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NTIS HC $4.25

X-553-72-472

PREPRINT

~~NASA TM X-~~ 66147

THE ACCURACY OF GEOPOTENTIAL SOLUTIONS FROM RESONANT SATELLITE DATA

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DECEMBER 1972

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GODDARD SPACE FLIGHT CENTER

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ABSTRACT

Tracking data from a significant number of strongly resonant satellites have not yet been incorporated into recent comprehensive geopotential solutions. This data furnishes excellent comparative and absolute tests of these solutions for resonant coefficients of order (m) 2, 3, 4, 9 and 14. Tracking arcs of from 1 month to 6 years are examined on seven satellites of 1 rev/day, three of 2 revs/day, and one each of 9 and 14 revs/day. Current values for these fully normalized resonant coefficients as judged by this independent and sensitive data, range in accuracy from 0.02×10^{-6} to 0.05×10^{-6}. This represents an increase in accuracy by a factor from 3 to 5 over solutions current in the mid 1960's.

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ACKNOWLEDGMENT

Chris Massey, of Computing and Software, Inc., was responsible for setting up and executing the many computer runs in this study. Her able assistance is gratefully acknowledged.

THE ACCURACY OF GEOPOTENTIAL SOLUTIONS FROM RESONANT SATELLITE DATA

INTRODUCTION

The National Satellite Geodesy Program (NGSP) has as one of its goals, the determination of the geopotential to degree and order 15. The accuracy with which this should be accomplished was not specified. However, a widely quoted error goal[1] of no more than 0.2 mgal at the earth's surface per spherical harmonic coefficient, would provide about 3 mgals error over the full set through 15th degree. These requirements are more severe for the high degree potential coefficients, because of the greater sensitivity of gravity anomalies to the short wavelength terms. For example, with the above error budget, the accuracy requirements range from 20×10^{-8} for 2nd degree to only 1.4×10^{-8} for 15th degree terms.[1] How severe is the total error budget? A simple calculation using Kaula's rule for the normalized potential coefficients (10^{-5}/degree^2) shows that the root mean square anomaly for this full set is only about 18 mgals. To leave an error of 3 mgals implies an overall determination to an accuracy of about 20%, which is quite modest. Yet comparisons of recent satellite-gravimetry combination models among themselves and with unused surface data show discrepancies of 8 mgals on average.[2],[3] In terms of potential coefficients (fully normalized), these comparisons show differences[4] of the order of 5×10^{-8} (rms) which meets the accuracy requirement for only as far as 5th degree harmonics. It is clear that overall, the determination of the (15,15) field is still far from adequate. Nevertheless, it is reasonable to enquire whether some parts of the field are better determined than others compared to the apparent overall average accuracy figure of 5×10^{-8}. But there are (in general) only two classes of harmonic terms for which we can make absolute accuracy judgments. These are the zonal harmonics which alone have secular effects on all satellite orbits, and the resonant harmonics (of specific orders) which have characteristic long period effects on orbits commensurate with the earth's rotation. A previous study by the author[5] has shown that the individual zonal harmonics of degree less than 11 are now known individually to an accuracy of better than 2×10^{-8}. This satisfies the stated accuracy requirements for the NGSP. The present study examines the absolute accuracy of certain well observed resonant harmonics on orbits and data which have not yet been incorporated into geopotential solutions.

The orbits examined are of 1, 2, 9 and 14 revolutions per day. The highest altitude orbits of these are dominated by resonance with the (2, 2) harmonic, though the effect of other higher degree and order terms are observable. The lower altitude 9 and 14 revs/day orbits have no single dominant resonant harmonic.

The geopotential fields evaluated in this study are representative of the best satellite and combination fields produced in the last 6 years using a variety and diversity of satellites, data types and solution methods. Their characteristics are summarized in table 1. The SAO SE 1 is the M1 field of the 1st Smithsonian Standard Earth.[6] It is complete through degree and order 8 with selected higher degree terms to (16, 14). This field includes the strong 9th order-resonance from Telstar 1, but only its influence on (15, 9). The APL 5.0 (1967) is perhaps the best doppler only field yet published.[7] It is nearly complete through (12, 12) with selected higher degree resonant terms (none of 9th order). The SAO SE2 field is the 2nd Smithsonian Standard Earth,[2] complete through (16, 16) with selected higher degree terms to (22, 14). However it does not contain data from the strong 9th order resonant Telstar 1 orbit. The (Goddard Earth Models) GEM 3 and 4 fields do contain this orbit information.[8] GEM 3 is complete through (12, 12) and GEM 4 through (16, 16), both with selected higher degree terms to (22, 14).

DATA PROCESSING

24 Hour Satellites

The data employed in this field accuracy study are mainly sets of long arcs of Kepler mean elements for the resonant satellites, determined by various agencies from different kinds of "raw" tracking data. Arc spans range from 1 month to over 6 years. The simple idea of the study was to see how much of the total resonant effect in this data could be explained by the 5 chosen geopotential fields. The unexplained amount, expressed as a percentage of the total, should be a direct measure of the error in the set of resonant coefficients for that field. For the deeply resonant (librating) 24 hour satellites, most of the arcs have nearly stationary ground tracks. Many of the best of these allow precise accelerations to be calculated by fitting the semimajor axis and longitude data to a model which adjusts the (2, 2) harmonic by a "least squares" process. The model trajectories are calculated by numerical integration of the LaGrange planetary equations for mean elements.[9] Subsequently, the longitude accelerations are calculated from the adjusted (2, 2) values by a formula for orbits with stationary ground tracks.[10] It should be noted that this formula shows that (2, 2) accounts for about 80% of the resonant acceleration on the 24 hour satellite. The longitude (λ) is defined as $(M + \omega)/n + N\text{-}\theta_e$, where M is the orbits mean anomaly, ω is its argument of perigee, n its mean motion integer in revolutions/day, N is its right ascension of the ascending node and θ_e is the hour angle of Greenwich.

The precisely measured accelerations were compared to values computed by the formula using the 5 fields. The results are given in table 2 and displayed in figure 1. The measured data on Skynet (from R. H. Merson) is from radar range and angle tracking using a numerical program which adjusts (by least squares) the (2, 2) harmonic directly to the tracking data.[11] The ATS 3 data was derived in a similar way directly from radar range and range rate data.[12]

Where the orbits are not sufficiently stationary, the full resonance effect is taken to be the rms residual in longitude, mean anomaly or semimajor axis from a trajectory fitted by least squares to these observed elements by a model without resonant geopotential coefficients. The results of these orbit determination tests on two very long (nonstationary) arcs of SYNCOM 2 and 3 are shown in table 3. The observed longitudes for these arcs are displayed in figures 2 and

4. For SYNCOM 2, the longitude in this arc librates over an amplitude of only 10° providing a fairly local test of the field. For SYNCOM 3 on the other hand, the longitude span is worldwide. Residuals in longitude for the best fitting GEM 4 and SAO SE2 trajectories are shown in Figures 3 and 5. In both of these tests (local and worldwide) the superiority of the GEM 4 solution is evident.

<u>12 Hour Satellites</u>

The satellites INTELSAT 2F1 (1966 96A) Cosmos 41 rocket (1964 49E) and Molniya 11 (1969 35A) are all well observed deeply resonant 12 hour satellites. Calculation shows (with the formula for stationary orbits[10]) that (2, 2) accounts for about 60% of the acceleration of the 18° inclined INTELSAT 2F1 and about 65% on the near critically inclined Cosmos and Molniya orbits. The INTELSAT orbit has been observed over a full range of both longitude and argument of perigee (figure 6). On the other hand the Cosmos 41 rocket orbit, while deeply librating over almost all longitudes has had only a limited perigee sampling (figure 8). The Molniya 11 orbit is even a more limited test of the field, librating over an amplitude of only 15° (figure 10) with an even more restricted apsidal rotation. The results of orbit determinations with the 5 historic fields for these long 12 hour arcs (using all 6 Kepler mean elements to fit the trajectories) are shown in tables 4 to 6. Only the rms residuals in the 2 elements most sensitive to the resonance (semimajor axis and mean anomaly) are shown. Also shown (as in table 3) are rms along track residuals, calculated as $\Delta M \times a$ where

ΔM is the rms mean anomaly residual in radians and a is the orbits semimajor axis in kilometers. It is clear, from this statistic, that a good set of resonant coefficients is essential to maintain long term tracking accuracy on these satellites. Figures 7 and 9 give residuals in mean anomaly for the best fitting GEM 4 and SAO SE2 trajectories to the INTELSAT 2F1 and Cosmos 41 rocket data. Again, the superiority of the GEM 4 solution to the SAO SE2 (as indicated in tables 4 and 5) is evident. The results for Molniya 11 are interesting, with poorer GEM solutions, relatively. They show that for local sampling the order of superiority of these fields is unpredictable. But this was also the case with the precise accelerations on 24 hour satellites, where a few measurements were best predicted by the earlier SAO SE 1 and APL 5. 0 fields.

2-2/3 Hour (9 Revs/Day) Satellite

The satellite 1970 103B, Cosmos 382 Rocket, was found to be in a deeply resonant orbit with 9th order terms in the geopotential. When the orbit mean motion was well established, a great number of strong effects with a wide range of distinct frequencies were estimated using first order perturbations[13] (table 7). The observed semimajor axis of this orbit over 1-1/2 years since launch is shown in figure 11A. Also shown is a trajectory for this orbit with only radiation pressure, drag, zonal geopotential and luni-solar gravity effects included. Radiation pressure is a significant influence on the eccentric orbit, but the estimated resonance effects (on the order of 50 m maximum) is certainly seen even in this relatively inaccurate NORAD data. Table 8 shows the results of best fitted trajectories through all the NORAD data for the 5 test fields including their 9th order resonant coefficients. Because of the high correlation of the radiation pressure with the resonant coefficients a fixed value of C_R was used in these tests. This value was determined (without prejudice to any field) simultaneously with a clean resonant solution for (9, 9) $\longrightarrow$ (12, 9) using this data. Figure 11B shows the semimajor axis evolution from the best fitted GEM 4 and SAO SE2 trajectories. The GEM 4 solution is dramatically closer to the data. The fact that GEM 4 contains data from TELSTAR 1, a fairly strong 9th order resonant orbit, while SAO SE2 does not, may account for this result.

14 Revolutions/Day Orbit

The recent launching and close tracking of ERTS 1 has provided very accurate mean element data to test the generally well represented 14th order resonant coefficients of the historic fields. This data was supplied by Arthur Fuchs of Goddard Space Flight Center. The orbits were determined (nearly every day) mainly from accurate unified S band two way range and range rate observations. Initial estimates gave the resonance effect on the semimajor axis at a level of about 15m (table 9). The observed variation (after drag is removed) is seen to be closer to 5m (figure 12). But as table 10 (and figure 12) show, the SAO SE2 field is able to remove all but about 1/3 of the resonant variation. The residuals also show the small effects of 28th order harmonics. The poorly represented 14th order harmonics of the APL field gives a result which is actually worse than a non resonant field. This is similar to the result for SAO SE2 and APL 5.0 on the previous 9th order resonant orbit. Figure 13 shows that the inclination, while not as well determined as the semimajor axis, also shows significant 14th order resonance effects which are fairly well modeled by the SAO SE2 field.

SUMMARY OF RESULTS

Precise 24-Hour Accelerations

The GEM 3/4 resonant terms explain all but about 0.3% of the data (table 2). If this error were distributed among the resonant terms according to their dominance, a shift of the order of 0.5×10^{-8} in the (2, 2) harmonic of these fields (in normalized coefficients), would be necessary to completely explain the accelerations. These accelerations are worldwide.

Syncom 2 and 3 Long Arcs

GEM 3/4 can explain all but about 1% of the resonant effect in these arcs. A shift of from 2 to 4×10^{-8} in (2, 2) is required to explain its proportion of the error (80%). The lower number applies to the worldwide sample on Syncom 3. However the Syncom 2 sample is for a very limited libration with small acceleration. In addition, the Syncom 3 data is so poor that the "noise only" (resonant) solution is only marginally superior to the GEM solutions. Therefore, the overall 24 hour satellite results seem to imply an error of no more than 2×10^{-8} in the dominant low order coefficients.

12-Hour Satellites

In the case of INTELSAT 2F1, a thorough worldwide sample of the field, GEM 3/4 can account for all but about 0.3% of the sensitive resonant data. Here, an error of less than 0.5×10^{-8} is implied in (2, 2), because it accounts for only 60% of the total acceleration.

In the case of the Cosmos 41 rocket data, a more limited but worldwide sample, GEM 3 can account for all but about 1.0% of the resonant data variation. An error of about 2×10^{-8} in (2, 2) (which accounts for 65% of the total acceleration) is implied.

In the case of the very limited field sample on Molniya11, GEM 3/4 can account for all but about 6.5% of the resonance. This would imply a shift of 11×10^{-8} in (2, 2) if 65% of the error were assigned to it. But since this is a shallow libration and a very small sampling of the field, this local error is not unreasonable.

12- and 24-Hour Satellites--Combined Solution

The overall conclusion from the tests on these orbits is that the low order resonant constants from the recent GEM solutions [(2,2) dominating, but including (3, 1) (3, 3), (4, 2), (4, 4) and others with less certainty] are accurate as a set to better than 3×10^{-8}. In particular (2, 2) is undoubtedly known alone to better than 2×10^{-8}. As a test of this estimate, all the 12- and 24-hour data has been processed through the Rapid Orbit Analysis and Determination (ROAD) program.[9] One simultaneous adjustment of all the resonant coefficients through (6, 6) which removed almost all of the resonant effects from the data, is shown in figure 14 for (2, 2) to (4, 4). There is unfortunately considerable correlation among the coefficients in this solution so that, for example, $S_{4,2}$ though close to the APL value, appears to be quite unrealistic. However, the other coefficients seem reasonable. In fact the rms of the differences between the GEM 4 coefficients and this set, through (4, 4), excluding $S_{4,2}$ is only 2.2×10^{-8}. It does appear that the set of low order and degree resonant coefficients to (4, 4) is known to better than 3×10^{-8}. It also seems reasonable to extend this judgment to all the nonzonal coefficients through (4, 4) since they are as well observed by the ordinary geodetic satellites as these special harmonics.[1] In fact, Ron Kolenkiewicz[14] has shown from analysis of short period effects in laser residuals on the BE-C satellite that they can be removed by an average adjustment of (4, 3) of only 1×10^{-8} from the SAO SE2 values. Since the correlations involving the low order and degree coefficients in the large geodetic solutions is small,[8] it is probably safe to generalize these results and say that each nonzonal coefficient through (4, 4) is now determined to better than 3×10^{-8}.

9th Order Resonant Satellite

From table 8, the GEM solutions explain all but about 40% of the resonant data. From table 7, the resonant coefficients from (10, 9) through (14, 9) dominate about equally on the orbit. Conservatively assigning the full data error to each such coefficient results in coefficient errors of from 4 to 6×10^{-8}. It seems safe to assume that the GEM 9th order coefficients are accurate to better than 5×10^{-8}.

14th Order Resonant Satellite

The latest geopotential solutions explain all but about 35% of the observed 14th order variation. Table 9 shows that almost all of this variation should be due to (15, 14). Assigning all of the data error to (15, 14) implies the SAO SE2 value is in error by 1.2×10^{-8}. Actually, the rms of order 14 coefficient differences between SAO SE2 and GEM 4, is 2×10^{-8}. The conclusion is clear that the well observed 14th order harmonics are known, at least as a set, to better than 2×10^{-8}.

Accuracy of Mid-1960's Solutions

The SAO SE1 and APL 5.0 fields are used to judge the general accuracy of solutions in this time period. For the precise 24-hour satellite accelerations, the SAO SE1 is accurate to within about 2.7% (table 2), implying an error of about 7×10^{-8} in (2, 2). An improvement of about 3-1/2 is noted for the more recent GEM solutions. For the long Syncom 2 and 3 arcs, SAO SE1 explains all but about 4% of the data (table 3). An improvement of about 4 is noted for the most recent solutions.

For the global 12 hour satellite arcs, SAO SE1 explains all but 1.6% of the INTELSAT 2F1 data and APL 5.0 explains all but 1.7% of the Cosmos 41 rocket libration (tables 4 and 5). The most recent solutions are 3 to 5 times improved over the mid 1960's fields. The 9th order resonant term (15, 9) of the SAO SE1 explains essentially none of the resonance of the Cosmos 382 rocket arc. It is unfortunate that more 9th order effects were not included in this field because they do influence the orbits of Midas 4 and especially TELSTAR 1, used in this solution. The GEM solutions using essentially the same orbital material have achieved a satisfactory reproduction of the Cosmos 382 rocket resonance. The SAO SE2 field includes no TELSTAR 1 data and, as a result, shows a poor recovery of the Cosmos resonance. Summarizing, the GEM fields represent an improvement of about 2-1/2 over the 1966 SAO SE1 solution with respect to 9th order coefficients.

The SAO SE1 was not tested on ERTS 1, but because it is well represented by 14th order resonant orbits (table 1) it would be expected to perform as well as the more recent fields.

CONCLUSIONS

If a goal of 3 mgal total error budget is accepted for the total gravity field through (15, 15), this goal has not yet been reached by the NGSP. However, significant progress has been made towards this goal since the mid 1960's and for certain sets of coefficients the goal has apparently been reached. These sets include the zonals to at least degree 10 and the low order and degree coefficients to at least (4, 4). In addition the resonant coefficients of 14th order, with many well observed satellite orbits, have been shown to probably just satisfy this goal. It is also probable that the equally well represented resonant coefficients of 13th order satisfy the above NGSP goal. An improvement in 2nd, 3rd, 4th and 9th order resonant coefficients of from 3 to 5 is noted over fields current in the mid 1960's. Nevertheless the bulk of the coefficients between 5th and 12th degree are still not sufficiently well determined in terms of the surface anomalies they give rise to.

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Table 1

Gravity Fields used in Orbit Comparisons

FIELD	DATA USED	SOLUTION METHOD	TOTAL NUMBER OF DISTINCT SATELLITE ORBITS	NUMBER OF RESONANT SATELLITES OF ORDER:	
				9	14
SAO SE 1 (1966)	SATELLITE-OPTICAL	ANALYTIC	14	1	7
APL 5.0 (1967)	SATELLITE-DOPPLER	NUMERIC–ANALYTIC	7	0	1
SAO SE 2 (1970)	SATELLITE-OPTICAL, LASER; GRAVIMETRIC	ANALYTIC	19	1	8
GEM 3 (1972)	SATELLITE-OPTICAL, LASER, ELECTRONIC	NUMERIC	25	2	9
GEM 4 (1972)	SATELLITE-OPTICAL, LASER, ELECTRONIC: GRAVIMETRIC	NUMERIC–ANALYTIC	25	2	9

Table 2

Precise Accelerations on 24 Hour Satellites

ORBITS: a = 6.6105 e.r., e < .001, $n \approx$ 1 REV./DAY

SATELLITE	λ (DEG'S)	I (DEG'S)	$\ddot{\lambda}$ (10^{-5} RAD/ SID. DAY^2)	$\sigma(\ddot{\lambda})$ (10^{-8} RAD/ SID. DAY^2)	WEIGHTED RESIDUALS, $(O-C)/\sigma$, COMPUTED FROM: GEM 4	GEM 3	SE 2	APL	SE 1	RESONANT FIELD
SKYNET 1,1	39.58	2.17	3.0553	5.0	4.0	2.9	6.4	5.4	15.4	.3
SKYNET 1,5	45.64	1.36	2.8409	1.6	11.4	8.1	16.2	8.4	32.1	.4
SKYNET 1,7	50.04	1.21	2.5865	1.8	7.4	4.6	9.8	-0.4	15.4	.5
ATS 3, 1	314.90	0.29	-2.3140	4.0	-6.3	-4.7	-10.0	-3.0	-31.2	.5
INTELSAT 2 F4, 1	181.20	0.90	1.9139	14.8	1.4	1.8	3.1	5.6	0.5	.0
ATS 5	105.04	1.05	-0.0112	0.4	12.0	-8.8	-1.5	-108.8	154.8	1.5
INTELSAT 2 F3, 1	350.00	1.00	0.1395	2.6	-1.1	1.2	8.0	14.0	17.5	-1.9
INTELSAT 2 F3, 2	347.50	1.10	-0.0925	4.6	1.0	2.4	5.7	9.3	9.1	1.1
SYNCOM 3, 11	167.40	0.60	0.6084	5.0	0.7	1.4	6.0	16.5	1.3	-2.6
SYNCOM 2, 8	65.90	31.85	0.9763	28.4	-1.0	-1.1	-1.3	-2.4	-2.2	-1.0
SYNCOM 3, 14	158.40	2.50	-0.3834	7.2	1.1	1.5	4.2	11.9	3.5	-.2
ATS 3, 4	265.20	0.50	-0.8367	2.4	-3.8	-7.3	-9.1	-21.7	4.0	-4.7
STATISTICS:	RMS MEASUREMENTS → 2.25			RMS RESID. (10^{-8} RAD./ DAY^2) →	7.4	5.9	10.0	41.9	59.9	2.2
COMMENTS:	RMS MEASUREMENT $\sigma = 1.3 \times 10^{-8}$ R/D^2 $= [12/\Sigma(1/\sigma)^2]^{1/2}$									
	RMS RESIDUAL = $[\Sigma(\text{WEIGHTED RESIDUALS})^2/\Sigma(1/\sigma)^2]^{1/2}$			RMS RESIDUAL x 100/ RMS MEAS.	.33	.26	.44	1.9	2.7	.10

SKYNET MEASUREMENTS ARE FROM R. H. MERSON. SYNCOM, INTELSAT AND ATS 5 MEASUREMENTS WERE COMPUTED BY THE ROAD PROGRAM FROM KEPLER ELEMENT DATA. ATS 3 MEASUREMENTS WERE COMPUTED BY THE GEODYN PROGRAM FROM RADAR RANGE AND RANGE RATE DATA. THE RESONANT FIELD INCLUDES ALL RELEVANT TERMS THROUGH (5,5) AND USES DATA FROM ALL THE ABOVE SATELLITES EXCEPT ATS 3,4.

Table 3

Field Tests on Two Long 24 Hour Satellite Arcs

FIELD USED IN ORBIT DETERMINATION	ORBITS TESTED: SYNCOM 2 (DATA SPAN: 1300 DAYS, I ≈ 31°) RMS LONGITUDE RESIDUAL (DEGREES)	SYNCOM 2 RMS x 100/ NON RES. RMS	SYNCOM 2 RMS ALONG TRACK RESIDUAL (KM)	SYNCOM 3 (DATA SPAN: 1900 DAYS, I ≈ 4°) RMS LONGITUDE RESIDUAL (DEGREES)	SYNCOM 3 RMS x 100/ NON RES. RMS	SYNCOM 3 RMS ALONG TRACK RESIDUAL (KM)
GEM 4 (1972)	0.096	1.2	71	0.187	.8	138
GEM 3 (1972)	0.101	1.3	74	0.200	.8	148
SAO SE 2 (1970)	0.236	3.1	174	0.307	1.2	227
APL 5.0 (1967)	0.502	6.5	370	0.750	3.0	554
SAO SE 1 (1966)	0.574	7.4	422	0.254	1.0	188
NON RESONANT	7.75	100.0	5680	25.0	100.0	10300
RESONANT	0.040	.52	30	0.134	.5	99

COMMENTS: ONLY SEMIMAJOR AXIS AND LONGITUDE $(M + \omega + N - \theta_e)$ DATA USED IN ORBIT DETERMINATIONS. ALL RESONANT EFFECTS THROUGH (5,5) USED EXCEPT FOR NON RESONANT FIELD TEST. RESONANT FIELD HAS ADJUSTMENT FOR TERMS THROUGH (4,4).

SYNCOM 2: DATA IS FROM DOD, RANGE & RANGE RATE TRACKING.

SYNCOM 3: DATA IS FROM DOD, RANGE & RANGE RATE TRACKING (MJD 39665-40175), NASA X-Y ANGLE TRACKING (OF BEACON), MJD 40833-41580.

Table 4

Field Tests on Intelsat 2F1, 1328 Day Arc

ORBIT: a = 4.165 e.r., e = .64, I = 18°, n ≈ 2 $\frac{\text{REVS}}{\text{DAY}}$

FIELD USED IN ORBIT DETERMINATION	RMS SEMIMAJOR AXIS RESIDUAL (m)	RMS $\bar{M}$EAN ANOMALY RESIDUAL (DEGREES)	RMS M RESID. x 100/ NON RES. RMS M RESID.	RMS ALONG TRACK RESIDUAL (KM)
GEM 4 (1972)	241	0.26	0.33	120
GEM 3 (1972)	237	0.25	0.32	118
SAO SE 2 (1970)	247	0.47	0.60	217
APL 5.0 (1967)	696	1.58	2.03	730
SAO SE 1 (1966)	507	1.25	1.60	580
NON RESONANT	9650	78.00	100.00	36100
RESONANT	228	.04	0.05	17

COMMENTS: DATA SPAN: MJD 40059-41387 ORBIT DATA USED: 131 SETS OF BROUWER MEAN ELEMENTS FROM MINITRACK OBSERVATIONS OVER ABOUT ONE WEEK OF OBSERVATIONS PER SET. RADIATION PRESSURE EFFECTS INCLUDED; CR = 1.08, A/M = .1 cm^2/gm. FOR RESONANT FIELDS, ALL GEOPOTENTIAL EFFECTS ARE INCLUDED GIVING AT LEAST 0.005 OF MAXIMUM ACCELERATION ($\dot{M}$) DUE TO (2,2): FROM (2,2)→(11,2), (4,4)→(11,4), AND (6,6)→(13,6). THE RESONANT FIELD IS GEM 4 WITH ADJUSTED COEFFICIENTS FOR (2,2), (3,2), (4,2) AND (5,2).

Table 5

Field Tests on Cosmos 41 Rocket [1966 493], 2298 Day Arc

ORBIT: a = 4.16 e.r., e = .69, I = 68°, $n \approx 2 \frac{\text{REVS.}}{\text{DAY}}$

FIELD USED IN ORBIT DETERMINATION	RMS SEMIMAJOR AXIS RESIDUAL (m)	RMS MEAN ANOMALY RESIDUAL (DEGREES)	RMS M RESID. x 100/ NON RES. RMS M RESID.	RMS ALONG TRACK RESIDUAL (KM)
GEM 3 (1972)	543	1.31	1.0	607
SAO SE 2 (1970)	797	2.43	1.9	1125
APL 5.0 (1967)	1148	4.43	2.5	2051
NON RESONANT	49112	127.5	100.00	118088
RESONANT	478	.20	0.2	37

COMMENTS: DATA SPAN: MJD 39157-41455. ORBIT DATA USED: 302 SETS OF NORAD MEAN ELEMENTS FROM RADAR SKIN TRACKING. RADIATION PRESSURE AND DRAG EFFECTS INCLUDED: A/M = .1 cm^2/gm. FOR RESONANT FIELDS, ALL GEOPOTENTIAL EFFECTS ARE INCLUDED GIVING AT LEAST 0.005 OF MAXIMUM ACCELERATION ($\dot{M}$) DUE TO (2,2): FROM (2,2) → (8,2), (4,4) → (9,4), (6,6) → (15,6), (8,8) → (16,8) AND (10,10) → (14,10). THE RESONANT FIELD IS GEM 4 WITH ADJUSTED COEFFICIENTS FOR (2,2), (4,4), (6,6) AND (8,8).

Table 6

Field Tests on Molniya 11, 519 Day Arc

ORBIT: $a = 4.16$, $e = .71$, $I = 65°$, $16° \leqslant \lambda \leqslant 48°$, $n \approx 2 \frac{\text{REVS}}{\text{DAY}}$

FIELD USED IN ORBIT DETERMINATION	RMS SEMIMAJOR AXIS RESIDUAL (m)	RMS MEAN ANOMALY RESIDUAL (DEGREES)	RMS M RESID. x 100/ NON RES. M RESID.	RMS ALONG TRACK RESIDUAL (KM)
GEM 4 (1972)	429	0.92	6.1	424
GEM 3 (1972)	498	1.04	6.9	480
SAO SE 2 (1970)	444	0.75	5.0	347
APL 5.0 (1967)	442	0.68	4.5	314
SAO SE 1 (1966)	494	1.03	6.9	475
NON RESONANT	6610	15.05	100.0	6970
RESONANT	359	0.03	0.2	15

COMMENTS: DATA SPAN: MJD 40556-41075. ORBIT DATA USED: 282 SETS OF NORAD ELEMENTS. RADIATION PRESSURE AND DRAG EFFECTS INCLUDED: CR = 1.09, CD = 4.06, A/M = .1 cm^2/gm. FOR RESONANT FIELDS, ALL EFFECTS ARE INCLUDED GIVING AT LEAST 0.005 OF MAXIMUM ACCELERATION ($\dot{M}$) DUE TO (2,2): FROM (2,2)→(12,2), (4,4)→(10,4), (6,6)→(8,6) AND (8,8). RESONANT FIELD IS GEM 4 WITH ADJUSTED COEFFICIENTS FOR (2,2), (4,4) AND (6,6). FAIRLY HIGH CORRELATIONS EXIST BETWEEN RESONANT, DRAG AND RADIATION COEFFICIENTS.

Table 7

Estimated Resonance Effects on Cosmos 382 Rocket

ORBIT: $a = 1.52313$ e.r., $e = .18$, $i = 51.5°$, $n \approx 9$ REVS./DAY
GRAVITY FIELD: $\bar{J}_{\ell,m} = 1.4 \times 10^{5}/\ell^{2}$: ONLY EFFECTS OVER 01° IN MEAN ANOMALY LISTED

TERM (ℓ, m, p, q)	BEAT PERIOD (DAYS)	PERTURBATION AMPLITUDES	
		SEMIMAJOR AXIS (meters)	MEAN ANOMALY (DEGREES)
14, 9, 8, 3	-704.7	7.0	0.40
12, 9, 7, 3	-704.7	6.3	0.35
16, 9, 9, 3	-704.7	3.1	0.17
10, 9, 6, 3	-704.7	2.3	0.13
13, 9, 7, 2	579.1	24.0	1.10
11, 9, 6, 2	579.1	23.0	1.10
15, 9, 8, 2	579.1	9.0	0.42
9, 9, 5, 2	579.1	7.1	0.25
12, 9, 6, 1	205.2	27.0	0.44
10, 9, 5, 1	205.2	24.0	0.42
14, 9, 7, 1	205.2	8.4	0.13
16, 9, 8, 1	205.2	2.1	0.03
11, 9, 5, 0	124.7	32.0	0.32
9, 9, 4, 0	124.7	25.0	0.25
13, 9, 6, 0	124.7	9.8	0.10
15, 9, 7, 0	124.7	2.5	0.02
10, 9, 4, -1	87.5	17.0	0.12
12, 9, 5, -1	87.5	6.8	0.05
14, 9, 6, -1	87.5	1.8	0.01
9, 9, 3, -2	68.6	3.6	0.02
11, 9, 4, -2	68.6	2.6	0.02
ROOT SUM OF SQUARES OF ALL TERMS:		73	1.5

Table 8

Field Tests on Cosmos 382 Rocket [1970 103B] Data

FIELD USED IN ORBIT DETERMINATION	RMS SEMIMAJOR AXIS RESIDUAL (m)	RMS MEAN ANOMALY RESIDUAL (DEGREES)	RMS ALONG TRACK RESIDUAL (KM)
GEM 4 (1972)	11.6	.162	27.5
GEM 3 (1972)	11.3	.158	26.8
SAO SE 2 (1970)	55.0	.690	116.5
APL 5.0 (1968)	68.5	1.460	247.0
SAO SE 1 (1966)	26.3	.438	74.2
NON RESONANT	26.4	.444	75.1
RESONANT	7.9	.023	3.9
COMMENTS:	DATA ARE MEAN KEPLER ELEMENTS FROM DOD (NORTH AMERICAN AIR DEFENSE COMMAND), MJD 40928-41355. "MEAN" OFF RESONANT BEAT PERIOD = 125 DAYS. ORBIT INCLINATION = 51.5° ECCENTRICITY = .18. MINIMUM PERIGEE HEIGHT = 1600 KM. RESONANT FIELD HAS ADJUSTED (9,9), (10,9), (11,9) AND (12,9) COEFFICIENTS (WITH SOME HIGH CORRELATIONS) AND GEM 4 (13,9)⟶(16,9) AND ZONAL COEFFICIENTS. SAO SE 1 HAS NO SIGNIFICANT RESONANT EFFECTS ON THIS ORBIT.		

Table 9

Estimated Resonance Effects on ERTS 1

ORBIT: a = 1.142, e = .0015, I = 99.1°, n ≈ 14 REVS./DAY
GRAVITY FIELD: $\bar{J}_{\ell,m} = 1.4 \times 10^{-5}/\ell^2$; ONLY EFFECTS OVER 0.1 m IN SEMIMAJOR AXIS LISTED ($\ell \leq 30$)

TERM (ℓ, m, p, q)	BEAT PERIOD (DAYS)	PERTURBATION AMPLITUDES	
		SEMIMAJOR AXIS (meters)	INCLINATION (10^{-4} DEGREES)
15, 14, 7, 0	-18.7	17.0	9.4
17, 14, 8, 0	-18.7	5.9	3.3
19, 14, 9, 0	-18.7	1.0	0.6
21, 14, 10, 0	-18.7	0.7	0.4
23, 14, 11, 0	-18.7	1.0	0.6
25, 14, 12, 0	-18.7	0.8	0.5
27, 14, 13, 0	-18.7	0.5	0.3
29, 14, 14, 0	-18.7	0.3	0.2
14, 14, 7, 1	-21.8	0.3	0.2
14, 14, 6, -1	-16.4	0.1	0.1
28, 28, 13, 0	- 9.4	0.4	0.2
30, 28, 14, 0	- 9.4	0.4	0.2
		RSS: 18.1	10.0

Table 10

Field Tests on ERTS 1 [58 day arc]

$n \approx 14$, PRIMARY BEAT PERIOD = 19 DAYS, a = 1.142, e = .0015, I = 99.1°

FIELD USED IN ORBIT DETERMINATION	RMS SEMIMAJOR AXIS RESIDUAL (m)	RMS A RESID. x 100/ NON RES. RMS A RESID.	RMS INCLINATION RESIDUAL (10^{-4} DEGREES)
GEM 4 (1972)	1.48	37	5.95
GEM 3 (1972)	1.49	37	5.95
SAO SE 2 (1970)	1.31	33	6.70
APL 5.0 (1967)	4.58	114	9.25
NON RESONANT	4.03	100	9.00
RESONANT	.30	8	—

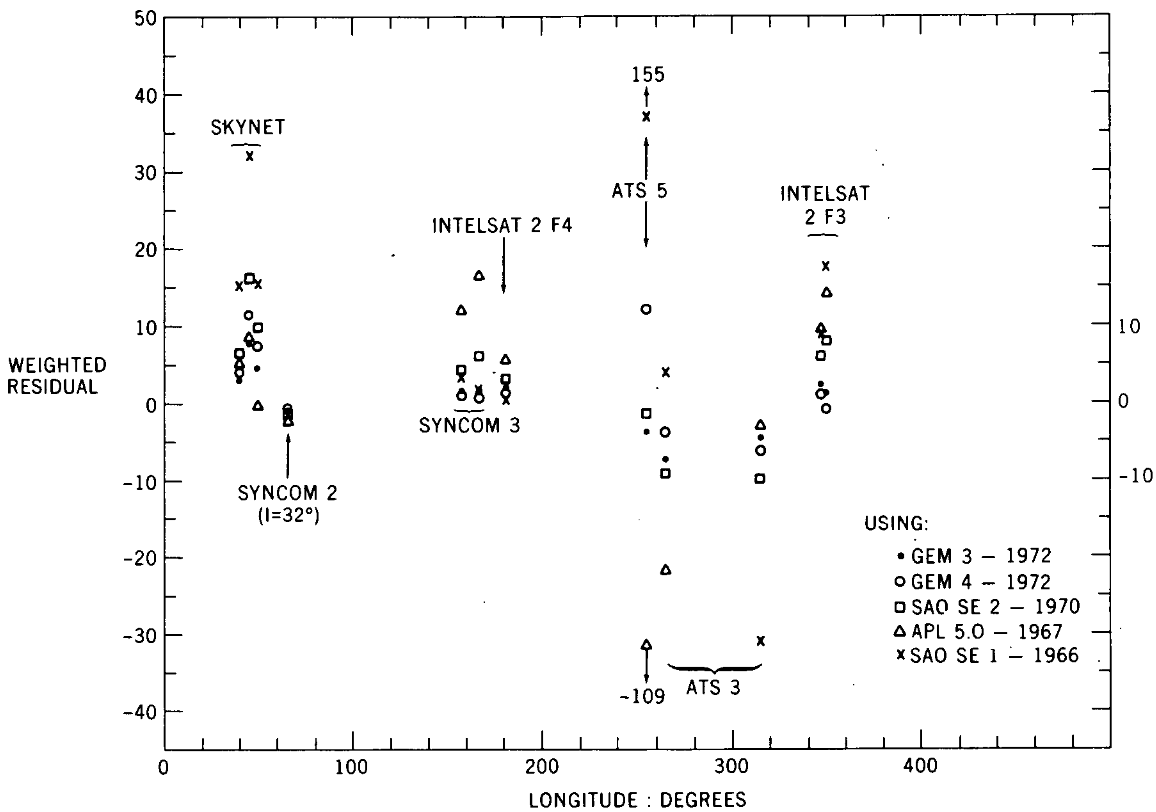

Figure 1. Weighted Residuals for Precise Accelerations on 24 Hour Satellites

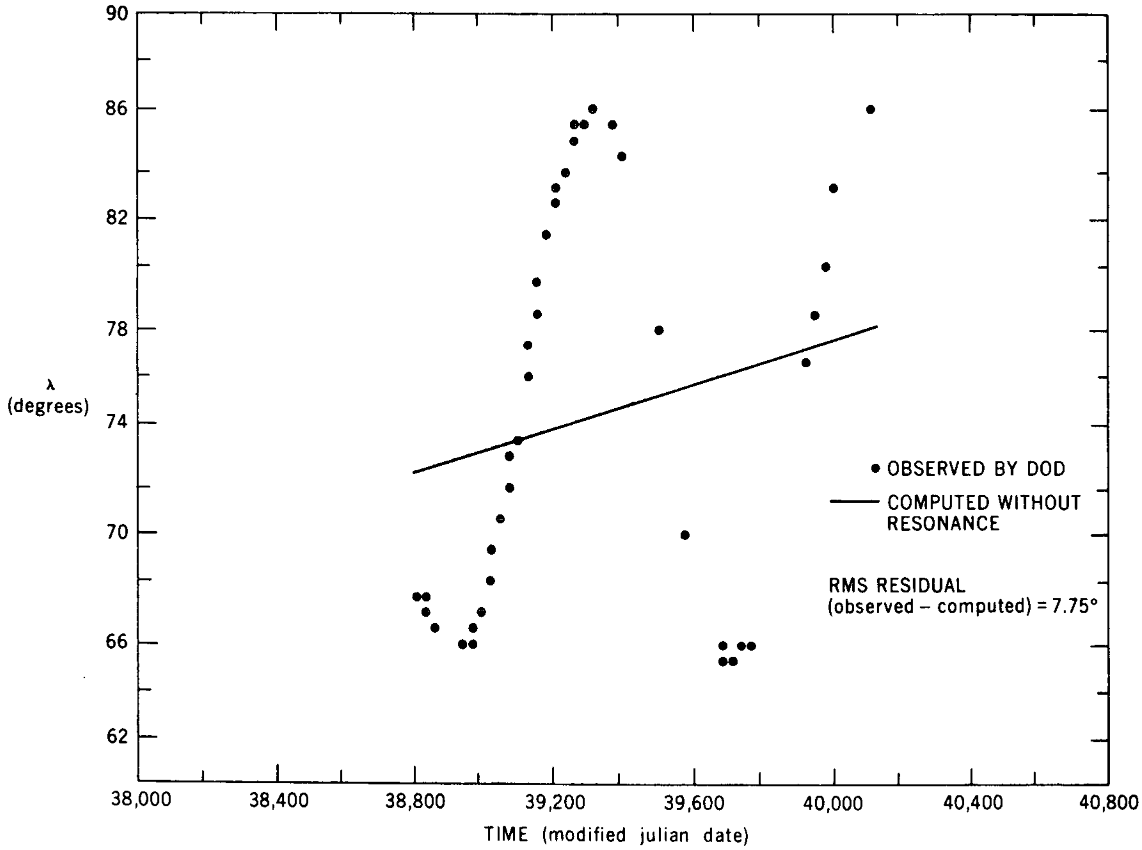

Figure 2. The Longitude of Syncom 2

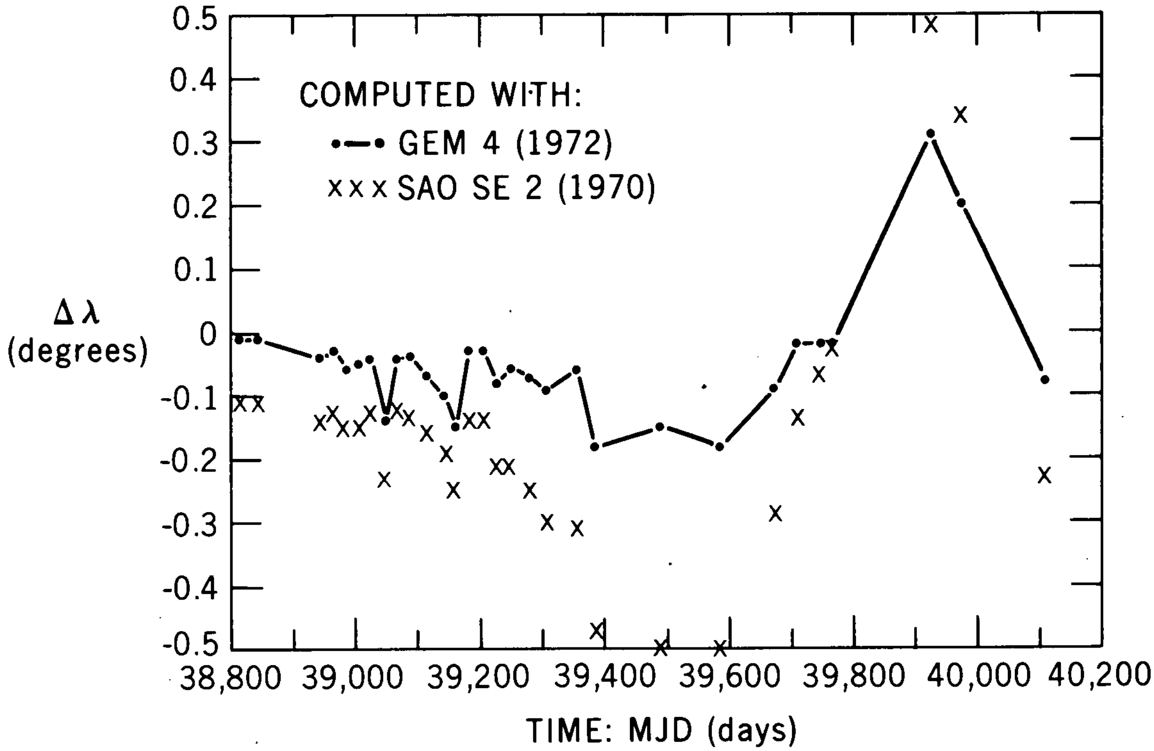

Figure 3. Longitude Residuals for Syncom 2

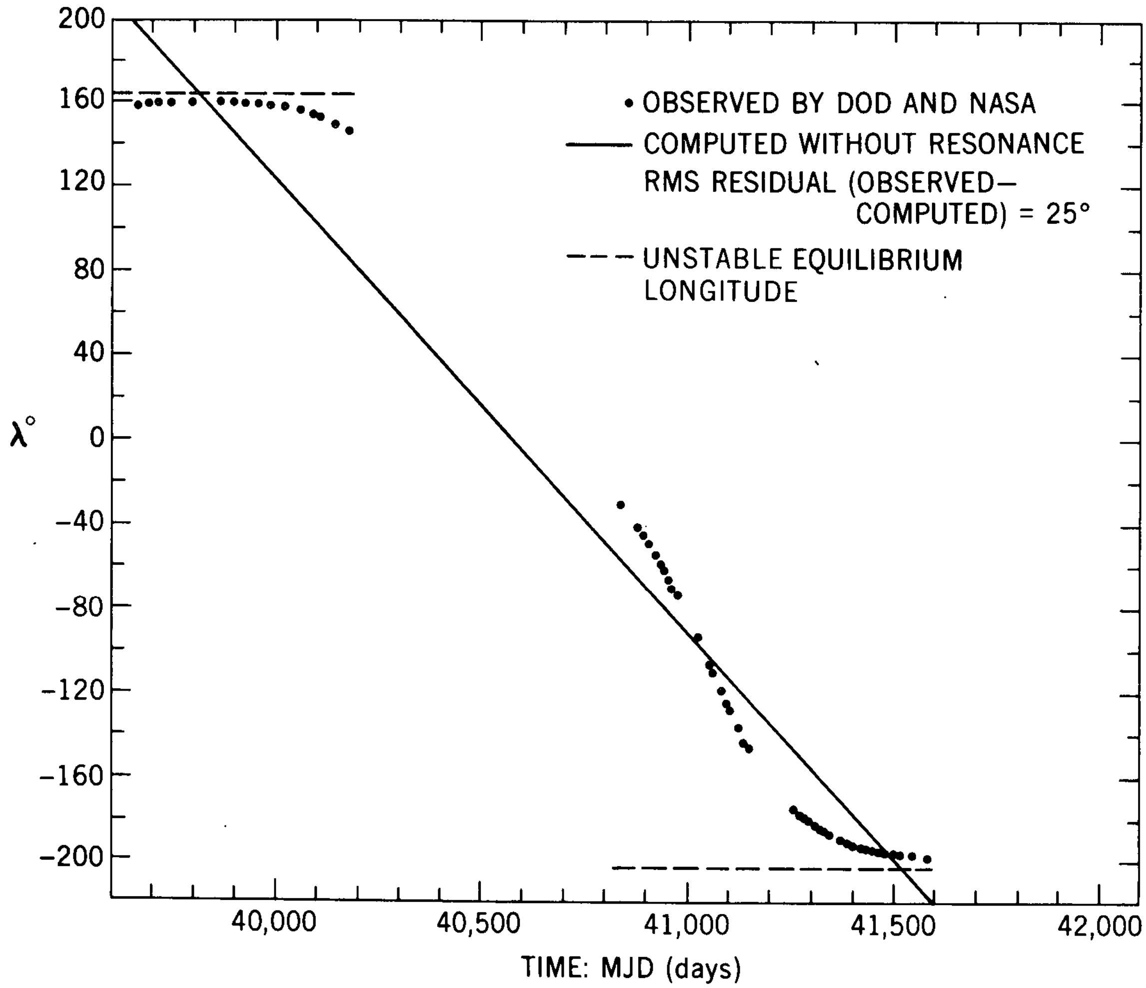

Figure 4. The Longitude of Syncom 3

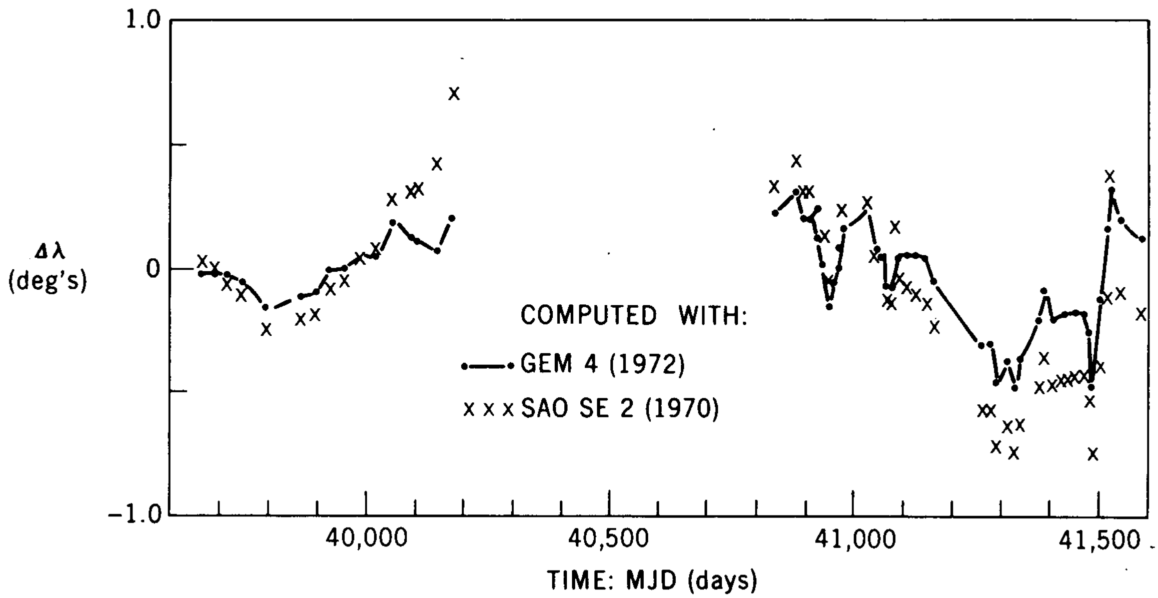

Figure 5. Longitude Residuals for Syncom 3

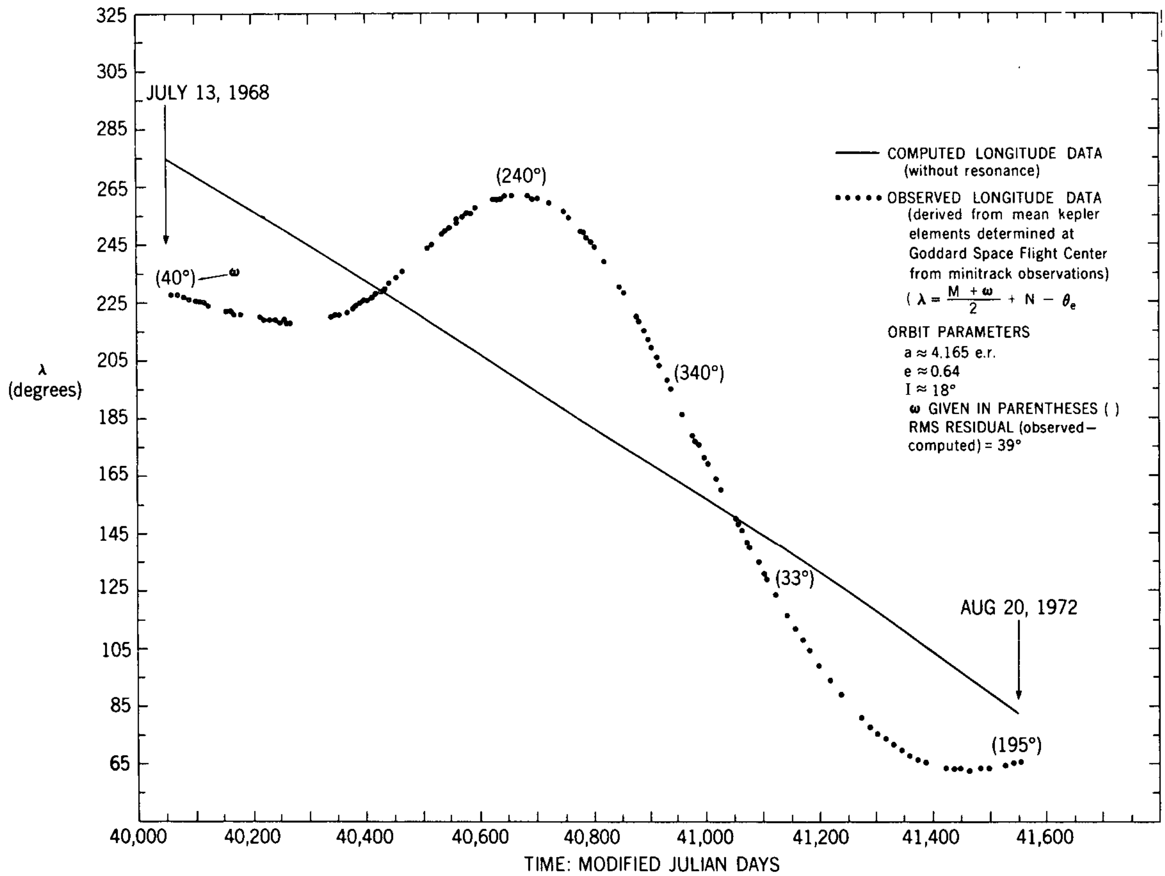

Figure 6. Mean Longitude for 1966 96 A [Intelsat 2 F1]

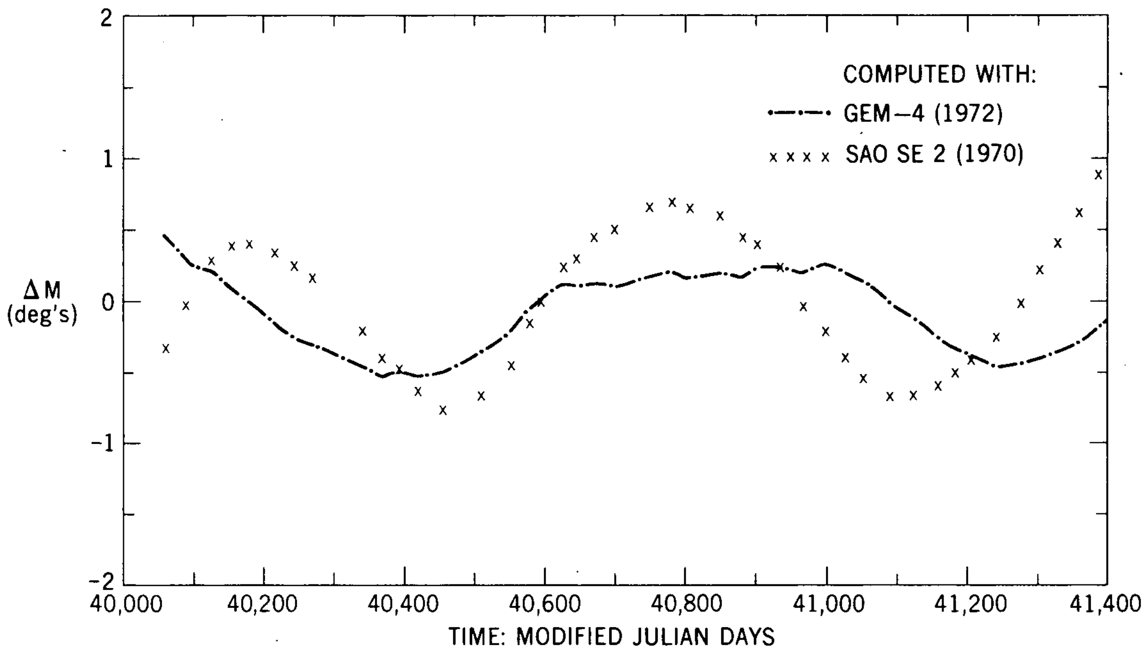

Figure 7. Mean Anomaly Residuals for Intelsat 2 F1

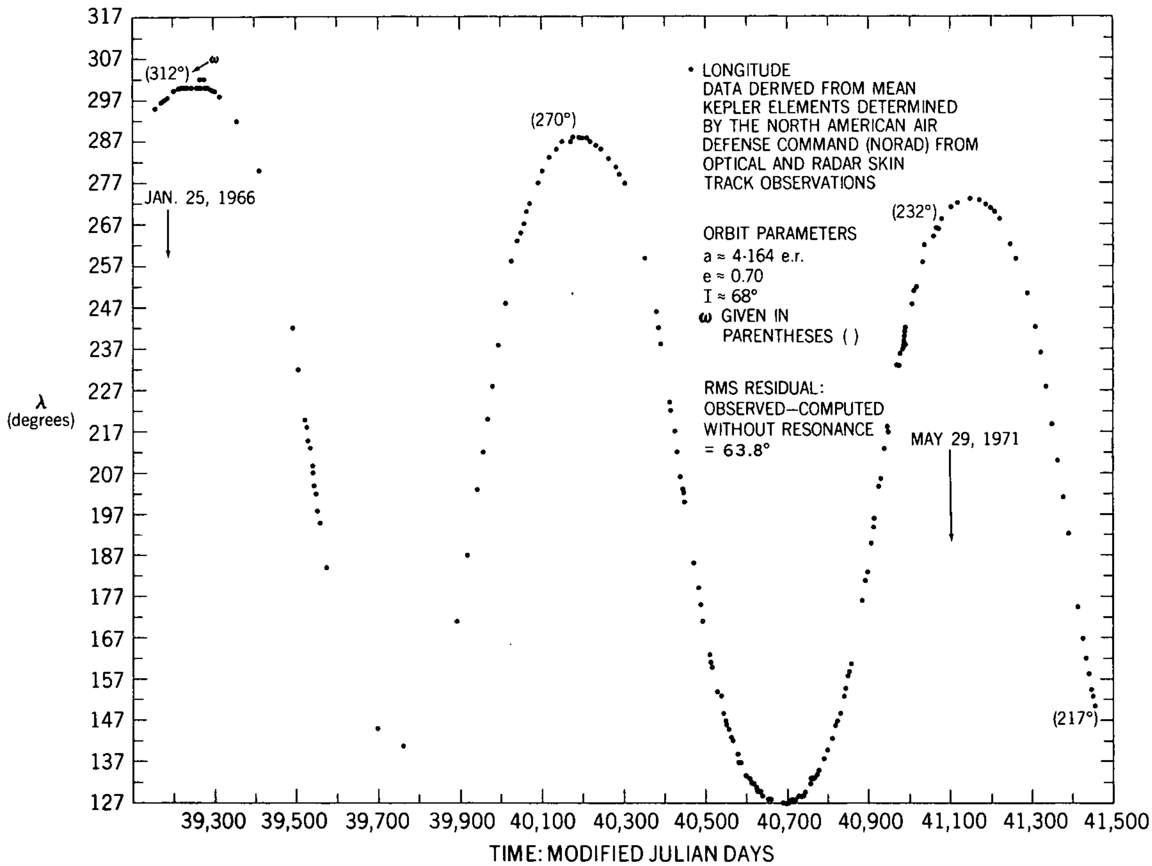

Figure 8. Mean Longitude for Cosmos 41 Rocket (1964 49 E)

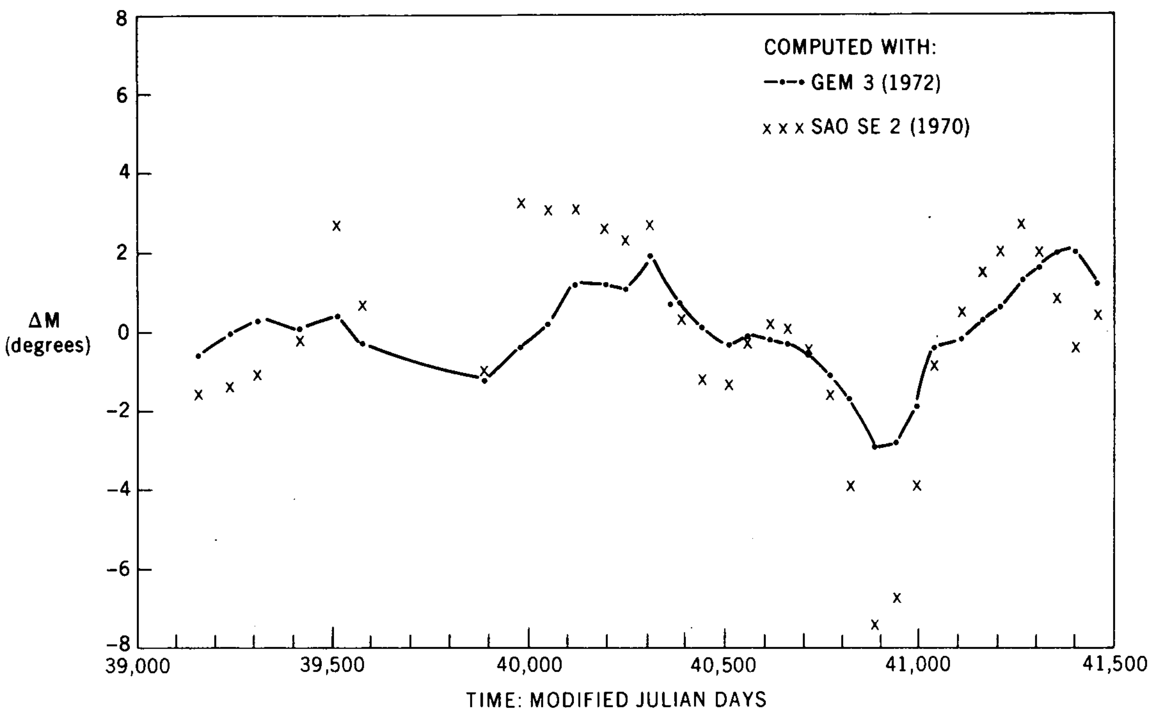

Figure 9. Mean Anomaly Residuals for Cosmos 41 Rocket

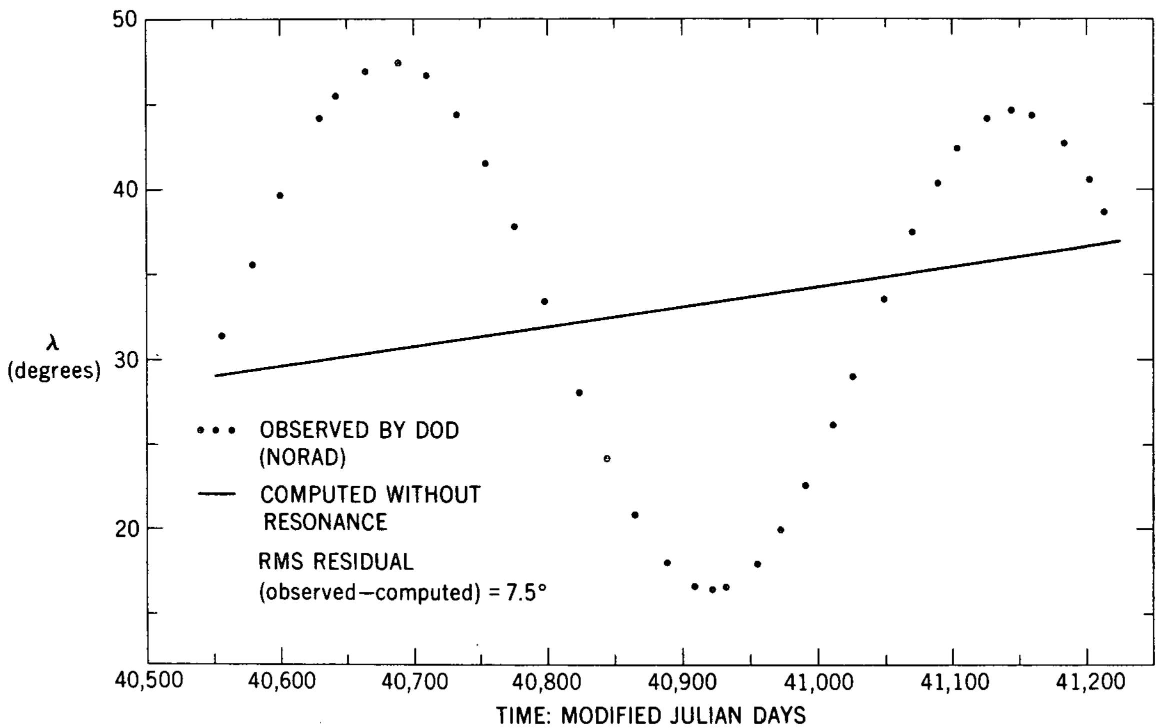

Figure 10. The Mean Longitude of Molniya 11

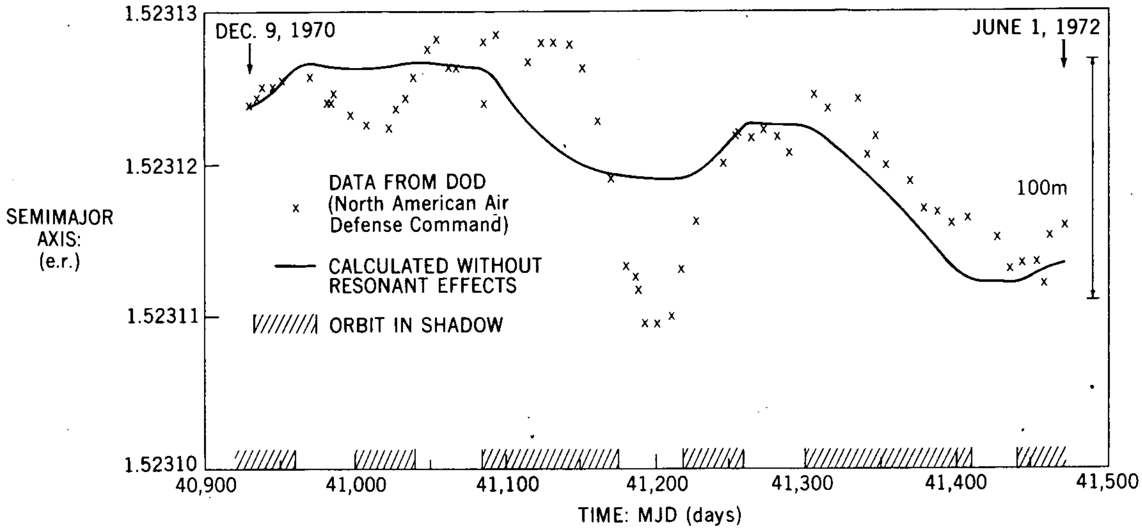

Figure 11A. The Semimajor Axis of Cosmos 382 Rocket

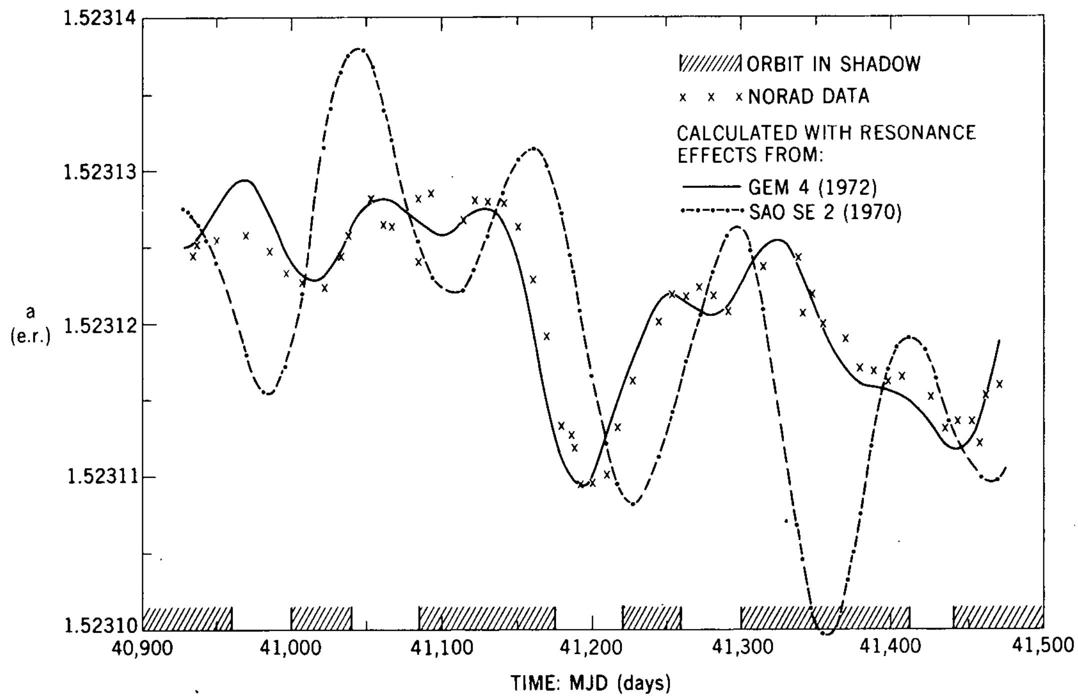

Figure 11B. The Semimajor Axis of Cosmos 382 Rocket

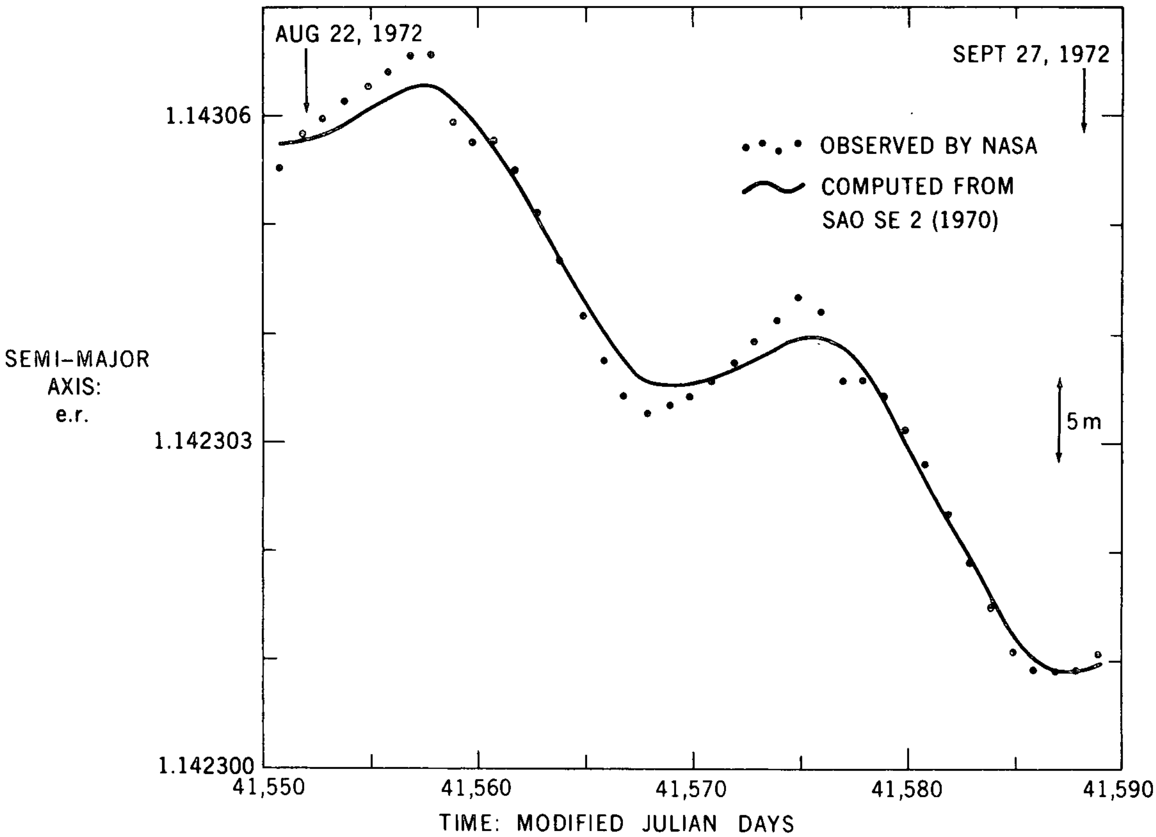

Figure 12. The Semimajor Axis of ERTS 1

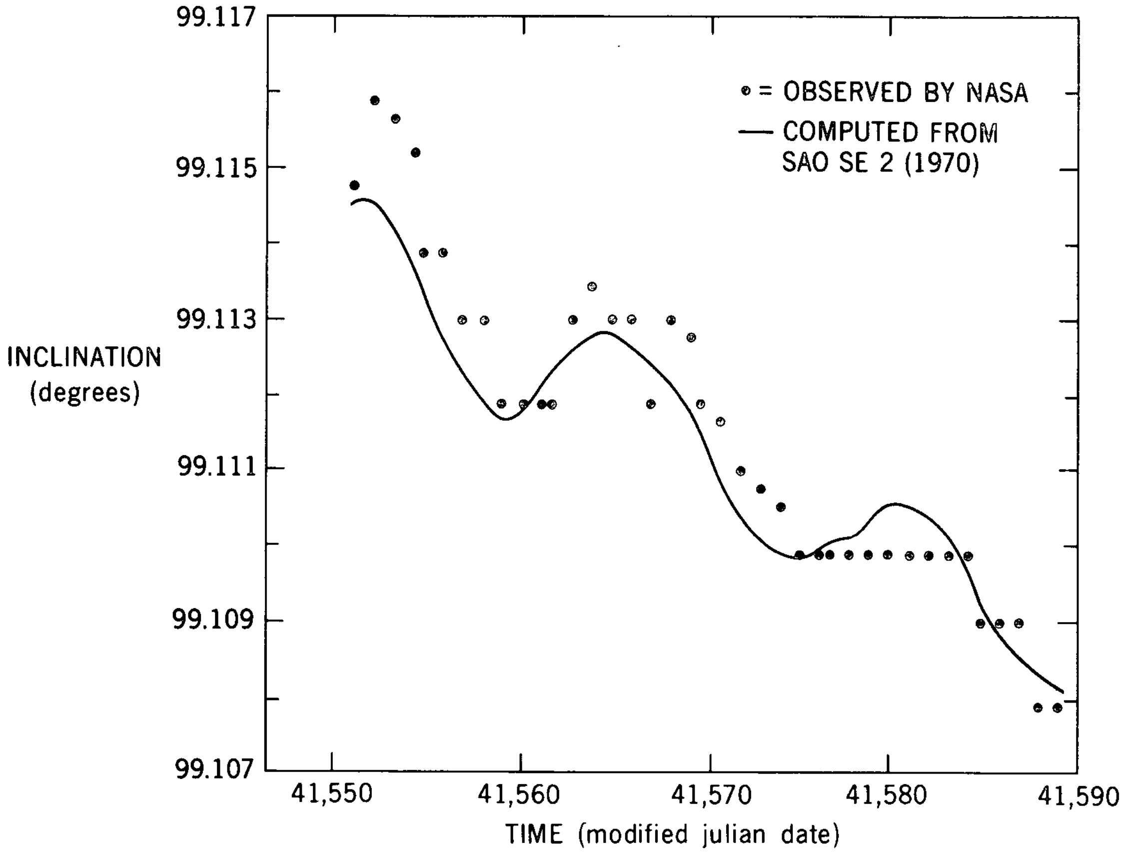

Figure 13. The Inclination of ERTS 1

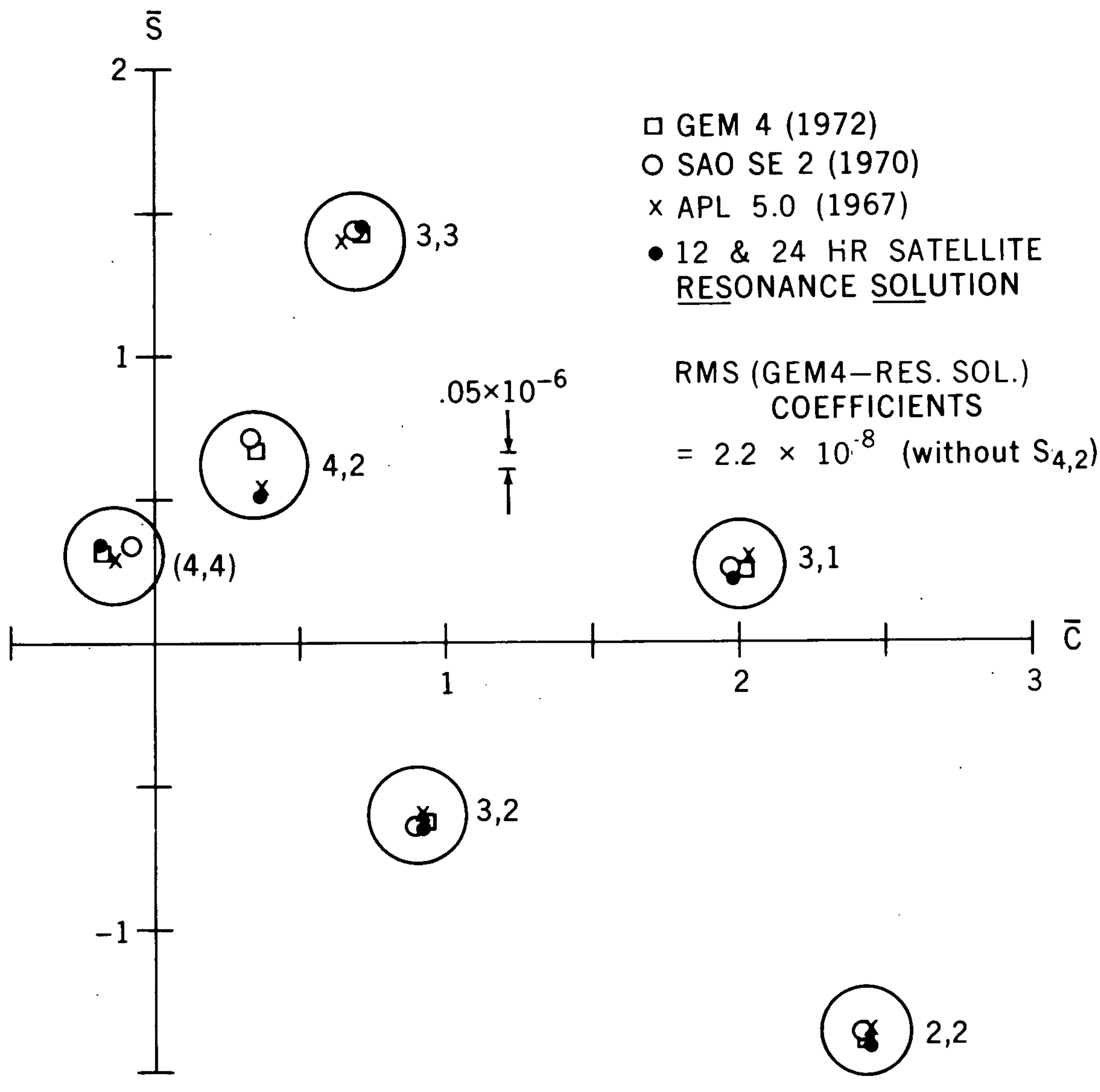

Figure 14. Normalized Gravity Coefficients from Recent Solutions [units of 10^{-6}]

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