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Lunar Electromagnetic Scattering

I. Propagation Parallel to the Diamagnetic Cavity Axis

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ABSTRACT

An analytic theory is developed for the time dependent magnetic fields inside the Moon and the diamagnetic cavity when the interplanetary electromagnetic field fluctuation propagates parallel to the cavity axis. The Moon model has an electrical conductivity which is an arbitrary function of radius. The lunar cavity is modelled by a nonconducting cylinder extending infinitely far downstream. For frequencies less than about 50 Hz, the cavity is a cylindrical waveguide below cutoff. Thus, cavity field perturbations due to the Moon do not propagate down the cavity, but are instead attenuated with distance downstream from the Moon. Far from the Moon, the cavity electromagnetic field is a cylindrical TE mode propagating downstream with the same frequency and wavelength as the interplanetary field. Thus the magnetic field in the far downstream cavity has a component parallel to the cavity axis which is 90° out of phase with the incident magnetic field. The far cavity field is the result of a surface wave on the cylindrical boundary induced by the interplanetary field and moving downstream with it. Cavity surface currents and charges accompany the far field surface wave.

INTRODUCTION

In a recent paper Schubert et al. [1973] took the first step in the development of an analytic theory for the asymmetric electromagnetic induction in the space defined by a spherical Moon and its downstream cylindrical cavity formed by the solar wind. Previous theories of lunar induction were spherically symmetric approximations [Schubert and Schwartz, 1969; Blank and Sill, 1969; Schwartz and Schubert, 1969; Schubert and Schwartz, 1972]. The asymmetric theory was developed in the quasistatic limit corresponding to large wavelength and low frequency of the oscillating interplanetary field. Thus Schubert et al. [1973] determined the asymmetric electromagnetic response of a two layer Moon model with an infinitely conducting core of radius b and an insulating layer of thickness (a-b) downstream of which was a nonconducting cylindrical cavity. The frequency dependence of the response was treated qualitatively by the rough correspondence between frequency and the ratio of the core radius to the lunar radius (b/a). Small values of b/a correspond to low frequencies while larger values of b/a correspond to higher frequencies. Despite the simplifications of this first asymmetric theory Smith et al. [1973] have obtained good agreement between theory and the experimental data of the Apollo 12 Lunar Surface Magnetometer.

In the quasistatic limit there are only two fundamental orientations of the interplanetary magnetic field, parallel and perpendicular to the lunar cavity axis. In the more complete time dependent scattering theory all directions for the plane wave propagation vector of the incident interplanetary field relative to the cavity axis must be considered. In the case of incident waves parallel to the cavity axis all linear polarizations are theoretically and physically equivalent since the incident electric and magnetic fields are both perpendicular to the cavity axis. However, when the propagation vector is not parallel to the cavity axis there are two distinct physical cases: the magnetic field may be perpendicular and the electric field may have a component parallel to the cavity axis, or the electric field may be perpendicular and the magnetic field may have a component parallel to the cavity axis. Thus there are three theoretical problems associated with the general solution for the electromagnetic scattering of a plane, linearly polarized wave incident on the asymmetric space defined by a spherical Moon and its downstream cavity.

In this paper we obtain the general analytic solution for the propagation vector parallel to the cavity axis. We develop the theory in terms of an arbitrary Moon model subject only to the condition that the lunar electrical conductivity σ , permittivity ε and permeability μ are functions of radius. The theory is also limited to solar wind velocities less than $0(10^6 \mathrm{m/s})$ and to frequencies less than or equal to 10 Hz. The former condition is of little consequence in the lunar environment while the latter one encompasses the total frequency range of the Apollo surface and subsatellite magnetometer experiments. In future papers we will extend the theory to the case of the incident wave propagating perpendicular to the cavity axis. The asymmetric scattering theory can then be used to invert Apollo surface and subsatellite magnetometer data to yield lunar electrical conductivity distributions with the advantage that all magnetometer data, whether characteristic of the sunlit or night side lunar environments, will be treated as a unified body of data to be inverted with a single theory.

THEORETICAL MODEL

In this section we lay the groundwork for the general analytic solution to the problem of electromagnetic scattering of a plane wave incident on a spherical Moon of arbitrary conductivity $\sigma(r)$ and its downstream nonconducting cylindrical cavity when the incident wave propagation vector is parallel to the cavity axis. Following the notation of Schubert and Schwartz [1969] we represent the incident field by

$$\left\{\begin{matrix}\underline{E}\\ \underline{H}\end{matrix}\right\}_{\text{incident}} = H_0 e^{i\left(\frac{2\pi z}{\lambda} - \omega t\right)} \left\{\begin{matrix}\mu v \underline{a}_x\\ \underline{a}_y\end{matrix}\right\} \quad , \tag{1}$$

where $\underline{E}$ and $\underline{H}$ are the electric and magnetic fields, $\underline{a}_x$, $\underline{a}_y$ and $\underline{a}_z$ are the unit vectors of a Cartesian coordinate system whose origin is at the center of the Moon, λ is the wavelength, ω is the circular frequency as measured by an observer fixed with the Moon, H_0 is the amplitude of the magnetic field oscillation, $v = \lambda\omega/2\pi$ and μ is the magnetic permeability of free space. Throughout this paper MKS units are used. Also the time dependence $e^{-i\omega t}$ will henceforth often be understood. The wave propagates in the positive z-direction which is also the axis of the cylindrical downstream diamagnetic cavity. The Moon-cavity geometry and the geometry of the incident electromagnetic field are shown in Figure 1. Both spherical coordinates (r, θ, φ) and cylindrical coordinates (ρ, φ, z) will be used. The space outside the Moon and the cylindrical cavity contains only the incident interplanetary electromagnetic field given by (1). The fields associated with the current systems induced within the Moon, on the lunar sunlit hemisphere and on the cavity boundary cannot penetrate into this external region. The confinement condition on the front side is approximately correct since magnetohydrodynamic waves cannot travel upstream in the supermagnetosonic solar wind plasma. Confinement to the interior of the cavity occurs because downstream travelling scattered waves in the supermagnetosonic solar wind are limited to the interior of the Mach cone extending downstream from the lunar limb. Since the Mach angle is small, the cylindrical surface is a good approximation to the Mach cone.

The confinement is accomplished by currents which flow in the interplanetary plasma at the sunlit lunar surface and the cylindrical surface of the cavity. In the theoretical model these currents are assumed to be ideal surface current distributions, i.e. the current layers have negligible thickness. The boundary conditions are the continuity of tangential electric fields and normal magnetic fields on the sunlit lunar hemisphere and cavity boundary.

THE MAGNETIC FIELD IN THE LUNAR INTERIOR

The electromagnetic field inside the Moon is most easily written in spherical coordinates. In general it consists of both transverse electric $TE(E_r = 0)$ and transverse magnetic $TM(H_r = 0)$ modes, however, we will neglect the TM field. At the frequencies of interest $f = \omega/2\pi < 1$ Hz, the TM induction is negligible because of the relatively high resistivity near the lunar surface.

On the sunlit lunar hemisphere, the radial component of the lunar magnetic field H_r must be equal to that of the interplanetary field (the permeability of the Moon will be assumed to be that of free space). The application of this boundary condition is facilitated by writing the TE part of the incident interplanetary field in multipole form. Also, this spherical harmonic expansion of the TE incident field serves to indicate the form of the expansion for the lunar TE field. For the TE part of the incident magnetic field (1) we have [Schubert and Schwartz, 1972]

$$\left\{\begin{matrix} H_r \\ H_\theta \\ H_\varphi \end{matrix}\right\}_{\substack{\text{incident}\\ \text{TE}}} = \frac{H_0\lambda}{2\pi i r} \cdot \left\{\begin{matrix} \sin\varphi \\ \sin\varphi \\ \cos\varphi \end{matrix}\right\} \sum_{\ell=1}^{\infty} \beta_\ell \left\{\begin{matrix} \ell(\ell+1)\, j_\ell\left(\frac{2\pi r}{\lambda}\right) P_\ell^1(\cos\theta) \\ \frac{d}{dr}\left[r j_\ell\left(\frac{2\pi r}{\lambda}\right)\right] \frac{d}{d\theta} P_\ell^1(\cos\theta) \\ \frac{d}{dr}\left[r j_\ell\left(\frac{2\pi r}{\lambda}\right)\right] \frac{P_\ell^1(\cos\theta)}{\sin\theta} \end{matrix}\right\}, \qquad (2)$$

where

$$\beta_\ell = \frac{i^\ell (2\ell+1)}{\ell(\ell+1)} \quad , \tag{3}$$

and $j_\ell(x)$ and $P_\ell^1(\cos\theta)$ are the spherical Bessel functions and associated Legendre functions, respectively.

The form of the TE magnetic field in the lunar interior is identical to that of incident field (2) except for the radially dependent functions. Following Schubert and Schwartz [1972] we write

$$\begin{Bmatrix} H_r \\ H_\theta \\ H_\varphi \end{Bmatrix}_{\substack{\text{lunar}\\ \text{TE}}} = \frac{H_0 a^2}{r} \begin{Bmatrix} \sin\varphi \\ \sin\varphi \\ \cos\varphi \end{Bmatrix} \sum_{\ell=1}^{\infty} \Gamma_\ell A_\ell \begin{Bmatrix} \ell(\ell+1) \dfrac{G_\ell(r)}{r} P_\ell^1 (\cos\theta) \\ \dfrac{dG_\ell}{dr} \dfrac{d}{d\theta} P_\ell^1 (\cos\theta) \\ \dfrac{dG_\ell}{dr} \dfrac{P_\ell^1(\cos\theta)}{\sin\theta} \end{Bmatrix} , \tag{4}$$

where

$$\Gamma_\ell = \frac{\lambda}{2\pi i a} \beta_\ell \, j_\ell \, (2\pi a/\lambda) \quad , \tag{5}$$

and A_ℓ are coefficients to be determined. The $G_\ell(r)$ are solutions of the second order ordinary differential equation

$$\frac{d^2 G_\ell}{dr^2} + \left[k^2 - \frac{\ell(\ell+1)}{r^2}\right] G_\ell(r) = 0 \quad , \tag{6}$$

subject to the conditions that G_ℓ are finite at the origin and $G_\ell(a) = 1$. The latter condition on G_ℓ is only a convenient normalization condition; the coefficients A_ℓ of each multipole contribution to $\underline{H}$ inside the Moon are as yet unknown. The propagation constant k is

$$k = (\omega^2\mu\epsilon + i\omega\mu\sigma)^{\frac{1}{2}} , \tag{7}$$

where, for simplicity, μ and ϵ have the free space values and σ is an arbitrary function of r (it is not any more difficult to allow μ and ϵ to also be functions of r). As an example, for the two layer Moon model with a perfectly conducting core of radius b and an insulating shell of thickness (a-b)

$$G_\ell(r) = \begin{cases} 0 & r \le b \\ \dfrac{(r/a)^{\ell+1} - (b/a)^{2\ell+1}\,(a/r)^{\ell}}{1 - (b/a)^{2\ell+1}} & b \le r \le a \end{cases} , \tag{7}$$

for frequencies of interest.

THE CAVITY AS A CYLINDRICAL WAVEGUIDE BELOW CUTOFF

Before we construct the general solution for the electromagnetic field in the nonconducting cylindrical cavity downstream of the Moon, it is instructive to consider the physics of wave propagation in this cavity. Electromagnetic wave propagation in a hollow cylindrical waveguide is a classical problem discussed, for example, in Stratton [1941]. A priori possible modes of propagation are transverse magnetic TM(H_z = 0) and transverse electric TE(E_z = 0). The TE and TM nomenclature in the cavity should not be confused with similar designations inside the Moon.

The form of the incident field can be used to restrict our consideration of the number of possible modes. In cylindrical coordinates the incident field (1) is

$$\left\{\begin{matrix}\underline{E}\\ \underline{H}\end{matrix}\right\}_{\text{incident}} = H_0 e^{i\left(\frac{2\pi z}{\lambda}-\omega t\right)} \left\{\begin{matrix}\mu v\cos\varphi\ \hat{\underline{\rho}} - \mu v\sin\varphi\ \hat{\underline{\varphi}}\\ \sin\varphi\ \hat{\underline{\rho}} + \cos\varphi\ \hat{\underline{\varphi}}\end{matrix}\right\} . \tag{8}$$

Thus we need only consider possible TE and TM modes of order 1, for which the lowest cutoff frequencies are 50.52 Hz and 105.15 Hz, respectively. The cutoff frequencies of the first order TE and TM modes are given by $c/2\pi a$ (c is the speed of light in vacuum) times the zeros of $dJ_1(x)/dx$ and $J_1(x)$, respectively ($J_1(x)$ is the Bessel function of first order).

Since the frequencies of interest are more than an order of magnitude smaller than the lower of the above two cutoff frequencies, disturbances caused by the Moon cannot propagate down the cavity. Magnetic field perturbations in the cavity, due to the presence of the Moon, are oscillatory, with amplitudes which decay exponentially with distance down the cavity. The first order TE disturbance decays essentially as $\exp(-1.84z/a)$ while the TM disturbance decays essentially according to $\exp(-3.83\, z/a)$. <u>The electromagnetic field far downstream $z \to \infty$ in the cavity is independent of the presence of the Moon.</u>

THE ELECTROMAGNETIC FIELD IN THE FAR DOWNSTREAM CAVITY

The electromagnetic field in the cylindrical cavity far downstream from the Moon can be obtained by a linear superposition of elementary cylindrical wave functions [see Stratton, 1941, pages 360-361]. From the form of the incident field (8) only first order TE and TM modes could contribute to the cavity field. Since there is no z-component in the incident electric field, continuity of the tangential electric field on the cylindrical boundary requires that the cavity field be a pure TE mode. Continuity of the radial magnetic field H_ρ on $\rho=a$ uniquely determines the TE solution in the far downstream cavity. The condition that the tangential electric field component E_φ be continuous on $\rho=a$ is automatically satisfied. The solution is

$$\begin{Bmatrix} H_\rho \\ \\ H_\varphi \\ \\ H_z \end{Bmatrix} = \frac{H_0 e^{i\left(\frac{2\pi z}{\lambda} - \omega t\right)}}{I_1'\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)} \begin{Bmatrix} \sin\varphi \\ \\ \cos\varphi \\ \\ \sin\varphi \end{Bmatrix} \begin{Bmatrix} I_1'\left(\frac{2\pi\rho}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right) \\ \\ \dfrac{I_1\left(\frac{2\pi\rho}{\lambda}\sqrt{1-v^2/c^2}\right)}{\frac{2\pi\rho}{\lambda}\sqrt{1-v^2/c^2}} \\ \\ i\sqrt{1-v^2/c^2}\; I_1\left(\frac{2\pi\rho}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right) \end{Bmatrix}, \quad (9)$$

$$\begin{Bmatrix} E_\rho \\ \\ E_\varphi \end{Bmatrix} = \frac{\mu v H_0 \, e^{i\left(\frac{2\pi z}{\lambda} - \omega t\right)}}{I_1'\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)} \begin{Bmatrix} \cos\varphi \; \dfrac{I_1\left(\frac{2\pi\rho}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)}{\frac{2\pi\rho}{\lambda}\sqrt{1-v^2/c^2}} \\ \\ -\sin\varphi \; I_1'\left(\frac{2\pi\rho}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right) \end{Bmatrix} ,$$

$$E_z = 0 \qquad , \tag{10}$$

where $I_1(x)$ is the modified Bessel function of order 1 and $I_1'(x) = dI_1(x)/dx$. The field in the far downstream cavity (9) and (10) reduces to the incident field (8) in the limit $\omega \to 0$ or $v \to c$.

The TE cavity field (9) and (10) is the result of a surface wave on the cavity boundary induced by the incident field. This is most easily understood by considering the surface wave on a plane $y=0$ induced by a travelling wave (1). In the region $y>0$, the field is everywhere the incident field and the half-space $y<0$ is vacuum. The field in the vacuum is independent of x , proportional to $e^{i\left(\frac{2\pi z}{\lambda} - \omega t\right)}$ and dependent on y. The vacuum electric field has only an x-component which is given by

$$\mu v H_0 \, e^{\frac{2\pi y}{\lambda}\sqrt{1-v^2/c^2}} \, e^{i\left(\frac{2\pi z}{\lambda} - \omega t\right)} \qquad . \tag{11}$$

It is a surface wave travelling in the z-direction with speed v. Its amplitude is exponentially damped with distance from the surface into the vacuum. The attenuation depth is on the order of a wavelength. Note that the electric field is continuous on y=0. The vacuum magnetic field has a y-component given by

$$H_0\, e^{\frac{2\pi y}{\lambda}\sqrt{1-v^2/c^2}}\, e^{i\left(\frac{2\pi z}{\lambda}-\omega t\right)} \quad , \tag{12}$$

which is continuous with the y-component of (1) on y=0. Since the normal magnetic field in the vacuum is attenuated with distance from the surface, $\nabla\cdot\underline{H} = 0$ requires that the surface wave possess a z-component of magnetic field

$$i\sqrt{1-\frac{v^2}{c^2}}\; H_0\, e^{\frac{2\pi y}{\lambda}\sqrt{1-v^2/c^2}}\, e^{i\left(\frac{2\pi z}{\lambda}-\omega t\right)} \quad . \tag{13}$$

The surface wave is thus not a transverse wave, however, it is TE. Clearly (9) and (10) represent waves on the cylindrical surface of the cavity which are induced by the passage of the interplanetary field. The magnetic field components are 90° out of phase.

The surface current density $\underline{K}$ on the cavity boundary ρ=a is

$$K_\varphi = i\sqrt{1-v^2/c^2}\; H_0\, e^{i\left(\frac{2\pi z}{\lambda}-\omega t\right)} \sin\varphi\, \frac{I_1\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)}{I_1'\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)} \quad , \tag{14}$$

$$K_z = H_0 e^{i(\frac{2\pi z}{\lambda} - \omega t)} \cos\varphi\left\{1 - \frac{I_1\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)}{\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\, I_1'\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)}\right\} \cdot \quad (15)$$

Note that K_φ and K_z approach zero as $\omega \to 0$ or $v \to c$. The surface charge density Σ on $\rho = a$ is

$$\Sigma = \frac{vH_0}{c^2} e^{i(\frac{2\pi z}{\lambda} - \omega t)} \cos\varphi\left\{1 - \frac{I_1\left(\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}\right)}{\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\, I_1'\left(\frac{2\pi a}{\lambda}\sqrt{1-\frac{v^2}{c^2}}\right)}\right\} \quad (16)$$

As in the case of the currents, $\Sigma \to 0$ as $\omega \to 0$ or $v \to c$.

Figure 2 shows H_ρ/H_0 and $-E_\varphi/\mu v H_0$ as functions of ρ/a with $\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}$ as a parameter. The dependence of H_ρ and E_φ on $\sin\varphi \cdot \exp[i(\frac{2\pi z}{\lambda} - \omega t)]$ has been suppressed. The field components H_ρ and E_φ have been normalized with respect to the amplitudes of the corresponding interplanetary field components. Thus the continuity of H_ρ and E_φ on $\rho = a$ makes the normalized field components unity on $\rho = a$, independent of the value of the wavelength parameter. When the wavelength parameter is zero, the normalized field components are identical to those of the incident field. As the wavelength parameter increases the field components of the surface wave are increasingly attenuated with distance from the cavity boundary.

The normalized cavity field components H_φ and E_ρ are shown in Figure 3 as functions of ρ/a with $\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}$ as a parameter. The φ, z and t dependences of H_φ and E_ρ have been suppressed. These field components are seen not to be continuous on $\rho=a$, except when the wavelength parameter approaches zero and in this limit H_φ and E_ρ become uniform and equal to the incident field components. As the wavelength parameter increases, the amplitudes of these surface wave components at $\rho=a$ decrease and the attenuation of these components with distance from the cavity boundary increases. At $\rho=0$, $E_\rho/\mu v H_0 = H_\varphi/H_0 = - E_\varphi/\mu v H_0 = H_\rho/H_0 = 0$.

Figure 4 shows the cavity field component $\dfrac{H_z}{i\, H_0\sqrt{1-v^2/c^2}}$ as a function of ρ/a with the same parameter as in Figures 2 and 3. Again the φ, z and t dependences have been suppressed. Note that the incident field has no z-component. Also the H_z in the cavity is 90° out of phase with all the other field components. For the wavelength parameter equal to zero, H_z vanishes. As the parameter increases the normalized surface amplitude of H_z increases, reaching a maximum value of about 1.1 at a parameter value about 2.8, and then asymptotically decreases to unity. As with the other components, the attenuation of H_z with distance from the cavity boundary increases with the wavelength parameter. Note that H_z vanishes as $\rho\to 0$.

The discontinuities in H_z and H_φ at $\rho=a$ are associated with the cavity surface current densities K_φ and K_z , respectively. The normalized surface current densities K_z/H_0 and $\frac{K_\varphi}{i\, H_0\sqrt{1-v^2/c^2}}$. are shown in Figure 5 as functions of the wavelength parameter (the dependences on z,t and φ are suppressed).. A surface charge density Σ is associated with the discontinuity in E_ρ at $\rho=a$. The normalized surface charge density $\Sigma\, c^2/vH_0$ is identical to K_z/H_0 . As the wavelength parameter approaches zero the surface currents and charge approach zero. The normalized K_z and Σ monotonically approach unity as the parameter increases, while the normalized K_φ exhibits a maximum before eventually approaching unity. The magnitude of the azimuthal cavity surface currents exceeds that of the axial surface currents by a factor which increases monotonically as the parameter approaches zero. As in the case of H_z , K_φ is 90° out of phase with K_z and Σ .

For a wavelength parameter smaller than 1/2 , the cavity field components H_ρ , H_φ , E_ρ , and E_φ are essentially the same as the corresponding components of the interplanetary field. The parameter value 1/2 corresponds to a frequency of 0.02 Hz , for an assumed wave propagation speed of 400 km/sec. The wavelength parameter must be smaller by a factor of 2 (frequency smaller than 0.01 Hz) for H_z in the cavity to be negligible.

For hydromagnetic waves and discontinuities in the solar wind $v \lesssim 10^6$ m/sec and $v^2/c^2 < 10^{-5}$. Using the smallness of v^2/c^2, the expression (9) for the cavity magnetic field as $z \to \infty$ can be rewritten as

$$\underline{H} = \nabla \left\{ \frac{H_0 e^{i(\frac{2\pi z}{\lambda} - \omega t)} \sin\varphi}{\frac{2\pi}{\lambda} I_1' \left(\frac{2\pi a}{\lambda}\right)} I_1 \left(\frac{2\pi \rho}{\lambda}\right) \right\} \quad . \tag{17}$$

In the limit of large wavelength i.e. $2\pi a/\lambda \ll 1$, (17) is nearly a plane travelling wave with the magnetic field in the y-direction. With the aid of an identity from Cooke [1956] , (17) can be re-written in a form appropriate for satisfying matching conditions on the night side lunar hemisphere

$$\underline{H} = \nabla \left\{ \frac{H_0 e^{-i\omega t} \sin\varphi}{I_1' \left(\frac{2\pi a}{\lambda}\right)} \; r \sum_{m=1}^{\infty} \frac{\left(\frac{2\pi i r}{\lambda}\right)^{m-1}}{(m+1)!} P_m^1 (\cos\theta) \right\} \quad . \tag{18}$$

MATCHING THE LUNAR AND FAR CAVITY MAGNETIC FIELDS

The far cavity field satisfies the boundary conditions on the cylindrical cavity surface exactly. To match the lunar TE magnetic field to the far cavity magnetic field it is only necessary to add to the far cavity magnetic field the decaying TE mode of the cylindrical waveguide. The decaying TE mode satisfies $H_\rho = E_\varphi = E_z = 0$ on $\rho=a$ and consequently the sum of the far cavity field and the TE waveguide mode match the incident field correctly on the cavity surface $\rho=a$. The magnetic field of the decaying TE waveguide mode can be written [Stratton, 1941, p. 541]

$$\begin{Bmatrix} H_\rho \\ H_\varphi \\ H_z \end{Bmatrix} = H_0 \sum_{m=1}^{\infty} C_m e^{-h_m z} \begin{Bmatrix} \frac{\gamma_m}{a} J_1' (\gamma_m \frac{\rho}{a}) \sin\varphi \\ (J_1(\gamma_m \rho/a)/\rho) \cos\varphi \\ (\frac{-\gamma_m^2}{h_m} J_1(\gamma_m \rho/a)/a^2) \sin\varphi \end{Bmatrix}, \tag{19}$$

where γ_m for $m=1,2,\cdots$ are the ascending zeros of $J_1'(x)$,

$$h_m = \sqrt{(\frac{\gamma_m}{a})^2 - (\frac{\omega}{c})^2} = \frac{\gamma_m}{a}\sqrt{1-\frac{\omega^2}{\omega_m^2}}, \tag{20}$$

and ω_m is the mth cutoff frequency $c\gamma_m/a$. As noted in our discussion of the cavity as a waveguide below cutoff, the frequencies of interest lie one or more orders of magnitude below the smallest cutoff frequency of about 50 Hz. Thus it is an excellent approximation to take $h_m \approx \gamma_m/a$ and rewrite the magnetic field (19) as

$$\underline{H} = \nabla\left\{H_0 \sum_{m=1}^{\infty} \sin\varphi \; C_m \; e^{-\gamma_m z/a} \; J_1\left(v_m \frac{\rho}{a}\right)\right\} \quad . \tag{21}$$

This is identical to the quasistatic cavity disturbance field determined by Schubert et al. [1973]. To match with the lunar TE magnetic field on the night side hemisphere, (21) must be rewritten in spherical coordinates. Using Cooke [1956] we find

$$\underline{H} = \nabla\left\{-\sin\varphi \; H_0 \sum_{m=1}^{\infty} C_m \sum_{\ell=1}^{\infty} \frac{\left(-\frac{\gamma_m r}{a}\right)^{\ell}}{(\ell+1)!} \; P_\ell^1 \; (\cos\theta)\right\} \quad . \tag{22}$$

Because we have used only the decaying TE waveguide mode to appropriately match the magnetic fields on the night side lunar surface, some mismatch of the electric fields is possible. This could be corrected by the inclusion of the decaying TM waveguide mode (and the lunar TM mode) which would contribute a magnetic field a factor of ω^2/ω_m^2 smaller than the TE magnetic field. Thus for the frequencies of interest we make a negligible error by matching with only the decaying TE waveguide mode.

COMPLETE ANALYTIC SOLUTION

In the previous sections we developed representations for the TE magnetic field in the lunar interior (4) and in the cavity, the sum of (17) and (21), for an incident linearly polarized plane wave propagating in the positive z direction with velocity $v \ll c (=3 \times 10^8 \text{m/s})$, frequency $f \ll 50$ Hz and with the magnetic field vector in the y direction. These forms contain two infinite sets of constants A_n and C_n which must be evaluated by using appropriate boundary conditions. By the manner in which we constructed the cavity field, the boundary conditions at the cylindrical cavity surface, $\rho=a$, are automatically satisfied. The continuity of the normal component of the magnetic field at $r=a$ over the entire Moon and the continuity of the tangential component of the magnetic field on $r=a$ in the cavity, $0 \leq \theta \leq \frac{\pi}{2}$, determine the doubly infinite set of constants. The algebraic detail of this procedure is given in the appendix, together with final formulae for the A_n and C_n.

SUMMARY

In this paper we have obtained the general analytic solution for the interaction of the Moon and its downstream cavity with a linearly polarized plane electromagnetic wave propagating in the direction parallel to the axis of the cavity. The solution is formulated in terms of a spherical Moon model with arbitrary radially dependent electromagnetic parameters and a nonconducting cylindrical downstream cavity. A number of approximations consistent with the physical circumstances of the Moon-solar wind interaction and our present ability to measure this interaction have been made. Outside the Moon and the cavity the electromagnetic field is the incident interplanetary field. The electrical conductivity of the near lunar surface is sufficiently low to suppress the TM lunar field. The incident wave velocity v is much less than the velocity of light in vacuum and the highest frequency of interest is of order 10 Hz. All but the first of these assumptions could be relaxed without presenting any special difficulty in carrying through the solution with the methods of this paper.

Of particular significance is the finding that for frequencies below about 50 Hz, the diamagnetic cavity is a cylindrical waveguide below cutoff, i.e. cavity electromagnetic field perturbations due to the Moon decay exponentially with downstream distance.

Thus the quasistatic representation of the cavity perturbation magnetic field due to the Moon given by Schubert et al. [1973] is valid for frequencies as high as 10 Hz. This is not to imply that the quasistatic solution for the lunar and cavity magnetic fields is valid at such high frequencies. Quite to the contrary, high frequency modifications in the lunar and cavity magnetic fields will be present at frequencies above about 0.01 Hz.

Far downstream from the Moon, the interplanetary field forces a surface wave to propagate down the cavity boundary. The far cavity field, which is determined by this propagating surface wave, is different from the interplanetary field, e.g. it has an axial magnetic field component. These differences become more significant as the frequency of the field increases. The far cavity field is independent of the presence of the Moon whose cavity field perturbations will be damped within several lunar radii downstream. Differences between the interplanetary and cavity fields measured farther than a few lunar radii downstream must be attributed to the cavity itself; they can yield no information about the lunar electrical conductivity.

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APPENDIX

To determine the unknown coefficients A_n and C_n first consider the continuity of the normal component of the magnetic field on r=a. We can write this condition in the form

$$\sum_{\ell=1}^{\infty} \ell(\ell+1)\ \Gamma_\ell (A_\ell - 1) P_\ell^1(\cos\theta) = \begin{cases} 0\ , & \pi \geq \theta \geq \pi/2\ , \\ \frac{1}{Q} \sum_{n=1}^{\infty} n i^n \xi^n P_n^1(\cos\theta) - \sum_{m=1}^{\infty} \left(\frac{C_m}{a}\right) \sum_{n=1}^{\infty} n(-1)^n \xi_m^n P_n^1(\cos\theta) - \sum_{\ell=1}^{\infty} \ell(\ell+1)\ \Gamma_\ell P_\ell^1(\cos\theta), & \frac{\pi}{2} \geq \theta \geq 0 \end{cases} \tag{A-1}$$

where

$$\xi^n = \frac{\left(\frac{2\pi a}{\lambda}\right)^n}{(n+1)!}\ , \tag{A-2}$$

$$\xi_m^n = \frac{\gamma_m^n}{(n+1)!}\ , \tag{A-4}$$

$$Q = \frac{2\pi i a}{\lambda} I_1' \left(\frac{2\pi a}{\lambda}\right)\ . \tag{A-4}$$

Multiplying (A-1) by $P^1_{\ell'}(\cos\theta)\sin\theta\, d\theta$ and integrating from 0 to π yields

$$\frac{2\ell'^2(\ell'+1)^2\Gamma_{\ell'}(A_{\ell'}-1)}{(2\ell'+1)} =$$

$$\sum_{\ell=1}^{\infty} \ell P^1_{\ell\ell'} \left\{ \frac{i^\ell \xi^\ell}{Q} - (\ell+1)\Gamma_\ell - \sum_{m=1}^{\infty} \left(\frac{C_m}{a}\right) \xi_m^\ell (-1)^\ell \right\} , \tag{A-5}$$

for $\ell'=1,2,3,\cdots$ where

$$P^1_{\ell\ell'} = \begin{cases} \dfrac{\ell'(\ell'+1)}{2\ell'+1} , & \ell'=\ell \\[2ex] \dfrac{P^1_{\ell'}(0)P^2_\ell(0)-P^1_\ell(0)P^2_{\ell'}(0)}{g_{\ell\ell'}} , & \ell'\neq\ell \end{cases} , \tag{A-6}$$

$$g_{\ell\ell'} = (\ell-\ell')\ (\ell+\ell'+1) = -\ g_{\ell'\ell} . \tag{A-7}$$

On the lunar night side, within the cavity $(\frac{\pi}{2} \geq \theta \geq 0)$ and on $r=a$, both H_θ and H_φ are continuous. It is sufficient to use only the continuity of H_φ . This leads to the equation

$$\sum_{\ell=1}^{\infty} \Gamma_\ell A_\ell u_\ell P^1_\ell(\cos\theta) = \sum_{\ell=1}^{\infty} \frac{i^\ell \xi^\ell}{Q} P^1_\ell (\cos\theta) -$$

$$\sum_{m=1}^{\infty} \left(\frac{C_m}{a}\right) \sum_{n=1}^{\infty} (-1)^n \xi_m^n P^1_n(\cos\theta) , \tag{A-8}$$

where

$$u_{\ell} = a\left(\frac{dG_{\ell}}{dr}\right)_{r=a} \quad . \tag{A-9}$$

To generate a set of equations for the unknown coefficients from (A-8) we note that on $0 \le \theta \le \pi/2$ the odd associated Legendre functions form a complete set. Expanding (A-8) in terms of this set gives the infinite set of equations,

$$\sum_{\ell=1}^{\infty} P^{1}_{\ell\ell'}\left\{\Gamma_{\ell} A_{\ell} u_{\ell} - \frac{i^{\ell}\xi^{\ell}}{Q}\right\} = - \sum_{m=1}^{\infty} \left(\frac{C_m}{a}\right) \sum_{\ell=1}^{\infty} (-1)^{\ell} \xi^{\ell}_{m} P^{1}_{\ell\ell'} \, , \tag{A-10}$$

where $\ell'=1,\ 3,\ 5,\cdots$

Equations (A-5) and (A-10) are sufficient for the determination of the A_ℓ and C_ℓ. After considerable algebraic manipulation a single set of equations can be derived for the C_ℓ ,

$$\sum_{m=1}^{\infty} \frac{C_m}{a} \left\{ \frac{\ell'(\ell'+1)\xi^{\ell'}_{m}}{(2\ell'+1)P^{1}_{\ell'}(0)} \left[\frac{u_{\ell'}}{2(\ell'+1)} - 1\right] + \right.$$

$$\sum_{n=2,\text{even}}^{\infty} \frac{\xi^{n}_{m} P^{2}_{n}(0)}{g_{n\ell'}} \left[1 - \frac{n}{2}\left(\frac{u_n}{n(n+1)} + \frac{u_{\ell'}}{\ell'(\ell'+1)}\right)\right] +$$

$$\left. \sum_{n=1,\text{odd}}^{\infty} n\xi^{n}_{m} P^{1}_{n}(0) \sum_{\ell=2,\text{even}}^{\infty} \frac{(2\ell+1)u_{\ell}\left[P^{2}_{\ell}(0)\right]^{2}}{2\ell^{2}(\ell+1)^{2} g_{\ell\ell'} g_{\ell n}} \right\} =$$

$$\frac{\ell' i^{\ell'} \xi^{\ell'}}{(2\ell'+1) Q\, P^1_{\ell'}(0)} \left[\ell'+1 - \frac{u_{\ell'}}{2}\right] - \frac{\ell'(\ell'+1)\Gamma_{\ell'} u_{\ell'}}{2(2\ell'+1) P^1_{\ell'}(0)} +$$

$$\sum_{\ell=2,\text{even}}^{\infty} \frac{P^2_\ell(0)}{g_{\ell\ell'}} \left\{ \frac{i^\ell \xi^\ell}{Q} \left[1 - \frac{\ell}{2}\left(\frac{u_\ell}{\ell(\ell+1)} + \frac{u_{\ell'}}{\ell'(\ell'+1)} \right)\right] - \right.$$

$$\left. \frac{\Gamma_\ell}{2}\left(u_\ell - \frac{\ell(\ell+1) u_{\ell'}}{\ell'(\ell'+1)} \right)\right\} + \sum_{\ell=2,\text{even}}^{\infty} \frac{[P^2_\ell(0)]^2 u_\ell (2\ell+1)}{2\ell^2(\ell+1)^2 g_{\ell\ell'}} \cdot$$

$$\sum_{n=1,\text{odd}}^{\infty} \frac{n P^1_n(0)}{g_{\ell n}} \left\{ (n+1)\, \Gamma_n - \frac{i^n \xi^n}{Q} \right\} \tag{A-11}$$

for $\ell'=1,3,5,\cdots$ In the limit $v \to c$ and $\lambda \to \infty$ (A-11) reduces exactly to (A-31) of Schubert et al. [1973] with the identification of u_ℓ as $\ell(\ell+1)\alpha_\ell$. Equation (A-11) represents an infinite set of linear equations (one for each ℓ') for the infinite set of unknowns C_m . Since the C_m eventually decrease in importance as m increases the set of equations can be solved numerically by truncating in m and ℓ'. This was done in the quasistatic limit by Schubert et al. [1973] and will be done for various Moon models using the present theory in a future paper. Once the C_m have been found, the A_m may be obtained from (A-5). After some algebraic manipulation we find

$$\Gamma_\ell A_\ell = \frac{\Gamma_\ell}{2} + \frac{1}{2(\ell+1)} \left\{ \frac{i^\ell \xi^\ell}{Q} - (-1)^\ell \sum_{m=1}^{\infty} \xi_m^\ell \left(\frac{C_m}{a}\right) \right\} + \frac{2\ell+1}{2\ell^2(\ell+1)^2} T_\ell \quad , \qquad \text{(A-12)}$$

where for ℓ odd

$$T_\ell = P_\ell^1(0) \sum_{n=2,\text{even}}^{\infty} \frac{nP_n^2(0)}{g_{n\ell}} \left\{ \frac{(-1)^{n/2}\xi^n}{Q} - (n+1)\,\Gamma_n - \sum_{m=1}^{\infty} \left(\frac{C_m}{a}\right) \xi_m^n \right\} \quad , \qquad \text{(A-13)}$$

and for ℓ even

$$T_\ell = P_\ell^2(0) \sum_{n=1,\text{odd}}^{\infty} \frac{nP_n^1(0)}{g_{\ell n}} \left\{ \frac{i^n \xi^n}{Q} - (n+1)\,\Gamma_n + \sum_{m=1}^{\infty} \left(\frac{C_m}{a}\right) \xi_m^n \right\} \quad . \qquad \text{(A-14)}$$

Again, in the limit of $v \to c$ and $\lambda \to \infty$, the equations for the quantities $\Gamma_\ell A_\ell$ formally reduce to equations (A.30) and (A.31) of Schubert et al. [1973] .

FIGURE CAPTIONS

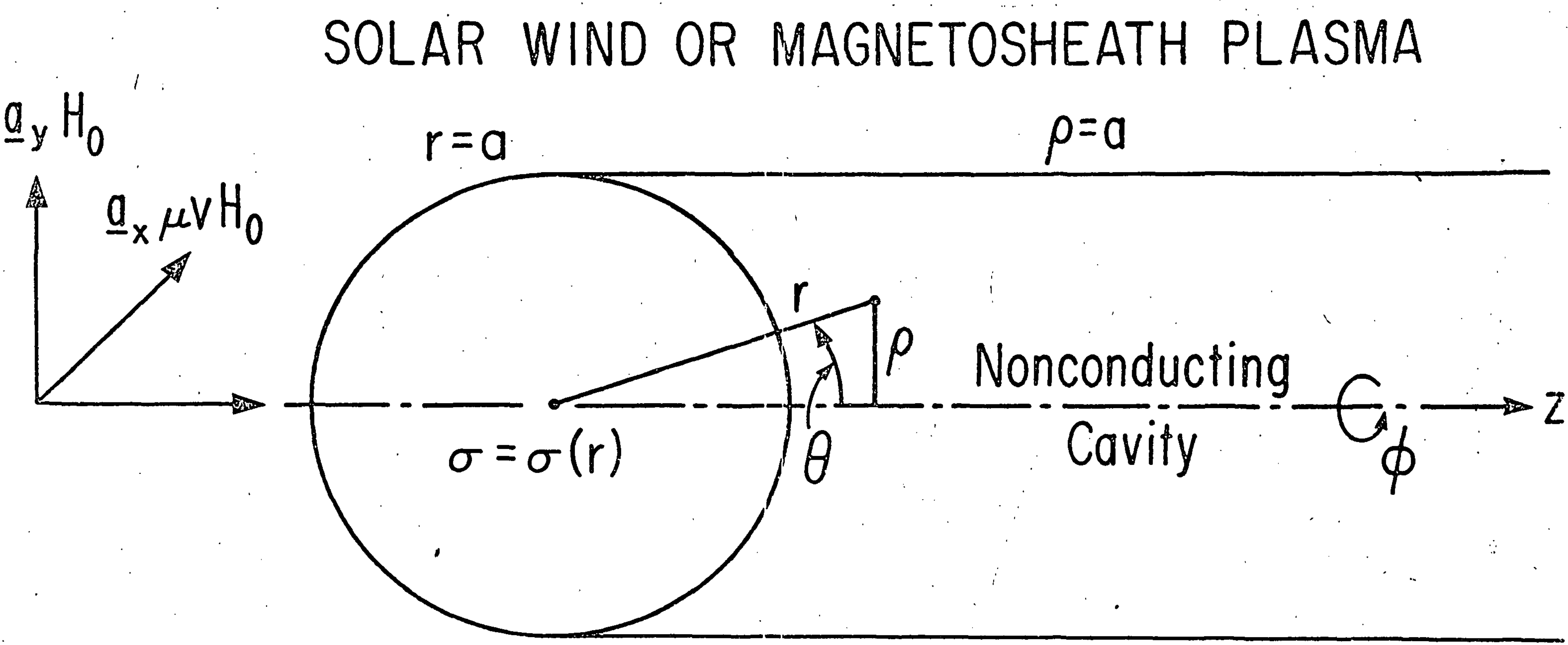
Figure 1. The geometry of the asymmetric lunar electromagnetic scattering problem.

Figure 2. The far downstream cavity field components H_ρ and E_φ normalized to the corresponding incident field components as a function of normalized distance from the cavity axis with $\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}$ as a parameter.

Figure 3. The far downstream cavity field components H_φ and E_ρ normalized to the corresponding incident field components as a function of normalized distance from the cavity axis with $\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}$ as a parameter.

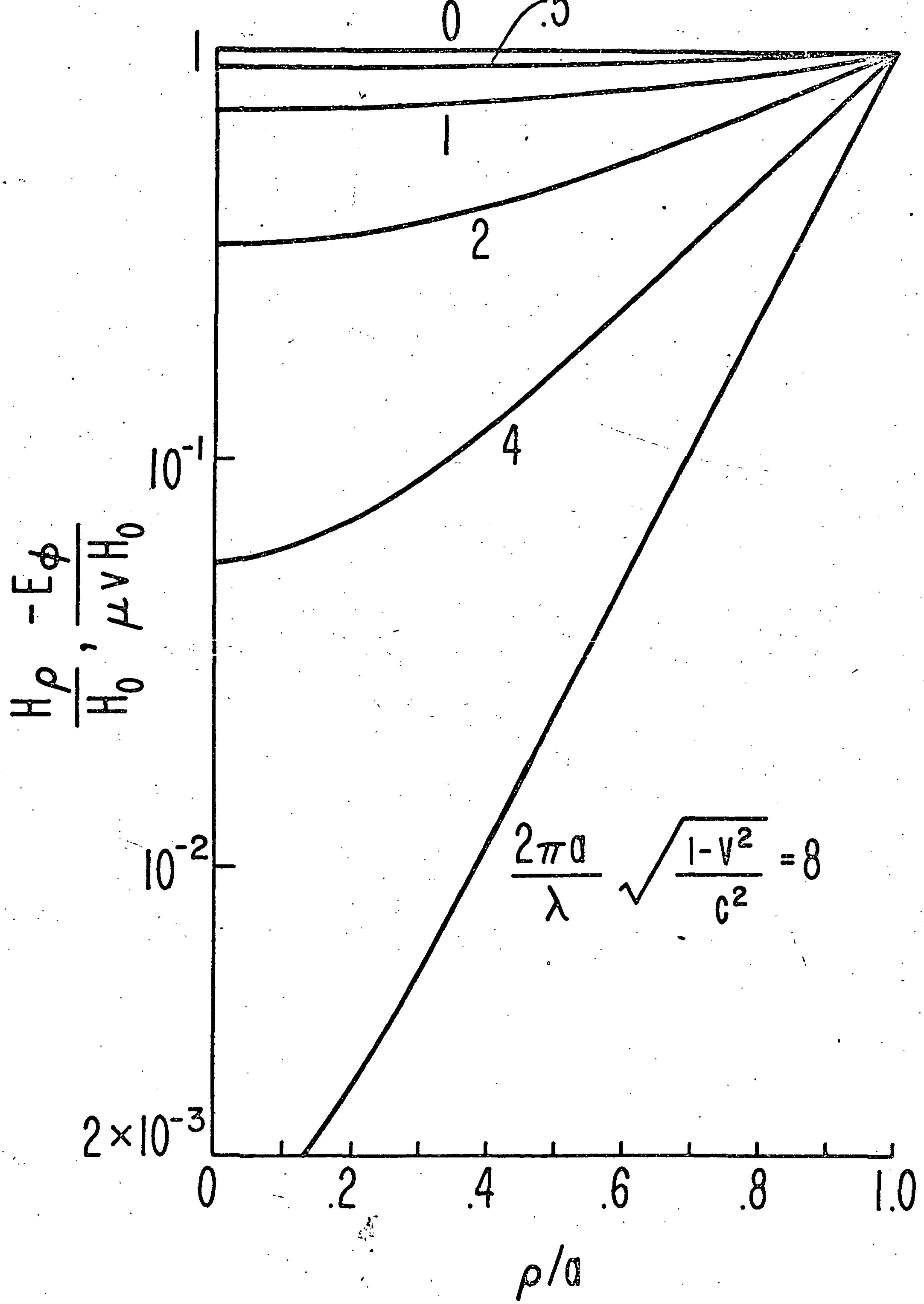
Figure 4. The far downstream cavity axial magnetic field $H_z/i\ H_0\sqrt{1-v^2/c^2}$ as a function of normalized distance from the cavity axis with $\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}$ as a parameter. The dependence of H_z on z, φ and t has been suppressed.

Figure 5. Normalized surface current density and charge density on the far downstream cavity boundary as a function of $\frac{2\pi a}{\lambda}\sqrt{1-v^2/c^2}$. The dependence on z, φ and t has been suppressed

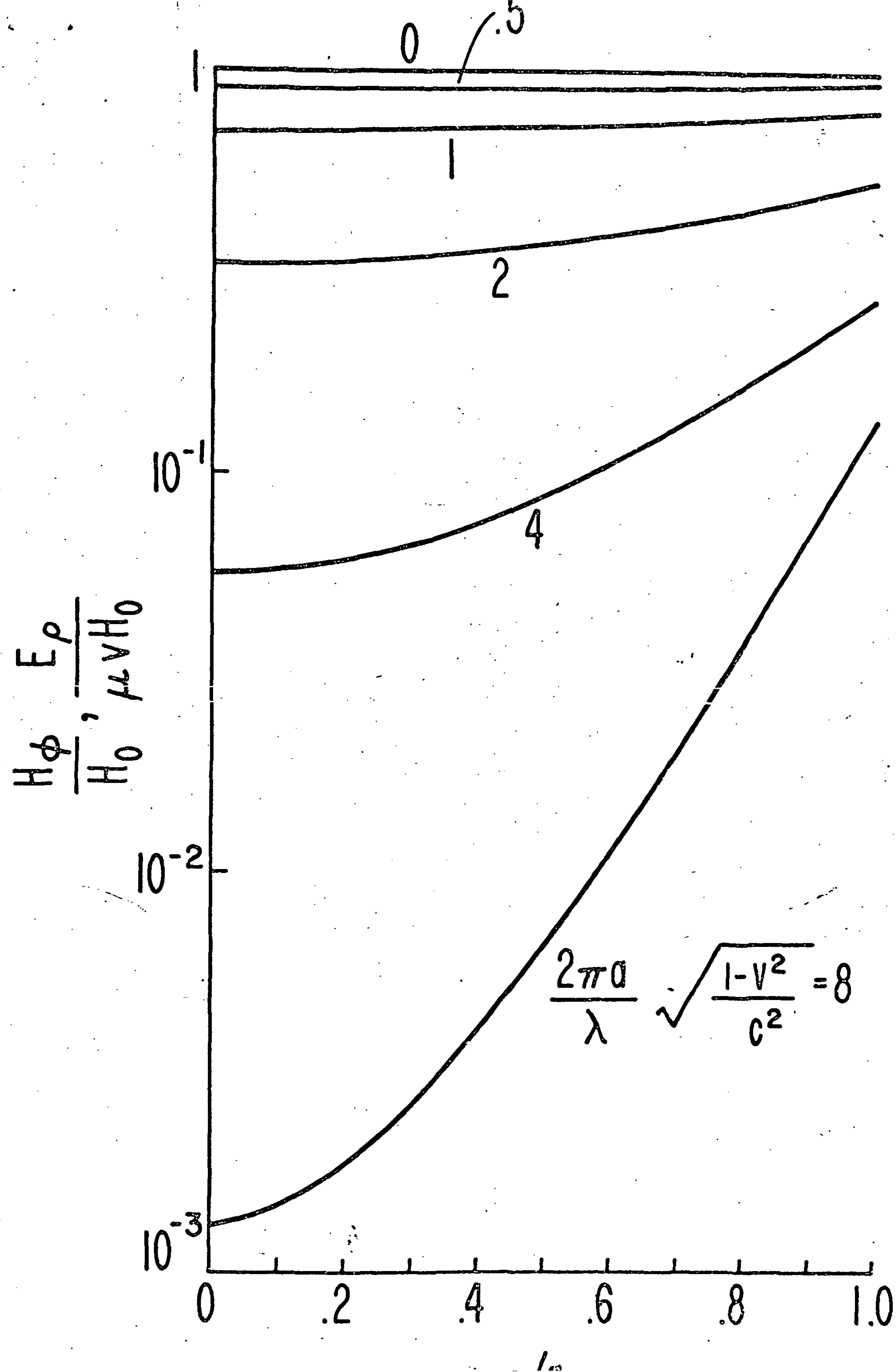


SOLAR WIND OR MAGNETOSHEATH PLASMA
$\underline{a}_y H_0$
$\underline{a}_x \mu v H_0$
$r=a$
$\rho=a$
r
ρ
Nonconducting
Cavity
z
ϕ
θ
$\sigma=\sigma(r)$

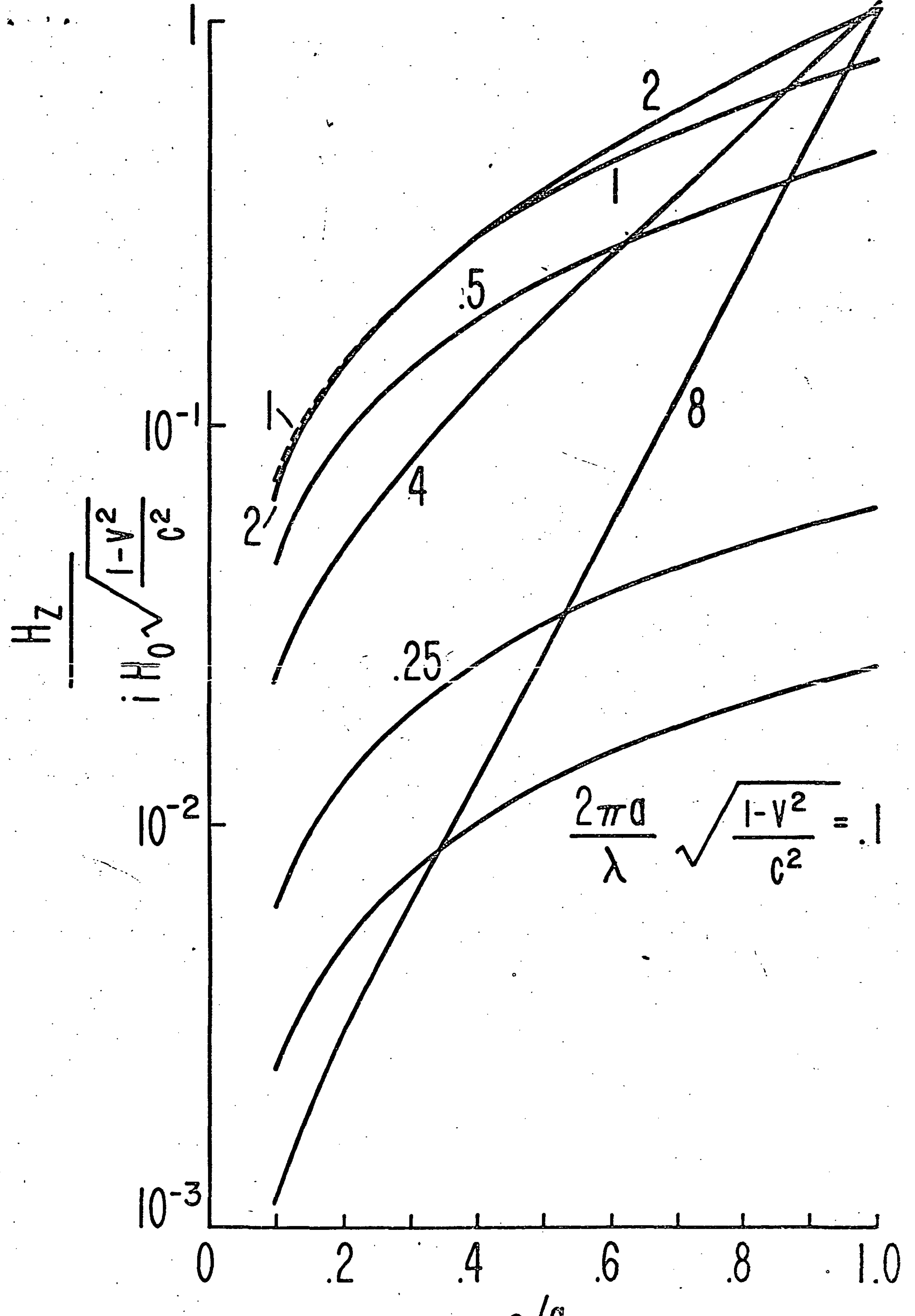
Fig.

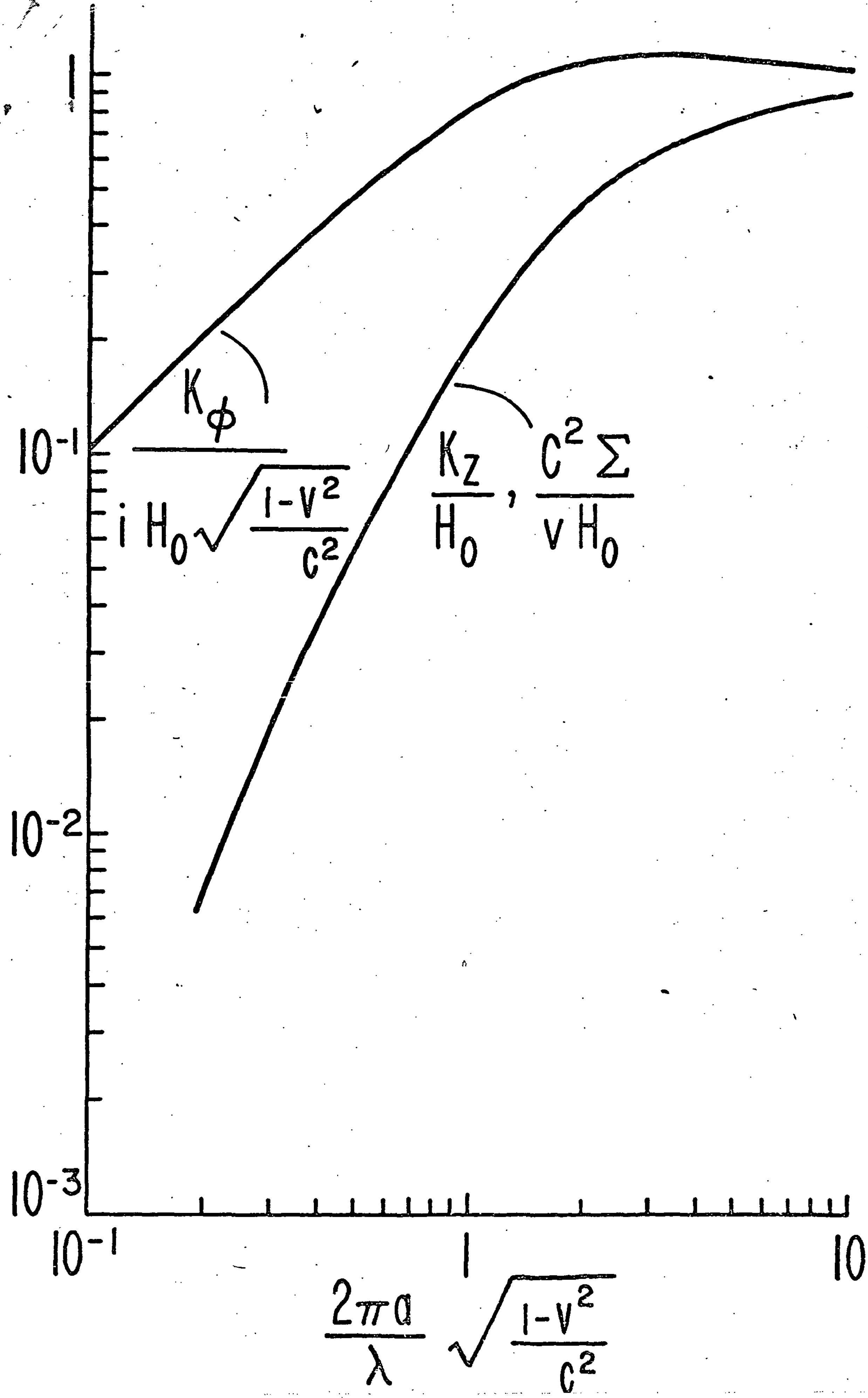


0
.5
1
2
4
$\frac{2\pi a}{\lambda}\sqrt{\frac{1-v^2}{c^2}} = 8$
1
10^{-1}
10^{-2}
2×10^{-3}
$\frac{H_\rho}{H_0}, \frac{-E_\phi}{\mu v H_0}$
0
.2
.4
.6
.8
1.0
ρ/a



0
.5
1
2
4
$\frac{2\pi a}{\lambda}\sqrt{\frac{1-v^2}{c^2}} = 8$
1
10^{-1}
10^{-2}
10^{-3}
$\frac{H_\phi}{H_0}, \frac{E_\rho}{\mu v H_0}$
0
.2
.4
.6
.8
1.0


1
10^{-1}
10^{-2}
10^{-3}
$\frac{H_z}{iH_0\sqrt{1-\frac{v^2}{c^2}}}$
2
1
.5
1
2
4
8
.25
$\frac{2\pi a}{\lambda}\sqrt{\frac{1-v^2}{c^2}} = .1$
0
.2
.4
.6
.8
1.0



1
10^{-1}
10^{-2}
10^{-3}
$\frac{K_\phi}{i H_0 \sqrt{\frac{1-v^2}{c^2}}}$
$\frac{K_Z}{H_0}, \frac{c^2 \Sigma}{v H_0}$
10^{-1}
1
10
$\frac{2\pi a}{\lambda} \sqrt{\frac{1-v^2}{c^2}}$