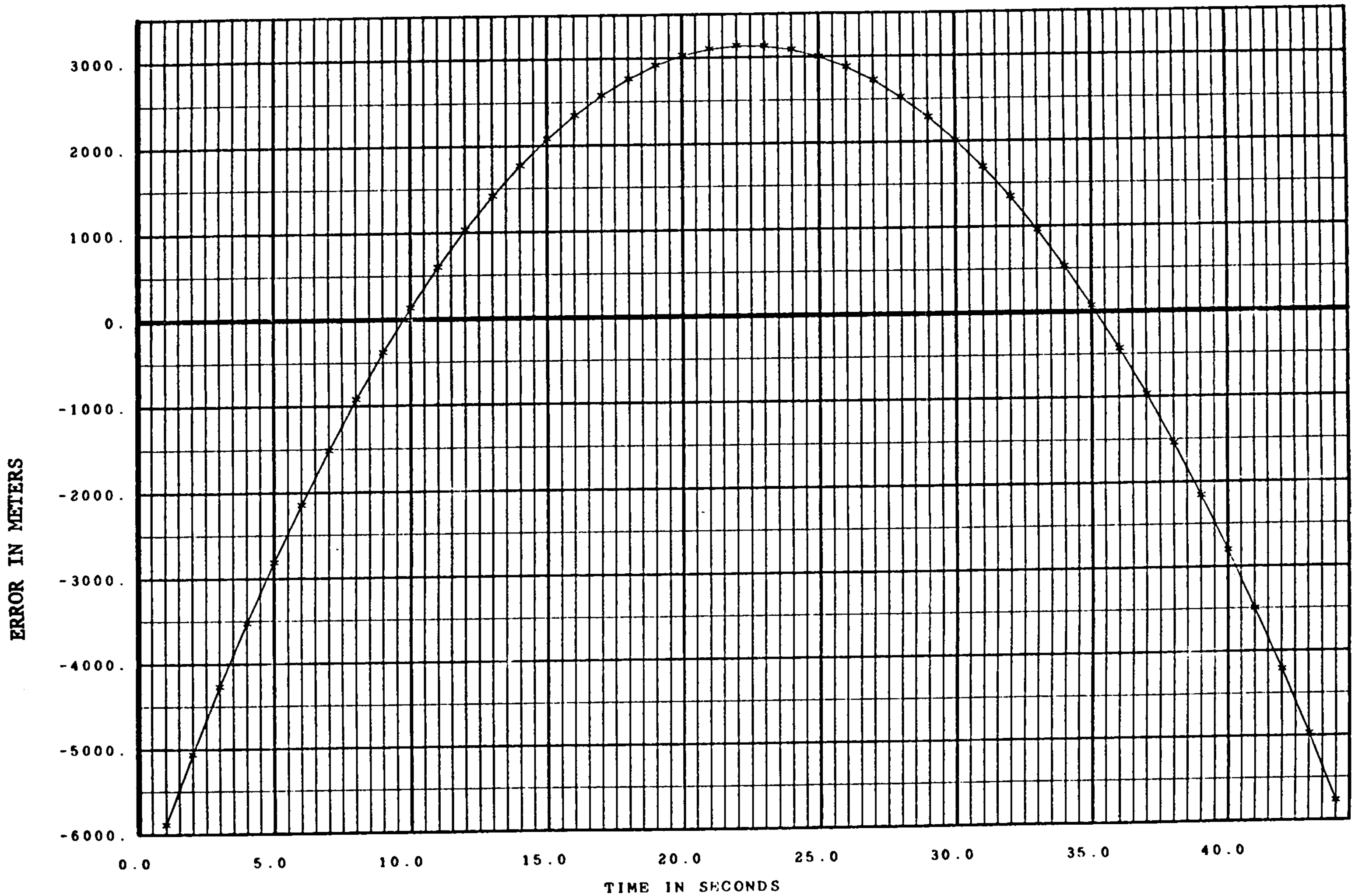

Figure A-1. Error (calculated polynomial minus true function) from using 1st degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

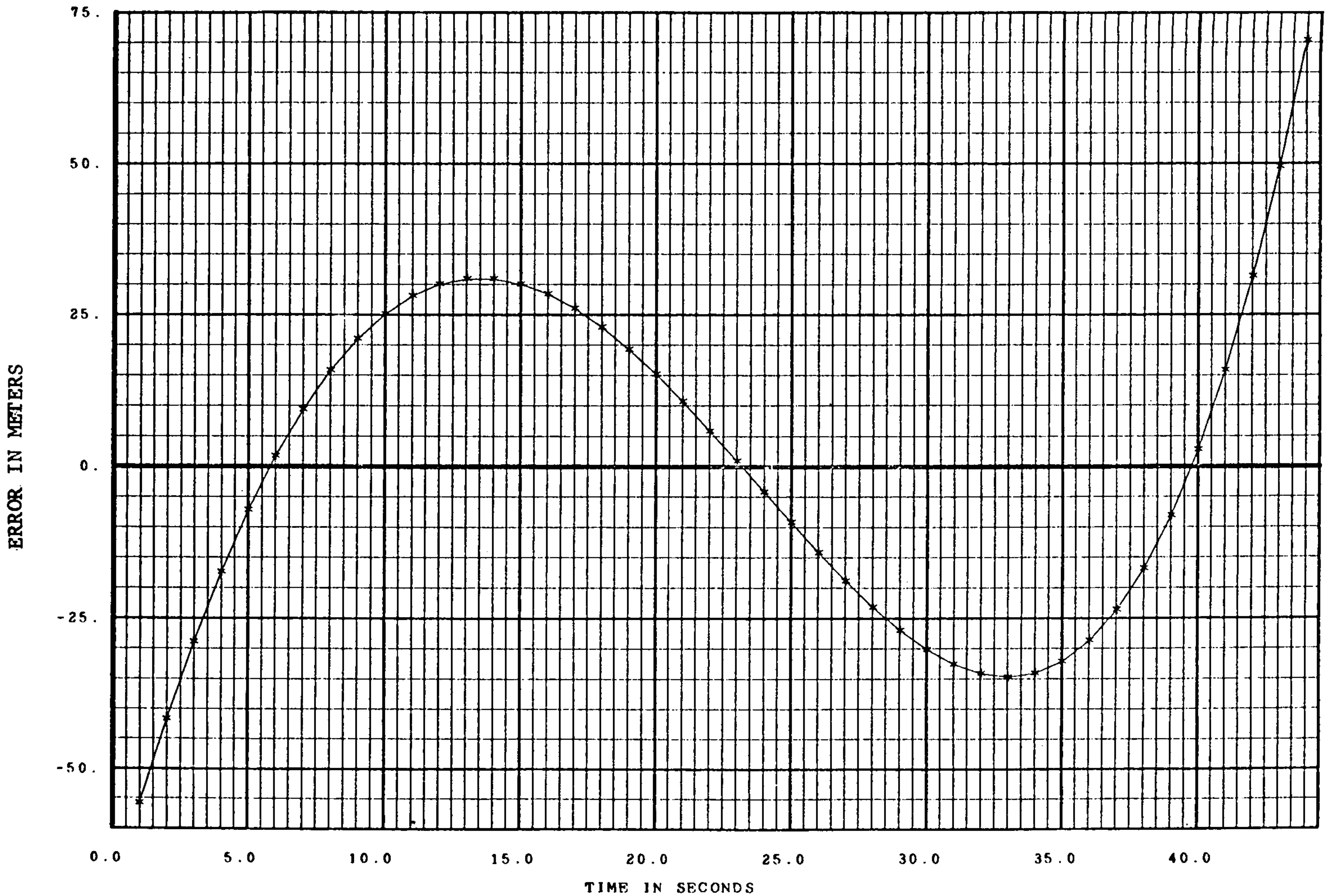

Figure A-2. Error (calculated polynomial minus true function) from using 2nd degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

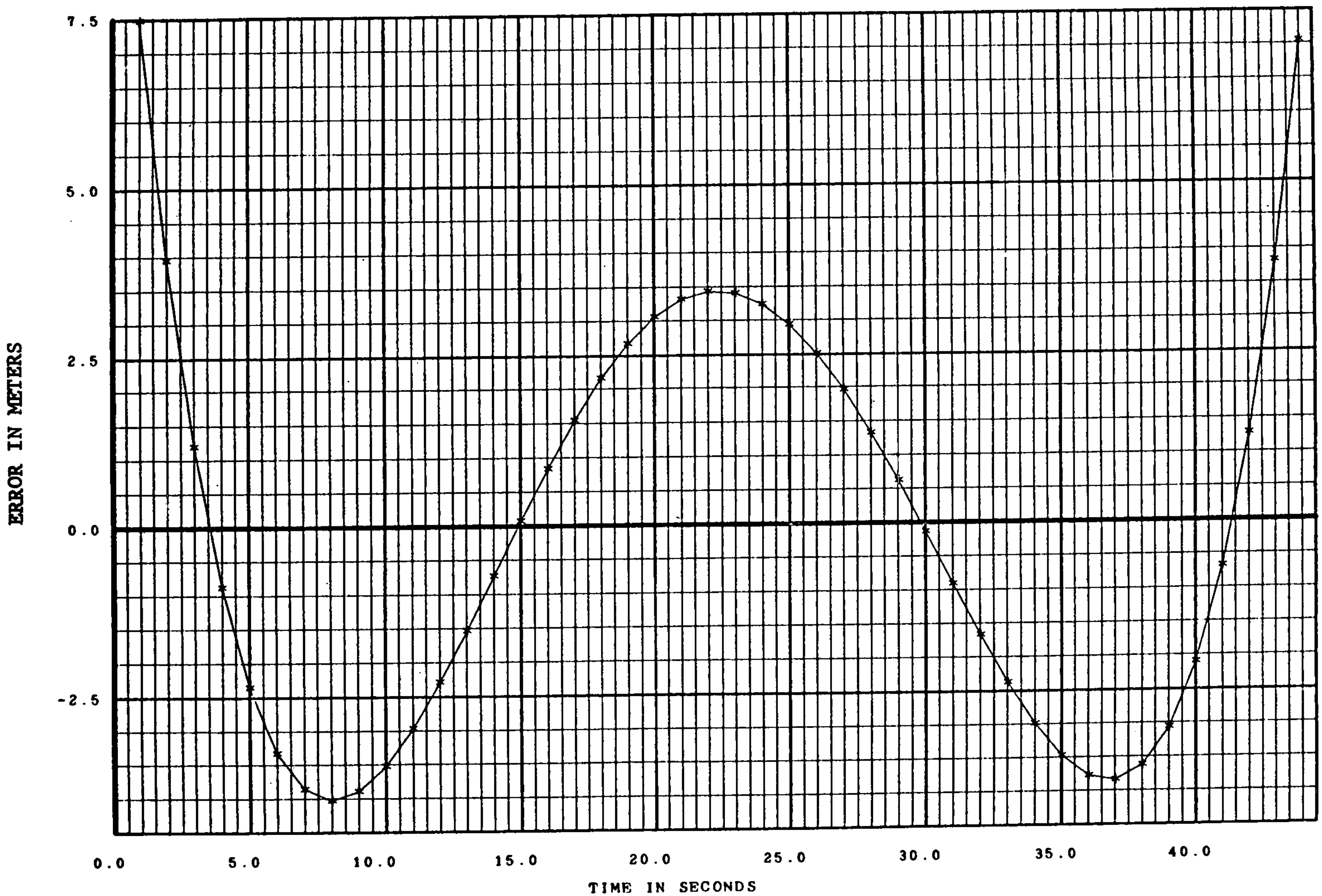

Figure A-3. Error (calculated polynomial minus true function) from using 3rd degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

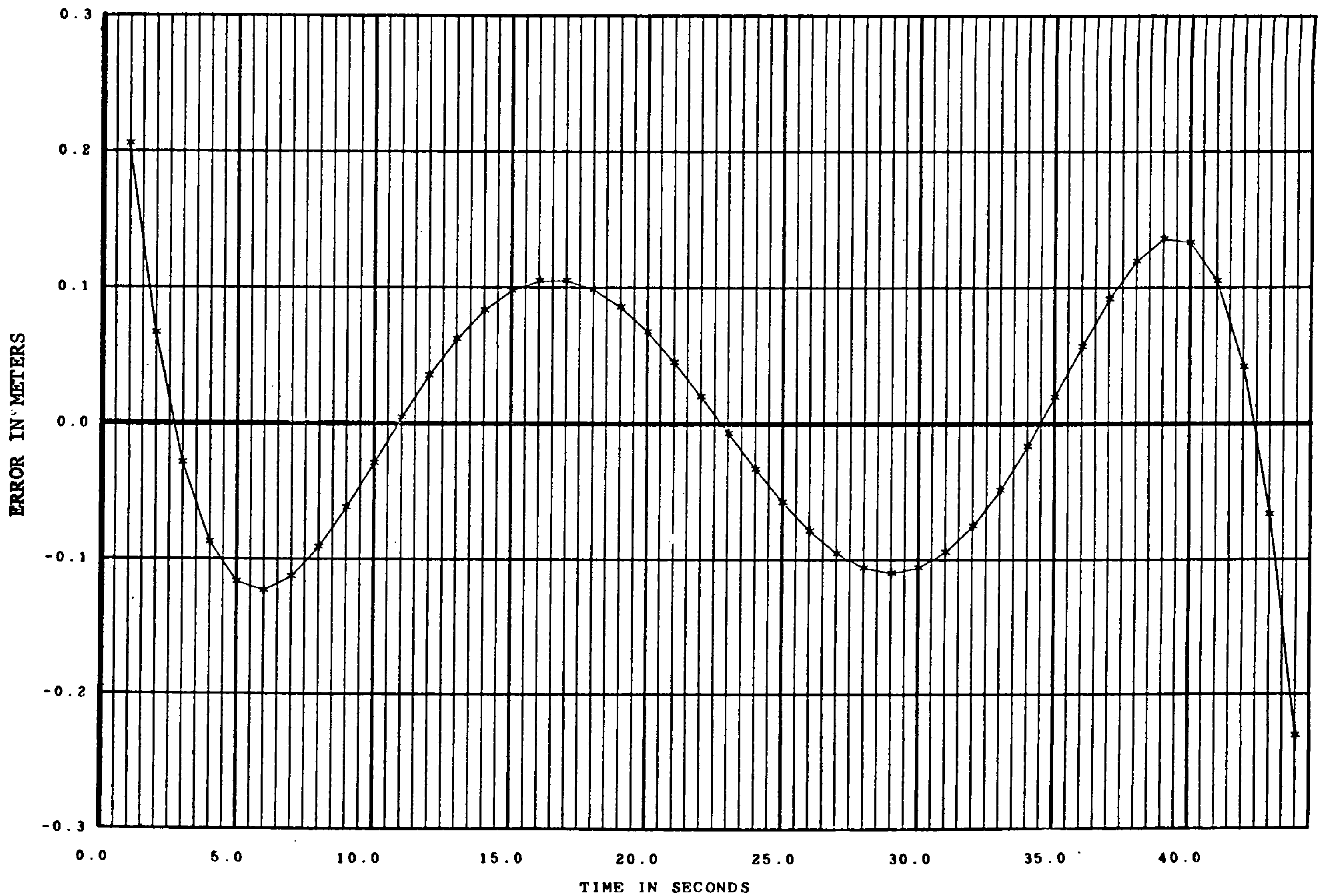

Figure A-4. Error (calculated polynomial minus true function) from using 4th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

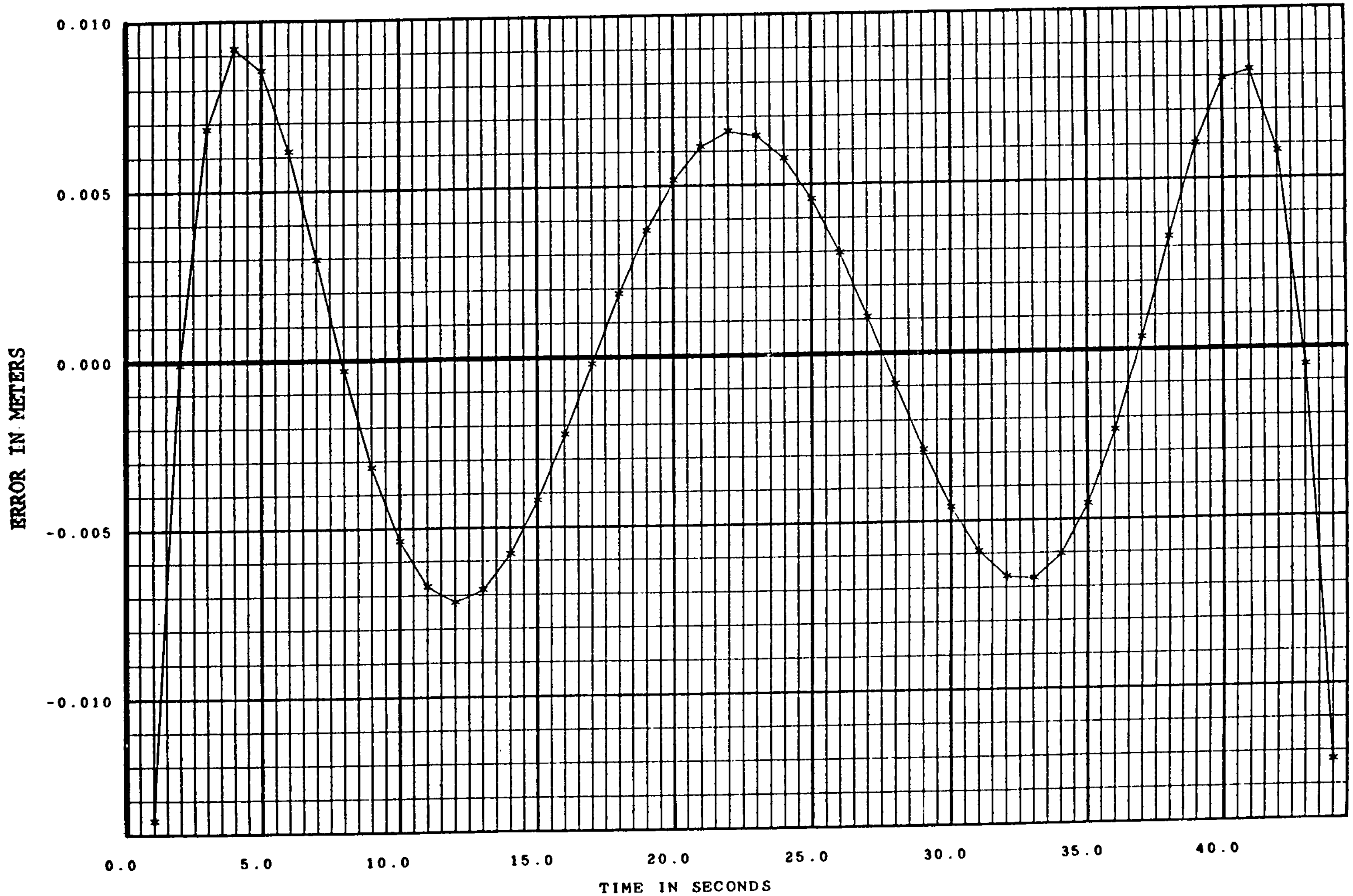

Figure A-5. Error (calculated polynomial minus true function) from using 5th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

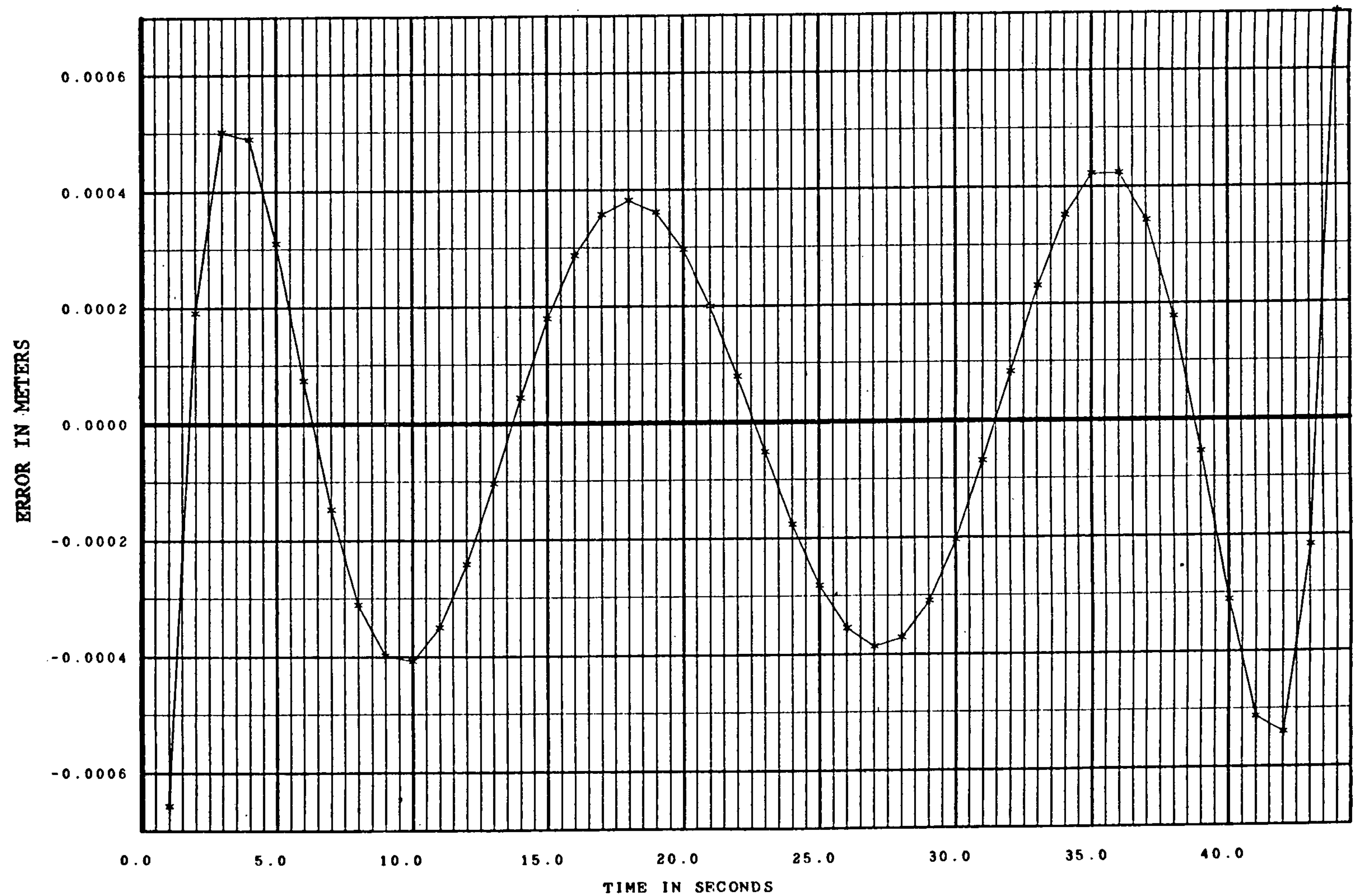

Figure A-6. Error (calculated polynomial minus true function) from using 6th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, $a = 7378$ km, $e = 0$, zero time at spacecraft crossing station zenith (overhead pass).

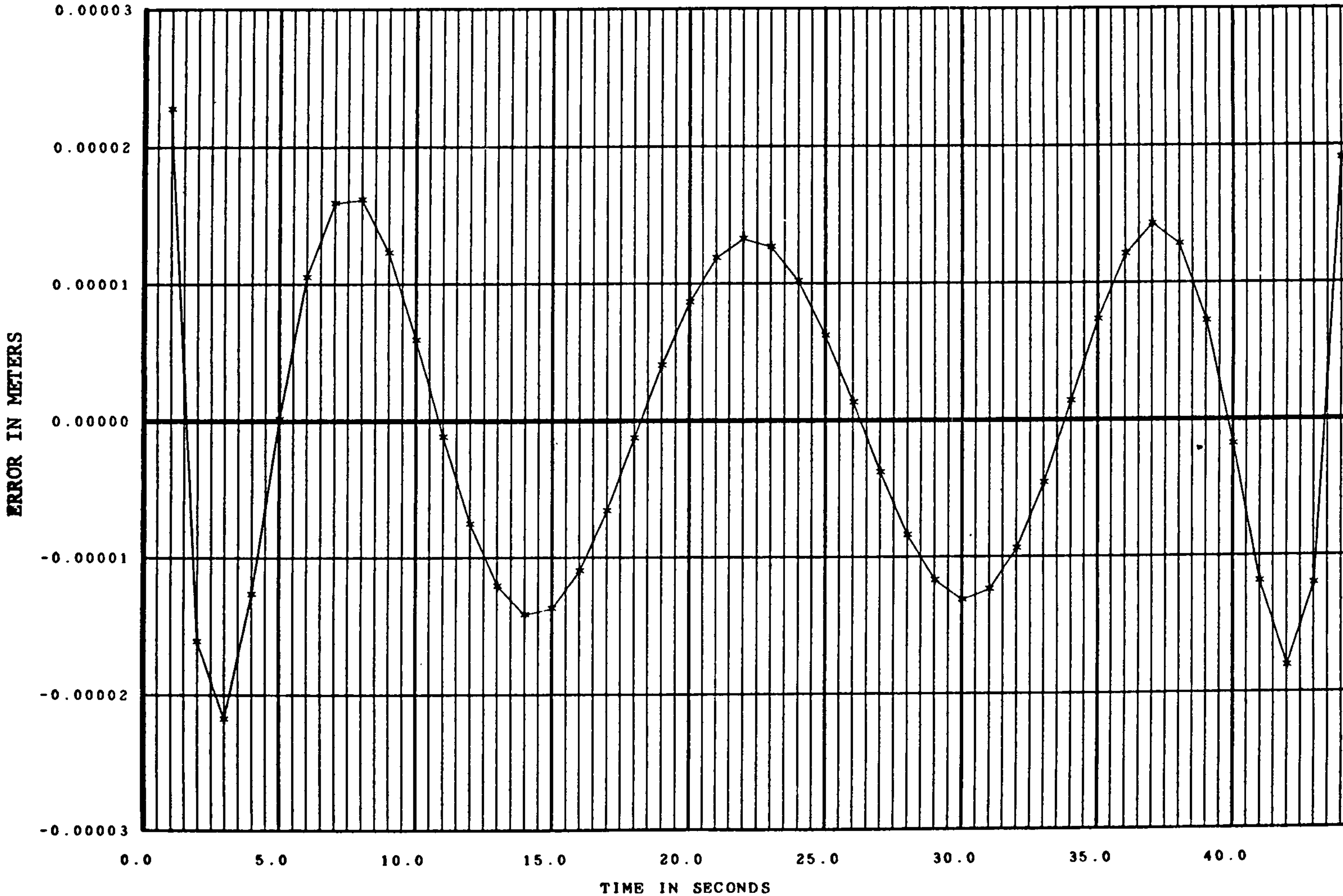

Figure A-7. Error (calculated polynomial minus true function) from using 7th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

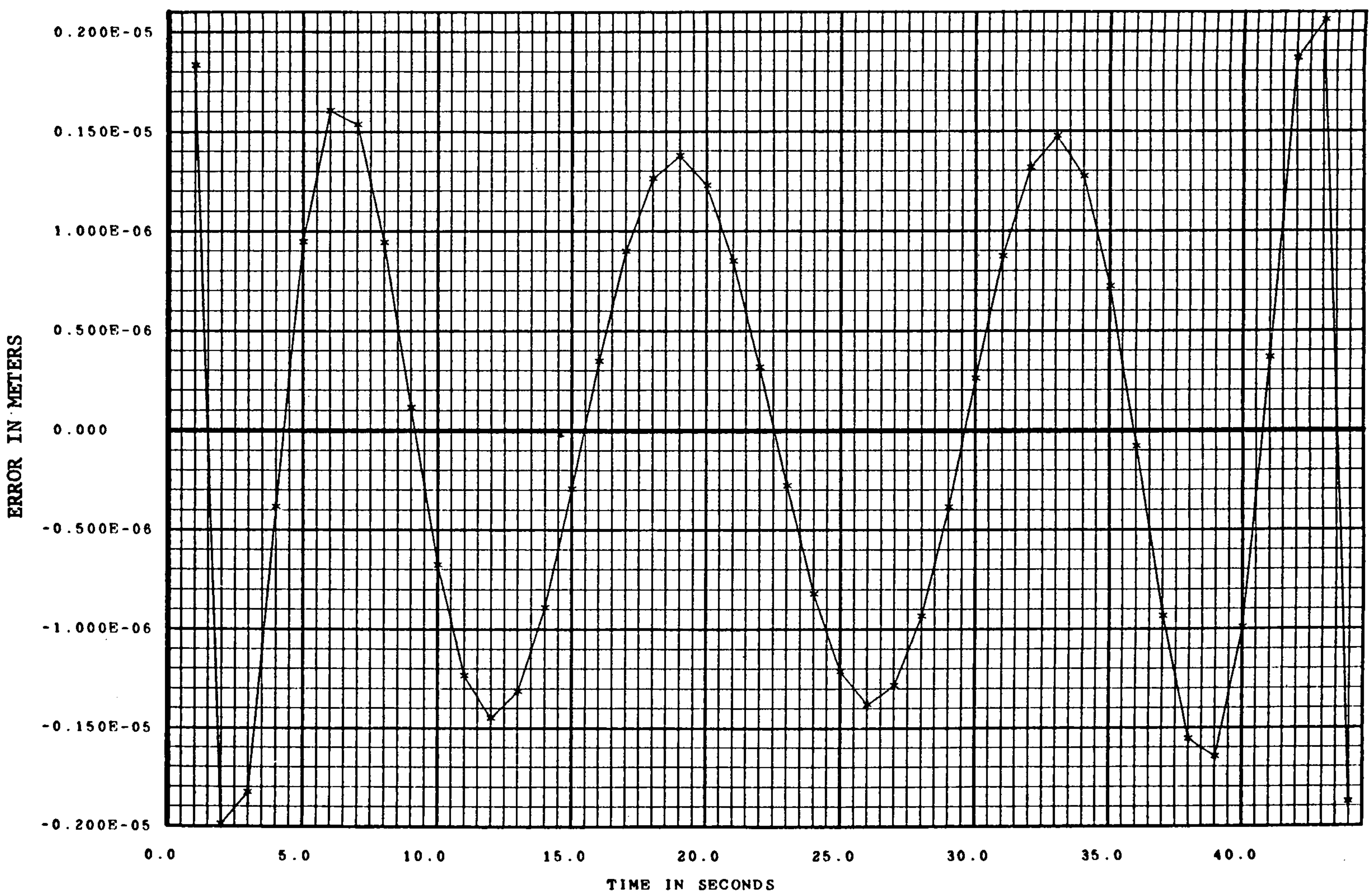

Figure A-8. Error (calculated polynomial minus true function) from using 8th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

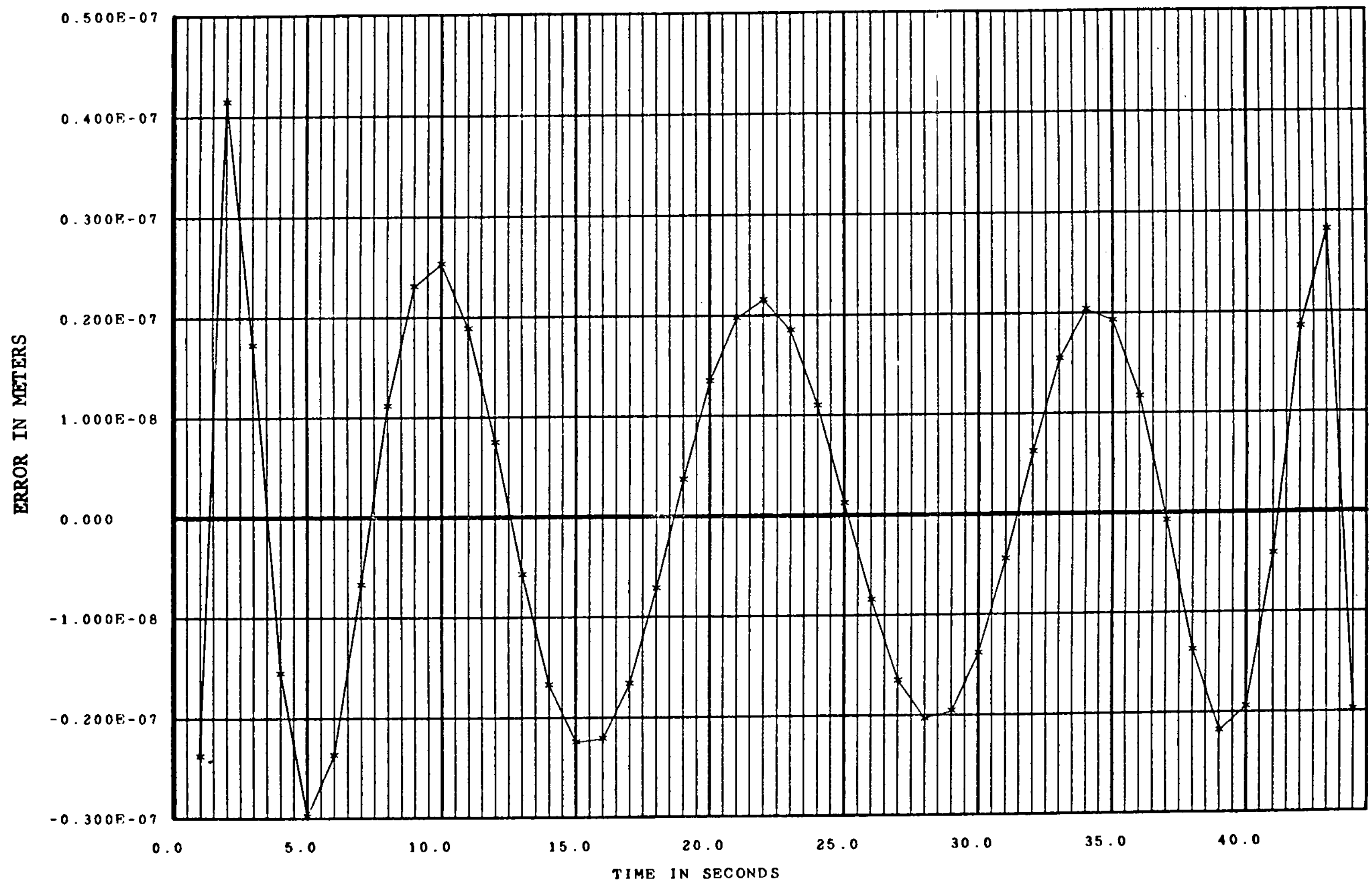

Figure A-9. Error (calculated polynomial minus true function) from using 9th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

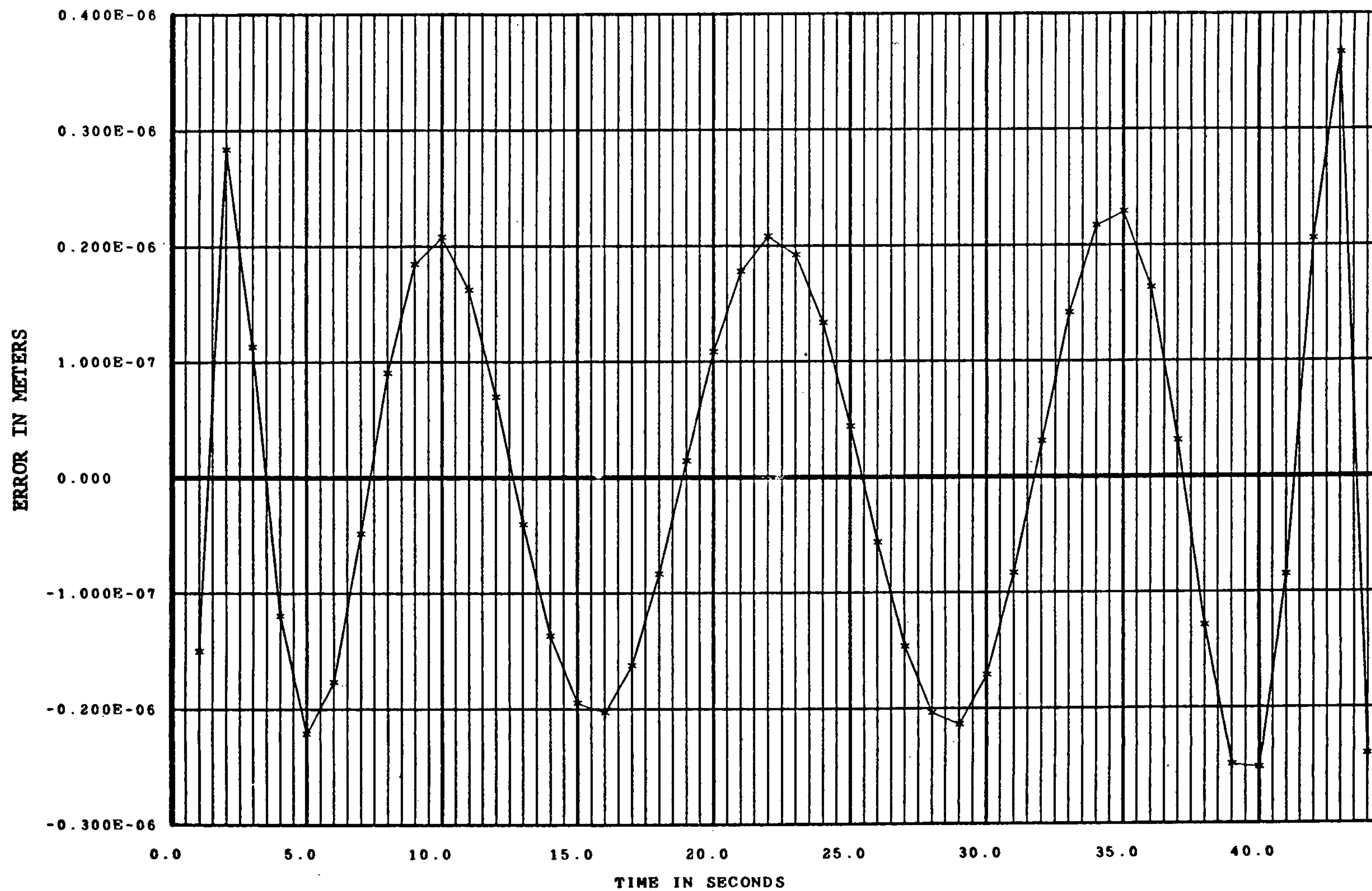

Figure A-10. Error (calculated polynomial minus true function) from using 10th degree polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

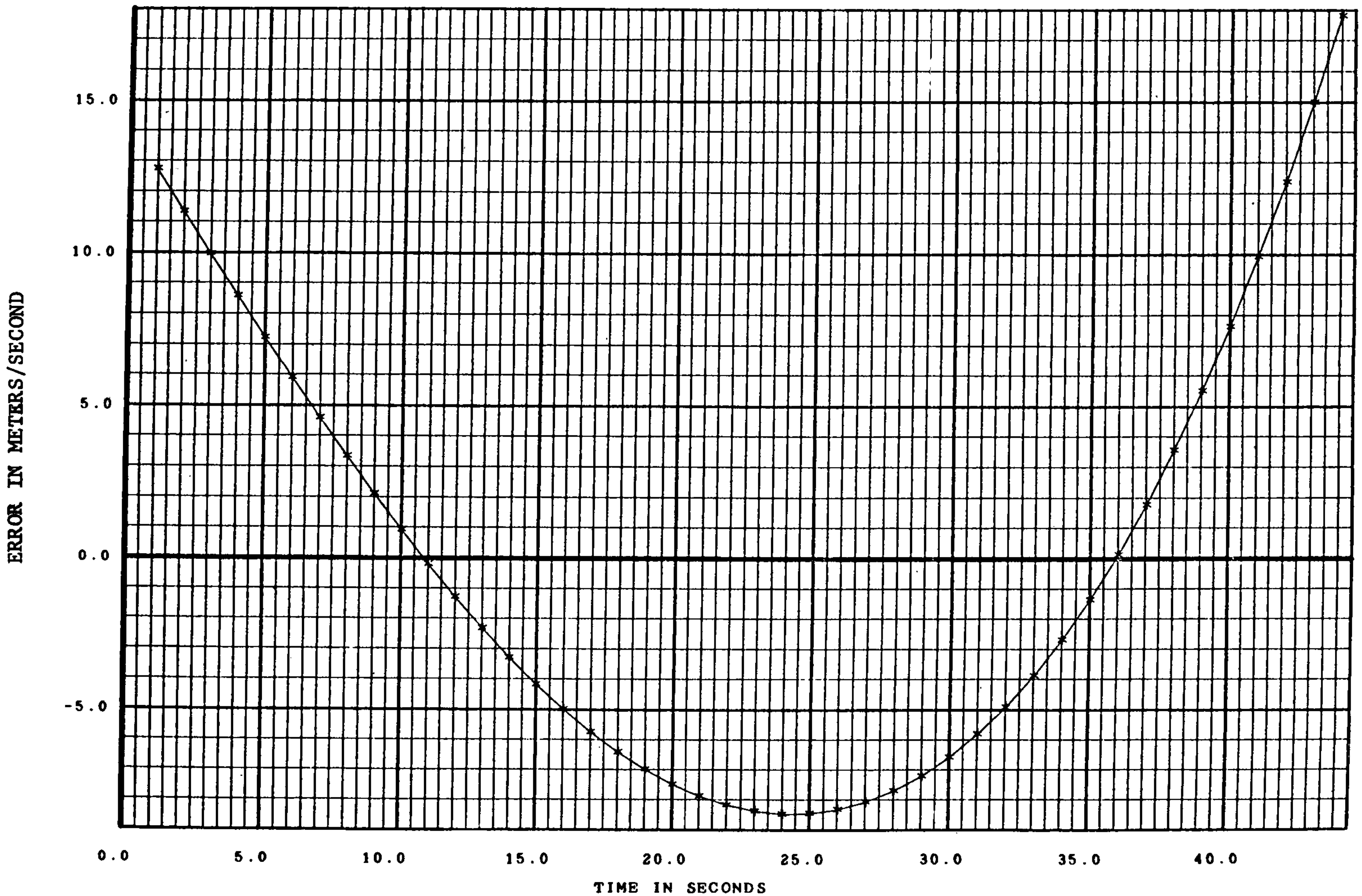

Figure A-11. Error (calculated polynomial minus true function) from using 1st degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

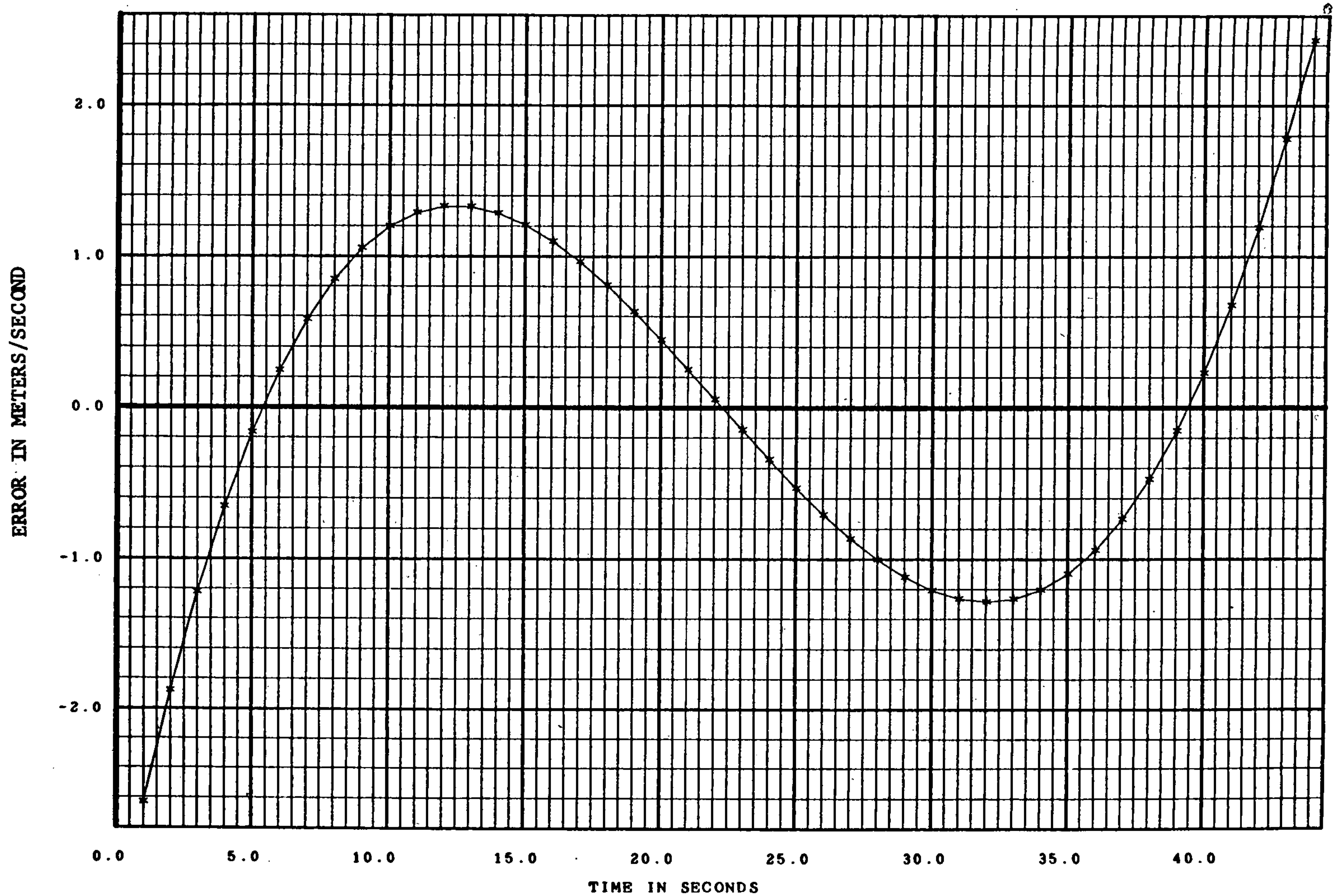

Figure A-12. Error (calculated polynomial minus true function) from using 2nd degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

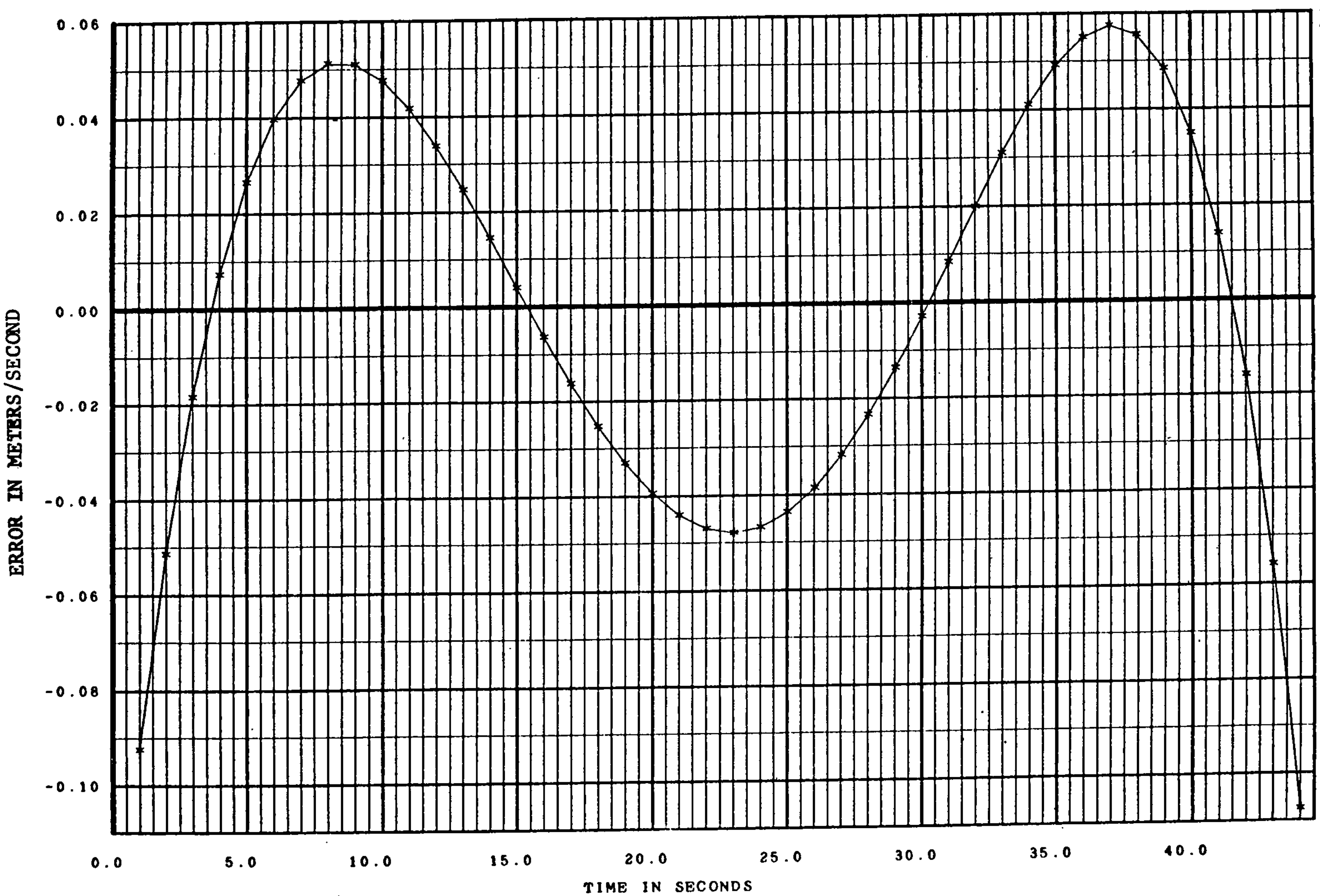

Figure A-13. Error (calculated polynomial minus true function) from using 3rd degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

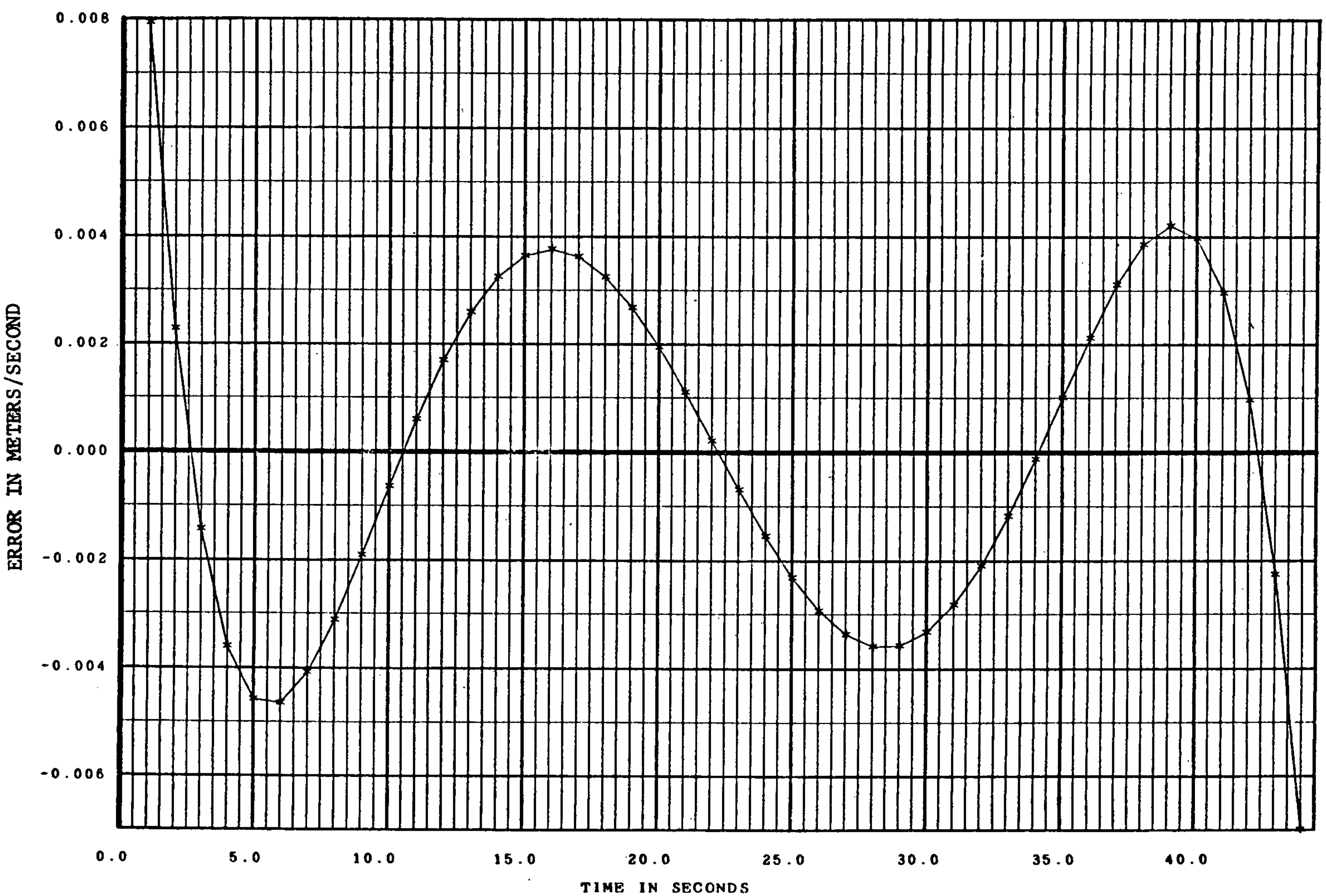

Figure A-14. Error (calculated polynimal minus true function) from using 4th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

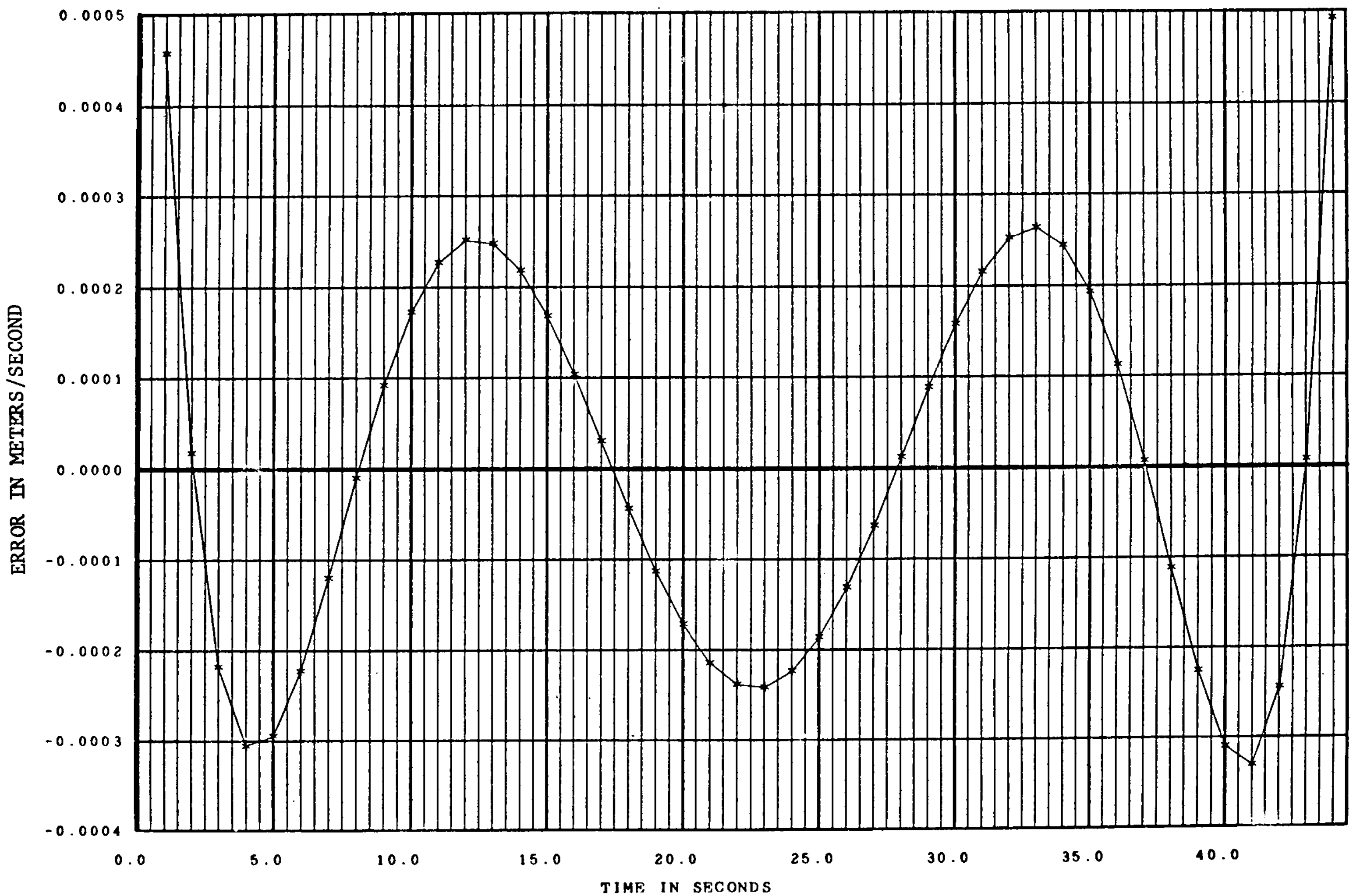

Figure A-15. Error (calculated polynomial minus true function) from using 5th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

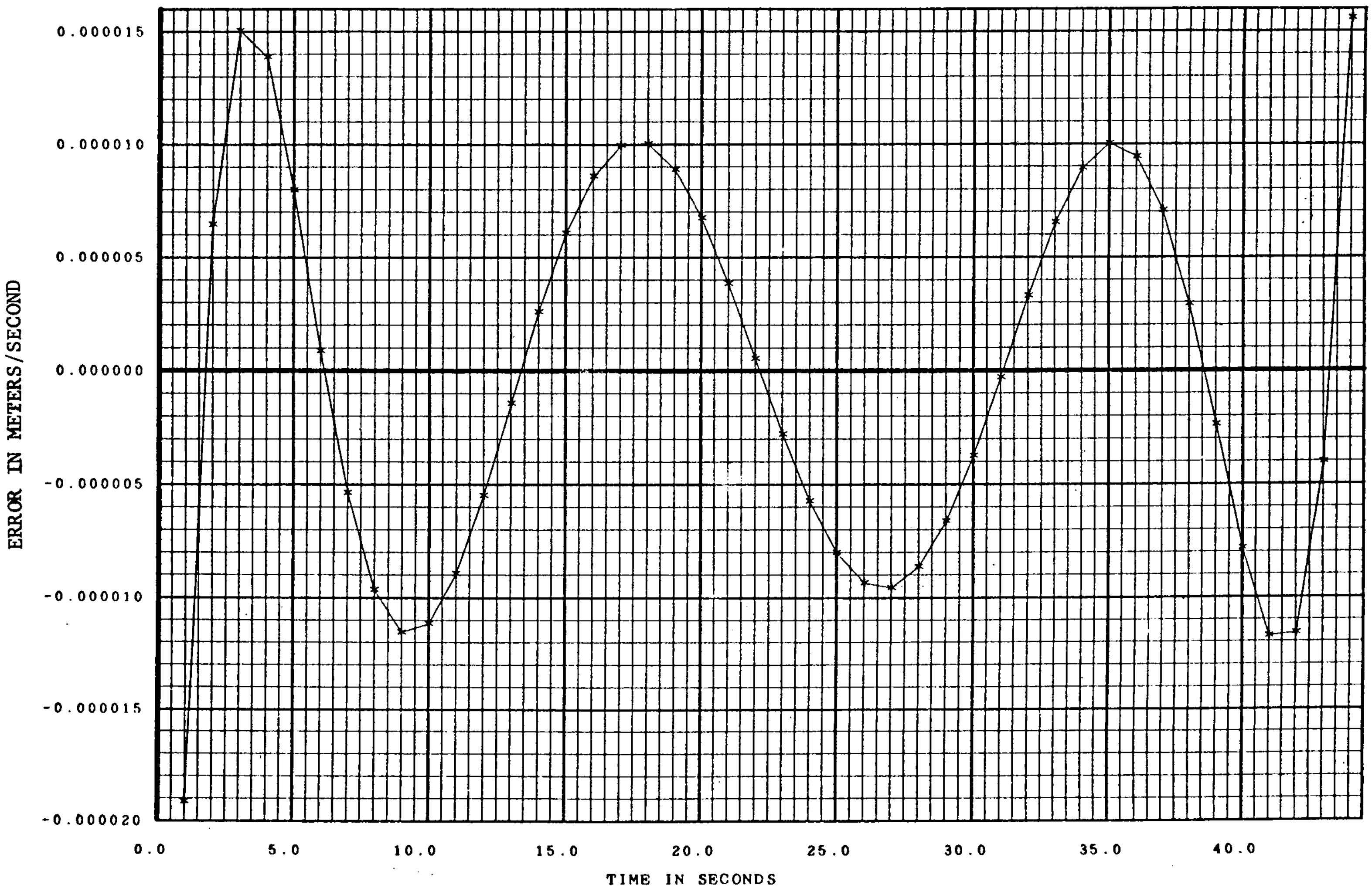

Figure A-16. Error (calculated polynomial minus true function) from using 6th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

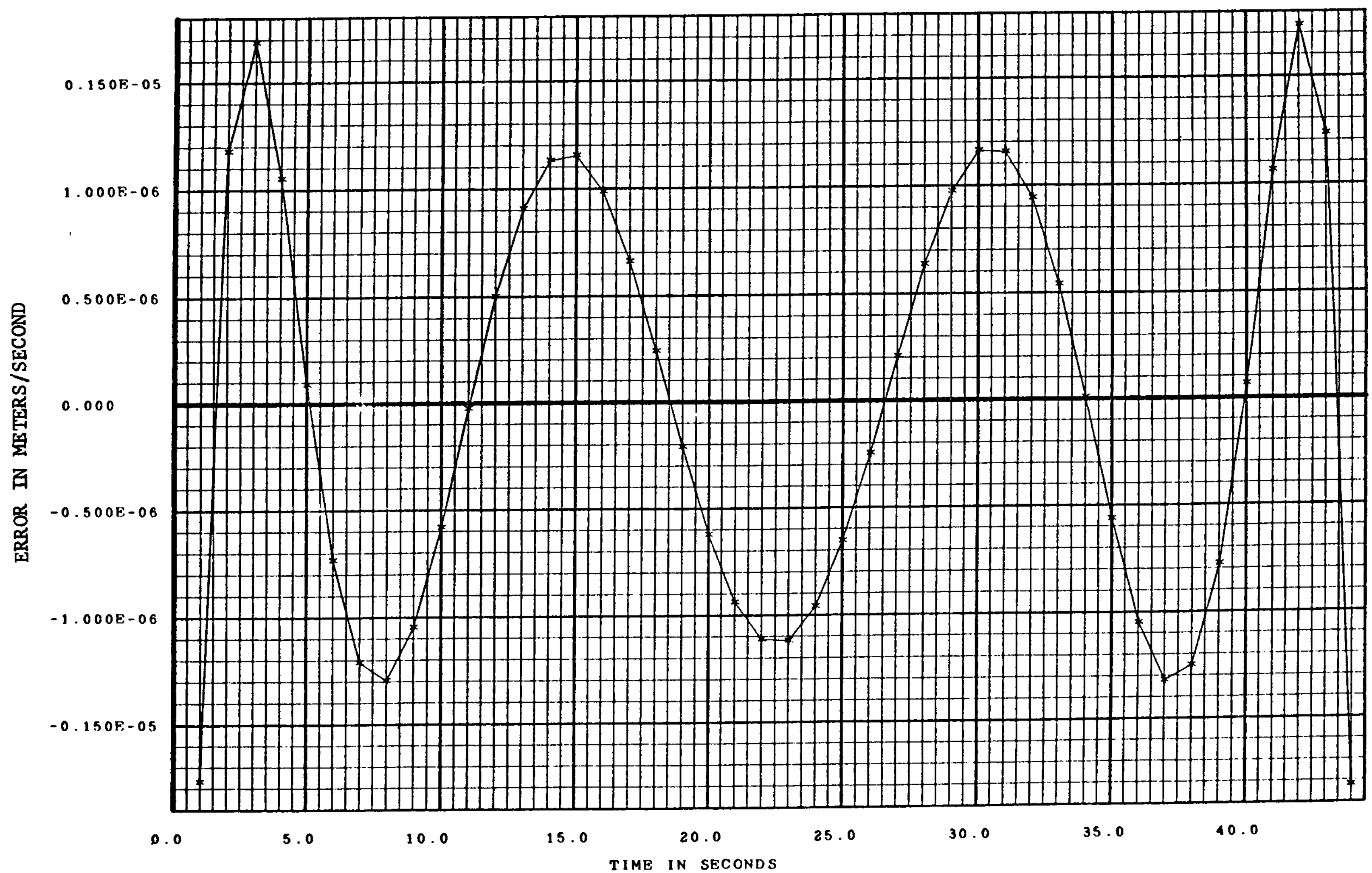

Figure A-17. Error (calculated polynomial minus true function) from using 7th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

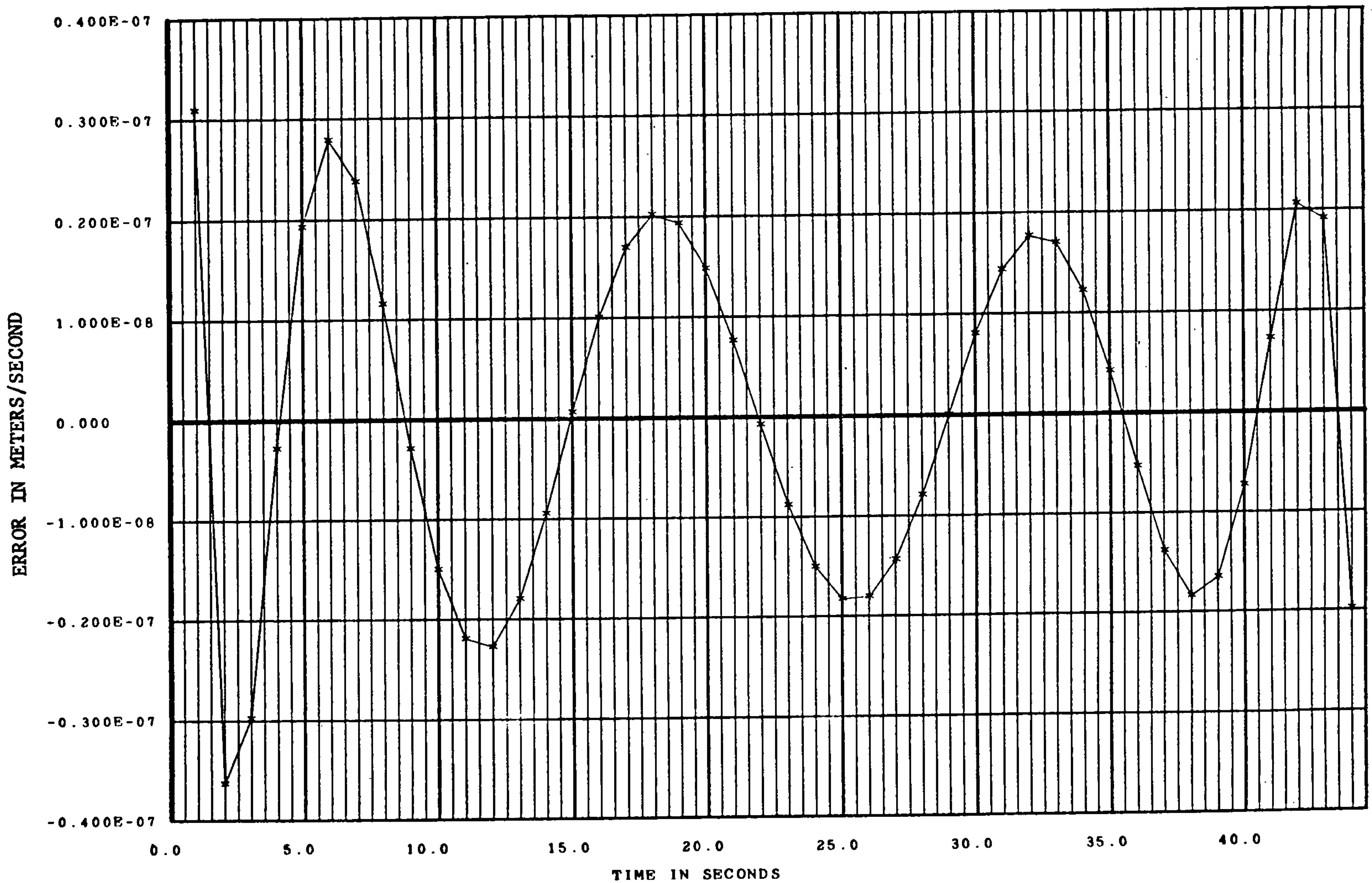

Figure A-18. Error (calculated polynomial minus true function) from using 8th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

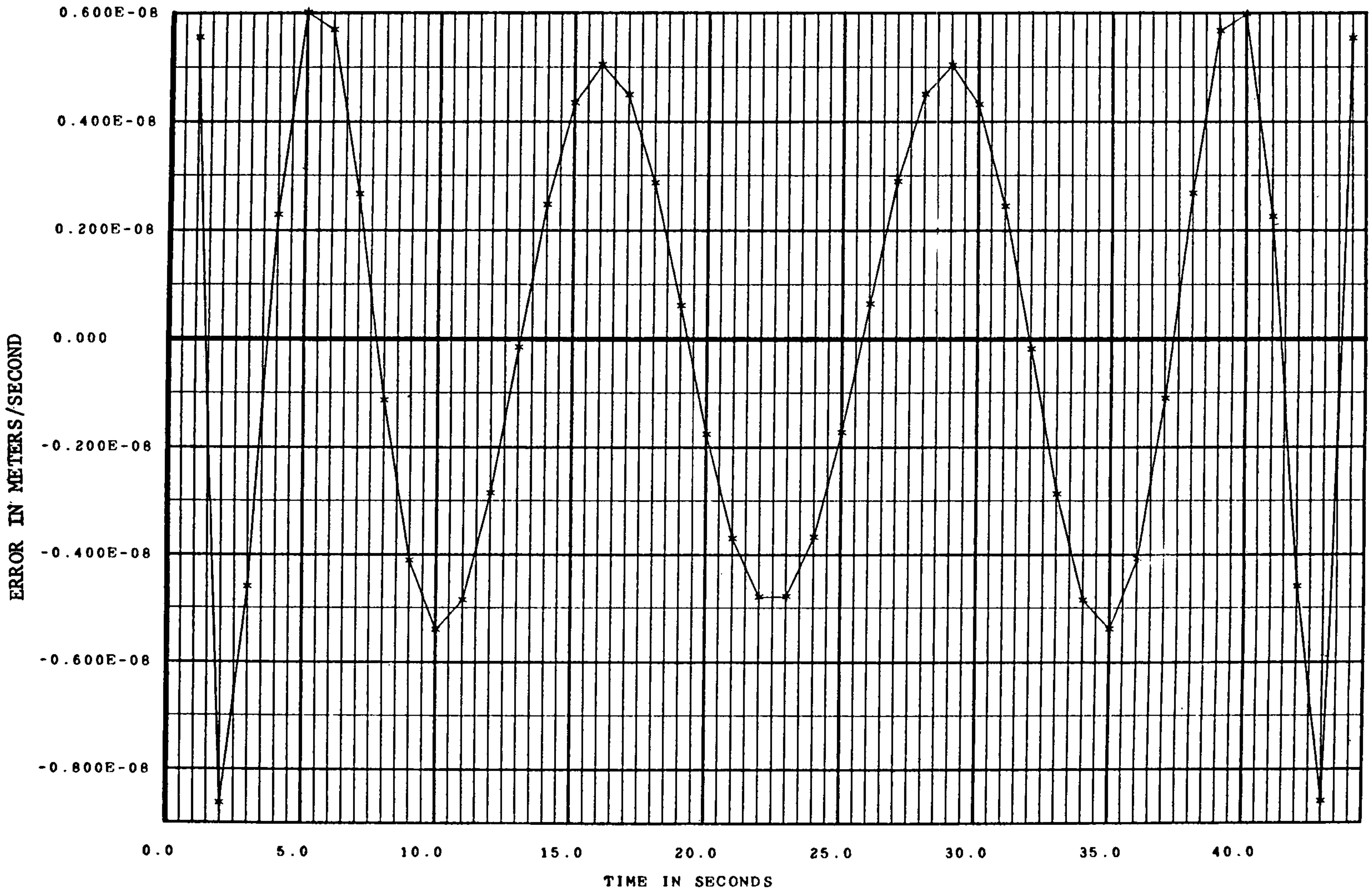

Figure A-19. Error (calculated polynomial minus true function) from using 9th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

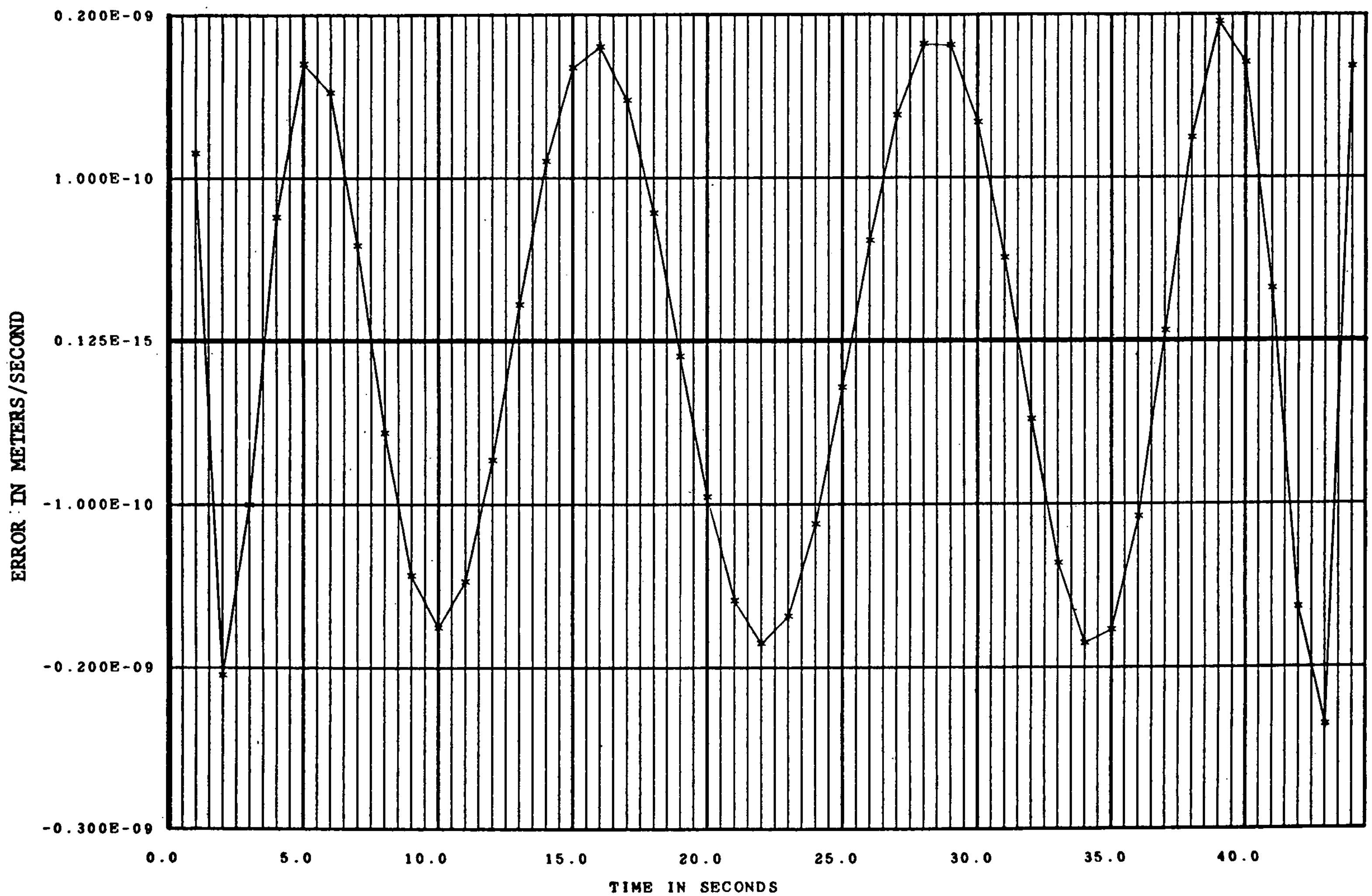

Figure A-20. Error (calculated polynomial minus true function) from using 10th degree polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

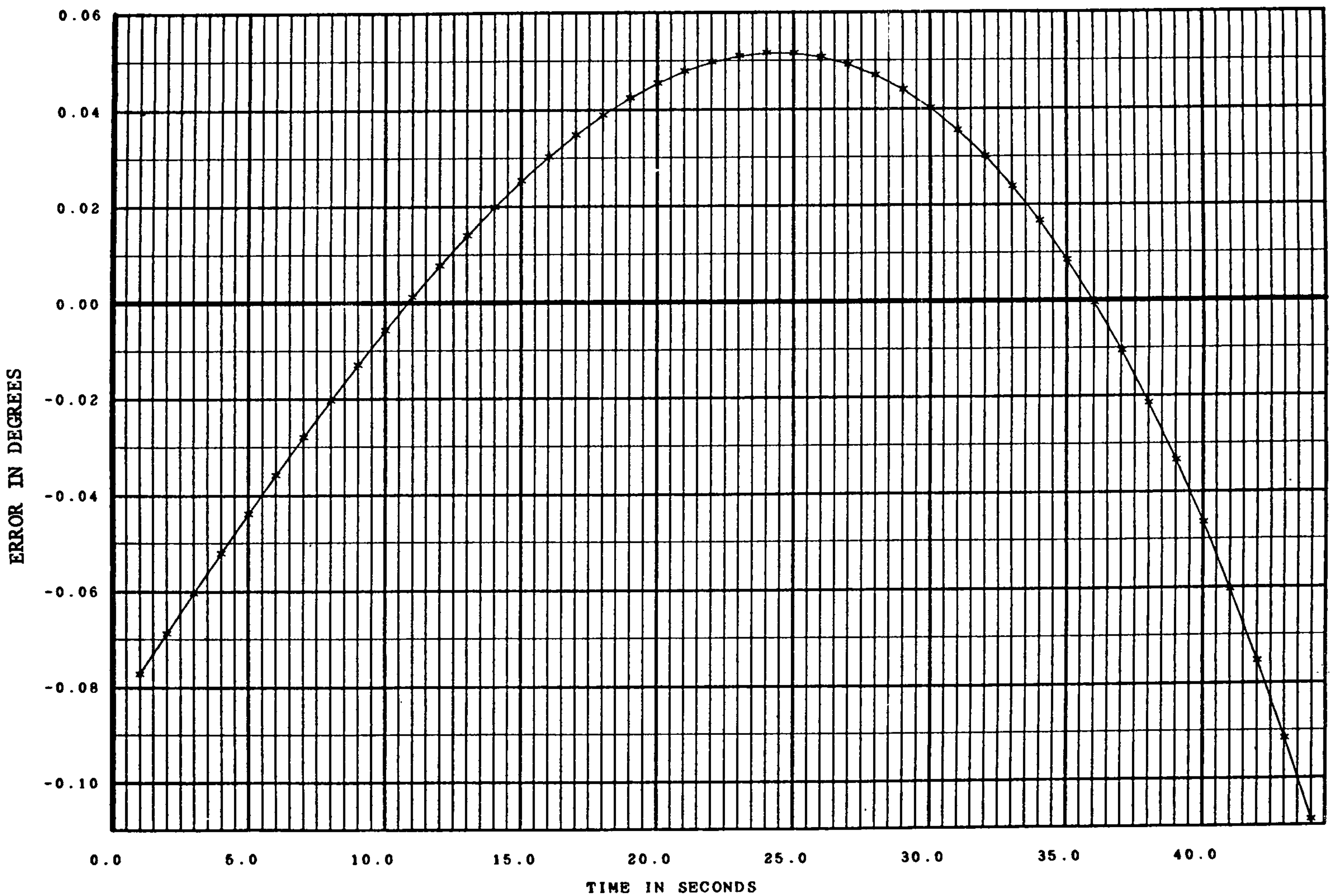

Figure A-21. Error (calculated polynomial minus true function) from using 1st degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

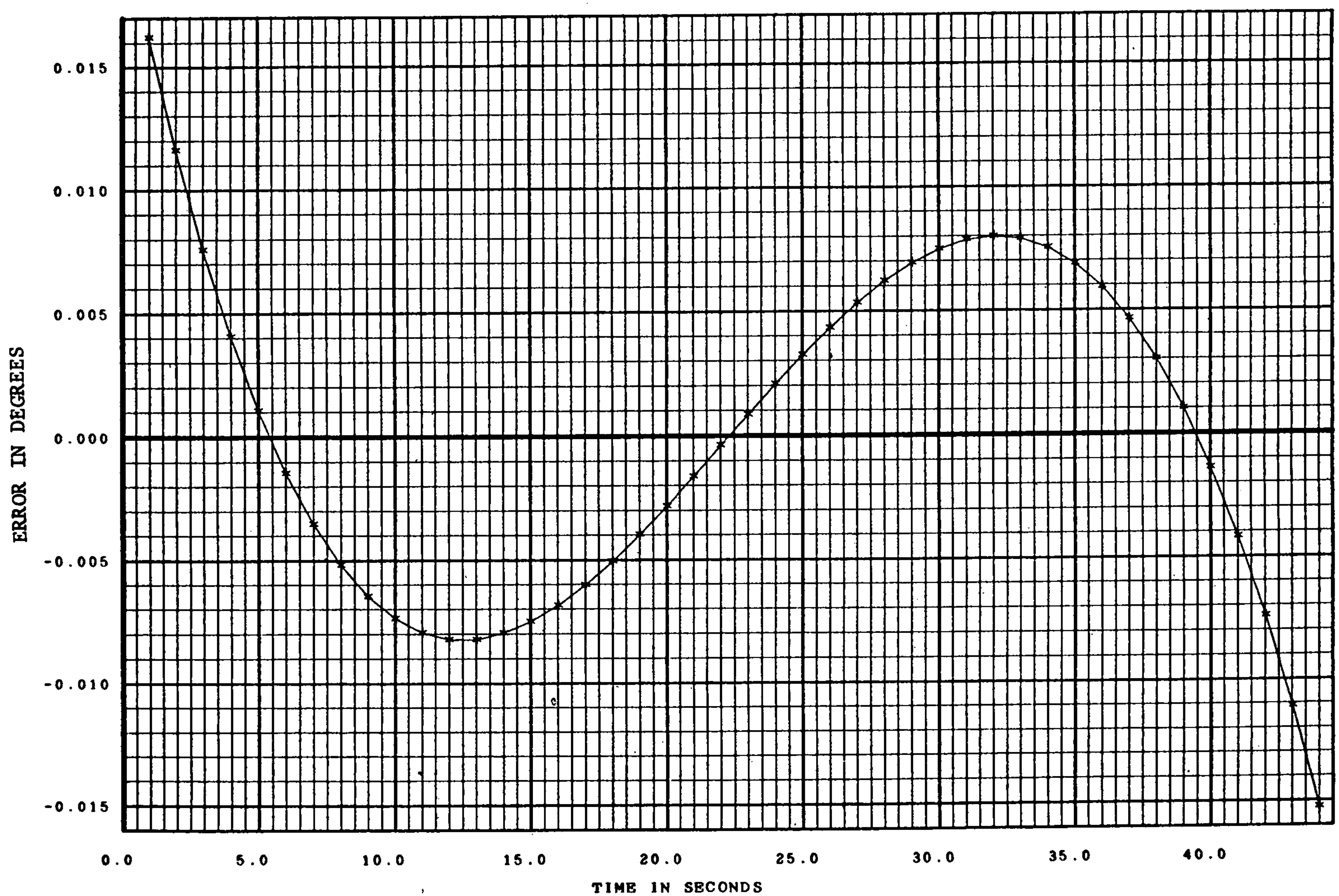

Figure A-22. Error (calculated polynomial minus true function) from using 2nd degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

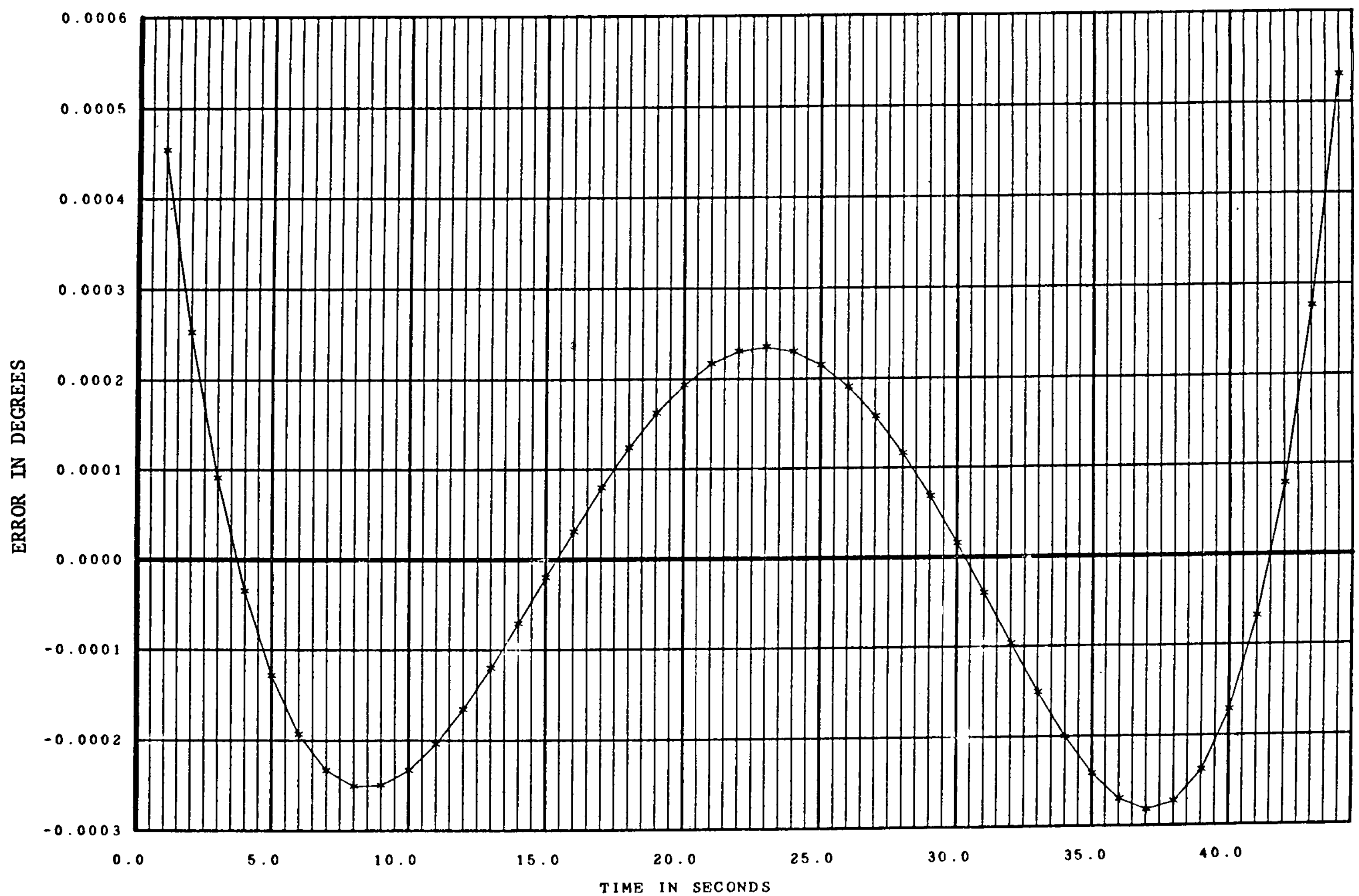

Figure A-23. Error (calculated polynomial minus true function) from using 3rd degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

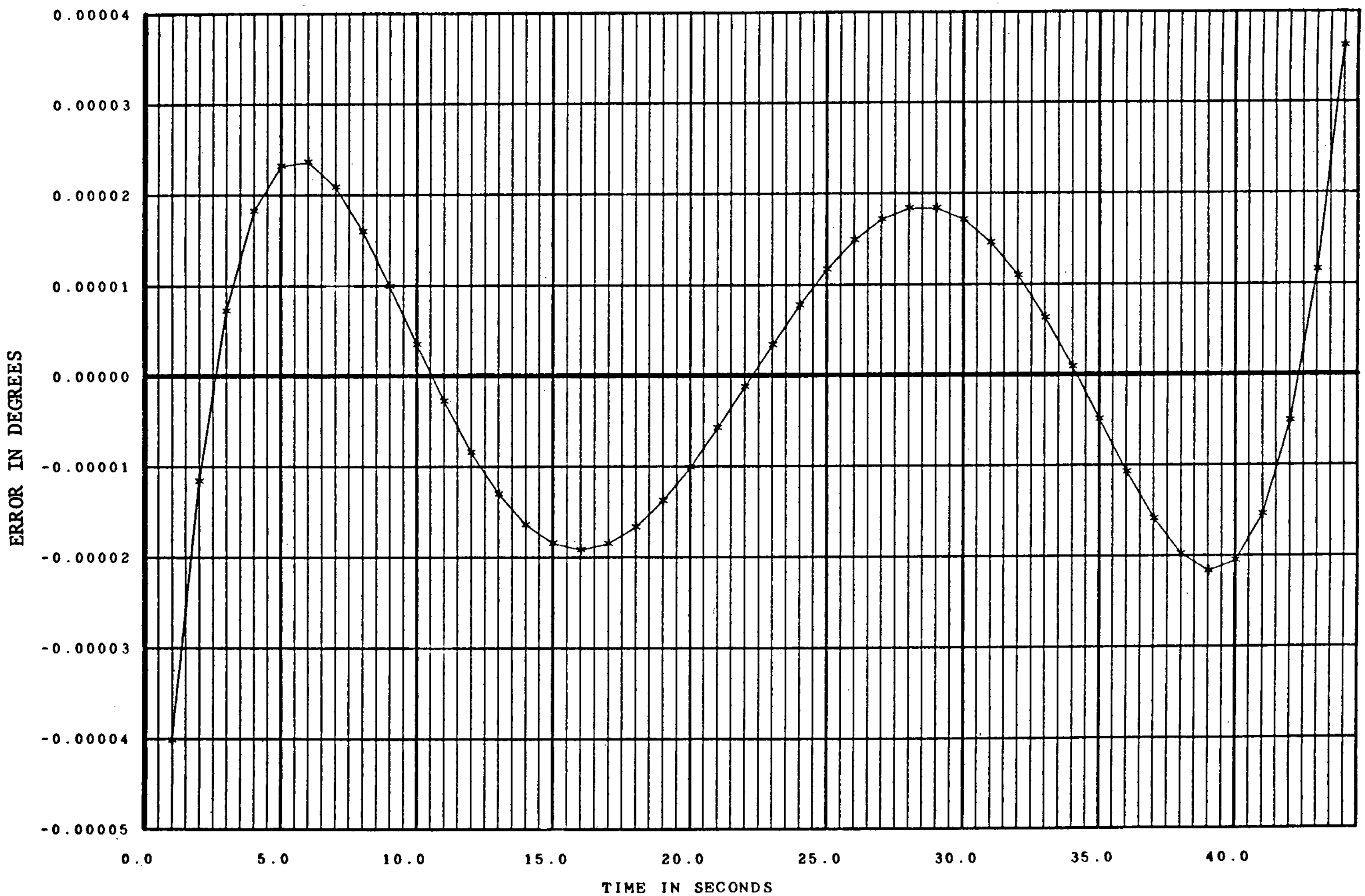

Figure A-24. Error (calculated polynomial minus true function) from using 4th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

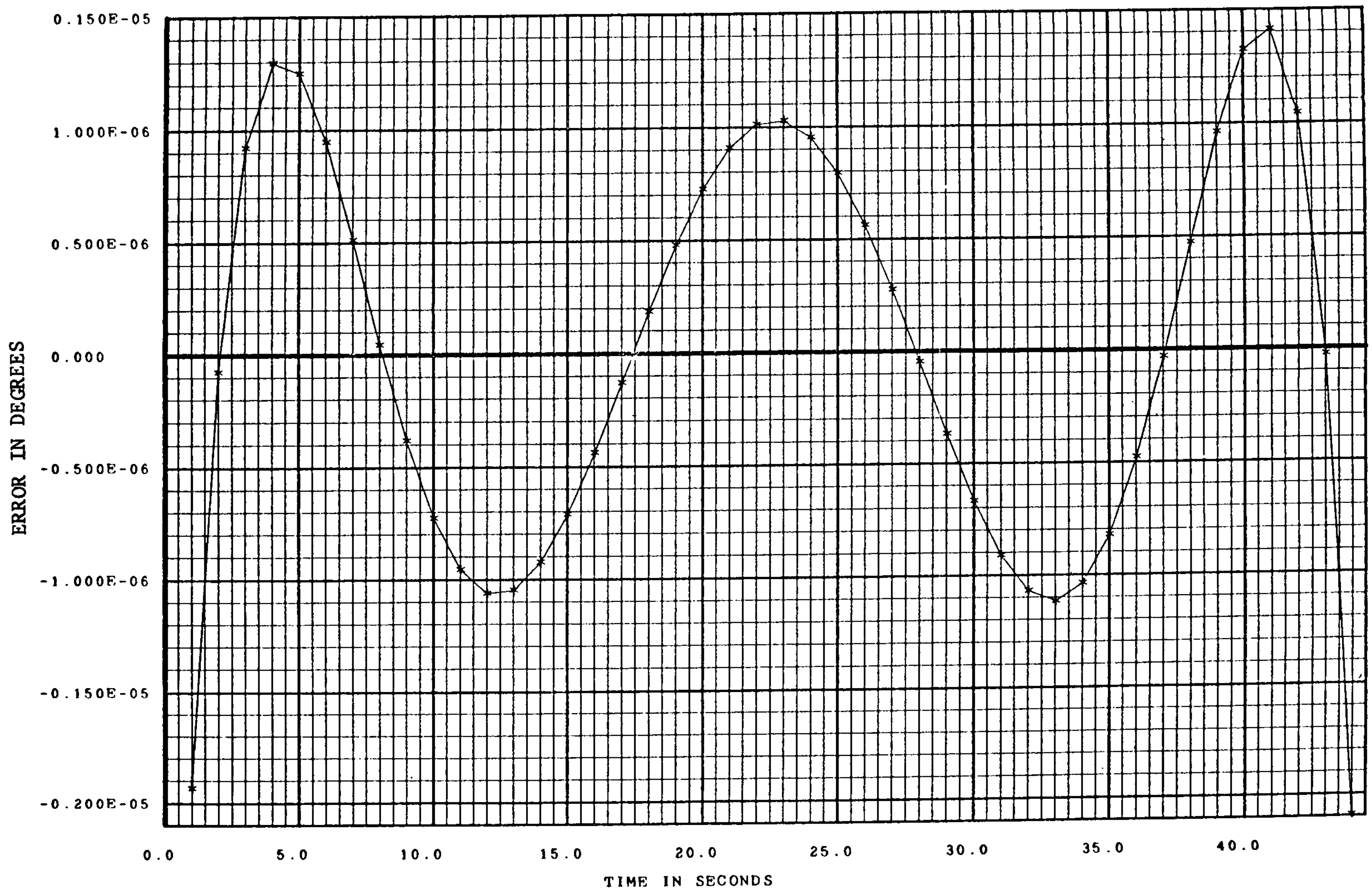

Figure A-25. Error (calculated polynomial minus true function) from using 5th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

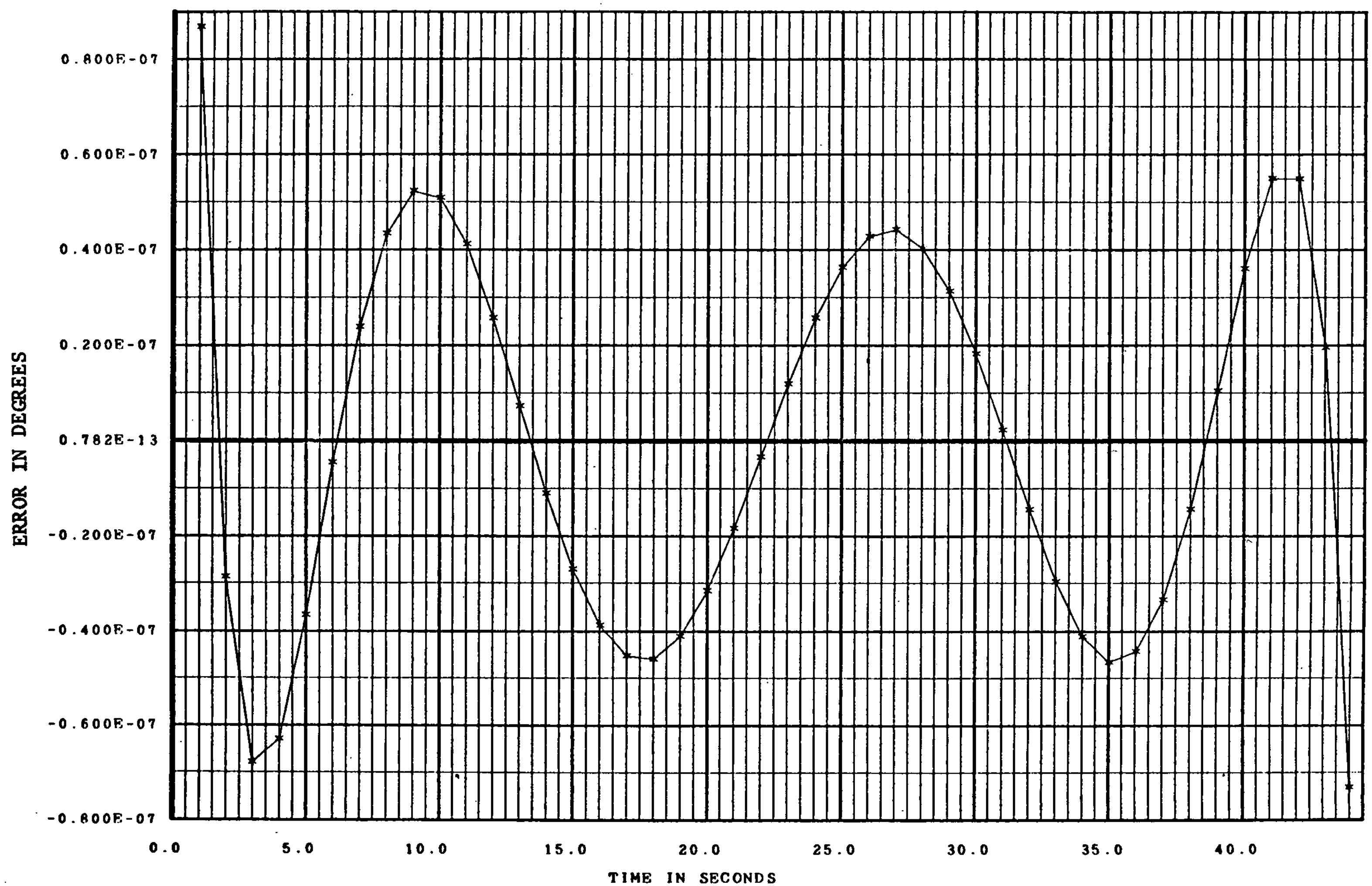

Figure A-26. Error (calculated polynomial minus true function) from using 6th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

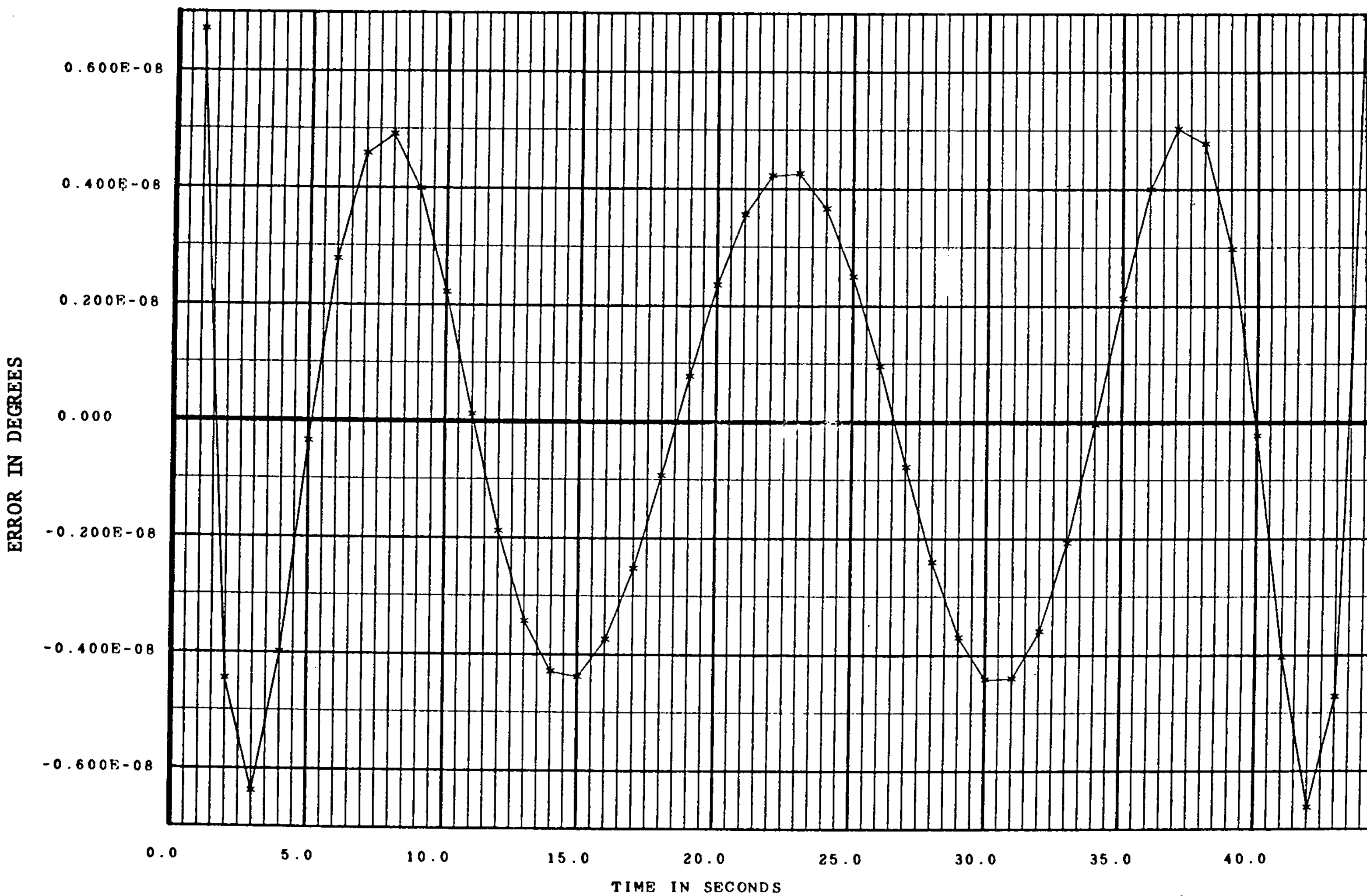

Figure A-27. Error (calculated polynomial minus true function) from using 7th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

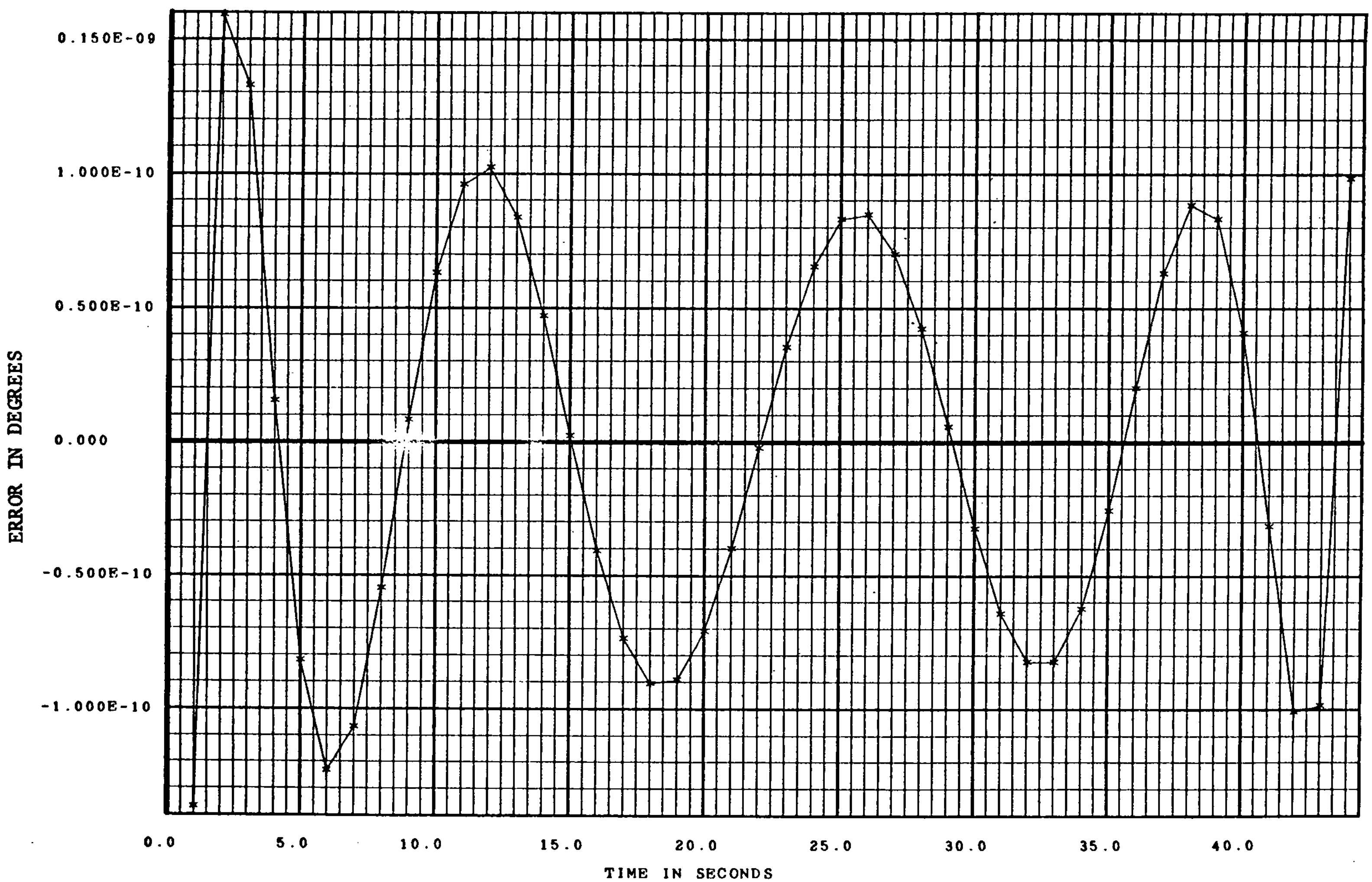

Figure A-28. Error (calculated polynomial minus true function) from using 8th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

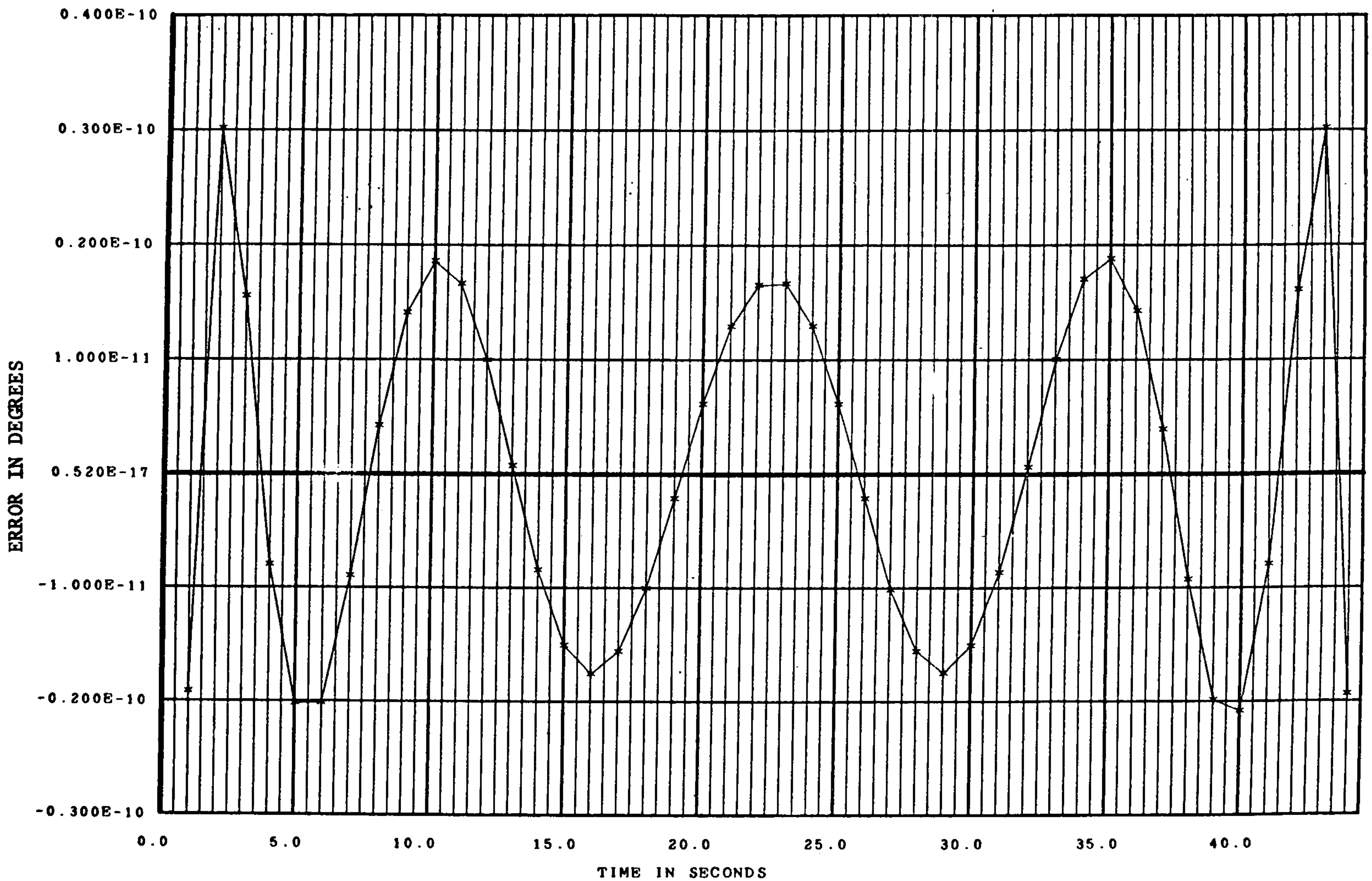

Figure A-29. Error (calculated polynomial minus true function) from using 9th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

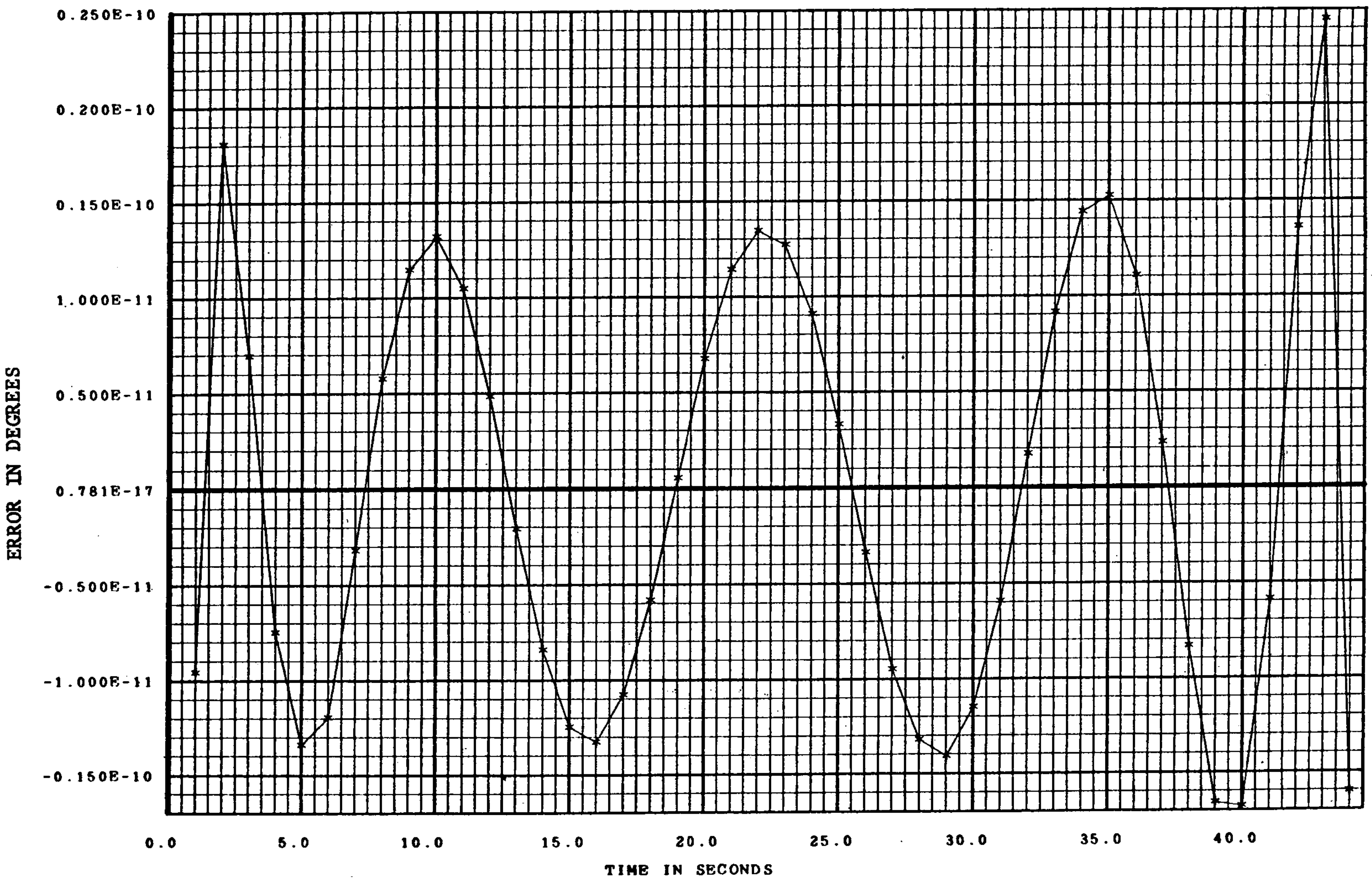

Figure A-30. Error (calculated polynomial minus true function) from using 10th degree polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

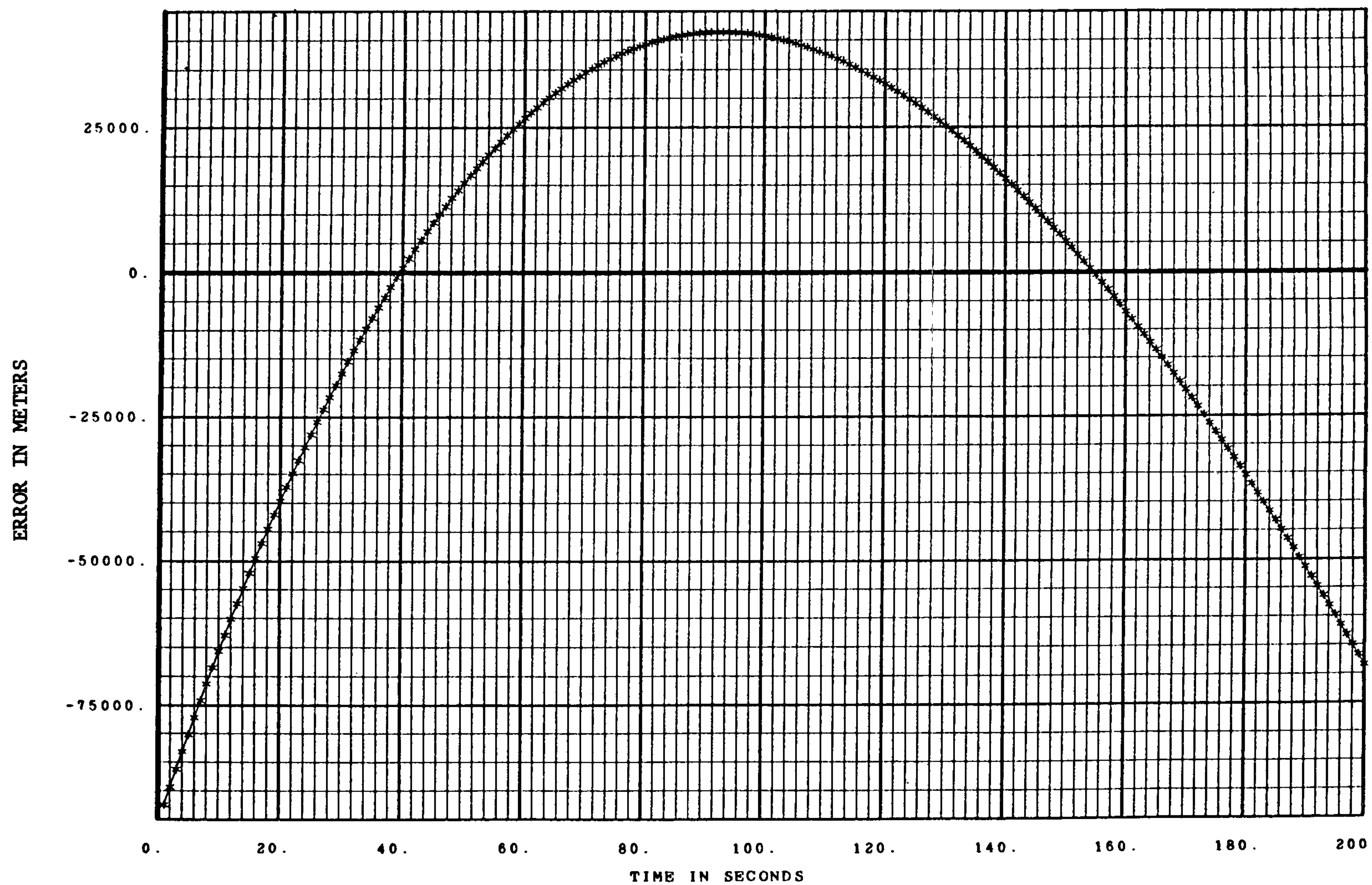

Figure A-31. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

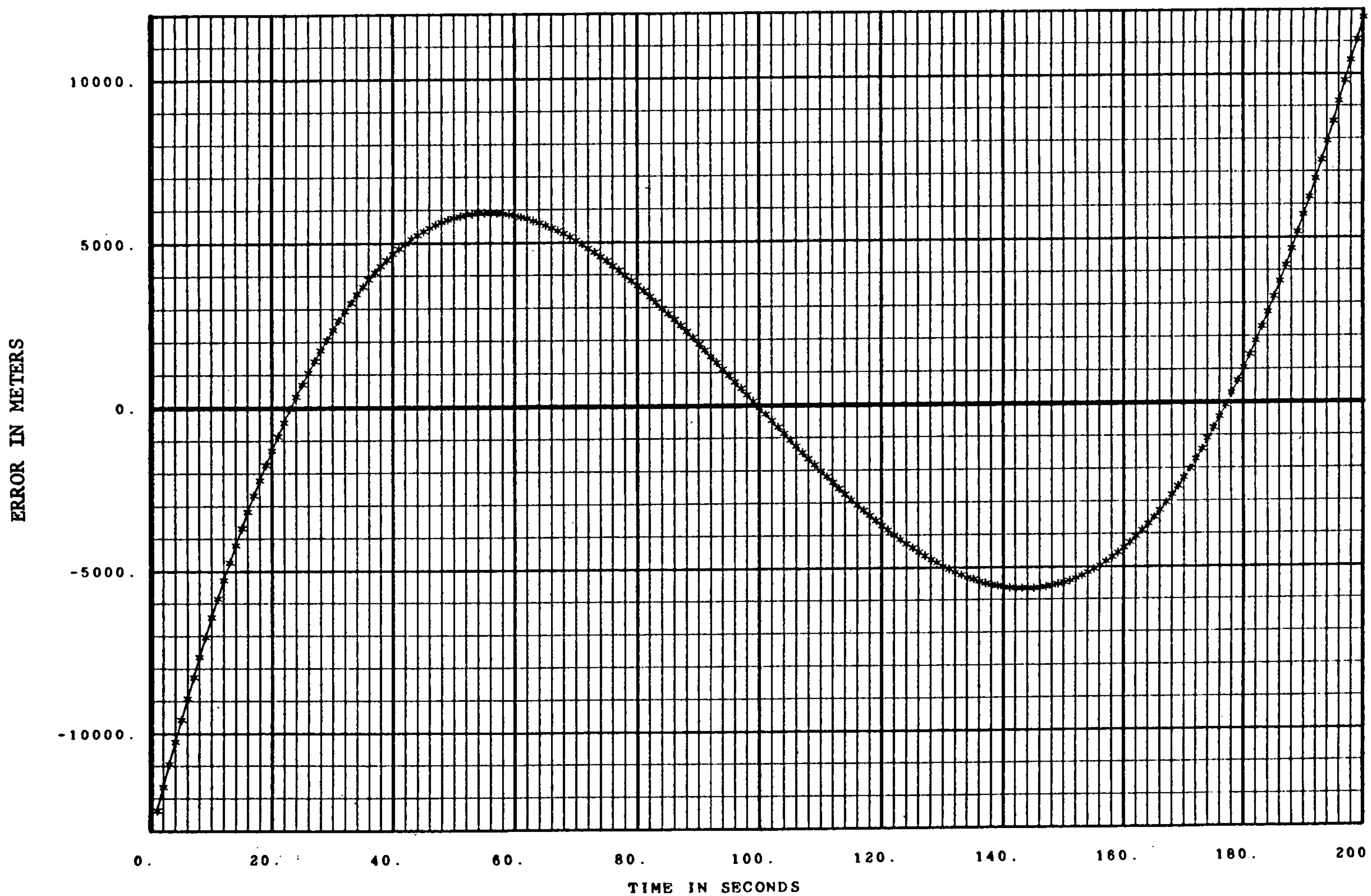

Figure A-32. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

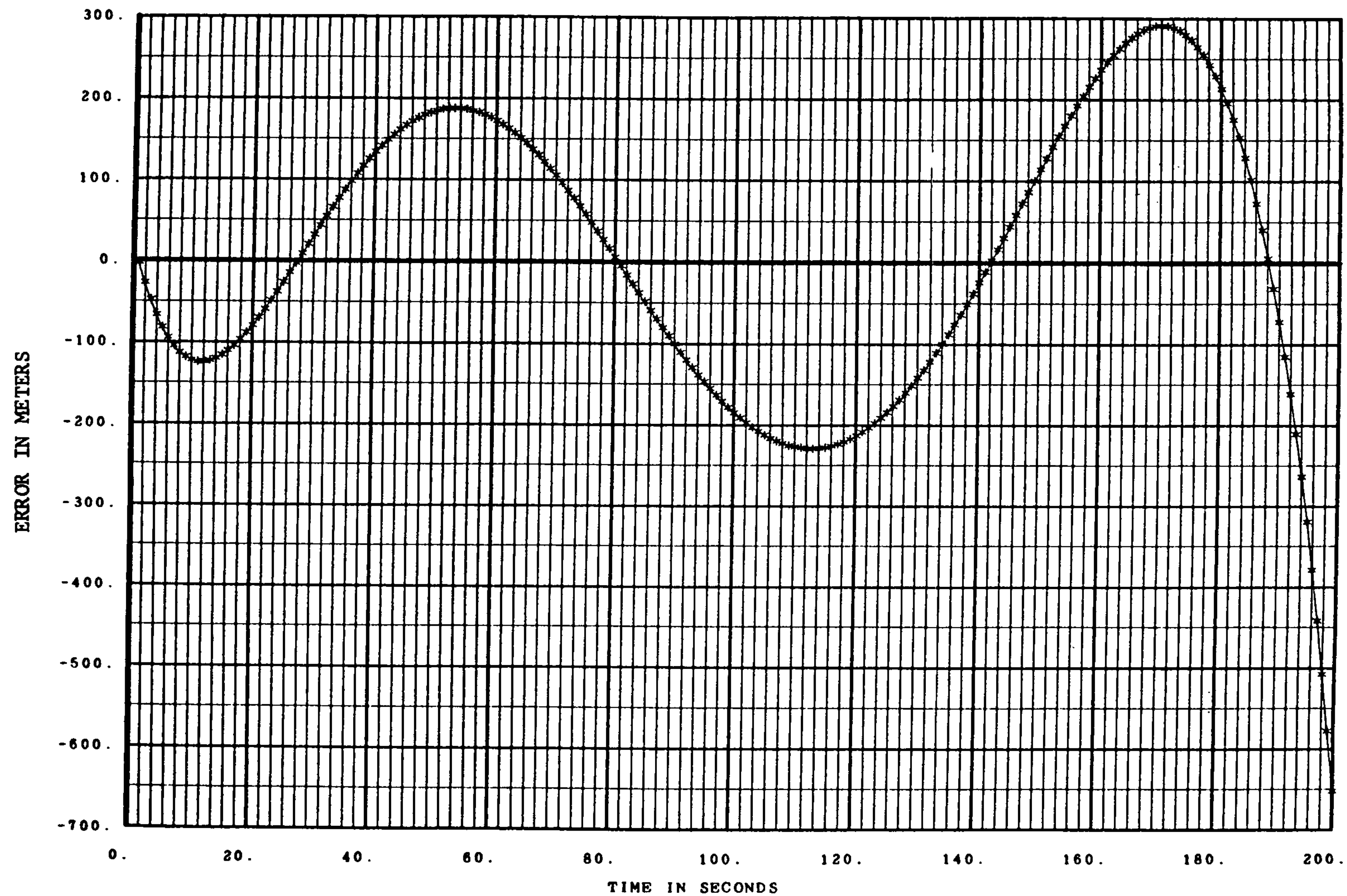

Figure A-33. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

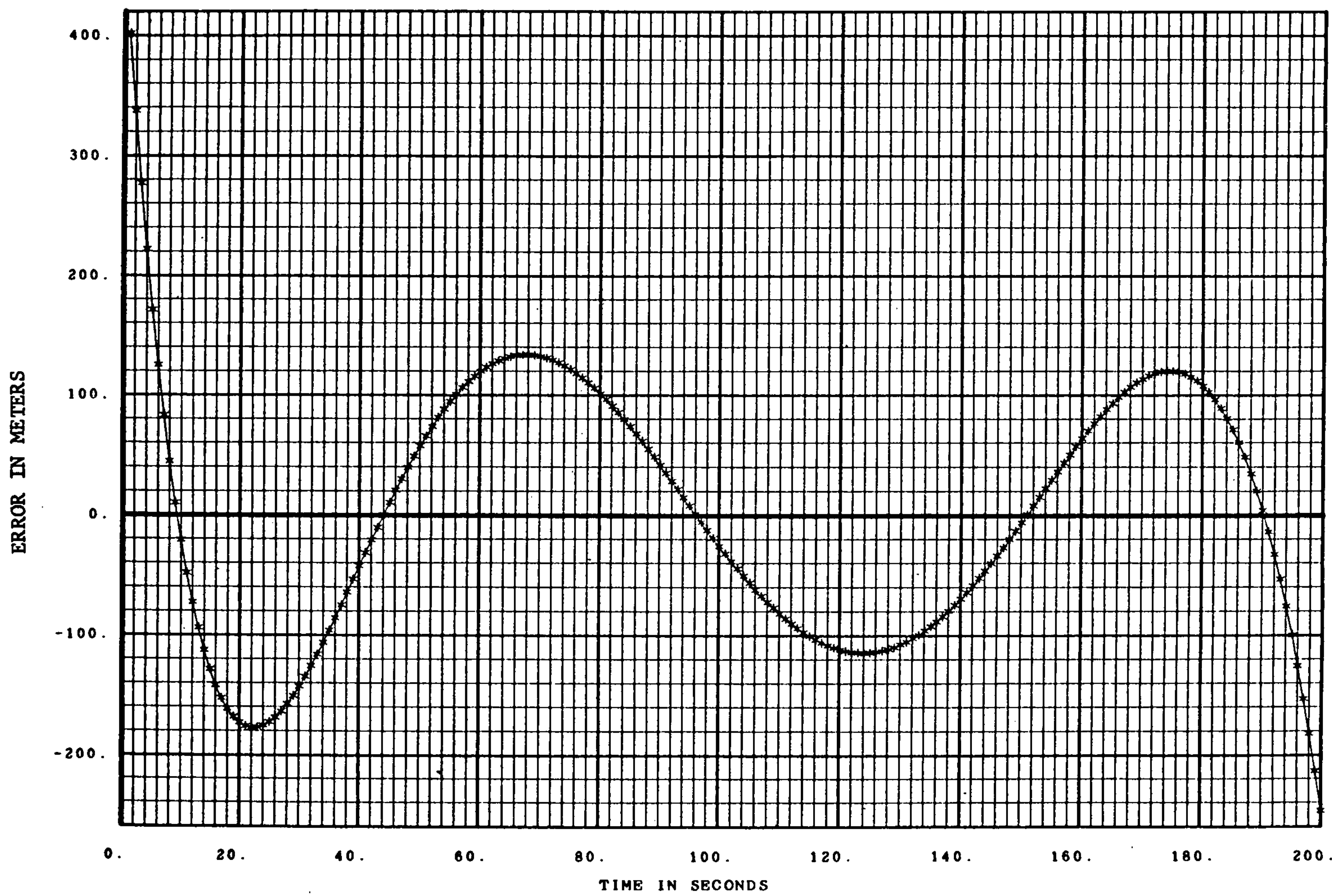

Figure A-34. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

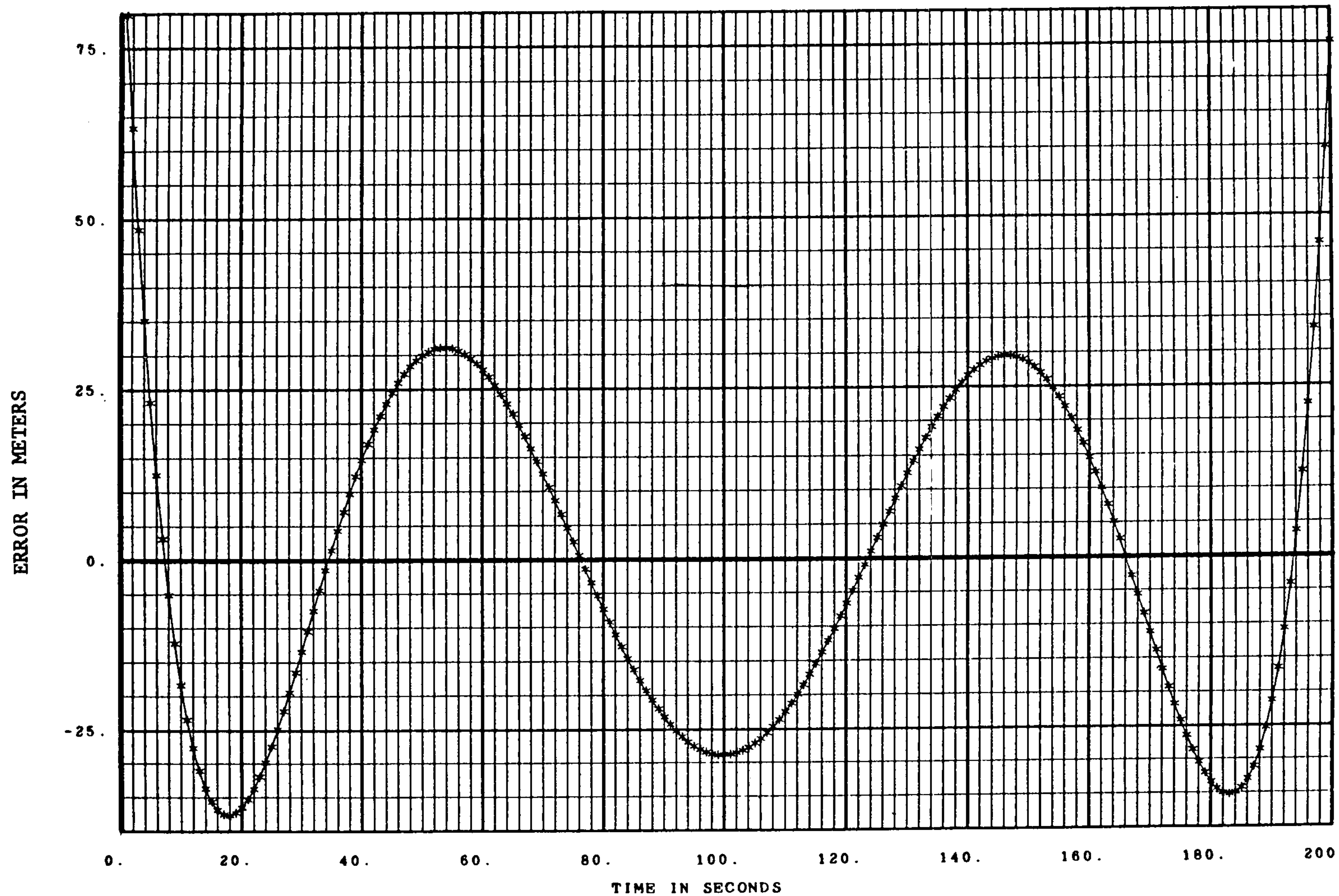

Figure A-35. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

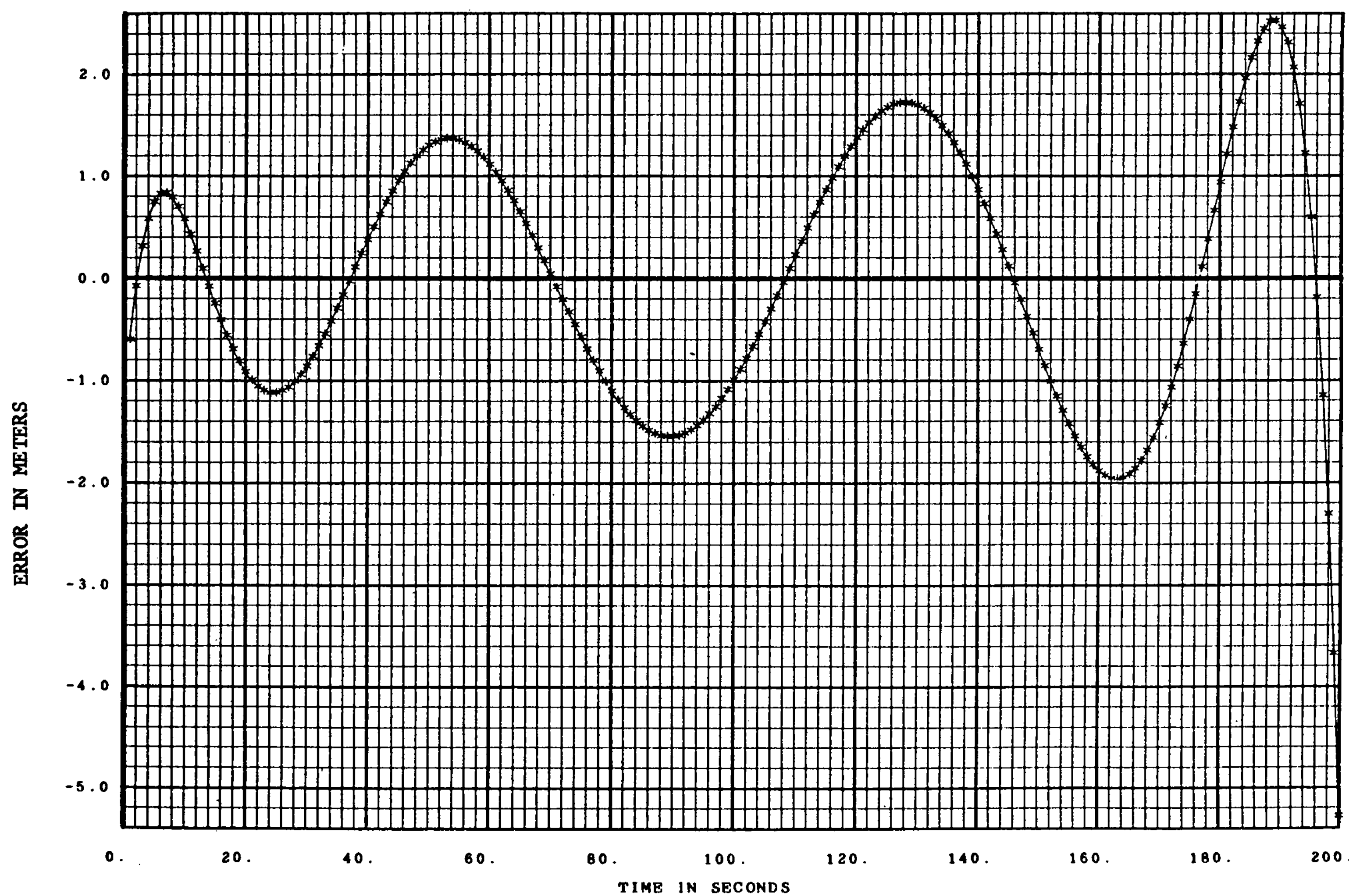

Figure A-36. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

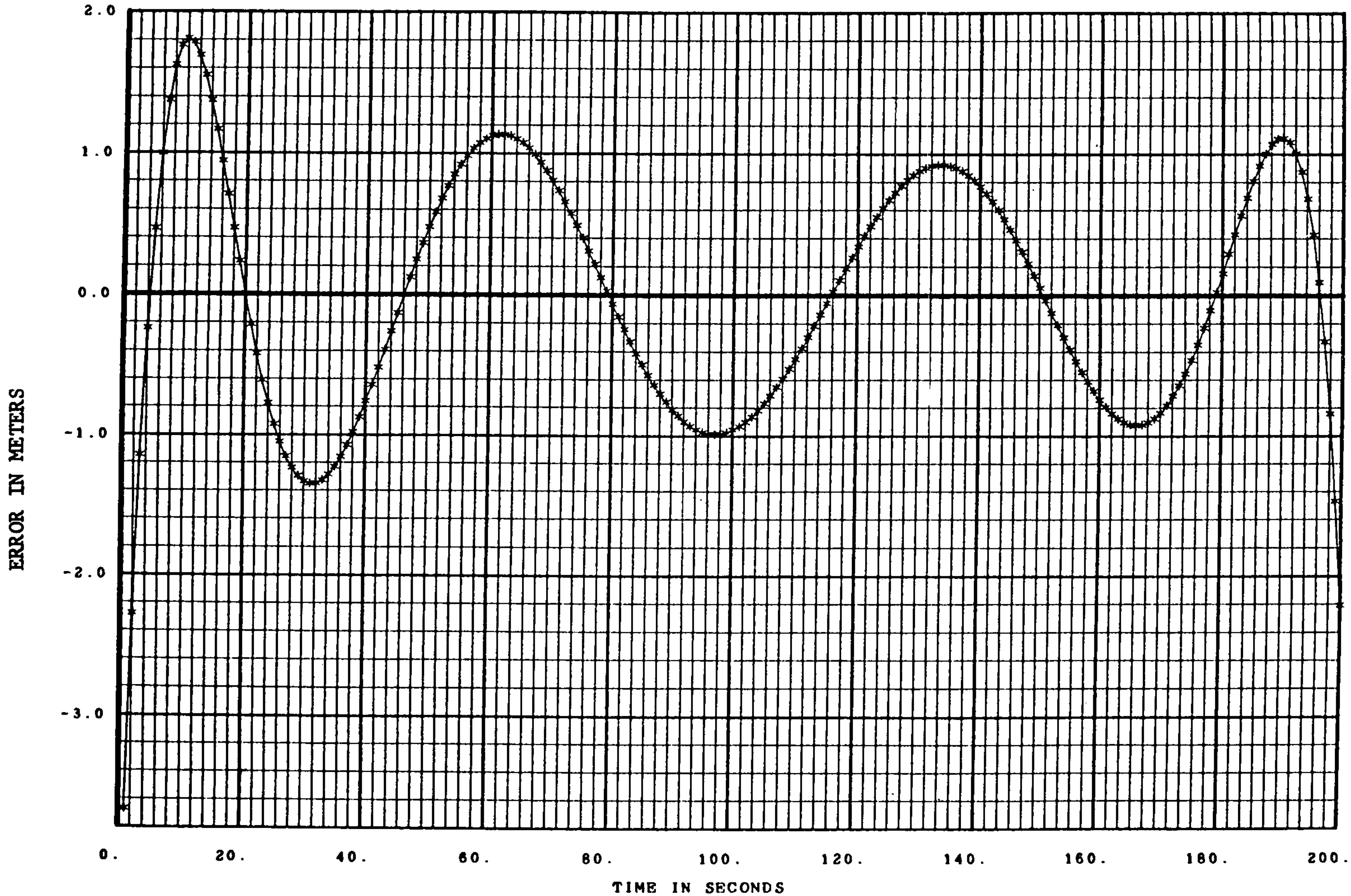

Figure A-37. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

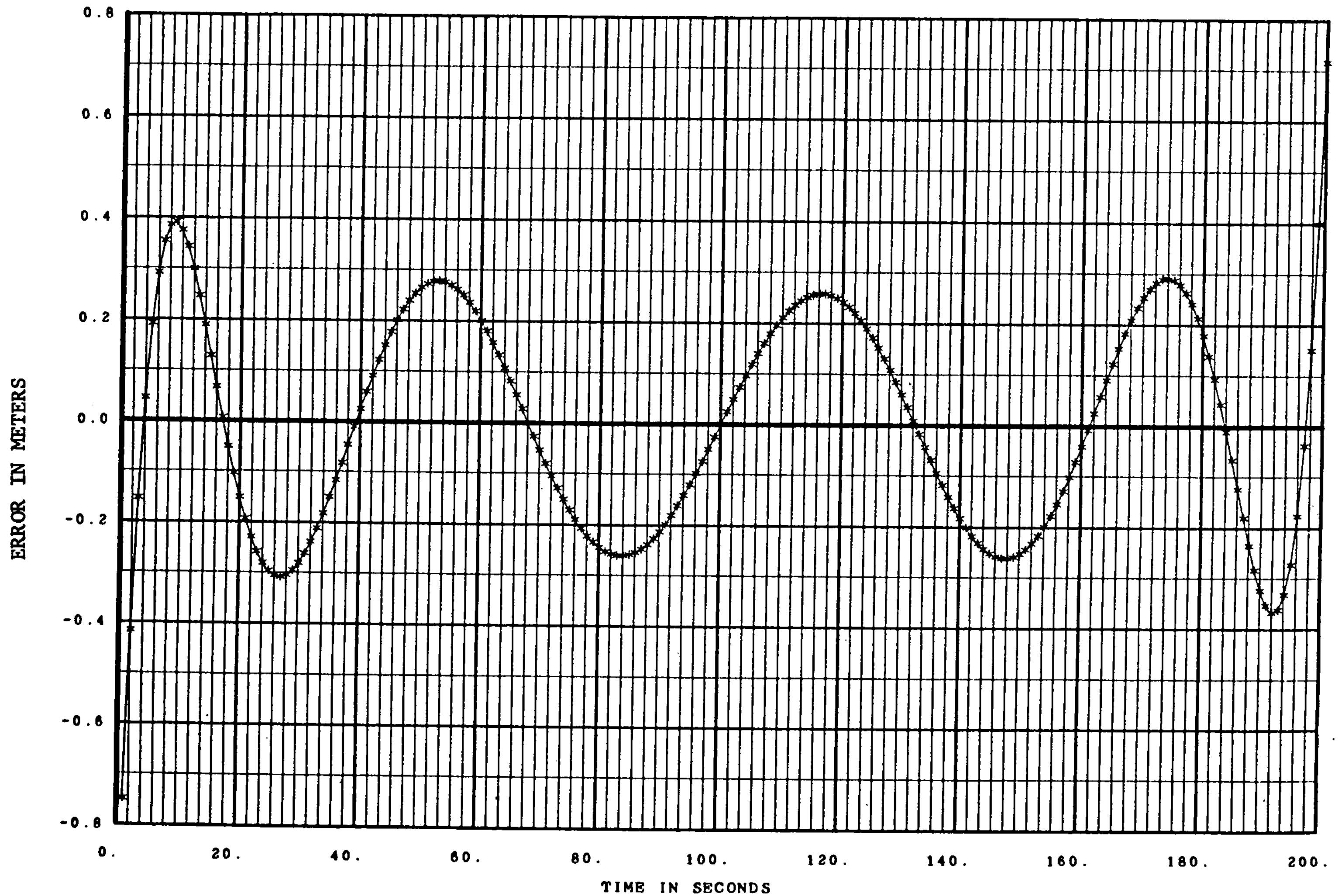

Figure A-38. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

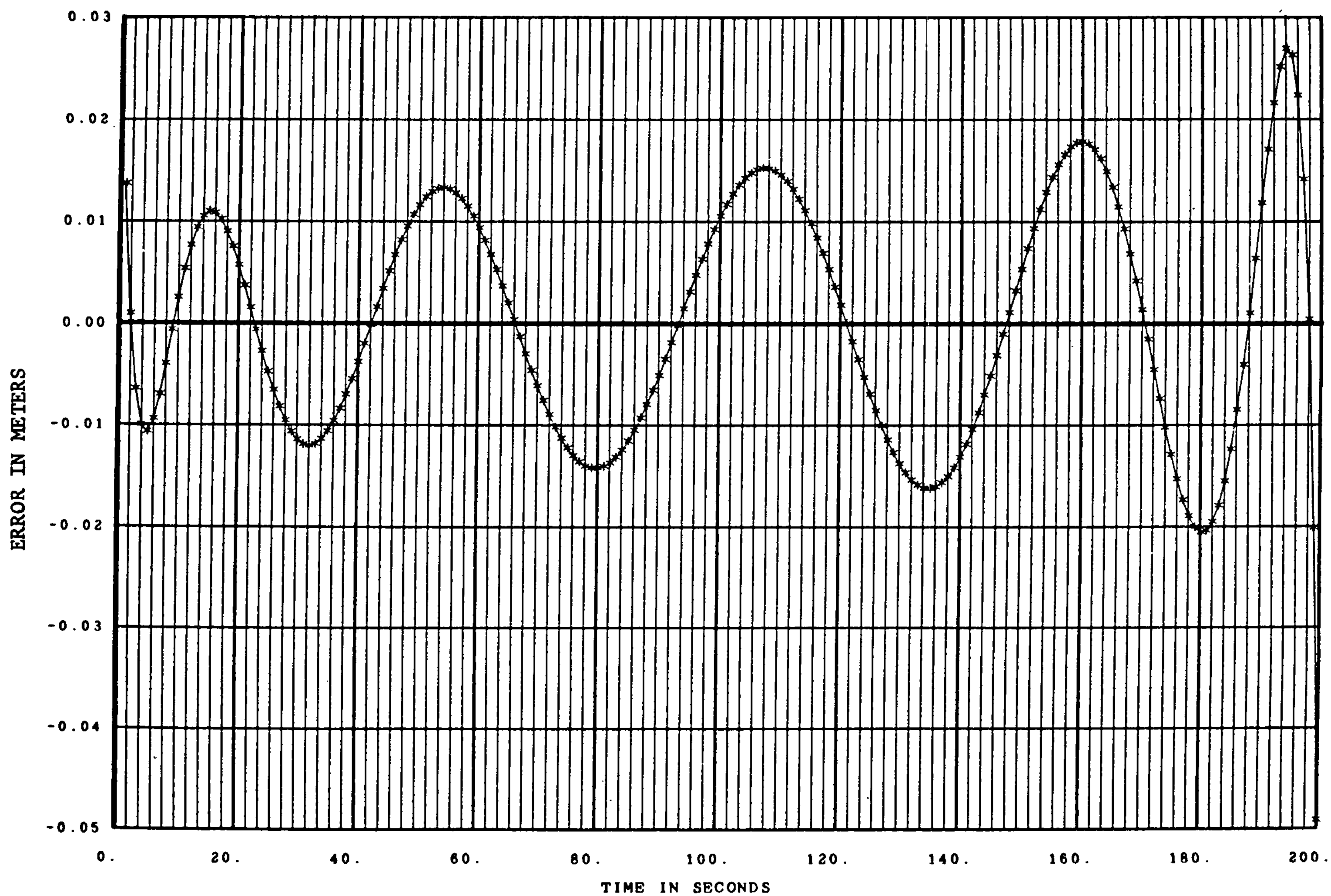

Figure A-39. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

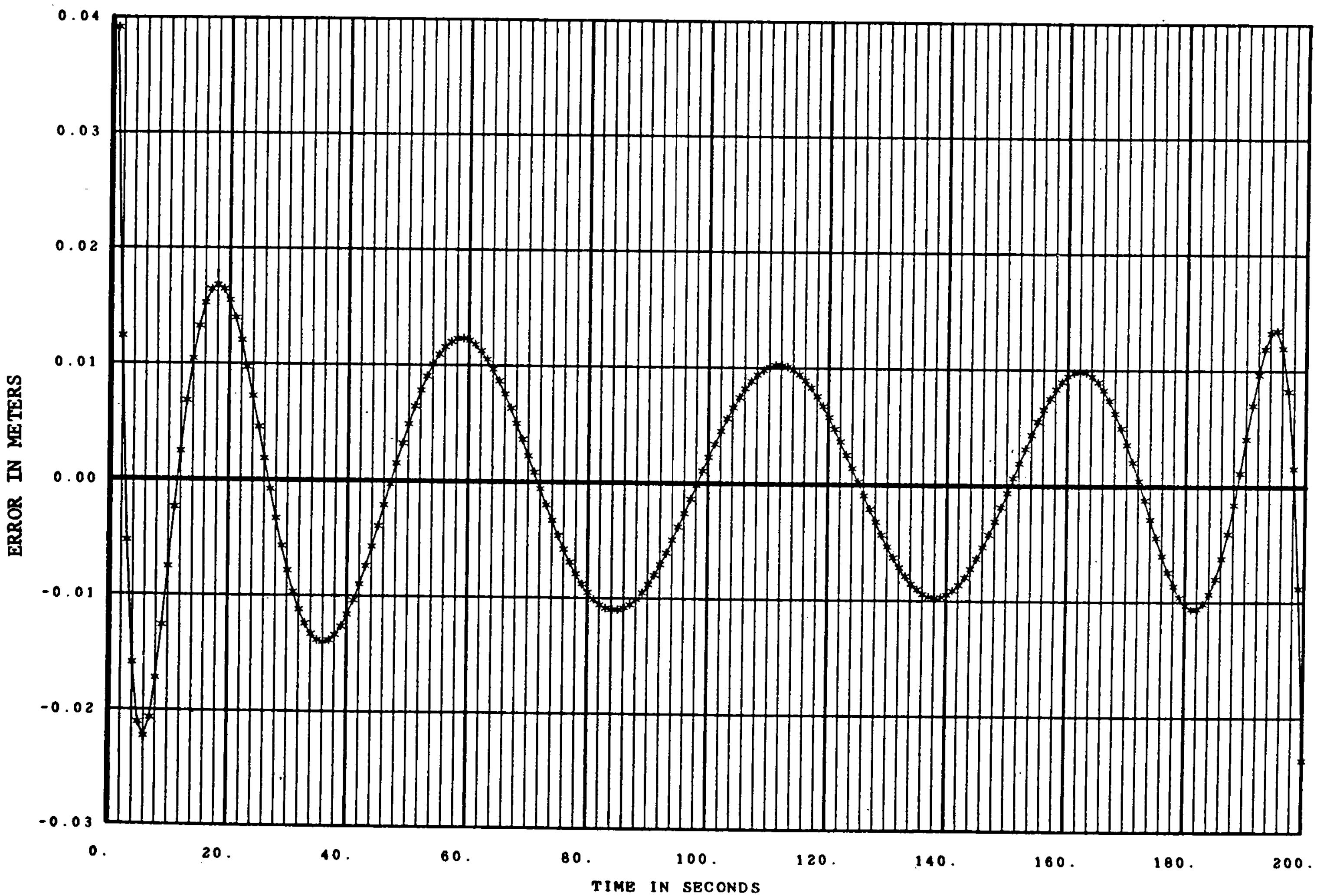

Figure A-40. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

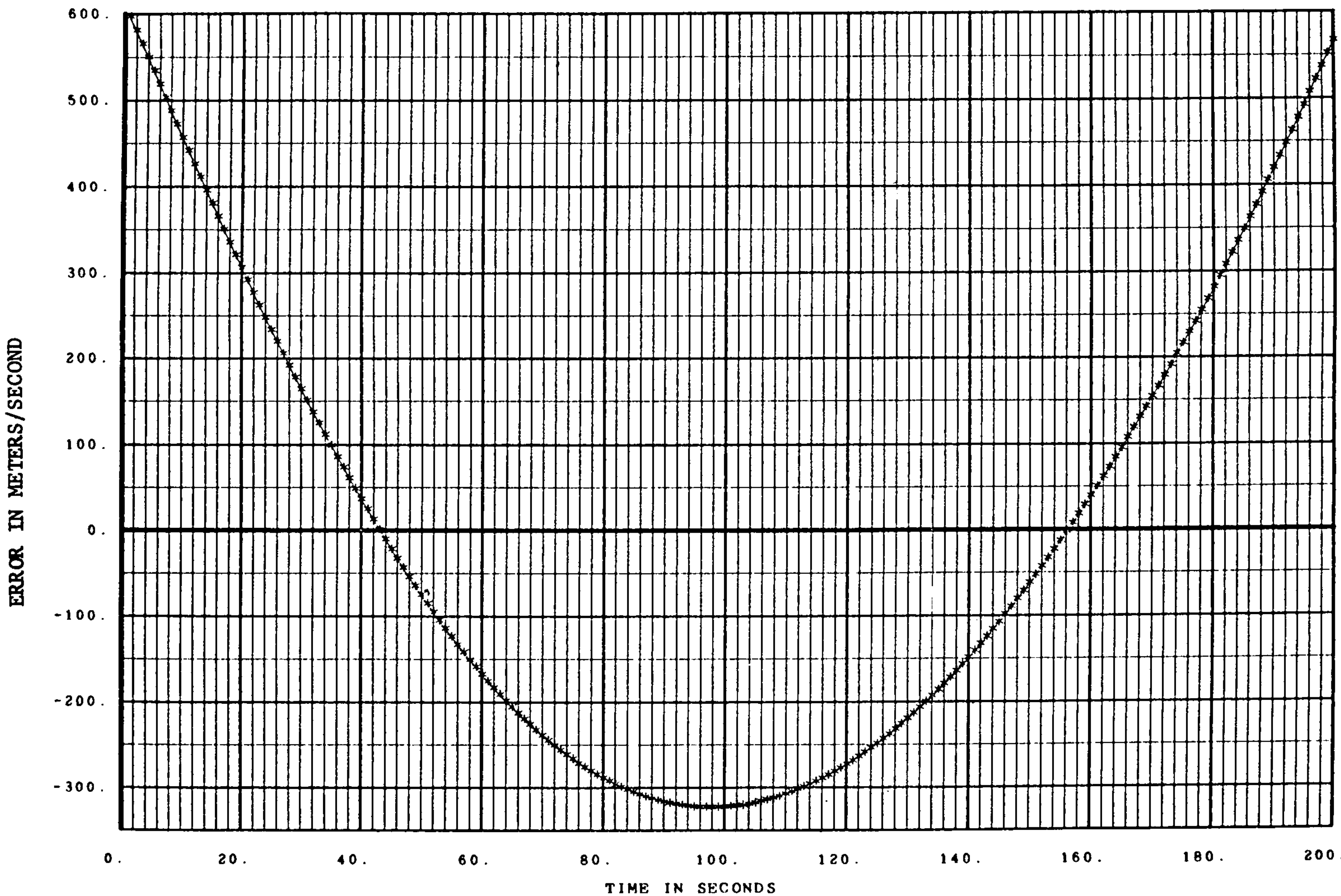

Figure A-41. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

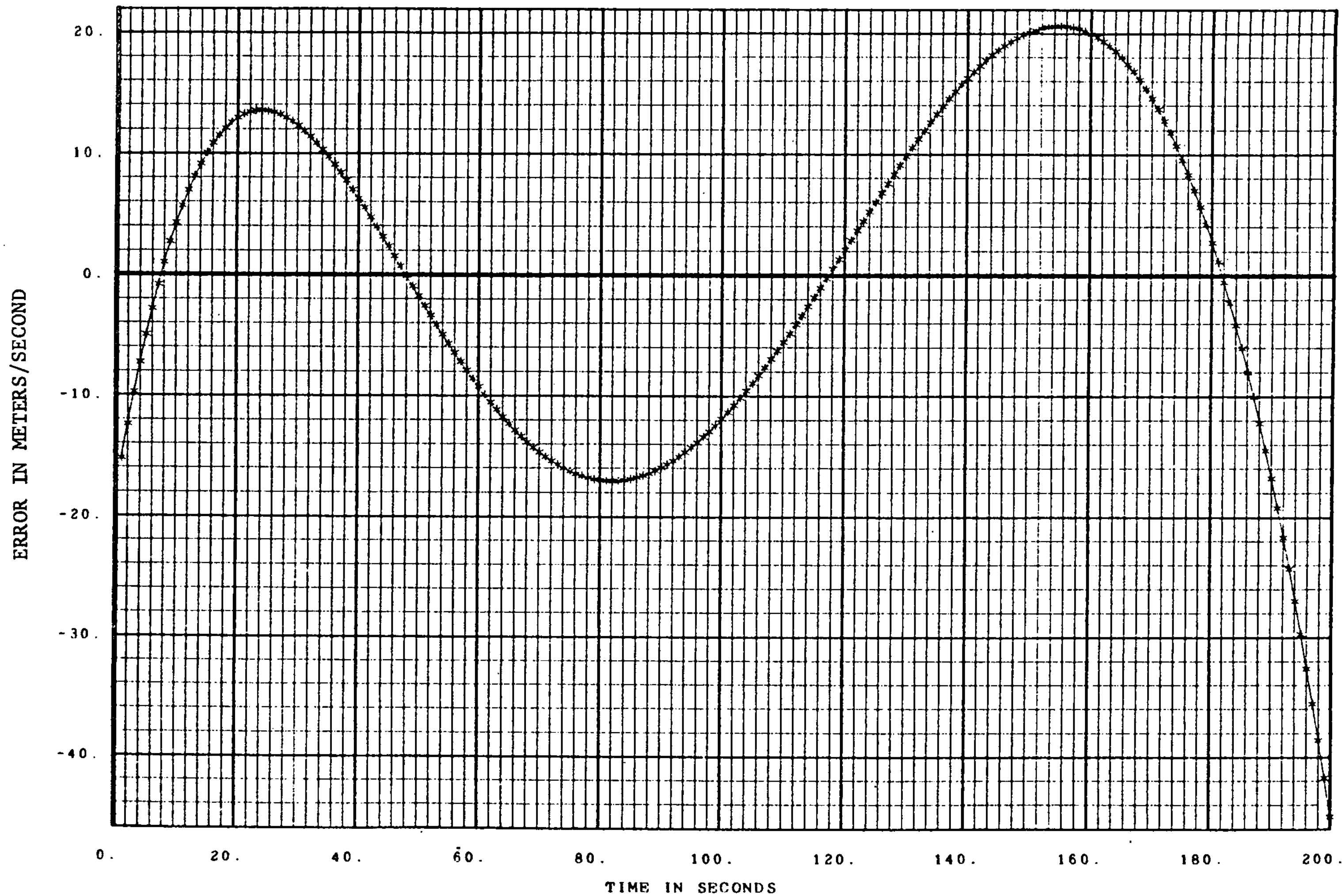

Figure A-42. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

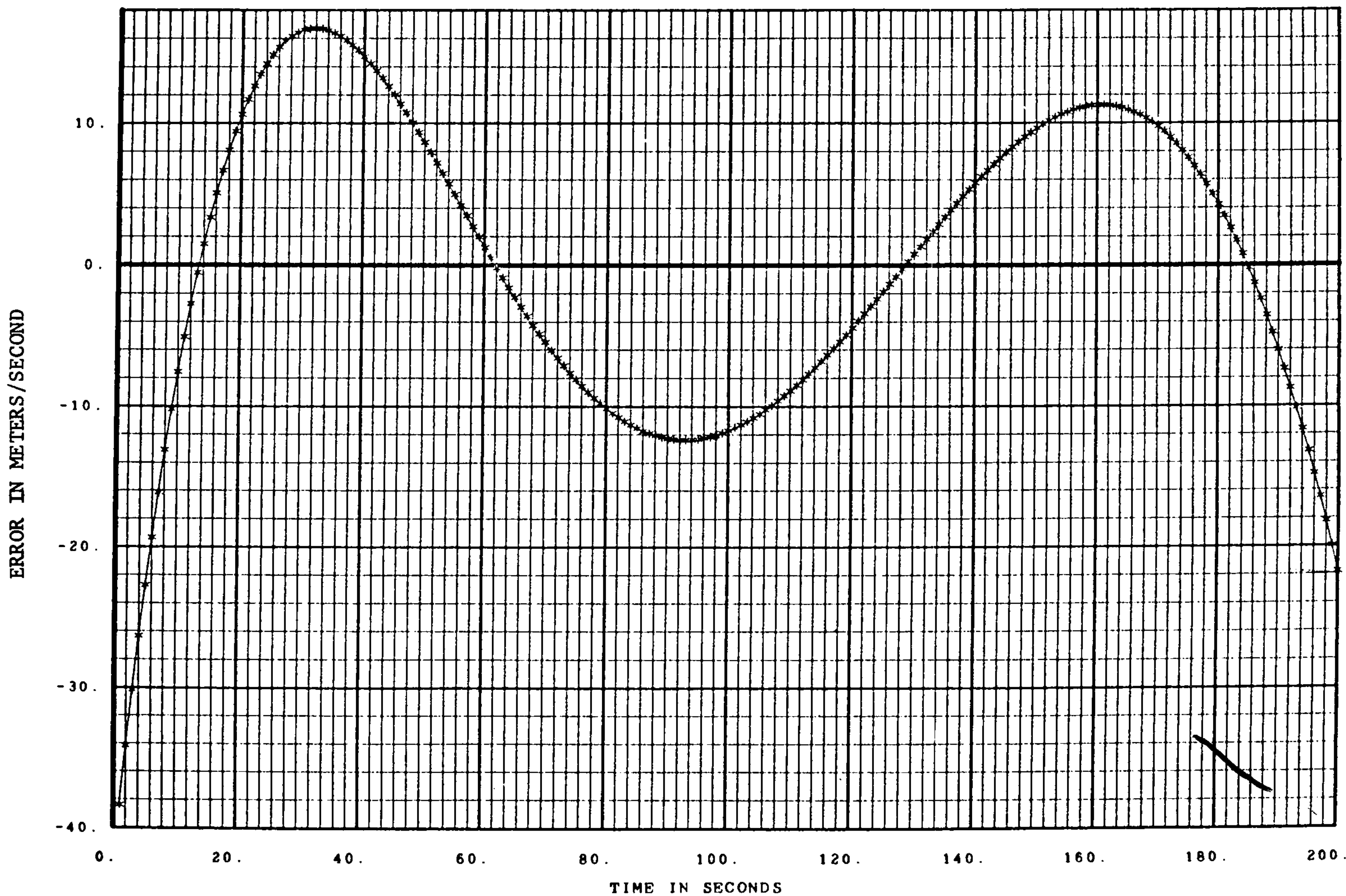

Figure A-43. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

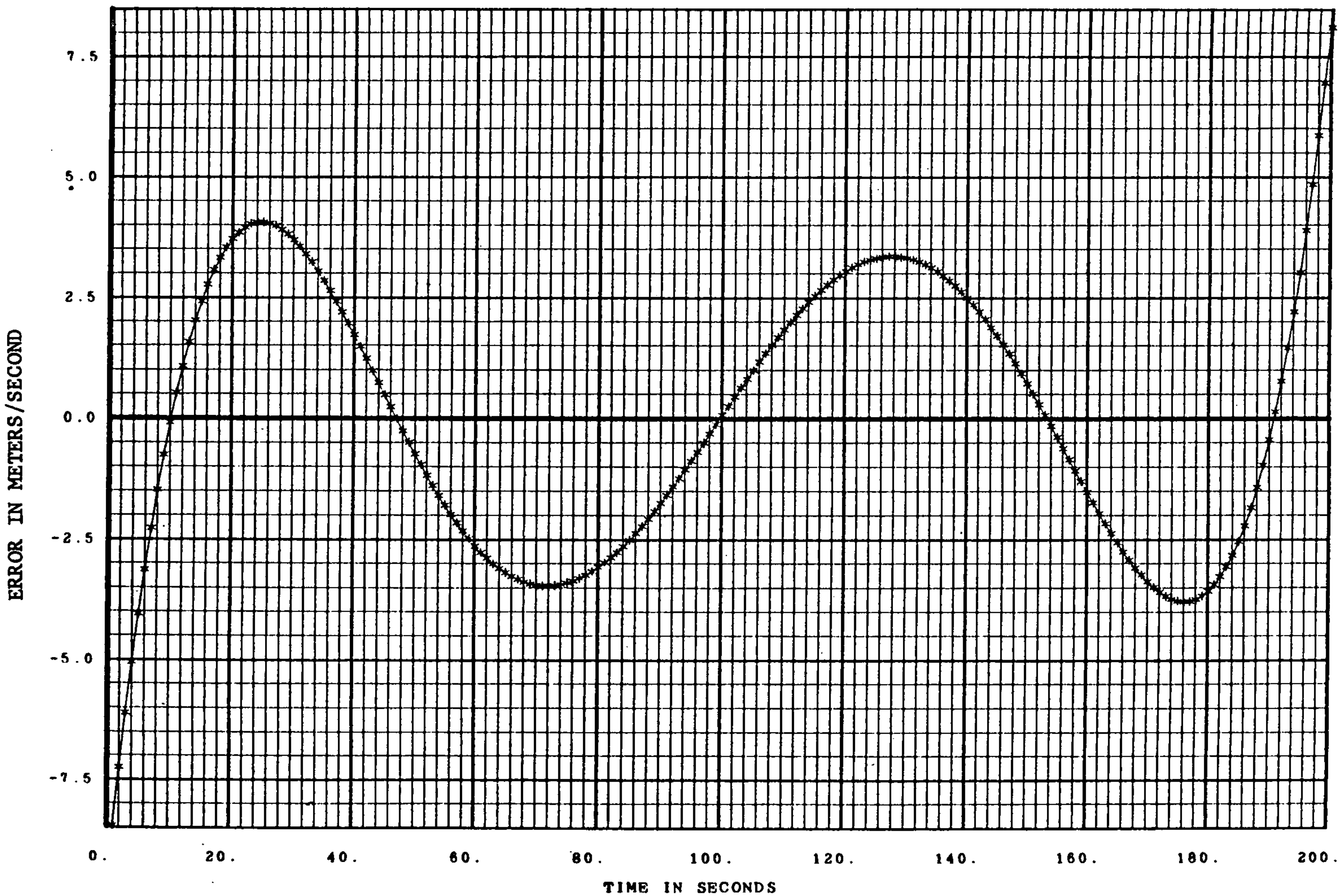

Figure A-44. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

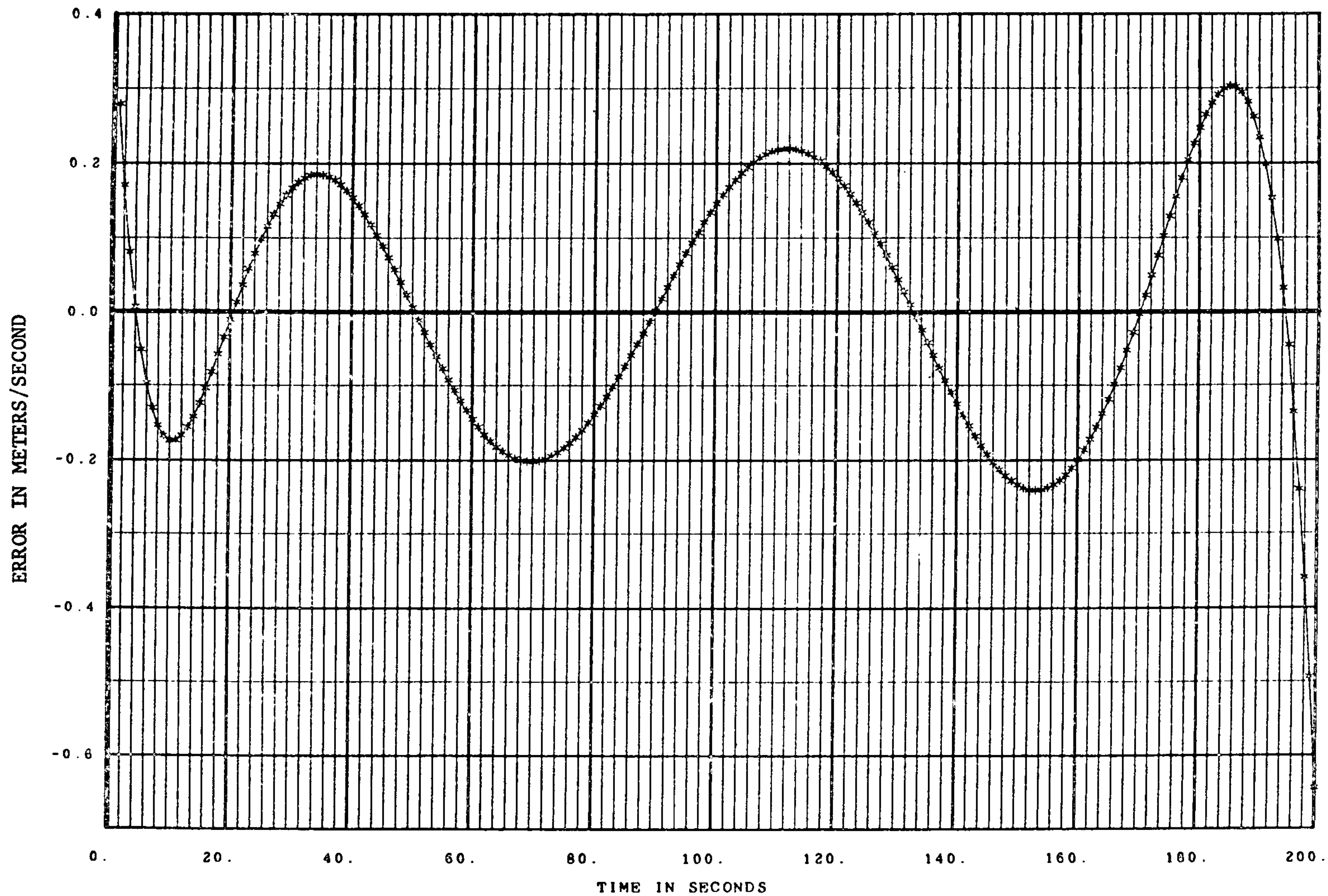

Figure A-45. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, $a = 7378$ km, $e = 0$, zero time at spacecraft crossing station zenith (overhead pass).

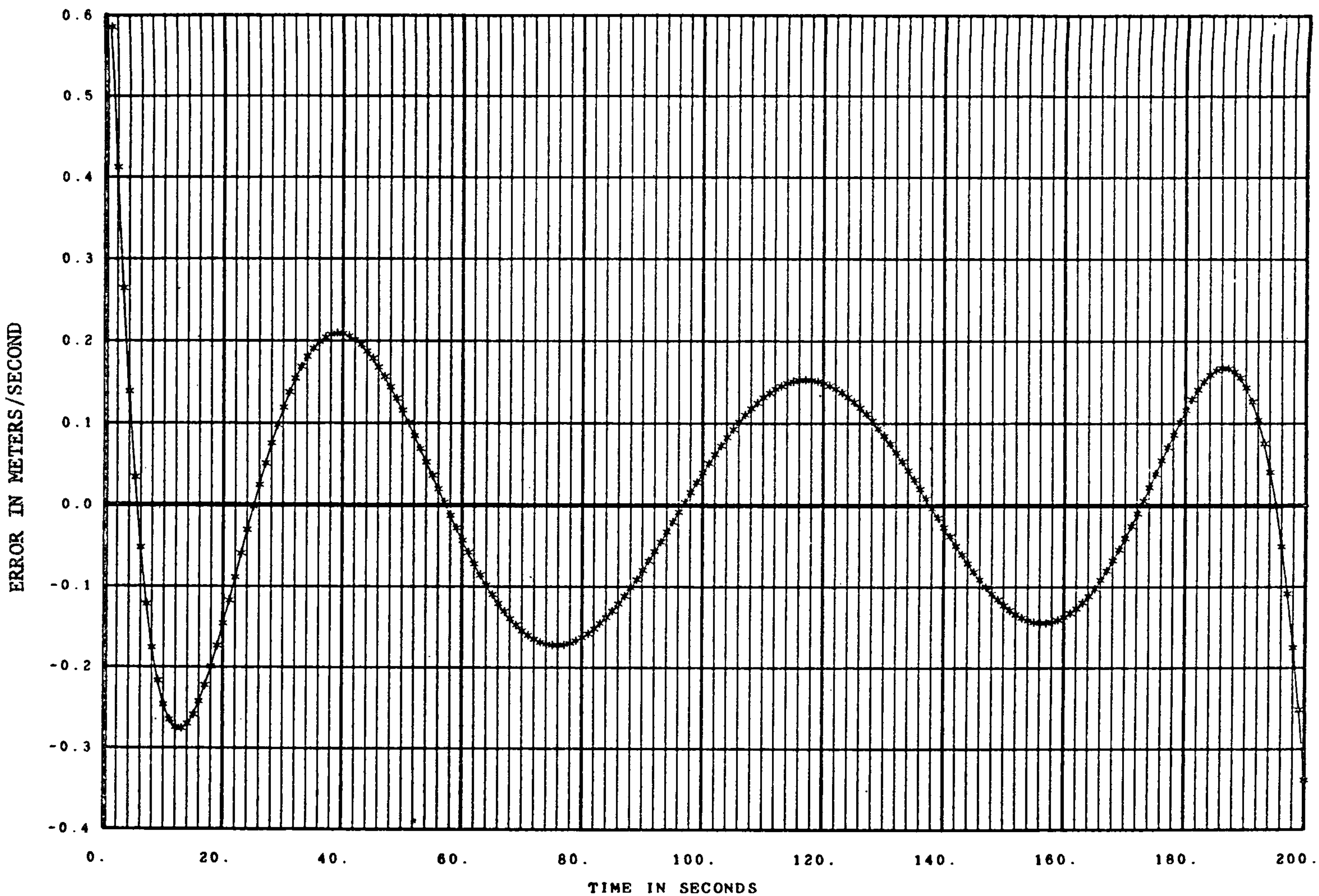

Figure A-46. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

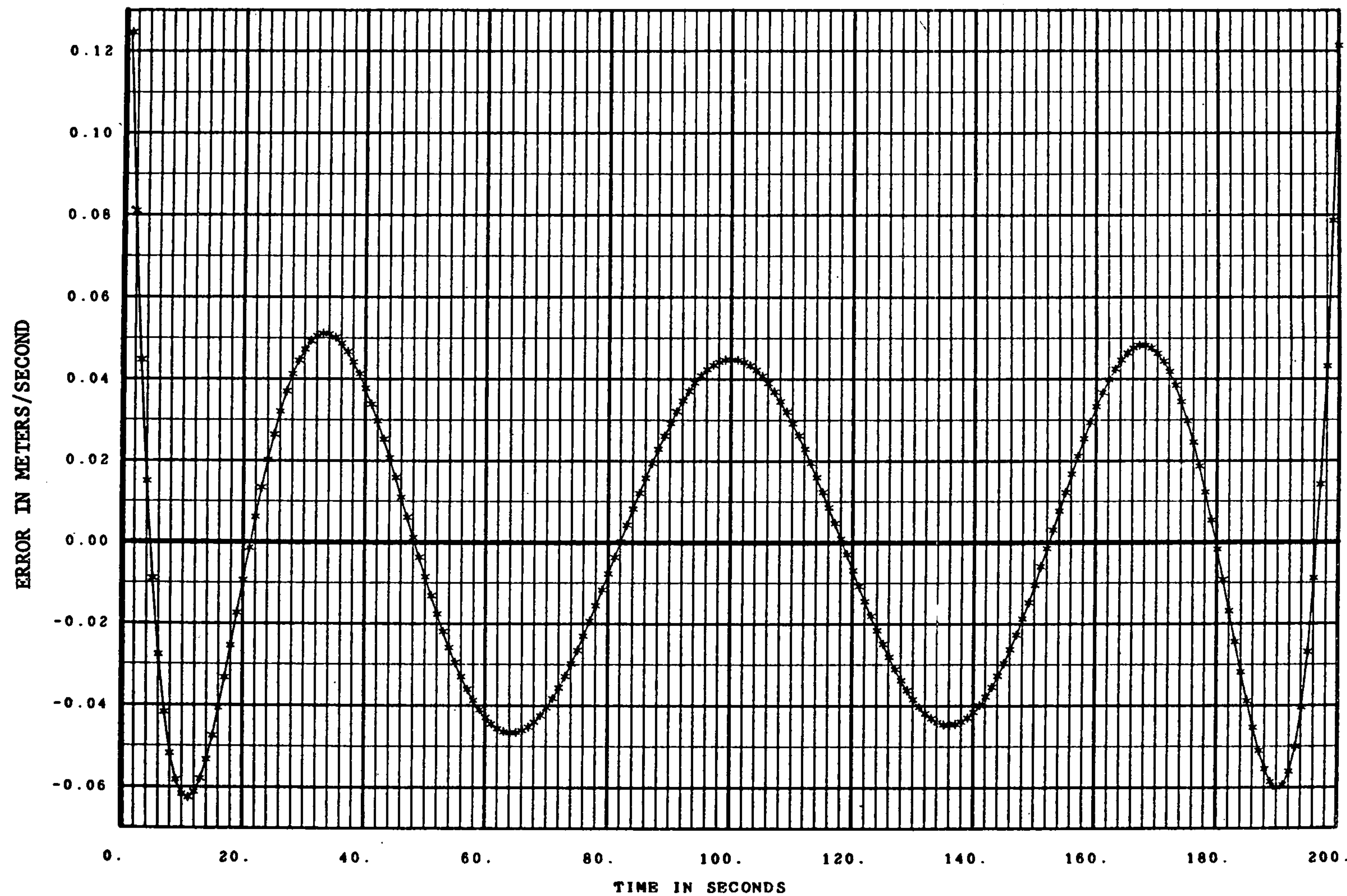

Figure A-47. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

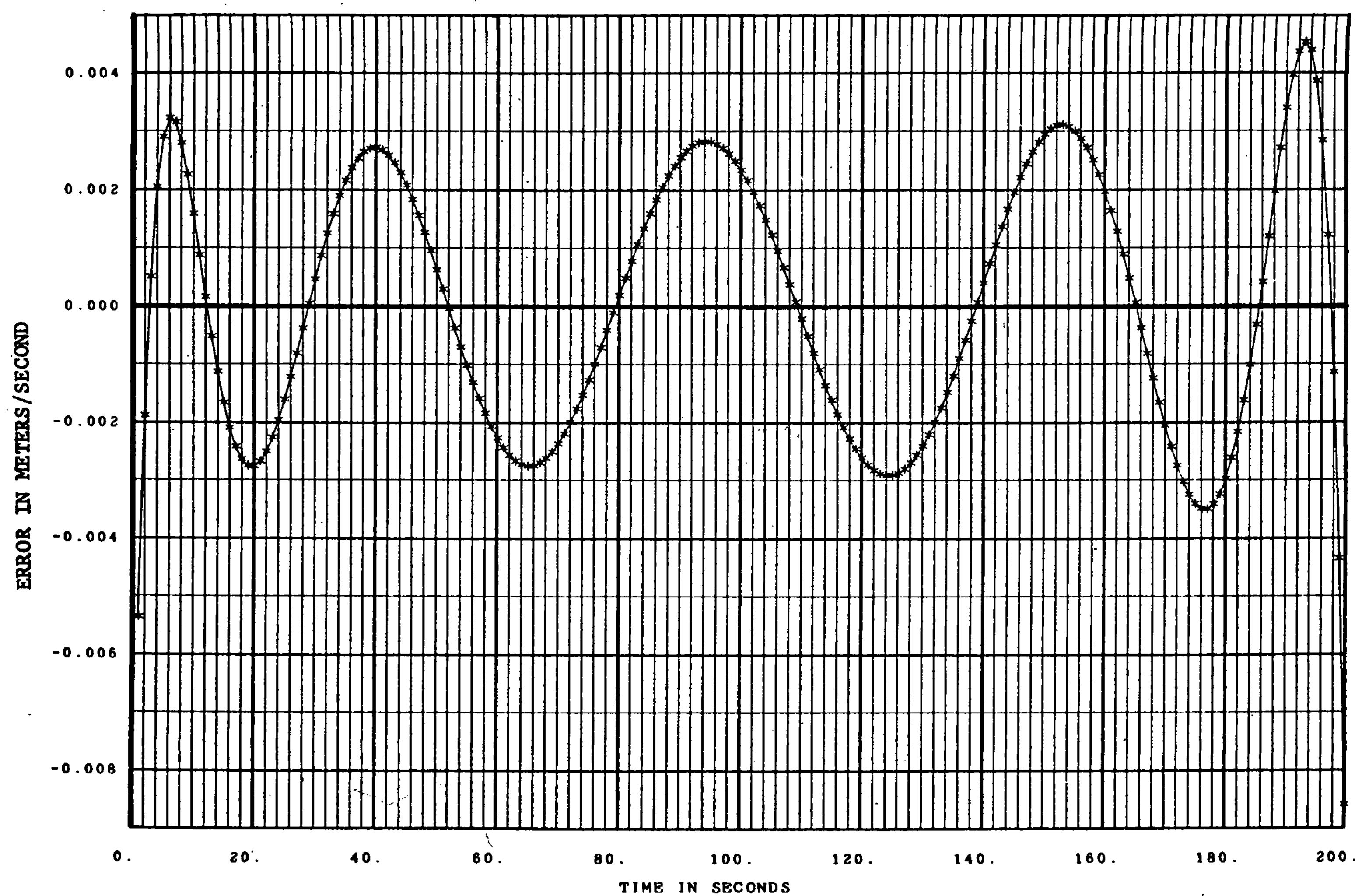

Figure A-48. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

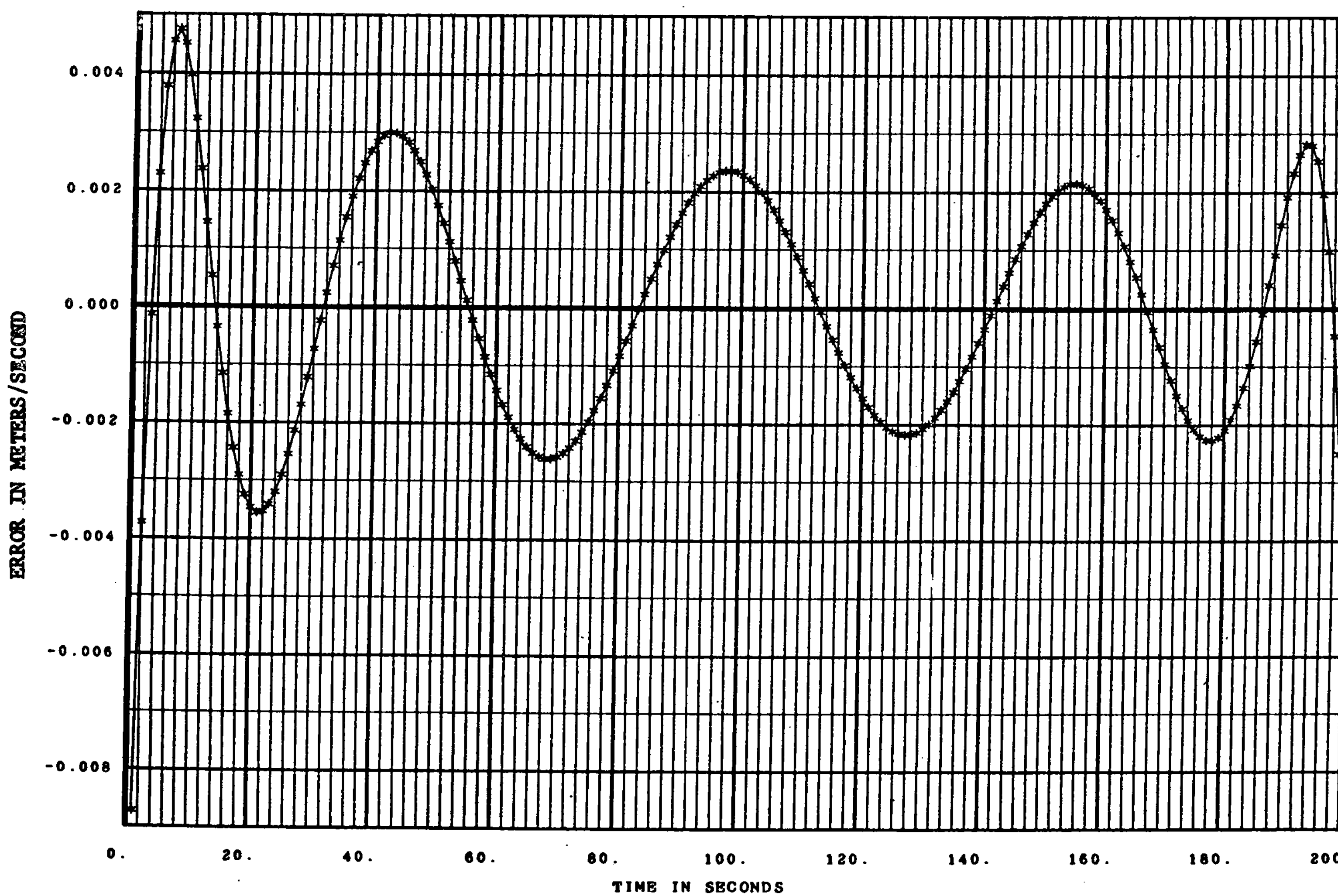

Figure A-49. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

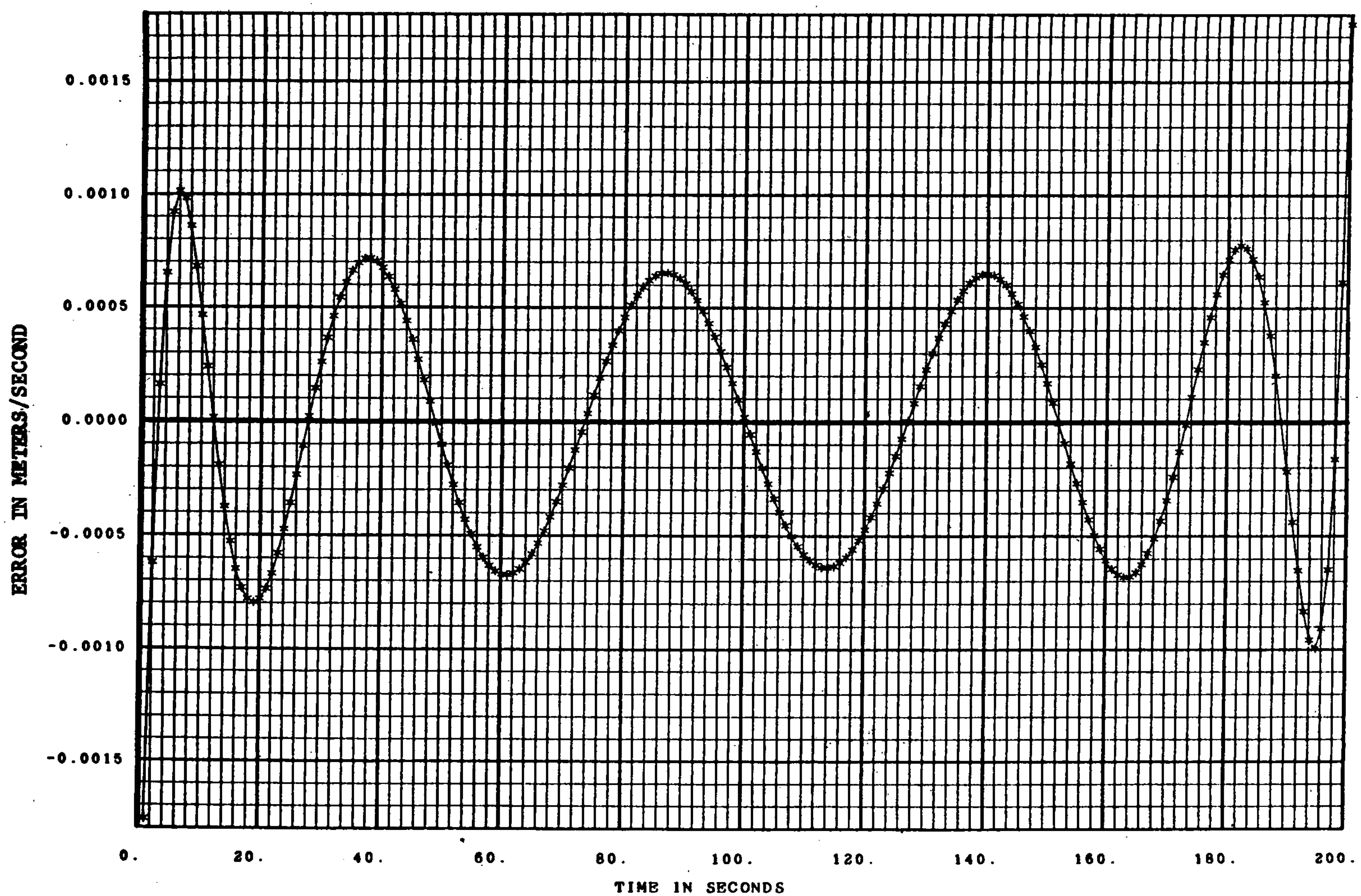

Figure A-50. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

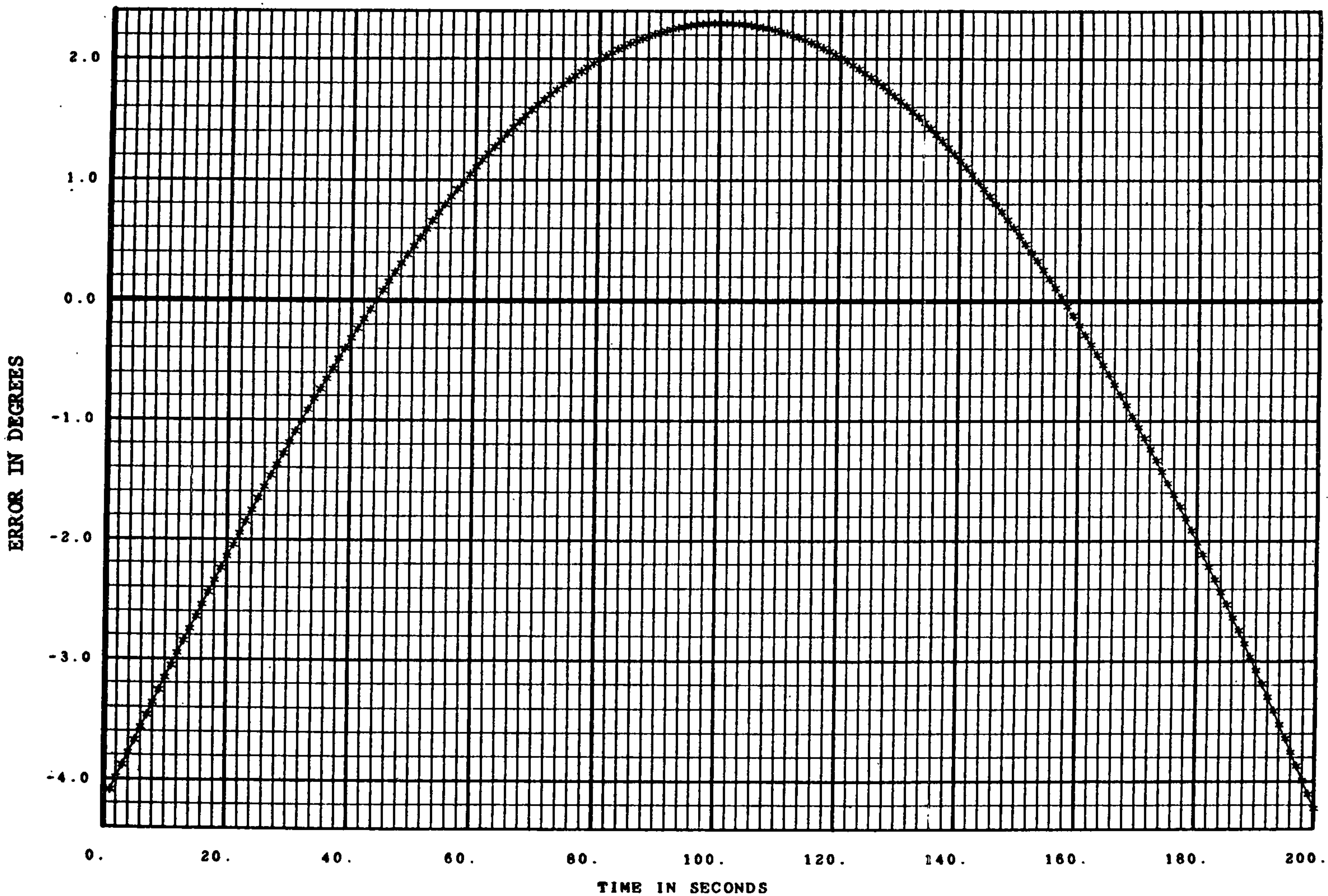

Figure A-51. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

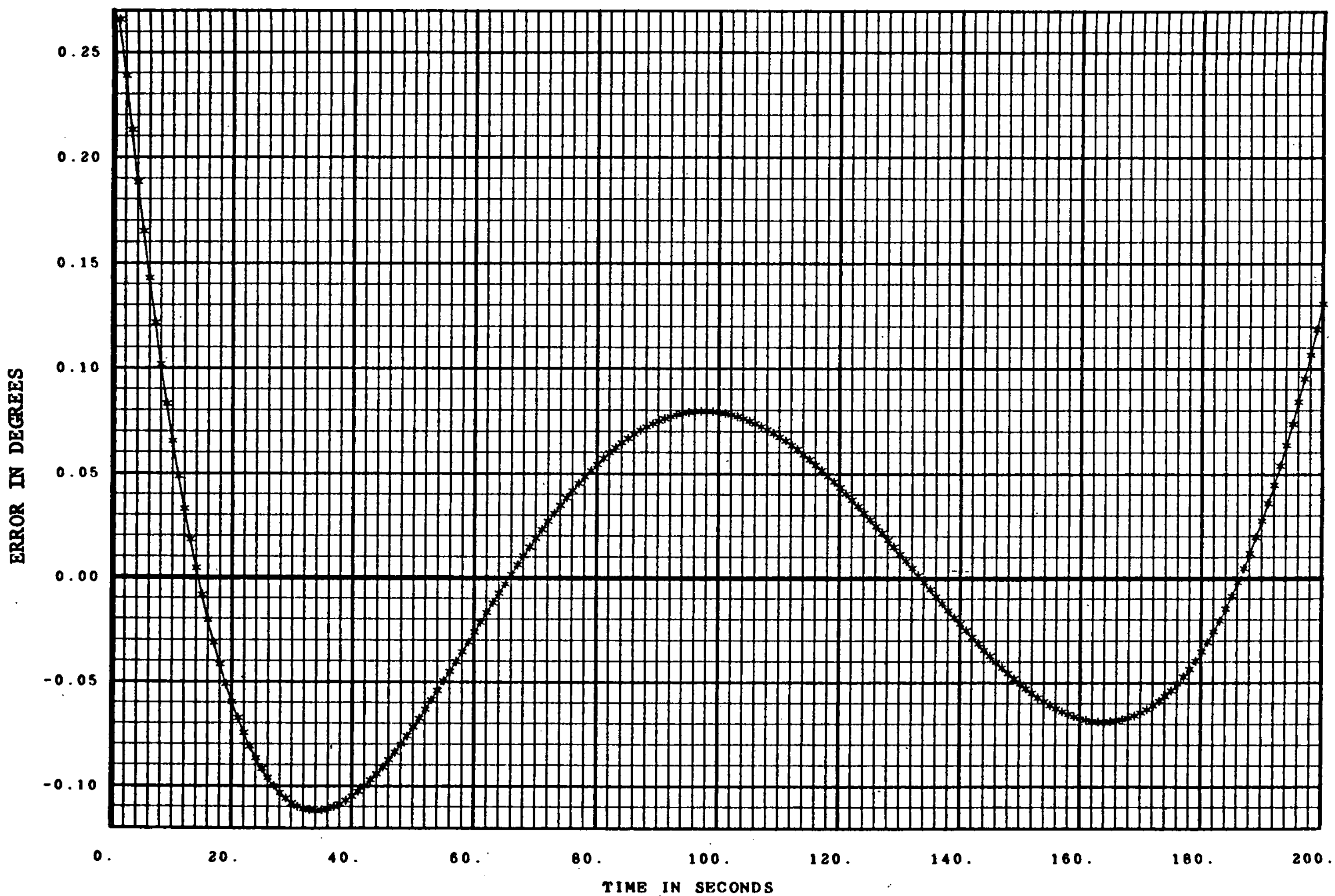

Figure A-52. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

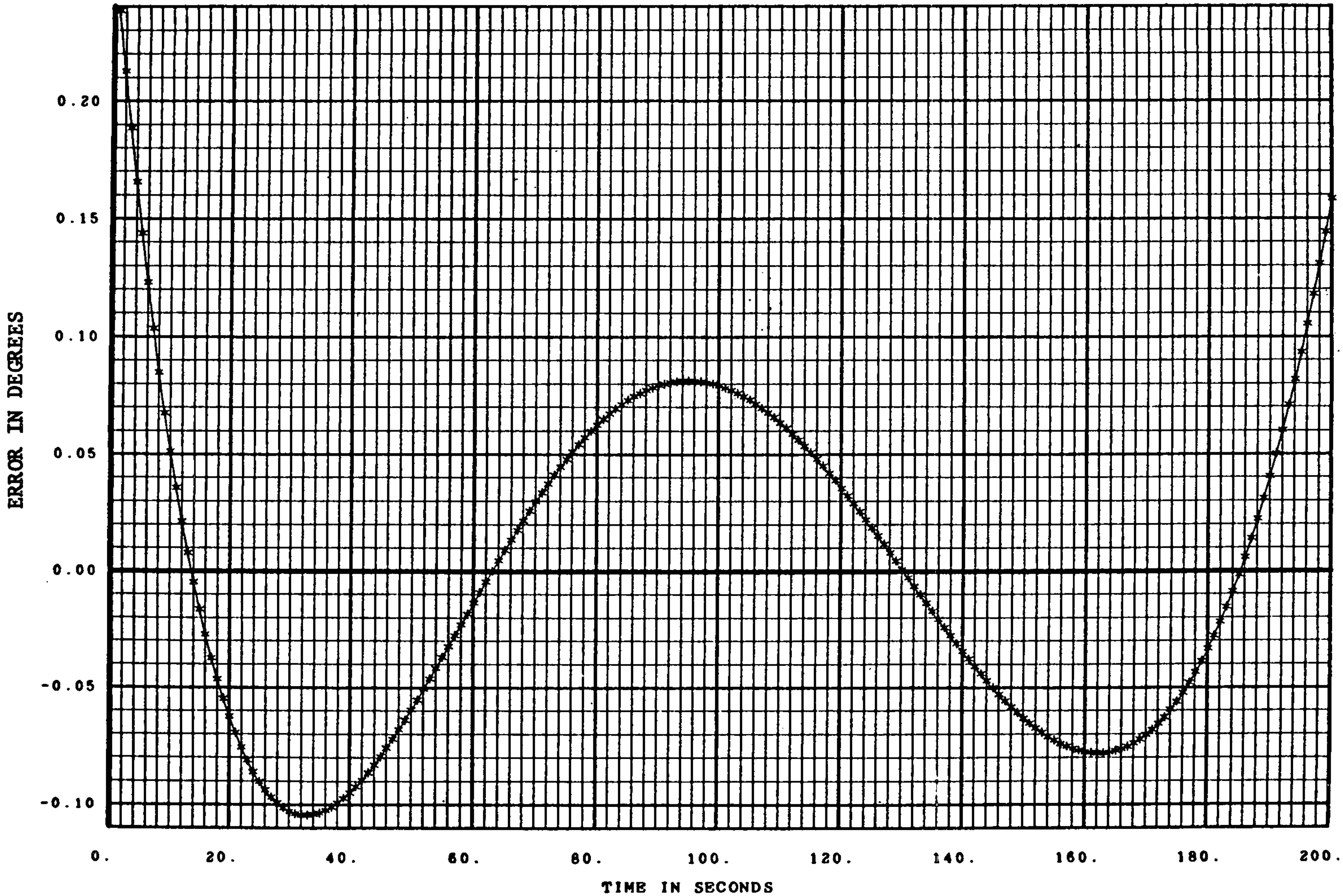

Figure A-53. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

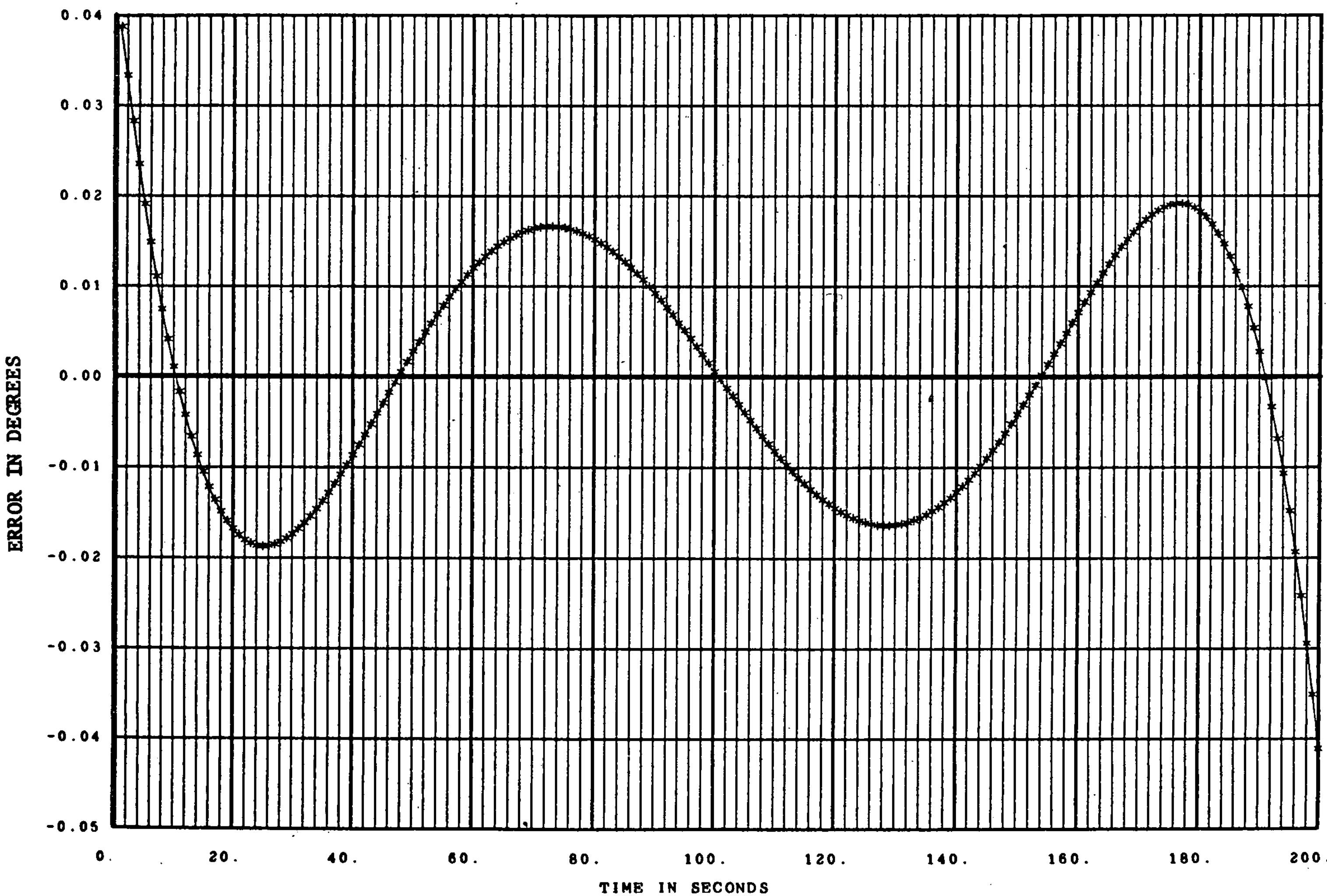

Figure A-54. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

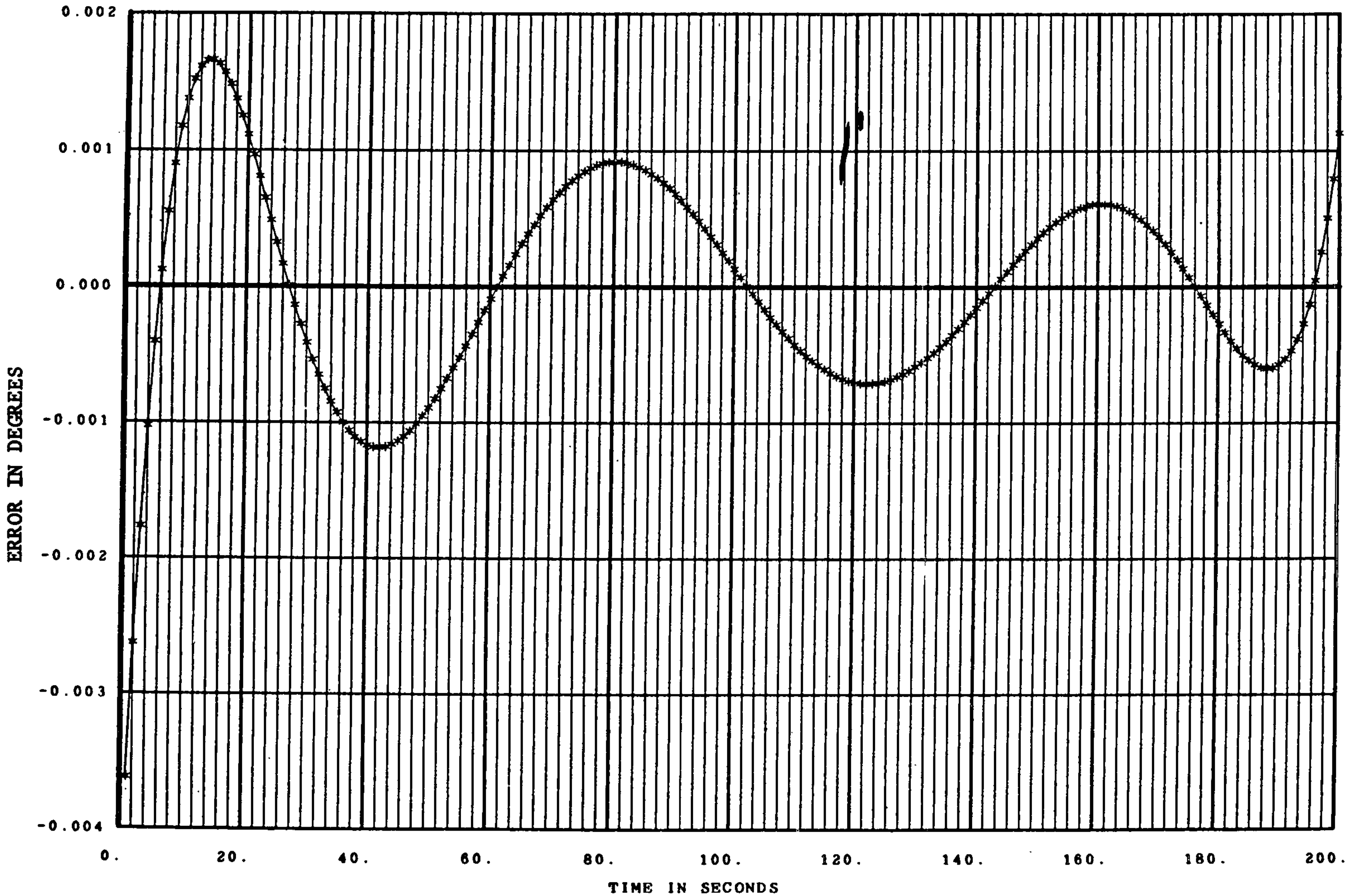

Figure A-55. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

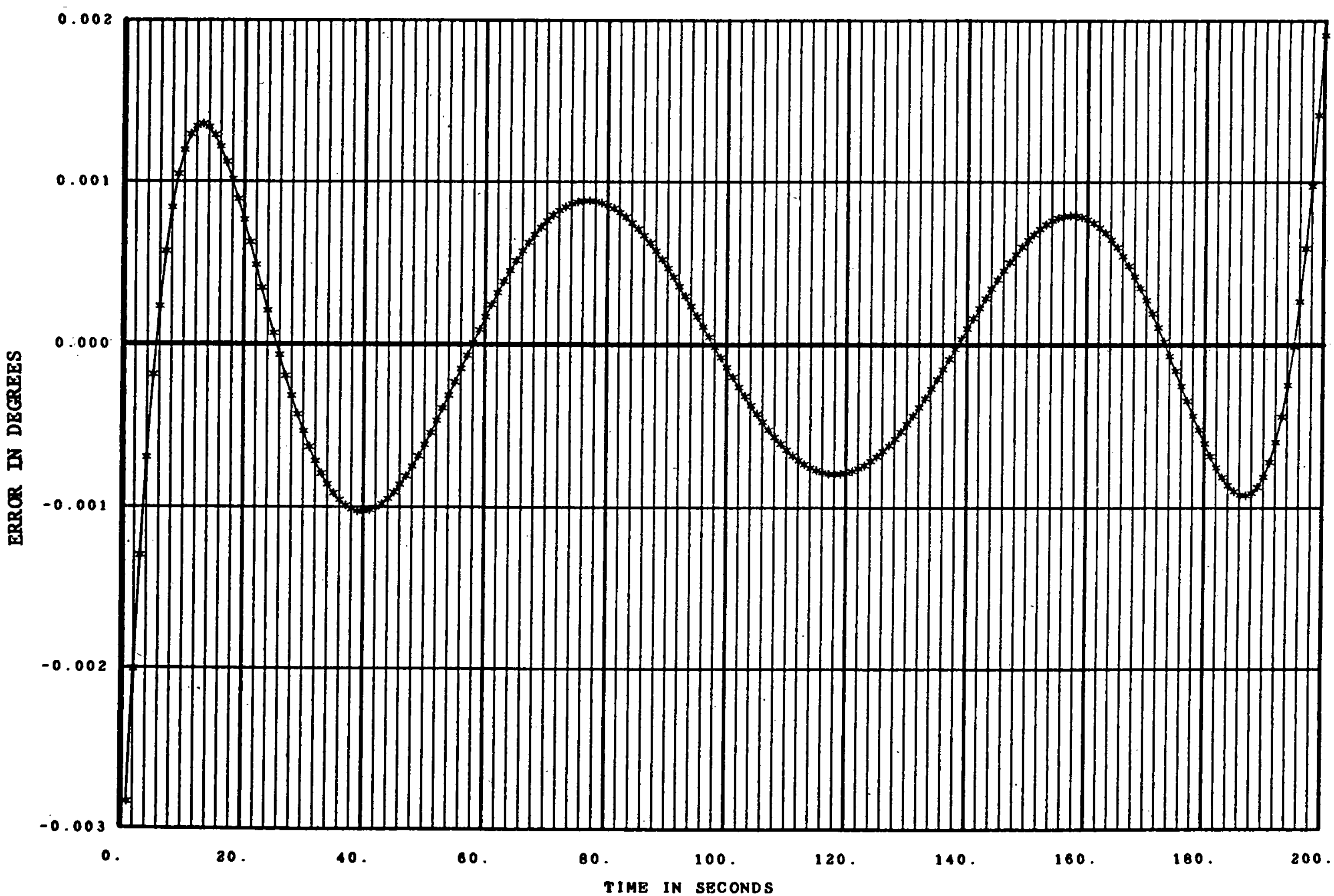

Figure A-56. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

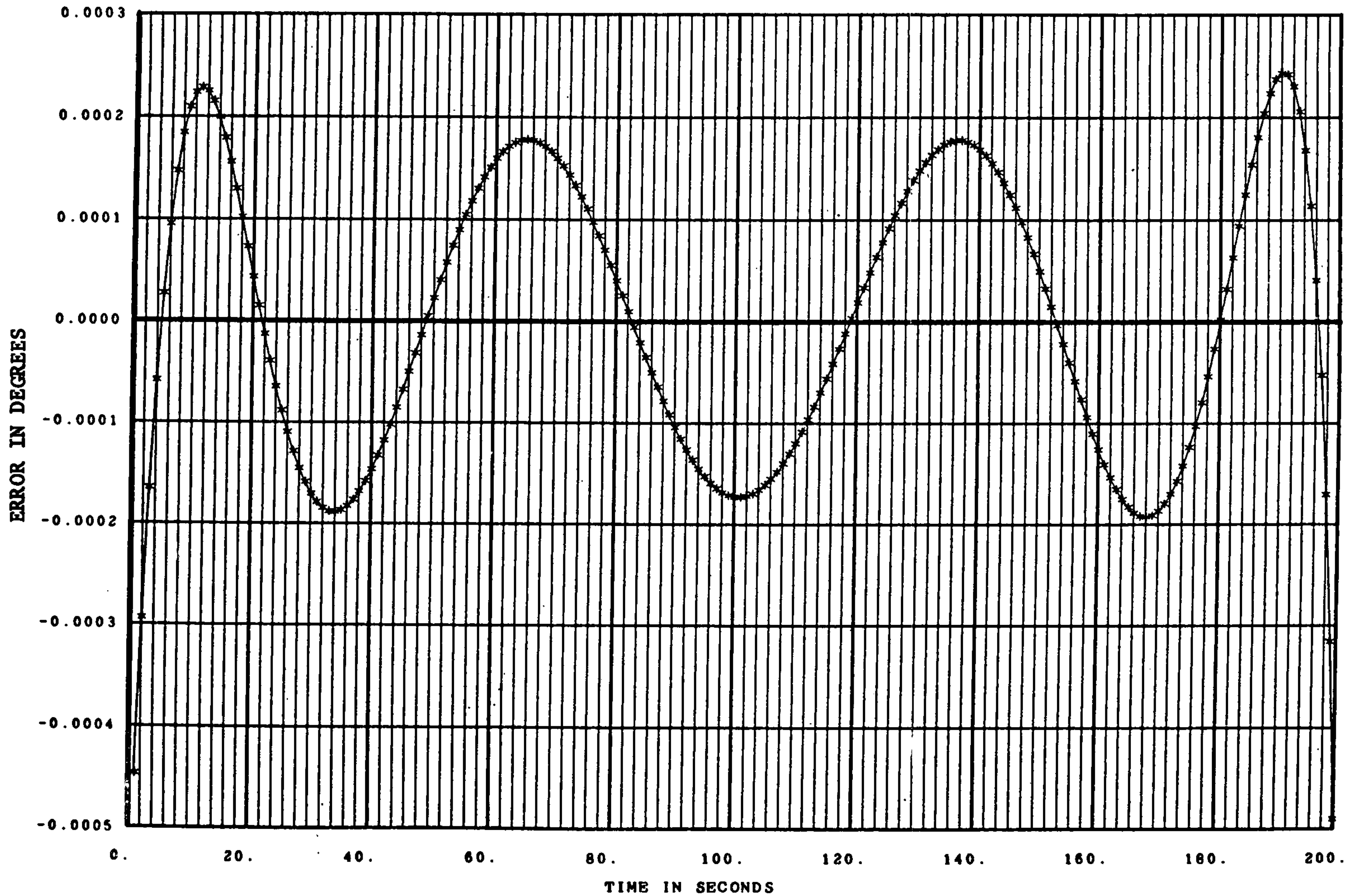

Figure A-57. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

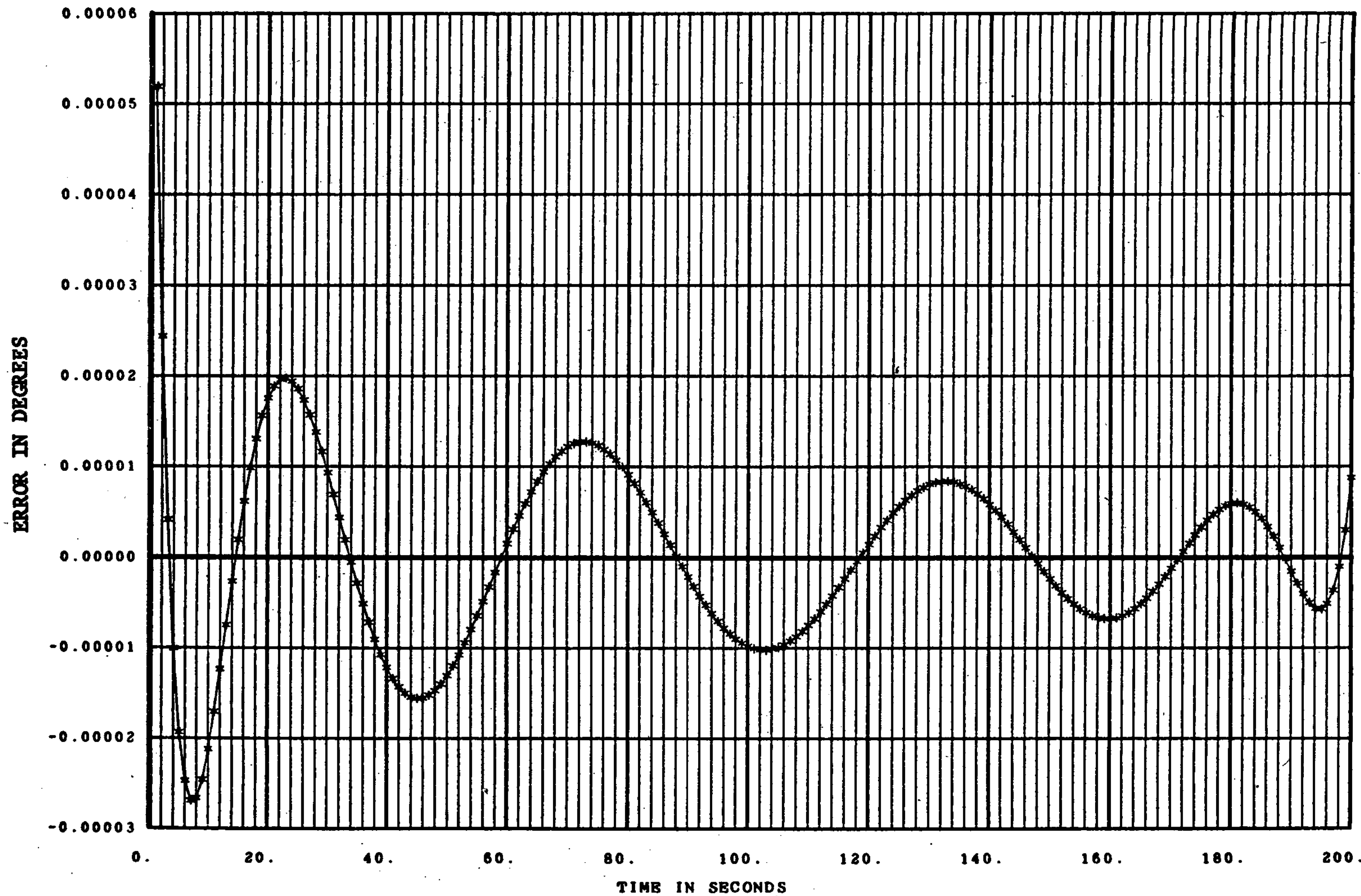

Figure A-58. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

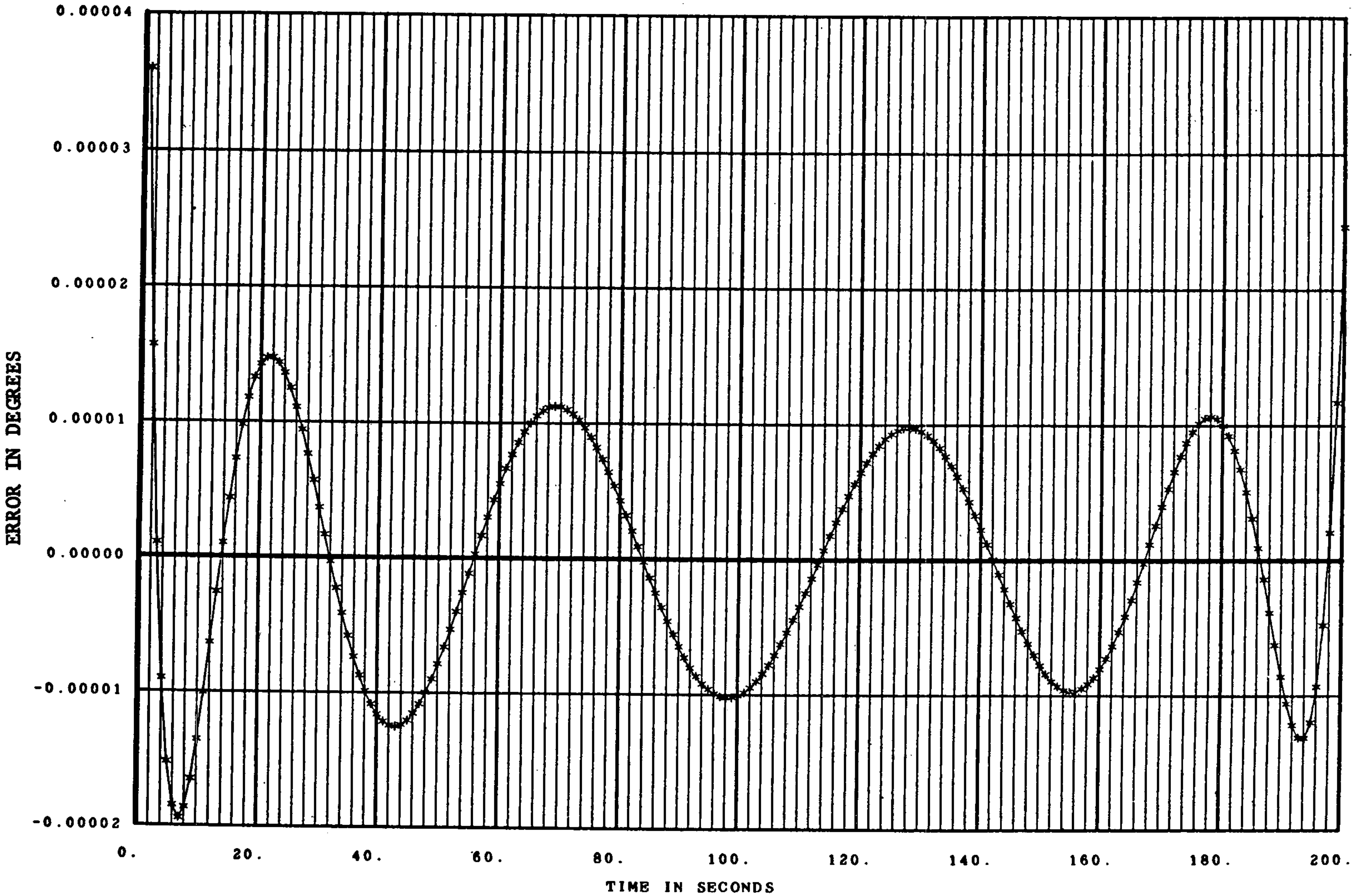

Figure A-59. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

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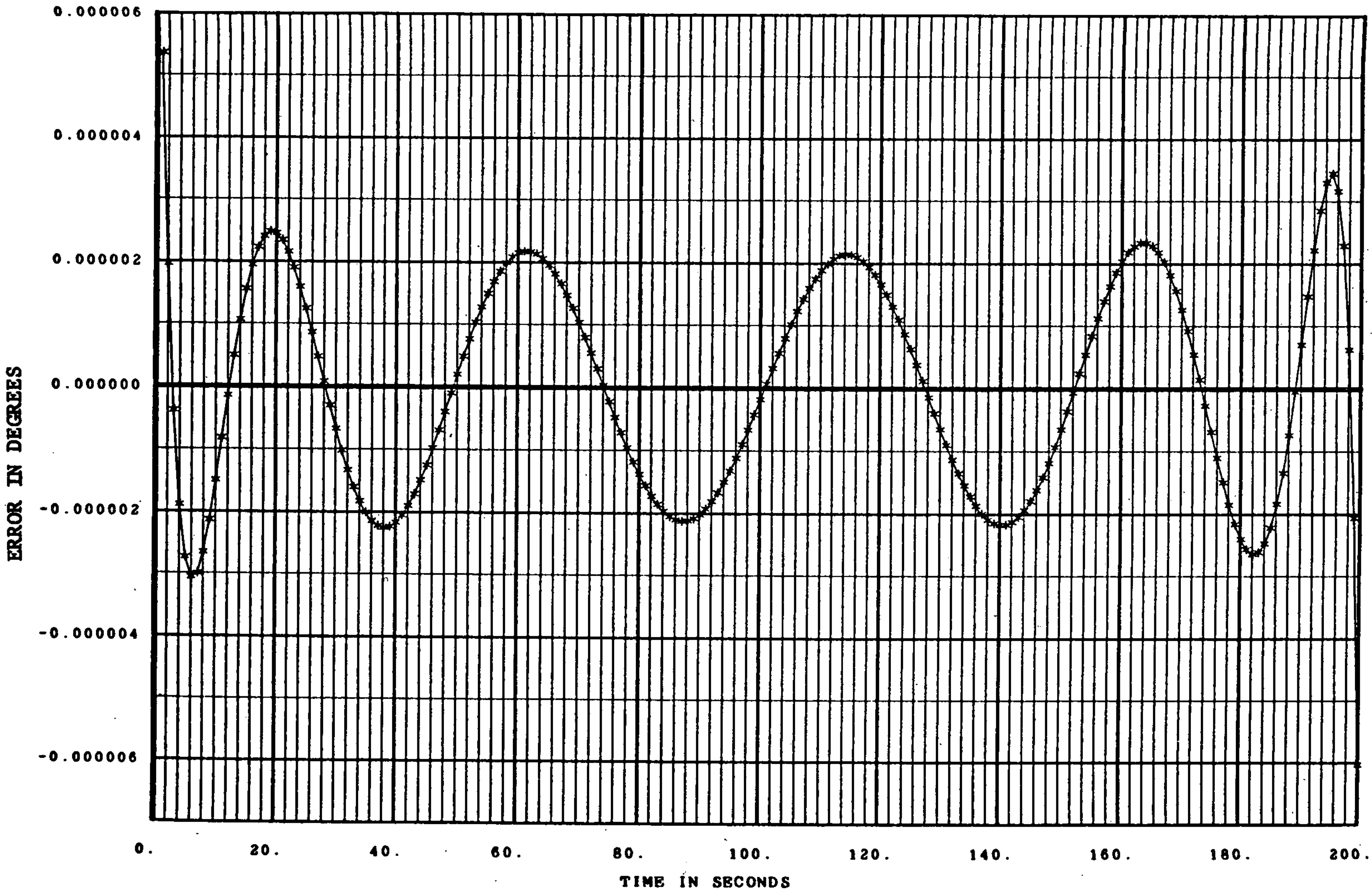

Figure A-60. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 7378 km, e = 0, zero time at spacecraft crossing station zenith (overhead pass).

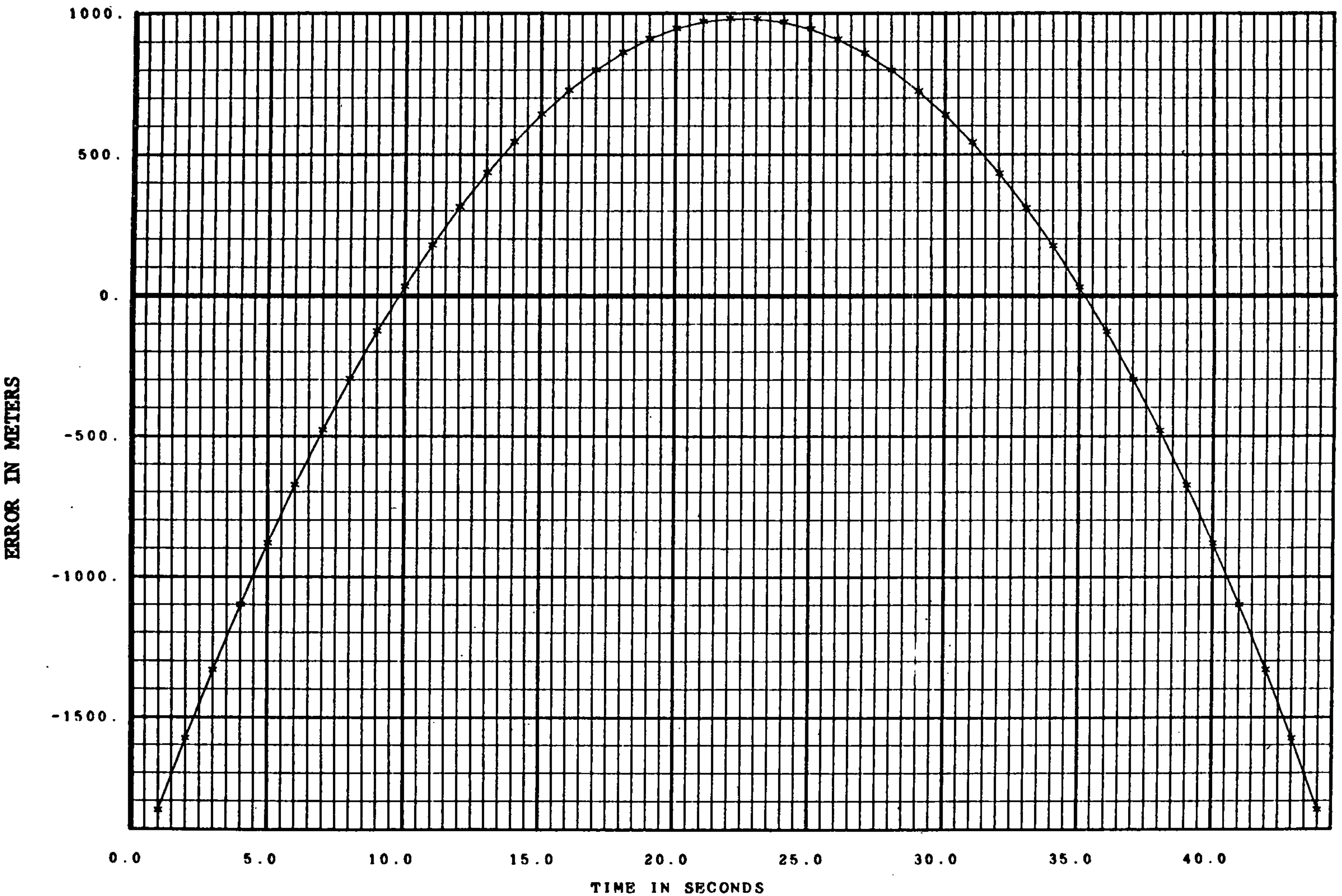

Figure A-61. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

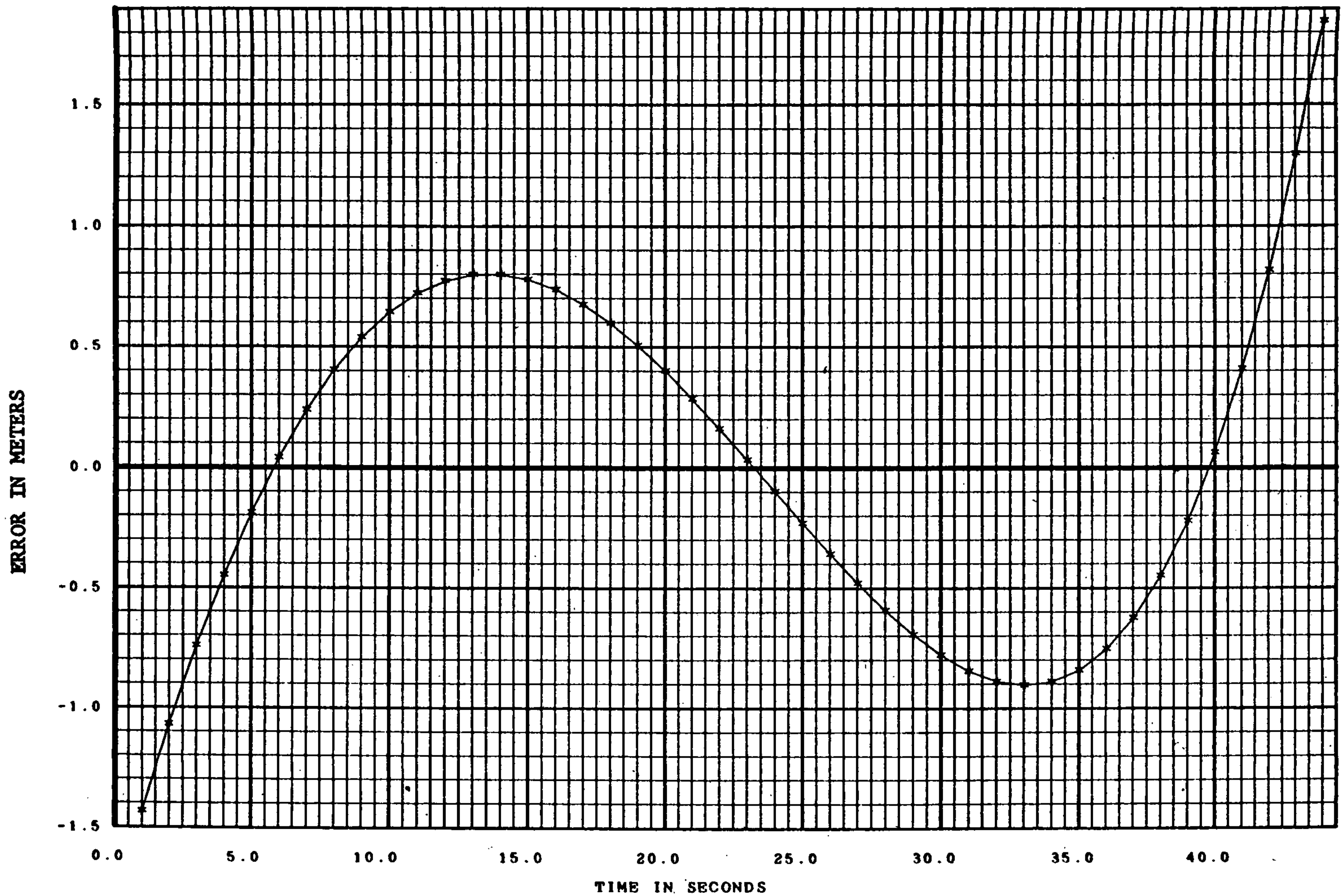

Figure A-62. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

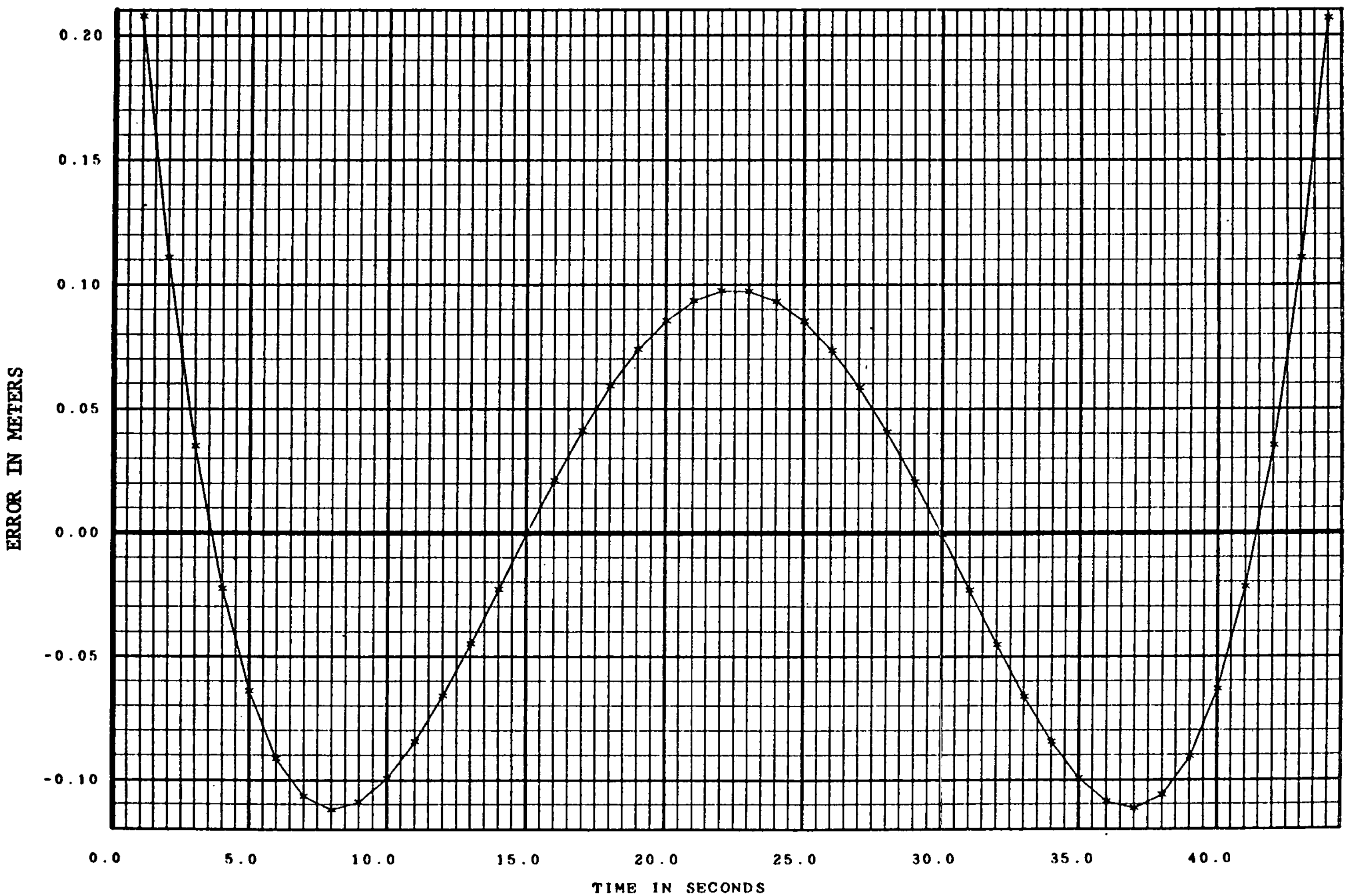

Figure A-63. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

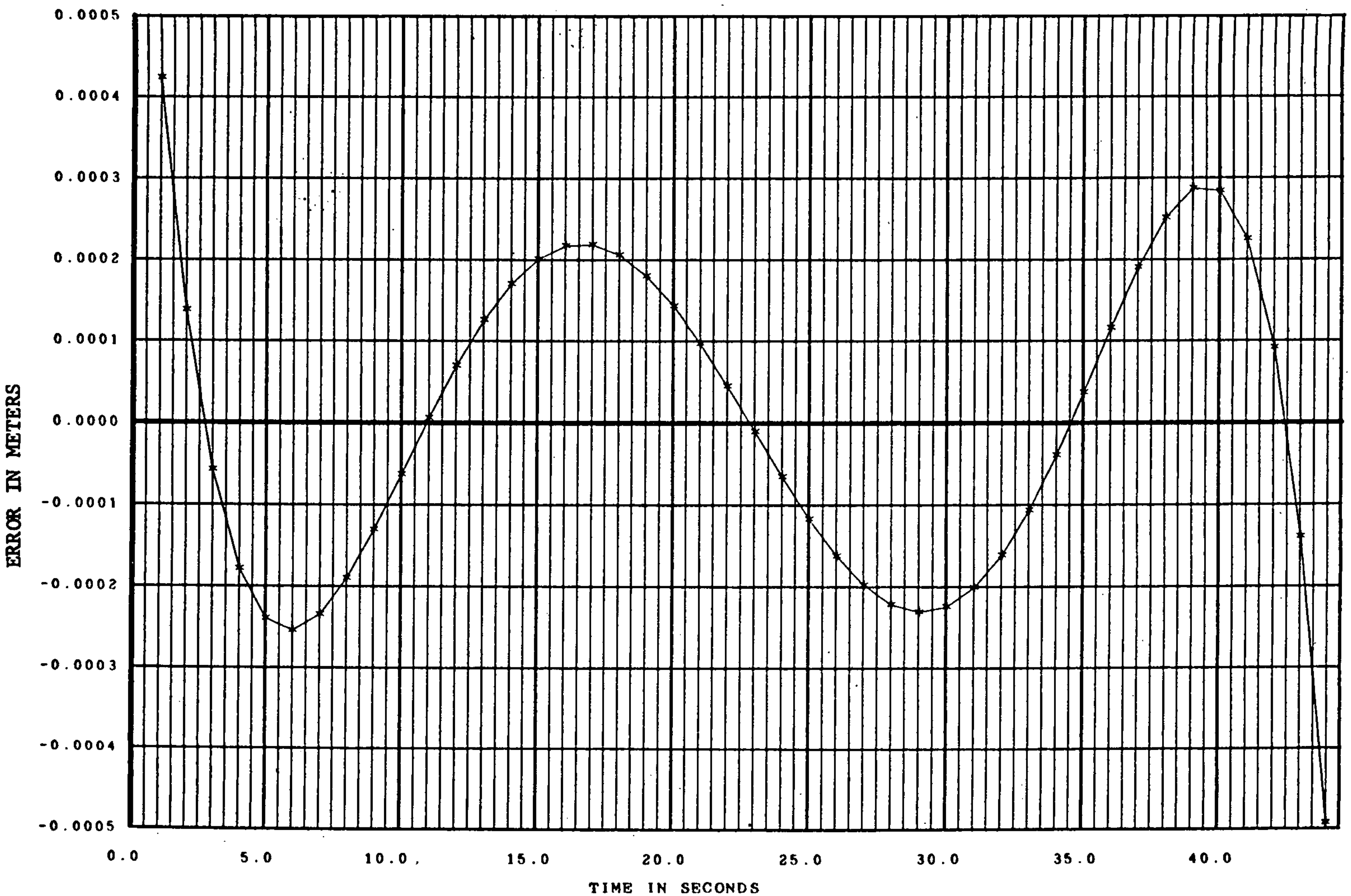

Figure A-64. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

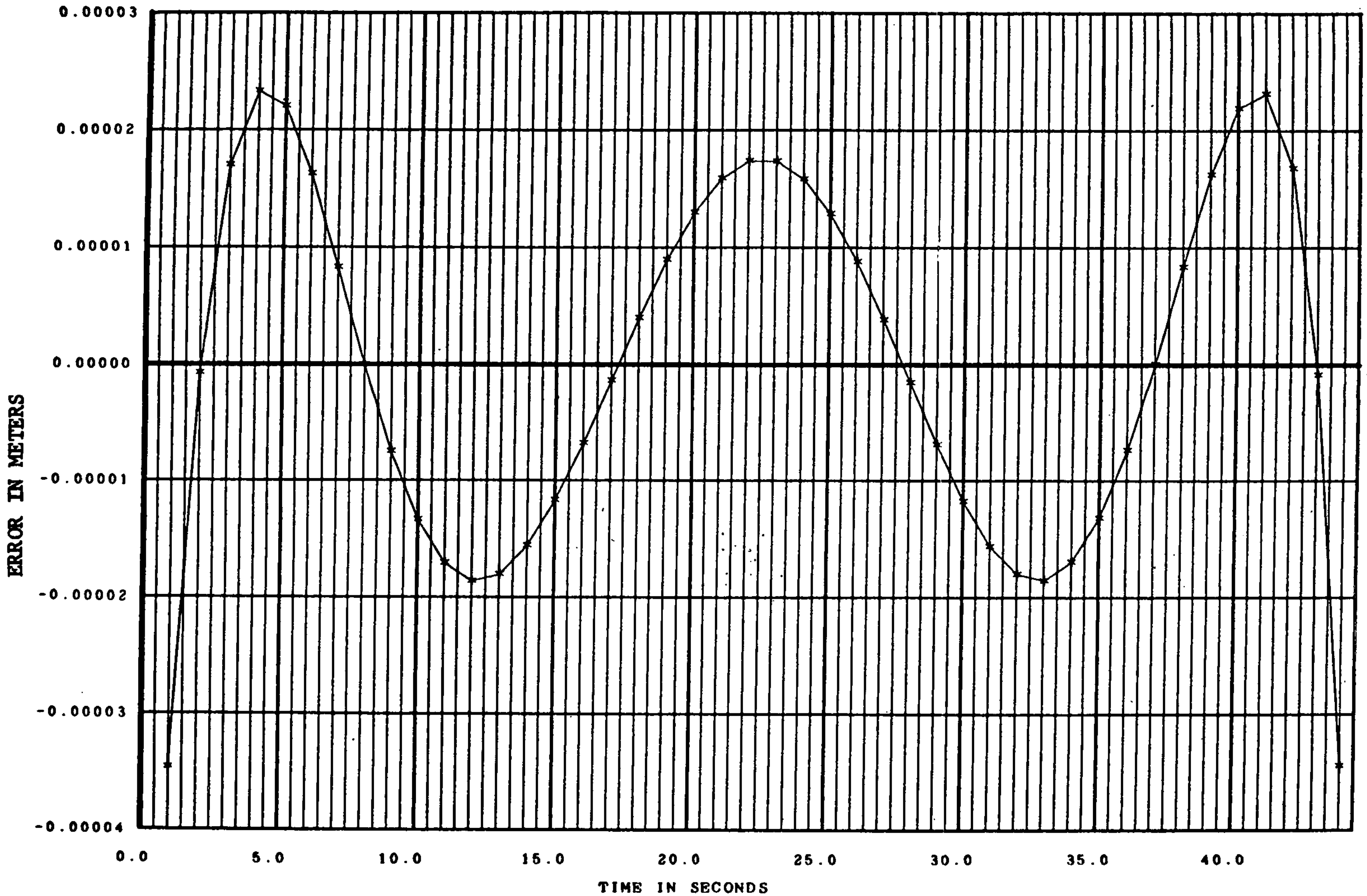

Figure A-65. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

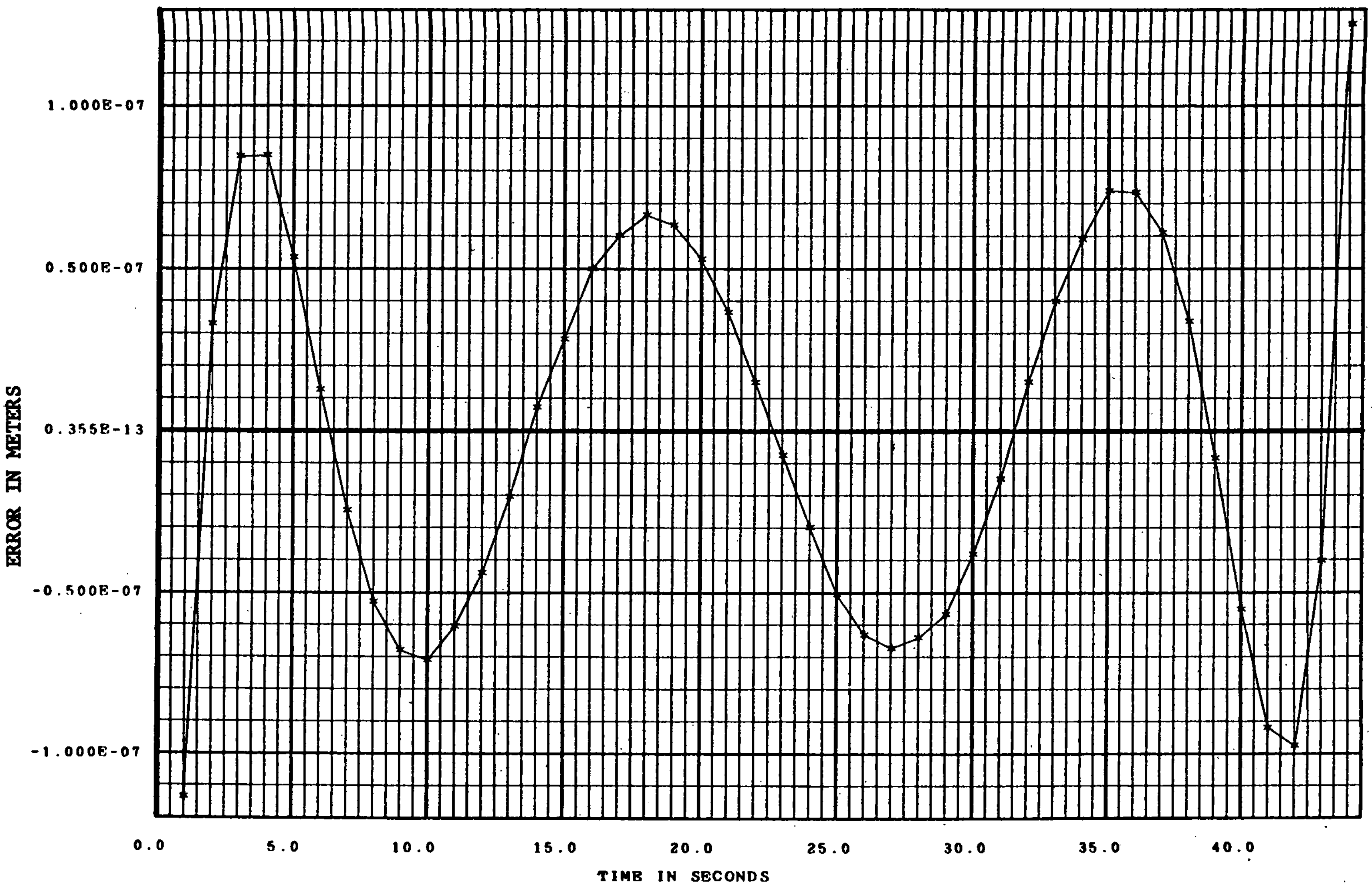

Figure A-66. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

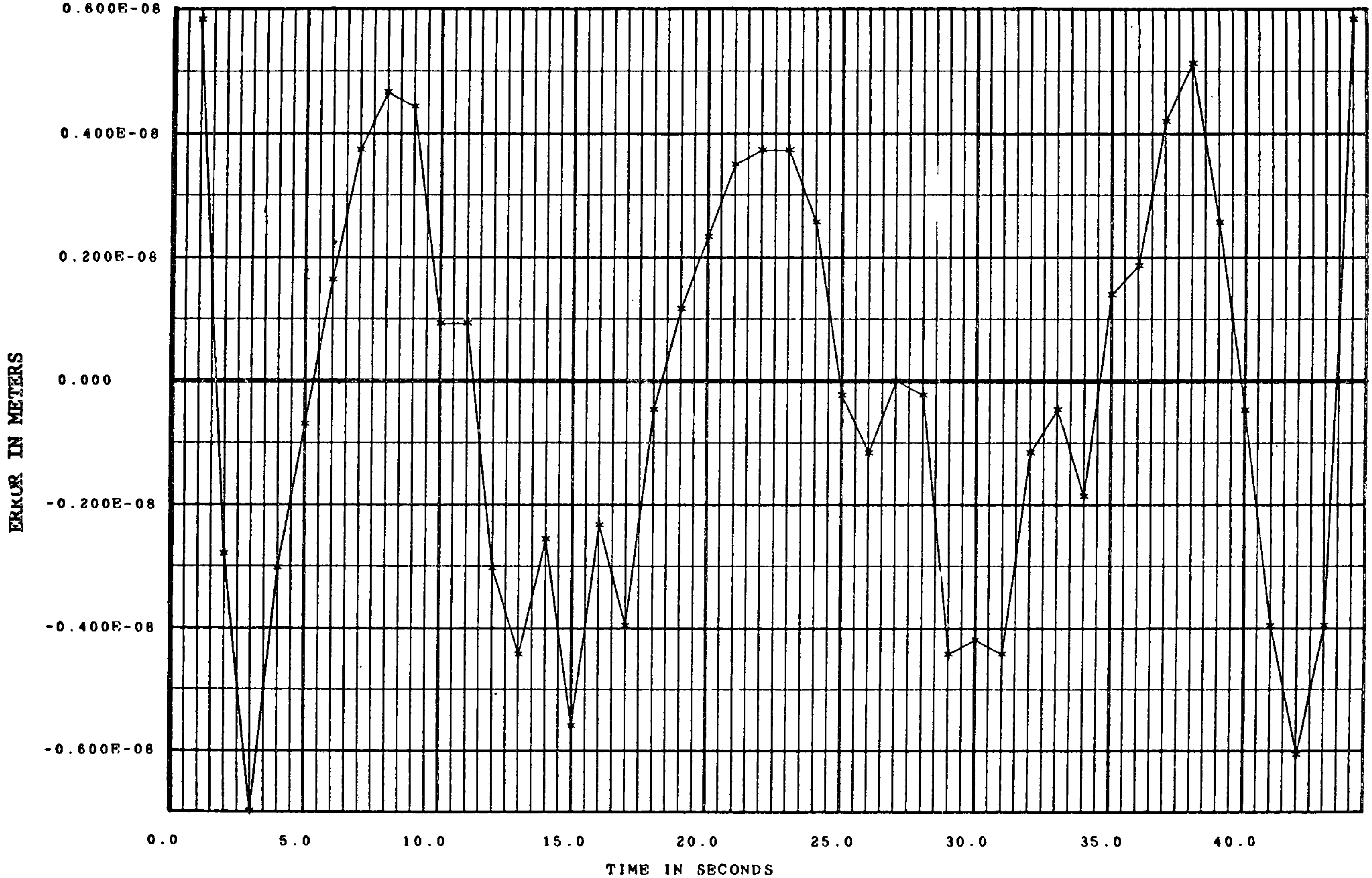

Figure A-67. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

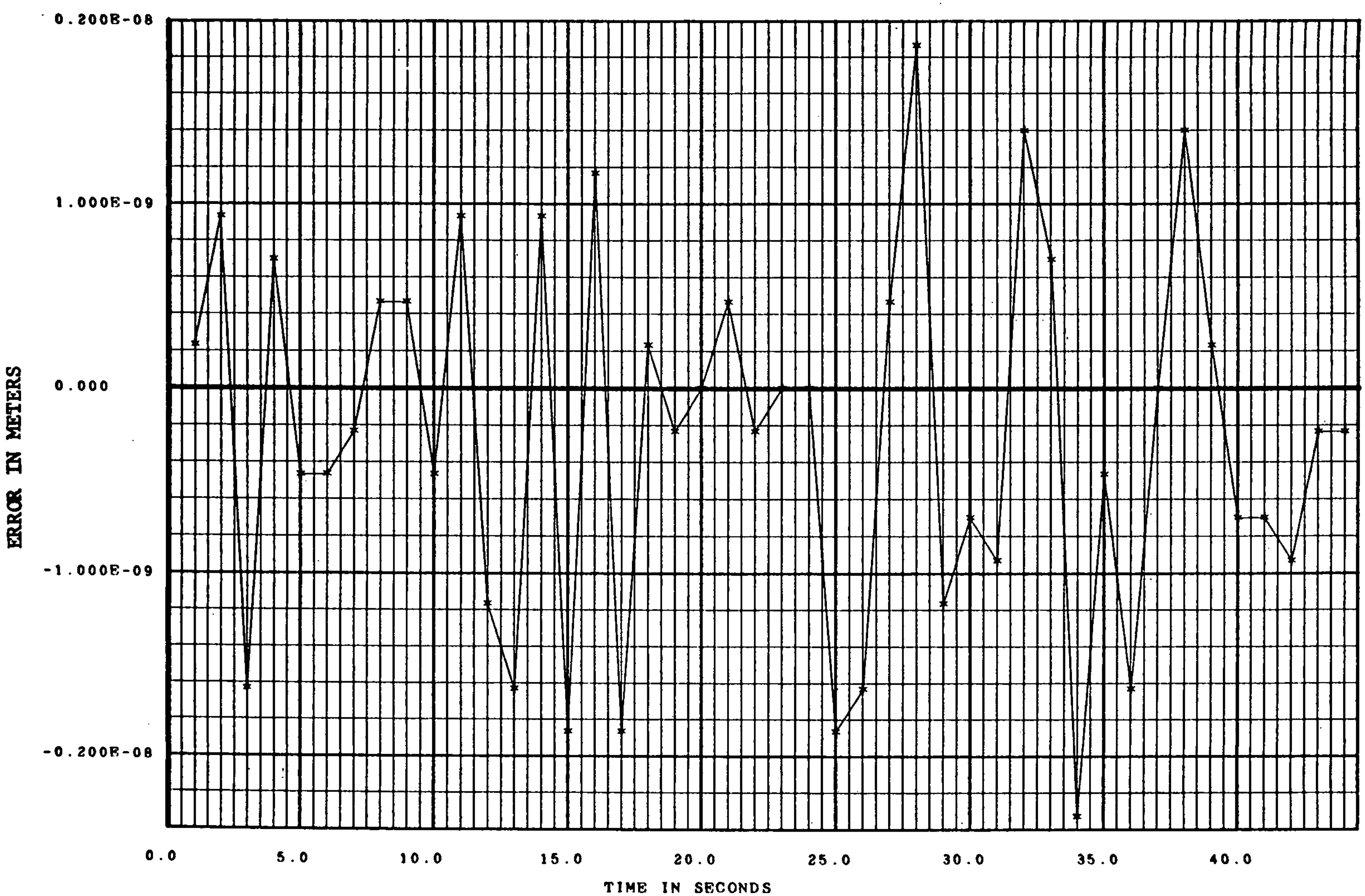

Figure A-68. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

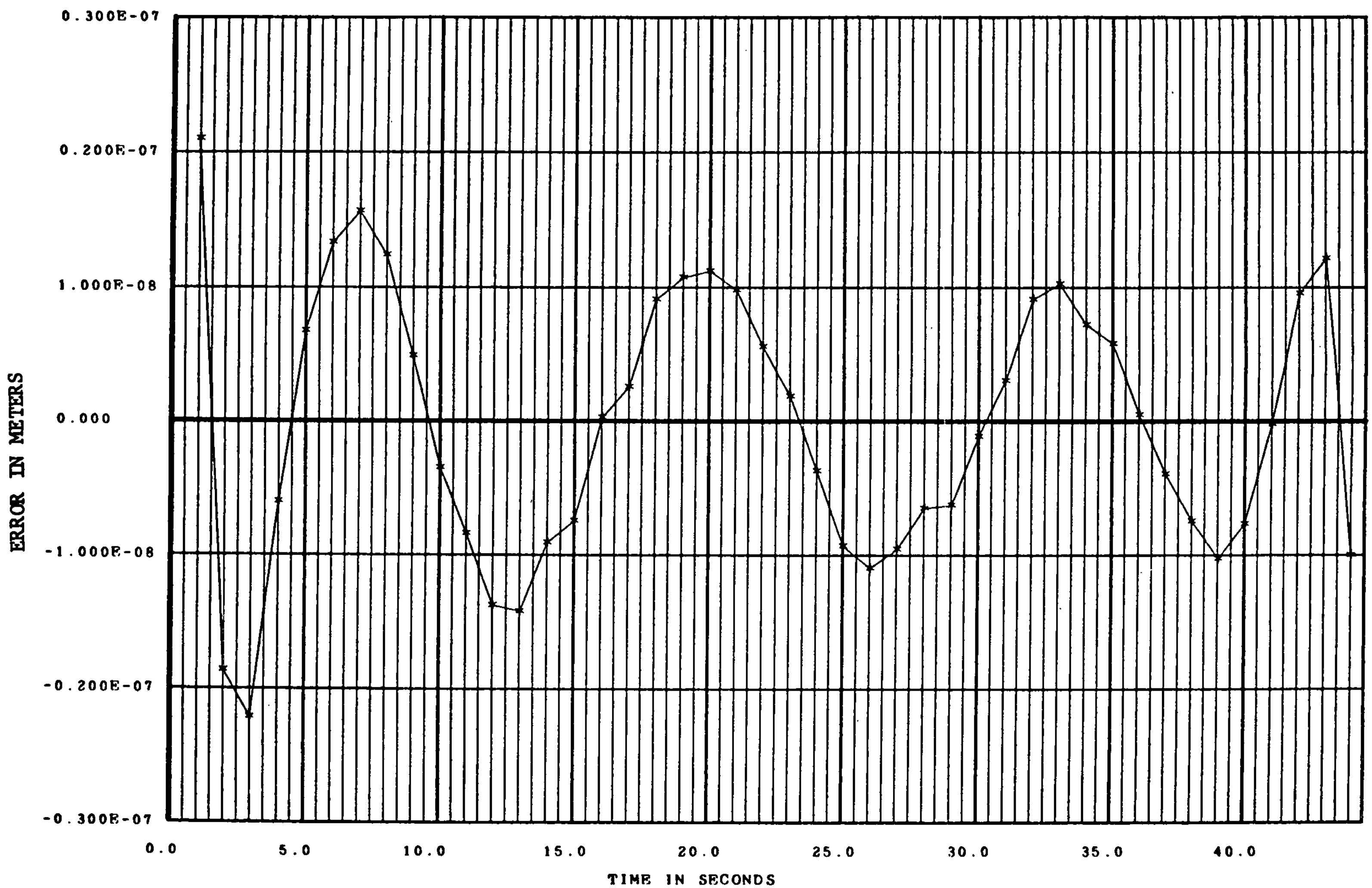

Figure A-69. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

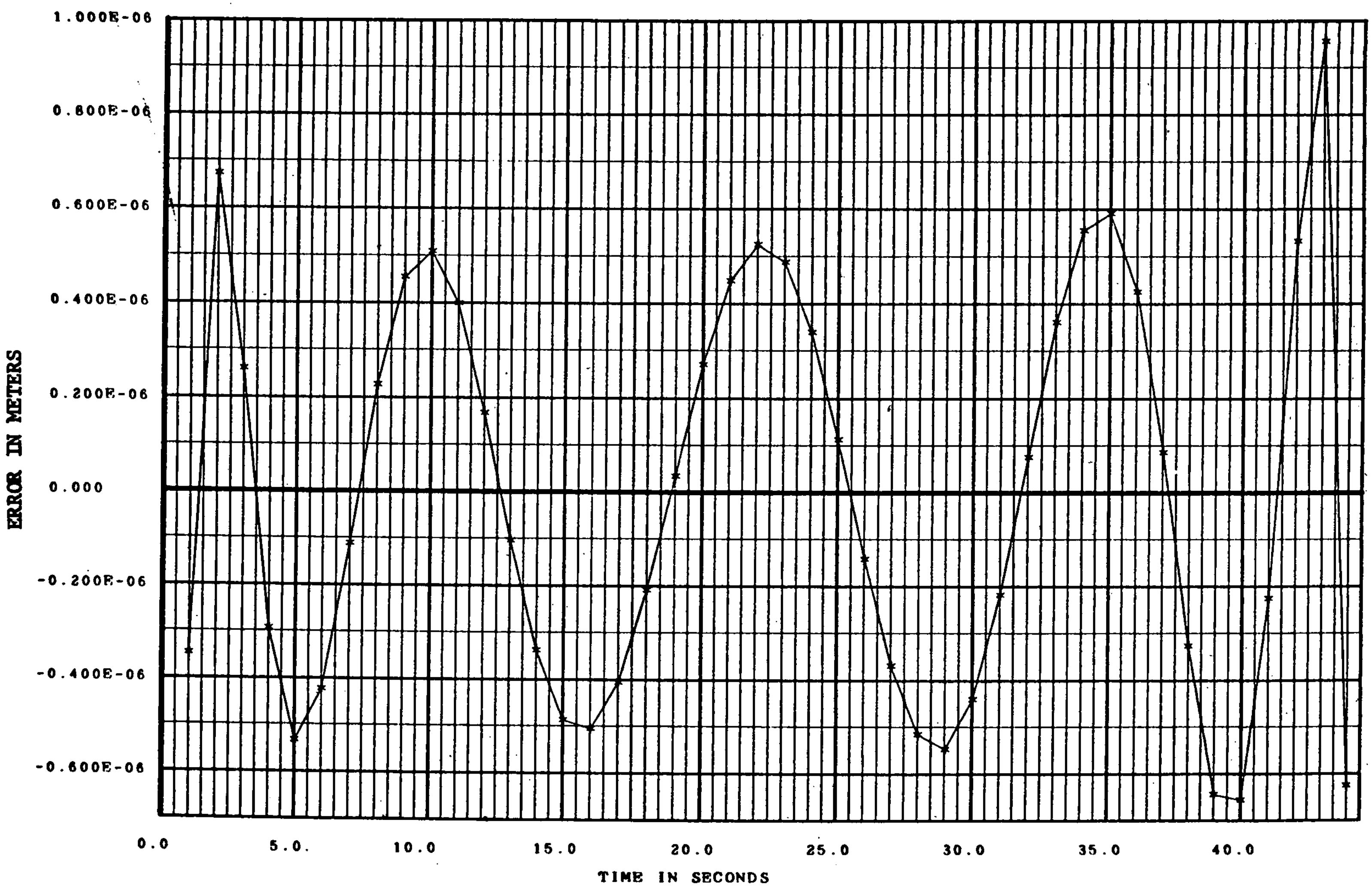

Figure A-70. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 44 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

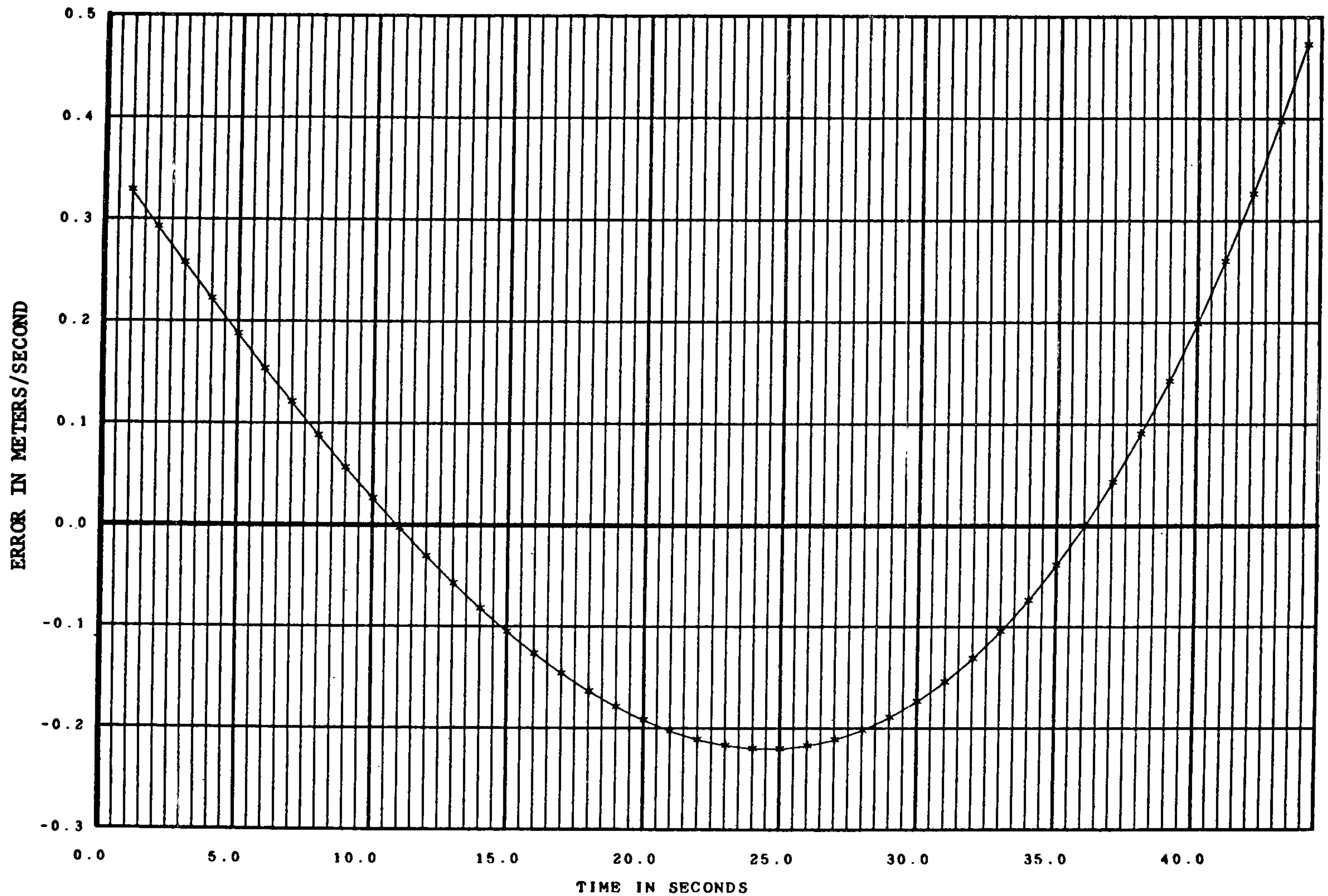

Figure A-71. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

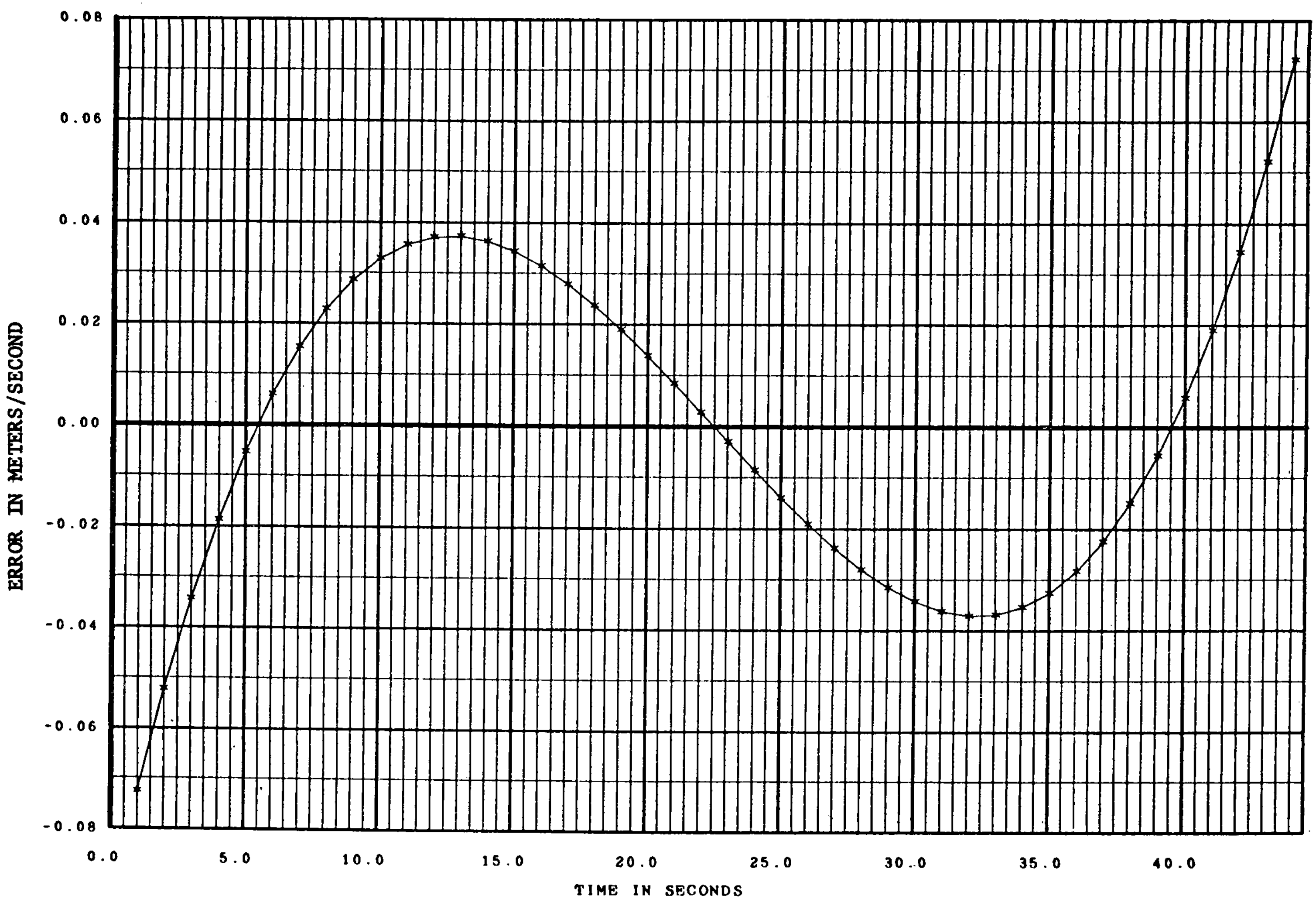

Figure A-72. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

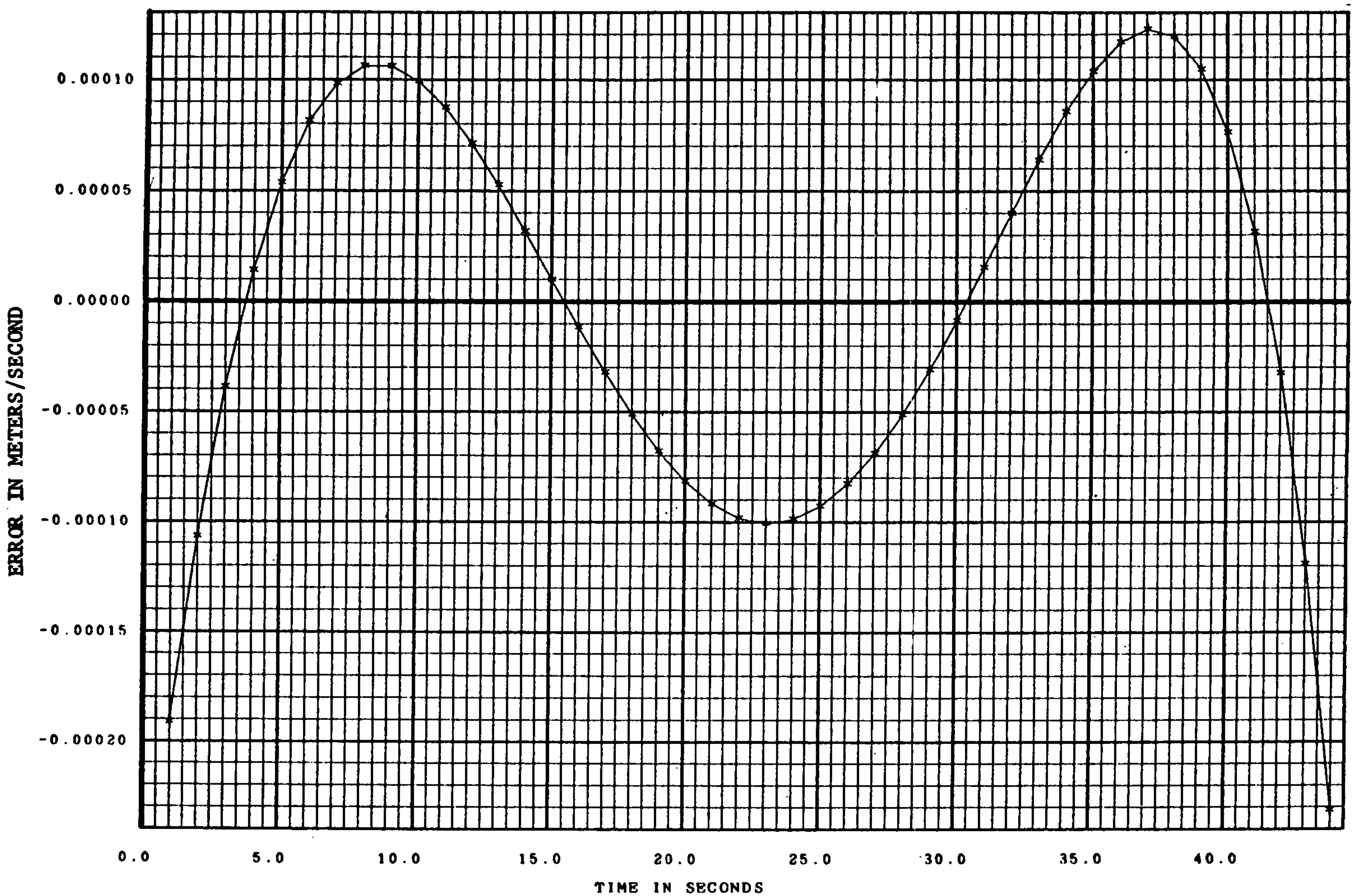

Figure A-73. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

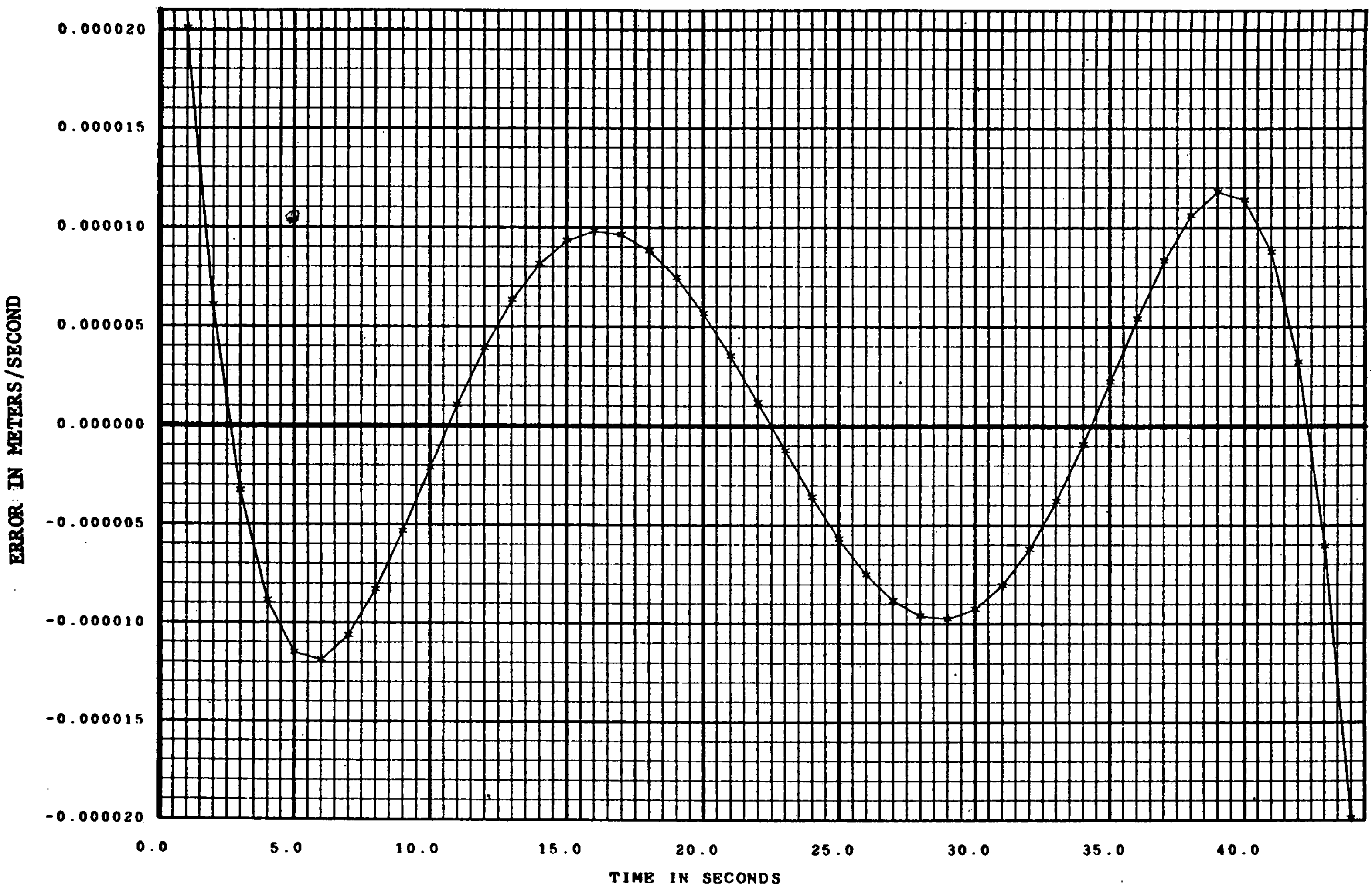

Figure A-74. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

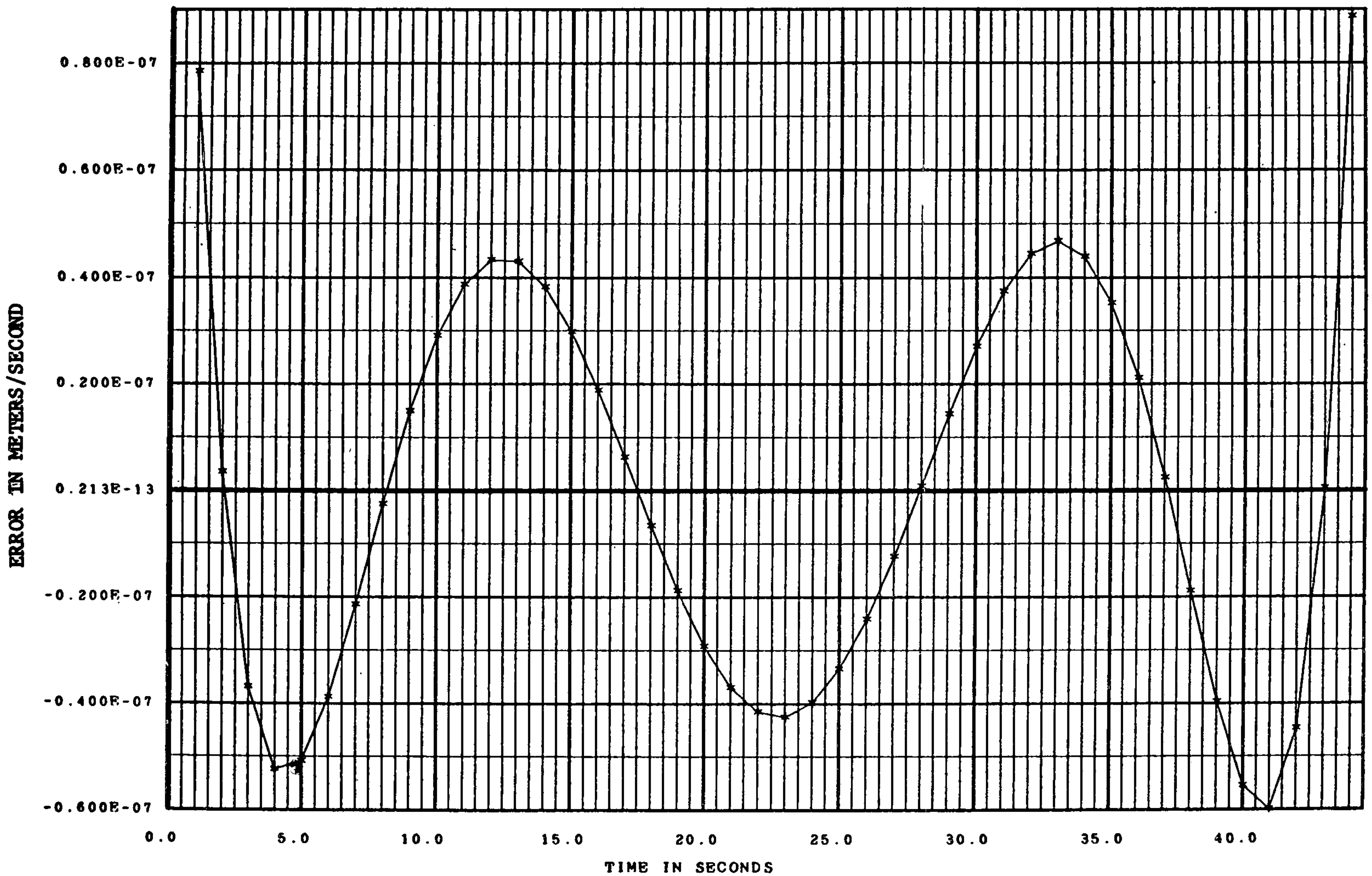

Figure A-75. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

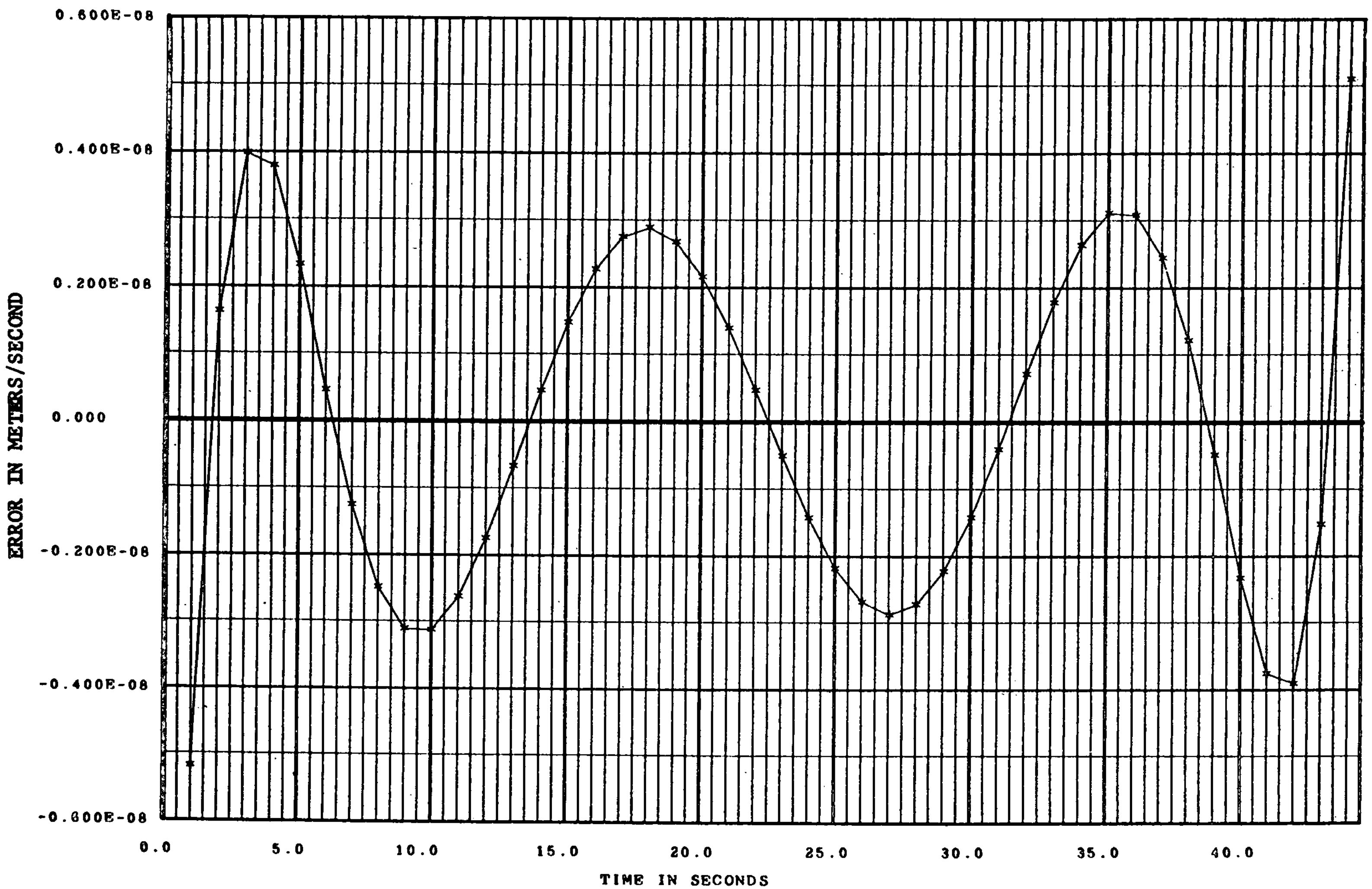

Figure A-76. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

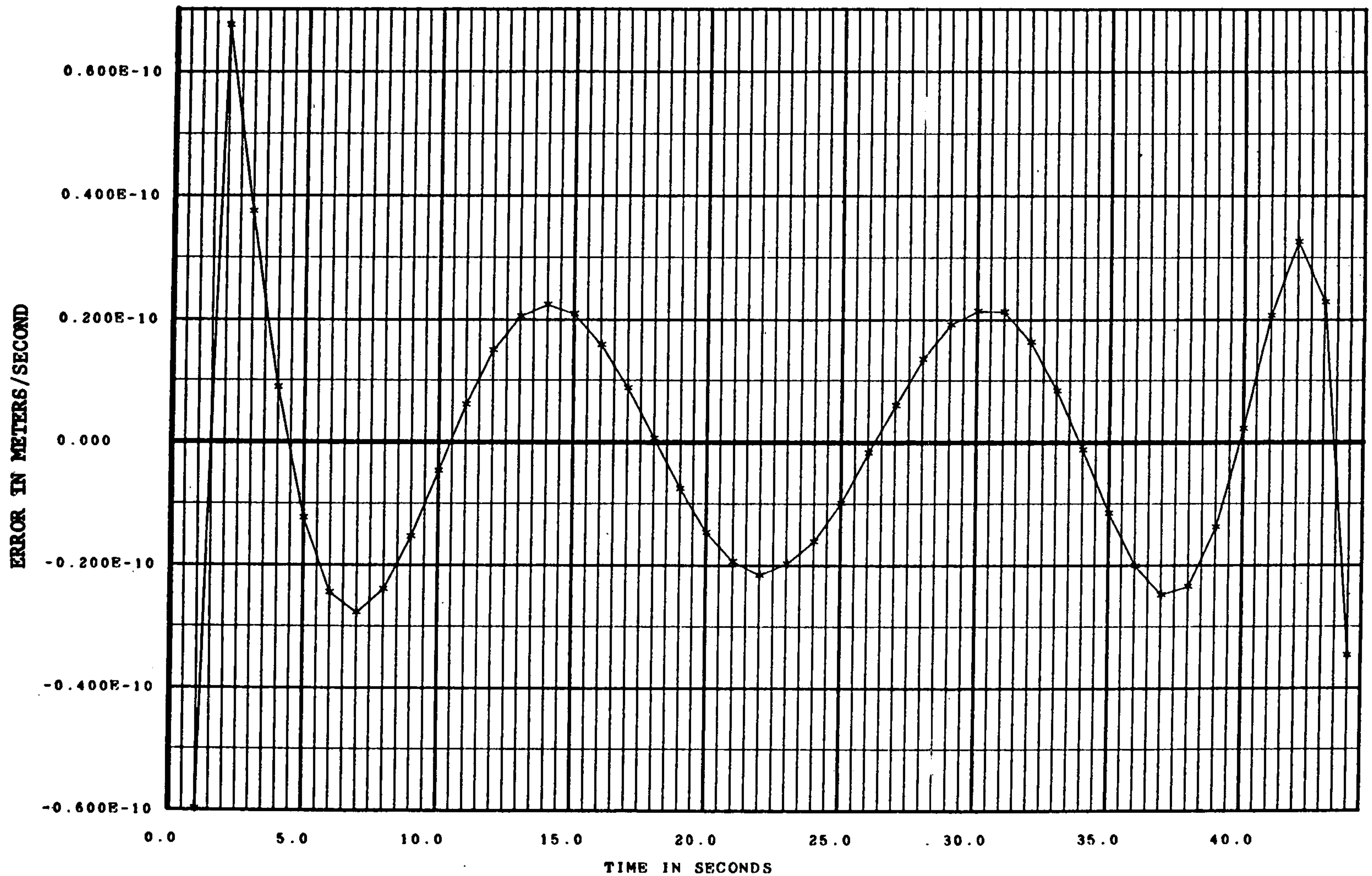

Figure A-77. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

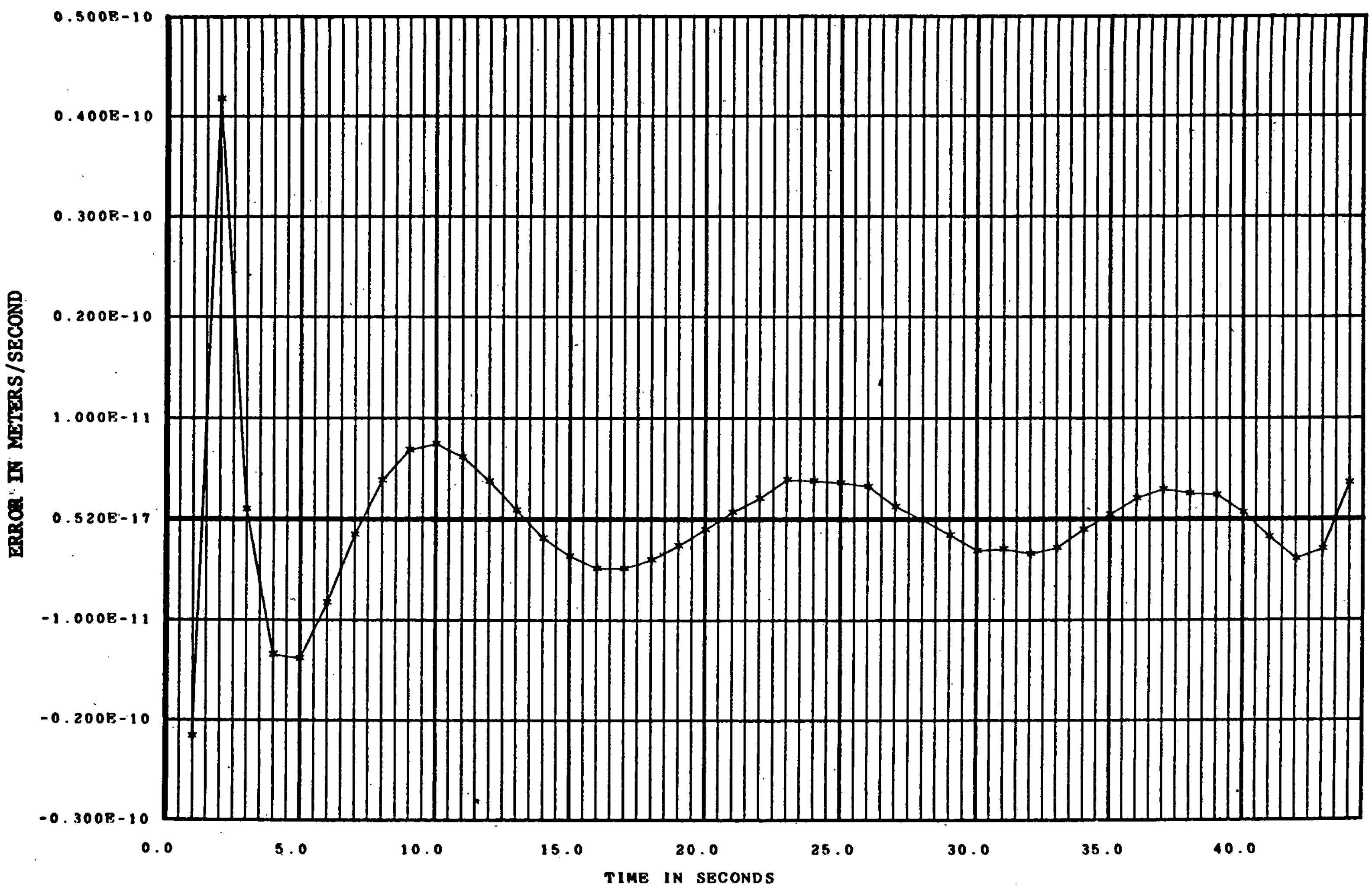

Figure A-78. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

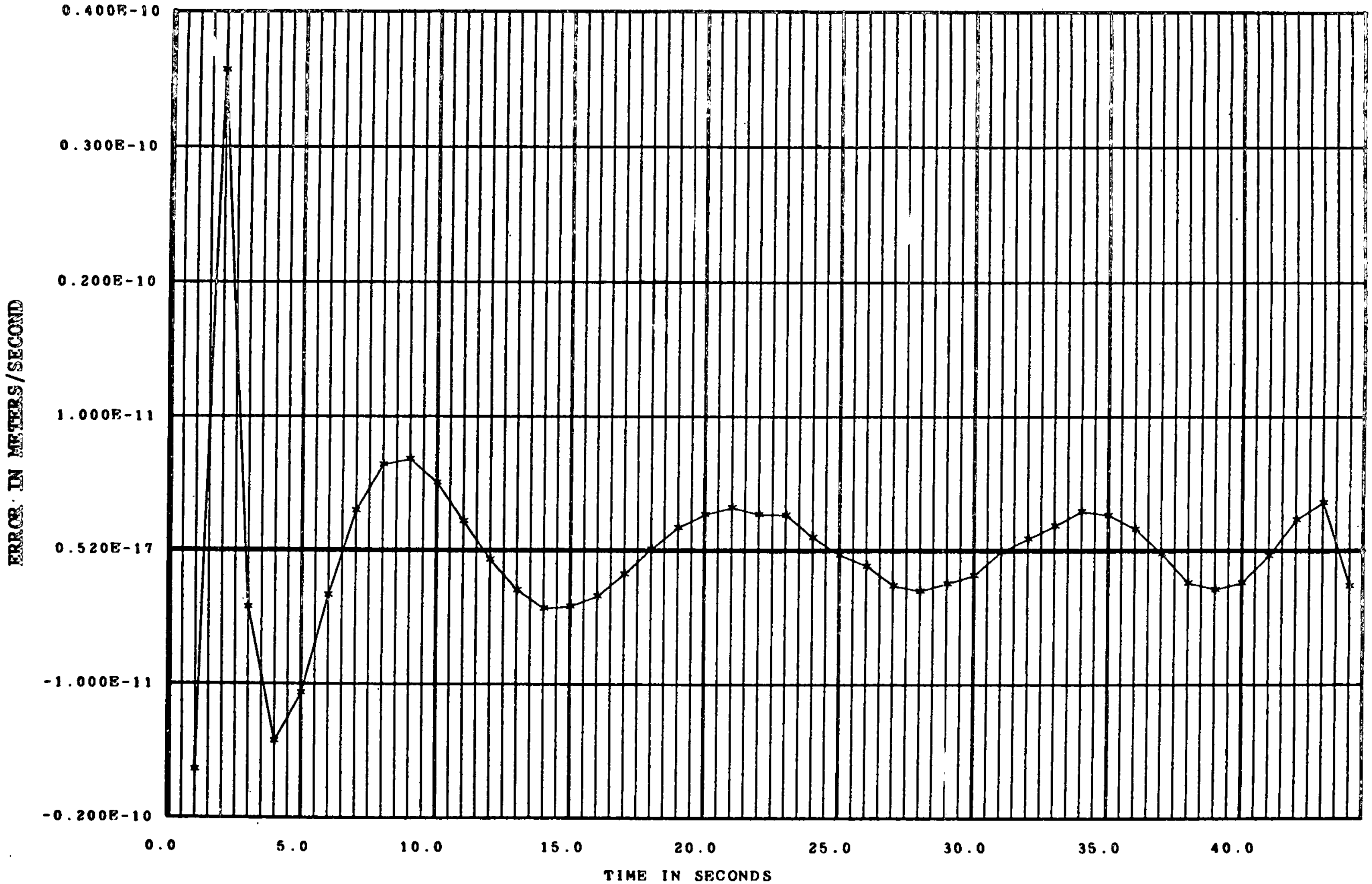

Figure A-79. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

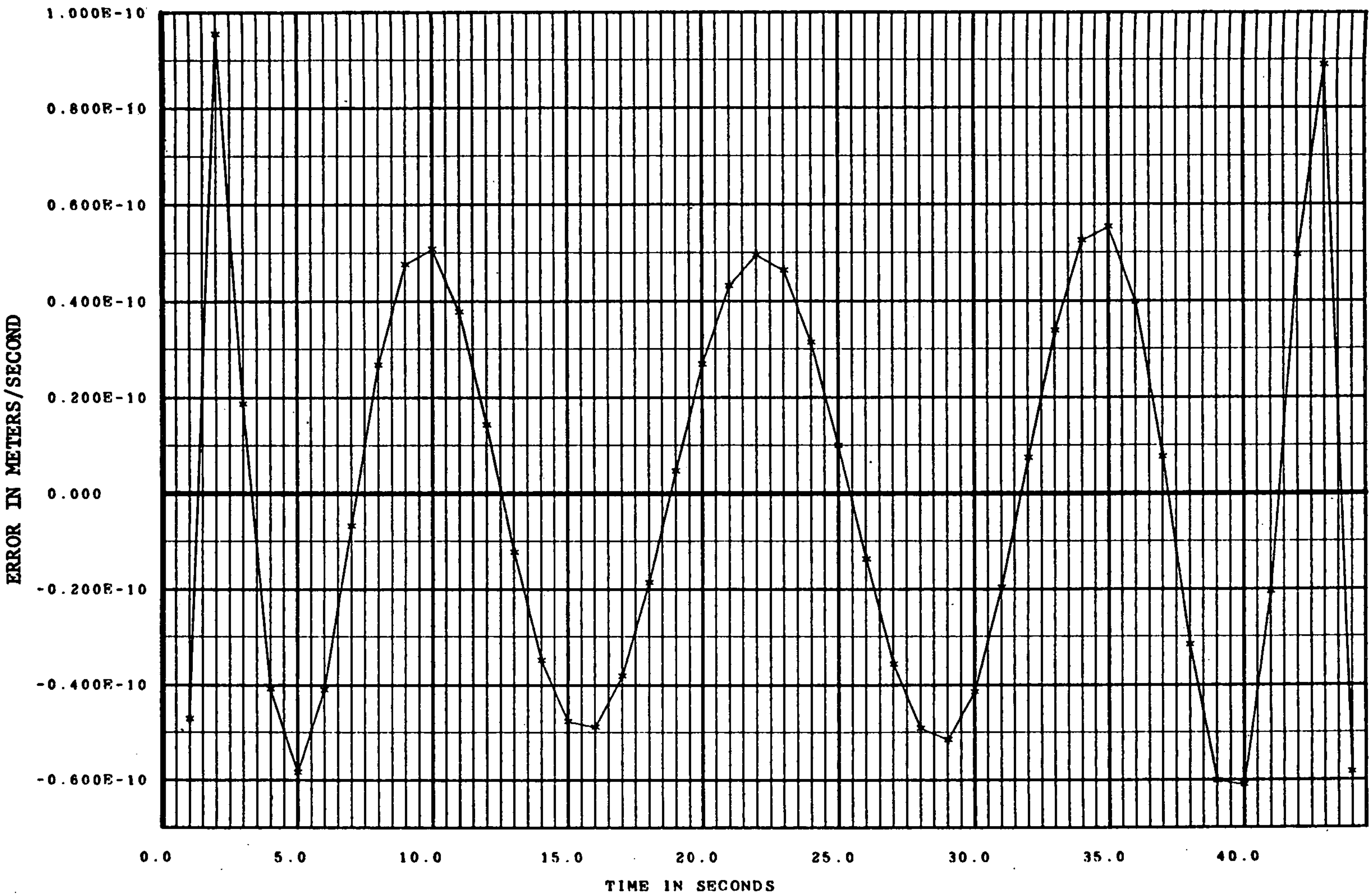

Figure A-80. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 44 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

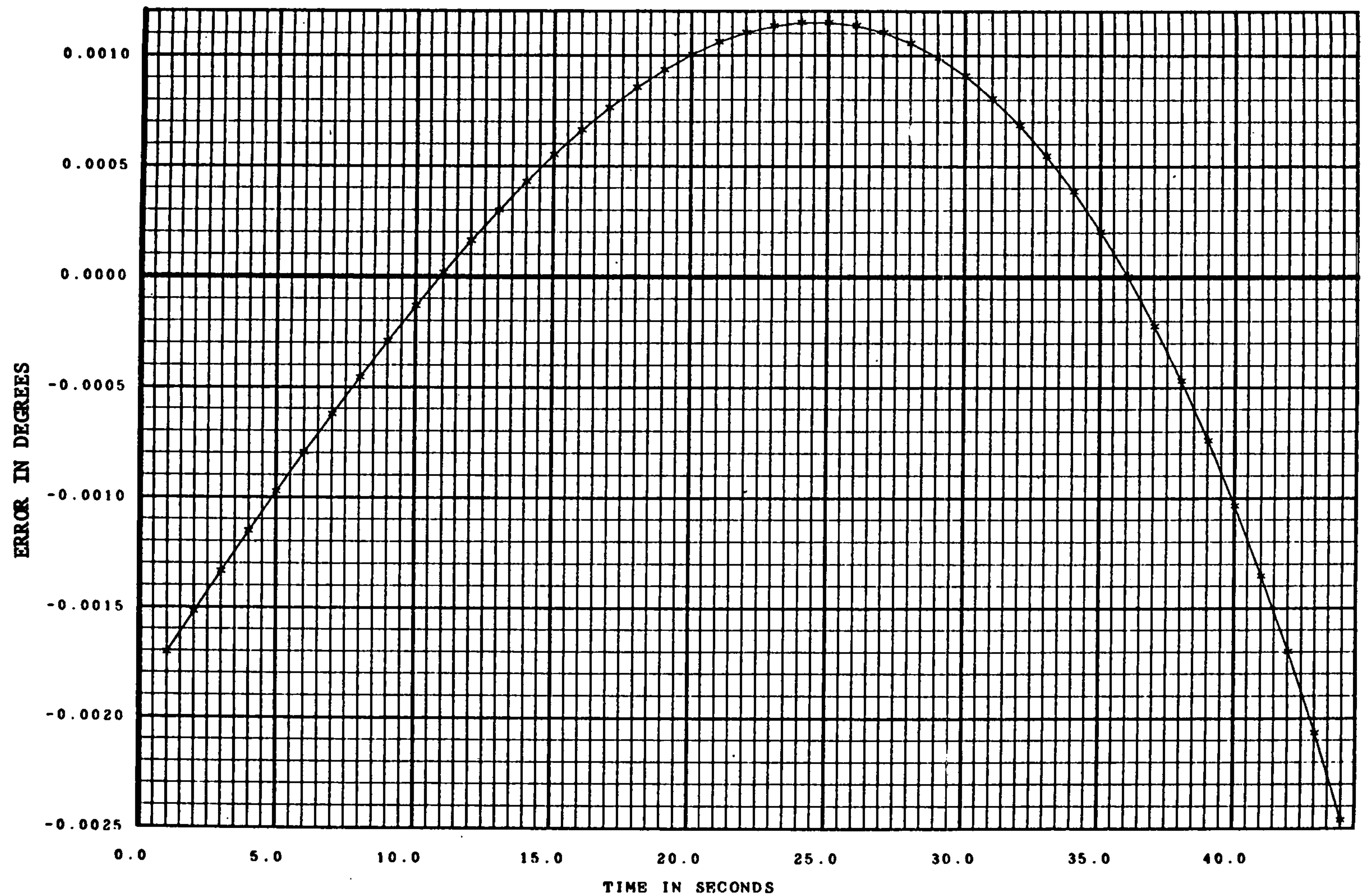

Figure A-81. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

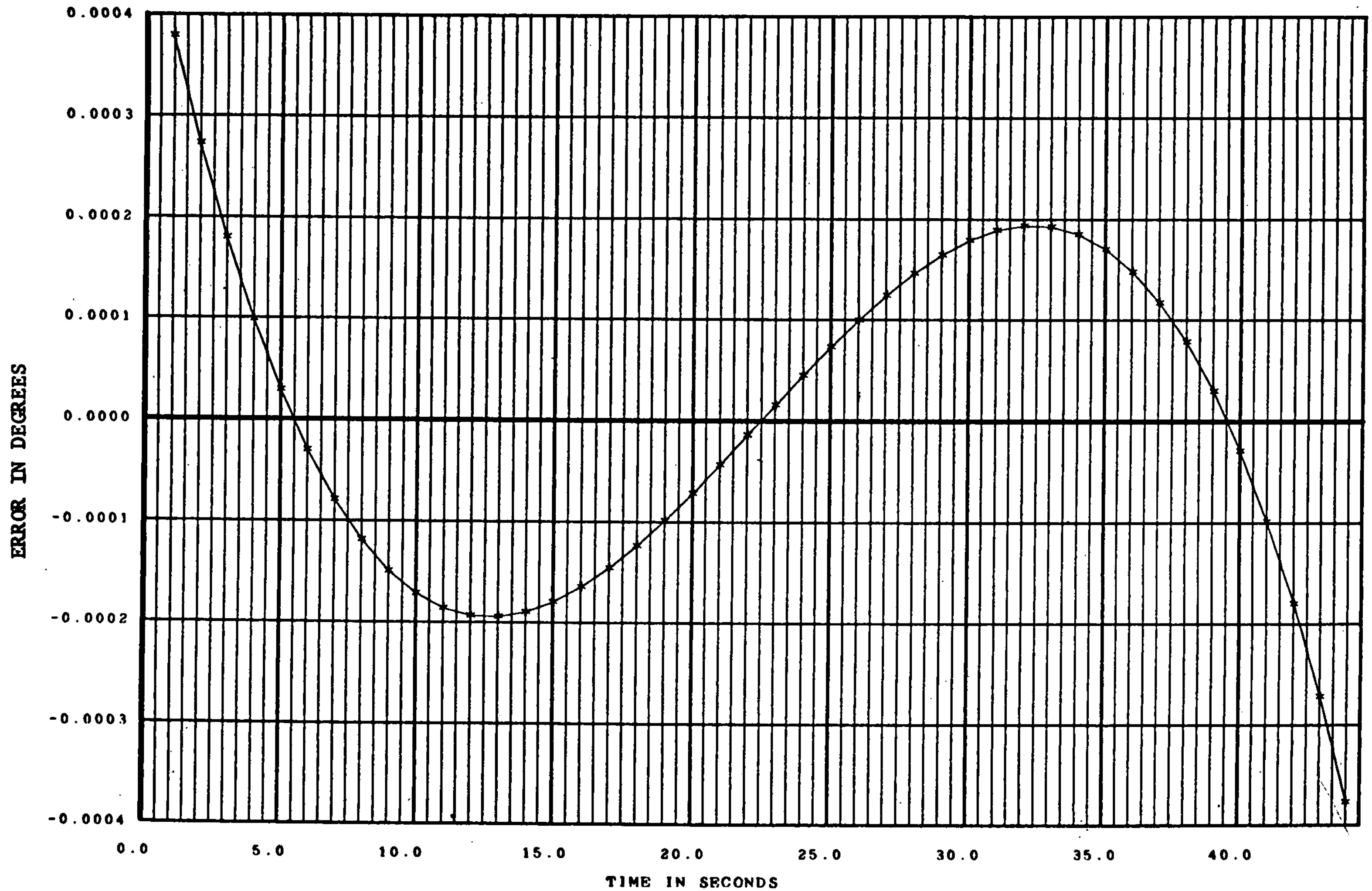

Figure A-82. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

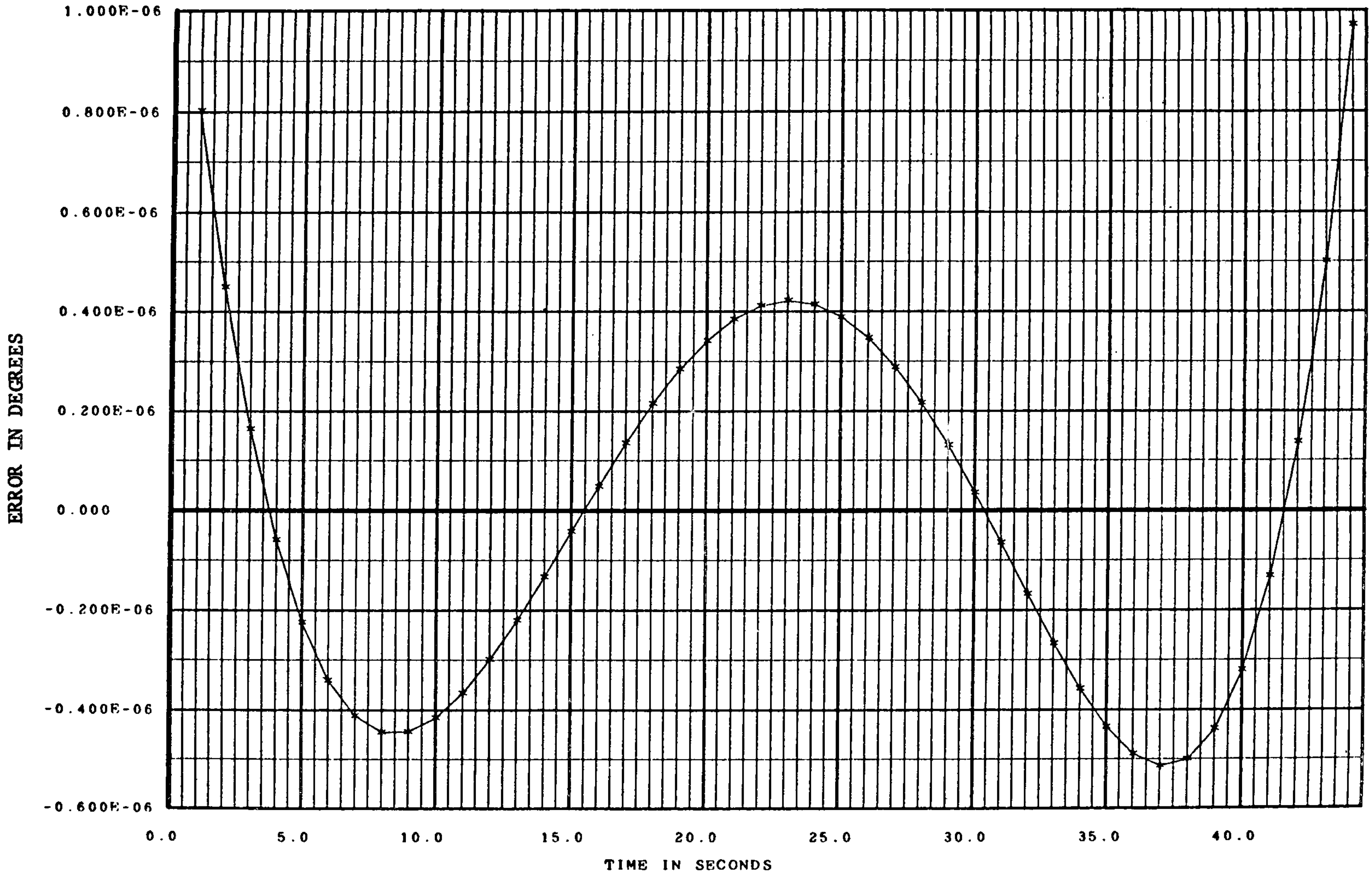

Figure A-83. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

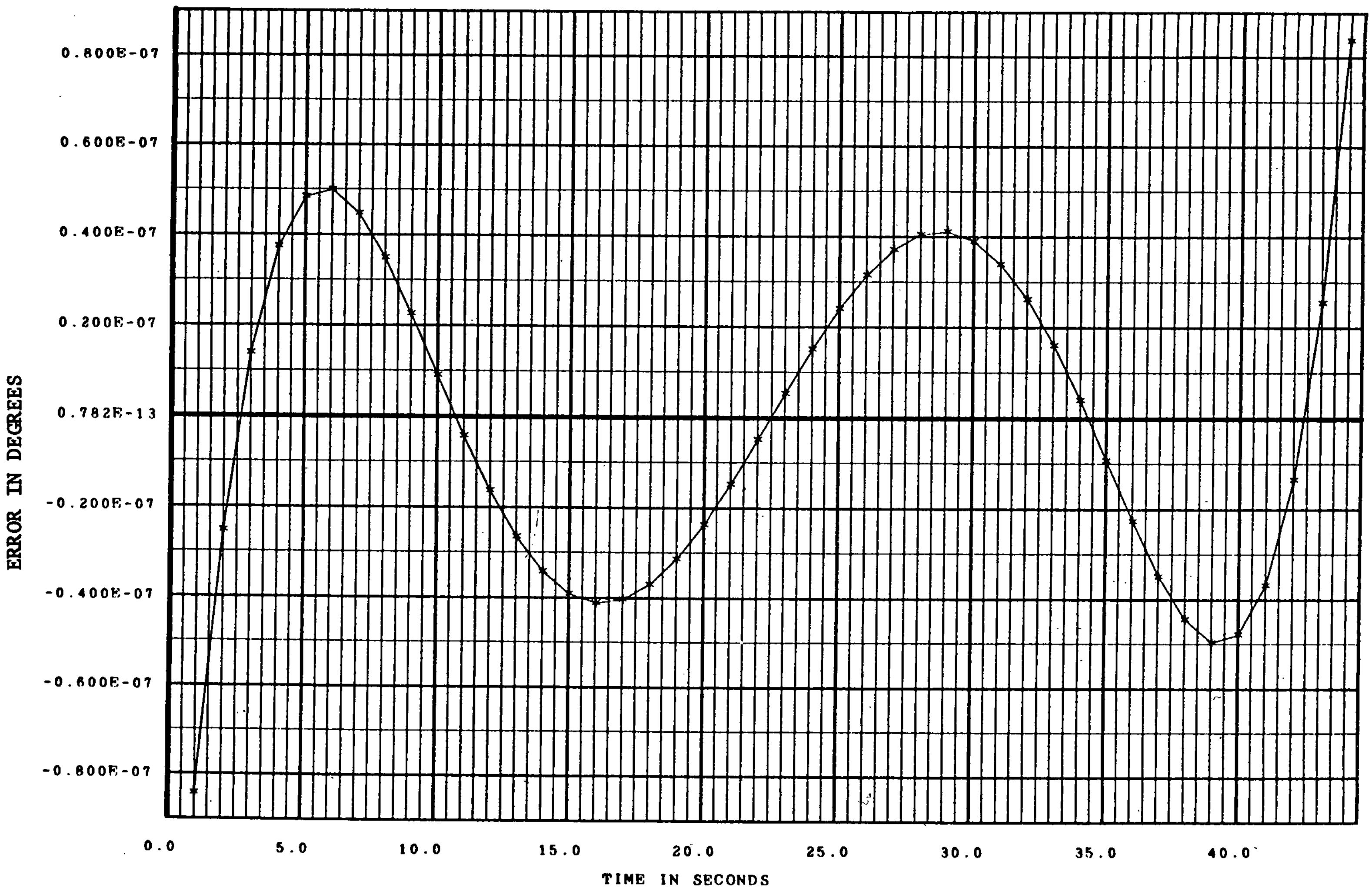

Figure A-84. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

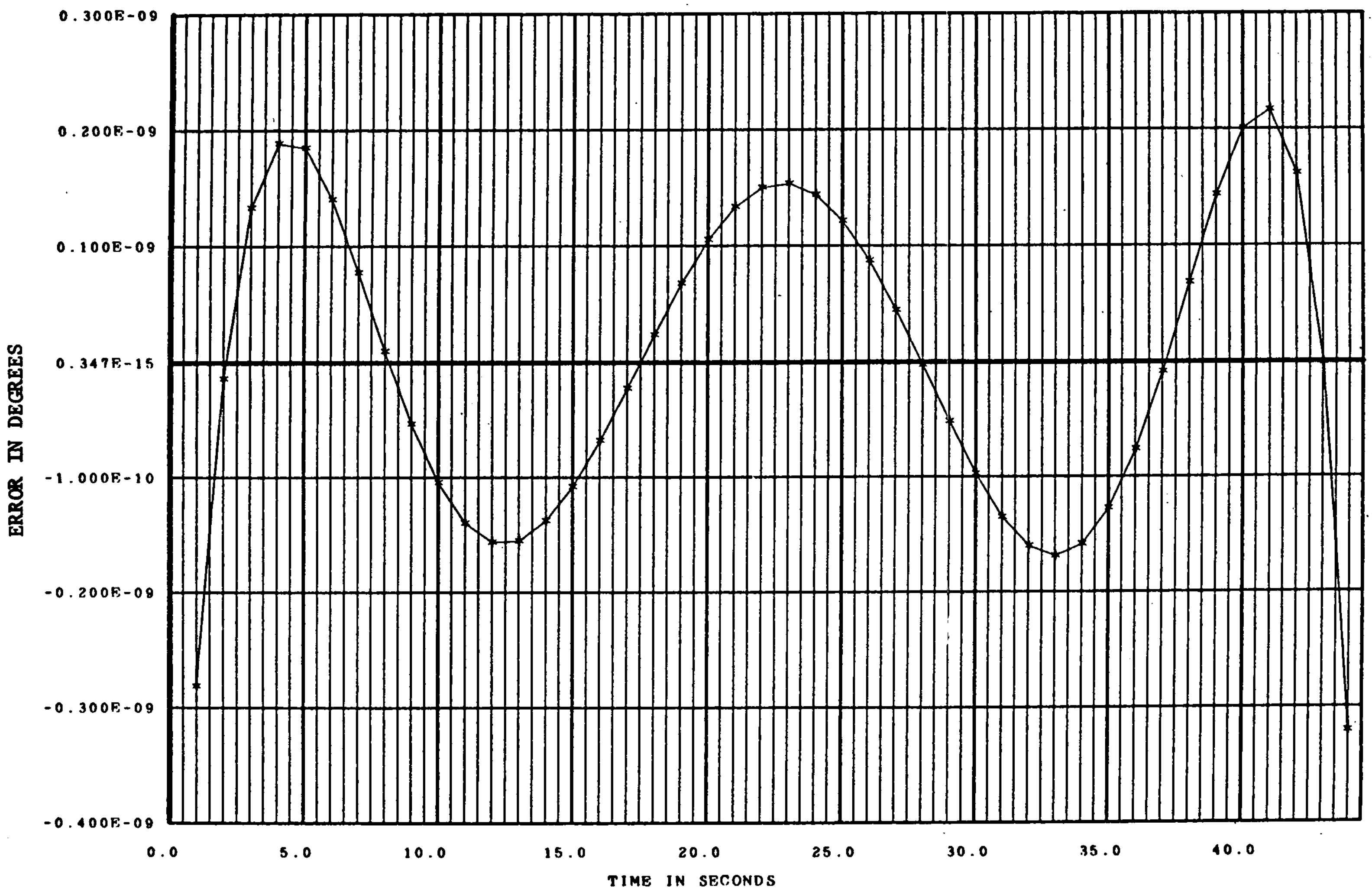

Figure A-85. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

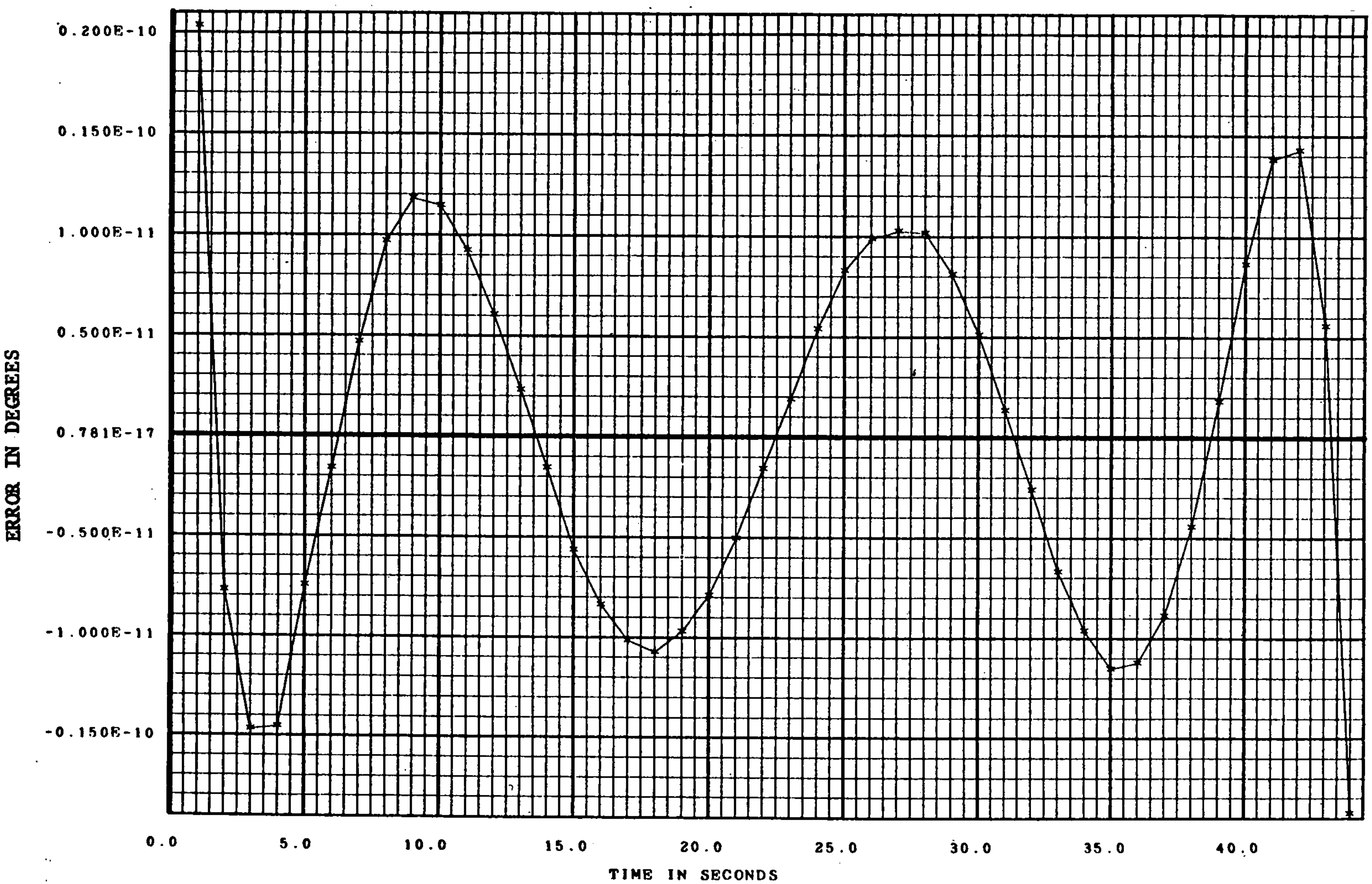

Figure A-86. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

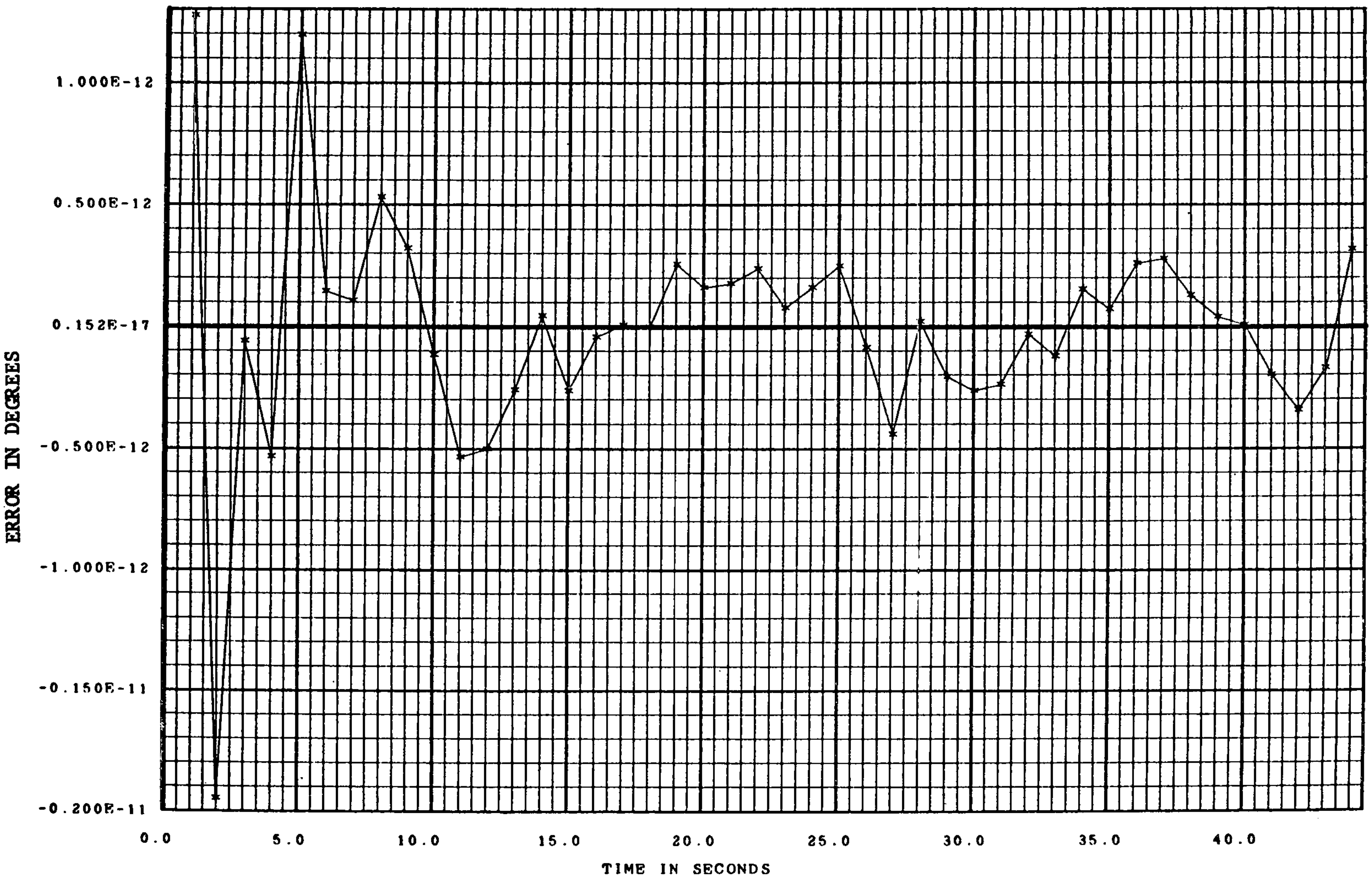

Figure A-87. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

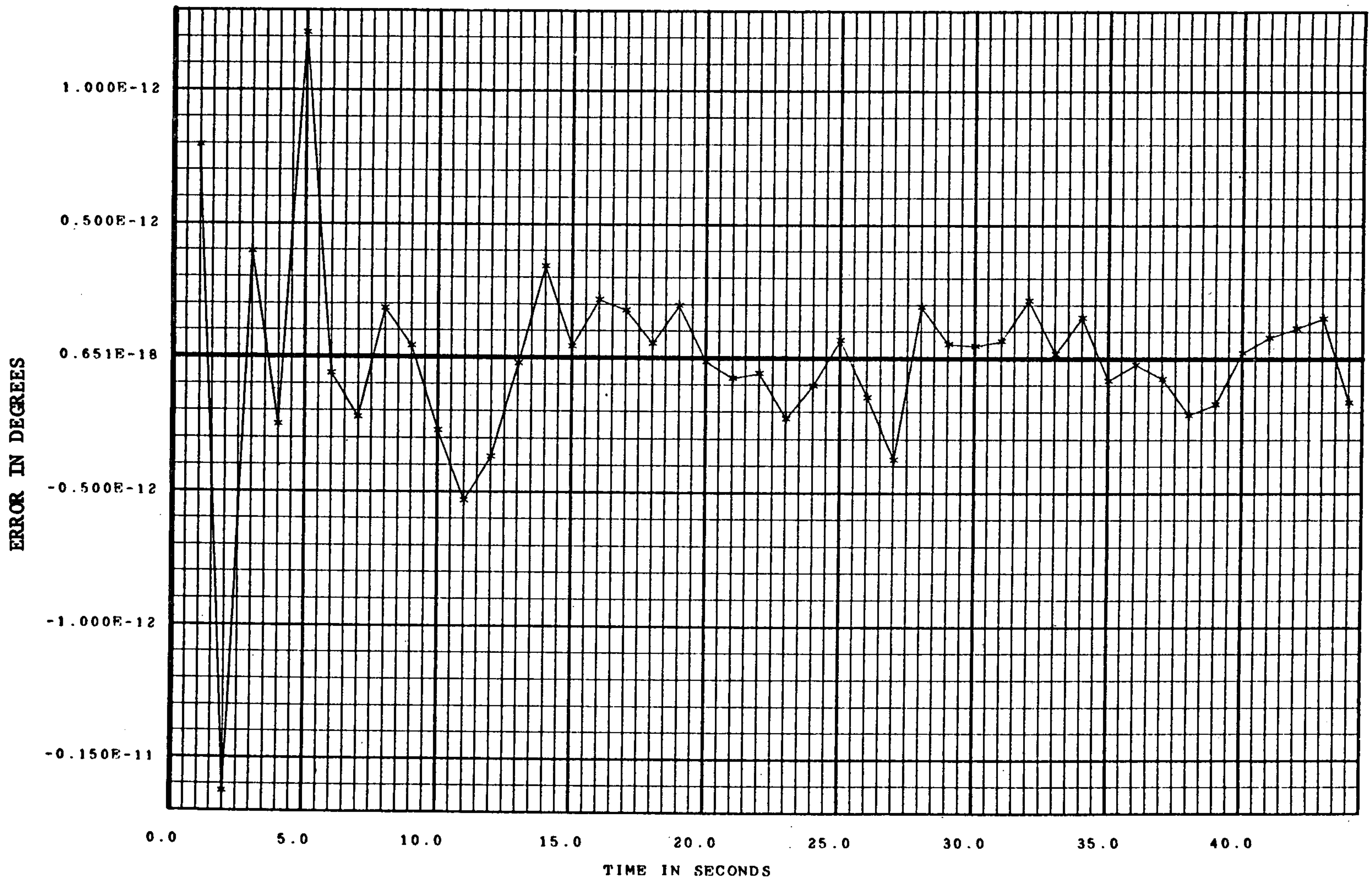

Figure A-88. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

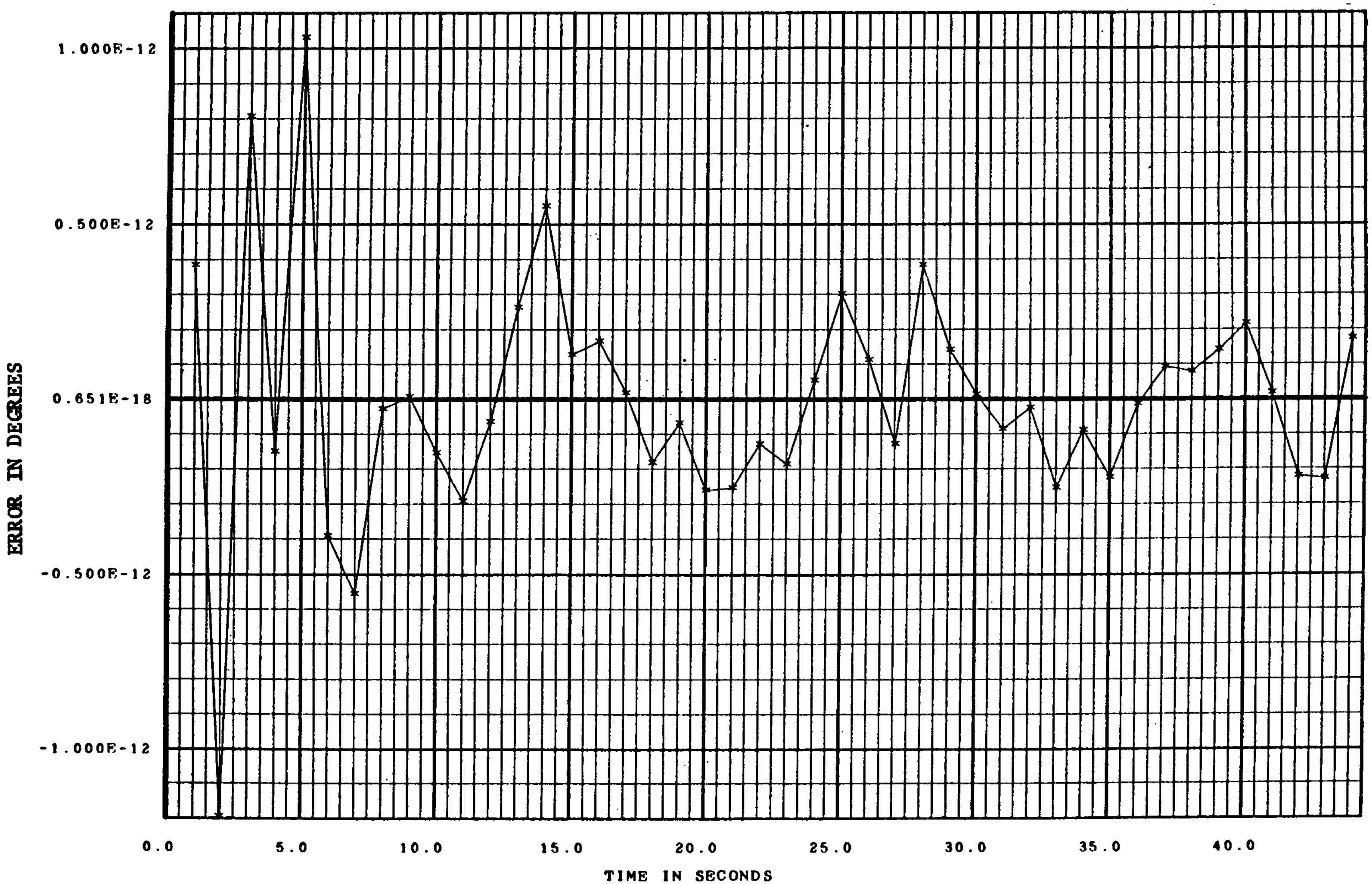

Figure A-89. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

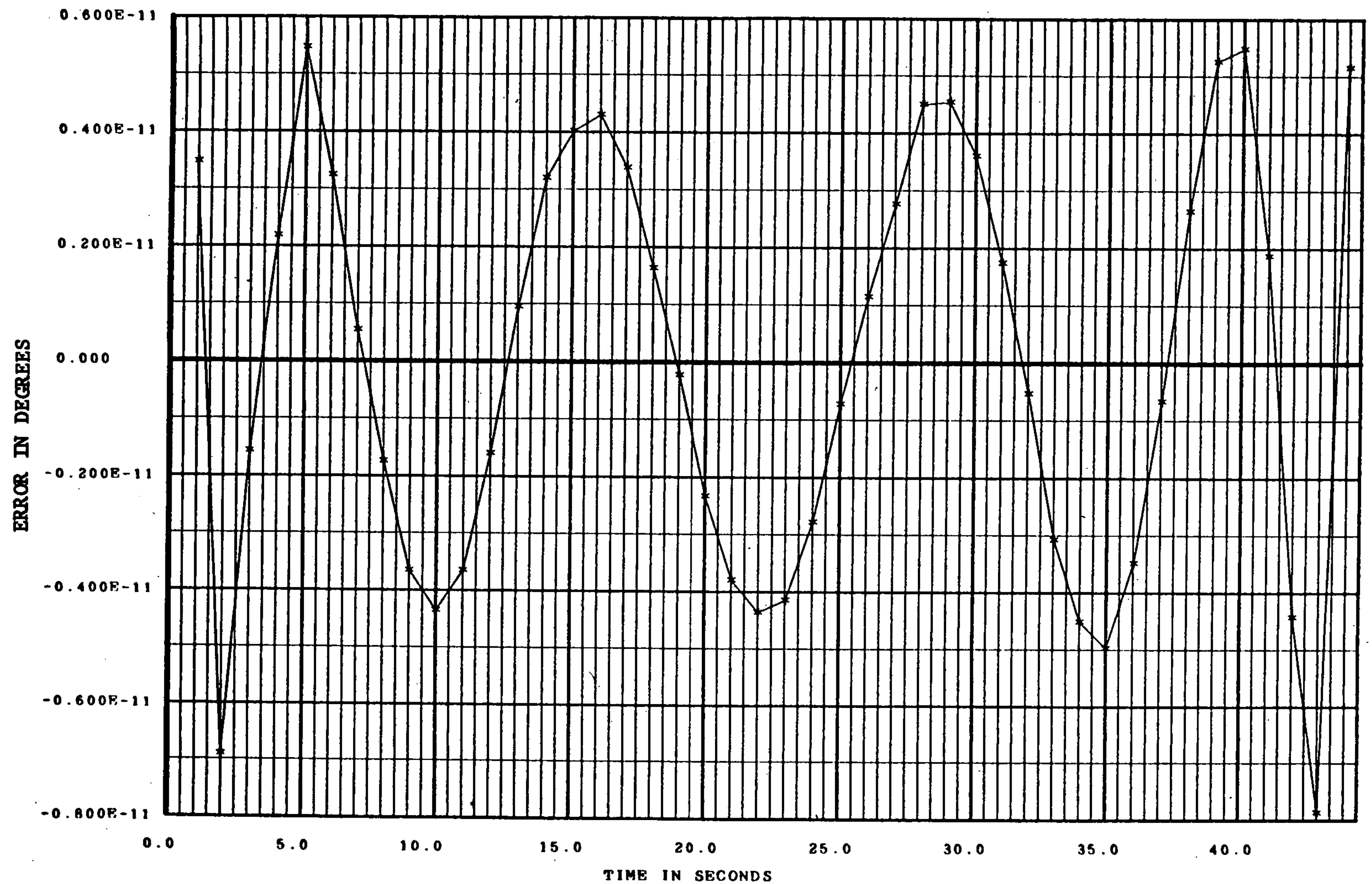

Figure A-90. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 44 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

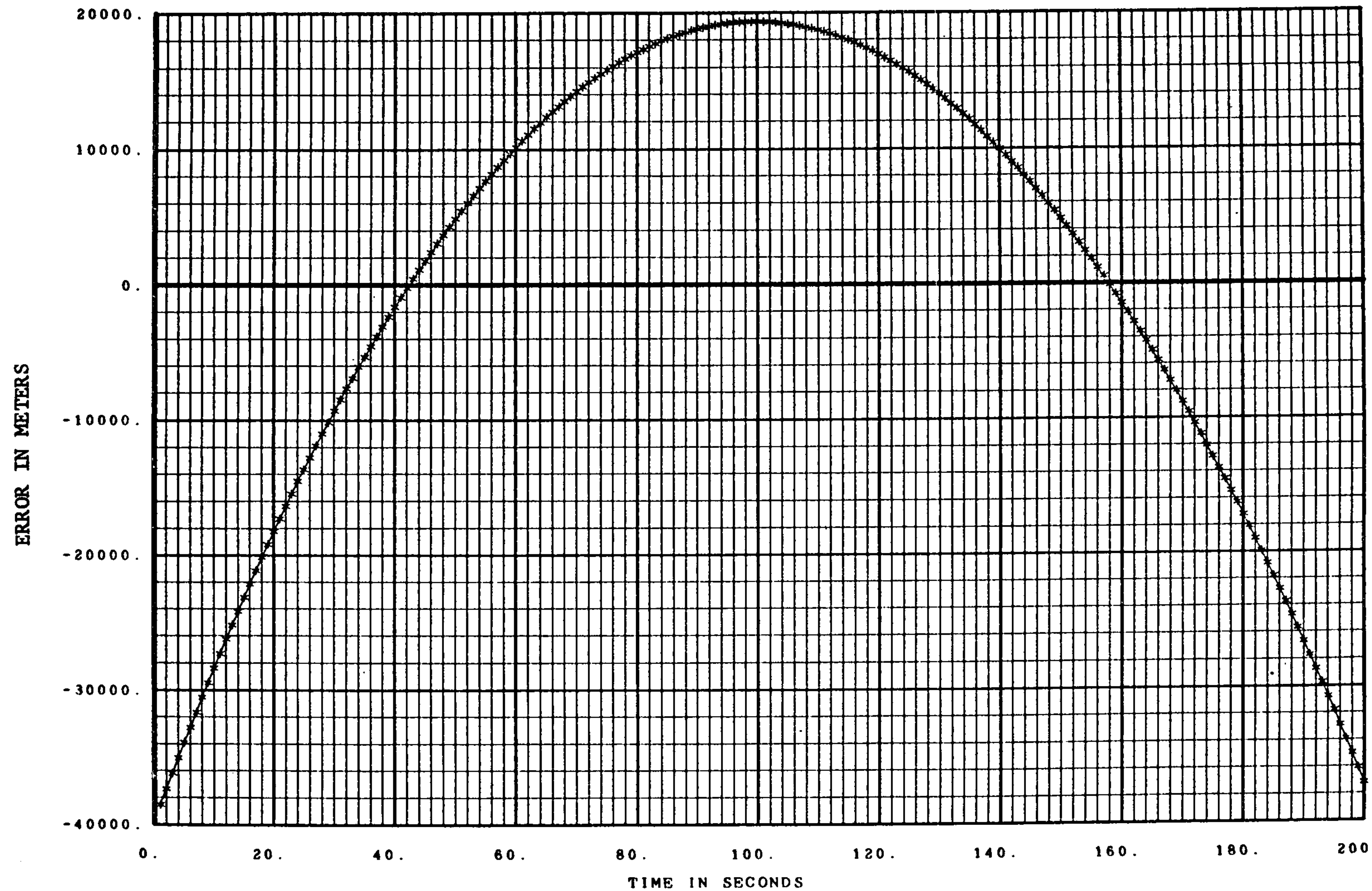

Figure A-91. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

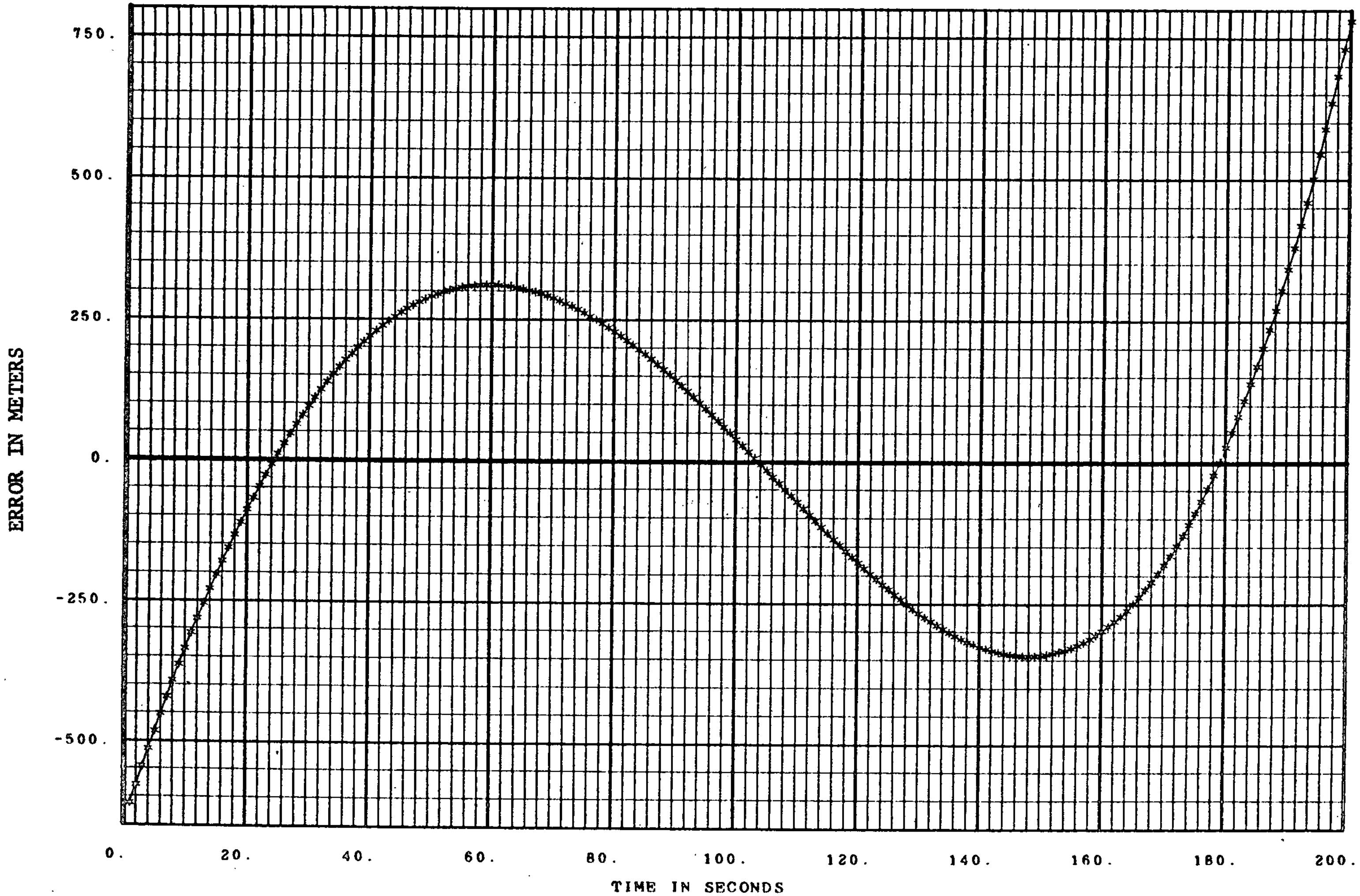

Figure A-92. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

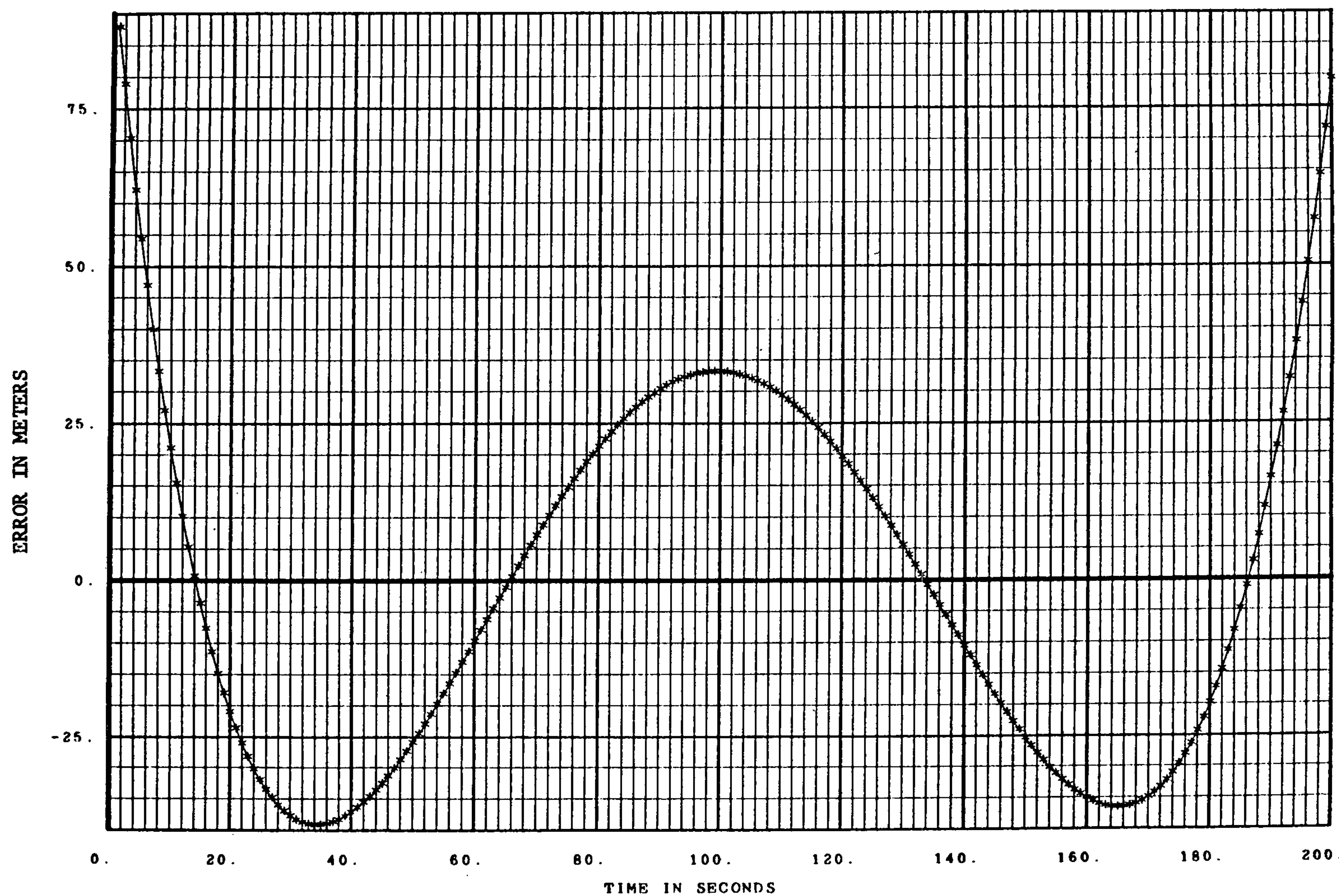

Figure A-93. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

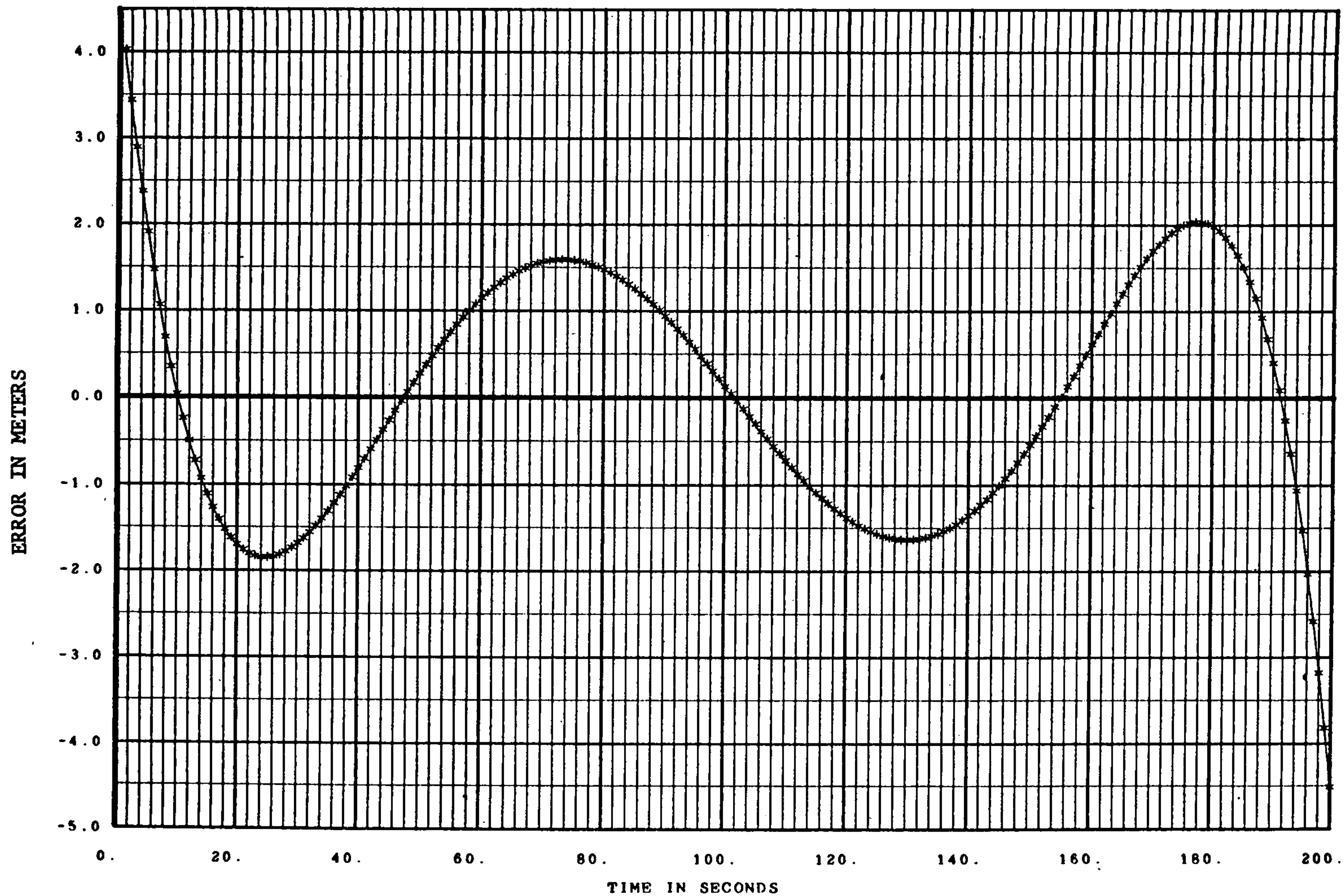

Figure A-94. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

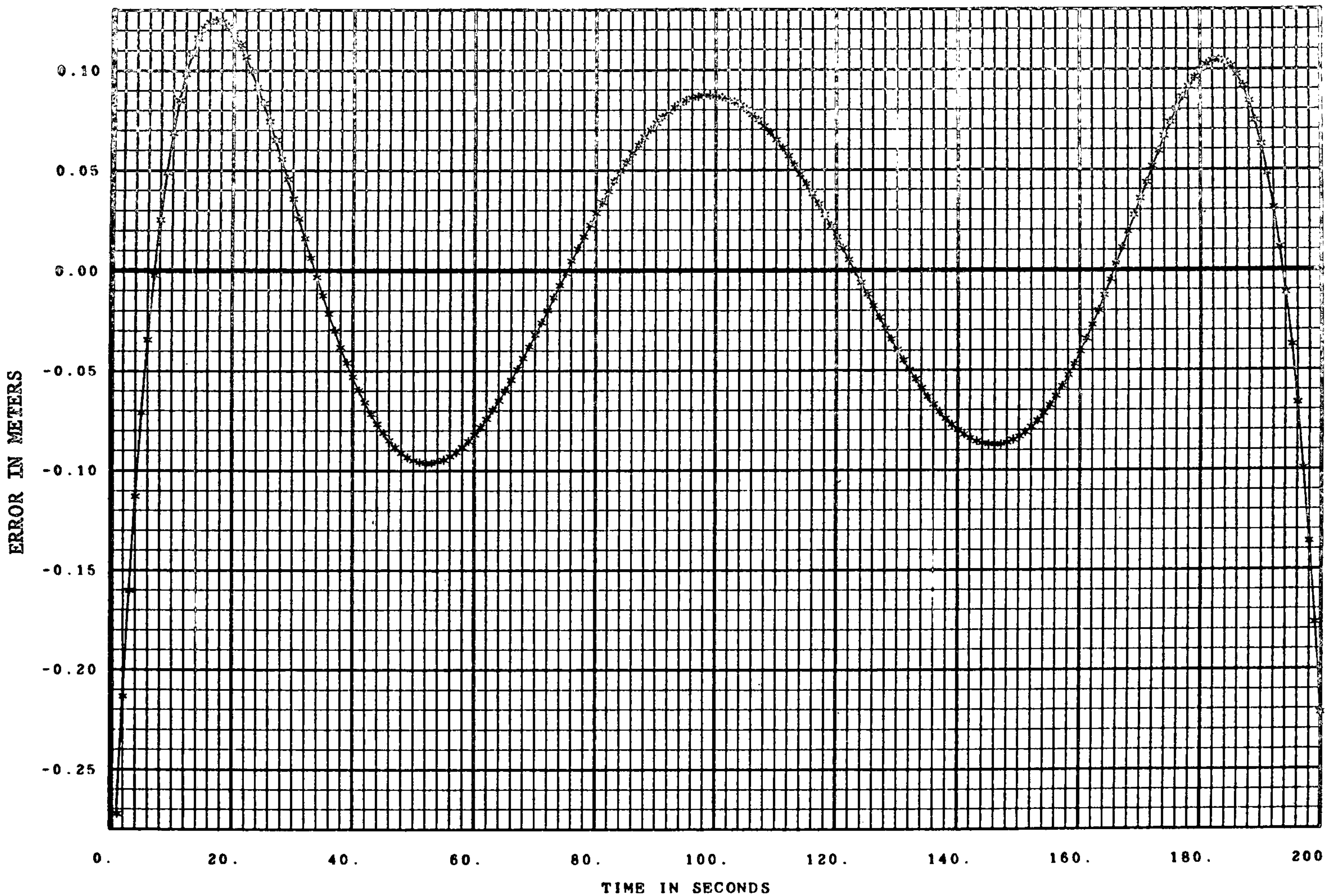

Figure A-95. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

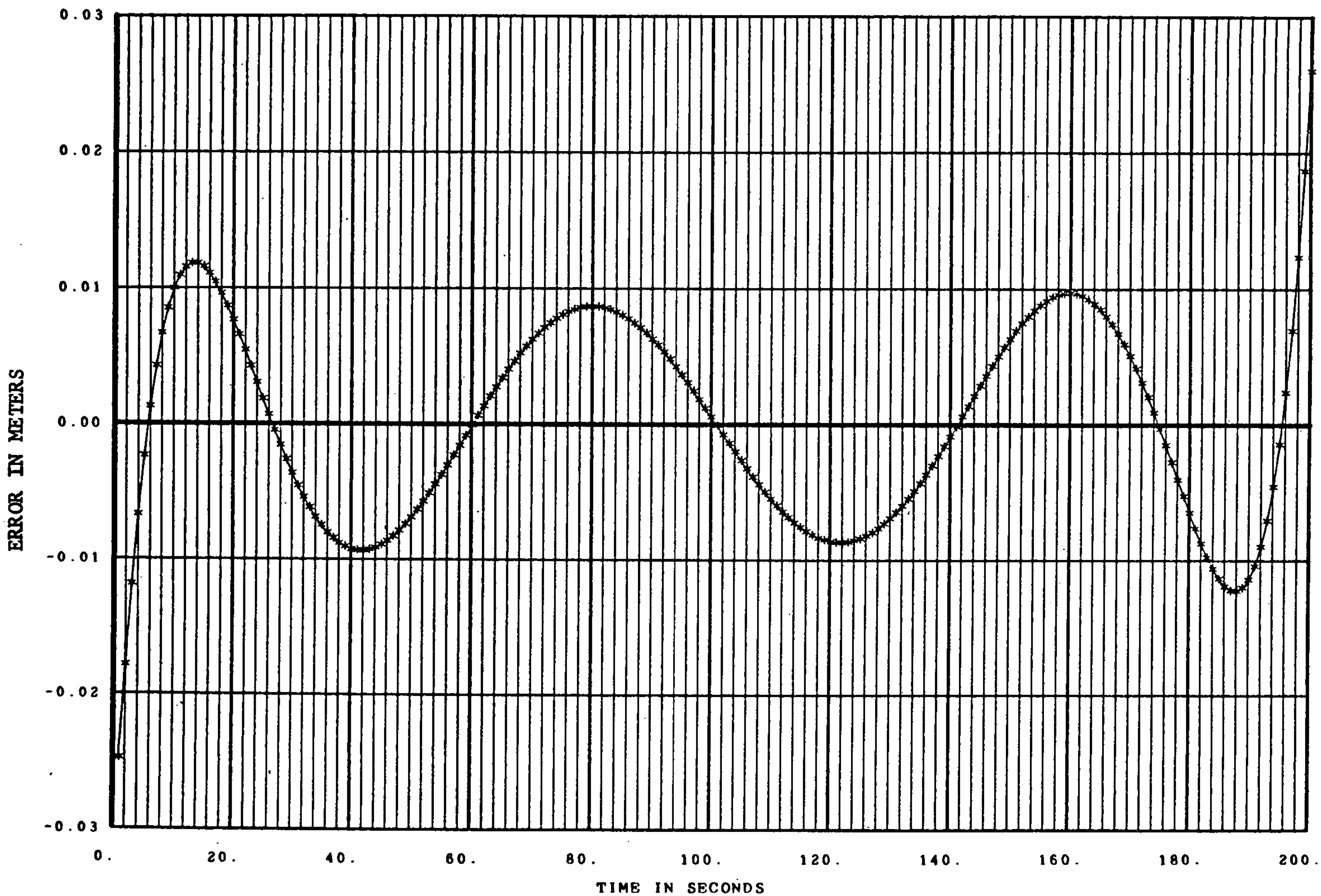

Figure A-96. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

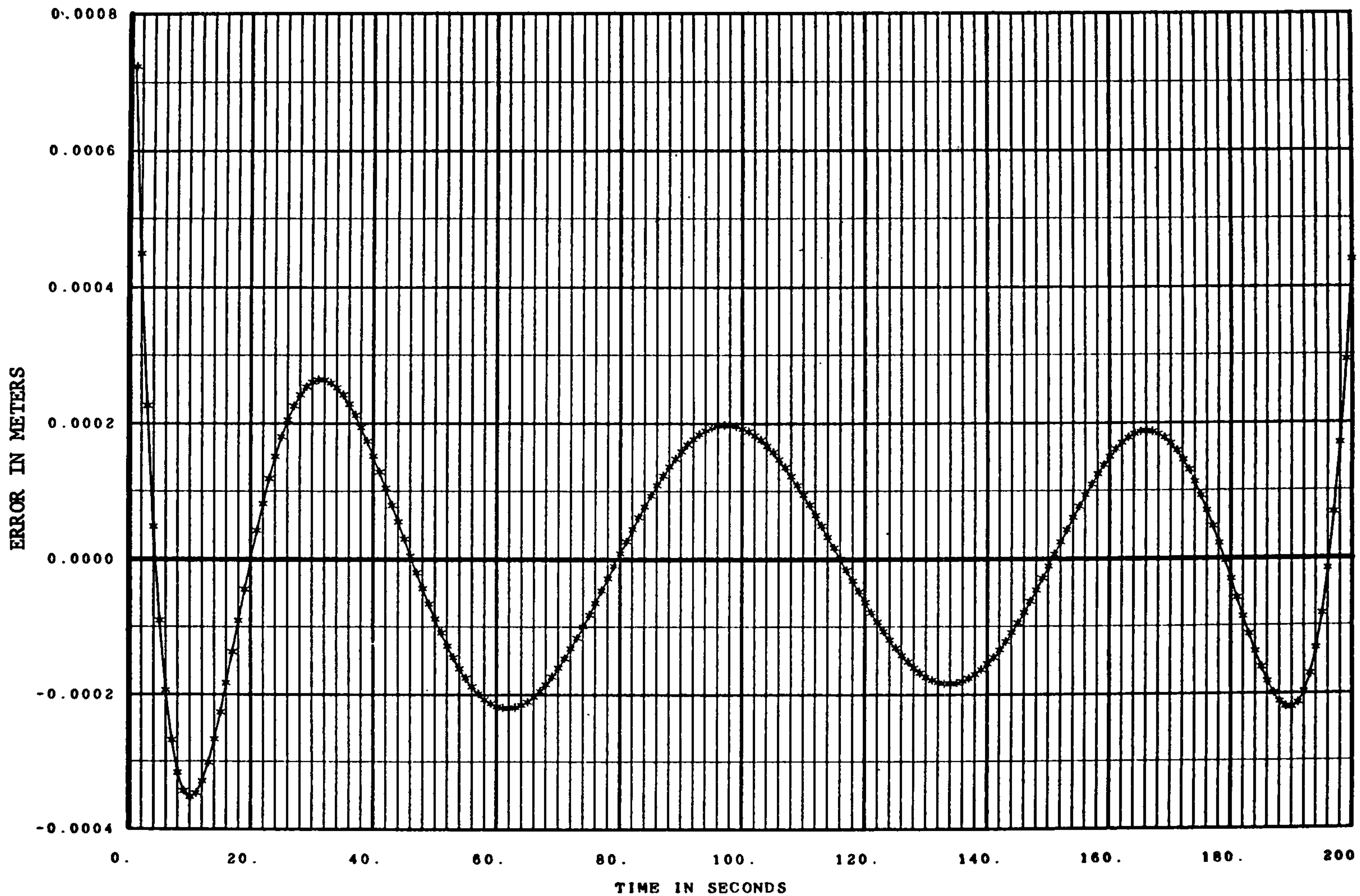

Figure A-97. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

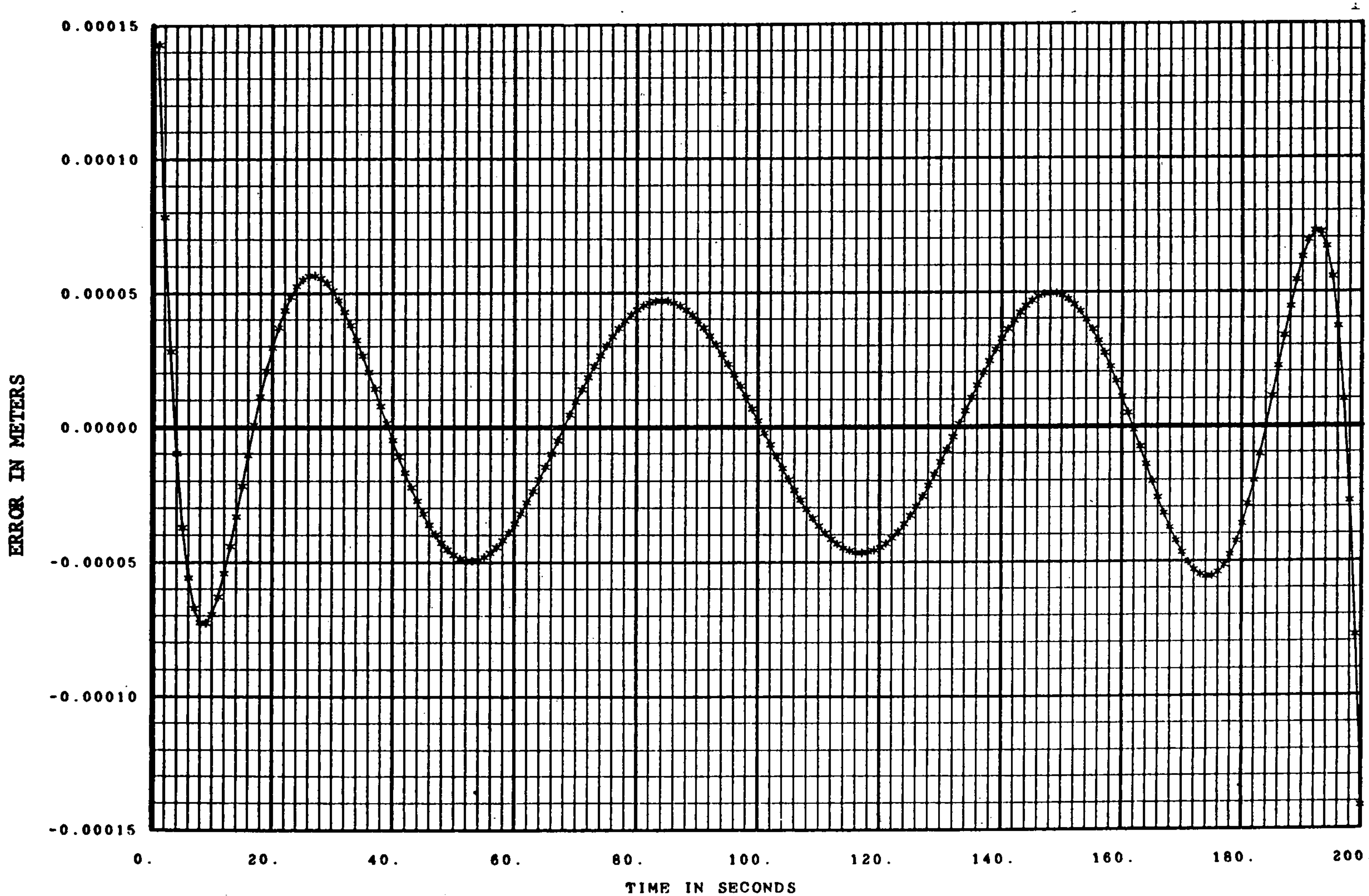

Figure A-98. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

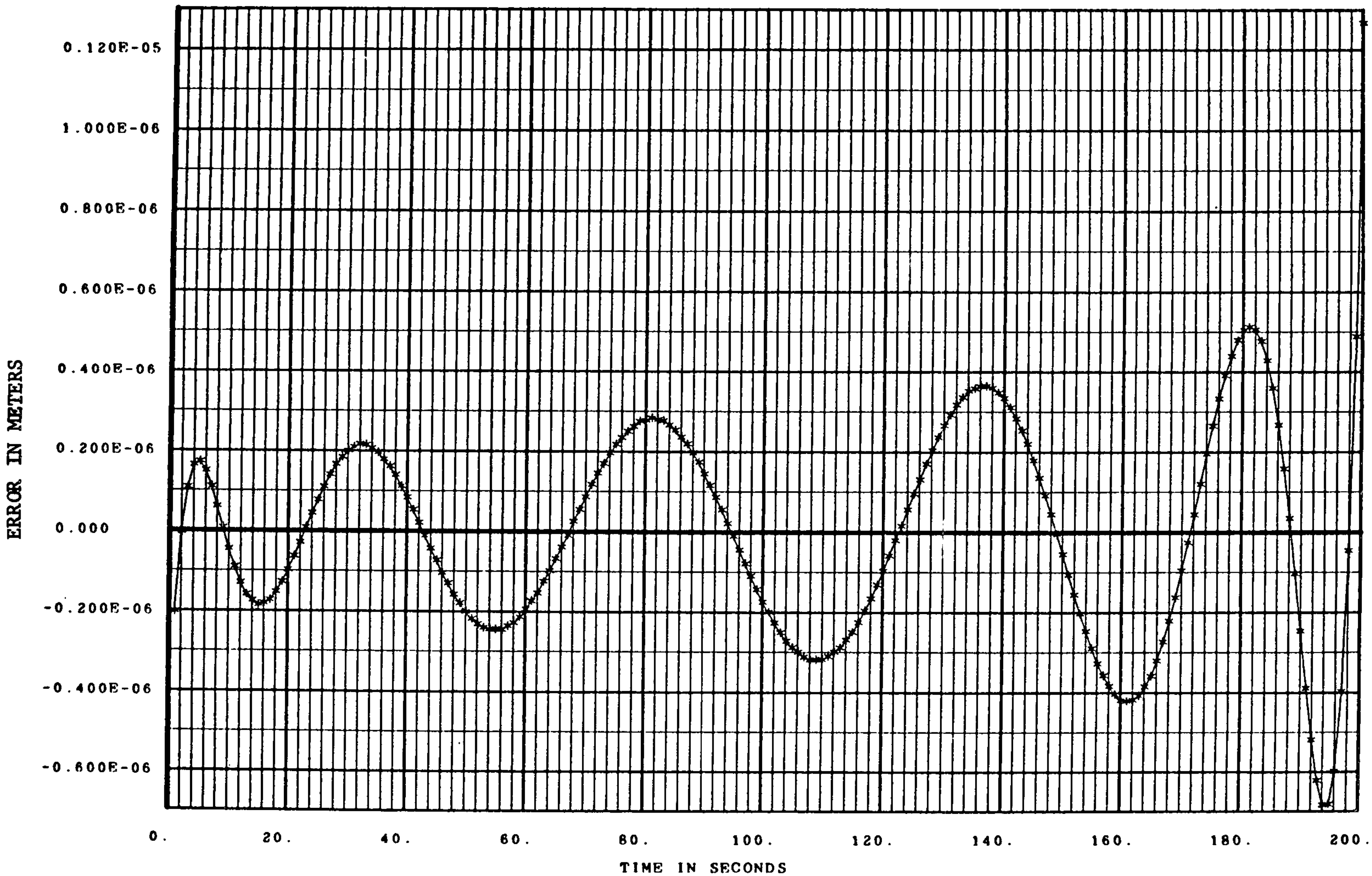

Figure A-99. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

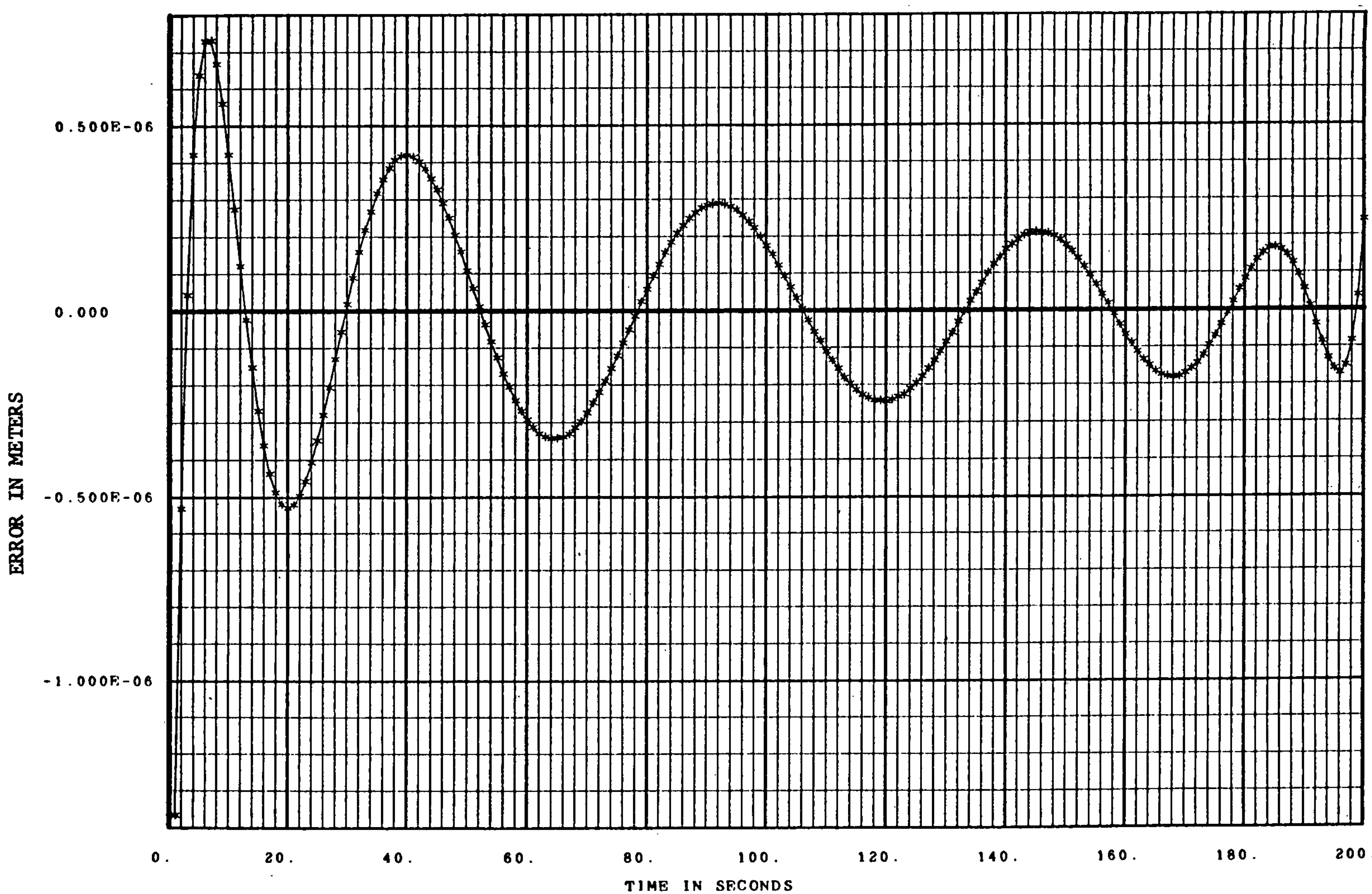

Figure A-100. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 200 points of simulated range data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

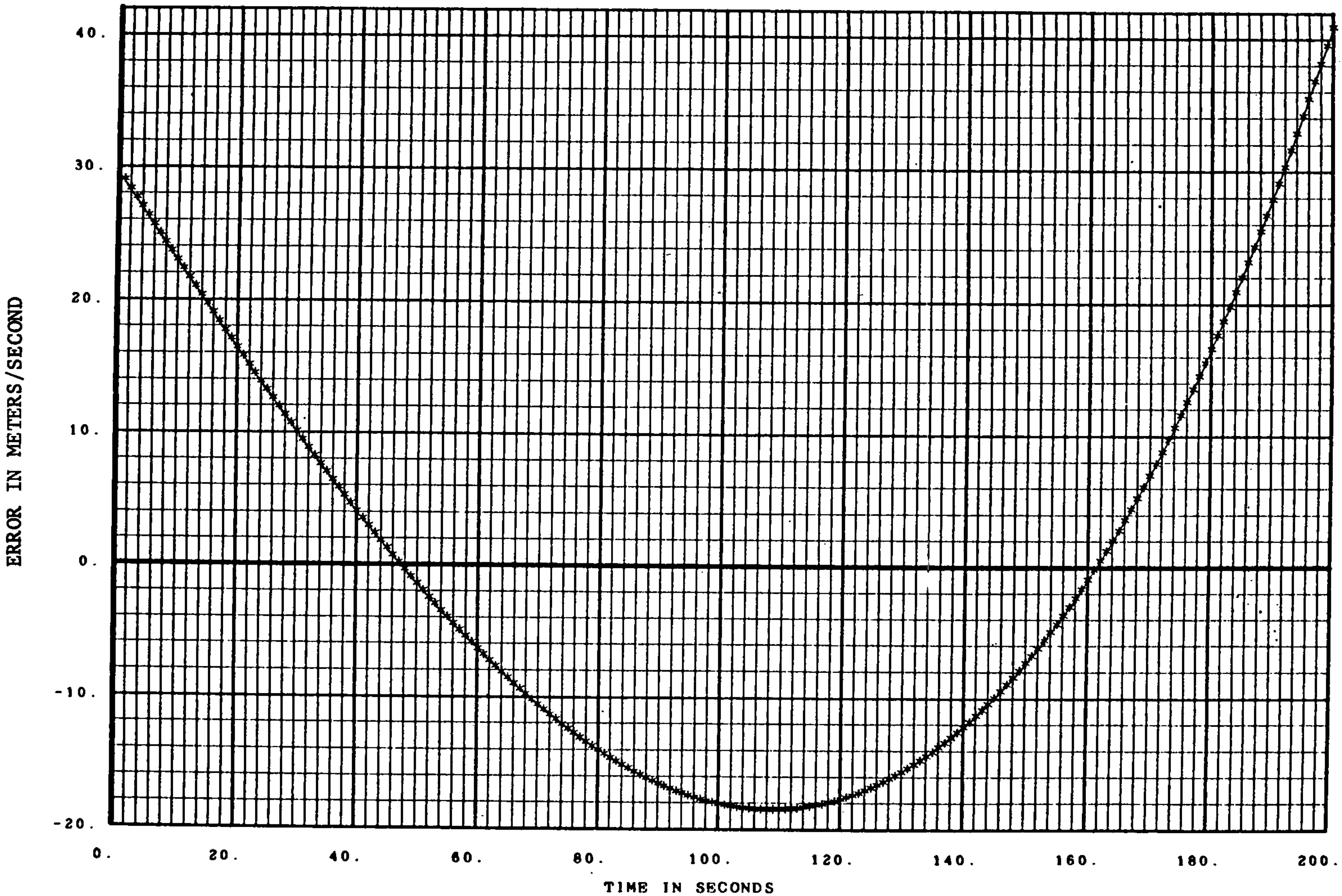

Figure A-101. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

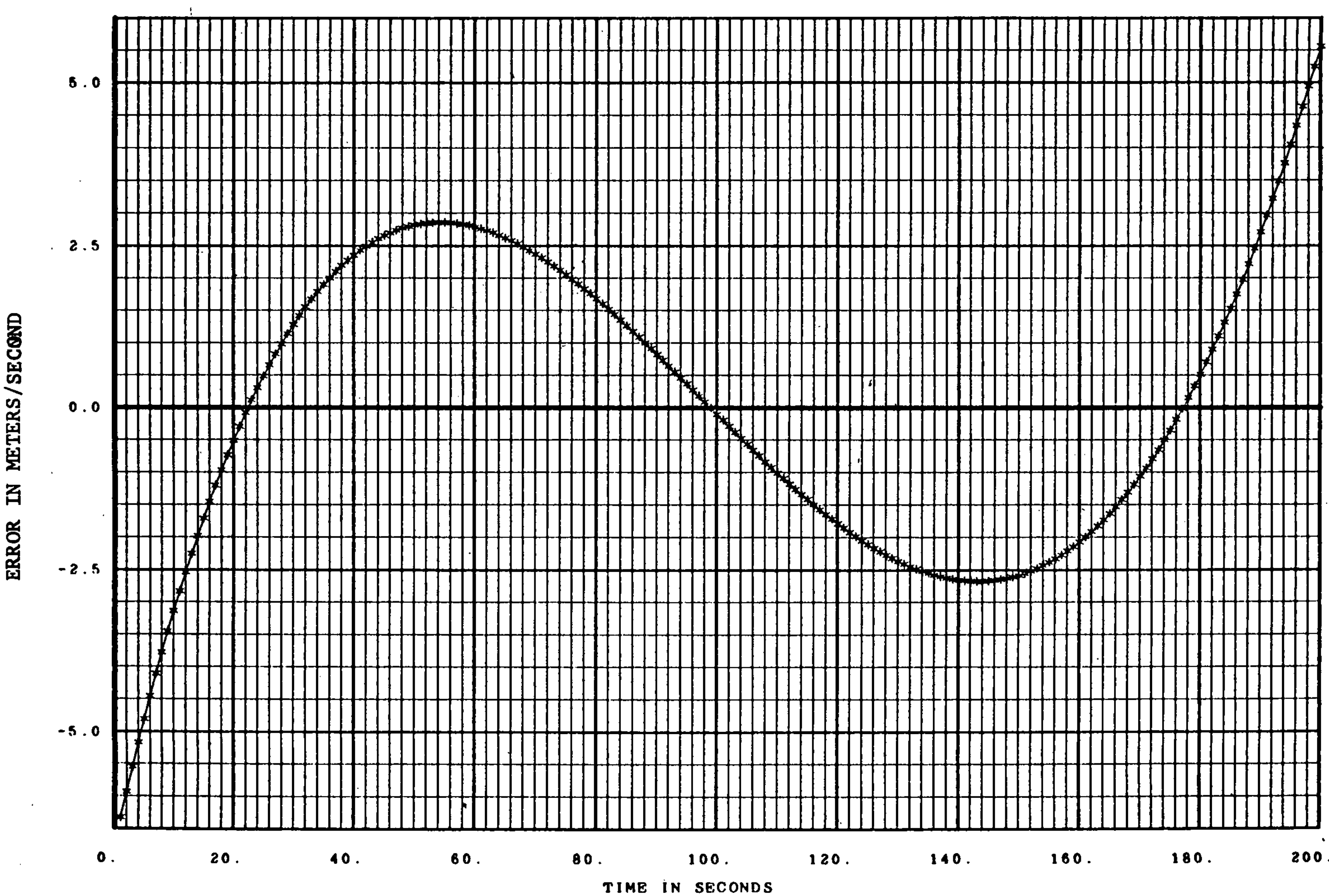

Figure A-102. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

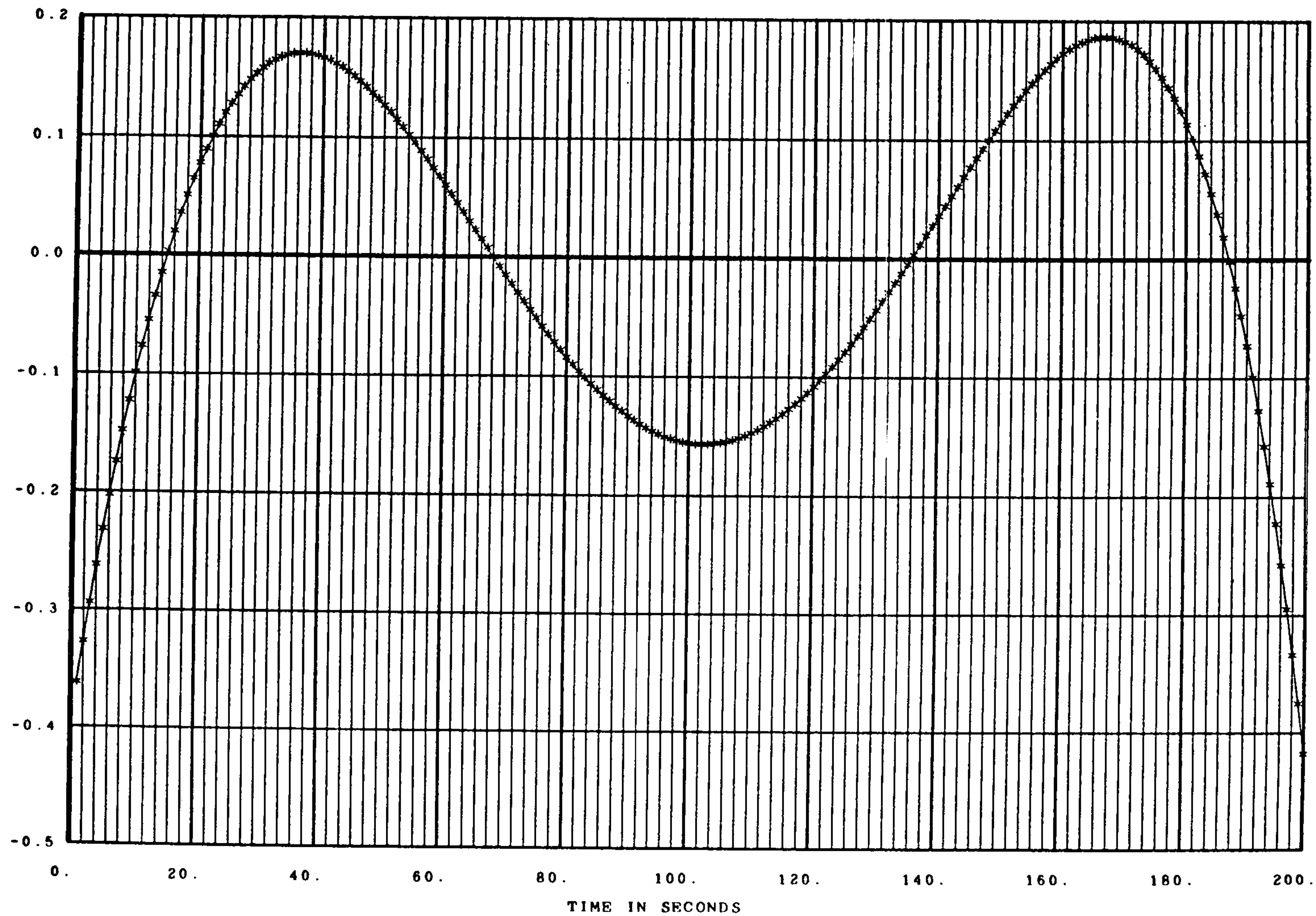

Figure A-103. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

3

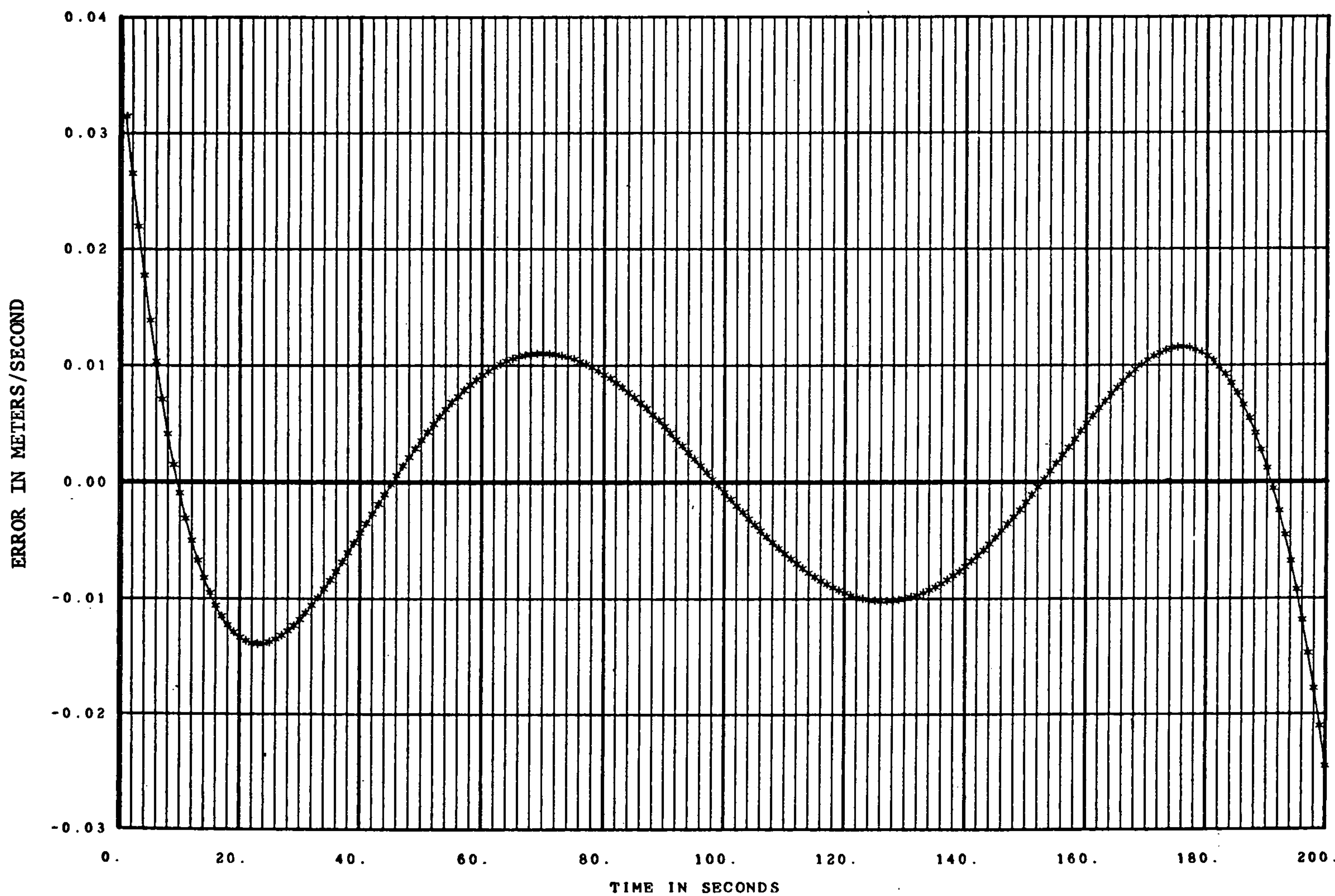

Figure A-104. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

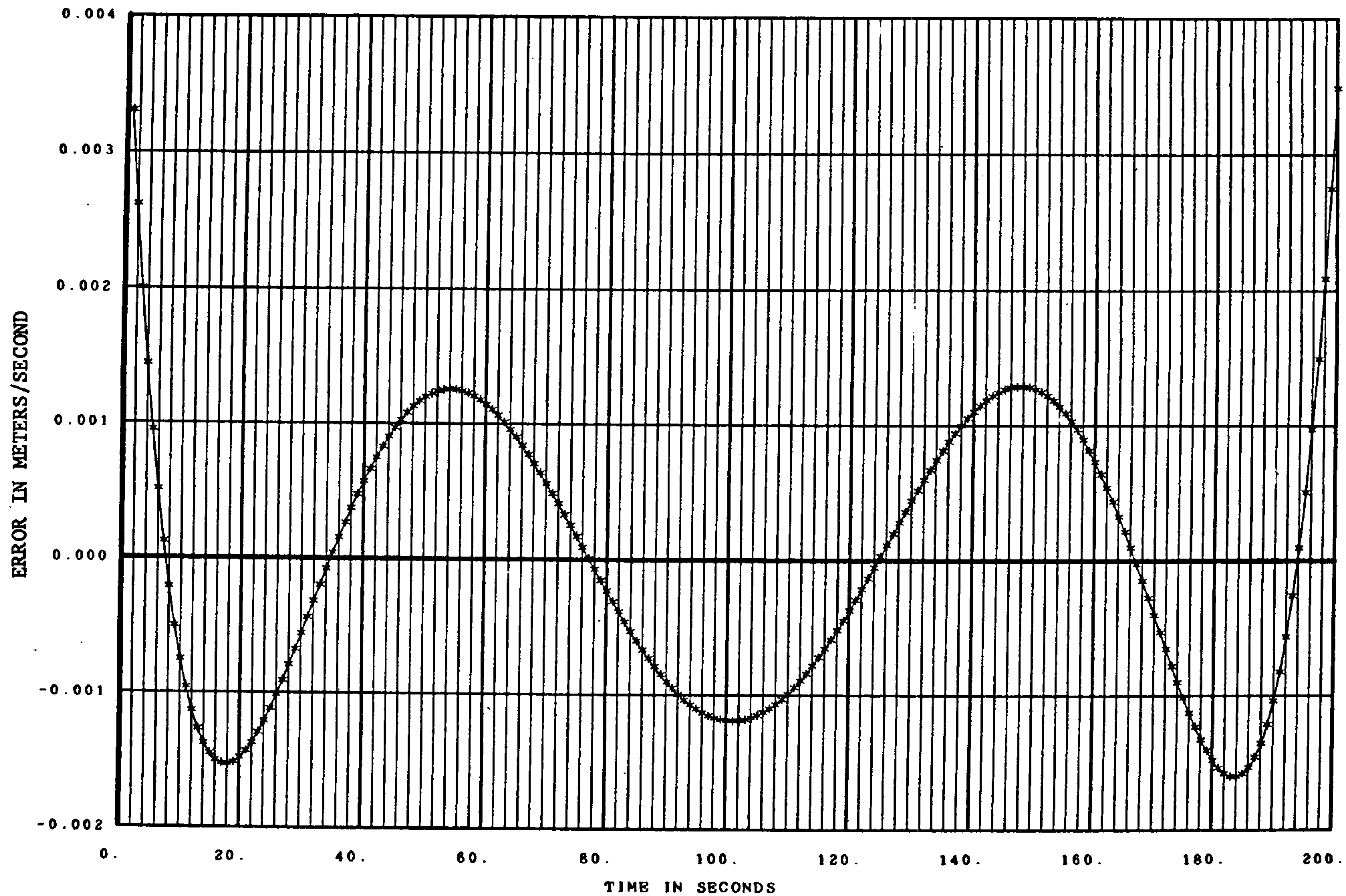

Figure A-105. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

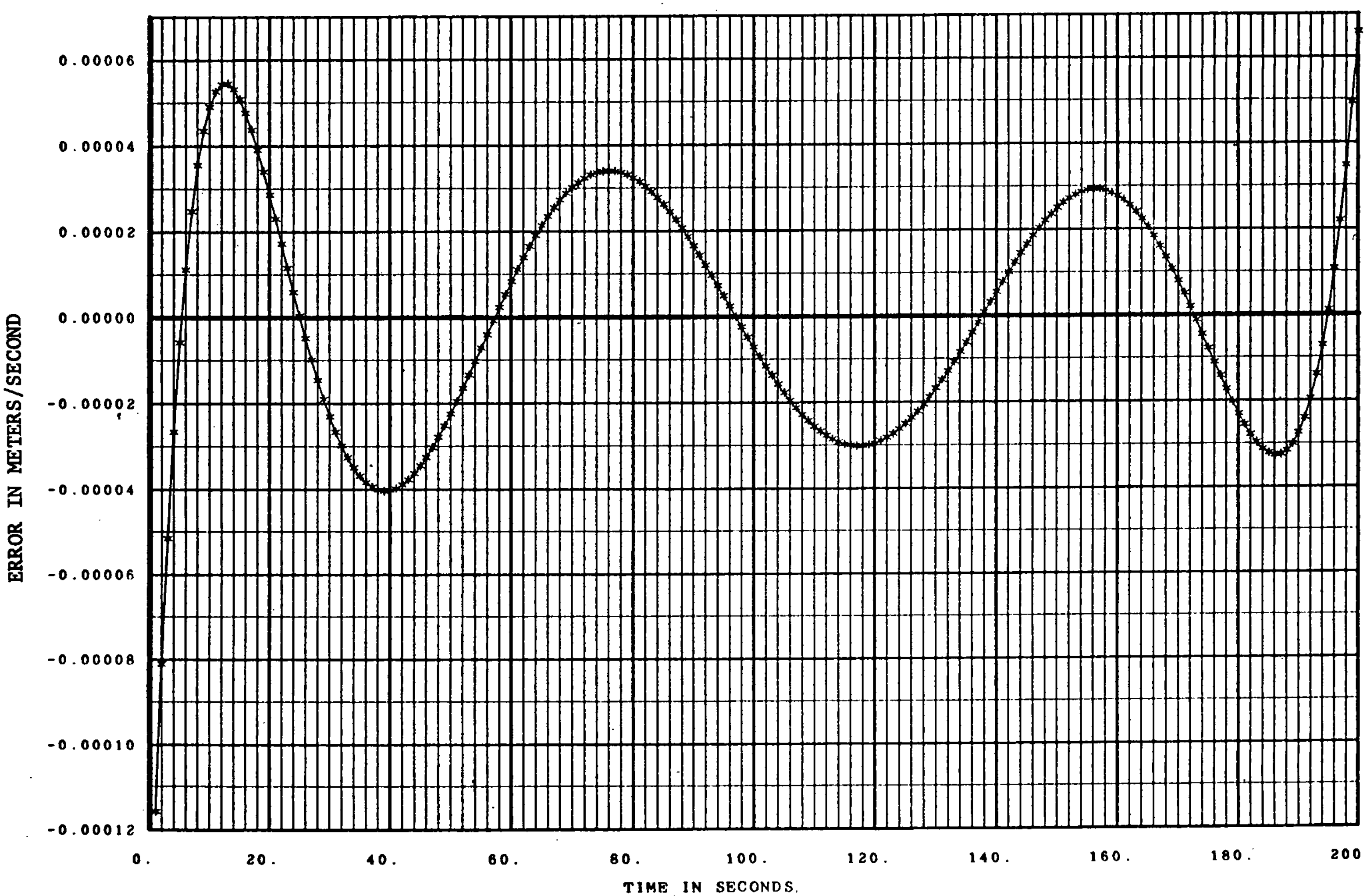

Figure A-106. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

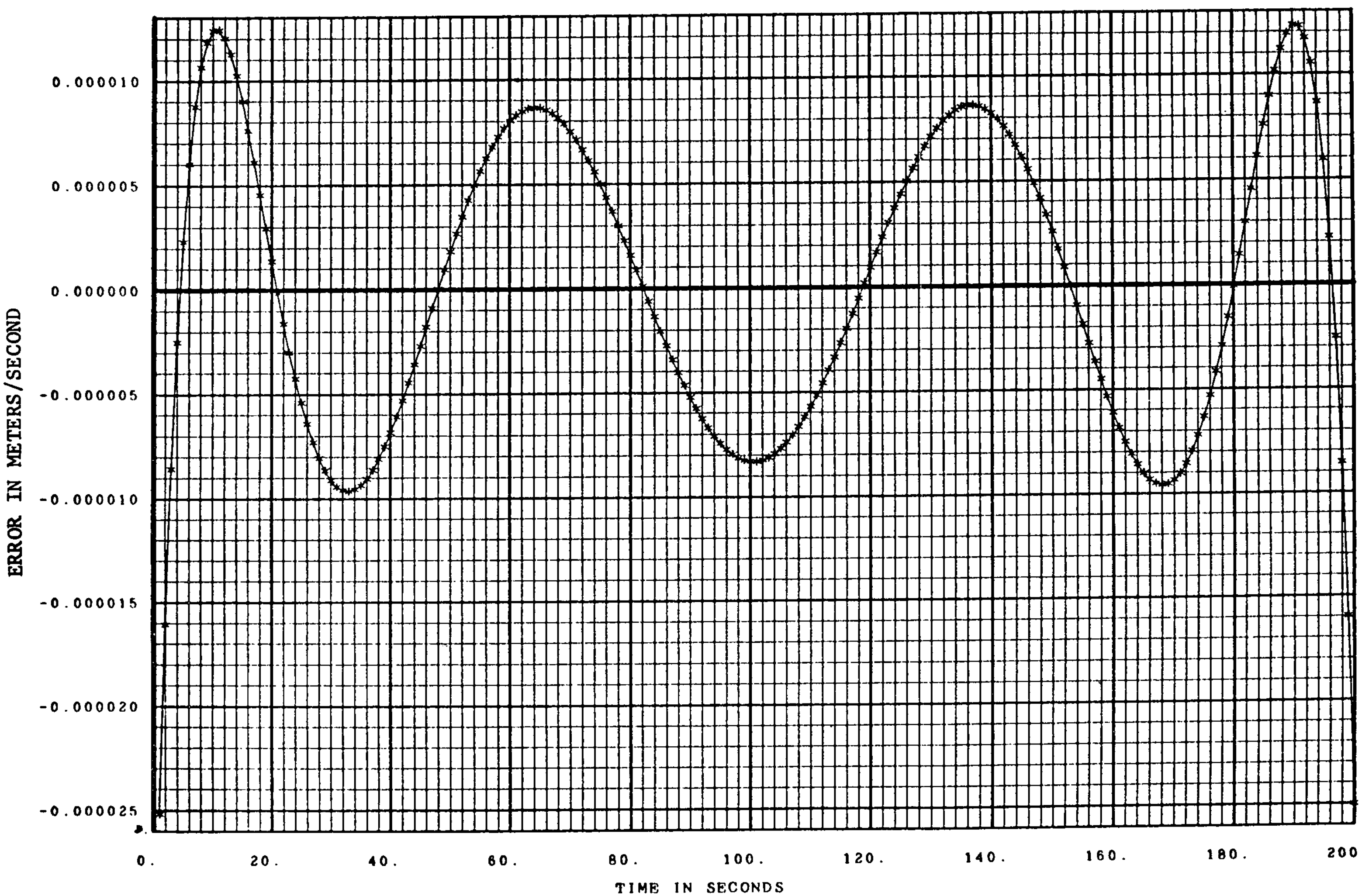

Figure A-107. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

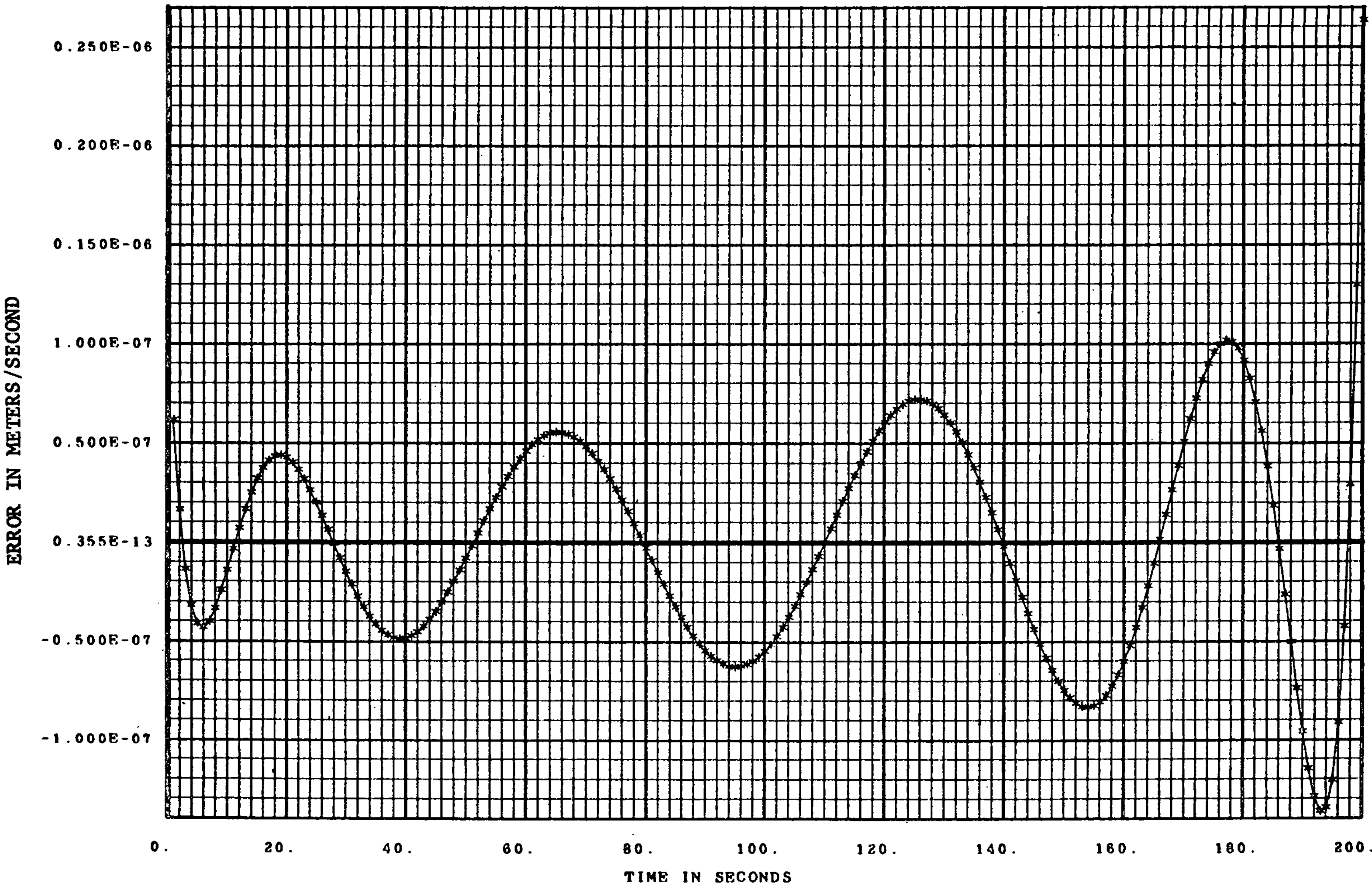

Figure A-108. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

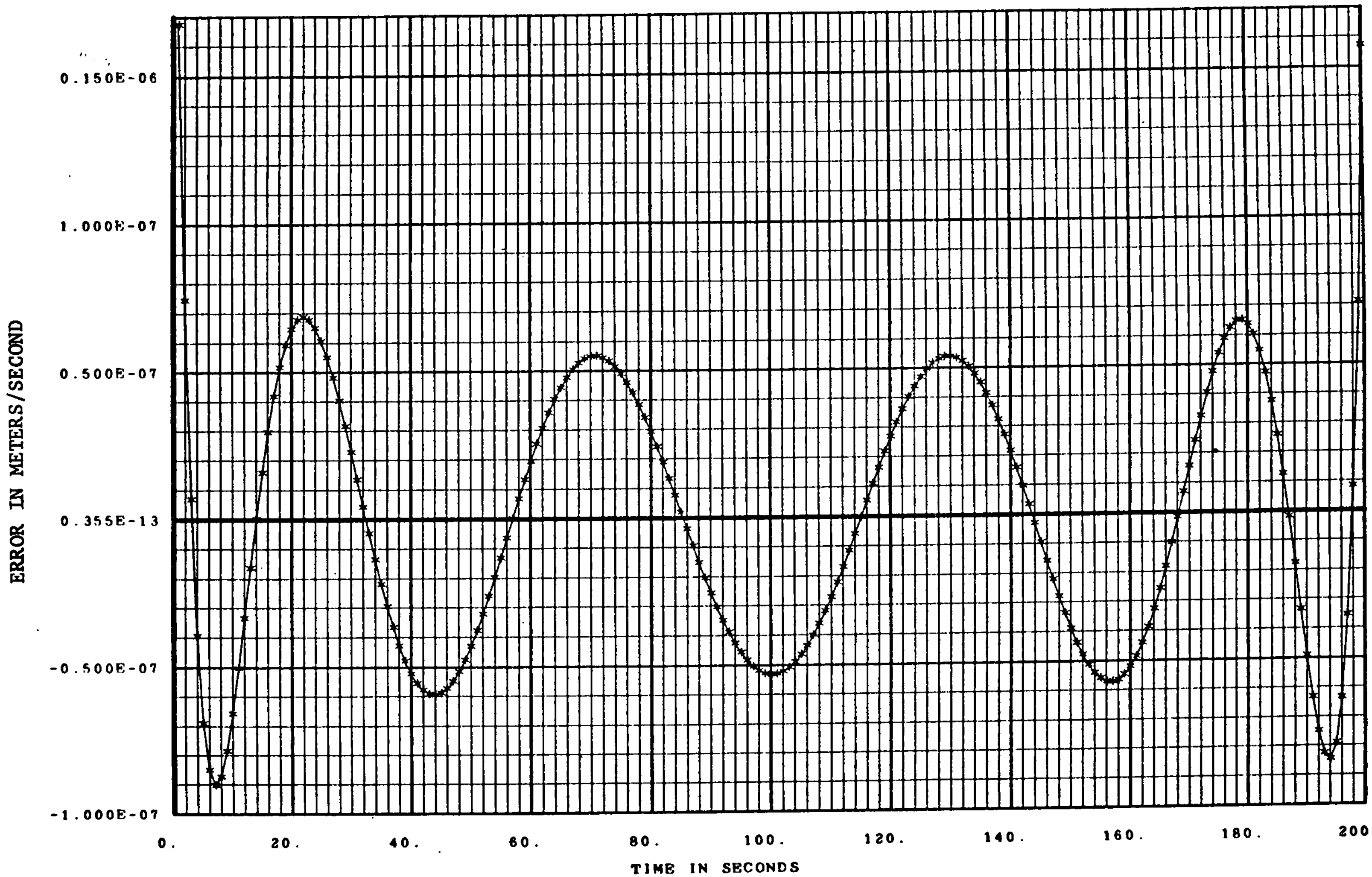

Figure A-109. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

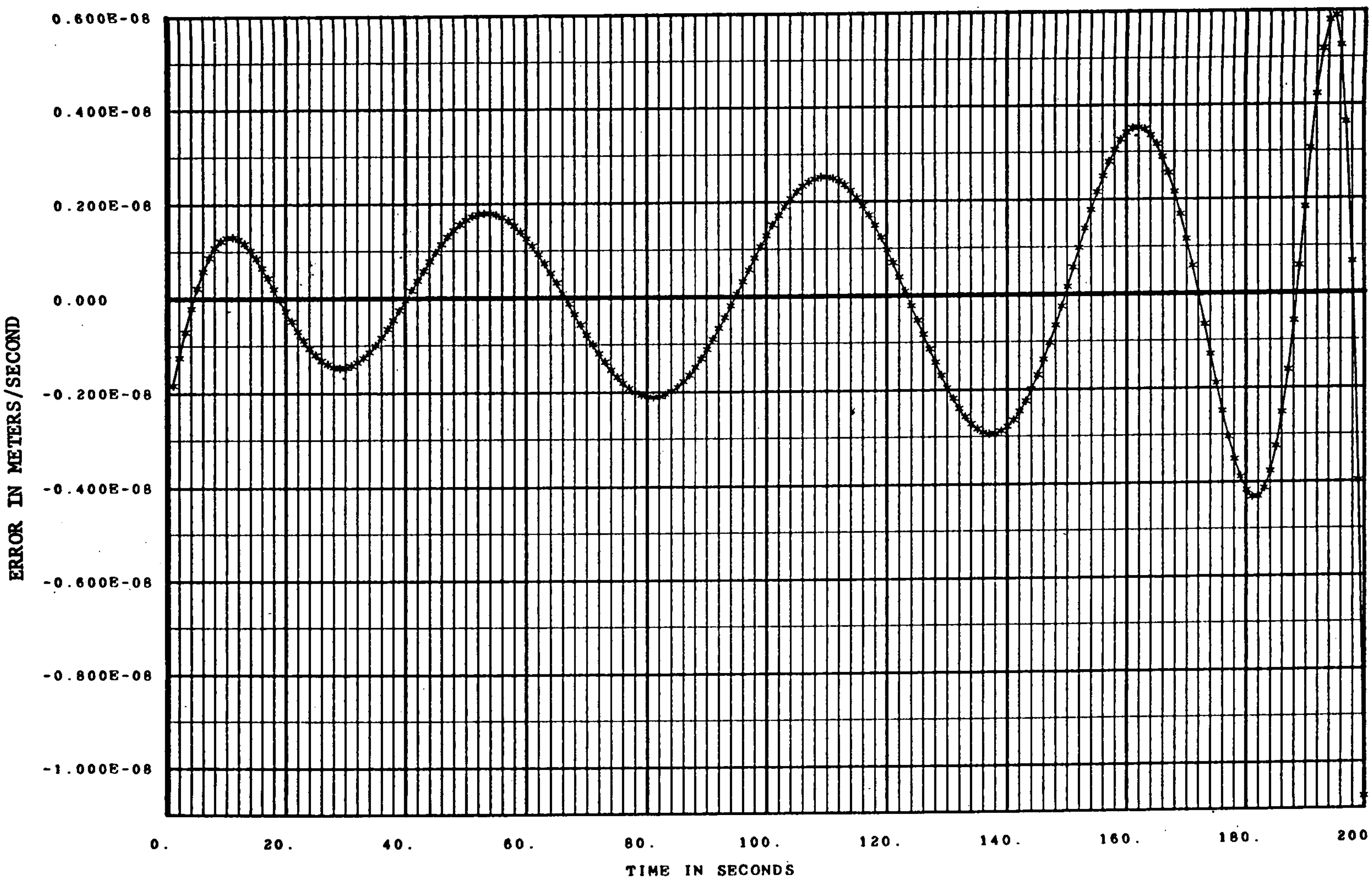

Figure A-110. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 200 points of simulated range rate data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

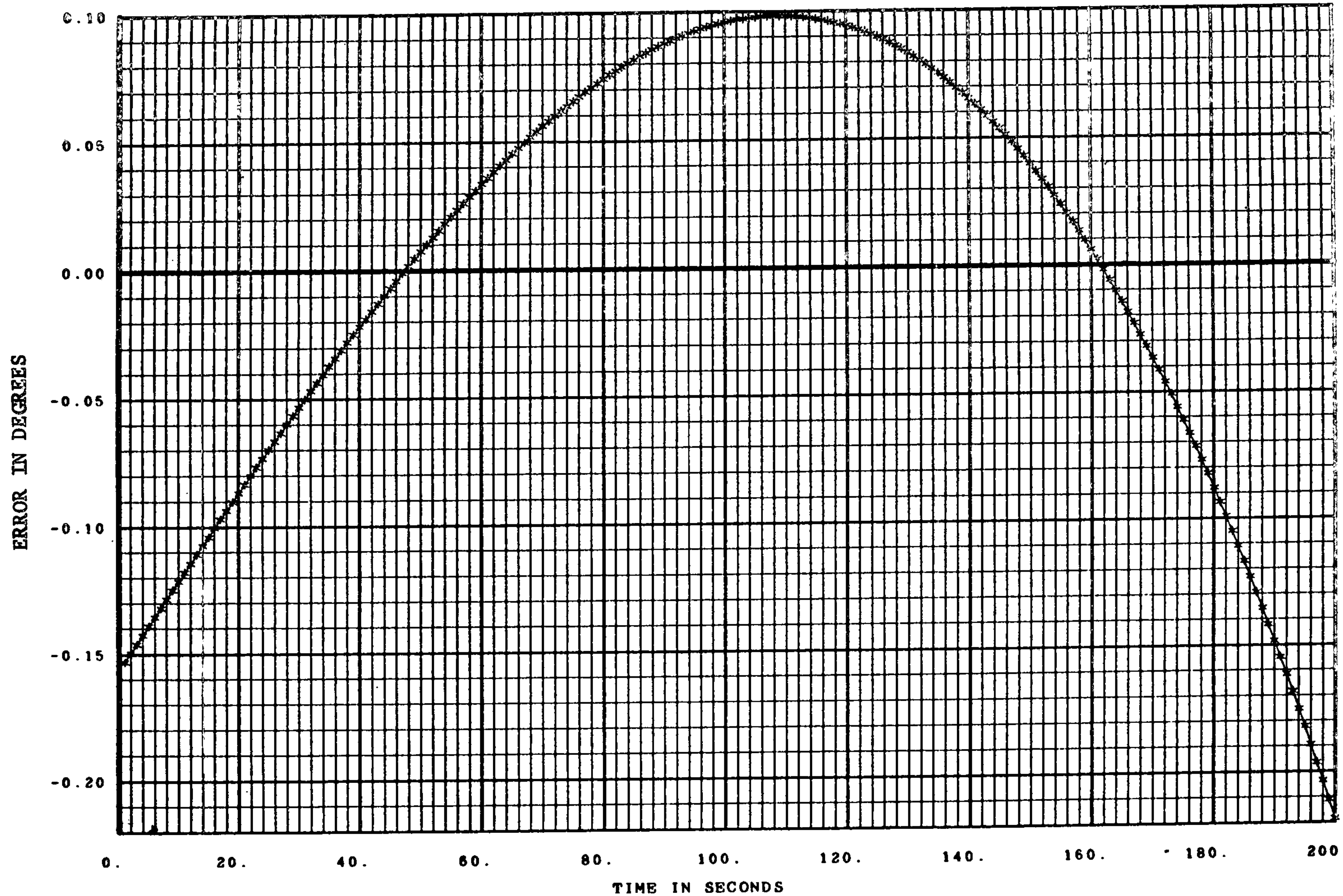

Figure A-111. Error (calculated polynomial minus true function) from using 1st degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

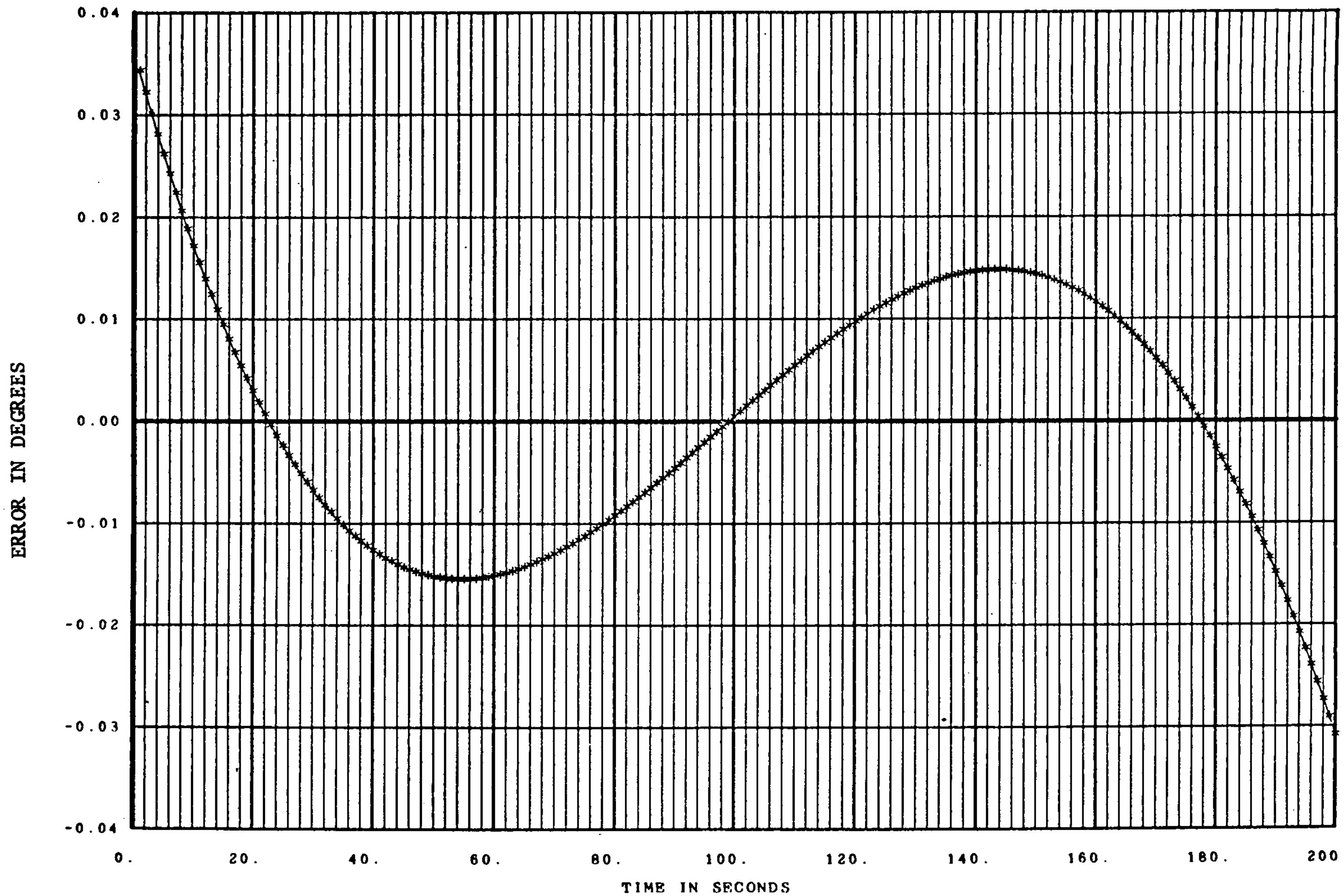

Figure A-112. Error (calculated polynomial minus true function) from using 2nd degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

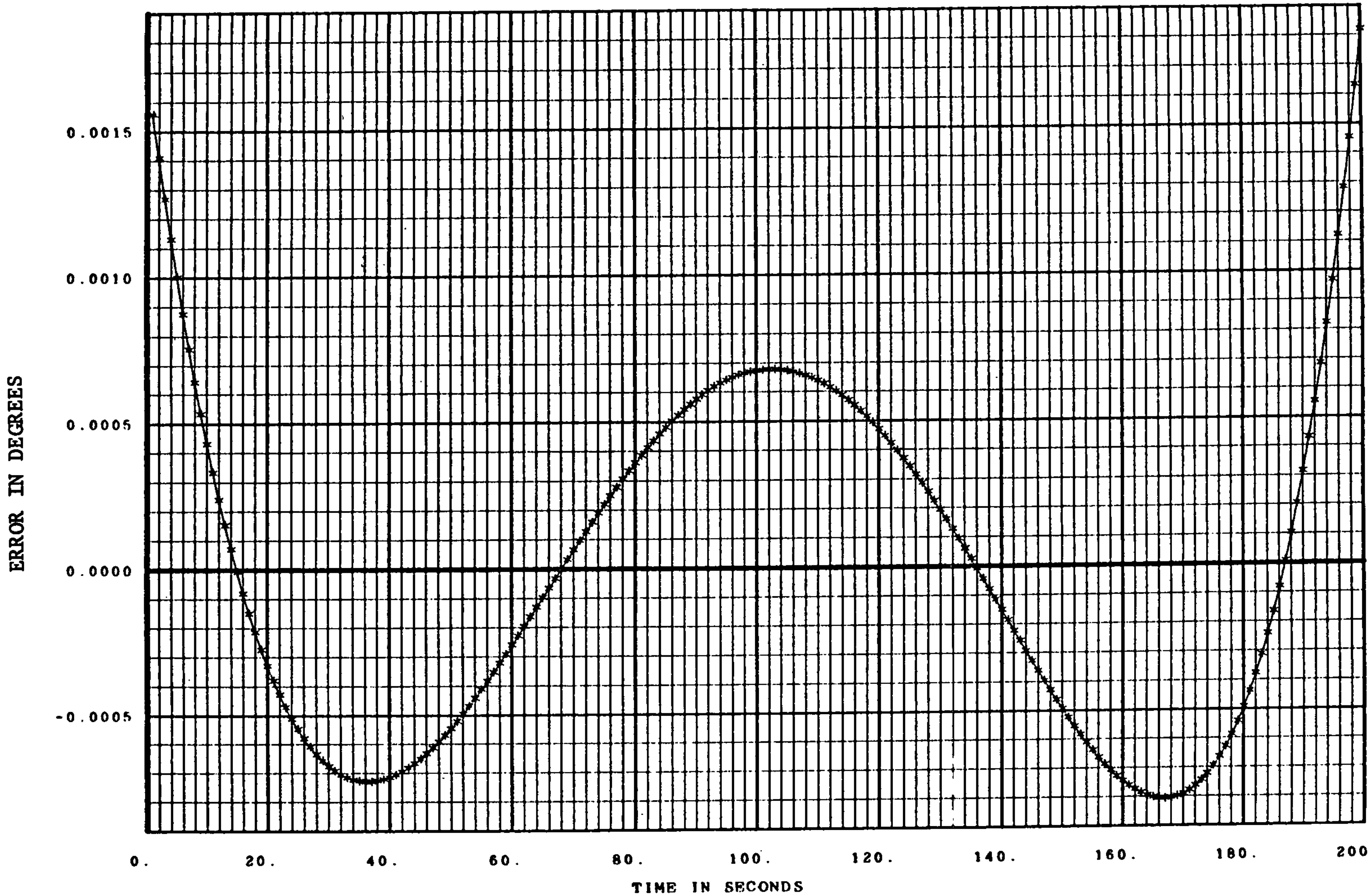

Figure A-113. Error (calculated polynomial minus true function) from using 3rd degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

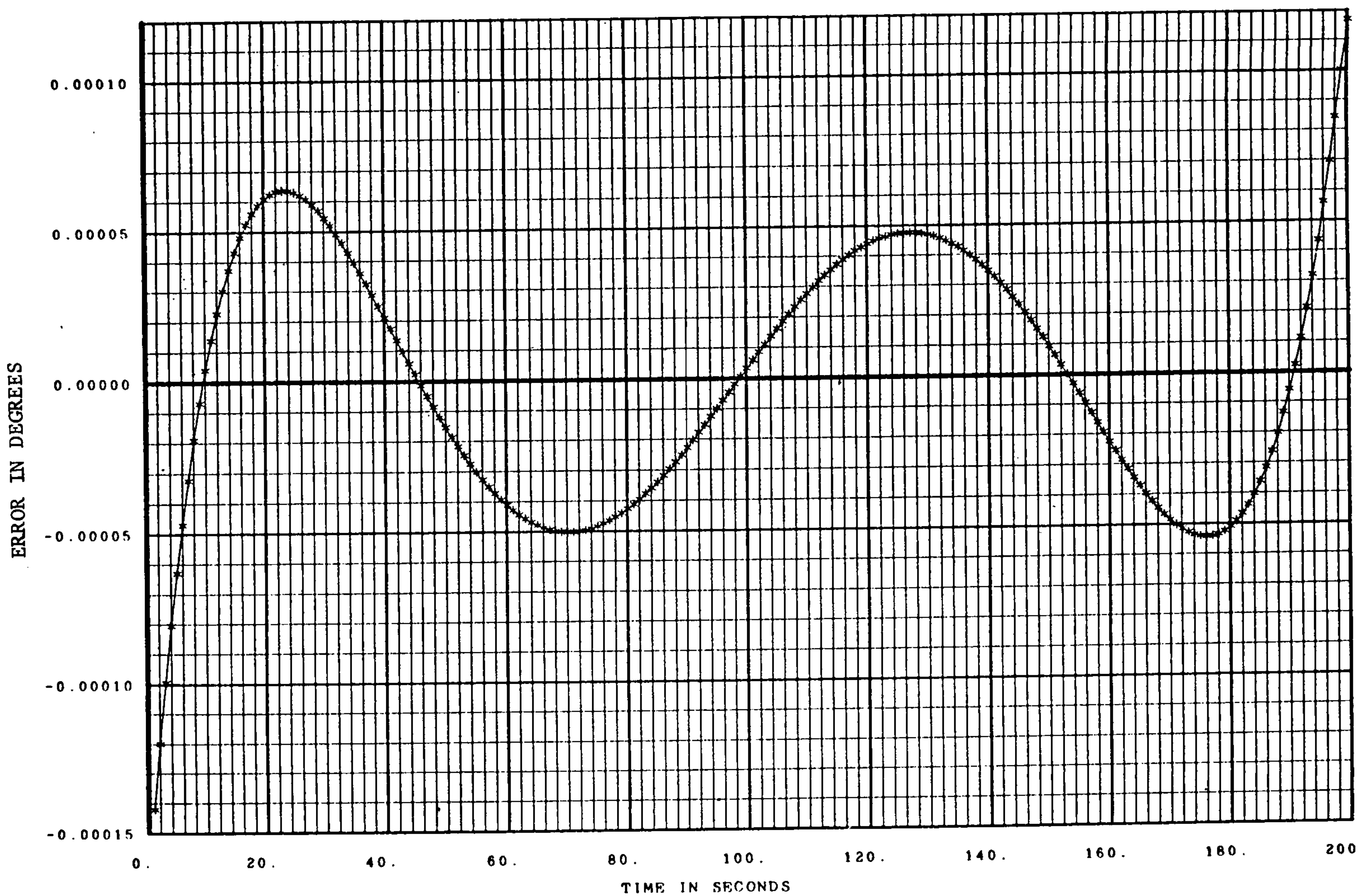

Figure A-114. Error (calculated polynomial minus true function) from using 4th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

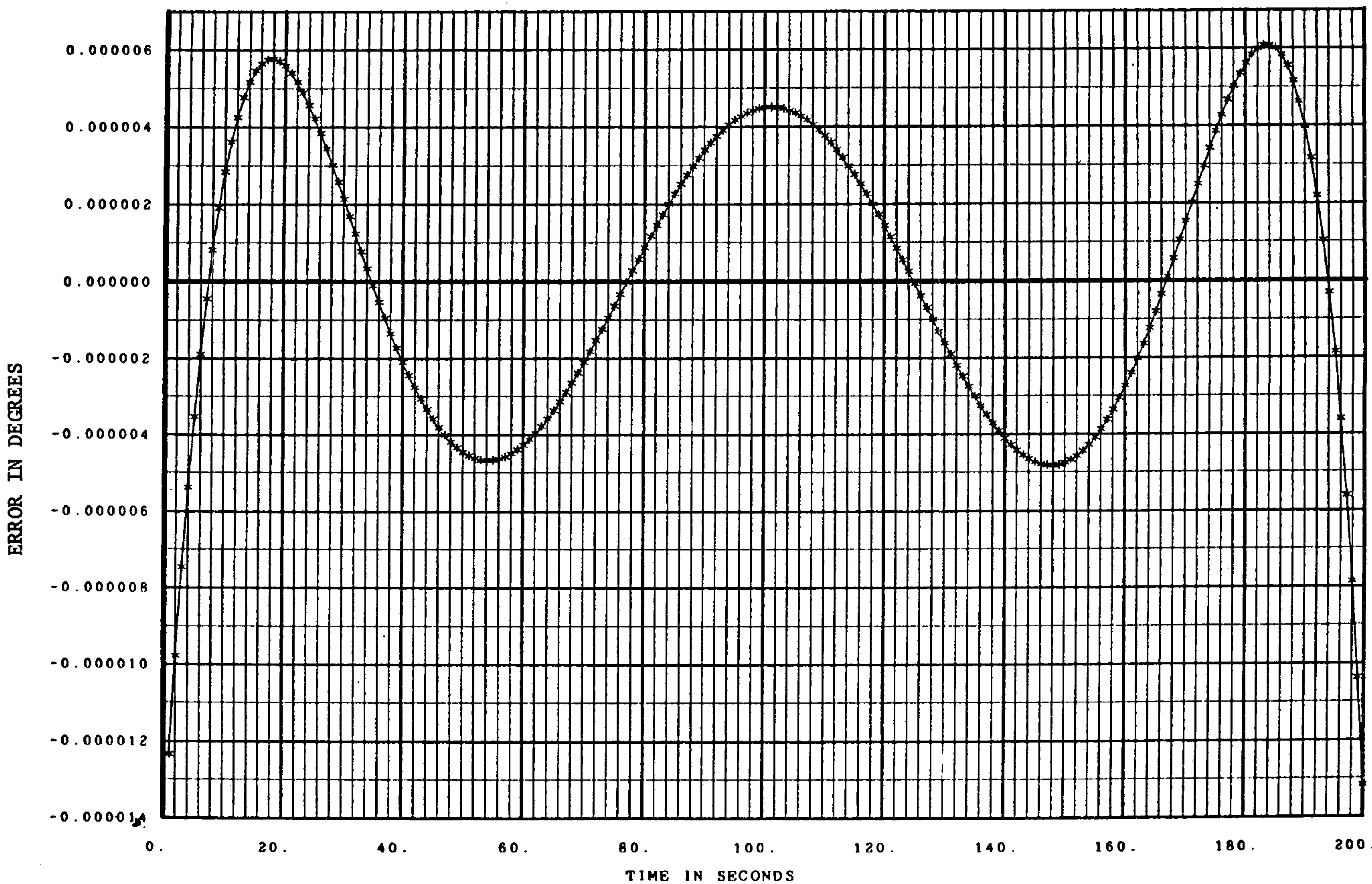

Figure A-115. Error (calculated polynomial minus true function) from using 5th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

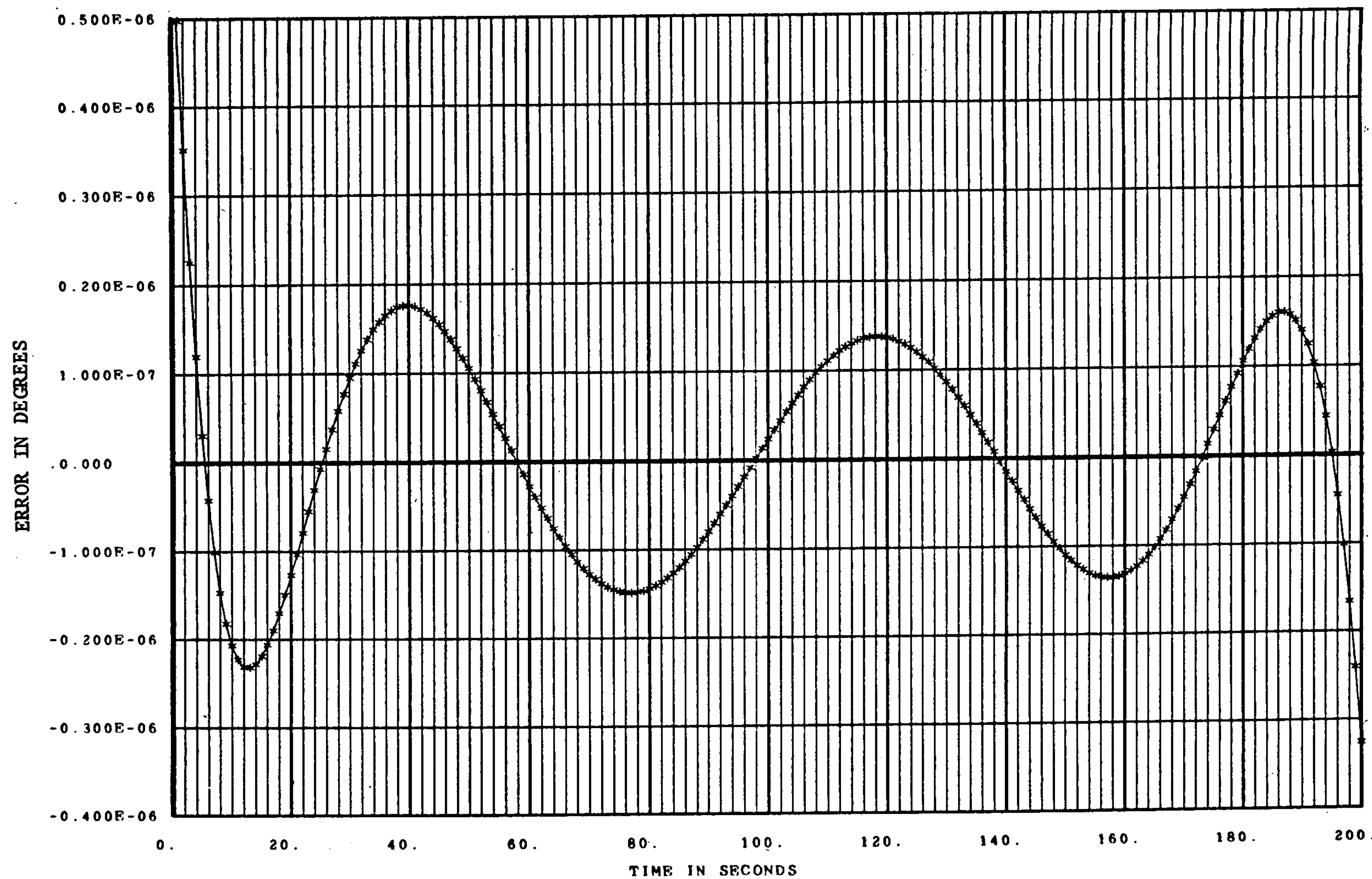

Figure A-116. Error (calculated polynomial minus true function) from using 6th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

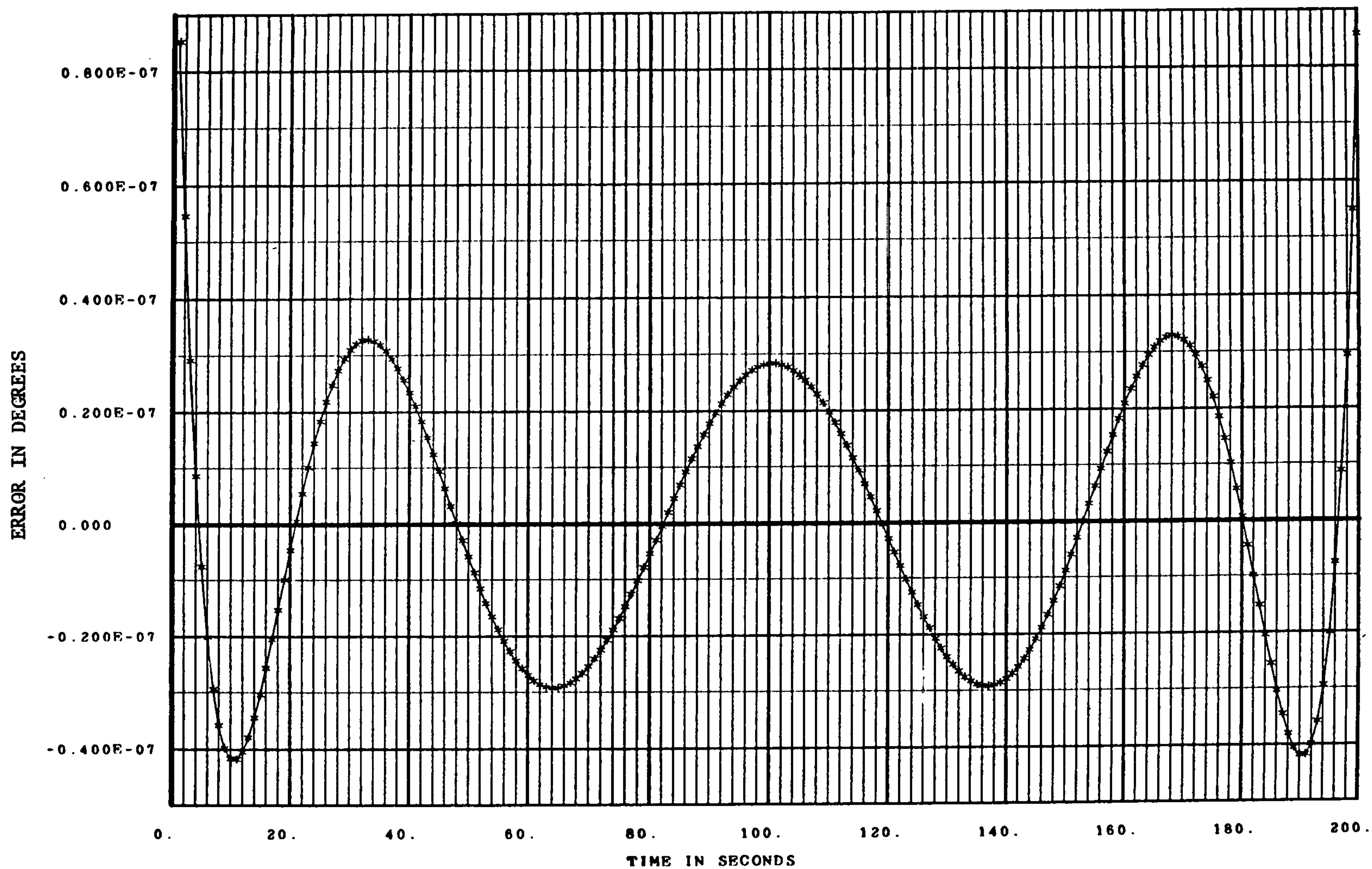

Figure A-117. Error (calculated polynomial minus true function) from using 7th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

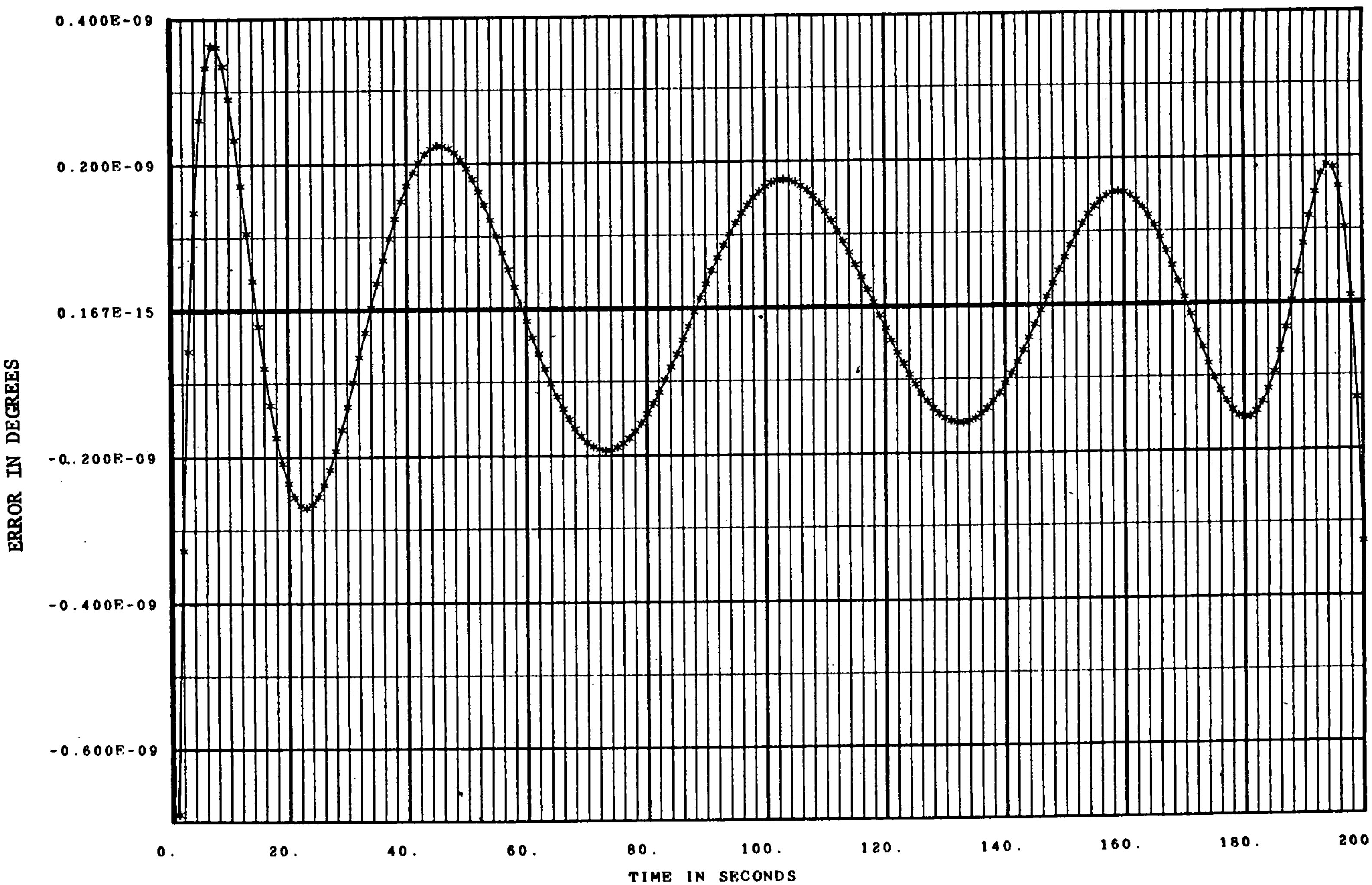

Figure A-118. Error (calculated polynomial minus true function) from using 8th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

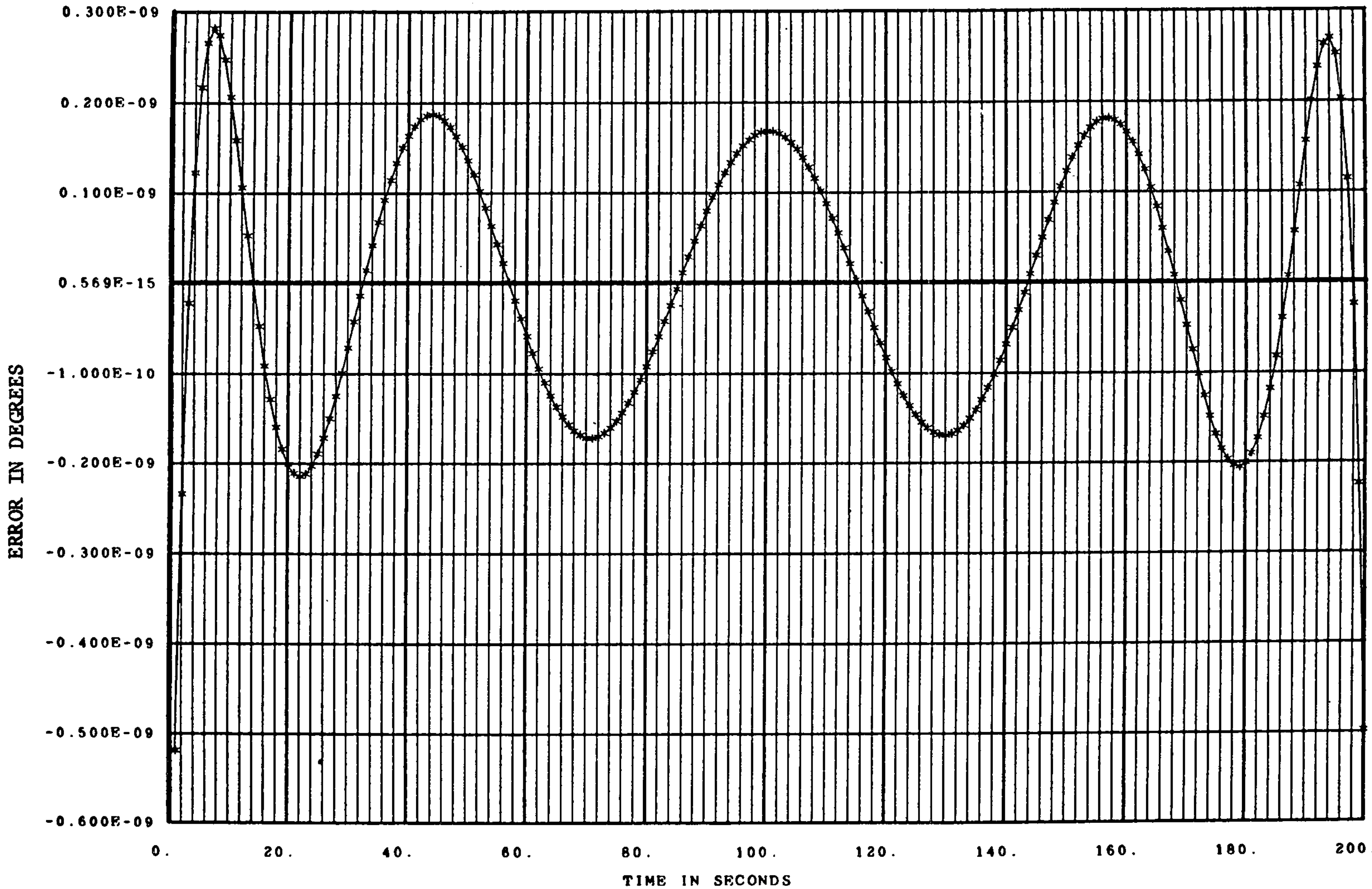

Figure A-119. Error (calculated polynomial minus true function) from using 9th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

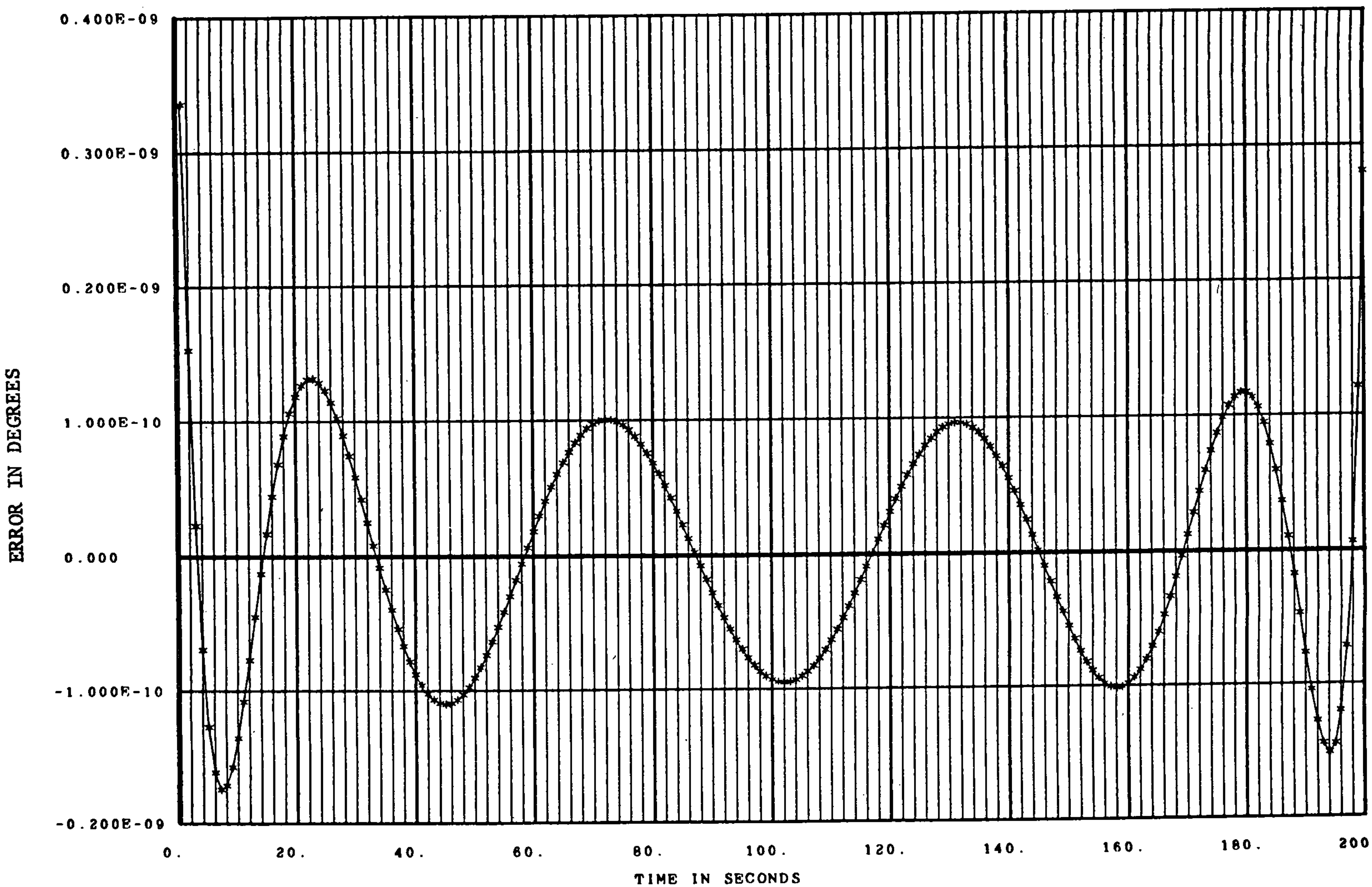

Figure A-120. Error (calculated polynomial minus true function) from using 10th degree, least squares fitted polynomial as approximation to 200 points of simulated elevation data at one per second data rate, a = 95,000 km, e = 0.89, zero time at spacecraft crossing station zenith (overhead pass).

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