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ABSTRACT

A Magnetic Control System for Attitude Acquisition

This report describes a spacecraft magnetic attitude acquisition system that is capable of automatically despinning a satellite from arbitrarily high rates around any axis, and provides terminal orientation that makes capture by conventional fine control attitude control systems routine. The system consists of a 3-axis magnetometer, a set of 3 orthogonal magnets, and appropriate control logic. No earth sensor is required.

Acquisition is treated in two phases. During the despin phase we are concerned with removing the tumbling motion of the satellite In this phase the performance of the system is unaffected by the presence or absence of a momentum wheel. Phase 2, that of orienting the spacecraft to the desired attitude, requires that a momentum bias wheel be present. In the terminal orientation, the axis of the momentum wheel is substantially perpendicular to the orbit plane (roll and yaw errors near zero) and the pitch rate of the satellite is at twice orbit rate.

This report describes the analysis and simulation that has been done in evaluating the performance of this system. A well-configured system will result in despin times of the order of 5 orbits per RPM for spacecraft in low earth orbits. Following despin, terminal orientation is achieved after another one to three orbits, depending on the capture range of the associated fine control system.

While this report does not describe the physical hardware, the system can be implemented inexpensively with weight less than 5 lbs. and power of about 3 watts.

W. Craig Stickler

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A MAGNETIC CONTROL SYSTEM FOR ATTITUDE ACQUISITION

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CAST OF CHARACTERS
(In Order of Appearance)

$\bar{v}$	A vector (general)
v or $\lvert\bar{v}\rvert$	Magnitude of that vector
v_i	$\underline{i}$ th component of $\bar{v}$
$\bar{\bar{I}}$	Inertia tensor
I_i	Moment of inertia about $\underline{i}$ th coordinate axis
$\bar{\tau}$	Torque
$\bar{H}$	System total angular momentum
$\bar{\omega}$	Angular velocity
$\bar{h}$	System angular momentum contribution from wheels
T	Kinetic energy
$\bar{M}$	Magnetic dipole moment
$\bar{B}$	Magnetic flux density
ω_o	Orbital rate
$\bar{e}_n$	Unit vector along the orbit normal
K, α	Constants
Δt	Time intervals
$\bar{P}_o$	Initial spin momentum (not due to wheels)
T_{sys}	A parameter
Ω	Angular velocity
δx	A "small" change in x
I_{3e}	An equivalent moment of inertia about the 3 axis. Defined by Eq. (C7)
α_1, α_2	Defined by Eq. (E10) & (E11)
A	A matrix (page E-2 & E-3)
$\bar{x}(t)$	A vector whose components are $\{\delta\omega_1, \delta\omega_2, \delta B_1, \delta B_2\}$
s	The Laplace Operator
$\bar{X}(s)$	Laplace transform of $\bar{X}(t)$
L^{-1}	Inverse Laplace transform

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INTRODUCTION

In this report we will be concerned with the description and evaluation of a magnetic acquisition/attitude control system. The work described here has been performed during the last several months, principally by the author, with the assistance and guidance of Messrs. David Sonnabend, and Robert Fowler. This work was done under contract #NAS5-21649, at the behest of the Systems Analysis Branch, GSFC, Greenbelt, Maryland. The original concept of this system was suggested by Seymor Kant, Head, Systems Analysis Branch, GSFC.

The attitude control system described here is designed to "despin" a satellite and then orient it to any preferred Earth referenced attitude. This is done thru interaction with the Earth's magnetic field. An interesting feature of the system is that it requires no Earth sensor.

In this report we will discuss the control law for despinning the satellite and the mechanism thru which some preferred attitude is attained. Then we will discuss stability considerations as they apply to this system. Next, system performance is evaluated in the light of both analytical results and simulation results. We will be concerned with both despin and attitude acquisition performance. Finally, some recommendations for further study are included in case any parties might wish to continue the work discussed here.

Appendix B displays a listing of the FORTRAN program used in the digital simulation. Appendix C reviews some of the dynamic stability theory for dual spin satellites, for convenient reference, but does so in a different fashion than is usually employed. The results obtained are, of course, the same as usual. In Appendix D we analyze the equilibrium points of our system; the stability of these points is briefly treated in Appendix E, using some of the results of Appendix C.

The author wishes to make one further remark before continuing. The (differential) equations describing the behavior of the system discussed herein are nonlinear and otherwise intractable. This being the case, it is not possible to obtain explicit analytical solutions and recourse to simulation must be made. Much of the results presented here were obtained via a digital simulation model. A program, previously written by the author and noted in the references (1), was used to this end. A listing of this program, and an index of computer runs, may be found in the appendices. A brief description of the simulated satellite is contained in the following section.

SIMULATED SATELLITE

Most of the simulation runs dealt with a satellite with the following characteristics: (1) moments of inertia of 47.5Kg-m^2 (35 slug-ft^2), 67.9Kg-m^2 (50 slug-ft^2), and 33.9Kg-m^2 (25 slug-ft^2) about, respectively, its yaw, roll and pitch axes; (2) a pitch momentum bias of 0.944 Nt-m-sec (0.7 lbf-ft-sec); and (3) switchable control magnets of 10^4 pole-cm strength. There are three such magnets, one each along the yaw, roll and pitch axes. We again note that these are principal axes for the satellite. This satellite is in a circular low altitude orbit with a period of approximately 100 min. We note however, that these parameters of orbit and vehicle are of no special significance, but were chosen merely to implement the digital simulation.

DESCRIPTION OF PROBLEM

In general, a satellite injected into orbit will be observed to be tumbling about with a certain residual angular velocity. This is so in spite of efforts made to release it "gently". Various devices have been utilized to dissipate this residual motion. In our case, the satellite will be launched by a spin stabilized rocket and hence will be inserted into orbit with an angular velocity of roughly 100 RPM about some (known) axis. A simple mechanism (a yo-yo) will then reduce that spin to the order of 1-10 RPM. However, this rate is, in general, still so high as to prevent the basic attitude control system from locking onto some target (the Earth, sun, stars, etc.). A further spin rate reduction is required, preferably to say 0.1 RPM or less. To this end, such active devices as rate gyros, accelerometers, and magnetometers coupled to gas jet systems, and passive devices such as eddy current and magnetic hysteresis rods have been employed.

Reverting to general terms, then, there are two distinct phases of the attitude acquisition procedure. The first is despin, of which we have just spoken, and the second is an attitude orientation phase which facilitates capture by the basic control system.

EQUATIONS OF MOTION

The equations describing the rotary motion of a satellite (or any other rigid body) are most conveniently written for a set of body fixed principal axes. They are the well known Euler's Equations of Motion, with terms added to account for momentum wheels. To wit,

$$\bar{\tau} = \frac{d}{dt}^{SPACE}(\bar{H}) = \frac{d}{dt}^{BODY}(\bar{H}) + \bar{\omega} \times \bar{H} \tag{1}$$

$$\text{where } \bar{H} = \bar{\bar{I}} \cdot \bar{\omega} + \bar{h} \tag{1'}$$

or, in scalar form

$$\tau_1 = \dot{h}_1 + I_1\dot{\omega}_1 + \omega_2 h_3 - \omega_3 h_2 + \omega_2\omega_3(I_3 - I_2) \tag{1a}$$

$$\tau_2 = \dot{h}_2 + I_2\dot{\omega}_2 + \omega_3 h_1 - \omega_1 h_3 + \omega_1\omega_3(I_1 - I_3) \tag{1b}$$

$$\tau_3 = \dot{h}_3 + I_3\dot{\omega}_3 + \omega_1 h_2 - \omega_2 h_1 + \omega_1\omega_2(I_2 - I_1) \tag{1c}$$

The reader is cautioned that great care should be exercised if these axes are not right handed. As mentioned above, these axes are principal axes passing thru the center of mass. Here, the I_i are the moments of inertia about the ith coordinate axis, $\bar{\tau}$ is the applied torque, and $\bar{h}$ is the momentum contribution of the wheels.

CONTROL THEORY

As noted above, the attitude acquisition system performs two principal functions. The first is despinning the tumbling satellite, and the second consists of providing some preferred orientation. Consider the former.

The rotational kinetic energy of the satellite, not including that of any reaction wheels, is

$$T = \frac{1}{2} \sum_i I_i \omega_i^2 \qquad (3)$$

We would like to diminish this quantity.

$$\frac{d}{dt}(T) = \dot{T} = \sum_i I_i \omega_i \dot{\omega}_i = \bar{\tau} \cdot \bar{\omega} \qquad (4)$$

where $\bar{\tau}$ is the external torque acting on the satellite, and $\bar{\tau} \cdot \bar{\omega}$ is the rate at which work is done on the satellite. For a dipole $\bar{M}$ in a magnetic field $\bar{B}$, the torque exerted by the field <u>on</u> the dipole is

$$\bar{\tau} = \bar{M} \times \bar{B} \qquad (5)$$

Combining (4) & (5)

$$\dot{T} = \bar{M} \times \bar{B} \cdot \bar{\omega} = \bar{B} \times \bar{\omega} \cdot \bar{M} \qquad (6)$$

Now consider the quantity $\frac{d^{BODY}}{dt}(\bar{B})$

$$\frac{d^{SPACE}}{dt}(\bar{B}) = \frac{d^{BODY}}{dt}(\bar{B}) + \bar{\omega} \times \bar{B} \qquad (7)$$

Suppose $\frac{d^{SPACE}}{dt}(\bar{B}) = \bar{0}$. That is, we say that this term is negligible in comparison with the other two terms. Then (7) becomes

$$\frac{d^{BODY}}{dt}(\bar{B}) = \bar{B} \times \bar{\omega} \qquad (8)$$

Substituting in (6) we obtain

$$\dot{T} = \dot{\bar{B}} \cdot \bar{M} \qquad (9)$$

Assumption - Important

The desired control scheme is now clear. We must simply measure $\dot{\bar{B}}$ along any axis in the satellite, and then change the polarity of a magnet lying along that axis, keeping the sense of $\bar{M}$ opposite that of $\dot{\bar{B}}$. This insures $\dot{T} \leq 0$ and decreasing T. The limitations of such a scheme are also clear. Equation (9) is valid only when (8) is valid, that is, if $\frac{d^{SPACE}(\bar{B})}{dt}$ is truly negligible in comparison with $\bar{\omega} \times \bar{B}$. Now, $\bar{B}$ varies in inertia space as $\sin 2\omega_o t$, where ω_o is the orbital frequency. Thus, $\frac{d^{SPACE}(\bar{B})}{dt}$ is equivalent to a body rate of $2\omega_o$, and as long as $\bar{\omega}$ is, say, an order of magnitude greater than $2\omega_o$ (about 0.002 rad/sec for low altitude orbits), we may safely neglect $\frac{d^{SPACE}(\bar{B})}{dt}$.

For reasons which will become clear later, the chosen control configuration involves three magnets. These lie along the principal axes of the vehicle and are controlled by magnetometers (which measure the field component) along these same axes. The controller operates in a flip-flop manner, switching the magnets' polarity to keep the M_i opposite in sense to B_i, as measured by magnetometers on board the satellite.

TERMINAL ORIENTATION

We now consider the second phase of attitude acquisition, orientation. We wish to attain some particular attitude. As long as this preferred orientation is with respect to the Earth, any desired attitude may be attained. In this section we will confine our discussion to a satellite having only a pitch momentum bias. Other momentum wheels, if any, are not yet activated at this point. The arguments advanced below are not as rigorous as might be desired but it is felt that their physical appeal more than compensates for that.

Consider the satellite to be orbiting the Earth at angular rate ω_o, and assume also that it has been completely despun. Then it must be that the satellite interprets this situation as though it were spinning backwards at angular velocity $-2\omega_o \bar{e}_n$, where $\bar{e}_n$ is a unit vector normal to the plane of orbit (and whose sense is determined by the direction of orbit). This is because the satellite senses the Earth's magnetic field passing by it twice per orbit. Note that the trajectory yaw and roll axis components of the Earth's field vary as $\sin 2\theta$, where θ is the angle into orbit from some reference point. Since the system "senses" an angular velocity whose direction is $-\bar{e}_n$, it responds with a corrective torque along the direction $+\bar{e}_n$. The net result of such a torque must be to gradually align the system momentum bias (which properly lies along the vehicle pitch axis) with the orbit normal. In this way the proper yaw-roll attitude is attained. It is the fact that the satellite is <u>not</u> spinning at $2\omega_o$ that results in the net torque along $\bar{e}_n$ which in turn aligns the momentum bias with the orbit normal. But what if it had been spinning at $2\omega_o$ originally, or in some way reached this state. It is apparent that this could occur in only two ways. That is, the system momentum must be either parallel or antiparallel to $\bar{e}_n$. If this is not so, then a component of this momentum lies in the orbit plane and a body fixed torque, also lying in the orbit plane and of the correct magnitude would be required to force the body to rotate about $\bar{e}_n$ at $2\omega_o$. Where is this torque to come from? The author does not believe in its existence and hence concludes that the system momentum bias must lie parallel or antiparallel to $\bar{e}_n$ if it is to rotate at $2\omega_o$. Both of these situations correspond to equilibrium points. We simply note here that the antiparallel situation is not a stable condition.

Our conclusions are then that the satellite will eventually align its momentum bias axis (in our case, the pitch axis) with the orbit normal and thus provide yaw-roll attitude acquisition. If the reader is not completely convinced of this by the rather unrigorous arguments advanced here, we reassure him by noting that considerable simulation experience is completely in accord with the above analysis. A further discussion concerning the amount of momentum bias required in the above scheme and the effects of more than one momentum wheel, etc., are reserved for later sections. Refer to the section titled "System Performance" and see Appendix C.

STABILITY CONSIDERATIONS

The desired action of the control law just developed is, of course, to reduce the angular velocity of the satellite to zero. Such a condition is not, properly, an equilibrium point for our system, since no provision has been made, as of yet, for a null state for the magnets. They are always on, having one polarity or the other, and the satellite always has some angular acceleration, hence the system possesses no equilibrium points. It would be easy however, to insert deadzones in the magnet controllers. Provided these deadzones are large enough to prevent magnet activation due to the relatively small term $\frac{dSPA}{dt}(\bar{B})$ in (7) (which the satellite experiences due to its orbital motion and the rotation of the Earth), then, the condition $\omega = 0$, $\bar{B}$ = any, is an equilibrium point. If this sort of provision is not made, then it would seem that the satellite should behave in some sort of quasi-oscillatory manner, at small angular rates. That this condition would, once attained, persist, seems quite plausible from the analysis above. It is not, however, a simple matter to talk about equilibrium points and stability. The equations describing this system are quite intractable to ordinary stability analysis. For one thing, the control law is binary. Even if we replace it with a linear law (the first term of its describing function) the resulting system equations consist of six first order nonlinear differential equations; three involve quadratic combinations, the other three cubic combinations of the state variables (ω_1, ω_2, ω_3, B_1, B_2, B_3).

We should note here that if the system has either a binary control law with deadzones or a linear control law (that is, $\bar{M} = -K\dot{\bar{B}}$), then there are an infinity of proper equilibrium points. These points belong to one of four distinct classes. However, in order to spare the reader, a discussion of these points and their stability is relegated to Appendix D. Suffice it to say that some of these points (other than $\omega = 0$) can be shown, by analysis, to be neutrally stable. Further, simulation results show that in some domain some of the points are stable.

There are other problems too. We are particularly interested in investigating the existence of any so called "psuedo equilibrium points" (PEP's for brevity). Since $\frac{dSPACE(\bar{B})}{dt} \neq 0$, and there are various disturbance torques acting on our satellite, and the system equations are nonlinear, there may be points which are not properly equilibrium (stationary) points but near which the system might "hang up". Here we are talking principally about limit cycle behavior, but must also be concerned with various (as yet unknown) forcing functions.

In view of these difficulties, it was decided that digital simulation was the most suitable tool for evaluating this system. A discussion of the results of this simulation work is reserved for the next section and Appendix E.

Although the actual stability analysis is put off until appendices C and E, the conclusions and implications of those analyses are reviewed here. First, we note that, for a vehicle configured as described in the section "Simulated Satellite" (that is to say, the proposed vehicle), the terminal condition (or behavior) corresponds to a stable equilibrium point. This is, of course, most desirable. We again note that the "upside down" orientation (that is, with the pitch axis flipped over) corresponds to an unstable equilibrium state; this too is soothing. Finally we note that, for our vehicle, a spin rate of up to 27.8×10^{-3} radians/sec. about the pitch axis also may be a stable equilibrium state.

This stability conclusion neglects the fact that the direction of $\bar{B}$ in space changes as the satellite orbits the Earth. This turns out to be a saving grace. Nonetheless, for strong enough magnets, the satellite may "track" the field. During such a period it is possible that the despin rate may be reduced by as much as a factor of fifty. For this reason it might be advisable to insert a mode switch for disabling the despin system for an eighth orbit or so if it seems as though this has occured. At the end of that time $\bar{\omega}$ and $\bar{B}$ would no longer be parallel and hence the despin system will again perform properly.

SYSTEM PERFORMANCE

Having convinced the reader (hopefully) that our system works, we now attempt to answer the question - how well? As before, we shall first be concerned with the despin phase.

As the system kinetic energy $\frac{1}{2}\bar{\omega}\cdot\bar{\bar{I}}\cdot\bar{\omega}$ tends towards zero (this was assured in the section titled "Control Theory"), so must $\bar{\omega}$ and the system momentum $\bar{\bar{I}}\cdot\bar{\omega}$ (this, of course, excludes the wheels' momenta). In fact, the despin process may be looked on as a reduction of the momentum term $\bar{\bar{I}}\cdot\bar{\omega}$ to zero. In the very best case this requires a time Δt_{opt} given by

$$\Delta t_{opt} = \frac{|\bar{\bar{I}}\cdot\bar{\omega}|}{\tau_{max}} = \frac{|\bar{P}_o|}{MB} \tag{10}$$

Here $\bar{P}_o = \bar{\bar{I}}\cdot\bar{\omega}_{init}$ is the initial momentum of the body other than that due to the wheels. It is useful to consider an "efficiency factor" α,

$$\alpha = \frac{\Delta t_{opt}}{\Delta t_{actual}} \tag{11}$$

where α of course lies between zero and one. α is a function of various parameters, such as $\bar{\omega}_{init}$, the inertia configuration of the satellite, and the altitude and inclination of orbit. That α is less than one is a reflection of the fact that $\bar{M}$ is not always perpendicular to $\bar{B}$ and that $\bar{\tau}$ does not always lie opposite $\bar{\bar{I}}\cdot\bar{\omega}$.

For a given system configuration ($\bar{\bar{I}}$ & $\bar{M}$) and a given orbit and initial conditions (determining $\bar{\omega}$, $\bar{B}$) we should be able to determine α and hence the time for despin. This was the goal of much of our early simulation, in which various orbits, initial conditions and system configurations were evaluated. This simulation shows that α generally lies in the range 0.55-0.80. For the worst case, a near equatorial orbit with initial spin along the field vector, α may be as poor as 0.15-0.20.

Another way to look at the performance of this system is from an energy point of view. Consider, for simplicity, rotation about a single axis.

$$T = \frac{1}{2}I\omega^2 \tag{12}$$

$$\dot{T} = I\omega\dot{\omega} = \omega\tau \tag{13}$$

$$\therefore \quad I\dot{\omega} = \tau = \alpha MB \qquad (0<\alpha<1) \tag{14}$$

The last of these equations says that ω, and the angular momentum $I\omega$, are always decreasing. The constancy of this rate of decrease depends on the constancy of α. We are of course assuming that τ has the proper sense.

The attitude acquisition part of the problem is more difficult to analyze. If, for example h approaches 0 we will not acquire. But also if h approaches infinity we will acquire, but it will take infinite time. Also, if M approaches 0 or infinity we will not attain the proper attitude. In view of these considerations it would seem that there might be some ratio of momentum bias to magnet torque which would provide optimum performance. By this we mean both minimum time to acquire and minimum roll-yaw error after acquisition. Consider the parameter

$$T_{sys} = \frac{h}{MB} \tag{15}$$

which has the dimensions of time. It is rather difficult to say anything about this problem analytically, but the following is observed from simulation runs.

We first note that there is a certain minimum bias required for terminal attitude acquisition. It is shown in Appendix C, eq. (C10), that

$$h_o > \Omega \max(I_1, I_2) \tag{C10}$$

for us, $h_o = h + I_3\Omega$ and $\Omega = 2\omega_o$ (for the terminal condition). Thus (C10) becomes

$$h > 2\omega_o\{\max(I_1, I_2) - I_3\} \tag{16}$$

For good performance this amount should be exceeded by a factor of four or more. Little is gained by increasing h beyond this point. If we now choose a particular orbit, the time and quality of acquisition (after despin has been completed) are functions of the magnet strength M. Acquisition time is also, of course, very much a function of attitude at the conclusion of despin. A (very) qualitative picture of this situation is presented in Figure 1. As pointed out previously, for very weak or very strong magnets we do not attain the desired attitude. The author cannot caution the reader too strongly against taking this curve too literally. Much more work would be needed to get a more exact picture of this phenomena. There are too many variables and too few points on the curve of Figure 1 to let the author rest comfortably. As for the quality of roll-yaw acquisition, it is interesting to note that a curve qualitatively similar to Figure 1 may be used to describe this as well. Figure 2 depicts this situation. Note that the minimum point is shifted to about 2000 secs. (This appears to be significant) and the curve is somewhat flatter. The best that we can do appears to be about 2°. Again, the author cannot over-emphasize the tentative nature of these findings. Considerable work remains to be done in this area.

For a well proportioned system, acquisition times are of the order of one orbit. This being the case, it is expected that optimum performance will generally be defined in terms of quality of roll-yaw acquisition rather than in terms of minimum time to acquire.

B

REPRODUCIBILITY OF THE ORIGINAL COPY IS POOR.

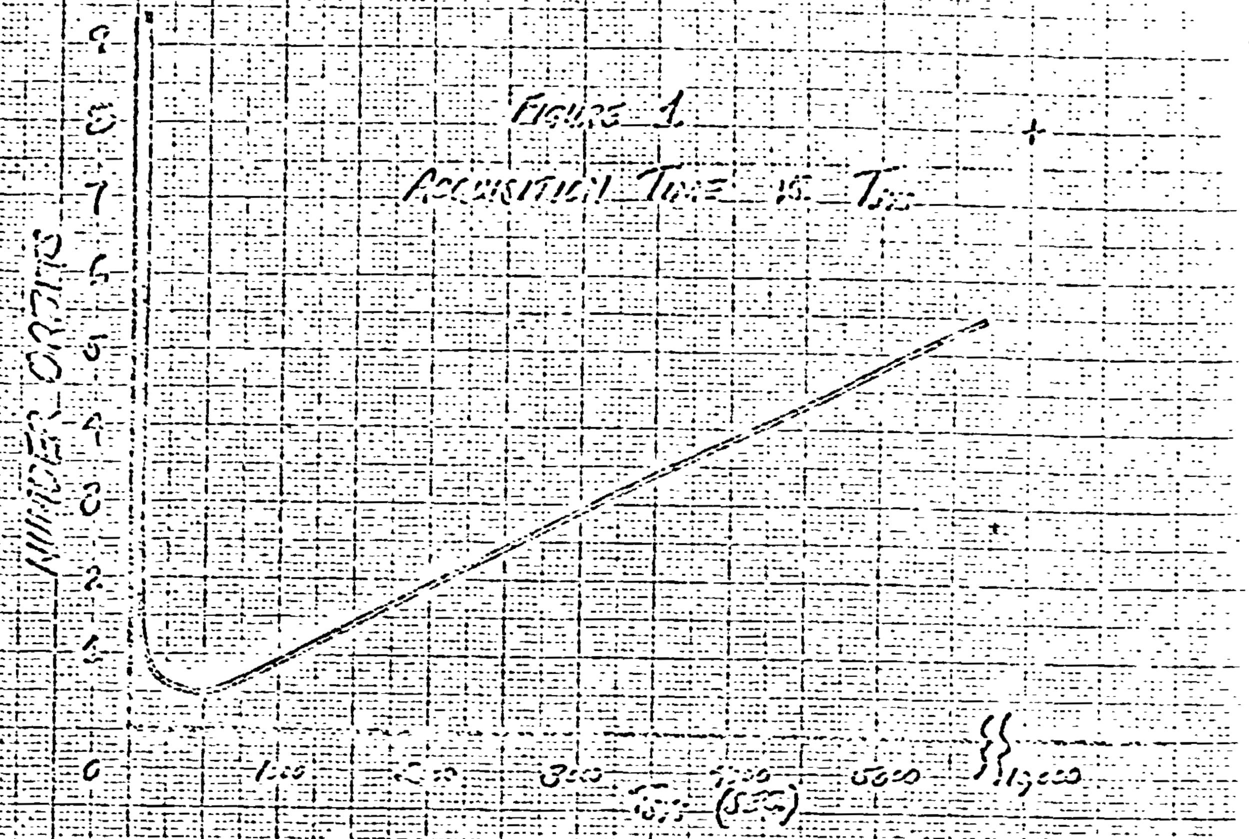

FIGURE 1
ACQUISITION TIME VS. TSYS

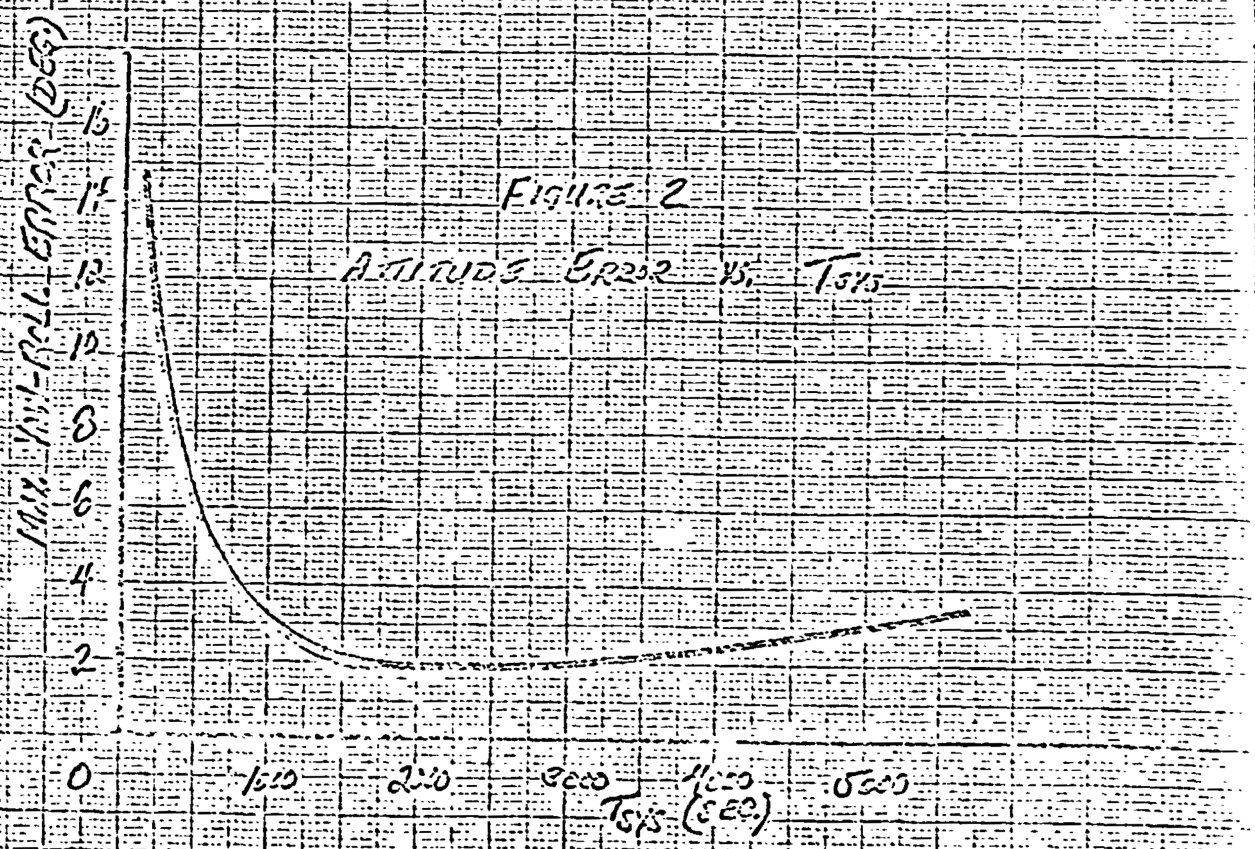

FIGURE 2
ATTITUDE ERROR VS. TSYS

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RECOMMENDATIONS FOR FURTHER WORK

The author feels that the work done to date definitely demonstrates the feasibility and desirability of this acquisition control system. Nevertheless, work remains to be done.

There is the matter of system stability. While it seems apparent that there are no stability problems which might interfere with the proper operation of the control system, nonetheless it would be good if we obtained some further analytical results. In particular, an analysis including the effects of the change in $\bar{B}$ due to orbital motion would be interesting.

As noted, we have not included the effects of disturbance torques on the performance of this system. Their effects on both limit cycle behavior and terminal attitude orientation should be investigated. Their neglect has been justified up to this point by the fact that the magnet torques are at least an order of magnitude greater than the disturbances.

Another area which needs refinement is the magnetic field model used in our simulation. So far, only the simple tilted dipole has been used. The effect of a more accurate field model on terminal attitude orientation needs to be investigated.

More investigation into the effects of deadbands, hysteresis loops, and inhibit controls placed in the magnet control loops would be desirable. There is also the matter of crosstalk between magnets and magnetometers. We must be sure that no unstable loop is set up in which switching magnets trigger magnetometers which in turn cause the magnets to switch again.

One other point which will bear further investigation is the potential of using this acquisition system as a complete attitude pointing system. Used with a pitch reaction wheel scanner and simple pitch control loop we would have complete three axis control. Some studies of pointing accuracies versus various system parameters would be useful in investigating this possibility.

BIBLIOGRAPHY

1. Stickler, A.C., "A General Purpose Digital Program for Solving Problems that Incorporates Euler Homogeneous Parameters", ITHACO Report No. 90288, July 13, 1971.

2. Stickler, A.C., "Variation of the Earth's Magnetic Field as Seen by a Satellite in Circular Orbit", ITHACO Report No. 90295, August 11, 1971.

3. Sonnabend, D., "Satellite Attitude Kinematics", ITHACO Report No. 90290, July 28, 1971.

4. Chen and Haas, Elements of Control Systems Analysis, Prentice-Hall, Englewood Cliffs, New Jersey, 1968.

5. Greenwood, D. P., Principles of Dynamics, Prentice-Hall, Englewood Cliffs, New Jersey, 1965.

6. Velman, J.R., "Attitude Dynamics of Dual Spin Satellites", Hughes Aircraft Company, Space Systems Division, Report No. SSD 60419R, September, 1966.

APPENDIX B

DIGITAL SIMULATION PROGRAM LISTING

This appendix consists of a listing of the digital computer program used to simulate control system/satellite behavior. The program is quite similar to the one discussed in reference 1, and the reader is referred there for a more detailed discussion, flowcharts, etc. Its operation may be briefly summarized as follows.

The main program performs the bookkeeping functions of input/output and maintains overall control. What we are doing here is, essentially, integrating equation (1), specialized for the vehicle described under "Simulated Satellite". This is done through subroutines WDOT and RK4. An additional problem we must deal with here is that the applied torques are functions of the vehicle's attitude. Namely, they are functions of the Earth's magnetic field components as measured in the vehicle. For this reason we must keep track of the vehicle's attitude. This is implemented through the use of Euler Homogeneous parameters in subroutines EHPQT, NORM, MAT1, and MULVEC. The Earth's magnetic field is first obtained in trajectory axis components (subroutine BFLDS1), and then transferred into body axis components, making use of the direction cosine matrix (subroutine EHPA). Subroutine TORQUE then computes the torques arising from the interaction between this field and the on board magnets. Please note that the subroutine displayed contains neither deadzones nor hysteresis loops, although some subroutines incorporating them have been used at times.

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*JOB
C   ****GENERAL PURPOSE KINEMATIC INTEGRATOR PACKAGE USES EULER            DAV20001
C   ****HOMOGENEOUS PARAMETERS IN ORDER TO SOLVE PROBLEMS INVOLVING        DAV20002
C   ****BODY RATE SPECIFIED ROTATION RATES. *******************            DAV20003
C   ****AUTHOR: ALBERT C. STICKLER                                         DAV20004
C   ***********************************************************************DAV20005
C   ****MAIN PROGRAM SKELETON.  FILL IN DETAILS, PARAMETERS, ETC.   ******DAV20006
C   ****ASSEMBLAGE INCLUDES SUBROUTINES NORM, MAT1, RK4, EHPA,             DAV20007
C   ****EHPQT, EANGLE, AND INTERP, AMONG OTHERS  **************************DAV20008
C                                                                          DAV20009
      REAL I1, I2, I3, QT(4,4), A(3,3),AXIS(3)                             DAV20010
      REAL*4 INCLIN,BT(3),BB(3)                                            DAV20011
      COMMON/RK/P(4),W(3),OT,DT,WO                                         DAV20012
      COMMON/WDUTS/ T(3), COMM(10), C1, C2, C3, C4, C5, C6                 DAV20013
      REAL TIME/0.0/,THETA/0.0/,WE/0.41784E-02/                            DAV20014
      INTEGER*4 I/-1/, IHEAD/-1/                                           DAV20015
      READ(5,90)I1,I2,I3,BIAS,FACTOR                                       DAV20016
      READ(5,90)    PRINT,BSTEP                                            DAV20017
      READ(5,90)W                                                          DAV20018
      READ(5,90)DT,TRUN                                                    DAV20019
      READ(5,90)INCLIN,EARTHR,ALT                                          DAV20020
      PERIOD=0.3151263*SQRT(0.1*ALT)**3                                    DAV20021
      WRITE(6,110) I1,I2,I3,BIAS,FACTOR                                    DAV20022
      WRITE(6,111) PRINT,BSTEP                                             DAV20023
      WRITE(6,112) DT,INCLIN,EARTHR,ALT,PERIOD                             DAV20024
      C1=(I2-I3)/I1                                                        DAV20025
      C2=(I3-I1)/I2                                                        DAV20026
      C3=(I1-I2)/I3                                                        DAV20027
      C4=1/I1                                                              DAV20028
      C5=1/I2                                                              DAV20029
      C6=1/I3                                                              DAV20030
C  READ AND WRITE INITIAL ATTITUDE                                         DAV20031
      READ(5,90)P                                                          DAV20032
      WRITE(6,91)P                                                         DAV20033
91    FORMAT(//,'  INITIAL EHPS WERE', 4F10.4)                             DAV20034
      CONST=0.001*FACTOR                                                   DAV20035
      COMM(1)=BIAS*C4                                                      DAV20036
      COMM(2)=BIAS*C5                                                      DAV20037
      IBSTEP=BSTEP/360.0*PERIOD/DT                                         DAV20038
      IPRINT=PRINT                                                         DAV20039
      DTHETA=360.0/PERIOD*DT                                               DAV20040
      CALL BFLDS1(INCLIN,ALT,BT)                                           DAV20041
      CALL BFLDS2(EARTHR,THETA,BT)                                         DAV20042
      WO=6 2832/PERIOD                                                     DAV20043
```

```
34          DTHAF=DT*0.5                                                          DAV20044
35       50 CALL NORM(P,PMAG)                                                     DAV20045
36          I=I+1                                                                 DAV20046
37          IF(I/IPRINT*IPRINT .NE. I) GO TO 75                                   DAV20047
38          ERROR=PMAG-1.0                                                        DAV20048
39          ENERGY=I1*W(1)*W(1)+I2*W(2)*W(2)+I3*W(3)*W(3)                         DAV20049
40          IHEAD=IHEAD+1                                                         DAV20050
41          IF(IHEAD/50*50 .EQ. IHEAD) WRITE(6,101)                               DAV20051
42          WRITE(6,100) TIME,W,BB,ERROR,ENERGY,A(1,3),A(2,3)                     DAV20052
43       75 TIME=TIME+DT                                                          DAV20053
44          CALL EHPA(P,A)                                                        DAV20054
45          THETA=THETA+DTHETA                                                    DAV20055
46          IF(I/IBSTEP*IBSTEP .EQ. I)CALL BFLDS2(EARTHR+WE*TIME,THETA,BT)        DAV20056
47          IF(ENERGY .LT. 0.01) CALL BFLDS3(THETA,BT)                            DAV20057
48          CALL MULVEC(A,BT,BB)                                                  DAV20058
49          CALL TORQUE(T,BB,CONST)                                               DAV20059
50          CALL RK4                                                              DAV20060
51          IF(TIME .GT. TRUN) STOP                                               DAV20061
52          GO TO 50                                                              DAV20062
53       90 FORMAT(5F10.3)                                                        DAV20063
54      100 FORMAT(1H ,F6.0, 2X,3(E10.3,1X),3(F6.3,1X), 2E15.5,2F7.3)             DAV20064
55      101 FORMAT(1H1,' TIME        OMEGA1        OMEGA2        OMEGA3      B1   BDAV20065
         &2       B3          ERROR             ENERGY     +ROLL   NEG. YAW')     DAV20066
56     1100FORMAT(/////,50X,'SYSTEM PARAMETERS ARE',//,60X,'INERTIA(1)=',         DAV20067
         1    F10.2,//,60X,'INERTIA(2)=',F10.2,//,60X,'INERTIA(3)=',F10.2,        DAV20068
         2.   //,60X,'MOMENTUM BIAS=',F10.3,//,60X,'MAGNET STRENGTH FACTOR=',DAV20069
         3    F10.2)                                                              DAV20070
57     1110FORMAT(/////,50X,'INTERVALS ARE',//,60X,'PRINT EVERY',F10.0,           DAV20071
         1    '    TIME STEPS',//,25X,'CALCULATE MAGNETIC FIELD EVERY',F10.2,     DAV20072
         2    '    DEGREES OF ORBIT')                                             DAV20073
58     1120FORMAT(/////,10X,'INTEGRATION STEP SIZE=',F5.2,'    SECONDS,-- RUNGDAV20074
         1E-KUTTA FOURTH ORDER INTEGRATION WITH EULER HOMOGENEOUS PARAMETERSDAV20075
         2',////,50X,'ORBIT SPECIFICATIONS',//,25X,'INCLINATION ANG.=',F10.2DAV20076
         3    ,'   DEGREES',//,25X,'ROTATION OF EARTH MINUS LONGITUDE OF THE    DAV20077
         4    NODES(INITIAL)=',F10.2,'    DEGREES',//,50X,'ORBIT ALTITUDE (CENDAV20078
         5TRAL DISTANCE)=',F10.0,'    KM.',//,25X,'PERIOD FOR THIS ORBIT=',   DAV20079
         6    F10.0,'    SECS.',///,25X,'ALL UNITS ARE IN M-K-S SYSTEM UNLESS DAV20080
         7    OTHERWISE NOTED')                                                   DAV20081
59          END                                                                   DAV20082
```

```
      SUBROUTINE EHPA(P,A)                                              DAV20083
      REAL*4 P(4), A(3,3)                                               DAV20084
C ****THIS ROUTINE RETURNS THE ROTATION MATRIX A CORRESPONDING          DAV20085
C ****TO THE EULER HOMOGENEOUS PARAMETERS P SUB I,  A*V(OLD)=V(NEW)     DAV20086
      P12=P(1)*P(1)                                                     DAV20087
      P22=P(2)*P(2)                                                     DAV20088
      P32=P(3)*P(3)                                                     DAV20089
      P42=P(4)*P(4)                                                     DAV20090
      A(1,1)=P42+P12-P22-P32                                            DAV20091
      A(1,2)=2.*(P(1)*P(2)+P(3)*P(4))                                   DAV20092
      A(1,3)=2.*(P(1)*P(3)-P(2)*P(4))                                   DAV20093
      A(2,1)=2.*(P(1)*P(2)-P(3)*P(4))                                   DAV20094
      A(2,2)=P42+P22-P32-P12                                            DAV20095
      A(2,3)=2.*(P(2)*P(3)+P(1)*P(4))                                   DAV20096
      A(3,1)=2.*(P(1)*P(3)+P(2)*P(4))                                   DAV20097
      A(3,2)=2.*(P(2)*P(3)-P(1)*P(4))                                   DAV20098
      A(3,3)=P42+P32-P22-P12                                            DAV20099
      RETURN                                                            DAV20100
      END                                                               DAV20101

      SUBROUTINE EHPQT(P,QT)                                            DAV20102
      REAL*4 P(4), QT(4,4)                                              DAV20103
C ****THIS SUBROUTINE SETS UP THE MATRIX Q TRANSPOSE FOR ANY GIVEN SET  DAV20104
C ****OF EULER'S HOMOGENEOUS PARAMETERS--- THE P SUB I                  DAV20105
      QT(1,1)=P(4)                                                      DAV20106
      QT(1,2)=-P(3)                                                     DAV20107
      QT(1,3)=P(2)                                                      DAV20108
      QT(2,1)=P(3)                                                      DAV20109
      QT(2,2)=P(4)                                                      DAV20110
      QT(2,3)=-P(1)                                                     DAV20111
      QT(3,1)=-P(2)                                                     DAV20112
      QT(3,2)=P(1)                                                      DAV20113
      QT(3,3)=P(4)                                                      DAV20114
      QT(4,1)=-P(1)                                                     DAV20115
      QT(4,2)=-P(2)                                                     DAV20116
      QT(4,3)=-P(3)                                                     DAV20117
      RETURN                                                            DAV20118
      END                                                               DAV20119
```

```
      SUBROUTINE NORM(P,MAG)                                            DAV20140
      REAL*4 P(4),MAG                                                   DAV20141
C ****THIS ROUTINE NORMALIZES P TO LENGTH = UNITY AND RETURNS ITS      DAV20142
C ****LENGTH PRIOR TO NORMALIZATION AS A CONVENIENT ERROR MEASURE.      DAV20143
      MAG=0.                                                            DAV20144
      DO 1 I=1,4                                                        DAV20145
    1 MAG=MAG+P(I)*P(I)                                                 DAV20146
      MAG=SORT(MAG)                                                     DAV20147
      DO 2 I=1,4                                                        DAV20148
    2 P(I)=P(I)/MAG                                                     DAV20149
      RETURN                                                            DAV20150
      END                                                               DAV20151
```

```
      SUBROUTINE WDOT(W,WD)                                             DAV20120
C EQUATIONS OF MOTION OF OUR BODY                                       DAV20121
C WD'S ARE OMEGA DERIVATIVES; T'S ARE TORQUES, C'S AND COMM'S ARE CONSTDAV20122
C AND MOMENTUM BIASES                                                   DAV20123
      REAL*4 W(3), WD(3)                                                DAV20124
      COMMON/WDOTS/ T(3), COMM(10), C1, C2, C3, C4, C5, C6              DAV20125
      WD(1)=T(1)*C4+C1*W(2)*W(3) -COMM(1)*W(2)                          DAV20126
      WD(2)=T(2)*C5+C2*W(1)*W(3) +COMM(2)*W(1)                          DAV20127
      WD(3)=T(3)*C6+C3*W(1)*W(2)                                        DAV20128
      RETURN                                                            DAV20129
      END                                                               DAV20130
```

```
      SUBROUTINE MATI(A,V1,V2)                                          DAV20131
C UTILITY MULTIPLICATION-- USED IN RELATING OMEGA'S TO P DOTS           DAV20132
      REAL*4 A(4,4), V1(3), V2(4)                                       DAV20133
      DO 1 I=1,4                                                        DAV20134
      V2(I)=0.                                                          DAV20135
      DO 1 J=1,3                                                        DAV20136
    1 V2(I)=V2(I)+A(I,J)*V1(J)                                          DAV20137
      RETURN                                                            DAV20138
      END                                                               DAV20139
```

```
      SUBROUTINE RK4                                                    DAV20152
      COMMON/RK/ P(4),W(3),QT(4,4),DT,WO                                DAV20153
      REAL*4 PDO(4),D(4)                                                DAV20154
      REAL*4 D1(3),D2(3),D3(3),D4(3),S(3),PD1(4),PD2(4),PD3(4),PD4(4)   DAV20155
C ****THIS SUBROUTINE INTEGRATES W (BODY REFERENCED ANGULAR VELOCITY)   DAV20156
C ****AND P (THE EULER HOMOGENEOUS PARAMETERS) FROM TIME T TO T+DT.     DAV20157
C ****TECHNIQUE IS A FOURTH ORDER RUNGE KUTTA SCHEME.                   DAV20158
C **********************************************************************DAV20159
C ****REQUIRES SUBROUTINES WDOT (USER SUPPLIED), MAT1, AND ARRAY QT.    DAV20160
      CALL EHPQT(P,QT)                                                  DAV20161
      CALL MAT1(QT,W,PD1)                                               DAV20162
      CALL WDOT(W,D1)                                                   DAV20163
      DO 10 I=1,3                                                       DAV20164
   10 S(I)=W(I)+0.5*DT*D1(I)                                            DAV20165
      DO 11 I=1,4                                                       DAV20166
   11 D(I)=P(I)+0.5*DT*PD1(I)                                           DAV20167
      CALL EHPQT(D,QT)                                                  DAV20168
      CALL MAT1(QT,S,PD2)                                               DAV20169
      CALL WDOT(S,D2)                                                   DAV20170
      DO 20 I=1,3                                                       DAV20171
   20 S(I)=W(I)+0.5*DT*D2(I)                                            DAV20172
      DO 21 I=1,4                                                       DAV20173
   21 D(I)=P(I)+0.5*DT*PD2(I)                                           DAV20174
      CALL EHPQT(D,QT)                                                  DAV20175
      CALL MAT1(QT,S,PD3)                                               DAV20176
      CALL WDOT(S,D3)                                                   DAV20177
      DO 30 I=1,3                                                       DAV20178
   30 S(I)=W(I)+     DT*D3(I)                                           DAV20179
      DO 31 I=1,4                                                       DAV20180
   31 D(I)=P(I)+     DT*PD3(I)                                          DAV20181
      CALL EHPQT(D,QT)                                                  DAV20182
      CALL MAT1(QT,S,PD4)                                               DAV20183
      CALL WDOT(S,D4)                                                   DAV20184
      DO 40 I=1,3                                                       DAV20185
   40 W(I)=W(I)+DT*(D1(I)+2.*(D2(I)+D3(I))+D4(I))/6.                    DAV20186
      PDO(1)=+WO*P(2)                                                   DAV20187
      PDO(2)=-WO*P(1)                                                   DAV20188
      PDO(3)=-WO*P(4)                                                   DAV20189
      PDO(4)=+WO*P(3)                                                   DAV20190
***ERROR***    MO-0
      DO 50 I=1,4                                                       DAV20191
   50 P(I)=P(I)+DT*(PD1(I)+PD4(I)+2.*PD2(I)+2.*PD3(I)+6.*PDO(I))/12.0   DAV20192
      RETURN                                                            DAV20193
      END
```

```
      SUBROUTINE TORQUE(T,B,CONST)                                       DAV20233
C   IDEAL MAGNET SWITCHER                                                DAV20234
C   SETS MAGNET POLARITIES OPPOSITE IN SIGN TO B DOTS                    DAV20235
C   SIGN OF B DOTS IS DETERMINED BY COMPARING PRESENT (B'S) VS. PREVIOUS DAV20236
C   (BOLD'S) BODY COMPONENTS OF MAGNETIC FIELD                           DAV20237
C   THEN, TORQUE= M CROSS B                                              DAV20238
      REAL*4 T(3), B(3), M1, M2, M3, B1OLD/0.0/, B2OLD/0.0/, B3OLD/0.0/  DAV20239
      B1=B(1)                                                            DAV20240
      B2=B(2)                                                            DAV20241
      B3=B(3)                                                            DAV20242
      BD1NEG=B1OLD-B1                                                    DAV20243
      BD2NEG=B2OLD-B2                                                    DAV20244
      BD3NEG=B3OLD-B3                                                    DAV20245
      M1=SIGN(CONST, BD1NEG)                                             DAV20246
      M2=SIGN(CONST, BD2NEG)                                             DAV20247
      M3=SIGN(CONST, BD3NEG)                                             DAV20248
      T(1)=M2*B3-M3*B2                                                   DAV20249
      T(2)=M3*B1-M1*B3                                                   DAV20250
      T(3)=M1*B2-M2*B1                                                   DAV20251
      B1OLD=B1                                                           DAV20252
      B2OLD=B2                                                           DAV20253
      B3OLD=B3                                                           DAV20254
      RETURN                                                             DAV20255
      END                                                                DAV20256
```

```
      SUBROUTINE MULVEC(A,V1,V2)                                         DAV20195
      REAL*4 A(3,3), V1(3), V2(3)                                        DAV20196
C   MULTIPLIES A*V1=V2                                                   DAV20197
      DO 1 I=1,3                                                         DAV20198
      V2(I)=0.0                                                          DAV20199
      DO 1 K=1,3                                                         DAV20200
    1 V2(I)=V2(I)+A(I,K)*V1(K)                                           DAV20201
      RETURN                                                             DAV20202
      END                                                                DAV20203
```

```
165         SUBROUTINE BFLDS1(INCLIN,A,B)                                   DAV202C4
        C   RETURNS MAGNETIC FIELD FOR VARIOUS POSITIONS IN ORBIT-- SEE SEPARATE DAV20205
        C   DOCUMENTATION                                                   DAV20206
166         REAL   B(3), INCLIN, PI/3.1416/                                 DAV20207
167         RADDEG=PI/180.                                                  DAV20208
168         BNOM=1.011E+12/(4.0*PI*A*A*A)                                   DAV20209
169         SN=0.2026                                                       DAV20210
170         CN=0.980                                                        DAV20211
171         SI=SIN(INCLIN*RADDEG)                                           DAV20212
172         CI=COS(INCLIN*RADDEG)                                           DAV20213
173         RETURN                                                          DAV20214
174         ENTRY BFLDS2(EARTHR,THETA,B)                                    DAV20215
        C   THETA IS THE ANGLE IN ORBIT FORWARD FROM ASCENDING NODE (DEGREES)  DAV20216
        C   EARTHR= W(EARTH)*T- LONGITUDE OF THE ASCENDING NODE  --(ALSO DEGREES)DAV20217
175         U=(69.0-EARTHR)*RADDEG                                          DAV20218
176         CU=COS(U)                                                       DAV20219
177         SU=SIN(U)                                                       DAV20220
178         B(3)=BNOM*(CI*CN+SI*SU*SN)                                      DAV20221
179         C1=-CU*SN                                                       DAV20222
180         C2=CI*SU*SN-SI*CN                                               DAV20223
181      10 V=THETA*RADDEG                                                  DAV20224
182         CV=COS(V)                                                       DAV20225
183         SV=SIN(V)                                                       DAV20226
184         B(1)=2.0*BNOM*(C1*CV+C2*SV)                                     DAV20227
185         B(2)=BNOM*(C1*SV-C2*CV)                                         DAV20228
186         RETURN                                                          DAV20229
187         ENTRY BFLDS3(THETA,B)                                           DAV20230
188         GO TO 10                                                        DAV20231
189         END                                                             DAV20232
```

APPENDIX C

DUAL SPIN EQUILIBRIUM CONSIDERATIONS

In this appendix we consider the equilibrium and stability of a dual spin configuration. Although this work has been done before (e.g. reference 6), we derive the results here in a simpler way. These results were obtained independently by the author before his acquaintance with reference 6 and are included here for completeness.

Consider a body with a single momentum bias wheel; the wheel spins at constant rate and carries angular momentum $\bar{h} = \{0,0,h\}$. The body is spinning about the same axis as is the rotor; for definiteness this is taken as the "3" axis. Suppose we consider the torque free motion of the body in the "neighborhood" of an equilibrium point, namely

$$\omega_1 = \omega_2 = 0, \ \omega_3 = \Omega \tag{C1}$$

In the neighborhood of this point

$$\omega_1 = \delta\omega_1, \quad \dot{\omega}_1 = \delta\dot{\omega}_1 \tag{C2,a}$$

$$\omega_2 = \delta\omega_2, \quad \dot{\omega}_2 = \delta\dot{\omega}_2 \tag{C2,b}$$

$$\omega_3 = \Omega + \delta\omega_3, \quad \dot{\omega}_3 = \delta\dot{\omega}_3 \tag{C2,c}$$

Now, the equations of motion, adapted from eq. (1) are

$$I_1\dot{\omega}_1 + \omega_2\{h + \omega_3(I_3 - I_2)\} = 0 \tag{C3,a}$$

$$I_2\dot{\omega}_2 - \omega_1\{h + \omega_3(I_3 - I_1)\} = 0 \tag{C3,b}$$

$$I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) = 0 \tag{C3,c}$$

Substituting eq. (C2) into these, and dropping second order terms (involving products of the $\delta\omega$'s), we obtain

$$I_1\delta\dot{\omega}_1 + \delta\omega_2\{h + (\Omega + \delta\omega_3)(I_3 - I_2)\} = 0 \tag{C4,a}$$

$$I_2\delta\dot{\omega}_2 - \delta\omega_1\{h + (\Omega + \delta\omega_3)(I_3 - I_1)\} = 0 \tag{C4,b}$$

$$I_3\delta\dot{\omega}_3 = 0 \tag{C4,c}$$

Thus we see immediately $\delta\dot{\omega}_3 = 0$, therefore $\omega_3 = \Omega =$ constant. Then, equations a and b can be combined to yield

$$\delta\ddot{\omega}_1 + \frac{\delta\omega_1}{I_1 I_2}\{h + \Omega(I_3 - I_2)\}\{h + \Omega(I_3 - I_2)\} = 0 \tag{C5}$$

C

What we have obviously obtained here is the equation for a simple harmonic oscillator. Consider one special case first. If Ω is "small", then ω_1 (and ω_2, which has the same form of solution) oscillates with circular frequency

$$\lambda_o = \frac{h}{\sqrt{I_1 I_2}} \tag{C6}$$

If Ω is not small, let us define an equivalent inertia

$$I_{3e} = I_3 + h/\Omega \tag{C7}$$

Then eq. (C5) reads

$$\delta\ddot{\omega}_1 + \frac{\delta\omega_1 \Omega^2 (I_{3e} - I_2)(I_{3e} - I_1)}{I_1 I_2} = 0 \tag{C8}$$

It is clear that if either

$$I_{3e} > I_1 \text{ and } I_{3e} > I_2 \tag{C9,a}$$

<u>or</u>

$$I_{3e} < I_1 \text{ and } I_{3e} < I_2 \tag{C9,b}$$

that is, if I_{3e} is a minimum or maximum inertia, then, our system is stable in the Liapunov sense. On the other hand, if I_{3e} is an intermediate inertia, the system is unstable. These results are of a physically appealing nature since they are of the same form as the results for spinning <u>rigid</u> bodies. In fact, if in eq. (C7) $h = 0$, $I_{3e} = I_3$, and those results are obtained directly.

For non-rigid (i.e., damped) bodies, these requirements are modified somewhat. First, there is generally little damping on the rotor, and that which is present has a destabilizing effect (almost always). Consider then a system with negligible damping on the rotor and a non-negligible amount on the main body. If the spin directions are such that the momenta of rotor and main body add ($\Omega>0$), it can be shown that it is necessary and sufficient for stability that I_{3e} be the major principal axis (maximum). Conversely, if the momenta oppose each other ($\Omega<0$), it is necessary and sufficient that

$$h_o = h + I_3\Omega > 0 \tag{C10}$$

$$\frac{-h}{I_3} < \Omega < 0 \tag{C11}$$

There is one additional region of stability for this system, and it exists only if I_3 is a major axis. The region defined by

$$\Omega < \frac{-h}{I_3 - \max(I_1, I_2)} \tag{C12}$$

corresponds to I_{3e} being a major principal axis while $\Omega<0$. As $h \rightarrow 0$ the above results reduce to the simple major spin axis requirements.

Definition (C7) may be combined with this observation into a simple rule, to wit

$$h_o > I\Omega \tag{C10}$$

where h_o is the system momentum bias (equal to $h + I_3\Omega$), and $I = \max(I_1, I_2)$. This same conclusion is obtained in reference 6 (page A-82).

The result (C10) has a direct application to the attitude acquisition phenomena noted under the section "Terminal Orientation". Namely, since the magnets will drive the satellite at $2\omega_o$ in the terminal orientation, and since we wish the terminal attitude to correspond to a stable condition, it is necessary that (C10) hold for $\Omega = 2\omega_o$. For a given orbit (which determines ω_o) and given set of inertias, this requirement then specifies a minimum h. The validity of this conclusion is supported by a number of simulation runs. In fact, it was observed that for good terminal attitude acquisition performance the left side of (C10) should be at least three or four times the right side.

APPENDIX D

SYSTEM EQUILIBRIUM POINTS

Consider the equilibrium states of the acquisition system described in this report under two conditions. (a) The magnetic field is fixed in trajectory axis coordinates, that is

$$\frac{d^{SPACE}}{dt}(\bar{B}) = 0 \tag{D1}$$

and (b), the magnet controllers employ a linear law

$$\bar{M} = -K\dot{\bar{B}} \tag{D2}$$

where $\dot{\bar{B}}$ is the time rate of change of $\bar{B}$ as measured in the vehicle. As mentioned in the body of this report, under the section titled "Stability", this system has no equilibrium points unless some modification is made to the flip-flop magnet control law. Now, let us consider the equilibrium points possible under the two conditions above.

The behavior of our system may be completely characterized by the six state variables $\omega_1, \omega_2, \omega_3, B_1, B_2, B_3$. Equilibrium points may be identified by setting the time derivatives of these variables to zero and solving the resulting six equations in six unknowns. That is, we require

$$\dot{\bar{\omega}} = 0 \tag{D3}$$

$$\dot{\bar{B}} = \bar{B} \times \bar{\omega} = 0 \qquad \text{eq. (8)=(D4)}$$

Starting with equation (1), we make the following substitutions: $\bar{\tau} = \underline{0}$ (torque free motion), $\dot{\bar{h}} = 0$ (constant speed wheel), and eq. (1′) $\bar{H} = \bar{\bar{I}}\cdot\bar{\omega} + \bar{h}$. We then obtain

$$\bar{\bar{I}}\cdot\dot{\bar{\omega}} = -\bar{\omega} \times (\bar{\bar{I}}\cdot\bar{\omega} + \bar{h}) = 0 \tag{D5}$$

The right hand side of (D5) equals zero according to (D3). An equilibrium point must then, by definition, satisfy equations (D4) and (D5). (D4) simply requires that

$$\bar{\omega} = \alpha(t)\bar{B} \tag{D6}$$

where $\alpha(t)$ may or may not be zero and may or may not be constant. Now, if

$$\bar{h} = \{0,\ 0,\ h\} = h \tag{D7}$$

That is, we have a pitch wheel only, then (D5) becomes, in scalar form

$$\omega_2\omega_3(I_3 - I_2) + h\omega_2 = 0 \tag{D8,a}$$

(D5, modified)

$$\omega_1\omega_3(I_1 - I_3) - h\omega_1 = 0 \tag{D8,b}$$

$$\omega_1\omega_2(I_2 - I_1) = 0 \tag{D8,c}$$

From the above discussion we now obtain the following results

(I) $\bar{\omega} = \{0, 0, 0\}$, $\bar{B}$ = any, $\alpha = 0$ is an equilibrium point, since it clearly satisfies eq. (D4) & (D8).

(II) $\bar{\omega} = \{0, \text{any}, h/(I_2 - I_3)\}$, $\bar{B} \parallel \bar{\omega}$ is an equilibrium point.

(III) $\bar{\omega} = \{\text{any}, 0, h/(I_1 - I_3)\}$, $\bar{B} \parallel \bar{\omega}$ is an equilibrium point.

(IV) $\bar{\omega} = \{0, 0, \text{any}\}$, $\bar{B} \parallel \bar{\omega}$ is the last equilibrium point.

These four classes contain all the equilibrium points of this system. In passing I note (for the mathematicians among us) that there are an infinity of equilibrium points. Note also that at equilibrium points, $\dot{\bar{B}} = 0$ so $\bar{M} = 0$ and so the fact that we set $\tau = 0$ at the outset does not affect the validity of the results obtained here.

There is, curiously, an alternate method of obtaining these same results. Starting with the equation of motion (1), the control law (D2), and the assumptions (D1) and (D7), we have

$$\tau_1 = I_1\dot{\omega}_1 + \omega_2 h \qquad \text{(D9,a)}$$

$$\tau_2 = I_2\dot{\omega}_2 - \omega_1 h$$

$$\tau_3 = I_3\dot{\omega}_3$$

where we have dropped the second order terms involving products of the ω's. Using eq. (D2) and (D4), we obtain

$$M_1 = K(\omega_2 B_3 - \omega_3 B_2) \qquad \text{(D10,a)}$$

$$M_2 = K(\omega_3 B_1 - \omega_1 B_3) \qquad \text{(D10,b)}$$

$$M_3 = K(\omega_1 B_2 - \omega_2 B_1)$$

Remembering eq. (5)

$$\bar{\tau} = \bar{M} \times \bar{B} \qquad (5)$$

and substituting $\bar{\tau}$ into eq. (D9), we obtain

$$I_1\dot{\omega}_1 + \omega_2 h = B_3 K(\omega_3 B_1 - \omega_1 B_3) - B_2 K(\omega_1 B_2 - \omega_2 B_1) \qquad \text{(D11,a)}$$

$$I_2\dot{\omega}_2 - \omega_1 h = B_1 K(\omega_1 B_2 - \omega_2 B_1) - B_3 K(\omega_2 B_3 - \omega_3 B_2) \qquad \text{(D11,b)}$$

$$I_3\dot{\omega}_3 = B_2 K(\omega_2 B_3 - \omega_3 B_2) - B_1 K(\omega_3 B_1 - \omega_1 B_3) \qquad \text{(D11,c)}$$

Using the definition of equilibrium points

$\dot{\bar{\omega}} = 0$

(here we are considering only $\bar{\omega}$ as a state variable, not $\bar{B}$)

we obtain three linear homogeneous equations in the three unknowns - ω_1, ω_2, and ω_3. For a solution to exist, the determinant of the coefficients must equal zero. That is

$$\begin{vmatrix} -(B_2^2 + B_3^2) & (B_1B_2 - h/K) & B_1B_3 \\ (B_1B_2 + h/K) & -(B_1^2 + B_3^2) & B_2B_3 \\ B_1B_3 & B_2B_3 & -(B_1^2 + B_2^2) \end{vmatrix} = 0 \qquad \text{(D12)}$$

This equation reduces to

$$\frac{-h^2}{K^2}(B_1^2 + B_2^2) = 0 \qquad \text{(D12')}$$

or

$$\bar{B} = \{0, 0, B_3\} \qquad \text{(D13)}$$

Substituting this back into eq. (D11) we have

$$I_1\dot{\omega}_1 + \omega_2 h = -B_3^2 K\omega_1 \qquad \text{(D14,a)}$$

$$I_2\dot{\omega}_2 - \omega_1 h = -B_3^2 K\omega_2 \qquad \text{(D14,b)}$$

$$I_3\dot{\omega}_3 = 0 \qquad \text{(D14,c)}$$

Since $\dot{\bar{\omega}} = 0$, we obtain the following equilibrium condition

$$\omega = \{0, 0, \text{any}\} \qquad \text{(D15)}$$

This result is not valid if ω_3 is large, since we neglected $\omega_3(I_3 - I_2)$ and $\omega_3(I_3 - I_1)$ in comparison to h in writing eq. (D9). To the extent that this is valid, (D15) is correct. Note that (D15) is the same as condition (IV) identified previously, and that (I) is a special case of (IV).

The only special thing about this alternate derivation is that we did not assume $\dot{\bar{B}} = 0$ at the outset. We were considering cases where $\bar{\omega} = 0$ but $\dot{\bar{B}}$ and hence $\bar{\tau}$ not necessarily equal to zero. Note that, in retrospect, we now have $\bar{B} = \{0, 0, B_3\}$, so $\dot{B}_1 = \dot{B}_2 = M_1 = M_2 = 0$. The result is then that $\bar{\tau} = 0$ anyway.

In the next section we consider the stability of the equilibrium points enumerated here.

APPENDIX E

STABILITY OF EQUILIBRIUM POINTS

In this appendix we investigate the stability of a particular equilibrium point, namely point IV of Appendix D. This stability investigation is carried out by linearizing the system equations about the equilibrium point and then forming the characteristic equation. A study of the roots of this equation then determines the stability of the system near the equilibrium point. This does not rule out the possibility of limit cycle like behavior but is nonetheless a useful technique. From the analysis that follows, certain conclusions may be drawn. These conclusions and their consequences are discussed at the end of this appendix.

Consider perturbations about the equilibrium point

$$\bar{\omega} = \{0, 0, \Omega\} \tag{E1}$$

$$\bar{B} = \{0, 0, B\}$$

Then

$$\bar{\omega} = \{\delta\omega_1, \delta\omega_2, \Omega + \delta\omega_3\} \tag{E2}$$

$$\bar{B} = \{\delta B_1, \delta B_2, B + \delta B_3\}$$

The following equations then apply

$$\dot{\bar{B}} = \bar{B} \times \bar{\omega} \tag{8}$$

$$\delta\dot{B}_1 = \Omega\delta B_2 - B\delta\omega_2$$

$$\delta\dot{B}_2 = B\delta\omega_1 - \Omega\delta B_1 \tag{E3}$$

$$\delta\dot{B}_3 = 0$$

where we have dropped second order terms in eq. (E3). Now, assuming a linear control law, i.e.,

$$\bar{M} = -K\dot{\bar{B}} \tag{E4}$$

$$M_1 = K(B\delta\omega_2 - \Omega\delta B_2)$$

$$M_2 = K(\Omega\delta B_1 - B\delta\omega_1) \tag{E5}$$

$$M_3 = 0$$

Then, since

$$\bar{\tau} = \bar{M} \times \bar{B} \tag{5}$$

We obtain

$$\tau_1 = KB(\Omega\delta B_1 - B\delta\omega_1)$$

$$\tau_2 = KB(\Omega\delta B_2 - B\delta\omega_2) \tag{E6}$$

$$\tau_3 = 0$$

Similarly, we may start with eq. (1), substitute from (E2) and drop second order terms in the result. Thus we obtain the linearized equations of motion about the equilibrium point

$$I_1 \delta\dot{\omega}_1 = \tau_1 - \delta\omega_2 (h + \Omega(I_3 - I_2)) \tag{E7}$$

$$I_2 \delta\dot{\omega}_2 = \tau_2 + \delta\omega_1 (h + \Omega(I_3 - I_1))$$

$$I_3 \delta\dot{\omega}_3 = \tau_3$$

These may be written in a way that makes them easier to interpret by using the following shorthand notation:

$$I_{3e} = \frac{h}{\Omega} + I_3 \tag{E8}$$

$$I_{3e}\Omega = h + I_3\Omega = h_o \tag{E9}$$

$$h + \Omega(I_3 - I_2) = \Omega(I_{3e} - I_2) = \alpha_1 \tag{E10}$$

$$h + \Omega(I_3 - I_1) = \Omega(I_{3e} - I_1) = \alpha_2 \tag{E11}$$

Using this notation and substituting for the τ_i from $(E\dot{6})$ we obtain

$$\delta\dot{\omega}_1 = \frac{-KB^2}{I_1}\delta\omega_1 - \frac{\alpha_1}{I_1}\delta\omega_2 + \frac{KB\Omega}{I_1}\delta B_1 + 0\delta B_2$$

$$\delta\dot{\omega}_2 = \frac{\alpha_2}{I_2}\delta\omega_1 - \frac{KB^2}{I_2}\delta\omega_2 + 0\delta B_1 + \frac{KB\Omega}{I_2}\delta B_2 \tag{E12}$$

$$\delta\dot{B}_1 = +0\delta\omega_1 - B\delta\omega_2 + 0\delta B_1 + \Omega\delta B_2$$

$$\delta\dot{B}_2 = B\delta\omega_1 + 0\delta\omega_2 - \Omega\delta B_1 + 0\delta B_2$$

The last two equations are from (E3). Note that the last equations in (E3) and (E7) immediately yield

$$\delta\dot{\tilde{B}}_3 = 0,\ \delta B_3 = \text{const} = 0,\ B_3 = B \tag{E13}$$

$$\delta\dot{\omega}_3 = 0,\ \delta\omega_3 = \text{const} = 0,\ \omega_3 = \Omega$$

Equations (E12) constitute four first order ordinary linear differential equations in four unknowns. They are of the form

$$\dot{\bar{X}} = A\,\bar{X} \tag{E12,a}$$

where A is a square matrix with constant coefficients.

Now, if A is negative (positive) definite it may be immediately shown that the system described by (E12,a) is assymptotically stable (unstable) by applying Liapunov's (Chetayev's) stability theorem to the function $v = \bar{x}\cdot\bar{x}$. Unfortunately, for our problem, as for many others, A is sign indefinite and the theorems noted are not so easily applied.

Instead, we Laplace transform (E12,a)

$$s\bar{X}(s) - \bar{x}_o = A\bar{X}(s) \tag{E14}$$

Thus

$$\bar{X}(s) = (A - Is)^{-1}(-\bar{x}_o) \tag{E15}$$

and

$$\bar{x}(t) = L^{-1}\{(A - Is)^{-1}(-\bar{x}_o)\} \tag{E16}$$

Note that, as shown by E15, all the components of $\bar{X}(s)$ (remember, $\bar{x}(t)$ and $\bar{X}(s)$ are column vectors) have a denominator D(s) and

$$D(s) = \text{Determinant}(A - Is) \tag{E17}$$

Now, D(s) may also be written as

$$D(s) = (s - s_1)(s - s_2)\cdots(s - s_n) \tag{E18}$$

where the s_i are the roots of $D(s) = 0$ or $\text{Det}(A - Is) = 0$. Alternately, the s_i are the eigenvalues of the matrix A. Obviously, for stability, it is necessary and sufficient that the real parts of the s_i be less than zero.

To determine the s_i we expand

$$D(s) = \text{Det}(A - Is) = 0 \tag{E19}$$

obtaining the (linearized system) characteristic equation in terms of the various system parameters. The matrix A from (E12) is

$$A = \begin{bmatrix} -KB^2/I_1 & -\alpha_1/I_1 & KB\Omega/I_1 & 0 \\ \alpha_2/I_2 & -KB^2/I_2 & 0 & KB\Omega/I_2 \\ 0 & -B & 0 & \Omega \\ B & 0 & -\Omega & 0 \end{bmatrix} \tag{E20}$$

and the expansion (E19) yields, after a little algebra

$$\begin{aligned} I_1I_2s^4+(I_1 + I_2)KB^2s^3 + (I_1I_2\Omega^2 + \alpha_1\alpha_2 + K^2B^4)s^2 \\ + KB^2\Omega(\alpha_1 + \alpha_2)s + \alpha_1\alpha_2\Omega^2 = 0 \end{aligned} \tag{E21}$$

We return now to conditions (a3) and (a4). ϕ_1 can always be made greater than zero for sufficiently large gain K. For K = 0, (a3) may be written as

$$2(I_1 + I_2) + \frac{(I_1 + I_2)(I_{3e} - I_2)(I_{3e} - I_1)}{I_1 I_2} > 2I_{3e} \quad \text{(E24)}$$

Various cases of this can now be considered. If Ω is small and less than zero (opposite in sense to $\bar{h}$), (E24) is easily satisfied. If Ω is small and positive this relation is always satisfied. For large values of Ω (either sign) $I_{3e} \to I_3$. If I_3 is a major or minor axis (E24) is always satisfied. If I_3 is an intermediate axis, (E24) may fail (for K = 0) for sufficiently large Ω (which is required to make $I_{3e} \to I_3$). Before this happens the criterion (a5) will fail, so we need not concern ourselves with (a3) except perhaps for some intermediate values of Ω, such that neither $I_{3e} >> I_1$, I_2 nor $I_{3e} \to I_3$. If we make some simplifying assumptions- namely $I_1 = I_2 = I$, and $I_{3e} = nI$, (E24) reduces to

$$(n - 1)^2 > (n - 2) \quad \text{(E25)}$$

which holds for all values of n, positive and negative. It would seem that except for a rather oddly configured satellite that $\phi_1 > 0$, and (a3) presents no problems. In fact, unless the ratio between I_1 and I_2 is greater than 5:1, a3 must be satisfied. Additionally, for K>0, ϕ_1 increases and (a3) presents even fewer problems. On the basis of this analysis we turn our attention to condition (a4).

$$\phi_2 = KB^2\Omega(\alpha_1 + \alpha_2) - KB^2\Omega^2\alpha_1\alpha_2(I_1 + I_2)/\phi_1 > 0 \quad \text{(a4)}$$

$$\phi_2 = KB^2\Omega^2[(2I_{3e}-I_1-I_2)-\Omega^2(I_1+I_2)(I_{3e}-I_1)(I_{3e}-I_2)/\phi_1]>0 \quad \text{(E26)}$$

Now, from condition (a5) we see that the second term of (E26) is less than zero. Thus, if the first term is less than zero, this system is unstable. It is a requirement then, for stability, that I_{3e} be the major principal axis of the satellite (ie, $I_{3e} > \max(I_1, I_2)$).

Now, for our system, I_3 is a minor axis, and for large values of Ω, $I_{3e} \to I_3$, so the system is unstable. For $\Omega > 0$, in fact, I_{3e} is always a minor axis (if I_3 is). The only stable values for operation are positive values not "large". In examining this problem we now turn to a numerical study.

For a numerical evaluation, the following values are substituted in conditions (a3), (a4) and (a5).

I_1 = 35 slug-ft^2 (47.5 Kg-m^2)

I_2 = 50 slug-ft^2 (67.9 Kg-m^2)

I_3 = 25 slug-ft^2 (33.9 Kg-m^2)

B = 0.5 Gauss

h = 0.7 lbf-ft-sec (0.944 Nt-m-sec)

K = 10^4 pole-cm/(5 x 10^{-4} Gauss/sec) = 2×10^{-8}Nt-m/(Tesla^2/sec)

K is chosen by calculating the describing function for a flip-flop control law and a system oscillation amplitude of about ω_0. Note that K decreases as ω grows and hence the system becomes less stable. Now, (a3) becomes

$$\phi_1 = 5019\Omega^2 - 97.7\Omega + 0.89 + K^2B^4 > 0 \tag{E27}$$

(E27) has no real zeros even for the worst case (K = 0), so we need not concern ourselves with it further. We next look at (a5) and determine the maximum value of Ω for which I_e is a major axis (recalling that this is required by ϕ_2)

$$\frac{h}{\Omega} + I_3 > \max(I_1, I_2) \tag{E28}$$

The result is $\Omega_{max} = 0.0278$ rad/sec.

Finally, we examine ϕ_2 (expression a4):

$$\frac{\phi_2}{KB^2} = \frac{-53315\Omega^4}{\phi_1} + \frac{5185\Omega^3}{\phi_1} - \left(47.6 + \frac{102.7}{\phi_1}\right)\Omega^2 + 1.89\Omega \tag{E29}$$

We are interested in determining whether $\phi_2 > 0$ for $.0278 > \Omega > 0$. This is required if we are to have any stable region of operation at all. This task is somewhat simplified by first examining ϕ_1. We see that ϕ_1 is relatively constant for this range of Ω. In fact, for K = 0,

$$(\phi_1 = 5015\Omega^2 - 97.7\Omega + 0.89) \tag{E30}$$

$$\phi_1(0) = 0.89$$

$$\phi_1(.01) = 0.515$$

$$\phi_1(.02) = 0.94$$

$$\phi_1(.0278) = 2.05$$

and

$$\phi_{1min} = 0.418 \text{ at } \Omega = 0.00973$$

K = 0 is surely the worst possible case, since as K increases, so does ϕ_1 and thus ϕ_2 will increase if we are in the range $0 < \Omega < 0.0278$.

Now, (E29) has only two real roots. One is obviously at $\Omega = 0$, the other lies at about $\Omega = 0.05$. Between these roots $\phi_2 > 0$. Thus we have ensured a stable region of operation for

$$0.0278 > \Omega > 0.0 \tag{E31}$$

One may convince oneself of the truth of this result by returning to equation (E21), substituting values for the coefficients and then solving the resulting equation numerically for the roots s_i. It is, of course, a necessary and sufficient condition for stability that all the s_i lie in the left half plane.

We require, for stability, that all the solutions of (E21) lie in the left half-plane. We may attack (E21) either numerically or analytically. Consider the analytical approach first.

If we apply Routh's Criterion to (E21) we obtain, as the necessary and sufficient conditions for stability, that the following quantities must be greater than zero if a_1 is

$$I_1 I_2 > 0 \tag{a1}$$

$$(I_1 + I_2)KB^2 > 0 \tag{a2}$$

$$\phi_1 = I_1 I_2 \Omega^2 + \alpha_1\alpha_2 + K^2B^4 - \frac{I_1 I_2 \Omega(\alpha_1 + \alpha_2)}{I_1 + I_2} > 0 \tag{a3}$$

$$\phi_2 = KB^2\Omega(\alpha_1 + \alpha_2) - KB^2\Omega^2\alpha_1\alpha_2(I_1 + I_2)/\phi_1 > 0 \tag{a4}$$

$$\alpha_1\alpha_2\Omega^2 > 0 \tag{a5}$$

Requirement (a1) is trivial, and so is (a2) as long as $K>0$. (a5) is satisfied as long as the body is dynamically stable (see appendix C) without the control system. We must address ourselves to (a3) and (a4). We first note, en passant, that if we set $K = 0$ in (E21) we obtain the characteristic equation for the system without control torques. This situation was examined in appendix C. Equation (E21) becomes

$$s^4 + as^2 + b = 0 \tag{E22}$$

where

$$b = \frac{\alpha_1\alpha_2\Omega^2}{I_1 I_2}$$

and

$$a = \Omega^2 + \frac{\alpha_1\alpha_2}{I_1 I_2} + K^2B^4$$

It is trivial to show that the requirements for stability of (E22) are that $a>0$, $b>0$. Obviously $b>0$ is the stronger condition; we obtain the requirement

$$\alpha_1\alpha_2 > 0 \tag{E23}$$

and this is always satisfied if the system is stable without the control system.

Substituting into (E21) we obtain

$$3225s^4 + 57.7s^3 + (3687\Omega^2 - 45\Omega + 1.14)s^2 + (-23.8\Omega^2 + 0.944\Omega)s + (462\Omega^4 - 45\Omega^3 + 0.89\Omega^2) = 0 \qquad \text{(E32)}$$

Numerical solutions for the roots of (E32) confirm the results (E31) obtained thru the Routhian analysis.

All of the results obtained in this appendix, up to this point, are only valid in a rigorous sense, for motion in the "neighborhood" of the equilibrium point. Otherwise the linearizations made in obtaining the system equations (E12) are not allowable. Nonetheless, considerable digital simulation experience indicates that the results obtained in this section are in fact applicable to the real (nonlinear) system. Let us consider then the significance of these results. First of all, they indicate that the terminal attitude of the satellite must be such that the momentum bias points along the orbit normal, rather than opposite it. We remember that in the terminal condition the satellite has an angular velocity $2\omega_o\bar{e}_n$. If the satellite were to acquire "backwards", then the angular velocity $2\omega_o\bar{e}_n$ would correspond to $\Omega<0$, and we have seen that this equilibrium point is an unstable one. We have seen that there are no stable equilibrium points with very high angular velocities; the greatest being $\Omega = 0.0278$ rad/sec. This means that, with the exception of possible limit cycle difficulties, this system should be well behaved and the control theory developed in the main body of this report provides for the positive despin of the vehicle. The author also wishes to note that he has observed no limit cycle behavior during extensive digital simulation of this system.

The reader is probably asking, at this point, "How about the equilibrium points II and III enumerated in Appendix D". The answer is that the author has not yet had time to perform a rigorous stability analysis for these points. However, on the basis of digital simulation it appears that they are not stable points unless $\omega_1 = \omega_2 = 0$, which is of course the case treated here.

Finally, we note that even the stable equilibrium states determined here present no real despin problems. The investigations here dealt, for simplicity, with a fixed external magnetic field. In fact, even in the worst case (an Equatorial orbit with the pitch wheel along the orbit normal) the Earth's field appears to cone about with a half angle of 11.7°. If the satellite is to remain in the equilibrium state, it too must cone about the orbit normal at frequency ω_o. The control torques maintaining the equilibrium state (i.e. damping out disturbances - which is how the coning of the $\bar{B}$ field appears to the system) are essentially dissipative in nature, since they oppose sensed velocities. The final result is that even the small residual spin $\Omega < .0278$ rad/sec along the momentum bias will be eliminated due to this previously unconsidered effect.