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ELASTOHYDRODYNAMIC ANALYSIS USING A POWER LAW PRESSURE-VISCOSITY RELATION

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and

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16. Abstract

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SUMMARY

An isothermal elastohydrodynamic (EHD) inlet analysis of the Grubin type which considers a power law pressure-viscosity relation ($\mu = \mu_0(1 + Kp)^n$) and a finite pressure at the inlet of the contact zone was performed. Numerical results from this analysis were compared to EHD film thickness data from X-ray measurements made with a synthetic paraffinic oil and to results from conventional EHD analysis.

In the high-contact stress regime; that is, above 1.04×10^9 newtons per square meter (150 000 psi), minimum film thickness as predicted by both present and conventional theories was consistently less sensitive to applied load (contact stress) than the measured data. The present EHD theory exhibits a slightly stronger load dependence than do previous isothermal EHD theories which consider a straight exponential pressure-viscosity relation but far less than that exhibited by the measured data.

INTRODUCTION

The effectiveness of an elastohydrodynamic (EHD) film lies in its ability to maintain separation between two loaded surfaces in moving contact. The size of this spacing or film thickness, as it is more commonly referred to, is primarily dependent on the rheological properties of the lubricant, the elastic properties of the contacting surfaces, and the conditions of operation. Because of its importance on the design life of contacting machine elements, prediction of EHD film thickness has been the focal point of many theoretical and experimental investigations. A compendium of the advancements made in EHD lubrication up to about 1964 appears in a text by Dowson and Higginson (ref. 1) and more recently in a report by McGrew et al. (ref. 2).

The bulk of the experimental work conducted in EHD lubrication has been confined to conditions of moderate speeds, that is, up to 25.4 meters per second (1000 in./sec), and moderate loads, that is, maximum Hertz stresses to 1.24×10^{9} newtons per square meter (180 000 psi) (refs. 3 to 6). Recently Parker and Kannel (ref. 7) have extended EHD film thickness measurements with an X-ray disk apparatus to a range of speed and load conditions, that is, surface speeds to 37.6 meters per second (1480 in./sec) and maximum Hertz stress to 2.42×10^{9} newtons per square meter (350 000 psi) which are more in line with current bearing and gear design requirements. Whereas, the film thickness measurements determined at conditions of moderate loads and speeds by previous investigators have shown reasonably good agreement with EHD theory, the recent X-ray data is at odds with theory. In particular, at high contact pressures the sensitivity of film thickness to load as determined experimentally is far greater than predicted by the theory of either Grubin (ref. 8) or Dowson and Higginson (ref. 9). Consequently, predicted film thickness as at high loads are substantially higher than those observed.

In an attempt to resolve this discrepancy and to determine whether the X-ray meassurements were themselves in error, Kannel and Bell (ref. 10) conducted a critical reexamination of the X-ray technique. Among the factors which may contribute to experimental inaccuracies, Kannel and Bell considered the influence of poor X-ray beam collimation, the effects of possible surface reflections, and the consequences of lubricant X-ray absorption. They concluded that none of the aforesaid factors were of such significance as to seriously alter the accuracy of the X-ray measurements.

Bell and Kannel (ref. 11) then examined several lubricant rheological factors which may help to account for the observed high-load effects. They considered in their analysis the effects of a non-Newtonian lubricant of the Ree-Eyring form whose viscosity exhibited a time-delay response to a sudden change of pressure. Their theory showed that for most practical cases, the time delay of pressure's effect on viscosity was sufficiently long as to not permit the lubricant's viscosity to change appreciably upon entering the contact zone. This resulted in a solution in which the film thickness exhibits a greater dependence upon load, $h \alpha (p_{Hz})^{-10/11}$, than that predicted by all previous theories. Although Bell and Kannel's solution is in better agreement with the observed trends, it still did not account for the rapid fall off of film thickness at the higher contact stress levels.

Cheng (ref. 12) has attempted to extend present EHD theory to include the region of extremely heavy loads and high rolling speeds. Cheng determined that the introduction of a composite exponential pressure-viscosity model ($\mu = \mu_o e^{\alpha p_1 + \gamma(p - p_1)}$) into the theory, as originally proposed by Allen, Townsend, and Zaretsky (ref. 13), in place of the conventional straight exponential pressure-viscosity model ($\mu = \mu_o e^{\alpha p}$) had negligible effect on the predicted film thickness. Furthermore, the introduction of a thermal reduction factor developed by Cheng in reference 14 to correct the isothermal theory for the influence of lubricant viscous heating at high-rolling speeds failed to significantly improve the agreement between theory and the experimental data within the heavy load regime.

The variation of the lubricant's viscosity with changes in hydrodynamic pressure is known to play a vital role in the extent of surface separation within an EHD contact. Yet, there has not been enough work done toward establishing this relation under the appropriate dynamic conditions. Even steady-state viscosity measurements have shown that not all lubricants obey the conventional straight exponential pressure-viscosity relation frequently used in classical EHD theory. A notable example of this can be found from the work of Chu and Cameron (ref. 15) in which it is shown that straight paraffinic mineral oils deviate substantially from the straight exponential relation (fig. 1). As an alternate, Chu and Cameron have developed a power law relation ($\mu = \mu_0(1 + Kp)^n$) to fit the experimental pressure-viscosity data for the paraffinic based oils from reference 16.

Dyson, Naylor, and Wilson (ref. 6) utilizing a similar power law pressure-viscosity relation developed a correction to be applied to the EHD theory of Grubin. However, a comparison between predicted film thicknesses with this correction and those obtained through capacitance measurement were not made due to uncertainties in

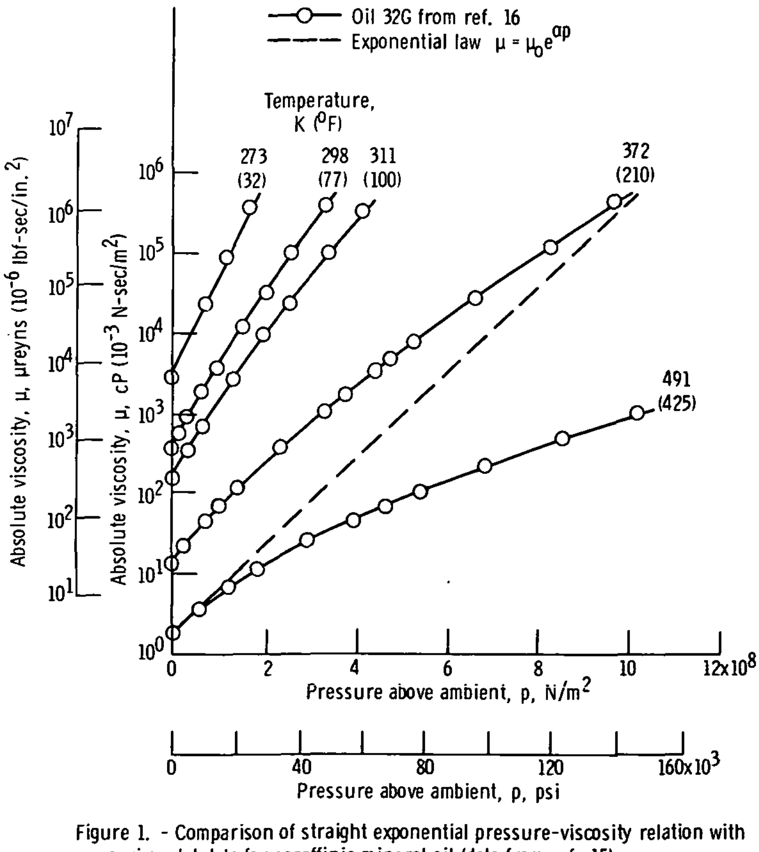

Figure 1. - Comparison of straight exponential pressure-viscosity relation with experimental data for paraffinic mineral oil (data from ref. 15).

estimating the value of the correction itself. Similarly, Cameron and Gohar (ref. 17) derived a generalized film thickness expression in which the power law pressure-viscosity relation could be used in place of the conventional straight exponential model.

The objectives of the analysis reported herein were: (1) to incorporate into elastohydrodynamic theory a pressure-viscosity relation which obeys the power law model ($\mu = \mu_o(1 + Kp)^n$) developed in reference 15, and (2) to compare the numerical results from this analysis with the X-ray measurements of film thickness reported in reference 7.

SYMBOLS

a	minor semi-axis of Hertzian contact, m (in.)
b	major semi-axis of Hertzian contact, m (in.)
b_h	half width of Hertzian contact zone, m (in.)
E_1, E_2	modulus of elasticity of elements 1 and 2, N/m^2 (psi)
E'	$\left(\dfrac{1 - \nu_1^2}{\pi E_1} + \dfrac{1 - \nu_2^2}{\pi E_2}\right)^{-1}$, N/m^2 (psi)
h	local film thickness, m (in.)
h_c	film thickness in Cheng's theory defined by eq. (29), m (in.)
h_g	film thickness in Grubin's theory defined by eq. (18), m (in.)
h_i	film thickness at inlet edge of Hertzian contact zone, m (in.)
h_m	film thickness at point of maximum Hertzian pressure, m (in.)
h_o	minimum film thickness, m (in.)
h_p	film thickness with power law lubricant, m (in.)
K	pressure-viscosity coefficient defined in eq. (3), m^2/N (psi^{-1})
k	constant defined indirectly in eq. (3)
n	pressure-viscosity exponent defined in eq. (3)
p	local pressure above ambient, N/m^2 (psi)
p_{Hz}	maximum Hertz stress, N/m^2 (psi)
p_i	pressure at inlet edge of Hertzian contact zone, N/m^2 (psi)
p_1	critical pressure in composite exponential pressure-viscosity model, N/m^2 (psi)

R_1, R_2	radius of elements 1 and 2 in rolling direction, m (in.)
R'_x	$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$, m (in.)
T	local temperature, K ($^{\circ}$F)
T_o	disk temperature, K ($^{\circ}$F)
u	$1/2(u_1 + u_2)$, m/sec (in./sec)
u_1, u_2	surface velocities of elements 1 and 2, m/sec (in./sec)
W	load parameter, $w/E'R'_x$
w	load per unit cylinder length, N/m (lbf/in.)
X	dimensionless distance parameter, x/b_h
x	distance from center of Hertzian contact zone, m (in.)
α	pressure-viscosity coefficient, m^2/N (in.^{2}/lbf)
α_c	equivalent pressure-viscosity coefficient defined in eq. (21), m^2/N (in.^{2}/lbf)
β	function of T defined in eq. (4)
β_o	constant defined indirectly in eq. (4)
γ	secondary pressure-viscosity coefficient in composite exponential pressure-viscosity model, m^2/N (in.^{2}/lbf)
λ	constant defined indirectly in eq. (4)
μ	local absolute viscosity, 10^{-3} N-sec/m^2 or cP (lb-sec/in.^{2} or reyns)
μ_o	ambient absolute viscosity, 10^{-3} N-sec/m^2 or cP (lb-sec/in.^{2} or reyns)
ν_1, ν_2	Poisson's ratio of elements 1 and 2
ξ	dimensionless pressure parameter, p_i/p_{Hz}

COMPARISON OF X-RAY DATA (REF. 7) WITH CONVENTIONAL ELASTOHYDRODYNAMIC THEORY

Elastohydrodynamic film thickness measurements were obtained in reference 7 using an X-ray rolling disk machine (fig. 2) with a synthetic paraffinic oil. The method of measuring film thickness with the X-ray technique comprises projecting X-rays between the surfaces of the two contacting disks and detecting the amount of X-rays which are transmitted through the contact. Since the greatest constriction occurs at the trail-

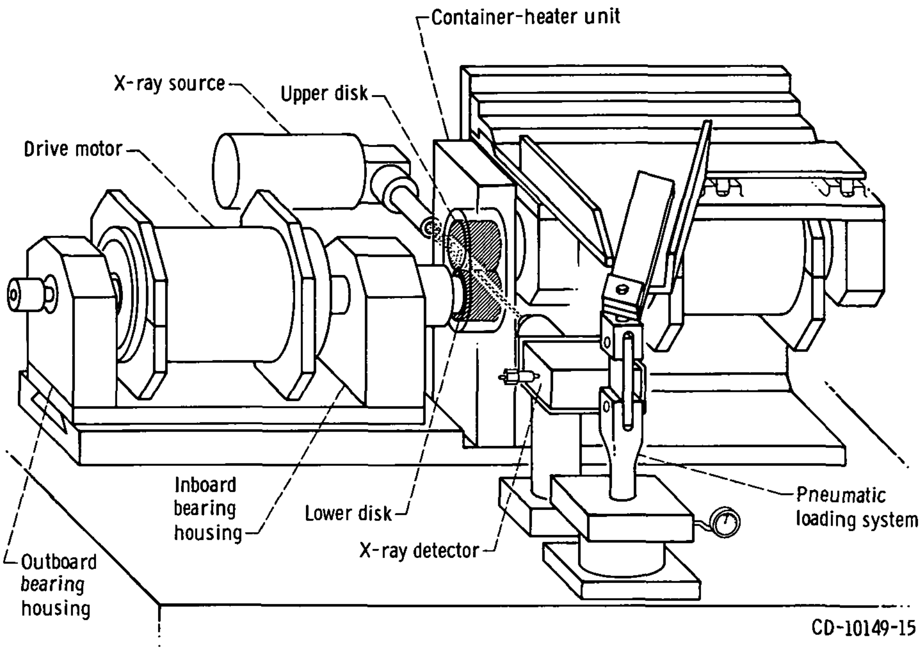

Figure 2. - X-ray rolling-contact disk machine (ref. 7).

ing edge of the contact, the X-ray count thus becomes a measure of the lubricant's minimum film thickness.

The range of test conditions consist of disk temperatures from 339 to 589 K (150° to 600° F), surface speeds from 9.4 to 37.6 meters per second (370 to 1480 in./sec) corresponding to disk rolling speeds from 5000 to 20 000 rpm, and maximum Hertz stresses from 1.04×10^{9} to 2.42×10^{9} newtons per square meter (150 000 to 350 000 psi). Two crowned AISI M-50 steel disks, each with a radius of 1.83 centimeters (0.72 in.) and a surface finish of 2.5×10^{-6} to 5.0×10^{-6} centimeter (1 to 2 μin.) rms were used as the test specimens. Both crowned disks and crowned-cone disks (with a cone angle of 10°) were tested and no significant differences were found between the two sets of data. All data reported herein were generated with the latter disk configuration. The properties of the synthetic paraffinic oil which was used in the course of this study are listed in table I.

A summary of the X-ray test results from reference 7 showing measured minimum film thickness h_o for a synthetic paraffinic oil plotted against maximum Hertz stress p_{Hz} for various mean surface speeds u and disk temperatures T_o is presented in figure 3. The marked deviation of the X-ray data from theory at the higher stresses can be clearly seen from the dotted line in figure 3(a) whose slope represents the stress exponent of -0.22 from Cheng's theory (ref. 12). Evidently, within a heavily loaded contact, the film thickness dependence upon load as evidenced by the X-ray data is far greater than current EHD theory predicts.

One possible explanation for this anomaly is that the lubricant's pressure varia-

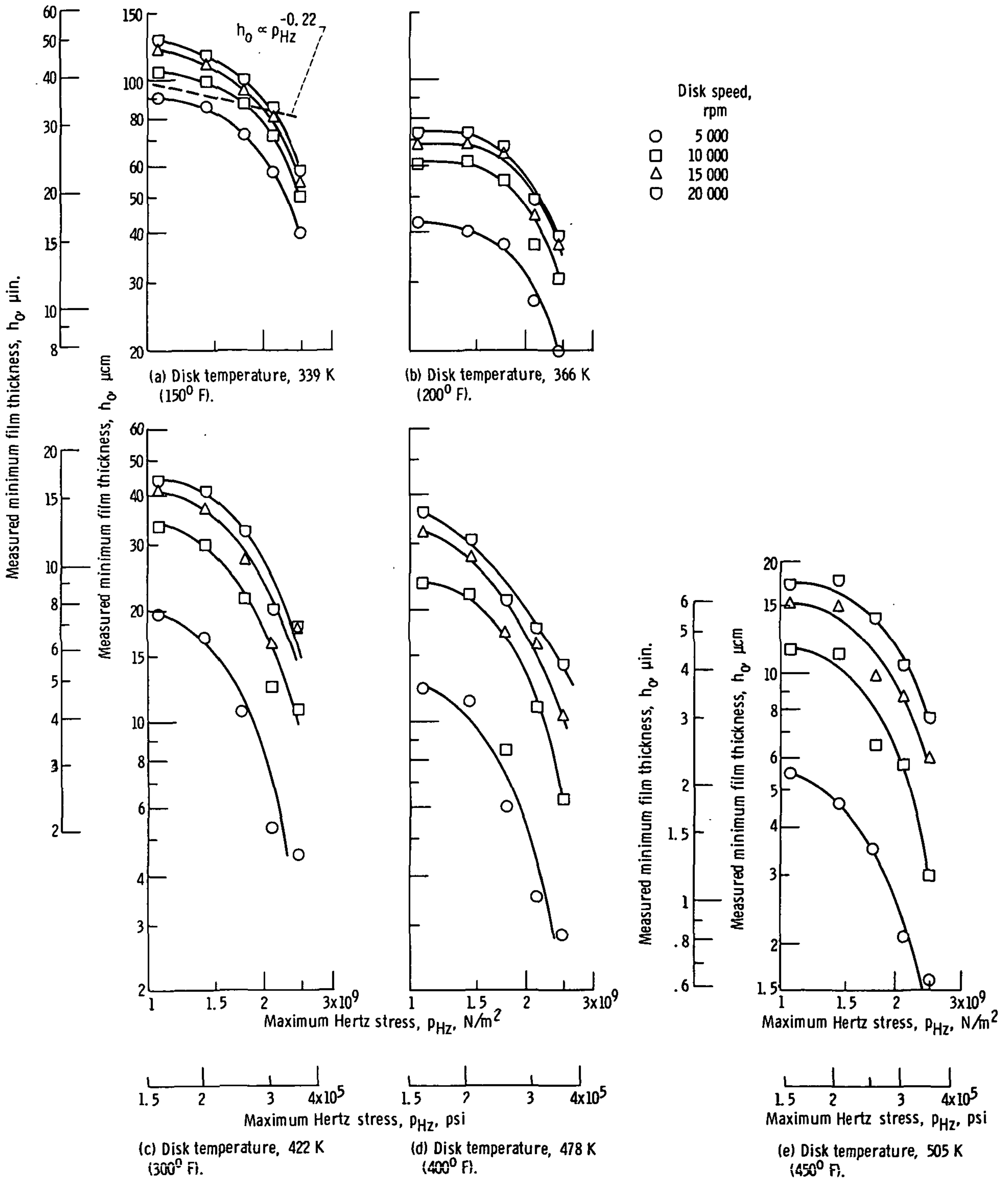

Figure 3. - X-ray test results showing effect of maximum Hertz stress on measured minimum film thickness with a synthetic paraffinic oil using crowned-cone disks (from ref. 7).

tion of viscosity does not follow the straight exponential model normally assumed by classical EHD theory. If the loss of pressure-viscosity dependence is to have an impact on the value of film thickness, it must occur at relatively low pressures. This is because the nominal film thickness is basically established at the inlet of the contact zone where the hydrodynamic pressures within the lubricant are relatively low, generally, just a fraction of the maximum contact pressure. This deduction has been recognized by Bell and Kannel (ref. 11). It is also supported by Cheng's work (ref. 12) which showed that the loss of the lubricant's pressure-viscosity dependence at even moderate pressures (i.e., 4.14×10^{8} N/m^2 (60 000 psi)) through the introduction of a composite experimental pressure-viscosity model ($\mu = \mu_o e^{\alpha p_1 + \gamma(p - p_1)}$) did not produce a significant difference upon the calculated value of film thickness.

The exact nature of the viscosity variation with pressure for the synthetic paraffinic oils is presently unknown. Lubricants are known to deviate from the familiar exponential form:

$$\mu = \mu_o e^{\alpha p} \tag{1}$$

where μ_o is the absolute viscosity at ambient pressure and p is the pressure above ambient. Chu and Cameron (ref. 15) reported that at high temperatures the straight paraffinic mineral oils do not obey the exponential relation shown in equation (1), even for low pressures as can be seen from figure 1. Based on the data from reference 16 which contains pressure-viscosity information for a large number of fluids, Chu and Cameron developed a power law relation to fit the pressure viscosity data of the following form:

$$\mu = \mu_o(1 + Kp)^n \tag{2}$$

where exponent n can be put equal to 16 and coefficient K is

$$K = k(10)^{\beta}\ \mu_o^{-0.062} \tag{3}$$

and where

$$\beta = -(\beta_o + \lambda T) \tag{4}$$

For constant K in units of $(\mathrm{N/m^2})^{-1}$, absolute viscosity μ_o in centipoises, and temperature T in K;

$$k = 8.99\times10^{-9}$$

$$\lambda = 0.0045$$

and constant β_0 was taken to be -0.75 for the straight paraffinic mineral oils.

However, for constant K in units of psi^{-1}, absolute viscosity μ_0 in reyns, and temperature T in ^{o}F;

$$k = 2.33\times10^{-5}$$

$$\lambda = 0.0025$$

and constant β_0 becomes 0.4.

It is of interest to examine the effects on film thickness of incorporating a power law viscosity variation with pressure of the kind described in equation (2) into an EHD inlet analysis of the Grubin type.

POWER LAW PRESSURE-VISCOSITY RELATION

Grubin (ref. 8) is credited with the first useful solution to the EHD problem of heavily loaded contacts. He had circumvented the lengthy numerical process attendant with the full EHD solution (e.g., see ref. 9) through a clever simplification. Grubin assumed that the elastic displacements of the contacting lubricated surfaces under heavy loads will be essentially unchanged from the deformed shape in dry Hertzian contact. Grubin's geometrical model appears in figure 4.

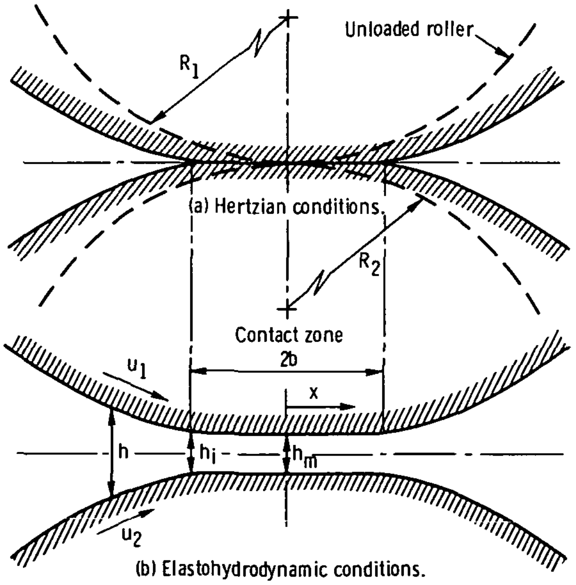

Figure 4. - Grubin's model of elastohydrodynamic contact.

With this assumption the film thickness distribution $h(x)$ outside of the contact zone could be represented by the following Hertzian expression:

$$\frac{h - h_m}{R'_x} = 2W\left[X\sqrt{X^2 - 1} - \ln\left(X + \sqrt{X^2 - 1}\right)\right] \tag{5}$$

where h_m is the film thickness at $dp/dx = 0$, that is, center film thickness in this instance, and W, the nondimensional load parameter, and X are defined as follows:

$$W = \frac{w}{E'R'_x} \quad \text{and} \quad X = \left|\frac{x}{b_h}\right| \tag{6}$$

Here b_h is the semi-width of the contact zone and is equal to

$$b_h = 2R'_x W^{1/2} \tag{7}$$

and w is the load per unit cylinder length. Parameters R'_x and E' are defined as

$$\frac{1}{R'_x} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{and} \quad \frac{1}{E'} = \frac{1 - \sigma_1^2}{\pi E_1} + \frac{1 - \sigma_2^2}{\pi E_2} \tag{8}$$

The hydrodynamic action of the lubricant within the contact zone is governed by the integrated form of the incompressible Reynolds equation as follows:

$$\frac{dp}{dx} = 12\,\mu u\left(\frac{h - h_m}{h^3}\right) \tag{9}$$

where dp/dx is the pressure gradient generated in the lubricant and u, the mean surface speed, is

$$u = \frac{u_1 + u_2}{2} \tag{10}$$

Equation (9) is developed on the assumption that the fluid is both Newtonian and incompressible. There is little to be lost by disregarding the effects of compressibility since the density increase normally experienced by the lubricant upon entering the contact zone is relatively small and consequently has little influence on the ultimate separation of the contacting surfaces (see ref. 1).

Using Grubin's simplifying assumption, that the film thickness is constant over most of the contact zone, (i.e., $h_i = h_m$, see fig. 4) and presuming that the lubricant's viscosity is solely a function of pressure, that is

$$\mu = \mu(p) \tag{11}$$

then equation (9) can be integrated from ambient pressure to the pressure at the inlet of the contact zone as follows:

$$\int_0^{p_i} \frac{dp}{\mu(p)} = 12u \int_{x=-\infty}^{-b_h} \left(\frac{h - h_i}{h^3}\right) dx \tag{12}$$

Utilizing the displacement expression shown in equation (5), Grubin numerically evaluated the integral appearing on the right-hand side of equation (12) for the practical range of h_i and found that

$$\int_{x=-\infty}^{-b_h} \left(\frac{h - h_i}{h^3}\right) dx = \frac{2}{R'_x} W^{-3/2} \times \text{Grubin's integral} \tag{13}$$

Dowson and Higginson (ref. 1) reported that Grubin's integral is fitted closely by

$$0.0986 \left(\frac{h_i/R'_x}{W}\right)^{-11/8} \tag{14}$$

therefore, equation (13) can be rewritten as

$$\int_{-\infty}^{-b_h} \left(\frac{h - h_i}{h^3}\right) dx = \frac{0.1972}{R'_x} W^{-1/8} \left(\frac{h_i}{R'_x}\right)^{-11/8} \tag{15}$$

and in terms of the inlet film thickness, rearranging equation (12) and combining with equation (15) yield

$$\frac{h_i}{R'_x} = 1.87\left(\frac{W'}{E'R'_x}\right)^{-1/11}\left(\frac{u_1 + u_2}{2R'_x}\right)^{8/11}\left[\int_0^{p_i}\frac{dp}{\mu(p)}\right]^{-8/11} \tag{16}$$

Equation (16) relates film thickness in terms of a generalized pressure-viscosity model. Grubin took the pressure variation of viscosity to be

$$\mu(p) = \mu_o e^{\alpha p} \tag{17}$$

He also assumed that the pressure developed at the inlet was very large, that is, $p_i \rightarrow \infty$. If these two conditions are applied, then equation (16) would reduce to the familiar Grubin formula

$$\frac{h_g}{R'_x} = 1.87\left(\frac{W'}{E'R'_x}\right)^{-1/11}\left[\frac{\alpha\mu_o(u_1 + u_2)}{2R'_x}\right]^{8/11} \tag{18}$$

noting that h_g has been substituted for h_i. Grubin's assumption that $p_i \rightarrow \infty$ yields reasonable values of film thickness for lubricants whose viscosity increases rapidly with pressure, that is, those with large values of α. This is true analytically because the $e^{-\alpha p}$ term which appears in the expression for film thickness when evaluating the pressure integral term in equation (16) is normally negligible for most practical situations. (An alteration to Grubin's theory for a finite inlet pressure can be found in ref. 11.) However, for those lubricants which display a reduced viscosity-pressure dependence, the incorporation of a finite inlet pressure into the theory can have an appreciable effect on film thickness. Substituting the viscosity relation shown in equation (2) into the pressure integral in equation (16) and performing the indicated integration yield

$$\int_0^{p_i}\frac{dp}{\mu(p)} = \frac{1}{K(n - 1)\mu_o}\left[1 - (1 + Kp_i)^{-n+1}\right] \tag{19}$$

Substituting equation (19) into equation (16) results in a complete film thickness formula. Such a formula is

$$\frac{h_i}{R'_x} = 1.87\left(\frac{W'}{E'R'_x}\right)^{-1/11}\left[\frac{\mu_o(u_1 + u_2)}{2R'_x}\right]^{8/11}\left\{\frac{1}{K(n - 1)}\left[1 - \frac{1}{(1 + Kp_i)^{n-1}}\right]\right\}^{-8/11} \tag{20}$$

If the condition $p_i \rightarrow \infty$ is applied to equation (20) and the result is then compared to Grubin's formula in equation (18), a quantity, say α_c, can be written as

$$\alpha_c = K(n - 1) \tag{21}$$

where α_c can be used in place of the customary α in Grubin's film thickness formula for lubricants obeying equation (2) in situations where the inlet pressure tends toward infinity. A similar expression was developed by Dyson, Naylor, and Wilson in reference 6 and Cameron and Gohar in reference 17.

For many practical EHD conditions, as will be shown later, the inlet pressure term in equation (20) can have an influence on the value of film thickness and should not be disregarded. For an approximate solution, it is convenient to express p_i as some fraction ξ of the maximum Hertz pressure. Thus, it is assumed that

$$p_i = \xi p_{Hz} \tag{22}$$

Now if exponent n in equation (2) is taken equal to its suggested value of 16 (ref. 15) and h_p is used to denote the film thickness for the power law lubricant, then equation (20) can be rewritten as

$$\frac{h_p}{R'_x} = 1.87\left(\frac{W'}{E'R'_x}\right)^{-1/11}\left[\frac{\mu_o(u_1 + u_2)}{2R'_x}\right]^{8/11}\left\{\frac{1}{15K}\left[1 - \frac{1}{\left(1 + \xi Kp_{Hz}\right)^{15}}\right]\right\}^{-8/11} \tag{23}$$

before numerical comparisons can be made with the previous film thickness relation, estimates are needed for coefficients ξ and K. The value of ξ is dependent upon the same parameters which influence film thickness, such as, surface speed, contact load, surface temperature, and type of lubricant as evidenced by the pressure distributions presented in reference 18. Nevertheless, taking $\xi = 0.3$, as an approximation, should yield informative results representative of the conditions under consideration. A similar approach was taken in reference 11.

For purposes of an approximate solution, a reasonable estimate for coefficient K can be made as follows: At low temperatures, less than about 100° F (311 K), reference 15 (fig. 1) shows that the viscosity predicted by the power law model in equation (2) does not deviate appreciably from the value predicted by the straight exponential model in equation (1) for pressures to 80 000 pounds per square inch (5.5×10^{8} N/m^2). Hence, at these conditions,

$$(1 + Kp)^{16} \approx e^{\alpha p} \tag{24}$$

or

$$(1 + Kp) \approx e^{\alpha p/16} \tag{25}$$

Expanding the right side of equation (25) in a Taylor series, we obtain

$$e^{\alpha p/16} = 1 + \frac{\alpha p}{16} + \frac{(\alpha p/16)^2}{2!} + \ldots + \frac{(\alpha p/16)^n}{n!} \tag{26}$$

Comparison of equations (25) and (26) shows that, for low pressures, a good first-order approximation for K is

$$K \approx \frac{\alpha}{16} \tag{27}$$

Currently, the only available published data for α of a synthetic paraffinic oil appears in reference 19 where α was reported to be 1.3×10^{-8} square meter per newton (0.92×10^{-4} psi^{-1}) at 298 K (77° F). Thus solving equations (3), (4), and (27) simultaneously at T = 298 K (77° F) one finds that

$$\beta_o = -0.48 \text{ for SI units} \tag{28}$$

or

$$\beta_o = 0.67 \text{ for U.S. customary units}$$

Using this value of β_o together with equations (3) and (4), one can calculate the value of K at the various disk temperatures as given in table II.

RESULTS AND DISCUSSION

A comparison of the power law film thickness formula developed in equation (23) with the experimental X-ray data (ref. 7) is presented in figures 5 to 7 and tabulated in table III. To contrast the present results with previous EHD theory, the isothermal film thickness equation of Cheng (ref. 20) has been included in these plots. Cheng's film thickness equation for line contact or more precisely elliptical contact with $b/a \geq 5$ can be written

$$\frac{h_c}{R_x'} = 1.47\left(\frac{\alpha\mu_o u}{R_x'}\right)^{0.74}\left(\frac{p_{Hz}}{E'}\right)^{-0.22} \tag{29}$$

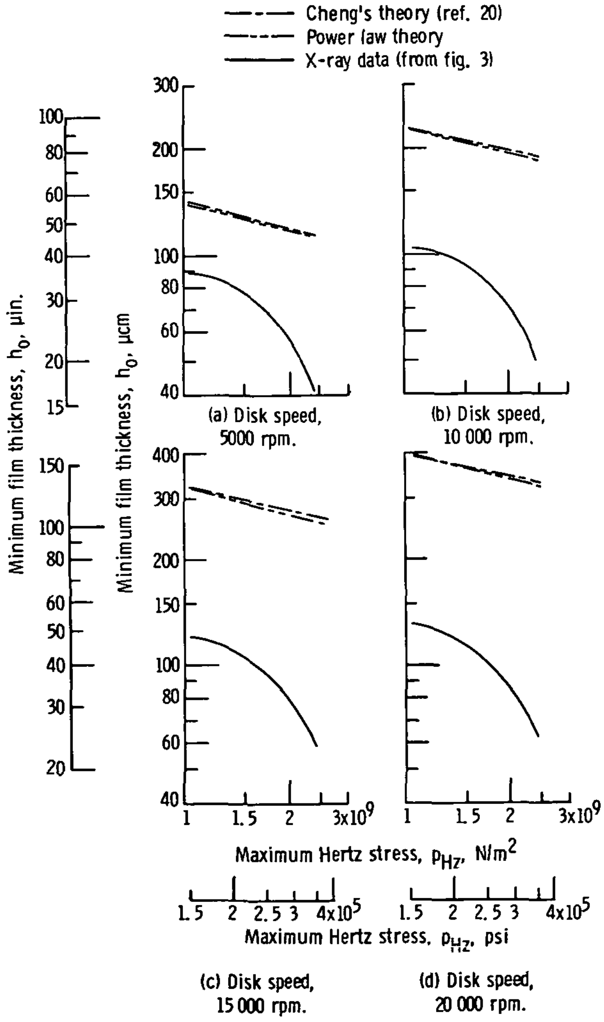

Figure 5. - Comparison of predicted minimum film thickness with X-ray data for synthetic paraffinic oil at disk temperature $T_0 = 339$ K (150° F).

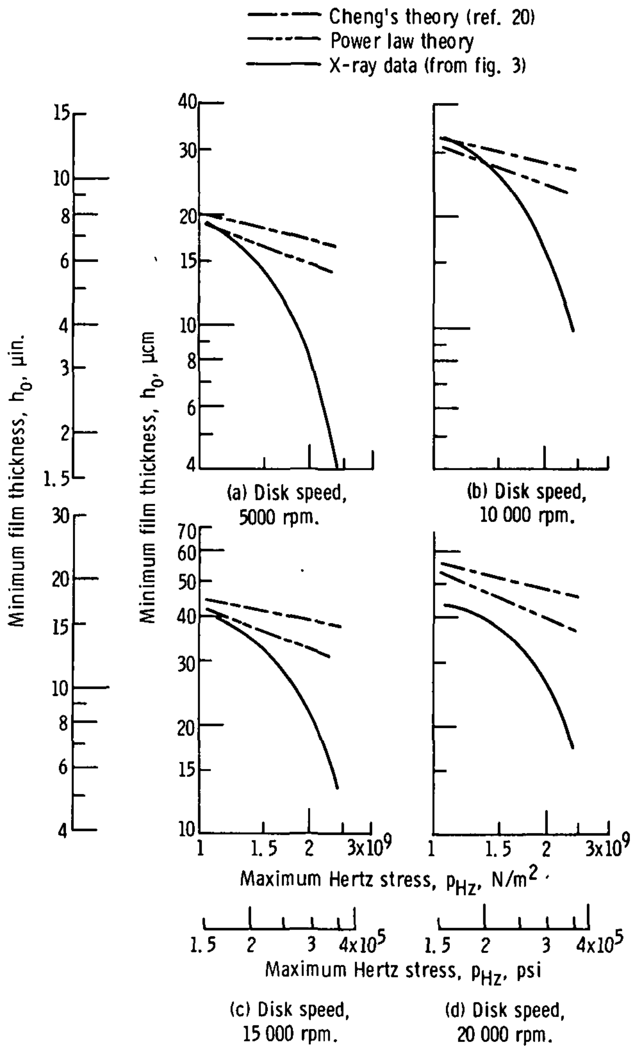

Figure 6. - Comparison of predicted minimum film thickness with X-ray data for synthetic paraffinic oil at disk temperature T_0 = 422 K (300° F).

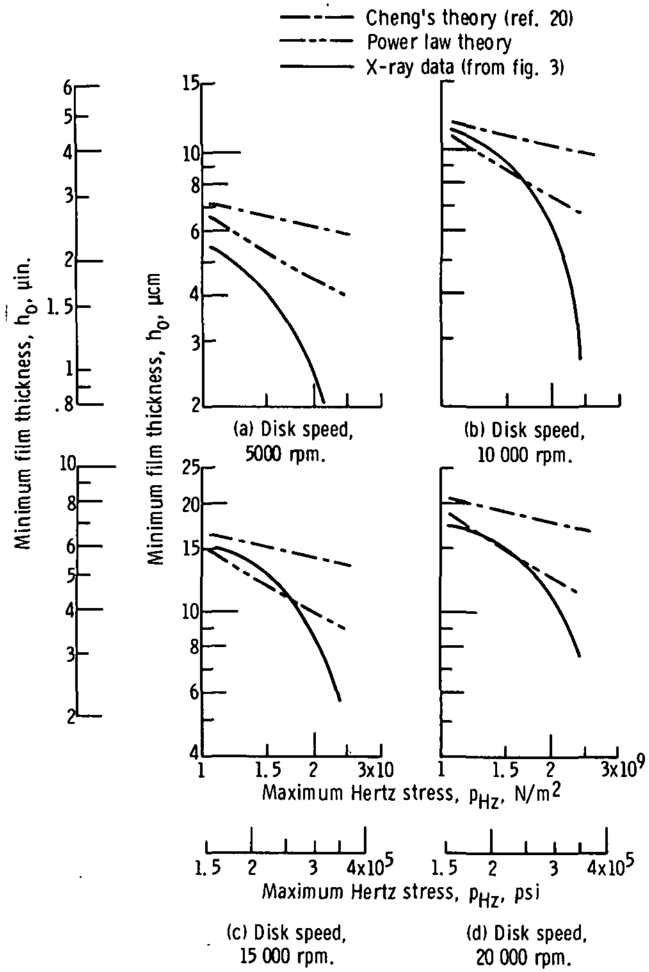

Figure 7. - Comparison of predicted minimum film thickness with X-ray data for synthetic paraffinic oil at disk temperature T_0 = 505 K (450° F).

In computing Cheng's minimum film thickness and the minimum film thickness from the present analysis, a correction factor of 0.8 has been applied to adjust the predicted center film thickness to minimum film thickness. The geometry of a typical EHD contact showing the relative size of the inlet, central, and minimum film thicknesses, is presented in figure 8.

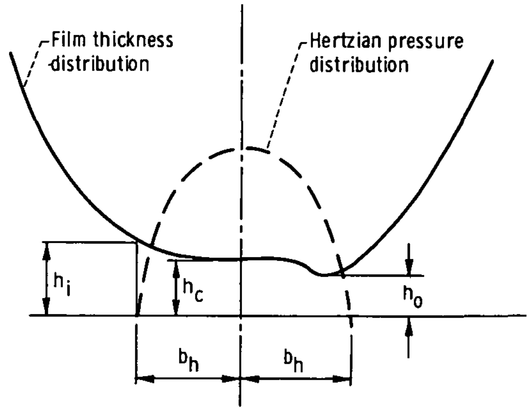

Figure 8. - Geometry of typical elastohydrodynamic contact showing relative size of inlet (h_i), central (h_c), and minimum (h_0) film thicknesses.

Before one can make a fair comparison of Cheng's formula with the other data, an estimate of the pressure-viscosity coefficient α at various temperatures is required. Unfortunately, data of this kind is not currently available for synthetic paraffinic oils. To overcome this obstacle, a first-order approximation of the variation of α for the synthetic paraffinic oil was modeled after the trend exhibited by the paraffinic mineral oils determined by the ASME Research Committee on Lubrication (ref. 16). The pressure-viscosity coefficients at ambient pressure for five paraffinic mineral oils labeled 31G to 35G were determined at temperatures of 273, 298, 311, 372, and 491 K (32°, 77°, 100°, 210°, and 425° F). Figure 9 shows a plot of α at the aforementioned temperatures normalized by α at 298 K (77° F) as a function of temperatures for the five oils. By applying this factor ($\alpha/\alpha_{298\ K}$ or $\alpha/\alpha_{77^\circ\ F}$) to the synthetic paraffinic oil's pressure-viscosity coefficient a curve of α against temperatures as presented in figure 10 can be obtained. A comparison of the exponential pressure-viscosity model (eq. (1)) with the power law pressure-viscosity model (eq. (2)) utilizing the estimated pressure-viscosity coefficients α and K appears in figure 11.

Incorporating the estimated value of α interpolated from figure 10 into Cheng's equation leads to a reasonably good trend and magnitude agreement with the X-ray data at moderate contact pressures, $p_{Hz} = 1.04\times10^9$ newtons per square meter (150 000 psi), at least for the higher temperatures as can be seen from figures 6 and 7. This agree-

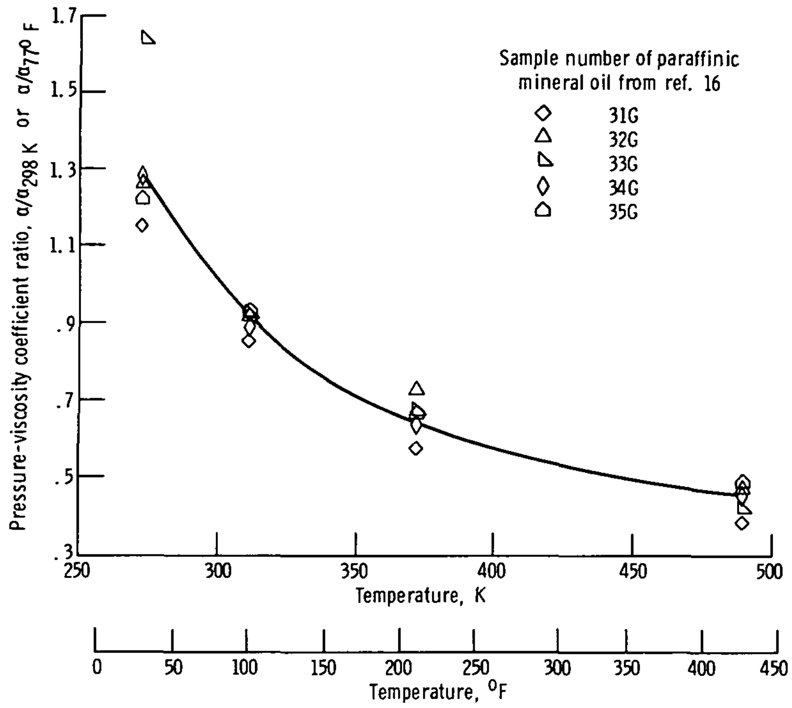

Figure 9. - Effect of temperature on pressure-viscosity coefficient for paraffinic mineral oils at ambient pressure.

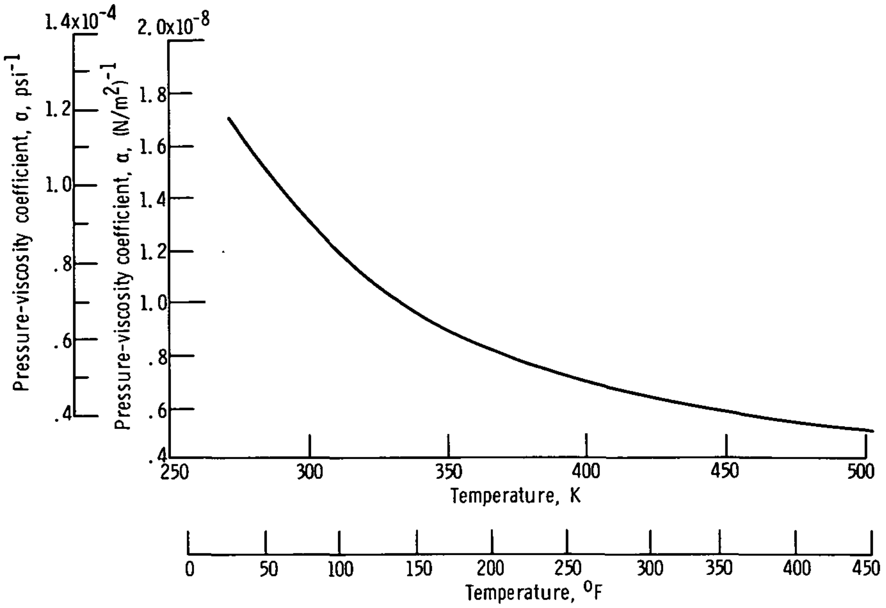

Figure 10. - Estimated pressure-viscosity coefficient variation with temperature for synthetic paraffinic oil.

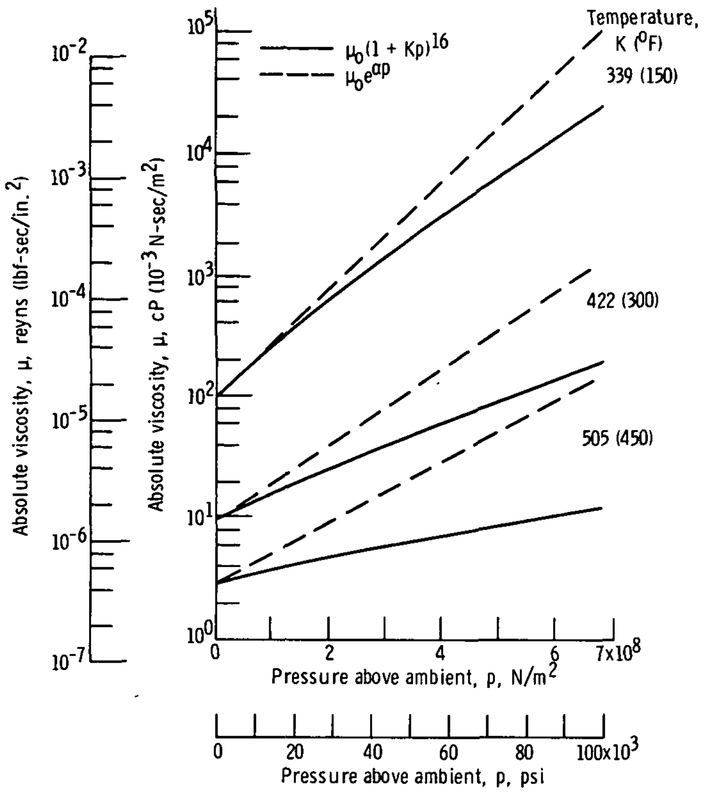

Figure 11. - Comparison of power law and straight exponential viscosity variation with pressure for synthetic paraffinic oil.

ment would not have been possible had the reduction of α with increasing temperature not been considered. Although the slopes of Cheng's film thickness formula are compatible with the X-ray results toward the lower load regime, they deviate substantially as the load increases.

The present theory from equation (23) differs little from previous theories at the lower temperatures. As previously discussed, the lubricant's pressure variation of viscosity can be adequately represented by either the exponential or power law models to moderate pressures at low disk temperatures as evidenced by figure 11. At the higher disk temperatures, the present theory displays a slightly stronger load dependence than previous isothermal EHD theories, but it also fails to account for the rapid reduction of film thickness at the higher loads.

The physical mechanism responsible for the high load dependence displayed by the measured minimum film thickness remains unexplained. There is some evidence (e.g., ref. 19 for the case of point contact) which suggests that the minimum film thickness may be inherently more sensitive to applied load than is the central film thickness. If this is the case, then a simple inlet-type EHD analysis would not be capable of reflecting this increased film thickness load sensitivity. Nevertheless, it is evident that the use of current EHD film thickness formulas, for most design purposes, will yield unrealistically high values of the minimum film thickness at maximum Hertz stress much above 1.04×10^9 newtons per square meter (150 000 psi).

SUMMARY OF RESULTS

An isothermal elastohydrodynamic (EHD) inlet analysis of the Grubin type which considers a power law viscosity variation with pressure ($\mu = \mu_0(1 + Kp)^n$) and a finite pressure at the inlet edge of the Hertzian contact zone was performed. Results from the present EHD inlet analysis were compared to the X-ray test data reported in NASA TN D-6411 and to the results from a conventional EHD analysis for line contact. Both present and previous EHD theories were numerically evaluated with estimated values of pressure-viscosity coefficients α and K for a synthetic paraffinic oil. The results of this comparison are as follows:

1. Analytical results from the elastohydrodynamic theory utilizing a power law pressure-viscosity relation display a slightly stronger film thickness dependence upon load than previous isothermal theories which assume a straight exponential pressure-viscosity relation. However, the model did not adequately define the decrease in film thickness with contact stress displayed by the X-ray test data.

2. The minimum film thickness predicted by conventional elastohydrodynamic theory exhibits reasonably good trend and magnitude agreement with the test data at moderate contact stresses (1.04×10^9 N/m^2 (150 000 psi)) for the higher disk temperatures (above 422 K (300° F)).

3. For contact stresses above 1.04×10^9 newtons per square meter (150 000 psi), the conventional elastohydrodynamic theory fails to account for the radical reduction of film thickness with increasing load displayed by the test data.

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National Aeronautics and Space Administration,
and
U.S. Army Air Mobility R&D Laboratory,
Cleveland, Ohio, September 19, 1972,
501-24.

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TABLE I. - PROPERTIES OF THE SYNTHETIC PARAFFINIC OIL

Kinematic viscosity, cS (or 10^{-6} m^2/sec), at -	
233 K (-40° F)	>100 000
311 K (100° F)	443
372 K (210° F)	40
478 K (400° F)	5.8
589 K (600° F)	[a]2.3
Flash point, K (°F)	541 (515)
Fire point, K (°F)	589 (600)
Autoignition temperature, K (°F)	703 (805)
Volatility (6.5 hr at 533 K (500° F)), wt. %	14.2
Specific heat at 533 K (500° F), J/(kg)(K) (Btu/(lb)(°F))	2910 (0.695)
Thermal conductivity at 533 K (500° F), J/(m)(sec)(K) (Btu/(hr)(ft)(°F))	0.12 (0.070)
Specific gravity at 533 K (500° F)	0.71
Pressure-viscosity coefficient at 298 K (77° F), m^2/N (psi^{-1})	[b]1.3×10^{-8} (0.92×10^{-4})

[a]Extrapolated.
[b]From ref. 19.

TABLE II. - ESTIMATED VALUES OF PRESSURE-VISCOSITY COEFFICIENT K FOR A SYNTHETIC PARAFFINIC OIL

Disk temperature		Pressure-viscosity coefficient, K	
K	°F	m^2/N	psi^{-1}
339	150	60.7×10^{-11}	41.9×10^{-7}
366	200	48.5	33.4
422	300	29.6	20.4
478	400	17.5	12.1
505	450	13.3	9.2

TABLE III. - COMPARISON OF PREDICTED MINIMUM FILM THICKNESS WITH X-RAY DATA

(a) SI units

Disk temperature, K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_o, μcm		
			X-ray data[a]	Power law theory	Cheng's theory (ref. 20)
339	5000	1.04×10^9	89	147	142
		1.38	86	135	134
		1.72	71	127	127
		2.07	58	123	122
		2.42	41	119	120
	10 000	1.04×10^9	107	242	240
		1.38	99	223	219
		1.72	86	211	212
		2.07	71	203	206
		2.42	51	197	198
	15 000	1.04×10^9	119	325	322
		1.38	109	300	302
		1.72	94	284	288
		2.07	81	271	277
		2.42	53	263	267
	20 000	1.04×10^9	127	401	399
		1.38	117	368	374
		1.72	102	350	356
		2.07	84	334	343
		2.42	58	325	330
366	5000	1.04×10^9	43	64	63
		1.38	41	58	59
		1.72	38	55	56
		2.07	25	52	53
		2.42	20	51	52
366	10 000	1.04×10^9	61	107	108
		1.38	61	97	98
		1.72	56	90	94
		2.07	38	86	89
		2.42	30	84	86
	15 000	1.04×10^9	66	142	140
		1.38	66	129	132
		1.72	64	121	126
		2.07	43	116	122
		2.42	38	112	117
	20 000	1.04×10^9	71	176	174
		1.38	71	160	164
		1.72	66	149	156
		2.07	48	143	150
		2.42	41	138	145
422	5000	1.04×10^9	20	20	20
		1.38	18	17	19
		1.72	10	16	18
		2.07	5	14	17
		2.42	5	14	17
	10 000	1.04×19^9	33	32	33
		1.38	30	28	32
		1.72	20	25	30
		2.07	12	24	28
		2.42	10	23	27

[a]Crowned-cone disk data from ref. 7.

TABLE III. - Continued. COMPARISON OF PREDICTED MINIMUM FILM THICKNESS WITH X-RAY DATA

(a) Concluded. SI units

Disk temperature K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_o, μcm		
			X-ray data[a]	Power law theory	Cheng's theory (ref. 20)
422	15 000	1.04×10^9	41	43	46
		1.38	35	38	43
		1.72	25	34	41
		2.07	15	32	39
		2.42	15	31	38
	20 000	1.04×10^9	43	53	56
		1.38	41	46	53
		1.72	30	42	51
		2.07	20	40	48
		2.42	20	38	47
478	5000	1.04×10^9	12	9	10
		1.38	10	8	9
		1.72	5	7	9
		2.07	2	6	9
		2.42	2	6	9
	10 000	1.04×10^9	23	15	16
		1.38	23	13	15
		1.72	8	11	14
		2.07	10	10	14
		2.42	8	10	13
	15 000	1.04×10^9	30	20	22
		1.38	28	17	21
		1.72	18	15	20
		2.07	15	14	19
		2.42	10	13	18
478	20 000	1.04×10^9	36	25	28
		1.38	30	21	25
		1.72	20	18	25
		2.07	18	17	24
		2.42	15	16	23
505	5000	1.04×10^9	5	7	7
		1.38	5	6	7
		1.72	2	5	7
		2.07	2	4	6
		2.42	2	4	6
	10 000	1.04×10^9	12	11	12
		1.38	10	9	11
		1.72	8	8	11
		2.07	5	7	11
		2.42	2	7	10
	15 000	1.04×10^9	15	15	17
		1.38	15	12	15
		1.72	10	11	15
		2.07	8	10	14
		2.42	5	9	14
	20 000	1.04×10^9	18	19	21
		1.38	18	15	19
		1.72	15	13	18
		2.07	10	12	18
		2.42	8	11	17

[a]Crowned-cone disk data from ref. 7.

TABLE III. - Continued. COMPARISON OF PREDICTED MINIMUM FILM THICKNESS WITH X-RAY DATA

(b) U.S. customary units

Disk temperature, K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_o, μcm		
			X-ray data[a]	Power law theory	Cheng's theory (ref. 20)
150	5000	1.5×10^5	35	58	56
		2.0	34	53	53
		2.5	28	50	50
		3.0	23	48	48
		3.5	16	47	47
	10 000	1.5×10^5	42	95	94
		2.0	39	88	88
		2.5	34	83	84
		3.0	28	80	81
		3.5	20	78	78
	15 000	1.5×10^5	47	128	127
		2.0	43	117	119
		2.5	37	112	113
		3.0	32	107	109
		3.5	21	103	105
	20 000	1.5×10^5	50	158	157
		2.0	46	145	147
		2.5	40	138	140
		3.0	33	132	135
		3.5	23	128	130
200	5000	1.5×10^5	17	25	25
		2.0	16	23	23
		2.5	15	22	22
		3.0	10	21	21
		3.5	8	20	20
200	10 000	1.5×10^5	24	42	41
		2.0	24	38	39
		2.5	22	36	37
		3.0	15	34	35
		3.5	12	33	34
	15 000	1.5×10^5	26	56	55
		2.0	26	51	52
		2.5	25	48	50
		3.0	17	46	48
		3.5	15	44	46
	20 000	1.5×10^5	28	69	69
		2.0	28	63	64
		2.5	26	59	61
		3.0	19	56	59
		3.5	16	54	57
300	5000	1.5×10^5	8	8	8
		2.0	7	7	7
		2.5	4	6	7
		3.0	2	6	7
		3.5	2	5	7
	10 000	1.5×10^5	13	13	13
		2.0	12	11	12
		2.5	8	10	12
		3.0	5	10	11
		3.5	4	9	11

[a]Crowned-cone disk data from ref. 7.

TABLE III. - Concluded. COMPARISON OF PREDICTED MINIMUM FILM THICKNESS WITH X-RAY DATA

(b) Concluded. U.S. customary units

Disk temperature, K	Disk speed, rpm	Maximum Hertz stress, N/m^2	Minimum film thickness, h_o, μcm		
			X-ray data[a]	Power law theory	Cheng's theory (ref. 20)
300	15 000	1.5×10^5	16	17	18
		2.0	14	15	17
		2.5	10	14	16
		3.0	6	13	15
		3.5	6	12	15
	20 000	1.5×10^5	17	21	22
		2.0	16	18	21
		2.5	12	17	20
		3.0	8	16	19
		3.5	8	15	18
400	5000	1.5×10^5	5	4	4
		2.0	4	3	4
		2.5	2	3	4
		3.0	1	2	4
		3.5	1	2	3
	10 000	1.5×10^5	9	6	6
		2.0	9	5	6
		2.5	3	4	6
		3.0	4	4	6
		3.5	3	4	5
	15 000	1.5×10^5	12	8	9
		2.0	11	7	8
		2.5	7	6	8
		3.0	6	5	7
		3.5	4	5	7
400	20 000	1.5×10^5	14	9	11
		2.0	12	8	10
		2.5	8	7	10
		3.0	7	6	9
		3.5	6	6	9
450	5000	1.5×10^5	2	3	3
		2.0	2	2	3
		2.5	1	2	3
		3.0	1	2	2
		3.5	1	2	2
	10 000	1.5×10^5	5	4	5
		2.0	4	4	5
		2.5	3	3	4
		3.0	2	3	4
		3.5	1	3	4
	15 000	1.5×10^5	6	6	7
		2.0	6	5	6
		2.5	4	4	6
		3.0	3	4	6
		3.5	2	4	5
	20 000	1.5×10^5	7	7	8
		2.0	7	6	8
		2.5	6	5	7
		3.0	4	5	7
		3.5	3	4	7

[a]Crowned-cone disk data from ref. 7.

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