N73-15705

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PROPAGATION OF WAVES OF ACOUSTIC FREQUENCIES IN CURVED DUCTS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1973

1. Report No. NASA TM X-2698	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle PROPAGATION OF WAVES OF ACOUSTIC FREQUENCIES IN CURVED DUCTS		5. Report Date January 1973
		6. Performing Organization Code
7. Author(s) Wojciech Rostafinski		8. Performing Organization Report No. E-7207
9. Performing Organization Name and Address Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio 44135		10. Work Unit No. 501-24
		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Waves Acoustics Motion in bends	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	12	$3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151

PROPAGATION OF WAVES OF ACOUSTIC FREQUENCIES IN CURVED DUCTS

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SUMMARY

The propagation of waves of acoustic frequencies in curved ducts is studied for the first four modes. The analysis makes use of Bessel functions of the order $(n + \frac{1}{2})$ to construct curves of wave number in the duct versus imposed wave number. The results apply to ducts of arbitrary width and arbitrary radii of curvature.

The characteristics of motion in a bend are compared with propagation of waves in a straight duct, and important differences in the behavior of waves are noted.

INTRODUCTION

Propagation of waves in curved ducts and pipes is characterized by wave patterns totally different from those known in straight ducts or in unlimited space (ref. 1). The curvilinear boundaries are responsible for the appearance of a continuous standing radial wave which in turn affects the transmitted tangential waves.

However, the method of the basic analysis on long waves (frequency less than 5 Hz) could not be extrapolated to higher frequencies, and a different analytical approach was needed.

Bessel functions of the order $(n + \frac{1}{2})$ are used to generate characteristic curves of the angular wave number versus imposed wave number. By using these curves one can obtain the angular wave number for any desired (imposed) frequency and examine the motion of waves in bends in detail.

Analysis of waves in the acoustic frequency range, at the present time up to 2500 hertz, is of interest in several areas of engineering. Besides problems of propagation and attenuation of noise in bends it has an application in studies of propagation of disturbances through compressor blade rows.

The analysis which follows concern a stabilized motion in bends with the exclusion of the transient effects in junctions between bends and straight lines or any other discontinuities. The physical systems considered consists of a two-dimensional circular bend with inner and outer wall radii R_1 and R_2, respectively. The walls will be assumed perfectly rigid.

ANALYSIS

The linearized wave equation in cylindrical coordinates is known to be separable in coordinates proper for the boundary, and its general solution may be written in the following form:

$$\varphi = \sum_{n=0}^{\infty} e^{i\omega t}\left[(iC_n - D_n)e^{-i\nu_n\theta} + (iC_n + D_n)e^{i\nu_n\theta}\right]\frac{F_{\nu_n}}{\sin(\pi\nu_n)}$$

where $k = \omega/c$ is the wave number of the imposed motion at the inlet and

$$F_{\nu_n} = J_{\nu_n}(kr)\,J'_{-\nu_n}(kR_1) - J_{-\nu_n}(kr)\,J'_{\nu_n}(kR_1) =$$

$$\sin(\pi_{\nu_n})\left(J_{\nu_n}(kr)\,Y'_{\nu_n}(kR_1) - Y_{\nu_n}(kr)\,J'_{\nu_n}(kR_1)\right)$$

where the primes indicate differentiation with respect to radius. The function F_{ν_n}, as shown, was established using the boundary condition $\partial\varphi/\partial r = 0$ at walls R_1 and R_2. The integration constants, still undetermined, may be used to match the flow in a bend with flow in any ducting upstream and downstream of the bend. In the present analysis the term depending on the angular position in the bend and containing these two constants will not be evaluated. Only the sustained motion in a bend of infinite length (like in a coil) will be examined. Consequently for the far end boundary condition no reflection of waves need be considered and we set $iC_n = -D_n$. The term in the bracket reduces to $-2\,D_n \exp i(-\nu_n\theta)$. The tangential and the radial vibrational velocities v_θ and v_r, respectively, are as follows:

$$v_\theta = \sum_{n=0} -2\,D_n\,(-i\nu_n)\,\frac{F_{\nu_n}}{r\,\sin(\nu_n\pi)}\,e^{i(\omega t-\nu_n\theta)}$$

$$v_r = \sum_{n=0} -2\, D_n \frac{F'_{\nu_n}}{\sin(\nu_n \pi)} e^{i(\omega t - \nu_n \theta)}$$

where F'_{ν_n} is the derivative of F_{ν_n} with respect to r

$$F'_{\nu_n}(kr, kR_1) = J'_{\nu_n}(kr)\, J_{-\nu_n}(kR_1) - J'_{-\nu_n}(kr)\, J'_{\nu_n}(kR_1) \tag{1}$$

If we set $r = R_2 = aR_1$ with $a = R_2/R_1$ and equate equation (1) to zero, we obtain

$$F'_{\nu_n}(akR_1, kR_1) = J'_{\nu_n}(akR_1)\, J'_{-\nu_n}(kR_1) - J'_{-\nu_n}(akR_1)\, J'_{\nu_n}(kR_1) = 0 \tag{2}$$

We are interested in finding the real roots of this characteristic equation, the ν_n's. They will be fractional numbers, functions of (kR_1) and of the parameter a. The ν_n's have the character of an angular wave number and determine the angular phase velocity $\dot{\theta} = \omega/\nu_n$ in curved ducts. Determination of the roots ν_n, not available in tables, required use of a special procedure. This procedure uses Bessel functions of the first and second kinds of the order $(n + \frac{1}{2})$ which are characterized by closed form expressions. Using published (ref. 2) expressions for spherical Bessel functions and the Raleigh's formula, F'_{ν_n} were calculated for $r = aR_1$, $\nu_n = n + \frac{1}{2}$ $(n = 0, 1, 2, \ldots)$ and for (kR_1), in small increments, ranging from 0.1 to 20. This study gave a series of roots (kR_1) satisfying characteristic equation (2). Solution of this inverse problem allowed construction of curves and tables which yield, by interpolation, values of ν_n for any given (kR_1). Typical results are shown on figure 1. The graph gives the angular wave numbers ν's for any arbitrary, imposed, wave number parameter (kR_1). The calculated curves are for $a = R_2/R_1 = 2.0$. The graph indicates that, up to $(kR_1) \cong 3.20$, only a single mode will be transmitted by a curved duct. At $3.20 \lesssim kR_1 \lesssim 6.30$ two modes may be transmitted provided that both modes are present at the inlet to the bend. In order to interpolate between $\nu_n = (n + \frac{1}{2})$, the function F'_{ν_n} was formed using general expansion of Bessel functions, limited to 15 terms, for the arbitrary, noninteger ν. The results of calculations are given in table I. To each ν_n correspond two columns of values of (kR_1), one calculated from the closed form expressions for the Bessel functions, the other using 15 terms of the Bessel series. It will be noted that all roots (kR_1) of the inverse problem, and consequently, the ν for the direct problem (when (kR_1) are imposed) are calculable by both expressions with high accuracy in the basic (zeroth) mode and in the first mode up to $\nu_1 = 3.5$.

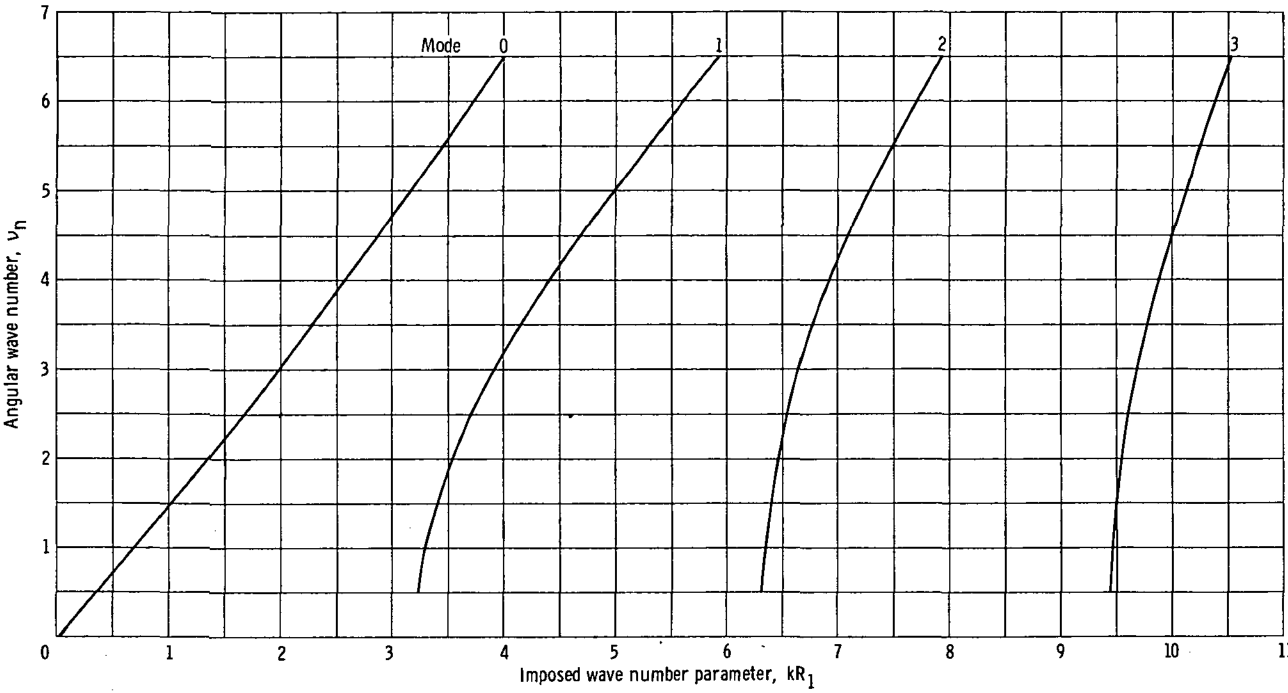

Figure 1. - Characteristics of motion in a bend for $a = R_2/R_1 = 2$.

The roots of the second and higher modes are not exactly calculable by the general series expression. The effect of the curvature of the duct on the behavior of waves will be more apparent by comparing figure 1 to figure 2, which illustrates the behavior of waves in straight ducts. Figure 2 was established using the theory given in several texts (refs. 3 and 4). The equation for the velocity potential is

$$\varphi = \sum_{n=0} G_n \cos\left[\frac{n\pi}{2L}(y + L)\right] \exp i(\omega t - k_n x)$$

where $k = \omega/c$ and $k_n = \pi\left[(k/\pi)^2 - (n/2L)^2\right]^{1/2}$. The 2L is the width of the duct. The motion in a straight duct is governed by the imposed wave number k and by the wave number k_n resulting from the interaction of the duct and the imposed motion at the inlet. The diagram of figure 2 is drawn for a straight duct of the same width as the curved duct of figure 1, if $R_1 = 0.2$ meter. The main difference between the two maps is in the basic mode. In the straight duct the characteristic is linear; in the curved it is not. More significant differences will be apparent by calculation of the vibrational velocities for the two systems. In the straight duct the vibrational velocities of the basic (zeroth) mode are axial and uniform across the duct; they do not depend on frequency. In the curved duct, the vibrational velocities (tangential and radial) calculated for the zeroth mode and shown (normalized) on figure 3, depend on frequency. At a

TABLE I. - ZEROS (kR_1) OF $F'_\nu(2kR_1, kR_1)$ FOR $a = R_2/R_1 = 2$

ν =	0.5		1.5		2.5		3.5		4.5		5.5		6.5	
	Values of kR_1 calculated by —													
Mode	Exact expression	Approximate expansion	Exact expression	Approximate expansion	Exact expression	Approximate expansion	Exact expression	Approximate expansion	Exact expression	Approximate expansion	Exact expression	Approximate expansion	Exact expression	Approximate expansion
0	0.339582	0.33958	1.011525	1.0115	1.6633	1.663	2.28692	2.287	2.8817	2.8815	3.45234	3.45233	4.00651	4.0065
1	3.2182	3.2182	3.38770	3.38770	3.70998	3.71	4.15752	4.15749	4.698	4.7011	5.2992	5.21	5.93281	-------
2	6.3226	6.018	6.40403	-------	6.56480	7.39	6.80112	5.78	7.108	------	7.481	6.96	7.91535	8.04
3	9.4512	9.4512	9.50483	9.068	9.61144	10.84	9.7698	8.63	9.978	9.92	10.234	11.05	10.535	11.74

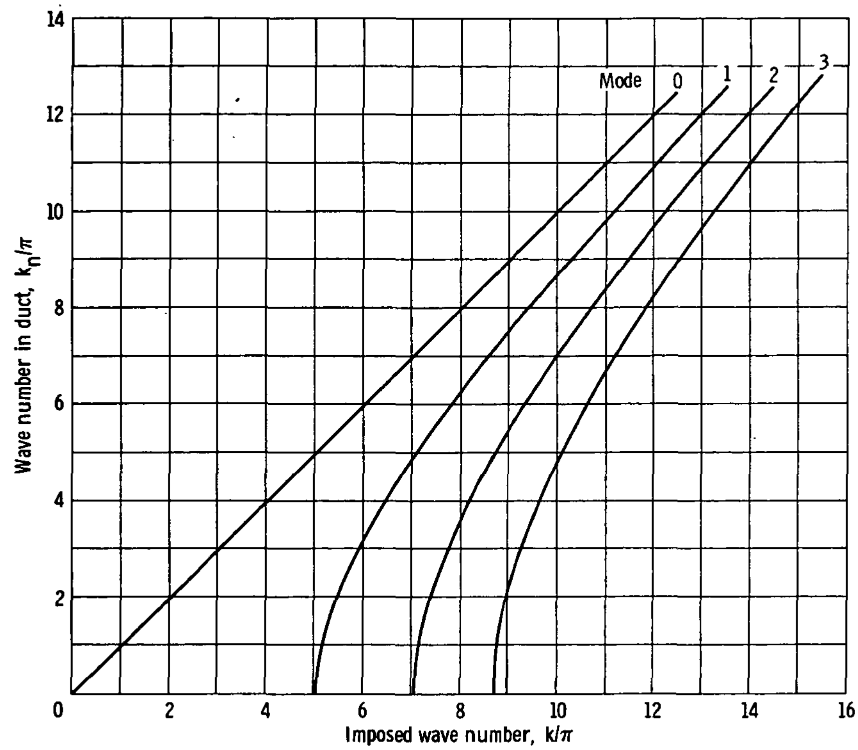

Figure 2. - Characteristics of motion in straight ducts for 2L = 0.2 meter.

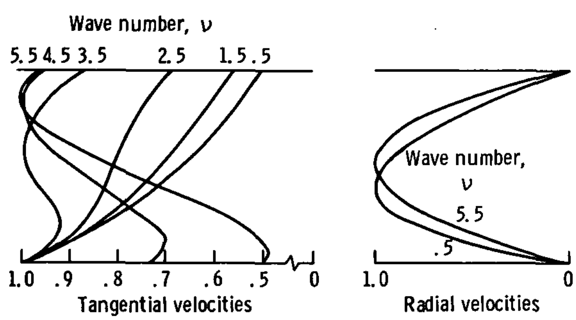

Figure 3. - Vibrational velocities of zeroth mode in curved duct of $a = R_2/R_1 = 2$ for a range of frequencies. All curves are normalized to maximum amplitude of 1.0.

low frequency $(\nu_o = \frac{1}{2})$, the distribution of tangential velocities across the duct's width is very nearly that of a potential vortex, inversely proportional to radius. At a higher frequency $(\nu_o = 5\frac{1}{2})$ the distribution becomes more of the forced vortex type, proportional to radius. However, the radial velocities exhibit little change in the same range of frequencies; a quasi-parabolic distribution remains and only the maximum is shifting continuously upwards with frequency. To continue evaluation of the data in figures 1 and 2 vibrational velocities have been calculated for four modes and for a single angular wave number $(\nu = \frac{1}{2})$. The results are shown in figure 4. In figure 5

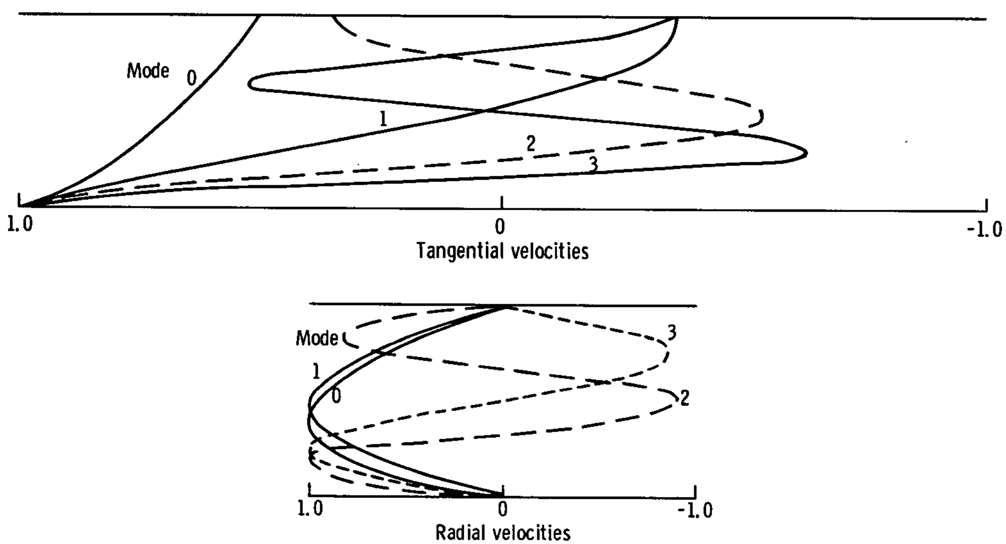

Figure 4. - Vibrational velocities in curved duct for $\nu_0 = 0.5$ for four consecutive modes. All curves are normalized to maximum amplitude of 1.0.

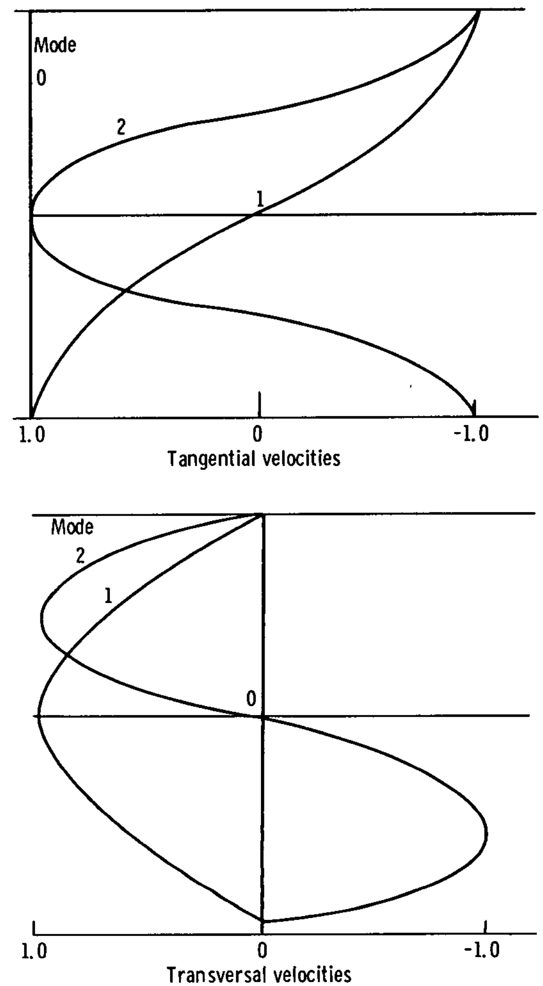

Figure 5. - Vibrational velocities in straight duct for one frequency and three consecutive modes.

the same type of data is presented for a straight duct. The vibrational velocities in a curved duct exhibit more pronounced changes than in a straight duct. It will be noted that the first and the higher modes in a straight duct are characterized by appearance of transversal velocities not present in the zeroth mode. In the case of a curved duct, the width of a duct or sharpness of the bend have a significant effect on the characteristic of the motion of waves. In figure 6, the data of figure 1 are supplemented by

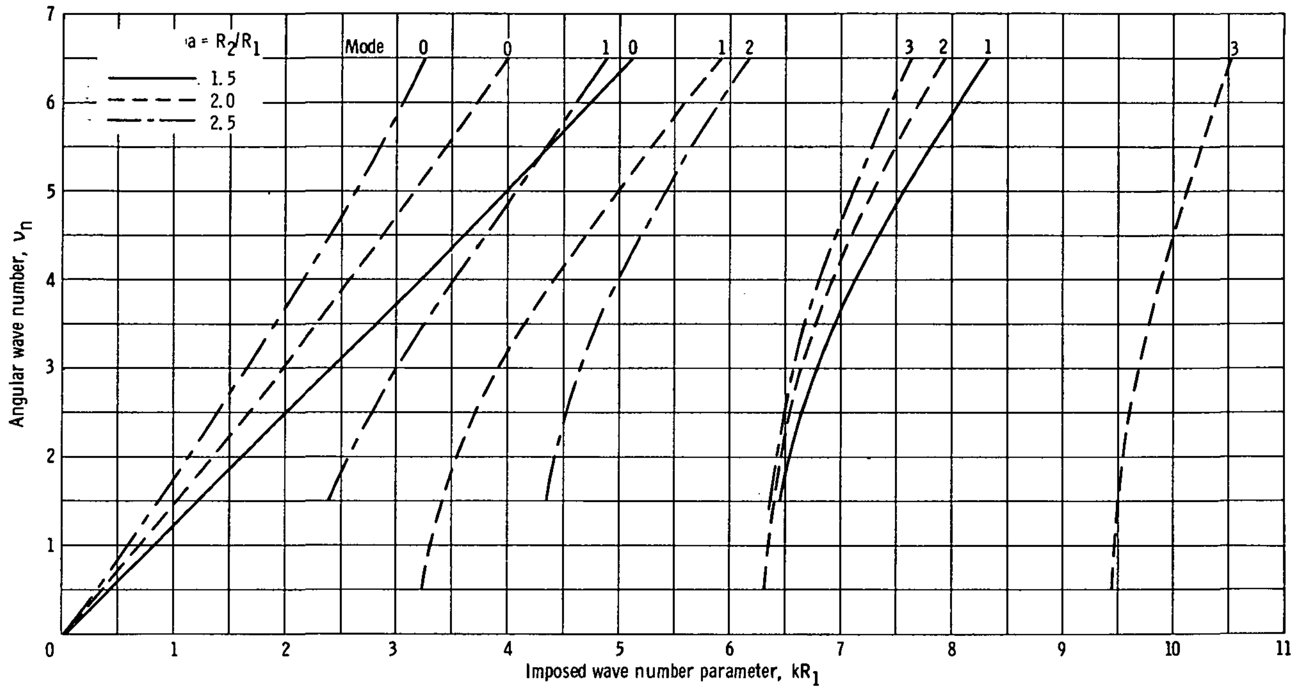

Figure 6. - Characteristics of motion in bends for three bends of different widths.

curves calculated for $a = 1.5$ and 2.5. In figure 7 the data of figure 2 are supplemented by data pertaining to a duct only half as wide as the duct of figure 1. Comparing the two maps we note that the width of a straight duct has no impact on the character of the basic mode. In the curved duct the width of the channel has a significant effect. The wave number will change, and the distribution of velocities will be altered. In both the straight and the curved ducts, the narrower the duct is the less chances are that higher modes will be transmitted. A wide duct admits higher modes much more easily. Roots of equation (2) for several values of the wave number ν_n and for two values of a are given in table II. They complete the data of table I. It was found that interpolation between data of the two tables was satisfactory provided that a curve is traced between three points taken from the tables.

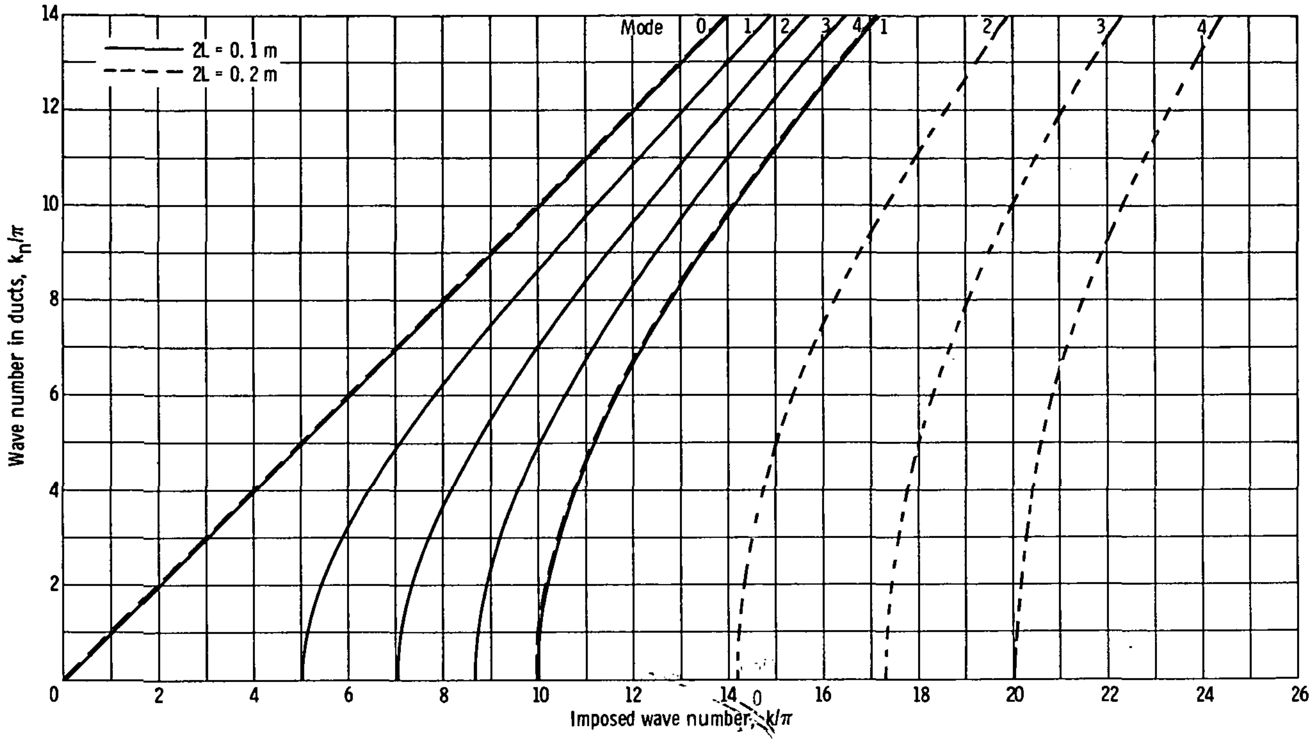

Figure 7. - Characteristics of motion in two straight ducts of different width.

TABLE II. - ZEROS (kR_1) OF $F'_{\nu_n}(akR_1, kR_1)$ FOR TWO VALUES OF $a = R_2/R_1$

$a = R_2/R_1$	Wave number, ν						
	0.5	1.5	2.5	3.5	4.5	5.5	6.5
	Values of kR_1						
1.5	0.4027 6.3355	1.207 6.445	2.008 6.658	2.804 6.966	3.593 7.360	4.373 7.830	5.145 8.365
2.5	0.2945 2.184 4.236 6.345	0.870 2.395 4.335 6.380	1.395 2.776 4.532 6.509	1.881 3.2717 4.8225 6.7015	2.3389 3.815 5.1991 6.9532	2.781 4.361 5.653 7.265	3.215 4.885 6.165 7.632

Finally as an application, let us evaluate motion in a curved duct of $R_1 = 0.1$ meter and $R_2 = 0.2$ meter (a = 2). We assume that, at the inlet $(kR_1) = 3.45$, frequency equals approximately 1900 hertz and that three modes are being generated. From figure 1 we learn that the duct will admit only two modes. The angular wave number of basic mode will be $\nu_0 = 5.5$ and that of the first mode is $\nu_1 = 1.74$. The distribution of the tangential and radial vibrational velocities is shown on figure 8. The vibrational velocities of the zeroth mode are positive everywhere. The tangential vibrational velocities of the first mode are negative in the outer half of the duct.

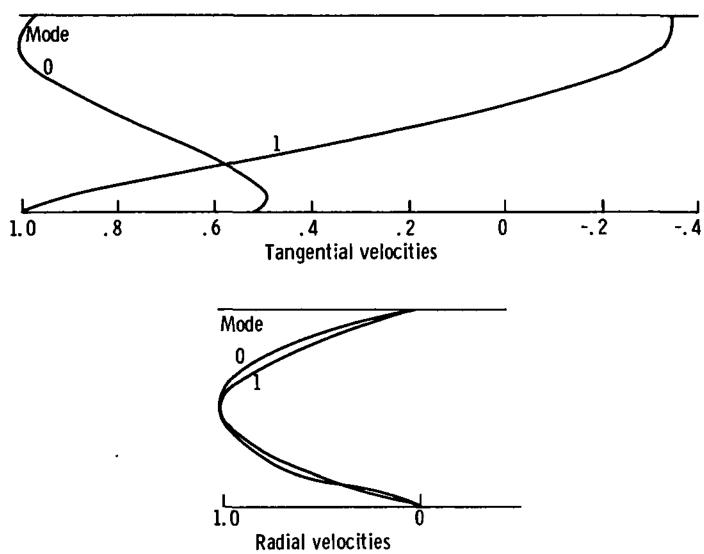

Figure 8. - Vibrational velocities in curved duct of $R_1 = 0.1$ meter with inlet $(kR_1) = 3.45$.

CONCLUDING REMARKS

Propagation of acoustic waves in a frequency range of 5 to 2500 hertz in curved ducts was examined. Using an inverse method (calculating the imposed wave number parameter (kR_1) for an assumed angular wave number ν_n instead of vice versa) correlation has been obtained between the wave number k of the imposed motion and the angular wave numbers in bends ν_n. The results have been tabulated for a range of parameters. Analysis indicates that interpolation between the tabulated data by means of series expansion of Bessel functions yields accurate results for the zeroth mode of motion in the range of arguments (kR_1) up to ~5.0.

Analysis showed that distribution of vibrational velocities in curved ducts strongly depends on frequency and the angular wave number depends on $a = R_2/R_1$, a measure of the bend's sharpness.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 30, 1972,
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