NASA TECHNICAL MEMORANDUM

NASA TM X-64714

THE BETA DISTRIBUTION: A STATISTICAL MODEL FOR WORLD CLOUD COVER

By Lee W. Falls
Aero-Astrodynamics Laboratory

January 10, 1973

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

MSFC - Form 3190 (Rev June 1971)

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA TM X-64714		

4. TITLE AND SUBTITLE	5. REPORT DATE
The Beta Distribution: A Statistical Model for World Cloud Cover	January 10, 1973
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Lee W. Falls	8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812	10. WORK UNIT NO.
	11. CONTRACT OR GRANT NO.
	13. TYPE OF REPORT & PERIOD COVERED Technical Memorandum
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D.C. 20546	
	14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

Prepared by Aero-Astrodynamics Laboratory, Science and Engineering

16. ABSTRACT

Much work has been performed in developing empirical global cloud-cover models. The investigation in this report was made to determine an underlying theoretical statistical distribution to represent worldwide cloud cover. The beta distribution with probability density function,

$$f(x) = \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\,\Gamma(\eta)} x^{\gamma-1} (1-x)^{\eta-1} \; ; \; 0 \leq x \leq 1 \; , \; \gamma > 0 \; , \; \eta > 0 \quad ,$$

is given to represent the variability of this random variable. It is shown that the beta distribution possesses the versatile statistical characteristics necessary to assume the wide variety of shapes exhibited by cloud cover. A total of 160 representative empirical cloud-cover distributions were investigated and the conclusion was reached that this study provides sufficient statistical evidence to accept the beta probability distribution as the underlying model for world cloud cover.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
Statistical Distributions Cloud Cover Random Variables Goodness of Fit Tests	Unclassified - unlimited E. D. Geissler Director, Aero-Astrodynamics Laboratory

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	52	NTIS

MSFC - Form 3292 (May 1969)

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF TABLES (Continued)

LIST OF TABLES (Continued)

LIST OF TABLES (Continued)

LIST OF TABLES (Continued)

LIST OF TABLES (Concluded)

TECHNICAL MEMORANDUM X-64714

THE BETA DISTRIBUTION: A STATISTICAL MODEL FOR WORLD CLOUD COVER

INTRODUCTION

The purpose of this report is to present an adequate underlying theoretical statistical distribution to represent worldwide cloud cover. Cloud cover is obviously very important in all earth–space experiments and much work has been performed in developing empirical global cloud cover models [1, 2, 3, 4]. In these studies the earth's surface has been divided into 29 regions, as shown by Figure 1. Meteorologists have chosen these regions to have homogeneous cloud-cover distributions. Conventional and satellite cloud-cover data were collected to establish that the regions were representative homogeneous cloud climatological regions. Five cloud-cover categories have been designated, as shown in Table 1. The corresponding ratios of cloud cover for each category are given in tenths of cloud cover.

TABLE 1. CLOUD CATEGORY DESIGNATION

Category	Tenths
1	0
2	1, 2, 3
3	4, 5
4	6, 7, 8, 9
5	10

It is obvious that "amount of cloud cover" may be considered a continuous variable in the interval (0, 100 percent). In practice, when a continuous variable is sampled, the variable is "discretized," that is, it is not possible to collect an infinite sample of a continuous variable. Therefore, some appropriate class interval is chosen to classify the data. The length of this class interval should be chosen in such a manner to preserve the attributes of the population being sampled as far as possible and to be representative of the population but to reduce the amount of data so that they can be analyzed efficiently. The five cloud-cover categories given in Table 1 should be considered

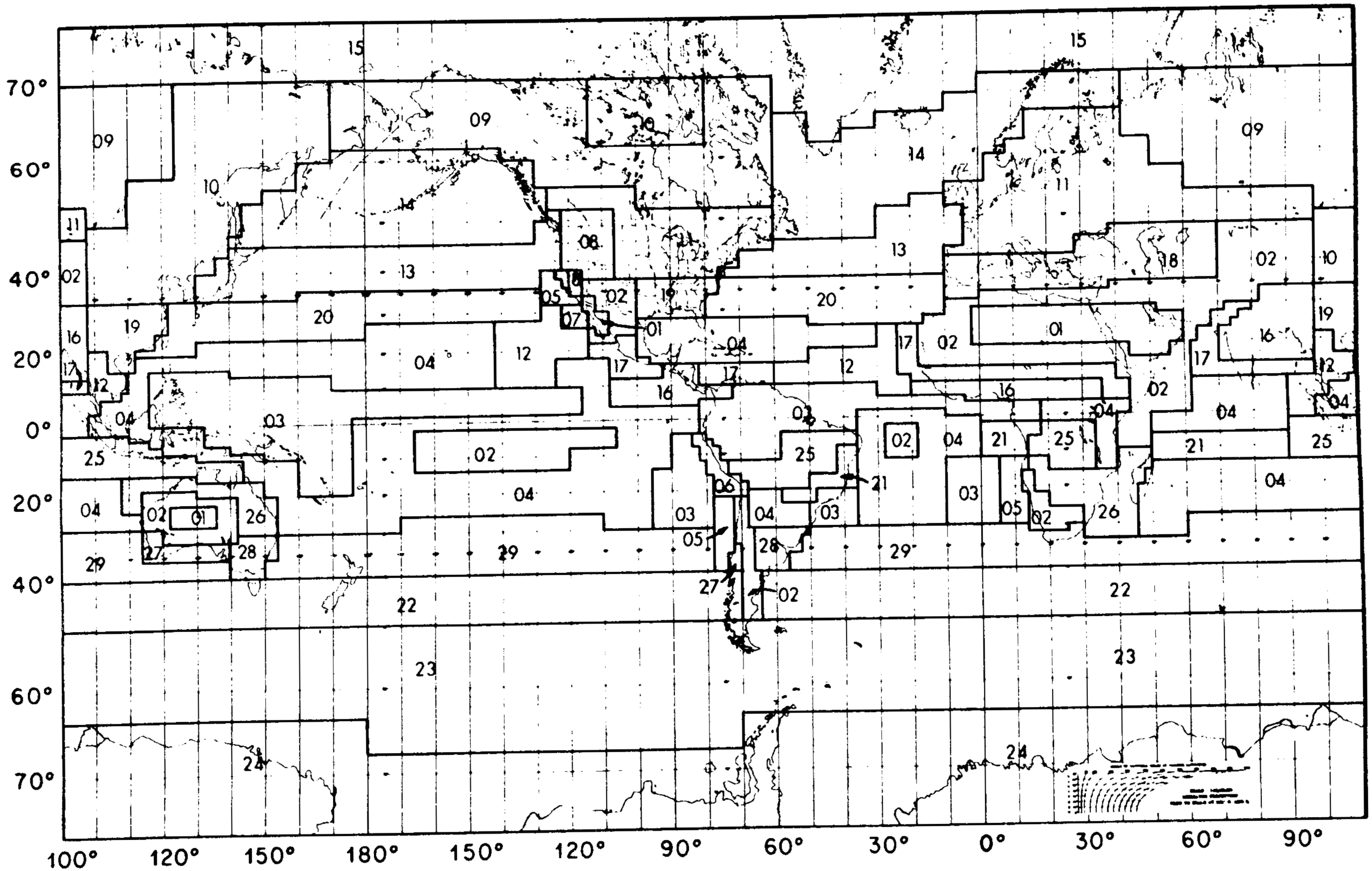

Figure 1. Cloud region location map for worldwide cloud cover data.

from this point of view. Although the categories are discrete (x = 1, 2, 3, 4, 5), they may be viewed as the midpoints of class intervals for the continuous variable described as "amount or tenths of cloud cover."

For this theoretical study 10 regions were selected which are of interest to NASA's Skylab program. These regions and their general geographical area are given in Table 2.

TABLE 2. TEN REPRESENTATIVE CLOUD-COVER REGIONS

Regions	Geographical Location
02, 04, 08, 11, 18, 19	United States
03	Brazil
16	Central America
25	South America, Australia
28	Australia

Because diurnal and monthly cloud-cover variations are important in most of the 29 regions, the data were subdivided into distributions for each month and for 3-hr intervals through the day. The beta distribution is given to represent the variation in cloud cover for the representative regions listed in Table 2.

STATISTICAL MODEL

An inspection of the empirical cloud-cover frequency curves [1, 2] shows the distributions whose shapes vary from U-shaped through transitional J shapes to bell-shaped distributions. The assumption was made [1, 2] that the weather observer can accurately record cloud-cover observations up to 27.80 km (15 n.mi.) i.e., the cloud cover data are for a circle whose diameter is 55.60 km (30 mi) with the station at the center. Consequently, the statistical inferences presented in this report apply only to a circle whose diameter is 55.60 km (30 mi) for the respective regions. Extensions to larger areas will be discussed later in this report.

A continuous distribution whose density function may assume all the shapes just described for cloud cover is the beta distribution. The beta distribution allows one to represent a wide variety of distributional shapes for physical variables such as cloud cover whose values are limited to a finite interval. The generalized beta probability density function that covers the interval (μ_0, μ_1) is

$$f(x') = \frac{1}{(\mu_1 - \mu_0)} \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \left(\frac{x' - \mu_0}{\mu_1 - \mu_0}\right)^{\gamma-1} \left(1 - \frac{x' - \mu_0}{\mu_1 - \mu_0}\right)^{\eta-1} ,$$

$$\mu_0 \leqslant x' \leqslant \mu_1 \quad , \quad \gamma>0 \quad , \quad \eta>0 \quad . \tag{1}$$

This form of the distribution is unwieldy and may be transformed to a beta variate over the interval (0, 1) by making the transformation,

$$x = \frac{x' - \mu_0}{\mu_1 - \mu_0} \quad .$$

The probability density function (1) now becomes

$$f(x) = \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\,\Gamma(\eta)} \; x^{\gamma-1} (1 - x)^{\eta-1} \; , \; 0 \leqslant x \leqslant 1 \; , \; \gamma>0 \; , \; \eta>0 \quad . \tag{2}$$

This is the form of the distribution used in this study. The shape of the density function depends upon the magnitude of the distribution's two parameters η and γ . Plots of beta distributions for various combinations of the two parameters are shown in Figure 2. Note that

1. When $\gamma > 1$ and $\eta > 1$, the distribution has one peak.

2. When $\gamma < 1$ and $\eta < 1$, the distribution is U-shaped.

3. When $\gamma < 1$ and $\eta \geqslant 1$, the distribution is reverse J-shaped and when $\eta < 1$ and $\gamma \geqslant 1$, it is J-shaped.

4. When $\gamma = \eta$, the distribution is symmetrical. The special case, $\gamma = \eta = 1$, is the rectangular distribution.

The parameters γ and η are shape parameters as both affect the distribution shape.

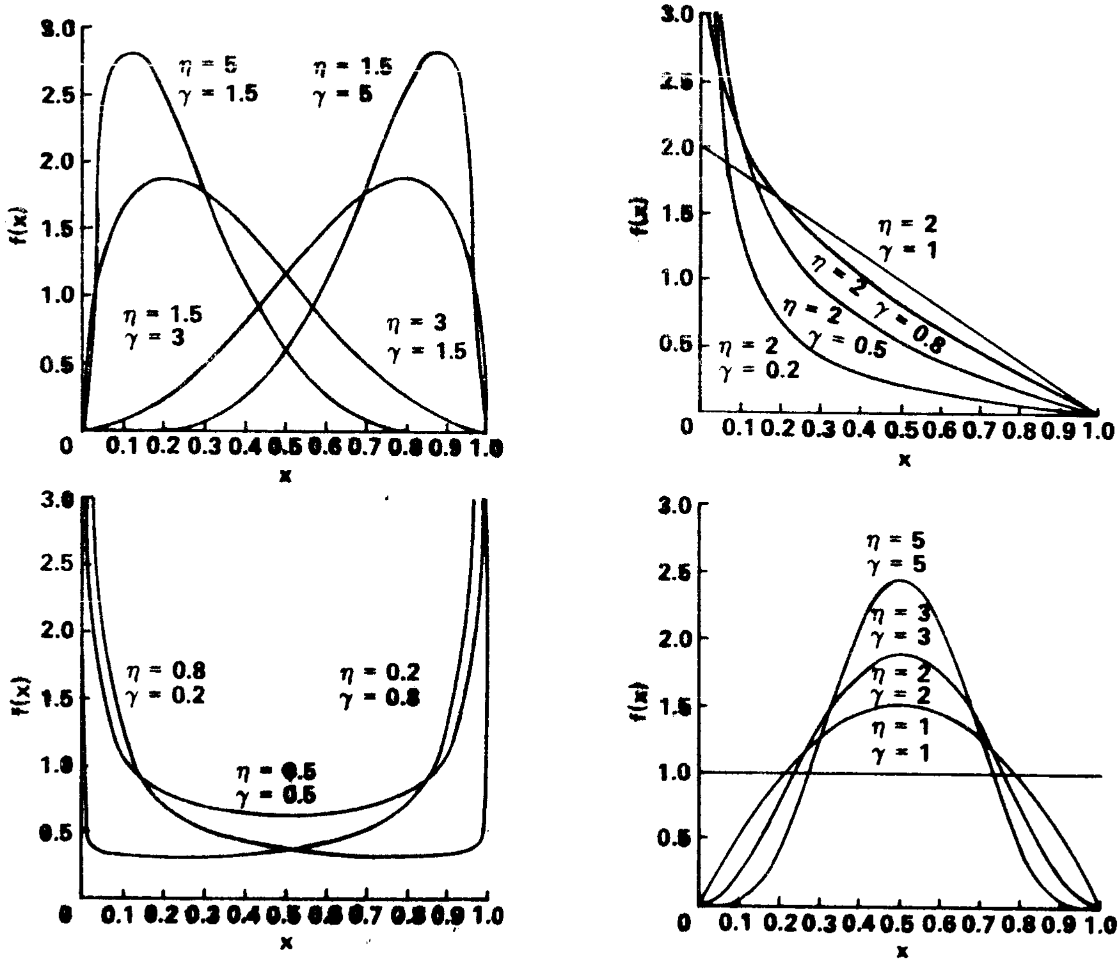

Figure 2. Beta distributions with different parameter values.

The cumulative beta distribution,

$$F(x) = \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\,\Gamma(\eta)} \int_0^x t^{\gamma-1}\,(1-t)^{\eta-1}\,dt\,,\quad 0 \leqslant x \leqslant 1\,, \tag{3}$$

is known as the incomplete beta function and is tabulated in Reference 5. F(x) gives the probability of obtaining a value of x less than or equal to some particular value of x .

Maximum-likelihood estimates of the parameters of the beta distribution are difficult to obtain. Using the method of moments for estimation, we have

$$\eta^* = \frac{(1-\bar{x})}{s^2}\,[\bar{x}(1-\bar{x}) - s^2] \tag{4}$$

and

$$\gamma^* = \frac{\bar{x}\eta^*}{1 - \bar{x}} \quad , \tag{5}$$

where $\bar{x}$ and s^2 are the sample mean and the sample variance, respectively.

ANALYSIS

For this study the 10 cloud regions given in Table 2 were analyzed for the months of January, April, July, and October. These four months may be considered to be representative of the annual changes in the cloud distributions in the earth's atmosphere. For each of these months the satellite cloud data, along with the cloud-cover data for hours 4, 10, and 16, were considered. These hours are likely selections for hours of the day that are significantly different over the 24-hr period, i.e., diurnal effects will be reflected in this choice of hours. Summaries for the regions that encompass the United States (02, 04, 08, 11, 18, and 19) are presented in Appendix A as Tables A-1 through A-96. Cloud regions 03, 16, 25, and 28 for other areas of the earth's surface were also analyzed but are not included here because of space limitations.

The notations used in Appendix A are as follows:

x = the cloud-cover category.

f_0 = the observed frequency for cloud-category x .

F_0 = the observed distribution function.

f_E = the expected frequencies using the beta distribution.

$F(x)$ = the beta-distribution function.

$\bar{x}$ = the sample mean.

s^2 = the sample variance.

η^*, γ^* = the parameter estimators of the beta distribution.

D_α = the tabulated Kolmogorov-Smirnov statistic for rejection level α = 0.05.

$|F_0 - F(x)|$ = the maximum absolute difference between the observed distribution function and the beta-distribution function.

Sample sizes for the 29 cloud regions (Fig. 1) were not recorded in the empirical studies [1, 2, 3]. The data were presented as relative frequencies for the five cloud categories. For this reason, in this study the sample size for all distributions was considered to be equal to 100 (the sum of the relative frequencies × 100). Statisticians consider a sample size of 100 to be sufficiently "large" to be representative of the population being sampled. Hence, valid statistical inferences and probability statements may be made. Lancaster [6] states that nonsignificant goodness-of-fit statistics at "moderate sample size" must be interpreted to mean that at this sample size, it is possible without loss of information to regard the data as having been generated by sampling from the hypothetical distribution.

The Kolmogorov-Smirnov (KS) test was used to determine the goodness of fit in this study for a number of reasons. This test is superior to the familiar chi-square (χ^2) test in several ways. When samples are of moderate size and adjacent categories must be combined before χ^2 may properly be computed, the χ^2 test is definitely less powerful than the KS test. In our cloud distributions we have a small number of class intervals, five in all cases. For the χ^2 test, when adjacent categories must be combined, we are sometimes faced with the impossible problem of zero or negative degrees of freedom for the test, which means simply that we have no test at all. Although not applicable here, for very small samples the χ^2 test is not appropriate at all but the Kolmogorov-Smirnov test is. Siegel [7] states that "the Kolmogorov-Smirnov test may in all cases be more powerful than its alternative, the χ^2 test."

Referring to Appendix A, let us consider Table A-2 as an example. This is the distribution of cloud cover for region 02 (southwestern United States) for the month of January, hour 4. There were 16 observations for cloud category 2 (1/10, 2/10 and 3/10 cloud cover). This means that 16 times out of 100 the observer recorded exactly 1/10 to 3/10 cloud cover. The observed distribution function (F_0) gives a probability of 0.540 of having cloud category 2 or less in region 02 at hour 4 during the month of January, or a probability of (1 – 0.540 = 0.460) of having more than cloud category 2 (more than 3/10 cloud cover). The beta distribution predicts 19.6 occurrences for category 2 and the probability [F(x)] is 0.488 of having cloud category 2 or less for this example, or a probability of (1 – 0.488 = 0.512) of having more than cloud category 2 (more than 3/10 cloud cover). Comparing F_0 with F(x) shows a maximum absolute difference in the distribution functions of 0.088 occurring at x = 1 . Since the Kolmogorov-Smirnov statistic D_α = 0.136, the value of 0.088 is not sufficiently large to reject the hypothesis at the 5-percent rejection level that this example may be considered to be a sample from a beta distribution population.

Estimates η^* and γ^* of the beta distribution parameters η and γ are given in Tables A-1 through A-96 for cloud-cover regions that encompass the United States. A tabulation of these parameters, along with those for the remaining cloud-cover regions given in Table 2, is provided in Appendix B.

CONCLUSIONS

It is a well-established fact in the science of mathematical statistics that there are many advantages in using a theoretical statistical model for predicting the behavior of random variables such as world cloud cover. Care must be taken in deciding which is a "best" model for the random variable being considered. If the physical constraints for the random variable are satisfied by a hypothetical distribution and after repeated sampling of the population of this random variable it is found that the hypothetical distribution cannot be rejected by an appropriate statistical test, then there is strong evidence that this hypothetical distribution is the underlying model for the random variable under investigation.

The random variable we have described as world cloud cover requires a versatile statistical distribution that is bounded at both ends and is capable of assuming a wide variety of shapes. We have shown that the beta distribution possesses these characteristics. Representative samples from the very large mass of empirical data collected on world cloud cover were selected and several statistical models, including the beta and normal distributions, were tried as prospective models [8]. A total of 160 representative empirical cloud-cover distributions were investigated and the results indicate that in all cases the beta distribution gave the "best" fit. Out of the 160 empirical distributions, in only seven cases was the hypothetical beta model rejected at the 5-percent rejection level (95-percent confidence level) by the Kolmogorov-Smirnov goodness-of-fit test. This author believes that this study provides sufficient statistical evidence to accept the beta distribution as the underlying model for world cloud cover.

RECOMMENDATIONS

As noted in the previous section entitled Statistical Model, the observed cloud-cover data in this study and in the empirical studies [1, 2, 3, 4] are for a circle whose diameter is 55.60 km (30 mi). It is frequently desired to predict cloud cover within much larger areas. Greaves, Spiegler, and Willand [2] suggest a method for doing this using the normal probability distribution. They state two reasons why this distribution was chosen. First, "It gives acceptable results," and second, "It is well known and widely tabulated." The normal distribution probably gives "acceptable results" because it is bell shaped. For a point or small area, the distribution of cloud cover is primarily U-shaped because of the average size of the "clouds" in relation to the observer. This is a bimodal distribution with the modes near zero and 100-percent cloud cover. As the area becomes larger, the distribution becomes unimodal and bell shaped because of the same reasoning, i.e., the average size of "clouds" relative to an observer at a point. The mode is now somewhere near 50-percent cloud cover with the tails of the distribution approaching 0 percent and 100 percent. In short, the physics of the cloud cover variable determine the transition of a U-shaped distribution within a small area or point to a bell-shaped distribution within a large area. It is recommended that the normal distribution not be used for "extrapolations" to larger areas for two reasons. This investigation has shown that the underlying distribution of cloud cover is definitely not normal and, furthermore, the normal distribution is unbounded at both ends, which violates the constraints of our random variable.

We have shown in this study that the underlying distribution of cloud cover is the beta distribution. As mentioned previously, this distribution may assume a wide variety of shapes, including the bell shape. For the reasons mentioned, this author suggests that the beta distribution, rather than the normal, be used in any scheme that provides for calculating the probabilities for areas larger than the observed 55.60-km (30-mi) area.

Orvel E. Smith, chief of the Terrestrial Environment Branch, Aerospace Environment Division of the Aero-Astrodynamics Laboratory at the Marshall Space Flight Center, has suggested that the beta distribution might be utilized either to confirm or to modify the existing global cloud model that is based upon empirical statistics. The beta distribution parameters would be determined for all worldwide stations. As shown in the section, Statistical Model, and in Figure 2 of this report, the magnitude of these two parameters determines the shape of the beta density function. Homogeneous cloud regions would subsequently be determined by classifying the beta parameters into nonsignificant (homogeneous) groups. This would be an interesting topic for future investigation.

Another suggestion relating to the computation of time and spatial conditional probabilities for cloud cover involves the use of the multivariate beta distribution as an appropriate model. This would be an alternative theoretical approach to the empirical methods presently used [1, 2, 3, 4] for predicting time and spatial cloud-cover relationships.

APPENDIX A

BETA DISTRIBUTIONS FOR CLOUD-COVER REGIONS 02, 04, 08, 11, 18, AND 19 GROUPED BY MONTH AND HOUR INCLUDING SATELLITE DATA [1]

1. In this appendix, $|F_0 - F(x)|$ marked with an asterisk (*) indicates rejection at the 5-percent rejection level. In all distributions (Tables A-1 through A-96), $D_\alpha = 0.136$.

TABLE A-1. REGION 02 – JANUARY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	32	0.320	26.3	0.263
2	21	0.530	22.1	0.484
3	11	0.640	21.6	0.700
4	26	0.900	18.6	0.886
5	10	1.000	11.4	1.000

$\bar{x} = 2.610$, $s^2 = 1.998$, $\eta^* = 1.458$, and $\gamma^* = 1.317$.

$|F_0 - F(x)| = 0.059$.

TABLE A-2. REGION 02 – JANUARY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	38	0.380	29.2	0.292
2	16	0.540	19.6	0.488
3	8	0.620	18.9	0.678
4	23	0.850	17.7	0.855
5	15	1.000	14.5	1.000

$\bar{x} = 2.610$, $s^2 = 2.358$, $\eta^* = 1.155$, and $\gamma^* = 1.044$.

$|F_0 - F(x)| = 0.088$.

TABLE A-3. REGION 02 – JANUARY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	18	0.180	15.7	0.157
2	21	0.390	19.0	0.347
3	10	0.490	22.5	0.572
4	35	0.840	23.5	0.807
5	16	1.000	19.3	1.000

$\bar{x} = 3.100$, $s^2 = 1.910$, $\eta^* = 1.263$, and $\gamma^* = 1.632$.

$|F_0 - F(x)| = 0.082$.

TABLE A-4. REGION 02 – JANUARY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	17	0.170	14.6	0.146
2	19	0.360	18.5	0.331
3	13	0.490	22.4	0.555
4	33	0.820	24.0	0.795
5	18	1.000	20.5	1.000

$\bar{x} = 3.160$, $s^2 = 1.894$, $\eta^* = 1.235$, and $\gamma^* = 1.668$.

$|F_0 - F(x)| = 0.065$.

TABLE A-5. REGION 02 – APRIL – SATELLITE

x	f_0	F_0	f_E	F(x)
1	36	0.360	29.2	0.292
2	21	0.570	22.6	0.518
3	10	0.670	21.0	0.728
4	24	0.910	17.3	0.901
5	9	1.000	9.9	1.000

$\bar{x} = 2.490$, $s^2 = 1.990$, $\eta^* = 1.514$, and $\gamma^* = 1.252$.

$|F_0 - F(x)| = 0.068$.

TABLE A-6. REGION 02 – APRIL – HOUR 4

x	f_0	F_0	f_E	F(x)
1	33	0.330	27.7	0.277
2	22	0.550	21.9	0.496
3	12	0.670	21.0	0.706
4	21	0.880	18.1	0.887
5	12	1.000	11.3	1.000

$\bar{x} = 2.570$, $s^2 = 2.045$, $\eta^* = 1.429$, and $\gamma^* = 1.253$.

$|F_0 - F(x)| = 0.054$.

TABLE A-7. REGION 02 – APRIL – HOUR 10

x	f_0	F_0	f_E	F(x)
1	23	0.230	20.6	0.206
2	23	0.460	19.5	0.401
3	11	0.570	21.2	0.613
4	25	0.820	21.1	0.824
5	18	1.000	17.6	1.000

$\bar{x} = 2.292$, $s^2 = 2.114$, $\eta^* = 1.203$, and $\gamma^* = 1.361$.

$|F_0 - F(x)| = 0.058$.

TABLE A-8. REGION 02 – APRIL – HOUR 16

x	f_0	F_0	f_E	F(x)
1	26	0.260	22.4	0.224
2	22	0.480	19.0	0.413
3	8	0.560	20.1	0.614
4	25	0.810	20.2	0.816
5	19	1.000	18.4	1.000

$\bar{x} = 2.890$, $s^2 = 2.258$, $\eta^* = 1.111$, and $\gamma^* = 1.230$.

$|F_0 - F(x)| = 0.067$.

TABLE A-9. REGION 02 – JULY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	60	0.600	47.4	0.474
2	20	0.800	28.8	0.762
3	8	0.880	16.7	0.929
4	11	0.990	6.3	0.992
5	1	1.000	0.8	1.000

$\bar{x} = 1.730$, $s^2 = 1.137$, $\eta^* = 3.246$, and $\gamma^* = 1.490$.

$|F_o - F(x)| = 0.126$.

TABLE A-10. REGION 02 – JULY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	65	0.650	50.4	0.504
2	17	0.820	27.3	0.777
3	8	0.900	15.5	0.932
4	7	0.970	6.0	0.992
5	3	1.000	0.8	1.000

$\bar{x} = 1.660$, $s^2 = 1.164$, $\eta^* = 3.124$, and $\gamma^* = 1.350$.

$|F_o - F(x)| = 0.146$.*

TABLE A-11. REGION 02 – JULY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	64	0.640	54.2	0.542
2	25	0.890	33.6	0.878
3	7	0.960	10.8	0.986
4	4	1.000	1.4	0.999
5	0	1.000	0.02	1.000

$\bar{x} = 1.510$, $s^2 = 0.630$, $\eta^* = 6.213$, and $\gamma^* = 2.351$.

$|F_o - F(x)| = 0.098$.

TABLE A-12. REGION 02 – JULY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	76	0.760	60.7	0.607
2	15	0.910	29.3	0.900
3	5	0.960	8.8	0.988
4	3	0.990	1.1	0.999
5	1	1.000	0.02	1.000

$\bar{x} = 1.380$, $s^2 = 0.636$, $\eta^* = 5.952$, and $\gamma^* = 1.994$.

$|F_o - F(x)| = 0.153$.*

TABLE A-13. REGION 02 – OCTOBER – SATELLITE

x	f_0	F_0	f_E	F(x)
1	36	0.360	29.8	0.298
2	23	0.590	23.9	0.538
3	11	0.700	21.7	0.755
4	23	0.930	16.6	0.921
5	7	1.000	7.9	1.000

$\bar{x} = 2.420$, $s^2 = 1.844$, $\eta^* = 1.704$, and $\gamma^* = 1.339$.

$|F_0 - F(x)| = 0.062$.

TABLE A-14. REGION 02 – OCTOBER – HOUR 4

x	f_0	F_0	f_E	F(x)
1	41	0.410	33.7	0.337
2	23	0.640	24.4	0.581
3	11	0.750	20.8	0.789
4	18	0.930	14.8	0.937
5	7	1.000	6.3	1.000

$\bar{x} = 2.270$, $s^2 = 1.797$, $\eta^* = 1.809$, and $\gamma^* = 1.271$.

$|F_0 - F(x)| = 0.073$.

TABLE A-15. REGION 02 – OCTOBER – HOUR 10

x	f_0	F_0	f_E	F(x)
1	21	0.210	19.3	0.193
2	26	0.470	23.5	0.428
3	15	0.620	25.1	0.679
4	30	0.920	21.4	0.893
5	8	1.000	10.7	1.000

$\bar{x} = 2.780$, $s^2 = 1.672$, $\eta^* = 1.743$, and $\gamma^* = 1.781$.

$|F_0 - F(x)| = 0.059$.

TABLE A-16. REGION 02 – OCTOBER – HOUR 16

x	f_0	F_0	f_E	F(x)
1	21	0.210	19.3	0.193
2	25	0.460	22.1	0.414
3	14	0.600	24.0	0.653
4	29	0.890	21.8	0.871
5	11	1.000	12.9	1.000

$\bar{x} = 2.840$, $s^2 = 1.794$, $\eta^* = 1.552$, and $\gamma^* = 1.658$.

$|F_0 - F(x)| = 0.053$.

TABLE A-17. REGION 04 – JANUARY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	32	0.320	26.3	0.263
2	21	0.530	21.6	0.479
3	10	0.630	21.2	0.691
4	26	0.890	18.7	0.878
5	11	1.000	12.2	1.000

$\bar{x} = 2.630$, $s^2 = 2.053$, $\eta^* = 1.397$, and $\gamma^* = 1.280$.

$|F_0 - F(x)| = 0.061$.

TABLE A-18. REGION 04 – JANUARY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	42	0.420	31.1	0.311
2	13	0.550	16.3	0.474
3	5	0.600	15.9	0.633
4	14	0.740	16.4	0.797
5	26	1.000	20.3	1.000

$\bar{x} = 2.690$, $s^2 = 2.894$, $\eta^* = 0.824$, and $\gamma^* = 0.788$.

$|F_0 - F(x)| = 0.109$.

TABLE A-19. REGION 04 – JANUARY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	21	0.210	18.9	0.189
2	21	0.420	16.8	0.357
3	10	0.520	18.9	0.546
4	20	0.720	20.9	0.756
5	28	1.000	24.4	1.000

$\bar{x} = 3.130$, $s^2 = 2.353$, $\eta^* = 0.928$, and $\gamma^* = 1.225$.

$|F_0 - F(x)| = 0.063$.

TABLE A-20. REGION 04 – JANUARY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	14	0.140	14.6	0.146
2	25	0.390	18.1	0.328
3	12	0.510	22.0	0.548
4	27	0.780	23.9	0.787
5	22	1.000	21.3	1.000

$\bar{x} = 3.180$, $s^2 = 1.928$, $\eta^* = 1.193$, and $\gamma^* = 1.635$.

$|F_0 - F(x)| = 0.062$.

TABLE A-21. REGION 04 – APRIL – SATELLITE

x	f_0	F_0	f_E	F(x)
1	29	0.290	24.4	0.244
2	22	0.510	21.8	0.461
3	11	0.620	21.9	0.680
4	27	0.890	19.5	0.875
5	11	1.000	12.5	1.000

$\bar{x} = 2.690$, $s^2 = 1.994$, $\eta^* = 1.426$, and $\gamma^* = 1.365$.

$|F_0 - F(x)| = 0.060$.

TABLE A-22. REGION 04 – APRIL – HOUR 4

x	f_0	F_0	f_E	F(x)
1	44	0.440	33.6	0.336
2	16	0.600	19.4	0.530
3	7	0.670	17.9	0.709
4	17	0.840	16.1	0.870
5	16	1.000	13.0	1.000

$\bar{x} = 2.450$, $s^2 = 2.428$, $\eta^* = 1.152$, and $\gamma^* = 0.926$.

$|F_0 - F(x)| = 0.104$.

TABLE A-23. REGION 04 – APRIL – HOUR 10

x	f_0	F_0	f_E	F(x)
1	17	0.170	16.4	0.164
2	23	0.400	19.4	0.358
3	16	0.560	22.6	0.584
4	25	0.810	23.2	0.816
5	19	1.000	18.4	1.000

$\bar{x} = 3.060$, $s^2 = 1.916$, $\eta^* = 1.285$, and $\gamma^* = 1.611$.

$|F_0 - F(x)| = 0.042$.

TABLE A-24. REGION 04 – APRIL – HOUR 16

x	f_0	F_0	f_E	F(x)
1	13	0.130	13.8	0.138
2	24	0.370	18.5	0.323
3	16	0.530	22.9	0.552
4	26	0.790	24.6	0.798
5	21	1.000	20.2	1.000

$\bar{x} = 3.180$, $s^2 = 1.828$, $\eta^* = 1.281$, and $\gamma^* = 1.756$.

$|F_0 - F(x)| = 0.047$.

TABLE A-25. REGION 04 – JULY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	21	0.210	18.3	0.183
2	22	0.430	20.6	0.389
3	12	0.550	23.0	0.619
4	32	0.870	22.3	0.842
5	13	1.000	15.8	1.000

$\bar{x} = 2.940$, $s^2 = 1.896$, $\eta^* = 1.382$, and $\gamma^* = 1.587$.

$|F_0 - F(x)| = 0.069$.

TABLE A-26. REGION 04 – JULY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	21	0.210	21.4	0.214
2	32	0.530	25.2	0.466
3	16	0.690	25.5	0.720
4	23	0.920	19.8	0.918
5	8	1.000	8.2	1.000

$\bar{x} = 2.650$, $s^2 = 1.588$, $\eta^* = 1.947$, and $\gamma^* = 1.810$.

$|F_0 - F(x)| = 0.064$.

TABLE A-27. REGION 04 – JULY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	1	0.010	3.3	0.033
2	19	0.200	13.7	0.170
3	22	0.420	27.8	0.448
4	40	0.820	35.1	0.800
5	18	1.000	20.0	1.000

$\bar{x} = 3.550$, $s^2 = 1.048$, $\eta^* = 1.989$, and $\gamma^* = 3.620$.

$|F_0 - F(x)| = 0.030$.

TABLE A-28. REGION 04 – JULY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	0	0.000	0.4	0.004
2	7	0.070	5.0	0.054
3	13	0.200	19.8	0.252
4	50	0.700	41.0	0.662
5	30	1.000	33.8	1.000

$\bar{x} = 4.030$, $s^2 = 0.709$, $\eta^* = 1.966$, and $\gamma^* = 5.389$.

$|F_0 - F(x)| = 0.052$.

TABLE A-29. REGION 04 – OCTOBER – SATELLITE

x	f_0	F_0	f_E	F(x)
1	30	0.300	25.4	0.254
2	23	0.530	22.4	0.478
3	11	0.640	22.0	0.698
4	26	0.900	19.0	0.887
5	10	1.000	11.3	1.000

$\bar{x} = 2.630$, $s^2 = 1.953$, $\eta^* = 1.495$, and $\gamma^* = 1.370$.

$|F_0 - F(x)| = 0.058$.

TABLE A-30. REGION 04 – OCTOBER – HOUR 4

x	f_0	F_0	f_E	F(x)
1	47	0.470	36.9	0.369
2	19	0.660	20.6	0.575
3	7	0.730	18.0	0.754
4	13	0.860	14.9	0.903
5	14	1.000	9.7	1.000

$\bar{x} = 2.280$, $s^2 = 2.242$, $\eta^* = 1.332$, and $\gamma^* = 0.943$.

$|F_0 - F(x)| = 0.101$.

TABLE A-31. REGION 04 – OCTOBER – HOUR 10

x	f_0	F_0	f_E	F(x)
1	20	0.200	20.3	0.203
2	29	0.490	21.0	0.413
3	12	0.610	22.6	0.639
4	23	0.840	21.4	0.853
5	16	1.000	14.7	1.000

$\bar{x} = 2.860$, $s^2 = 1.940$, $\eta^* = 1.388$, and $\gamma^* = 1.503$.

$|F_0 - F(x)| = 0.077$.

TABLE A-32. REGION 04 – OCTOBER – HOUR 16

x	f_0	F_0	f_E	F(x)
1	11	0.110	13.2	0.132
2	28	0.390	19.5	0.326
3	15	0.540	24.3	0.570
4	28	0.820	25.2	0.822
5	18	1.000	17.8	1.000

$\bar{x} = 3.140$, $s^2 = 1.700$, $\eta^* = 1.441$, and $\gamma^* = 1.917$.

$|F_0 - F(x)| = 0.064$.

TABLE A-33. REGION 08 – JANUARY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	19	0.190	15.4	0.154
2	17	0.360	17.3	0.327
3	9	0.450	20.7	0.534
4	34	0.790	23.1	0.765
5	21	1.000	23.5	1.000

$\bar{x} = 3.210$, $s^2 = 2.066$, $\eta^* = 1.065$, and $\gamma^* = 1.493$.

$|F_0 - F(x)| = 0.084$.

TABLE A-34. REGION 08 – JANUARY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	18	0.180	13.0	0.130
2	10	0.280	11.2	0.243
3	5	0.330	13.7	0.380
4	14	0.470	18.2	0.562
5	53	1.000	43.8	1.000

$\bar{x} = 3.740$, $s^2 = 2.532$, $\eta^* = 0.512$, and $\gamma^* = 1.088$.

$|F_0 - F(x)| = 0.092$.

TABLE A-35. REGION 08 – JANUARY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	7	0.070	5.4	0.054
2	11	0.180	9.1	0.145
3	5	0.230	14.4	0.289
4	21	0.440	22.2	0.511
5	56	1.000	48.9	1.000

$\bar{x} = 4.080$, $s^2 = 1.674$, $\eta^* = 0.636$, and $\gamma^* = 1.826$.

$|F_0 - F(x)| = 0.071$.

TABLE A-36. REGION 08 – JANUARY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	8	0.080	6.7	0.067
2	13	0.210	10.0	0.167
3	5	0.260	15.0	0.317
4	21	0.470	22.0	0.537
5	53	1.000	46.3	1.000

$\bar{x} = 3.980$, $s^2 = 1.820$, $\eta^* = 0.642$, and $\gamma^* = 1.682$.

$|F_0 - F(x)| = 0.067$.

TABLE A-37. REGION 08 – APRIL – SATELLITE

x	f_0	F_0	f_E	F(x)
1	24	0.240	19.7	0.197
2	19	0.430	19.3	0.391
3	10	0.530	21.2	0.603
4	31	0.840	21.5	0.818
5	16	1.000	18.2	1.000

$\bar{x} = 2.960$, $s^2 = 2.098$, $\eta^* = 1.193$, and $\gamma^* = 1.390$.

$|F_0 - F(x)| = 0.073$.

TABLE A-38. REGION 08 – APRIL – HOUR 4

x	f_0	F_0	f_E	F(x)
1	33	0.330	25.1	0.251
2	14	0.470	15.9	0.409
3	6	0.530	16.5	0.574
4	18	0.710	18.0	0.754
5	29	1.000	24.6	1.000

$\bar{x} = 2.960$, $s^2 = 2.798$, $\eta^* = 0.779$, and $\gamma^* = 0.908$.

$|F_0 - F(x)| = 0.079$.

TABLE A-39. REGION 08 – APRIL – HOUR 10

x	f_0	F_0	f_E	F(x)
1	13	0.130	11.7	0.117
2	17	0.300	13.5	0.252
3	8	0.380	17.4	0.426
4	23	0.610	22.3	0.650
5	39	1.000	35.0	1.000

$\bar{x} = 3.580$, $s^2 = 2.144$, $\eta^* = 0.770$, and $\gamma^* = 1.436$.

$|F_0 - F(x)| = 0.048$.

TABLE A-40. REGION 08 – APRIL – HOUR 16

x	f_0	F_0	f_E	F(x)
1	8	0.080	7.3	0.073
2	15	0.230	12.1	0.194
3	9	0.320	18.0	0.374
4	28	0.600	24.9	0.624
5	40	1.000	37.6	1.000

$\bar{x} = 3.770$, $s^2 = 1.757$, $\eta^* = 0.853$, and $\gamma^* = 1.859$.

$|F_0 - F(x)| = 0.054$.

TABLE A-41. REGION 08 – JULY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	48	0.480	38.5	0.385
2	22	0.700	25.8	0.643
3	9	0.790	19.8	0.841
4	17	0.960	12.1	0.962
5	4	1.000	3.8	1.000

$\bar{x} = 2.070$, $s^2 = 1.605$, $\eta^* = 2.135$, and $\gamma^* = 1.288$.

$|F_0 - F(x)| = 0.095$.

TABLE A-42. REGION 08 – JULY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	59	0.590	45.4	0.454
2	18	0.770	24.2	0.696
3	6	0.830	17.3	0.869
4	11	0.940	10.0	0.969
5	6	1.000	3.1	1.000

$\bar{x} = 1.870$, $s^2 = 1.613$, $\eta^* = 2.117$, and $\gamma^* = 1.091$.

$|F_0 - F(x)| = 0.135$.

TABLE A-43. REGION 08 – JULY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	51	0.510	41.0	0.410
2	23	0.740	26.0	0.670
3	7	0.810	19.1	0.861
4	15	0.960	10.9	0.970
5	4	1.000	3.0	1.000

$\bar{x} = 1.980$, $s^2 = 1.540$, $\eta^* = 2.257$, and $\gamma^* = 1.270$.

$|F_0 - F(x)| = 0.100$.

TABLE A-44. REGION 08 – JULY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	39	0.390	32.8	0.328
2	26	0.650	26.1	0.590
3	10	0.750	21.8	0.808
4	21	0.960	14.3	0.951
5	4	1.000	4.9	1.000

$\bar{x} = 2.250$, $s^2 = 1.628$, $\eta^* = 2.064$, and $\gamma^* = 1.429$.

$|F_0 - F(x)| = 0.062$.

TABLE A-45. REGION 08 – OCTOBER – SATELLITE

x	f_0	F_0	f_E	F(x)
1	36	0.360	29.0	0.290
2	20	0.560	22.0	0.510
3	10	0.660	20.7	0.717
4	24	0.900	17.6	0.893
5	10	1.000	10.7	1.000

$\bar{x} = 2.520$, $s^2 = 2.050$, $\eta^* = 1.443$, and $\gamma^* = 1.221$.

$|F_0 - F(x)| = 0.070$.

TABLE A-46. REGION 08 – OCTOBER – HOUR 4

x	f_0	F_0	f_E	F(x)
1	52	0.520	38.3	0.383
2	13	0.650	19.0	0.573
3	6	0.710	16.7	0.740
4	13	0.840	14.6	0.886
5	16	1.000	11.4	1.000

$\bar{x} = 2.280$, $s^2 = 2.462$, $\eta^* = 1.160$, and $\gamma^* = 0.822$.

$|F_0 - F(x)| = 0.137$.*

TABLE A-47. REGION 08 – OCTOBER – HOUR 10

x	f_0	F_0	f_E	F(x)
1	30	0.300	24.0	0.240
2	18	0.480	18.6	0.426
3	8	0.560	19.3	0.619
4	25	0.810	19.5	0.814
5	19	1.000	18.6	1.000

$\bar{x} = 2.850$, $s^2 = 2.367$, $\eta^* = 1.055$, and $\gamma^* = 1.135$.

$|F_0 - F(x)| = 0.060$.

TABLE A-48. REGION 08 – OCTOBER – HOUR 16

x	f_0	F_0	f_E	F(x)
1	28	0.280	23.1	0.231
2	19	0.470	18.3	0.414
3	9	0.560	19.2	0.606
4	23	0.790	19.8	0.804
5	21	1.000	19.6	1.000

$\bar{x} = 2.900$, $s^2 = 2.370$, $\eta^* = 1.031$, and $\gamma^* = 1.150$.

$|F_0 - F(x)| = 0.056$.

TABLE A-49. REGION 11 – JANUARY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	24	0.240	19.2	0.192
2	17	0.410	18.2	0.374
3	9	0.500	20.3	0.577
4	31	0.810	21.5	0.791
5	19	1.000	20.9	1.000

$\bar{x} = 3.040$, $s^2 = 2.198$, $\eta^* = 1.074$, and $\gamma^* = 1.328$.

$|F_0 - F(x)| = 0.077$.

TABLE A-50. REGION 11 – JANUARY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	32	0.320	22.3	0.223
2	8	0.400	12.2	0.345
3	4	0.440	13.1	0.476
4	9	0.530	15.9	0.635
5	47	1.000	36.5	1.000

$\bar{x} = 3.310$, $s^2 = 3.234$, $\eta^* = 0.494$, and $\gamma^* = 0.747$.

$|F_0 - F(x)| = 0.105$.

TABLE A-51. REGION 11 – JANUARY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	14	0.140	10.6	0.106
2	12	0.260	11.2	0.218
3	4	0.300	14.5	0.363
4	19	0.490	19.8	0.562
5	51	1.000	43.8	1.000

$\bar{x} = 3.810$, $s^2 = 2.254$, $\eta^* = 0.571$, and $\gamma^* = 1.286$.

$|F_0 - F(x)| = 0.072$.

TABLE A-52. REGION 11 – JANUARY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	15	0.150	11.5	0.115
2	12	0.270	11.8	0.233
3	6	0.330	15.1	0.384
4	19	0.520	20.1	0.586
5	48	1.000	41.4	1.000

$\bar{x} = 3.730$, $s^2 = 2.297$, $\eta^* = 0.603$, and $\gamma^* = 1.271$.

$|F_0 - F(x)| = 0.066$.

TABLE A-53. REGION 11 – APRIL – SATELLITE

x	f_0	F_0	f_E	F(x)
1	22	0.220	17.9	0.179
2	18	0.400	18.7	0.366
3	10	0.500	21.3	0.580
4	33	0.830	22.3	0.802
5	17	1.000	19.8	1.000

$\bar{x} = 3.050$, $s^2 = 2.068$, $\eta^* = 1.165$, and $\gamma^* = 1.500$.

$|F_0 - F(x)| = 0.080$.

TABLE A-54. REGION 11 – APRIL – HOUR 4

x	f_0	F_0	f_E	F(x)
1	29	0.290	22.0	0.220
2	13	0.420	14.6	0.366
3	6	0.480	15.8	0.523
4	17	0.650	18.2	0.705
5	35	1.000	29.5	1.000

$\bar{x} = 3.160$, $s^2 = 2.834$, $\eta^* = 0.684$, and $\gamma^* = 0.924$.

$|F_0 - F(x)| = 0.070$.

TABLE A-55. REGION 11 – APRIL – HOUR 10

x	f_0	F_0	f_E	F(x)
1	15	0.150	10.9	0.109
2	11	0.260	12.5	0.234
3	6	0.320	16.4	0.398
4	27	0.590	21.7	0.615
5	41	1.000	38.5	1.000

$\bar{x} = 3.680$, $s^2 = 2.158$, $\eta^* = 0.696$, and $\gamma^* = 1.408$.

$|F_0 - F(x)| = 0.078$.

TABLE A-56. REGION 11 – APRIL – HOUR 16

x	f_0	F_0	f_E	F(x)
1	12	0.120	8.9	0.089
2	11	0.230	12.5	0.214
3	9	0.320	17.5	0.389
4	29	0.610	23.6	0.624
5	39	1.000	37.6	1.000

$\bar{x} = 3.720$, $s^2 = 1.922$, $\eta^* = 0.792$, and $\gamma^* = 1.654$.

$|F_0 - F(x)| = 0.069$.

TABLE A-57. REGION 11 – JULY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	24	0.240	20.6	0.206
2	22	0.460	21.2	0.418
3	12	0.580	22.7	0.645
4	30	0.880	21.2	0.857
5	12	1.000	14.3	1.000

$\bar{x} = 2.840$, $s^2 = 1.934$, $\eta^* = 1.405$, and $\gamma^* = 1.500$.

$|F_0 - F(x)| = 0.065$.

TABLE A-58. REGION 11 – JULY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	26	0.260	23.0	0.230
2	23	0.490	19.9	0.429
3	12	0.610	20.7	0.637
4	21	0.820	20.1	0.838
5	18	1.000	16.2	1.000

$\bar{x} = 2.820$, $s^2 = 2.168$, $\eta^* = 1.212$, and $\gamma^* = 1.275$

$|F_0 - F(x)| = 0.061$.

TABLE A-59. REGION 11 – JULY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	13	0.130	11.6	0.116
2	18	0.310	16.2	0.278
3	12	0.430	21.2	0.489
4	32	0.750	25.0	0.739
5	25	1.000	26.1	1.000

$\bar{x} = 3.380$, $s^2 = 1.876$, $\eta^* = 1.087$, and $\gamma^* = 1.733$.

$|F_0 - F(x)| = 0.059$.

TABLE A-60. REGION 11 – JULY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	6	0.060	7.2	0.072
2	22	0.280	17.0	0.242
3	17	0.450	26.1	0.504
4	38	0.830	29.8	0.801
5	17	1.000	19.9	1.000

$\bar{x} = 3.380$, $s^2 = 1.376$, $\eta^* = 1.622$, and $\gamma^* = 2.587$.

$|F_0 - F(x)| = 0.054$.

TABLE A-61. REGION 11 – OCTOBER – SATELLITE

x	f_0	F_0	f_E	F(x)
1	34	0.340	27.5	0.275
2	20	0.540	21.6	0.491
3	10	0.640	20.8	0.699
4	25	0.890	18.2	0.881
5	11	1.000	11.9	1.000

$\bar{x} = 2.590$, $s^2 = 2.082$, $\eta^* = 1.386$, and $\gamma^* = 1.234$.

$|F_0 - F(x)| = 0.065$.

TABLE A-62. REGION 11 – OCTOBER – HOUR 4

x	f_0	F_0	f_E	F(x)
1	50	0.500	37.0	0.370
2	14	0.640	17.5	0.546
3	4	0.680	15.9	0.705
4	11	0.790	15.0	0.854
5	21	1.000	14.5	1.000

$\bar{x} = 2.390$, $s^2 = 2.718$, $\eta^* = 0.981$, and $\gamma^* = 0.754$.

$|F_0 - F(x)| = 0.130$.

TABLE A-63. REGION 11 – OCTOBER – HOUR 10

x	f_0	F_0	f_E	F(x)
1	34	0.340	26.1	0.261
2	15	0.490	17.1	0.432
3	7	0.560	17.4	0.607
4	20	0.760	18.2	0.789
5	24	1.000	21.1	1.000

$\bar{x} = 2.850$, $s^2 = 2.648$, $\eta^* = 0.893$, and $\gamma^* = 0.960$.

$|F_0 - F(x)| = 0.079$.

TABLE A-64. REGION 11 – OCTOBER – HOUR 16

x	f_0	F_0	f_E	F(x)
1	29	0.290	23.0	0.230
2	17	0.460	17.9	0.410
3	8	0.540	18.9	0.599
4	25	0.790	19.7	0.796
5	21	1.000	20.4	1.000

$\bar{x} = 2.920$, $s^2 = 2.414$, $\eta^* = 0.995$, and $\gamma^* = 1.126$.

$|F_0 - F(x)| = 0.060$.

TABLE A-65. REGION 18 – JANUARY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	28	0.280	22.1	0.221
2	17	0.450	19.2	0.412
3	9	0.540	20.4	0.616
4	30	0.840	20.4	0.820
5	16	1.000	18.0	1.000

$\bar{x} = 2.890$, $s^2 = 2.218$, $\eta^* = 1.139$, and $\gamma^* = 1.262$.

$|F_0 - F(x)| = 0.076$.

TABLE A-66. REGION 18 – JANUARY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	32	0.320	22.3	0.223
2	8	0.400	12.7	0.349
3	4	0.440	13.6	0.485
4	12	0.560	16.3	0.649
5	44	1.000	35.1	1.000

$\bar{x} = 3.280$, $s^2 = 3.162$, $\eta^* = 0.526$, and $\gamma^* = 0.777$.

$|F_0 - F(x)| = 0.097$.

TABLE A-67. REGION 18 – JANUARY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	22	0.220	15.3	0.153
2	9	0.310	12.0	0.273
3	3	0.340	14.1	0.414
4	19	0.530	18.1	0.595
5	47	1.000	40.5	1.000

$\bar{x} = 3.600$, $s^2 = 2.680$, $\eta^* = 0.536$, and $\gamma^* = 1.016$.

$|F_0 - F(x)| = 0.074$.

TABLE A-68. REGION 18 – JANUARY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	23	0.230	16.9	0.169
2	11	0.340	13.4	0.303
3	6	0.400	15.5	0.459
4	20	0.600	19.1	0.650
5	40	1.000	35.0	1.000

$\bar{x} = 3.430$, $s^2 = 2.645$, $\eta^* = 0.634$, and $\gamma^* = 1.050$.

$|F_0 - F(x)| = 0.061$.

TABLE A-69. REGION 18 – APRIL – SATELLITE

x	f_0	F_0	f_E	F(x)
1	34	0.340	27.2	0.272
2	19	0.530	21.1	0.483
3	10	0.630	20.5	0.688
4	25	0.880	18.4	0.872
5	12	1.000	12.8	1.000

$\bar{x} = 2.620$, $s^2 = 2.136$, $\eta^* = 1.326$, and $\gamma^* = 1.207$.

$|F_0 - F(x)| = 0.068$.

TABLE A-70. REGION 18 – APRIL – HOUR 4

x	f_0	F_0	f_E	F(x)
1	35	0.350	25.6	0.256
2	11	0.460	14.7	0.403
3	6	0.520	15.2	0.555
4	14	0.660	17.1	0.726
5	34	1.000	27.4	1.000

$\bar{x} = 3.010$, $s^2 = 3.010$, $\eta^* = 0.675$, and $\gamma^* = 0.815$.

$|F_0 - F(x)| = 0.094$.

TABLE A-71. REGION 18 – APRIL – HOUR 10

x	f_0	F_0	f_E	F(x)
1	25	0.250	19.9	0.199
2	16	0.410	15.3	0.352
3	6	0.470	17.0	0.522
4	21	0.680	19.5	0.717
5	32	1.000	28.3	1.000

$\bar{x} = 3.190$, $s^2 = 2.614$, $\eta^* = 0.764$, and $\gamma^* = 1.055$.

$|F_0 - F(x)| = 0.058$.

TABLE A-72. REGION 18 – APRIL – HOUR 16

x	f_0	F_0	f_E	F(x)
1	32	0.320	24.6	0.246
2	15	0.470	17.0	0.417
3	7	0.540	17.7	0.594
4	22	0.760	18.7	0.781
5	24	1.000	21.9	1.000

$\bar{x} = 2.910$, $s^2 = 2.602$, $\eta^* = 0.893$, and $\gamma^* = 1.004$.

$|F_0 - F(x)| = 0.074$.

TABLE A-73. REGION 18 – JULY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	57	0.570	44.4	0.444
2	19	0.760	26.4	0.708
3	8	0.840	17.9	0.888
4	13	0.970	9.1	0.979
5	3	1.000	2.1	1.000

$\bar{x} = 1.860$, $s^2 = 1.420$, $\eta^* = 2.493$, and $\gamma^* = 1.274$.

$|F_o - F(x)| = 0.126$.

TABLE A-74. REGION 18 – JULY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	34	0.340	24.5	0.245
2	10	0.440	13.8	0.383
3	5	0.490	14.5	0.528
4	13	0.620	16.8	0.696
5	38	1.000	30.4	1.000

$\bar{x} = 3.110$, $s^2 = 3.098$, $\eta^* = 0.608$, and $\gamma^* = 0.791$.

$|F_o - F(x)| = 0.095$.

TABLE A-75. REGION 18 – JULY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	56	0.560	42.3	0.423
2	16	0.720	21.3	0.637
3	6	0.780	17.2	0.808
4	11	0.890	12.7	0.935
5	11	1.000	6.5	1.000

$\bar{x} = 2.050$, $s^2 = 2.048$, $\eta^* = 1.539$, and $\gamma^* = 0.915$.

$|F_o - F(x)| = 0.136$.

TABLE A-76. REGION 18 – JULY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	71	0.710	52.9	0.529
2	13	0.840	24.3	0.772
3	4	0.880	14.7	0.920
4	7	0.950	6.7	0.987
5	5	1.000	1.3	1.000

$\bar{x} = 1.620$, $s^2 = 1.336$, $\eta^* = 2.615$, and $\gamma^* = 1.092$.

$|F_o - F(x)| = 0.181.*$

TABLE A-77. REGION 18 – OCTOBER – SATELLITE

x	f_0	F_0	f_E	F(x)
1	44	0.440	34.5	0.345
2	19	0.630	22.9	0.574
3	9	0.720	19.8	0.772
4	20	0.920	15.1	0.923
5	8	1.000	7.7	1.000

$\bar{x} = 2.290$, $s^2 = 1.966$, $\eta^* = 1.599$, and $\gamma^* = 1.141$.

$|F_0 - F(x)| = 0.095.$

TABLE A-78. REGION 18 – OCTOBER – HOUR 4

x	f_0	F_0	f_E	F(x)
1	46	0.460	32.1	0.321
2	8	0.540	14.4	0.465
3	4	0.580	14.0	0.605
4	9	0.670	15.2	0.757
5	33	1.000	24.3	1.000

$\bar{x} = 2.750$, $s^2 = 3.267$, $\eta^* = 0.657$, and $\gamma^* = 0.657$.

$|F_0 - F(x)| = 0.139.$*

TABLE A-79. REGION 18 – OCTOBER – HOUR 10

x	f_0	F_0	f_E	F(x)
1	38	0.380	28.3	0.283
2	13	0.510	16.4	0.447
3	6	0.570	16.4	0.611
4	17	0.740	17.2	0.784
5	26	1.000	21.6	1.000

$\bar{x} = 2.800$, $s^2 = 2.820$, $\eta^* = 0.825$, and $\gamma^* = 0.856$.

$|F_0 - F(x)| = 0.097.$

TABLE A-80. REGION 18 – OCTOBER – HOUR 16

x	f_0	F_0	f_E	F(x)
1	50	0.500	37.5	0.375
2	15	0.650	20.2	0.577
3	6	0.710	17.6	0.753
4	16	0.870	14.7	0.900
5	13	1.000	10.0	1.000

$\bar{x} = 2.270$, $s^2 = 2.297$, $\eta^* = 1.287$, and $\gamma^* = 0.905$.

$|F_0 - F(x)| = 0.125.$

TABLE A-81. REGION 19 – JANUARY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	23	0.230	18.1	0.181
2	16	0.390	17.8	0.359
3	9	0.480	20.2	0.560
4	32	0.800	21.8	0.779
5	20	1.000	22.1	1.000

$\bar{x} = 3.100$, $s^2 = 2.190$, $\eta^* = 1.046$, and $\gamma^* = 1.351$.

$|F_0 - F(x)| = 0.080$.

TABLE A-82. REGION 19 – JANUARY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	30	0.300	20.2	0.202
2	6	0.360	11.6	0.319
3	4	0.400	12.7	0.446
4	10	0.500	15.9	0.605
5	50	1.000	39.5	1.000

$\bar{x} = 3.440$, $s^2 = 3.166$, $\eta^* = 0.464$, and $\gamma^* = 0.774$.

$|F_0 - F(x)| = 0.105$.

TABLE A-83. REGION 19 – JANUARY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	16	0.160	10.8	0.108
2	8	0.240	10.4	0.212
3	4	0.280	13.3	0.345
4	16	0.440	18.4	0.529
5	56	1.000	47.1	1.000

$\bar{x} = 3.880$, $s^2 = 2.346$, $\eta^* = 0.495$, and $\gamma^* = 1.185$.

$|F_0 - F(x)| = 0.089$.

TABLE A-84. REGION 19 – JANUARY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	16	0.160	11.5	0.115
2	10	0.260	11.8	0.233
3	6	0.320	15.1	0.384
4	21	0.530	20.1	0.586
5	47	1.000	41.4	1.000

$\bar{x} = 3.730$, $s^2 = 2.297$, $\eta^* = 0.603$, and $\gamma^* = 1.271$.

$|F_0 - F(x)| = 0.064$.

TABLE A-85. REGION 19 – APRIL – SATELLITE

x	f_0	F_0	f_E	F(x)
1	25	0.250	20.5	0.205
2	19	0.440	19.4	0.399
3	10	0.540	21.1	0.610
4	30	0.840	21.1	0.821
5	16	1.000	17.9	1.000

$\bar{x} = 2.930$, $s^2 = 2.125$, $\eta^* = 1.189$, and $\gamma^* = 1.355$.

$|F_0 - F(x)| = 0.070$.

TABLE A-86. REGION 19 – APRIL – HOUR 4

x	f_0	F_0	f_E	F(x)
1	30	0.300	21.5	0.215
2	10	0.400	13.1	0.346
3	4	0.440	14.2	0.488
4	14	0.580	17.0	0.658
5	42	1.000	34.2	1.000

$\bar{x} = 3.280$, $s^2 = 3.042$, $\eta^* = 0.563$, and $\gamma^* = 0.831$.

$|F_0 - F(x)| = 0.085$.

TABLE A-87. REGION 19 – APRIL – HOUR 10

x	f_0	F_0	f_E	F(x)
1	18	0.180	12.6	0.126
2	9	0.270	12.2	0.247
3	6	0.330	15.2	0.399
4	22	0.550	19.9	0.598
5	45	1.000	40.2	1.000

$\bar{x} = 3.670$, $s^2 = 2.381$, $\eta^* = 0.606$, and $\gamma^* = 1.215$.

$|F_0 - F(x)| = 0.069$.

TABLE A-88. REGION 19 – APRIL – HOUR 16

x	f_0	F_0	f_E	F(x)
1	18	0.180	13.2	0.132
2	11	0.290	13.9	0.271
3	8	0.370	17.3	0.444
4	28	0.650	21.7	0.661
5	35	1.000	33.9	1.000

$\bar{x} = 3.510$, $s^2 = 2.250$, $\eta^* = 0.761$, and $\gamma^* = 1.343$.

$|F_0 - F(x)| = 0.074$.

TABLE A-89. REGION 19 – JULY – SATELLITE

x	f_0	F_0	f_E	F(x)
1	25	0.250	21.6	0.216
2	23	0.480	21.8	0.434
3	12	0.600	22.8	0.662
4	29	0.890	20.7	0.869
5	11	1.000	13.1	1.000

$\bar{x} = 2.780$, $s^2 = 1.912$, $\eta^* = 1.462$, and $\gamma^* = 1.494$.

$|F_0 - F(x)| = 0.062$.

TABLE A-90. REGION 19 – JULY – HOUR 4

x	f_0	F_0	f_E	F(x)
1	34	0.340	28.8	0.288
2	23	0.570	20.9	0.497
3	10	0.670	20.0	0.697
4	18	0.850	17.8	0.874
5	15	1.000	12.5	1.000

$\bar{x} = 2.570$, $s^2 = 2.185$, $\eta^* = 1.303$, and $\gamma^* = 1.143$.

$|F_0 - F(x)| = 0.073$.

TABLE A-91. REGION 19 – JULY – HOUR 10

x	f_0	F_0	f_E	F(x)
1	12	0.120	11.9	0.119
2	22	0.340	18.9	0.308
3	17	0.510	24.4	0.553
4	32	0.830	25.9	0.812
5	17	1.000	18.8	1.000

$\bar{x} = 3.200$, $s^2 = 1.660$, $\eta^* = 1.436$, and $\gamma^* = 1.998$.

$|F_0 - F(x)| = 0.043$.

TABLE A-92. REGION 19 – JULY – HOUR 16

x	f_0	F_0	f_E	F(x)
1	4	0.040	6.3	0.063
2	23	0.270	17.0	0.233
3	21	0.480	27.3	0.507
4	35	0.830	30.9	0.815
5	17	1.000	18.5	1.000

$\bar{x} = 3.380$, $s^2 = 1.276$, $\eta^* = 1.780$, and $\gamma^* = 2.838$.

$|F_0 - F(x)| = 0.037$.

TABLE A-93. REGION 19 – OCTOBER – SATELLITE

x	f_0	F_0	f_E	F(x)
1	35	0.350	28.5	0.285
2	21	0.560	22.1	0.507
3	10	0.660	21.0	0.716
4	24	0.900	17.7	0.893
5	10	1.000	10.7	1.000

$\bar{x} = 2.530$, $s^2 = 2.029$, $\eta^* = 1.460$, and $\gamma^* = 1.243$.

$|F_0 - F(x)| = 0.065$.

TABLE A-94. REGION 19 – OCTOBER – HOUR 4

x	f_0	F_0	f_E	F(x)
1	51	0.510	36.9	0.369
2	11	0.620	17.4	0.543
3	6	0.680	15.8	0.702
4	11	0.790	15.0	0.852
5	21	1.000	14.8	1.000

$\bar{x} = 2.400$, $s^2 = 2.740$, $\eta^* = 0.967$, and $\gamma^* = 0.749$.

$|F_0 - F(x)| = 0.141$.*

TABLE A-95. REGION 19 – OCTOBER – HOUR 10

x	f_0	F_0	f_E	F(x)
1	33	0.330	25.8	0.258
2	16	0.490	17.3	0.431
3	8	0.570	17.7	0.608
4	19	0.760	18.4	0.792
5	24	1.000	20.8	1.000

$\bar{x} = 2.850$, $s^2 = 2.608$, $\eta^* = 0.914$, and $\gamma^* = 0.983$.

$|F_0 - F(x)| = 0.072$.

TABLE A-96. REGION 19 – OCTOBER – HOUR 16

x	f_0	F_0	f_E	F(x)
1	30	0.300	25.8	0.258
2	23	0.530	20.4	0.462
3	10	0.630	20.4	0.666
4	21	0.840	19.0	0.856
5	16	1.000	14.4	1.000

$\bar{x} = 2.700$, $s^2 = 2.190$, $\eta^* = 1.248$, and $\gamma^* = 1.204$.

$|F_0 - F(x)| = 0.068$.

APPENDIX B

PARAMETER ESTIMATES η^* AND γ^* OF THE BETA DISTRIBUTION FOR CLOUD-COVER REGIONS 02, 03, 04, 08, 11, 16, 18, 19, 25, AND 28.

TABLE B-1. REGION 02

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.458	1.317	1.155	1.044	1.263	1.632	1.235	1.668
April	1.514	1.252	1.429	1.253	1.203	1.361	1.111	1.230
July	3.246	1.490	3.124	1.350	6.213	2.351	5.952	1.994
October	1.704	1.339	1.809	1.271	1.743	1.781	1.552	1.658

TABLE B-2. REGION 03

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.300	1.618	1.194	1.475	1.270	2.466	1.523	3.523
April	1.413	1.576	1.571	1.515	1.648	2.817	2.030	3.610
July	1.108	1.969	0.994	2.969	1.630	7.611	1.619	8.276
October	1.175	1.736	1.077	2.124	1.202	3.660	1.328	4.609

TABLE B-3. REGION 04

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.397	1.280	0.824	0.788	0.928	1.225	1.193	1.635
April	1.426	1.365	1.152	0.926	1.285	1.611	1.281	1.756
July	1.382	1.587	1.947	1.810	1.989	3.620	1.966	5.389
October	1.495	1.370	1.332	0.943	1.388	1.503	1.441	1.917

TABLE B-4. REGION 08

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.065	1.493	0.512	1.088	0.636	1.826	0.642	1.682
April	1.193	1.390	0.779	0.908	0.770	1.436	0.853	1.859
July	2.135	1.288	2.117	1.091	2.257	1.270	2.064	1.429
October	1.443	1.221	1.160	0.822	1.055	1.135	1.031	1.150

TABLE B-5. REGION 11

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.074	1.328	0.494	0.747	0.571	1.286	0.603	1.271
April	1.165	1.500	0.684	0.924	0.696	1.408	0.792	1.654
July	1.405	1.500	1.212	1.275	1.087	1.733	1.622	2.587
October	1.386	1.234	0.981	0.754	0.893	0.960	0.995	1.126

TABLE B-6. REGION 16

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.578	1.467	3.299	1.402	1.955	2.058	2.107	2.702
April	1.262	1.679	1.554	1.455	1.626	3.544	1.543	3.760
July	1.078	2.042	1.155	2.375	1.254	6.161	1.970	10.778
October	1.136	1.911	1.232	2.731	1.881	8.570	1.669	8.764

TABLE B-7. REGION 18

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.139	1.262	0.526	0.777	0.536	1.016	0.634	1.050
April	1.326	1.207	0.675	0.815	0.764	1.055	0.893	1.004
July	2.493	1.274	0.608	0.791	1.539	0.915	2.615	1.092
October	1.599	1.141	0.657	0.657	0.825	0.856	1.287	0.905

TABLE B-8. REGION 19

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.046	1.351	0.464	0.774	0.495	1.185	0.603	1.271
April	1.189	1.355	0.563	0.831	0.606	1.215	0.761	1.343
July	1.462	1.494	1.303	1.143	1.436	1.998	1.780	2.838
October	1.460	1.243	0.967	0.749	0.914	0.983	1.248	1.204

TABLE B-9. REGION 25

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.078	2.042	1.155	2.375	1.254	6.161	1.970	10.778
April	1.136	1.911	1.232	2.730	1.881	8.570	1.669	8.764
July	1.803	1.193	3.299	1.402	1.677	1.214	1.545	1.144
October	1.262	1.679	1.554	1.455	1.626	3.544	1.543	3.760

TABLE B-10. REGION 28

Month	Satellite		Hour 4		Hour 10		Hour 16	
	η^*	γ^*	η^*	γ^*	η^*	γ^*	η^*	γ^*
January	1.462	1.494	1.303	1.143	1.436	1.998	1.780	2.838
April	1.460	1.243	0.967	0.749	0.914	0.983	1.248	1.204
July	1.046	1.351	0.464	0.774	0.495	1.185	0.603	1.271
October	1.188	1.355	0.563	0.831	0.606	1.215	0.761	1.343

REFERENCES

1. Sherr, P.E.; Glaser, A.H.; Barnes, J.C.; and Willand, J.H.: World-Wide Cloud Cover Distributions For Use in Computer Simulations. NASA CR-61226, Marshall Space Flight Center, Huntsville, Ala., June 14, 1968.

2. Greaves, J.R.; Spiegler, D.B.; and Willand, J.H.: Development of a Global Cloud Model For Simulating Earth-Viewing Space Missions. NASA CR-61345, Marshall Space Flight Center, Huntsville, Ala., April 7, 1971.

3. Chang, D.T.; and Willand, J.H.: Further Developments in Cloud Statistics For Computer Simulations. NASA CR-61389, Marshall Space Flight Center, Huntsville, Ala., July 5, 1972.

4. Brown, S. C.: Simulating the Consequence of Cloud Cover on Earth-Viewing Space Missions. Bulletin of the American Meteorological Society, Vol. 51, No. 2, February 1970.

5. Pearson, Karl: Tables of the Incomplete Beta-Function. Cambridge University Press, Cambridge, England, 1956.

6. Lancaster, H.O.: The Chi-Squared Distribution. John Wiley and Sons, Inc., New York, 1969.

7. Siegel, Sidney: Nonparametric Statistics. McGraw-Hill Book Co., New York, 1956.

8. Falls, L.W.: A Computer Program For Standard Statistical Distributions. NASA TM X-64588, Marshall Space Flight Center, Huntsville, Ala., April 30, 1971.

APPROVAL

THE BETA DISTRIBUTION: A STATISTICAL MODEL FOR WORLD CLOUD COVER

By Lee W. Falls

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

O. E. SMITH
Chief, Terrestrial Environment Branch

W. W. VAUGHAN
Chief, Aerospace Environment Division

E. D. GEISSLER
Director, Aero-Astrodynamics Laboratory

NASA—MSFC