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THE CORRELATION LENGTH FOR INTERPLANETARY MAGNETIC FIELD FLUCTUATIONS

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ABSTRACT

It is argued that it is appropriate to consider two correlation lengths for interplanetary magnetic field fluctuations. For particles with gyro-radii large enough to encounter and be scattered by large-scale tangential discontinuities in the field (particles with energies $\gtrsim$ several GeV/nucleon) the appropriate correlation length is simply the mean spatial separation between the discontinuities, $L \simeq 2\text{x}10^{11}$cm. Particles with gyro-radii much less than this mean separation (energies $\lesssim$ 100 MeV/nucleon) appear to be unaffected by the discontinuities and respond only to smaller-scale field fluctuations. For these particles the correlation length is shown to be $L \simeq 2\text{x}10^{10}$cm. With this system of two correlation lengths the cosmic-ray diffusion tensor may be altered from what was predicted by, for example, Jokipii and Coleman, and the objections raised recently by Klimas and Sandri to the diffusion analysis of Jokipii may apply only at relatively low energies ($\sim$ 50 MeV/nucleon).

Introduction

The characteristic length over which fluctuations in the interplanetary magnetic field are correlated, the so-called correlation length, is an important parameter in most theories that attempt to express the cosmic-ray diffusion tensor in terms of the power spectrum of field fluctuations. In the theory due to Jokipii (1966, 1967, 1968, 1972) the rigidity-dependence of the diffusion tensor is substantially different depending on whether the particle gyro-radius is greater than or less than the correlation length. Klimas and Sandri (1971, 1973) take exception to the work by Jokipii and argue that there is at this time no firm theoretical basis for the use of a diffusion equation when the particle gyro-radius becomes comparable to the correlation length (i.e. the transport of cosmic rays may be a non-Markovian process in this limit). Klimas and Sandri, however, are in agreement with Jokipii when the gyro-radius is much greater than the correlation length.

It is the purpose of this Report to point out that it is necessary to consider two correlation lengths for the field fluctuations. As we discuss below, high energy particles (energies $\gtrsim$ 1 GeV/nucleon) should be scattered mainly by large-scale tangential discontinuities in the interplanetary magnetic field (a discontinuity where the field vector undergoes an abrupt change across a surface that has a normal pointing perpendicular to the field direction). For these high energy particles the appropriate correlation length is simply the mean spatial separation between the discontinuities, $L \simeq 2 \times 10^{11}$cm (Burlaga, 1969). On the other hand, low energy particles (energies $\lesssim$ 100 MeV/nucleon) appear to be unaffected by the large-scale tangential discontinuities (Sari, 1972, 1973) and respond only to smaller-scale field fluctuations that are mainly propagating wave disturbances and rotational discontinuities (a discontinuity that propagates at the Alfven speed and has a component of the field normal to the surface of the discontinuity). For these low energy particles, the appropriate correlation length is $L \simeq 2x10^{10}$cm. With this system of two correlation lengths, the rigidity-dependence of the cosmic-ray diffusion tensor will be difficult to calculate, but presumably will be altered from what was predicted by, for example, Jokipii and Coleman (1968). Further, in the scheme developed here, the objections raised by Klimas and Sandri (1971, 1973) to the validity of diffusion theory may apply only at relatively low energies (~ 50 MeV/ nucleon).

Analysis

Jokipii and Coleman (1968) noted that the correlation length L can be estimated from the shape of the power spectrum of field fluctuations according to the formula

$$L \simeq \frac{V_w}{2\pi f_c} \qquad (1)$$

Here V_w is the solar wind speed and f_c is the frequency at which the observed power spectrum shows a break from a flat spectrum to a fairly steep inverse power law dependence on frequency. For a typical power spectrum in 1965, Jokipii and Coleman (1968) find that $f_c \sim 3x10^{-5}$Hz, which implies that $L \sim 2x10^{11}$cm, on taking $V = 3.5x10^7$cm/sec. This value for L is roughly equal to the gyro-radius of a 3 GV particle in a 5γ magnetic field.

In the analysis of Jokipii and Coleman (1968) the correlation length is determined using all of the observed power, including the contribution to the power from directional discontinuities (either rotational or tangential) in the magnetic field. Recently Sari (1972, 1973) has analyzed the role that such discontinuities play in the scattering of low energy cosmic rays ($\leqslant$ 100 MeV/nucleon).

In the analysis by Sari (1971, 1973) directional discontinuities are identified in the data using the selection criteria established by Burlaga (1969). Discontinuities are defined as a change in the field occurring within the 30 second averaging period of the Pioneer 6 data and resulting in more than a 30° change in the magnetic field direction. The field both before and after the discontinuity is required to be relatively undisturbed, and thus the discontinuity takes on the appearance of a 'step-function'. These criteria differ from some of those used by other workers, and thus the relative number of tangential to rotational discontinuities within the sample may differ from the relative number in other samples. However, Burlaga (1971) has shown using a limited sample of Pioneer 6 data that the majority of the directional discontinuities identified with the criteria used by Burlaga (1969) and Sari (1972, 1973) are in fact tangential. As we discuss below, the particle data supports this conclusion (Sari, 1972, 1973).

Sari (1972, 1973) evaluates the role that directional discontinuities play in the scattering of low energy cosmic rays (< 100 MeV/nucleon) by comparing short-term fluctuations in the intensity of

$\sim$ 70 MeV protons (3-day running averages) with variations in the standard modulation parameter $V_w\tilde{R}/K_{||}$ (cf. Fisk and Axford, 1969) determined over comparable time periods. Here, $\tilde{R}$ is a characteristic heliocentric distance (normally taken to be $\tilde{R}$ = 1 AU for observations made near earth), and $K_{||}$ is the diffusion coefficient describing the propagation of $\sim$ 70 MeV particles parallel to the mean magnetic field. The diffusion coefficient is computed from the power spectra of magnetic field fluctuations according to the techniques developed by Jokipii (1966, 1968).

In the study by Sari (1972, 1973) the magnetic field power spectra are computed from Pioneer 6 data for consecutive 24 hour periods from December 1965 through April 1966 using the finite Fourier transform technique outlined by Blackman and Tukey (1958) (Sari and Ness 1969, 1970). Computed first are the power spectra using all of the observed fluctuations including the directional discontinuities. These power spectra are denoted by P_{real}. The data in a given 24 hour period is next fitted by a series of step-functions to simulate the directional discontinuities (see Sari and Ness, 1969), and then the power spectrum due to the discontinuities (denoted by P_{dis}) is computed from the resulting time series of the step-functions. The power spectrum due to fluctuations between discontinuities (denoted by P_{bet}) is computed from the observed data with the directional discontinuities artificially removed (see Sari (1972, 1973) for the details of this technique). Small-scale tangential discontinuities (changes in direction by < 30°)

contribute to P_{bet}; however, as we discuss below, the main contribution to the power appears to come from propagating waves and rotational discontinuities. The sum P_{dis} and P_{bet} is equal to P_{real} to within the error measures; a result that we assume holds at all frequencies considered in the Report.

The three different power spectra (P_{real}, P_{dis}, and P_{bet}) are used to compute the corresponding diffusion coefficients denoted respectively by $K_{||}$(real), $K_{||}$(dis), and $K_{||}$(bet). Changes in the modulation parameter $V_W\tilde{R}/K_{||}$, determined using alternately these three values for $K_{||}$, are then compared with simultaneous changes in the IMP III, 70 MeV proton intensity. The results are shown in Figure 1 (after Sari (1972, 1973)). It is expected that variations in $V_W\tilde{R}/K_{||}$ are anticorrelated with fluctuations in the particle intensity. As is evident, however, only for $K_{||}$ (bet) is the expected anticorrelation clearly exhibited, and analysis (Sari, 1972, 1973) shows that the poorest correlation is for $K_{||}$ (dis). Hence, these results suggest that the directional discontinuities identified by Sari (1972, 1973) do not contribute significantly to the scattering of ~ 70 MeV protons.

Admittedly, the theory developed by Jokipii (1966, 1967, 1968, 1972) and used by Sari (1972, 1973) may be inadequate for describing the interaction of cosmic rays with large-scale changes in the field such as rotational discontinuities, and this inadequacy may account for the lack of correlation between the intensity fluctuations and $V_W\tilde{R}/K_{||}$ (dis). It is more reasonable, however, to conclude that the directional discontinuities identified by Sari (1972, 1973) are mainly tangential and that the low energy particles rarely encounter such discontinuities.

The mean separation between discontinuities as defined by Burlaga (1969) and Sari (1972, 1973) is $\sim 2\times10^{11}$cm (Burlaga, 1969), which is many times the gyro-radius of $\sim$ 70 MeV proton ($\sim 10^{10}$ cm in a 5γ field). Thus, the $\sim$ 70 MeV protons will propagate essentially unaffected by the large-scale tangential discontinuities, provided that there is no significant cross-field diffusion.

Since the directional discontinuities identified by Sari (1972, 1973) evidently do not scatter low energy cosmic rays ($\lesssim$ 100 MeV/nucleon), only the power due to fluctuations between discontinuities (P_{bet}) should be used for computing the correlation length or the diffusion tensor of these particles. Substantially higher energy particles ($\gtrsim$ a few GeV/nucleon), however, will have a gyro-radius large enough to encounter and be scattered frequently by the discontinuities, and for these particles P_{real} should be used.

In Figure 2-4 we have plotted some representative power spectra. Unlike the earlier work by Sari (1972, 1973) the power spectra here are taken over 2.5-day intervals (as opposed to 24 hour intervals), and P_{real} is determined not by the finite Fourier transform technique but rather by the nested variance technique as used by Jokipii and Coleman (1968). At relatively high frequencies ($\gtrsim 3\times10^{-4}$Hz) both P_{dis} and P_{bet} are determined as outlined above; however, this technique breaks down at frequencies comparable to or less than the frequency at which the discontinuities occur, $\sim$ 1/hour $\simeq 3\times10^{-4}$Hz (Sari, 1972). The power spectrum due to discontinuities is extrapolated to lower frequencies using the theoretical prediction of P_{dis} given by Siscoe et al. (1968):

$$P_{dis}(f) = \frac{4\lambda\overline{A}^2}{\lambda^2+(2\pi f)^2} \qquad (2)$$

where $\overline{A}^2$ is the variance of the magnetic field fluctuations and $1/\lambda$ is the average time-separation between discontinuities (~ 1 hour). It is assumed in deriving expression (2) that the field maintains a constant value for varying periods of time, jumping discontinuously from one constant value to another (This description of the field is equivalent to assuming that the discontinuities can be simulated by fitting to the data a series of step-functions, as was done by Sari (1972, 1973) for computing P_{dis} at higher frequencies). The power spectrum P_{bet} is extrapolated to lower frequencies simply by subtracting the extrapolated values of P_{dis} from P_{real}. The error measures shown for P_{bet} at low frequencies are estimated by assuming that the errors in the extrapolated values of P_{dis} are the same as the errors in P_{real} at the same frequency; the errors shown for P_{real} represent 95% confidence levels.

Siscoe et al. (1968) considered the extrapolation that we have used here for extending P_{dis} to low frequencies, and suggested that it may be inadequate at very low frequencies. In the sample of discontinuities identified by Siscoe et al. (1968) there is supposedly a tendency for the discontinuities to occur in associated groups of two or more in which the change due to one discontinuity is largely cancelled by a following discontinuity. If this is the case, the total change in the field before and after the discontinuities occur is small, and thus the contribution to the power density at low frequencies is smaller than predicted by equation (2). However, in the sample of discontinuities identified by Burlaga (1969) and Sari (1972, 1973) there is no apparent tendency for one discontinuity to cancel the change in the field due to previous discontinuities (Burlaga, 1969; private communication). Also, any tendency for discontinuities to cancel each other would only be important for the power spectrum at frequencies that correspond to time periods longer than the mean temporal separation between discontinuities (~ 1 hour), i.e. at frequencies $\leqslant 10^{-5}$ Hz. The conclusions of this Report concerning the correlation length are not critically sensitive to these low frequencies.

The examples shown in Figures 2-4 were chosen because they are a representative sample of possible interplanetary power spectra and also because P_{real} is relatively flat at low frequencies; with the latter criteria the correlation length for P_{real} and P_{bet} is particularly easy to determine from (1). In Figure 2, P_{dis} is comparable to or in excess of P_{bet} up to the highest frequencies observed, whereas in Figure 3, P_{dis} is substantially less than P_{bet} at the higher frequencies. In Figure 4, the frequency at which the discontinuities occur is quite high (1.35/hour) with the result that the extrapolated power density at low frequencies for P_{dis} is relatively low (cf. equation (2)). Despite the differences in these examples, in all three cases P_{dis} appears to dominate P_{real} at low frequencies, to within the error measures. The data is consistent with the expected flattening in P_{bet} at low frequencies setting in at a relatively high frequency, $\sim 3 \times 10^{-4}$ Hz.

Note that the slope of P_{bet} at high frequencies is generally < 2, and is thus not characteristic of a spectra where the main source of the power is tangential discontinuities. This slope implies then that wave disturbances and rotational discontinuities are the main contributors to P_{bet}, not small-scale, tangential discontinuities (change in field direction by < 30°).

In Figures 2-4, the flattening at low frequencies in P_{bet} appears to occur at a frequency roughly an order of magnitude higher than where a similar flattening occurs in P_{real}. From equation (1) this implies that the correlation length for fluctuations between discontinuities is roughly an order of magnitude smaller than the correlation length determined by Jokipii and Coleman (1968) for P_{real}. That is, the correlation length for fluctuations between discontinuities is $L \sim 2\text{x}10^{10}$cm, which is comparable to the gyro-radius of a 50 MeV proton in a 5γ magnetic field. The correlation length for P_{real} is of course roughly equal to the mean spatial separation between large-scale tangential discontinuities, $L \sim 2\text{x}10^{11}$cm, since as shown in Figs. 2-4, P_{dis} dominates P_{real} at low frequencies.

The relatively small value for P_{bet} at low frequencies predicted in Figures 2-4 has implications for cross-field diffusion of low energy cosmic rays. The only efficient mechanism for transporting cosmic rays across the mean interplanetary field is via field-line random walk (Jokipii and Parker, 1969). A measure of the random walk is the power density at zero or low frequencies provided that this power is uncontaminated by other producers of low frequency power such as temporal variations, sector structure, or tangential discontinuities. Previous estimates of the perpendicular diffusion coefficient for the random-walk mechanism (Jokipii and Parker, 1969) have used P_{real}, which as we saw above is dominated by P_{dis} at low frequencies and thus is inappropriate. The more appropriate measure is P_{bet}, which at low frequencies appears in Figures 2-4 to be at least one-to-two orders of magnitude smaller than P_{real}, implying a comparable reduction in the perpendicular diffusion coefficient. The smaller perpendicular diffusion coefficient that we predict here is consistent with the recent observations of Krimigis et al. (1971), where coherent structures in the intensity of low energy protons (~ 1 MeV) are seen from several widely-separated spacecraft, apparently unattenuated by cross-field diffusion. The lack of significant cross-field diffusion is also consistent with our conclusion above that particles with gyro-radii much less than the mean separation between tangential discontinuities will not encounter the discontinuities.

Conclusions

In summary, we argue in this Report that it is appropriate to consider two correlation lengths for interplanetary field fluctuations. For particles with gyro-radii large enough to encounter large-scale tangential discontinuities (particles with energies $\gtrsim$ a few GeV/nucleon) the appropriate correlation length is simply the mean spatial separation between discontinuities, $L \simeq 2 \times 10^{11}$ cm. This value for the correlation length is the one found by Jokipii and Coleman (1968). However, particles with gyro-radii much less than the mean spatial separation between tangential discontinuities (particles with energy $\leqslant$ 100 MeV/nucleon) appear to be unaffected by the discontinuities and respond only to other, smaller-scale field fluctuations (Sari, 1972, 1973). For these particles the appropriate correlation length is $L \simeq 2 \times 10^{10}$cm.

Most theories for determining the cosmic-ray diffusion tensor (Jokipii, 1967; Klimas and Sandri, 1972, 1973) predict that for particles with gyro-radii in excess of the correlation length the diffusion coefficient for propagation along the mean field direction has a rigidity-squared dependence. Such a diffusion coefficient is expected for particles with gyro-radii greater than $\sim 2 \times 10^{11}$cm (energies above a few GeV/nucleon). The diffusion tensor for particles with gyro-radii approximately equal to or less than $\sim 2 \times 10^{10}$cm may exhibit the rigidity-dependence predicted according to the theory by Jokipii (1966, 1967, 1968, 1972). It is also possible that for these low energy particles the objections raised by Klimas and Sandri (1971, 1973) to Jokipii's analysis may apply (i.e. the predicted diffusion coefficients may be substantially different and/or the propagation may be a non-Markovian process). For particles in the energy range 100 MeV/nucleon to several GeV/nucleon the diffusion tensor (if it exists) may prove difficult to determine since here we should anticipate a partial interaction of the particles with the discontinuities; presumably, this interaction increases with increasing energy. In fact, to determine the diffusion tensor at these intermediate energies particle, as well as magnetic field data, may be required.

Acknowledgements

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Figure Captions

Figure 1. Changes in the intensity of ~ 70 MeV interplanetary protons over ~ 3 day periods vs. corresponding changes in the modulation parameter (after Sari (1972, 1973)). The modulation parameter is computed using alternately $K_{||}$ (real), $K_{||}$ (bet), and $K_{||}$ (dis).

Figure 2. Representative spectra for the total observed power density (P_{real}), the power density due to directional discontinuities (P_{dis}), and the power due to field fluctuations between the discontinuities (P_{bet}). The solar wind speed during this time period was ~ 400 km/sec.

Figure 3. Representative spectra for P_{real}, P_{dis} and P_{bet}. The solar wind speed during this time period was ~ 500 km/sec.

Figure 4. Representative spectra for P_{real}, P_{dis}, and P_{bet}. The solar wind speed during this time period was ~ 375 km/sec.

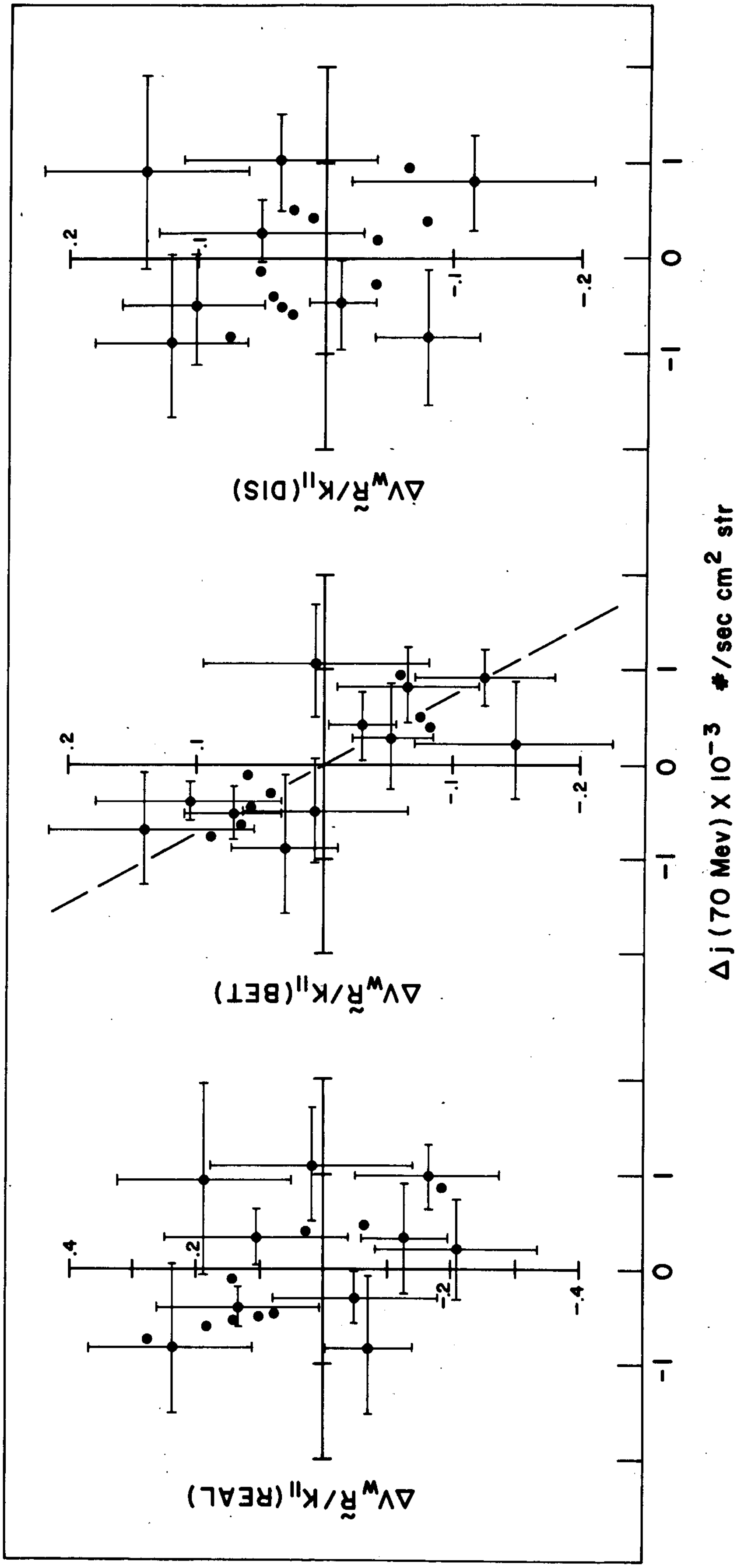

Figure 1

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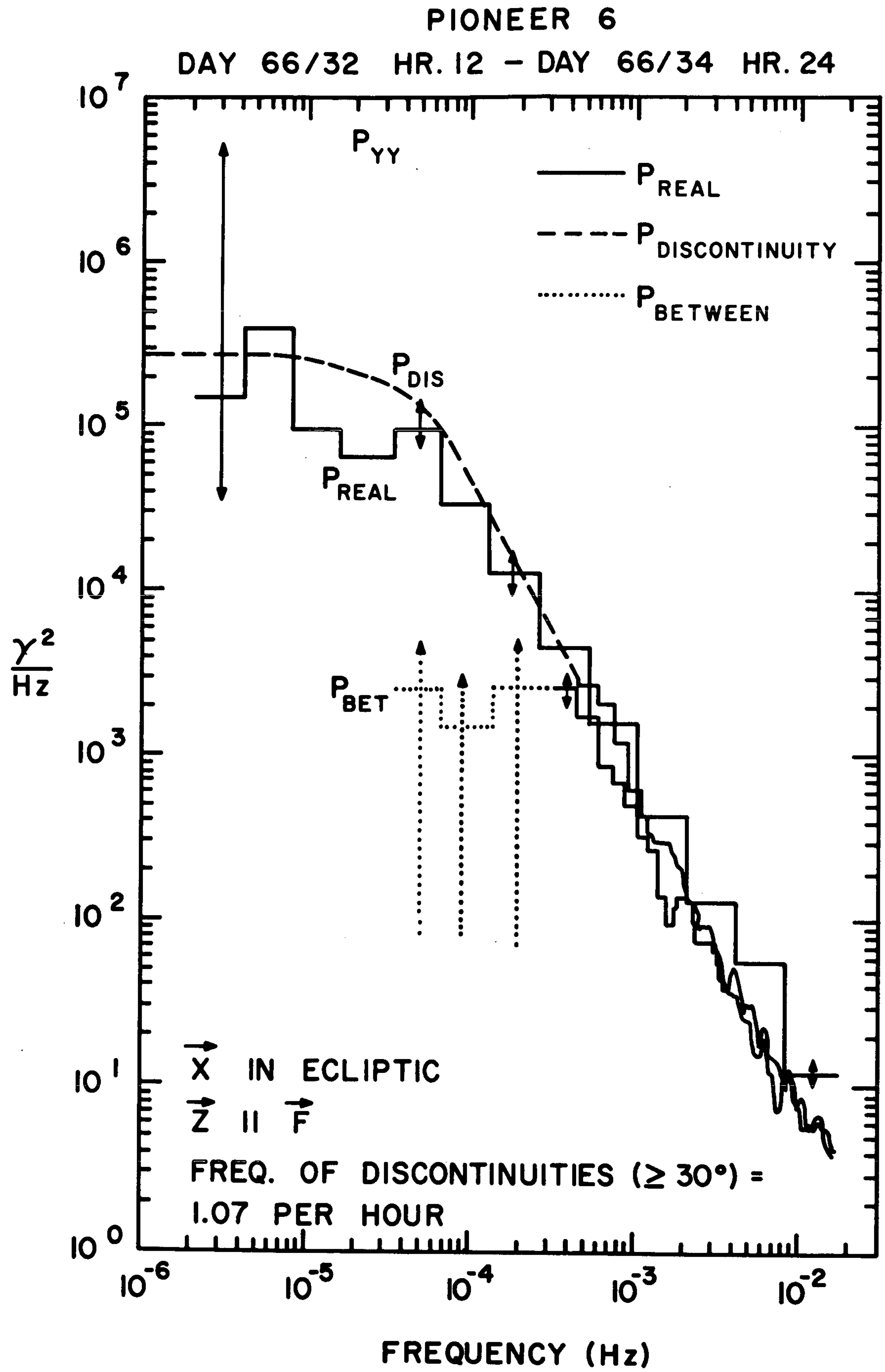
PIONEER 6
DAY 66/32 HR. 12 - DAY 66/34 HR. 24
P_{YY}
P_{REAL}
$P_{DISCONTINUITY}$
$P_{BETWEEN}$
P_{DIS}
P_{REAL}
P_{BET}
$\vec{X}$ IN ECLIPTIC
$\vec{Z}$ ll $\vec{F}$
FREQ. OF DISCONTINUITIES (≥ 30°) =
1.07 PER HOUR
$\frac{\gamma^2}{Hz}$
FREQUENCY (Hz)

Figure 2

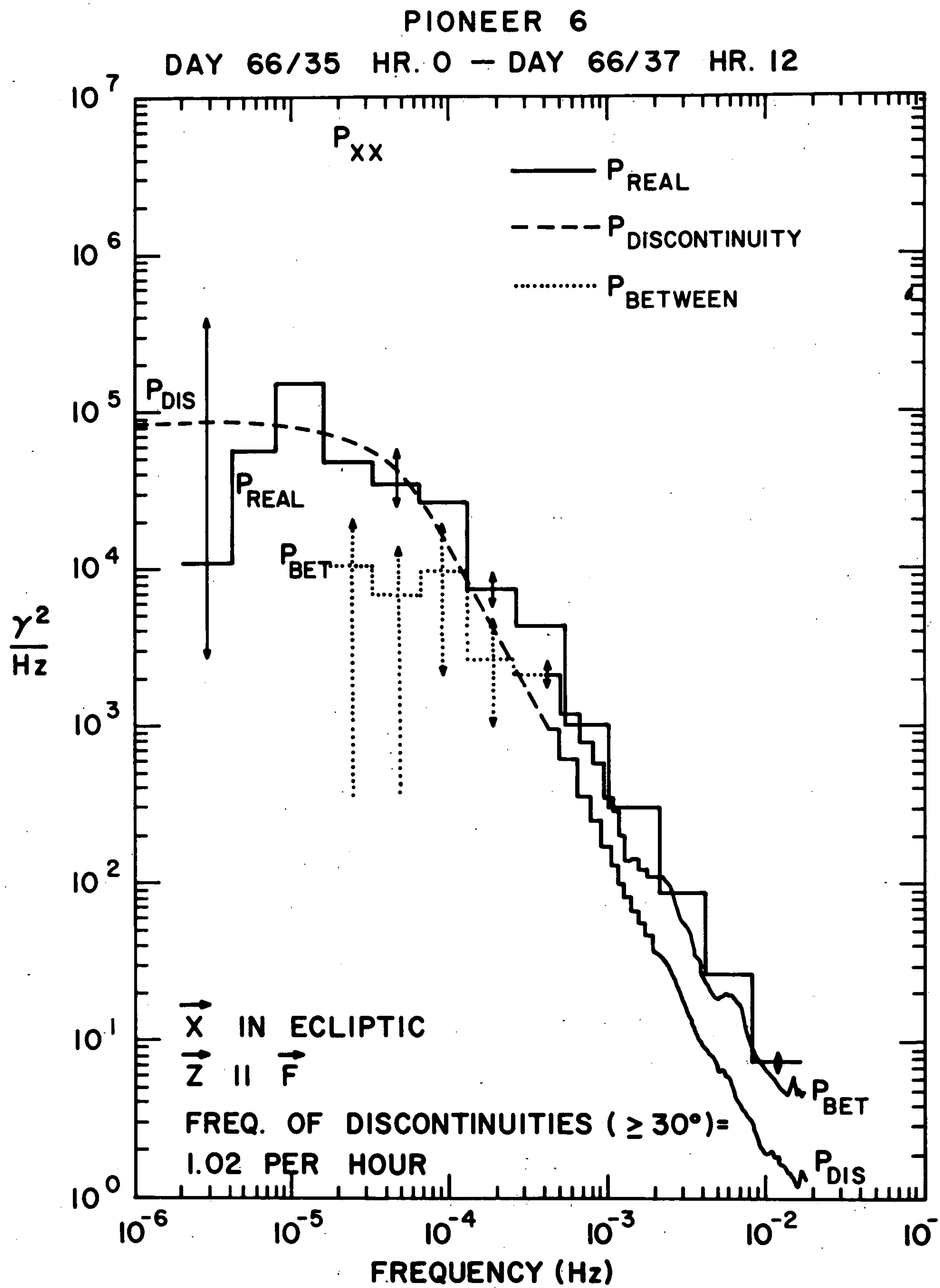
PIONEER 6
DAY 66/35 HR. 0 — DAY 66/37 HR. 12
P_{XX}
P_{REAL}
$P_{DISCONTINUITY}$
$P_{BETWEEN}$
P_{DIS}
P_{REAL}
P_{BET}
P_{BET}
P_{DIS}
$\vec{X}$ IN ECLIPTIC
$\vec{Z}$ II $\vec{F}$
FREQ. OF DISCONTINUITIES (≥ 30°)=
1.02 PER HOUR
$\frac{\gamma^2}{Hz}$
FREQUENCY (Hz)

Figure 3

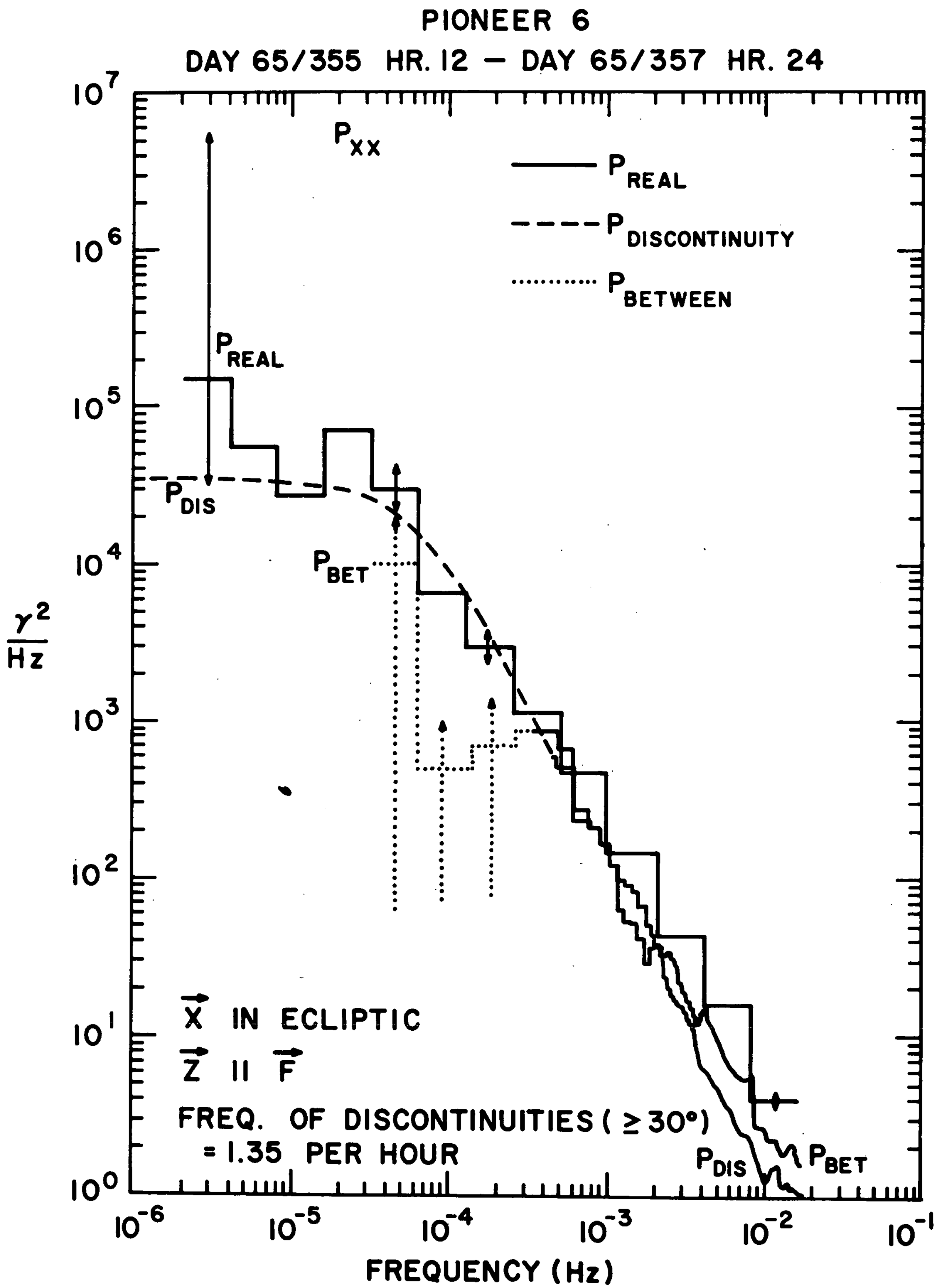

Figure4

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