UNIVERSITY OF CALIFORNIA, LOS ANGELES

ERROR ANALYSIS OF EARTH PHYSICS SATELLITE SYSTEMS

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PART I

William M. Kaula, Professor of Geophysics

Principal Investigator

Department of Planetary and Space Science

University of California

Los Angeles, California 90024

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SUMMARY

Error analyses of distant-satellite-to-close-satellite range-rate, satellite-to-sea altimetry, and ground station to satellite range are made by simulations in which observational variances are assumed, observation equations are formed, and normal equations incremented. The final normal equation matrix is inverted to obtain standard deviations and correlation coefficients.

The natural parameters solved for are the broad variations of the gravity field, represented by harmonic coefficients; local variations of gravity, represented by point masses; and the departure of the sea level from the geoid, represented by area means.

A standard case of a low (263 km) polar close satellite, three equatorial geosynchronous satellites, and eight ground tracking stations is set up. Sigmas of ±0.5 mm for range-rate, ±0.5 m for range, and ±2 m for altimetry are assumed. Twenty-one variations from this standard case are tested.

The principal conclusions:

1. Given ±0.5 mm/sec range-rate tracking from distant satellites and ±2 m altimetry, the range-rate is considerably more effective in determining variations of the gravity field.

2. For a satellite altitude of 260 km and ±0.5 mm/sec range-rate, the average resolution of determination of gravity variations will be about 3°, or 350 km.

3. The accuracy is sensitive to satellite altitude: a change of 60 km makes a difference of a factor of 50% in accuracy of determination of a given size element in the gravity field.

4. The accuracy is sensitive to the number of geosynchronous satellites from which the close satellite is tracked; apparently it is advantageous for the effect of a given element in the gravity field to be sensed from more than one direction.

5. A second close satellite at a lower inclination (say 45°) in addition to a polar satellite improves the accuracy by a factor of about 30%.

6. The accuracy of determination of the gravity variations is insensitive to the number and distribution of ground tracking stations, the accuracy of tracking station location and even the accuracy of the distant satellite orbit determination.

7. Altimeter accuracy of ±20 cm or better is needed to determine departures of mean sea level from the geoid.

I INTRODUCTION

Purpose

The purpose of this work is to estimate the accuracies with which earth parameters can be determined by new tracking techniques for given tracking station and satellite orbit configurations. It therefore is intended to aid in the optimization of satellite orbit specifications, tracking station locations, etc.

The tracking techniques considered in this report are: satellite-to-satellite range-rate; satellite-to-sea altimetry; and ground station-to-satellite range. There is no limitation on the satellite orbits considered, other than that, in a satellite-to-satellite tracking pair, one satellite is assumed to be significantly perturbed, while the other is not: i.e., one is much lower than the other.

Method

The method employed is to assume standard deviations for the tracking, construct a hypothetical series of observations, form observation equations and thence normal equations, and invert the final normal equation coefficients to obtain a covariance matrix for the parameters. Since the observations are hypothetical rather than real, for purposes of the error analysis orbits can be assumed to be secularly precessing ellipses, and the perturbations used to determine earth parameters linearly superimposed thereon.

Allowance is made in the procedure for a priori sigmas (standard deviations) for the parameters. These sigmas are estimates of the present accuracies to which the parameters are known. If the systems analyzed can make a significant contribution to the determination of a parameter, then the sigma obtained by taking the square root of the final covariance matrix should be appreciably less than the a priori sigma. While this procedure may seem less ideal than assuming no a priori sigmas whatsoever, it has the practical advantage of avoiding computational difficulties with ill-conditioned matrices.

Provision was made for the modeling of biases and other systematic errors. However, experience with such modeling to an elaborate extent in error analyses of lunar ranging under grant NGL 05-007-283 [Kaula, 1972] indicated that the results still give optimistically low sigmas. Hence such an effort is not made in the present analysis. The absolute sigmas in the results are therefore not to be accepted at face value. The changes in sigmas with changes in configuration are probably quite meaningful. Hence, given that the tracking results in significant improvement from rather simple considerations, the error analysis aids in choosing the optimum configuration.

II SYSTEM COMPONENTS

Locations

The system which the computer programs can analyze includes four types of sites for tracking instrumentation:

1) ground tracking stations,
2) close artificial satellites,
3) distant artificial satellites,
4) retro-reflectors on the moon.

The functional distinction between a "close" and a "distant" artificial satellite is that the former is measurably perturbed by longitudinal variations in the earth's gravitational field, while the latter is not.

Parameters pertaining to a ground station location are the three components of its position. Parameters pertaining to the satellites are their constants-of-integration at epoch.

Tracking

Between the four types of locations listed above, three types of tracking were assumed:

1) range from ground tracking stations to:
   - close satellites,
   - distant satellites,
   - retro-reflectors on the moon;
2) range-rate from distant satellites to close satellites;
3) altimetry from close satellites to the sea surface.

The accuracies usually assumed for the tracking types were those felt to be feasible with present technology at the 1969 Williamstown study: ±0.5 m for ground-to-satellite range, ±0.5 mm/sec for satellite-to-satellite range-rate, and ±2 m for radar altimetry. For one run, accuracies of ±0.05 mm/sec and ±0.10 m were assumed for the latter two types [MIT, 1970].

Although ranging to retro-reflectors on the moon is included in the capability of the program, the analyses described below do not include it because of the emphasis on determination of the gravitational field and mean sea level.

Consideration was also given to the inclusion of radio interferometry (VLBI) among the tracking types. An interferometer measures the difference in time of receipt of a signal at two stations. Hence, for objects at distances not extraordinarily larger than the baseline length the observation is best represented as a difference in the ranges from the two baseline ends. As such, the VLBI is the same as coupled range devices, with consequently fewer tracking opportunities. Hence for a given accuracy the error analysis will show the VLBI to be poorer than ordinary ranging.

Whether the VLBI can attain higher accuracy depends, of course, on atmospheric refractional effects. These will be larger than for laser ranging because of water vapor and higher zenith angles. It was concluded that a meaningful comparison of VLBI and ranging in an error analysis would first require a detailed study of the relative atmospheric refraction effects, hence VLBI was not included in the studies described in this report.

III PARAMETERS

Classification

The natural parameters the program is designed to test the system for ability to determine can be grouped into four classes:

1) Earth-geometrical: rotation, polar wobble, station position and drift.

2) Earth-long periodic gravitational: tides, mass shifts, zonal harmonics.

3) Earth-short periodic gravitational: spherical harmonic coefficients, mass distributions, mean sea level.

4) Moon-geometrical: lunar ephemaris, retro-reflector locations, physical librations.

Analyses pertaining to class 4) were carried out under grant NGL 05-007-283 (Error Analysis of Lunar Ranging) [Kaula, 1972]. Some work was done on class 2) in furtherance of the objectives of grant NSR 005-020-379 (Definition of a Drag-Free Satellite for Geodynamics) [Stanford, 1970]. An error analysis was made in which the parameters determined from satellite perturbations were the fixed zonal harmonics of the gravitational field, north-south variations in the tidal Love numbers and phase lags, and north-south seasonal mass shifts. The results were somewhat discouraging; for variations of wavelength shorter than representable by a 2nd degree harmonic, the sigmas were larger than the physically plausible magnitudes.

Definition and Form

The emphasis of this study is therefore on the determination of parameters in class 3), short periodic gravitational, with some investigation of their interaction with station position, in class 1). The functional distinction between "spherical harmonic coefficients" and "mass distributions" in class 3) is that the former represent variations in the gravitational field of wavelengths long enough that their integrated effect on the satellite position is large enough to be determined by tracking from ground stations alone, while the latter require either satellite-to-satellite range-rate or altimetry to be determinable. Mean sea level is here taken to be the displacement of the sea level from the geoid due to temperature, salinity, and steady-state air pressure and wind drag.

To economize in computer storage, the spherical harmonics considered were limited to the highest degrees which are well-determined by "classical" satellite geodesy techniques [Gaposchkin & Lambeck, 1971]. These we estimate to be the 11th and 12th degrees. Furthermore, only a limited number of orders n were taken. The total "long wavelength" gravity field was thus assumed to be represented by harmonics ℓ,m = 11,0; 11,3; 11,6; 11,9; 12,0; 12,3; 12,6; 12,9; and 12,12.

To economize in partial derivative calculation, the shorter wavelength mass distribution was assumed to be represented by point masses at uniform intervals. These intervals are necessarily less than $180^\circ/12 = 15^\circ$ if the point masses are to be non-zero. To economize in computer storage, these point masses were assumed to exist only within a test area covering a limited portion of the earth's surface: $30^\circ \times 30^\circ$, so as to encompass two wavelengths of the spherical harmonics. To further economize, on computer runs testing aspects other than how finely the gravity field can be resolved, the spacing was assumed to be 10°.

A Priori Sigmas

On those runs where interaction with tracking station position was taken into account, the error in each coordinate of the station position was assumed to be ±30 m, probably triple the actual uncertainty.

The a priori sigmas for the spherical harmonic coefficients were taken to be those given by the $10^{-5}/\ell^2$ rule-of-thumb.

The a priori sigma for a mass point was obtained as follows. Given the rms coefficient of potential, $\delta_\ell(V)$, the power spectrum of the potential is

$$\sigma_\ell^2(V) = (2\ell+1)\delta_\ell^2(V) \qquad (1)$$

and of a surface mass distribution ρ

$$\sigma_\ell^2(\rho) = (2\ell+1)^3 \left(\frac{M}{4\pi R^2}\right)^2 \delta_\ell^2(V) \qquad (2)$$

[Kaula, 1968, p. 67].

The mean square total anomalous mass $\sigma_s^2(m)$ for a square of side length s will be

$$\sigma_s^2(m) = s^4 \sum_{\ell=2}^{\pi/s} \sigma_\ell^2(\rho) \qquad (3)$$

The amount residual to a specific harmonic degree ℓ_x therefore will be

$$\Delta\sigma_s(m) = s^2 \left[\sum_{\ell=\ell_x+1}^{\pi/s} \sigma_\ell^2(\rho) \right]^{\frac{1}{2}} \qquad (4)$$

Numerical values for various size squares, using $\ell_x = 12$

s	$\Delta\sigma_s(m)$
2.5°	0.15×10^{-8}
5°	0.60×10^{-8}
10°	0.5×10^{-7}
15°	0.0

The unit of $\Delta\sigma_s(m)$ is the earth's mass.

The a priori sigma used for the departure of mean sea level from the geoid is based on calculation of the mean sea level from temperature and salinity data [Stommel, 1966, p. 180]. This appears to be about ±20 cm. This value was used in the analyses; it is probably an underestimate, but the results obtained did not indicate anything other than the obvious conclusion that an oceanographically valuable altimeter should have an accuracy small compared to the expected variations.

Partial Derivatives

The Kepler elements of the orbit were used as an intermediary in writing the partial derivatives with respect to spherical harmonic coefficients and the orbital constants-of-integration. The procedures in Chapter 4 of Kaula [1966] were followed for all calculations relating Kepler elements to Cartesian coordinates, ranges, etc. The principal new development related to the range-rates. To keep the partial derivatives with respect to the mass points simple, the satellite-to-satellite Doppler was treated as an acceleration: i.e., range-rates $\dot{r}$ at time intervals δt are replaced by the acceleration

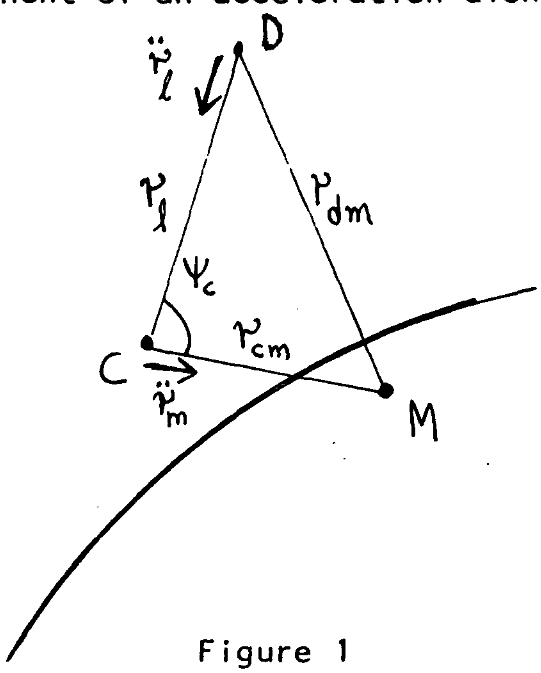
$$\ddot{r} = \dot{r}/\delta t \tag{5}$$

For a mass point spacing s (angular), the time interval δt is taken to be s/n, where n is the mean motion of the satellite.

Given distant satellite inertial coordinates $\underset{\sim}{x}_d$, close satellite inertial coordinates $\underset{\sim}{x}_c$, and mass point body fixed coordinates $\underset{\sim}{u}_m$, the mass point coordinates must first be converted to inertial by applying the negative (clockwise) rotation through the Greenwich Sidereal Time θ

$$\underset{\sim}{x}_m = \underset{\sim}{R}_3(-\theta)\underset{\sim}{u}_m \tag{6}$$

using the convention that the rotation axis is the 3-axis. Then letting r_ℓ, r_{cm}, r_{dm} be the three distances calculated from the inertial coordinates, the partial derivative will be for the component of an acceleration along the line cm in the direction dc



$$\frac{\partial \ddot{r}_\ell}{\partial m} = -\ddot{r}_m \cos\psi_c = -\frac{G}{r_{cm}^2}\cos\psi_c$$

$$= \frac{G}{r_{cm}^2}\,\frac{r_{cm}^2 + r_\ell^2 - r_{dm}^2}{2 r_{cm} r_\ell} \qquad (7)$$

Figure 1

using the cosine law to calculate the angle at the close satellite, ψ_c. The negative sign appears because a positive acceleration is counted to be away from the distant satellite. Alternatively, $\cos\psi_c$ can be expressed in terms of the vector dot product

$$\frac{\partial \ddot{r}_\ell}{\partial m} = -\frac{G}{r_{cm}^2}\,\frac{\left(x_{ci} - x_{di}\right)\left(x_{ci} - x_{mi}\right)}{r_{cm} r_\ell} \qquad (8)$$

In (8) and hereafter, summation over subscripts repeated in a product, i, j, or k, applies.

To obtain the component of acceleration $\ddot{r}_\ell$ along the line-of-sight between the two satellites, define

$$x_{\ell i} = x_{ci} - x_{di} \tag{9}$$

Then

$$r_\ell = \left[x_{\ell i} x_{\ell i}\right]^{\frac{1}{2}} \tag{10}$$

$$\dot{r}_\ell = x_{\ell i}\dot{x}_{\ell i}/r_\ell \tag{11}$$

$$\ddot{r}_\ell = \left(\dot{x}_{\ell i}\dot{x}_{\ell i} + x_{\ell i}\ddot{x}_{\ell i}\right)/r_\ell - \dot{r}_\ell x_{\ell i}\dot{x}_{\ell i}/r_\ell^2$$

$$= \left(\dot{x}_{\ell i}\dot{x}_{\ell i} + x_{\ell i}\ddot{x}_{\ell i}\right)/r_\ell - \left(x_{\ell i}\dot{x}_{\ell i}\right)^2/r_\ell^3 \tag{12}$$

For the partial derivatives of the acceleration (12) with respect to the constants-of-integration and the spherical harmonic coefficients, partial derivatives with respect to osculating Kepler elements s_{cj} and s_{dj} are needed

$$\frac{\partial \ddot{r}_\ell}{\partial s_{ij}} = \left[-\left(\dot{x}_{\ell i}\dot{x}_{\ell i} + x_{\ell i}\ddot{x}_{\ell i}\right)/r_\ell^2 + 3\left(x_{\ell i}\dot{x}_{\ell i}\right)^2/r_\ell^4\right]\frac{\partial r_\ell}{\partial s_{cj}}$$

$$+ \left[\frac{\ddot{x}_{\ell i}}{r_\ell} - \frac{2\dot{x}_{\ell i}\left(x_{\ell k}\dot{x}_{\ell k}\right)}{r_\ell^3}\right]\frac{\partial x_{cc}}{\partial s_{cj}}$$

$$+ \left[\frac{2\dot{x}_{\ell i}}{r_\ell} - \frac{2x_{\ell i}\left(x_{\ell k}\dot{x}_{\ell k}\right)}{r_\ell^3}\right]\frac{\partial \dot{x}_{ci}}{\partial s_{cj}}$$

$$+ \frac{x_{\ell i}}{r_\ell}\frac{\partial \ddot{x}_{ci}}{\partial s_{cj}} \tag{13}$$

where, from (10)

$$\frac{\partial r_\ell}{\partial s_{cj}} = \frac{x_{\ell i}}{r_\ell} \cdot \frac{\partial x_{ci}}{\partial s_{cj}} \tag{14}$$

The partial derivatives $\partial x_{ci}/\partial s_{cj}$ and $\partial \dot{x}_{ci}/\partial s_{cj}$ are given by Kaula [1966, pp. 67-68]

$$\frac{\partial x_{ci}}{\partial(\Omega, I, \omega)} = \frac{\partial R_{ij}}{\partial(\Omega, I, \omega)} q_j$$

$$\frac{\partial x_{ci}}{\partial(a, e, M)} = R_{ij} \frac{\partial q_j}{\partial(a, e, M)} \tag{15}$$

$$\frac{\partial \dot{x}_{ci}}{\partial(\Omega, I, \omega)} = \frac{\partial R_{ij}}{\partial(\Omega, I, \omega)} \dot{q}_j$$

$$\frac{\partial \dot{x}_{ci}}{\partial(a, e, M)} = R_{ij} \frac{\partial \dot{q}_j}{\partial(a, e, M)} \tag{16}$$

where R_{ij} is an element of the rotation matrix $\underset{\sim}{R}_{xq}$ [Kaula, 1966, p. 18]

$$\underset{\sim}{R}_{xq} = \underset{\sim}{R}_3(-\Omega)\ \underset{\sim}{R}_1(-I)\ \underset{\sim}{R}_3(-\omega) \tag{17}$$

and q_j, $\dot{q}_j$ are components of position and velocity in a coordinate system with the 3-axis normal to the osculating orbital plane and the 1-axis toward perigee [Kaula, 1966, p. 24]

$$\underset{\sim}{q} = \begin{Bmatrix} a(\cos E - e) \\ a\sqrt{1-e^2}\ \sin E \\ 0 \end{Bmatrix} \tag{18}$$

$$\dot{\underset{\sim}{q}} = \begin{Bmatrix} -\sin E \\ \sqrt{1-e^2}\cos E \\ 0 \end{Bmatrix} \frac{na}{1 - e\cos E} \tag{19}$$

Similarly,

$$\frac{\partial \ddot{x}_{ci}}{\partial(\Omega, I, \omega)} = \frac{\partial R_{ij}}{\partial(\Omega, I, \omega)} \ddot{q}_j$$

$$\frac{\partial \ddot{x}_{ci}}{\partial(a, e, M)} = R_{ij} \frac{\partial \ddot{q}_j}{\partial(a, e, M)} \tag{20}$$

where the acceleration vector $\ddot{\underset{\sim}{q}}$ can be taken as derived from the central term $-\mu/r^2$

$$\ddot{\underset{\sim}{q}} = -\mu \underset{\sim}{q}/r^3$$

$$\frac{\partial \ddot{q}_j}{\partial s_k} = \frac{\mu}{r^3}\left[\frac{3q_j}{r^2}q_m - \delta_{jm}\right]\frac{\partial q_m}{\partial s_k} \tag{21}$$

The notation used here for Kepler elements is the same as in Kaula [1966]: Ω, I, ω are, respectively, nodal longitude, inclination, and argument of perigee; a, e, M are semi-major axis, eccentricity, and mean anomaly; n, E are mean motion and eccentric anomaly. The partial derivatives of the Kepler elements with respect to the orbital constants-of-integration and spherical harmonic coefficients of the gravity field are given in Kaula [1966, pp. 69-70].

The other new development not discussed in Chapter 4 of Kaula [1966] is radar altimetry. For the satellite altitude above the sea we can write

$$h = r - \left(r_e(\varphi) + N + \nu\right) \tag{22}$$

where r is the orbital radius, r_e is the radius of the reference ellipsoid, N is the geoid height, and ν is the departure of the sea level from the geoid. For the purposes of an error analysis we can assume the flattening of the reference ellipsoid to be negligible, and thus concentrate on the quantities dependent on significantly unknown parameters. These include the orbital constants-of-integration, $\underset{\sim}{s}_o$; the long wavelength variations of the gravitational field, expressed by the harmonic ceofficients $\bar{C}_{\ell m}$, $\bar{S}_{\ell m}$; the short wavelength variations of the gravitational field, expressed by the mass points m_i; the departure of the tide height from the equilibrium value, $\delta\xi$; and the departure η of sea level from the tide height because of temperature, salinity, wind stress, and short term pressure variations. Thence

$$r = r\left(\underset{\sim}{s}_o\right) + \frac{\partial r}{\partial s_{oi}} \delta s_{oi} + \frac{\partial r}{\partial\left(\bar{C}_{\ell m}, \bar{S}_{\ell m}\right)} \delta\left(\bar{C}_{\ell m}, \bar{S}_{\ell m}\right)$$

$$+ \delta r(\underset{\sim}{m}) \tag{23}$$

assuming that the direct and equilibrium tide effects on the orbit are adequately known, and that the tidal $\delta\xi$ and oceanographic perturbations η are of negligible effect. These assumptions are all quite safe, at least for orbit durations of a few days. A larger effect will be $\delta r(\underset{\sim}{m})$, the perturbation of the orbital radius by the mass points representing the shorter wavelength gravitational variations. However, it is awkward to calculate $\delta r(\underset{\sim}{m})$, and furthermore it is certainly smaller than the mass point effect on the geoid height, $\delta N(\underset{\sim}{m})$, or on the range-rate $\delta \dot{r}_{\ell}(\underset{\sim}{m})$. Hence we assume it also to be negligible. For the partial derivatives of the radius with respect to the constants-of-integration and spherical harmonic coefficients, we again use the osculating Kepler elements as intermediaries. From

$$r = a(1 - e \cos E) \tag{24}$$

we get

$$\frac{\partial r}{\partial(a,\ e,\ M)} = \left\{\frac{r}{a},\ \frac{a^2}{r} e \sin E - a \cos E,\ \frac{a^2}{r} e \sin E\right\} \tag{25}$$

The geoid height N at φ_i, λ_i can be written as

$$N = R \sum_{\ell,m} \overline{P}_{\ell m}(\sin\varphi_i)\left[\overline{C}_{\ell m}\cos m\lambda_i + \overline{S}_{\ell m}\sin m\lambda_i\right]$$

$$+ \frac{G}{g}\sum \frac{m_j}{r_{ij}} \tag{26}$$

where r_{ij} is the distance from φ_i, λ_i to the mass point location, R is the radius of the earth, and g is the gravitational acceleration. Thence

$$\frac{\partial N}{\partial\left(\bar{C}_{\ell m},\bar{S}_{\ell m}\right)} = R\bar{P}_{\ell m i}\left[\cos m\lambda_i,\ \sin m\lambda_i\right] \tag{27}$$

$$\frac{\partial N}{\partial m_j} = \frac{G}{g r_{ij}} \approx \frac{R^2}{M r_{ij}} \tag{28}$$

A final assumption made was that the number of passes over each square of sea surface was sufficient to discriminate its mean departure from an equipotential, $\partial\eta_o$, from the time varying components of ν.

To summarize, the partial derivatives pertaining to the altitude over a subsatellite point location φ_i,λ_i are

$$\frac{\partial h}{\partial s_{oi}} = \frac{\partial r}{\partial s_i}\cdot\frac{\partial s_i}{\partial s_{oj}} \tag{29}$$

$$\frac{\partial h}{\partial\left(\bar{C}_{\ell m},\bar{S}_{\ell m}\right)} = \frac{\partial r}{\partial s_i}\cdot\frac{\partial s_i}{\partial\left(\bar{C}_{\ell m},\bar{S}_{\ell m}\right)} - R\bar{P}_{\ell m}\left[\cos m\lambda_i,\ \sin m\lambda_i\right] \tag{30}$$

$$\frac{\partial h}{\partial m_j} = -\frac{R^2}{M r_{ij}},\quad i \neq j \tag{31}$$

$$\approx -\frac{2R^2}{s},\quad i \neq j \tag{32}$$

where s is point spacing.

$$\frac{\partial h}{\partial \eta_{oj}} = -\delta_{ij},\ \text{the Kronecker delta.} \tag{33}$$

Standard Case and Variations

To keep computer time within reasonable bounds, and to minimize confusion, it is desirable (as in most error analyses) to choose one set of parameter values to define a standard configuration. The effects of changing parameters are then tested by one-at-a-time variations from the standard case.

The parameters can be classified into four groups for general discussion. We give here only the significant parameters and their values. The full set required for a computer run are best learned from the source program and sample input available on request.

1. Stations

The number, capabilities, percent clear weather, and locations of ground tracking stations can be specified. For the standard case, we assumed eight well-distributed stations with capability of laser ranging to both close and distant satellites and 50% probability (random day-to-day) of clear weather. The locations:

38° N, 23° E: Greece
35° N, 139° E: Japan
20° N, 204° E: Hawaii
35° N, 282° E: North Carolina
33° S, 18° E: Cape of Good Hope
31° S, 136° E: South Australia
26° S, 216° E: Rapa (south of Tahiti)
33° S, 290° E: Central Chile

2. Satellites

The number and orbital elements of each type of satellite can be specified. For a close satellite of specified a, e, I, the minimum adjustment of the semi-major axis is made and the nodal longitude is calculated so as to assure optimum coverage of the point mass and sea level test area. For the standard case, one close satellite and three distant satellites, all geosynchronous, were assumed. Their elements

a/R	e	I	λ_o
1.413	0.01	90°	--
6.61	0.00	0°	212.8°
6.61	0.00	0°	248.8°
6.61	0.00	0°	284.8°

The altitude of the close satellite is thus 263 km.

3. Tracking

Parameters affecting the entire program are the duration of tracking and the relative weights of different tracking types. The duration used should be the minimum to get good coverage of all elemental blocks in the test area. A close satellite will have a mean motion near 16 cycles/day. If the period is not an exact integral fraction of a day, then low latitude squares of side length s° will be crossed about s/11 times a day. Durations of 0.8 s days were used in the test runs, thus giving about seven crossings for each elemental block. For the standard case, this duration was eight days.

The weights for all tracking types were unity for the standard case.

The station-to-satellite laser ranging had a rather elaborate error model carried over from the lunar ranging error analysis, with allowance for components of random error and bias dependent on both zenith angle and sun-satellite angle. For the earth physics satellite error analysis the bias was omitted and the random error sigma in the standard case was taken to be

$$\sigma(r) = \left[1.0^2 + 0.1^2 \sec^2 z + 0.5^2 \cos^2\psi\right]^{\frac{1}{2}} \text{m} \qquad (34)$$

where z is zenith angle and ψ is sun-satellite angle. The tracking intervals assumed in the standard case were 0.5 days for the close satellite and 0.889 days for the distant satellite. The program determines when the satellite will be closest to the zenith for each station within the specified interval, 0.5 days, and assumes the range observed at this time. The tracking interval for geosynchronous satellites was deliberately made a non-integer fraction of a day to obtain a greater variety of geometries. For tracking over N days, the interval was normally taken to be $N^2/(N+1)/8$: i.e., enough to get at least nine ranges to determine the six orbital constants-of-integration.

The satellite-to-satellite range-rate had only a simple random error; ± 0.5 mm/sec was assumed in the standard case. The tracking interval Δt depended on the spacing s between mass points in the test area and the distance r_{cm} from the nearest square in the test area

$$\Delta t = k r_{cm}^{m} \tag{35}$$

Based on the criteria that the maximum Δt anywhere corresponds to a 45° step along the orbit, and that the maximum Δt in or into the test area corresponds to a step equal to the spacing s, values of k and m are

s	k	m
2.5°	0.00716	1.084
5°	0.00755	1.014
10°	0.00793	0.944
15°	0.00810	0.918

for Δt in days and r_{cm} in earth radii.

The satellite-to-sea altimetry was assumed to have a random error sigma of ±2 m. It was assumed that on every pass over the test area one observation was made on every elemental square crossed.

4. Natural Parameters

As described above, the long wavelength part of the gravity field was assumed to be represented by a set of 16 11th and 12th degree harmonics, and the short wavelength part by a set of nine mass points at 10° intervals over the 30° x 30° square. The departure of mean sea level from an equipotential was assumed to be represented by the mean values for nine corresponding $10^\circ \times 10^\circ$ squares.

In the calculation, only one set of orbital constants-of-integration for the distant satellite was carried at a time, by using the technique of normal matrix partitioning [Kaula, 1966, p. 105]. Hence the total number of parameters for the standard case was $16 + 2 \times 9 + 2 \times 6 = 46$.

Runs were designated by a numbering system S - Q, where S was the mass point & sea level spacing and Q the number of a question to be answered by a variation from the standard case. If a question had a minor variant, the variation number was followed by a letter. The standard case was designated 10-1. The variations, and the corresponding changed input:

5-2 Closer resolution than 10°.

Close satellite $a/R = 1.0407$

Satellite-to-satellite range-rate interval parameters:

$k = 0.00755$, $m = 1.014$

2.5-2 Close satellite $a/R = 1.0404$.

Internal parameters:

$k = 0.00716$, $m = 1.084$

10-3 Changed tracking station array.

A. eight stations, two Pacific stations replaced by near polar stations:

20°N, 204°E by 64°N, 212°E: Alaska

26°S, 216°E by 78°S, 195°E: Antarctica

B. four stations, three most distant from the test area omitted, leaving:

38° N, 23° E

35° N, 202° E

33° S, 18° E

33° S, 290° E

C. 0 stations tracking close satellites -- but all eight tracking distant satellites. The corresponding weight is set zero.

10-4 Omitting the satellite-to-satellite range-rate. The corresponding weight is set zero.

10-5 Omitting the altimetry. The corresponding weight is set zero.

10-6 Tracking station positions uncertain.

A. tracking station coordinates are added to the parameters set, and assumed to have a priori sigmas of ±30 m

B. the same as 6A, plus a second close satellite of inclination 45°

10-7 Only one geosynchronous satellite. The number of distant satellites is reduced to one at a longitude over the test area.

10-8 A second close satellite, inclination 45°.

10-9 Tracking distant satellites at exactly daily intervals, instead of 8/9 per day.

10-10 Improvement in accuracy of satellite-to-satellite range-rate and altimetry by an order of magnitude.

10-11 Distant satellites are four in polar orbit instead of three in geosynchronous equatorial orbit

10-12 Altitude of close satellites is varied:

A. lowered by 60 km

B. raised by 60 km

10-13 Test area at high latitude: $80^\circ \pm 50^\circ$.

10-14 Test area at high latitude <u>and</u> four distant polar satellites replace three equatorial geosynchronous satellites.

10-15 A. One distant satellite, in an orbit such that it drifts 360° in eight days: a = 621R.

B. Three distant satellites drifting 360° in eight days.

IV ALGORITHM

Before giving the results of computations carried out with the standard case and variations therefrom, a description of the program procedure may help to understand the nature of the analysis.

The program has five main components: a main program for read-in and conversion of data, matrix manipulation common to all derivation types, and write-out of the results; plus four principal subroutines, one for each observation type: station-to-distant satellite range (GRD); satellite-to-satellite range-rate (DDC); station-to-close satellite range (GRC); and close satellite-to-sea altimetry (CAG).

The main program reads in all input data, and converts it to "planetary" units: i.e., such that the earth's radius and mass and the gravitational constant are unity. All angles are converted to radians, and rotation matrices used repeatedly are calculated. About the only preliminary adjustments of input data are to correct the nodal longitude and semi-major axis of the close satellites so that the test area is sampled as much as possible. The motion of perigee $\dot{\omega}$ and the perturbation of the mean motion $\Delta\dot{M}$ by the oblateness J_2 are calculated in the usual manner [Kaula, 1966, p. 39]. Luni-solar contributions are also included. Then defining

$$k = \text{integer part of } [(n + \Delta\dot{M} + \dot{\omega})/\dot{\theta} + \tfrac{1}{2}] \tag{36}$$

the optimized semi-major axis is

$$a = \mu^{1/3}[(k - s/2\pi)\dot{\theta} - \Delta\dot{M} - \omega]^{-2/3} \tag{37}$$

where $\dot{\theta}$ is the sidereal rotation rate of the earth and s is the size of mean sea level elements in radians. To start the satellite off over the test area, the argument of perigee ω is calculated by

$$\sin\omega = \sin\varphi_s / \sin I, \quad -\pi/2 < \omega < \pi/2 \tag{38}$$

and the longitude of the node is calculated by

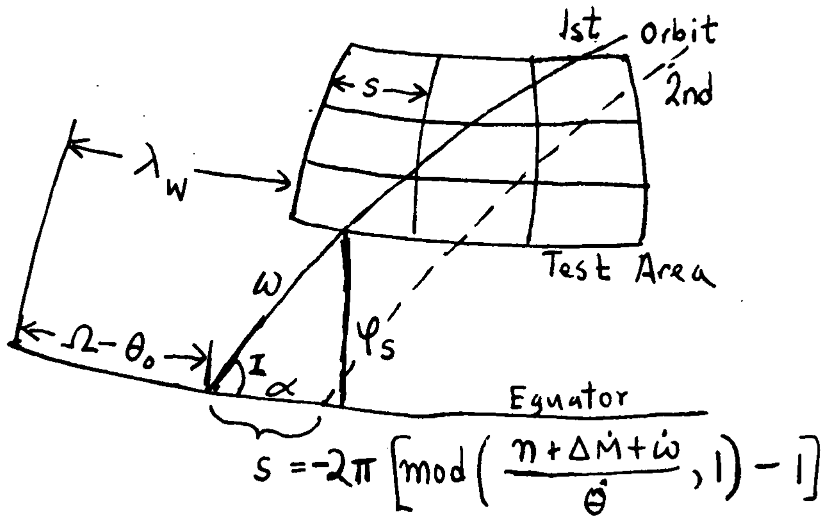


Figure 2

$$\cos \alpha = \cos \omega / \cos \varphi_s$$
$$\Omega = \lambda_W + s/2 + \theta_o - \alpha \tag{39}$$

where λ_W is the longitude of the western border of the test area, φ_s is the latitude of the south border, and θ_o is the Greenwich sidereal time at the starting epoch. The quadrant of α is the same as that of ω.

The heart of the program is a nest of two "DO" loops, the outer over all distant satellites and the inner over all close satellites. In the outer, the subroutine for station-to-distant satellite range (GRD) is called first. Then the inner loop over close satellites is executed, and for each close satellite the satellite-to-satellite range-rate (DDC), station-to-satellite range (GRC) and satellite-to-surface altimetry (CAG) subroutines are called. At the end of the outer loop the normal equation block corresponding to the distant satellite is reversed by the partitioned matrix technique [Kaula, 1966, p. 105].

In the case of the tracking type subroutines (GRD, DDC, GRC, CAG) the entire orbit histories are run through for the prescribed duration, determining when observations are possible, forming the observation equations at these times, and incrementing the normal equations.

For station-to-satellite ranging, the possibility of ranging depends not only on geometrical conditions, but also on clear weather, as determined by selection of a random number between 0.0 and 1.0. If this number exceeds the percent clear days, then that station does not observe that day. Determination of the combination of Greenwich sidereal time θ and eccentric anomaly E for observability is by the iterative technique in Kaula [1966, pp. 87-88]. The formation of partial derivatives with respect to the orbit constants-of-integration and spherical harmonic coefficients of the gravitational field is in accord with Kaula [1966, pp. 63 & 67-71]. Partial derivatives with respect to the mass points are neglected, since the ranging will be much less sensitive thereto than are the satellite-to-satellite range-rate and the altimetry.

For satellite-to-satellite range-rate, after initial calculations (such as rectangular coordinates of mass points) a starting time Δt is calculated from (35) using the minimum possible r_{cm}, a-R. Then intervisibility of the distant and close satellites is calculated as the condition that the angle ψ subtended at the earth's center between the two satellites is less than the angle ψ_x corresponding to a grazing line-of-sight (see Figure 3)

$$\cos \psi > \cos \psi_x$$

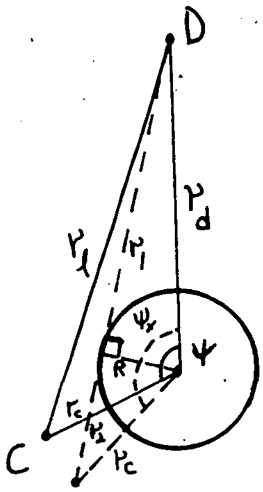


Figure 3

$$\cos \psi = \frac{r_c^2 + r_d^2 - r_\ell^2}{2 r_c r_d}$$

$$\cos \psi_x = \frac{r_c^2 + r_d^2 - (r_1 + r_2)^2}{2 r_c r_d}$$

$$-r_\ell^2 > -\left[(r_c^2 - R^2)^{\frac{1}{2}} + (r_d^2 - R^2)^{\frac{1}{2}}\right]^2$$

$$r_\ell < (r_c^2 - R^2)^{\frac{1}{2}} + (r_d^2 - R^2)^{\frac{1}{2}} \qquad (40)$$

If the satellites satisfy condition (40), the acceleration and its derivatives are calculated by (7) - (16). Partial derivatives with respect to constants-of-integration of both satellite orbits, spherical harmonic coefficients of the gravitational field, and mass points within a maximum range (usually about 0.4R) are calculated. The minimum r_{cm} is determined for the purpose of computing the time increment Δt to the next step by (35) and the cycle repeated until the end time is reached.

For satellite-to-sea altimetry, the mean anomaly and Greenwich sidereal time when the satellite will be over the test area must be calculated. Let α_{Nk} and α_{sk} be calculated in the same manner as α in (38) - (39), but using the latitudes φ_k of each of the latitudinal bands of elements in the test area. The times of initial crossings, both southward and northward, of the central latitude φ_c of the test area are calculated by (refer to Figure 2)

$$\sin u_{Nc} = \sin \varphi_c / \sin I, \qquad -\pi/2 < u_N < \pi/2$$

$$u_{sc} = \pi - u_N$$

$$f_{N,s} = u_{N,sc} - \omega_o$$

$$\cos E_{N,s} = \frac{1}{e}\left(1 - \frac{\left(1-e^2\right)}{1 + e \cos f_{N,s}}\right)$$

$$M_{N,s} = E_{N,s} - e \sin E_{N,s}$$

$$t_{N,s} = t_o + \left(M_{N,s} - M_o\right) / \dot{M} \tag{41}$$

The node $\Omega_{N,s}$ and Greenwich sidereal times $\theta_{N,s}$ are then calculated, and the longitudes when crossing the central latitude

$$\lambda_{N,sc} = \alpha_{N,sc} + \Omega_{N,s} - \theta_{N,s} \tag{42}$$

If the longitude is within a band making it possible for any of the test area elements to be crossed, then the mean anomalies for each latitude band are calculated and the same process (41) - (42) repeated to determine the exact elements crossed. The partial derivatives of altitude with respect to mean sea level are calculated by (33). After taking the other partial derivatives (29) - (32) and incrementing the normal equations, the time is advanced by $2\pi/(\dot{M}+\dot{\omega})$ and the process repeated until the whole duration is completed.

After the loops over all close and distant satellites are completed, the main program adds the inverse squares of the a priori sigmas to the normal matrix main diagonal; inverts the normal matrix; calculates the resulting standard deviations and correlation matrix; and prints out these results.

V RESULTS

The complete output for the standard case, 10-1A, is given on the next five pages. The output sigmas only for all cases are given in Table 1. The test area was assumed to be between longitudes 310°E and 340°E and latitudes 0° and 30°N. The odd odd longitudes of the distant satellites given for the standard case, 212.8°, 248.8°, and 284.8°, are the consequence of an erroneous omission of degree-to-radian conversion which was not realized until all the cases had been run. A re-run was made with the corrected program, which is designated as 10-1B in Table 1. The locations of the geosynchronous satellites with respect to the test area for 10-1A and 10-1B are shown in Figure 4. The 1B set given one distant satellite directly

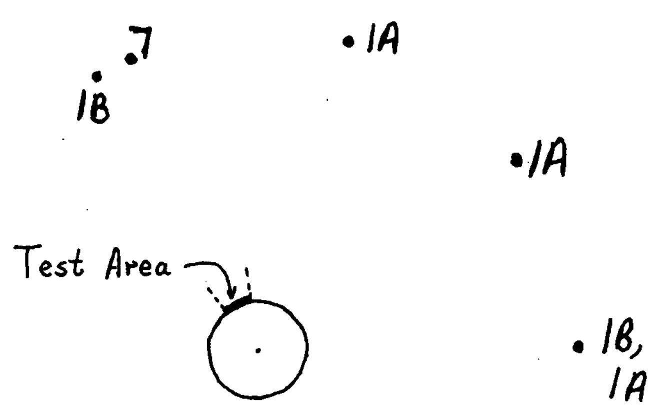


Figure 4

ERROR ANALYSIS OF EARTH PHYSICS SATELLITE SYSTEM

INPUT

NUMBER OF PARAMETERS OF EACH TYPE

MOON ELEM.	FORCED LIB.	FREE LIB.	LAT. VAR.	ROT. VAR.	ORB. ACCEL.	STA. COORD.	STA. DRIFT	REFL. COORD.	FIXED BIAS	SEC Z BIAS	SUN BIAS	AFB
0	0	0	0	0	0	0	0	0	0	0	0	0

CL.SAT.	D.SAT.	HARM.	M.PTS.	MSL	TIDES	M.SHIFT
6	6	16	9	9	0	0

STARTING MJD =41000.0, ENDING MJD =41008.0

GROUND-TO-SATELLITE RANGE
IN METERS, INSTR SIGMA= 0.500, INSTR BIAS= 0.0 , REFRACTION SIGMA= 0.100, REFRACTION BIAS= 0.0
SUN-MOON ANGLE SIGMA = 0.100, BIAS = 0.0
ASSUMED FORM OF DEPENDENCE ON SUN MOON ANGLE X--0.50+0.50COS X +0.0 COS X**0 +0.0 COS X**0 +0.0COS X**0
PERCENT DAYS ASSUMED CLEAR WEATHER BY STATION
0.50 0.50 0.50 0.50 0.50 0.50 0.50 0.50
DISTANT SATELLITE TRACKING TIME INTERVAL = 0.889000 DAYS
CLOSE SATELLITE TRACKING TIME INTERVAL = 0.500000 DAYS
MAX. ZEN. DIST. =75.0 MIN. SUN-MOON ANGLE =10.0
CLOSE SATELLITE RANGING WEIGHT =1.00
DISTANT SATELLITE RANGING WEIGHT =1.00

SATELLITE-TO-SATELLITE RANGE-RATE
IN MM/SEC, INSTR SIGMA = 0.500 INTERVAL PARAMETERS XK = 0.008, VMA = 0.944, RMAX = 0.400
WEIGHT =1.00

RADAR ALTIMETRY
IN METERS, INSTR SIGMA = 2.000, INSTR BIAS = 0.0
WEIGHT =1.00
ALTIMETRY MEASURING TIME INTERVAL = 0.100000E 00DAYS

8 TRACKING STATIONS

LONG	LAT	RANGE	VLBI
23.00	38.00	1	0
139.0	35.00	1	0
204.0	20.00	1	0
282.0	35.00	1	0
18.00	-33.00	1	0
136.0	-31.00	1	0
216.0	-26.00	1	0
290.0	-33.00	1	0

3 DISTANT SATELLITES WITH ORBIT ELEMENTS

A	E	I	NODE	PER.	M
6.6100	0.0010	0.0010	0.0	0.0	0.0
6.6100	0.0010	0.0010	0.0	0.0	36 ~~120~~.0000
6.6100	0.0010	0.0010	0.0	0.0	72 ~~240~~.0000

INDICES FOR FIXED SPHERICAL HARMONICS

L	M
11	0
11	3
11	6
11	9
12	0
12	3
12	6
12	9
12	12

MASS POINT SPACING =1100. KILOMETERS

LIMITS OF AREA FOR RESIDUAL MASS POINTS
LAT.-- 30. DEG. TO 0. DEG., LONG.--340. DEG. E. TO 310. DEG. E.

SEA LEVEL AREA ELEMENT SIZE =1100. KILOMETERS
LIMITS OF AREA FOR RESIDUAL MEAN SEA LEVEL
LAT.-- 30. DEG. TO 0. DEG., LONG.--340. DEG. E. TO 310. DEG. E.

1 CLOSE SATELLITES WITH ORBIT ELEMENTS

A	E	I	NODE	PER.	M
1.0413	0.0100	59.9000	102.2240	0.0	0.0

RESULTS

OBSERVATIONS PER STATION

8	6	15	15	5	6	13	15

SIGMAS		A PRIORI	FINAL	NOBS
GRAVITATIONAL HARMONICS				
NORMALIZED COEFF.				
	1	0.1000E-06	0.1905E-11	1980
	2	0.1000E-06	0.4917E-09	1980
	3	0.1000E-06	0.2778E-09	1980
	4	0.1000E-06	0.6262E-09	1980
	5	0.1000E-06	0.2822E-09	1980
	6	0.1000E-06	0.3879E-09	1980
	7	0.1000E-06	0.1508E-09	1980
	8	0.1000E-06	0.5602E-10	1980
	9	0.1000E-06	0.9223E-09	1980
	10	0.1000E-06	0.6115E-09	1980
	11	0.1000E-06	0.6721E-09	1980
	12	0.1000E-06	0.4285E-09	1980
	13	0.1000E-06	0.5857E-09	1980
	14	0.1000E-06	0.2179E-09	1980
	15	0.1000E-06	0.1041E-09	1980
	16	0.1000E-06	0.4576E-10	1980
SURFACE MASS POINTS				
EARTH MASS				
	17	0.5000E-07	0.5453E-09	407
	18	0.5000E-07	0.9842E-09	407
	19	0.5000E-07	0.4143E-09	407
	20	0.5000E-07	0.4125E-09	407
	21	0.5000E-07	0.7368E-09	407
	22	0.5000E-07	0.3735E-09	407
	23	0.5000E-07	0.4654E-09	407
	24	0.5000E-07	0.7331E-09	407
	25	0.5000E-07	0.4040E-09	407
SEA LEVEL ELEVATIONS				
METERS				
	26	0.1911E 00	0.1863E 00	6
	27	0.1911E 00	0.1839E 00	9
	28	0.1911E 00	0.1863E 00	6
	29	0.1911E 00	0.1855E 00	7
	30	0.1911E 00	0.1846E 00	8
	31	0.1911E 00	0.1862E 00	6
	32	0.1911E 00	0.1846E 00	8
	33	0.1911E 00	0.1846E 00	5
	34	0.1911E 00	0.1862E 00	6

DISTANT SATELLITE ORBITAL ELEMENTS

(NODE.I.PER..A.E.M)

RADIANS AND EARTH RADII

41	0.0	0.1436E-03	717
42	0.0	0.4800E-07	717
43	0.0	0.1436E-03	717
44	0.0	0.6834E-09	717
45	0.0	0.5829E-08	717
46	0.0	0.5674E-05	717

CLOSE SATELLITE ORBITAL ELEMENTS

(NODE, I, PER., A, E, M)
RADIANS AND EARTH RADII

35	0.0	0.2336E-07	1916
36	0.0	0.1781E-07	1980
37	0.0	0.3948E-05	1916
38	0.0	0.1573E-09	1980
39	0.0	0.1141E-07	1980
40	0.0	0.3948E-05	1980

CORRELATION COEFFICIENTS

```
 1  1.0

 2 -0.0 1.0

 3  0.2 0.0 1.0

 4  0.0 0.0-0.0 1.0

 5  0.3-0.1 0.6 0.0 1.0

 6  0.0-0.0 0.1-0.2 0.1 1.0

 7  0.2-0.0 0.5-0.0 0.4 0.1 1.0

 8  0.2-0.0-0.1 0.0 0.2 0.0 0.1 1.0

 9 -0.1-0.1-0.1-0.0 0.0-0.1-0.0 0.0 1.0

10  0.3 0.0 0.4 0.1 0.3 0.0 0.1-0.1-0.1 1.0

11 -0.0-0.0 0.1-0.1 0.0-0.1 0.1 0.1 0.1-0.0 1.0

12  0.1-0.1-0.2 0.0 0.4 0.0 0.2 0.3 0.2-0.2 0.0 1.0

13 -0.0 0.0 0.0 0.0-0.0-0.1 0.0 0.0 0.0 0.0 0.3-0.1 1.0

14  0.1 0.0 0.0 0.0-0.5-0.1-0.1-0.6-0.1 0.1-0.1-0.5-0.0 1.0
```

```
15  0.0 0.0 0.0-0.1 0.0-0.1 0.0 0.0 0.2-0.0 0.0 0.0-0.1-0.0 1.0

16 -0.1-0.0-0.1 0.0-0.3-0.1 0.1-0.1-0.0 0.4-0.1 0.1-0.0 0.2-0.0 1.0

17 -0.0-0.1 0.1-0.0 0.0-0.1-0.0 0.0-0.2-0.1-0.1-0.1 0.1-0.1 0.0-0.2 1.0

18  0.0 0.1 0.0 0.0 0.1-0.1 0.0-0.0-0.0 0.0 0.0 0.0 0.1-0.0-0.0-0.0-0.1 1.0

19 -0.1-0.0-0.1-0.0-0.1 0.0-0.0 0.0 0.0-0.1 0.1 0.0-0.0 0.0-0.1-0.0-0.0 0.2 1.0

20 -0.0-0.1 0.0-0.1-0.0-0.0-0.0 0.0-0.1-0.0-0.0-0.0 0.0-0.0 0.0-0.1 0.0-0.1 0.0 1.0

21  0.0 0.0 0.0-0.0 0.1-0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.1-0.0-0.1 1.0

22 -0.0-0.0-0.0 0.1-0.1 0.0 0.0 0.0 0.0-0.0 0.1-0.0-0.0 0.0-0.1 0.0 0.0-0.0 0.0 0.0 0.1 1.0

23 -0.0 0.0-0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0 1.0

24  0.0 0.0 0.1 0.0 0.1-0.0-0.0-0.0 0.0 0.1 0.0-0.0-0.0-0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.1 0.0-0.0 1.0

25 -0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0 1.0

26  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0
    1.0

27  0.0-0.0 0.0-0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0
    0.0 1.0

28  0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0
    0.0 0.0 1.0

29  0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0
    0.0 0.0 0.0 1.0

30 -0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0
   -0.0 0.0-0.0-0.0 1.0

31  0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0
    0.0 0.0 0.0 0.0-0.0 1.0

32  0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0-0.0-0.0 0.0 0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0
    0.0-0.0 0.0 0.0-0.0 0.0 1.0

33 -0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0-0.0-0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0
   -0.0 0.0-0.0-0.0 0.0-0.0-0.0 1.0
```

```
34  0.0-0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0-0.0-0.0-0.0
    0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 1.0

35  0.1-0.0 0.2 0.0 0.3 0.1 0.0-0.1-0.1 0.3-0.0 0.2-0.0-0.0-0.0 0.1-0.0 0.0-0.1-0.0 0.0-0.0-0.0 0.0-0.0
    0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 1.0

36 -0.1-0.0-0.1-0.0-0.2 0.0 0.0-0.2-0.1-0.2-0.0 0.1-0.1 0.1-0.0 0.1-0.0 0.0 0.0-0.0 0.0-0.0 0.0-0.0-0.0
   -0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.4 1.0

37 -1.0 0.0-0.2-0.1-0.3-0.0-0.2-0.2 0.1-0.3 0.0-0.1 0.0-0.1-0.0 0.1 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0
   -0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.1 0.1 1.0

38  0.2-0.0-0.1 0.0 0.2 0.0 0.1 0.9 0.0-0.1 0.1 0.3 0.0-0.5-0.0-0.1 0.1 0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0
    0.0 0.0 0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.1-0.1-0.2 1.0

39  0.2-0.1 0.1 0.0 0.2-0.0-0.1-0.0-0.0-0.0 0.0 0.1-0.0 0.0 0.0-0.1-0.1 0.0-0.0-0.1 0.1-0.0-0.1 0.1-0.0
   -0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.2-0.0 1.0

40  1.0-0.0 0.2 0.1 0.4 0.0 0.2 0.2-0.1 0.3-0.0 0.1-0.0 0.1 0.0-0.1-0.0 0.0-0.0-0.0 0.0 0.0-0.0 0.0 0.0
    0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.1-0.1-1.0 0.2 0.2 1.0

41 -0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0-0.0-0.0 0.0-0.0-0.0
   -0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0 0.0 0.0-0.0-0.0 1.0

42  0.0-0.0 0.1 0.0 0.1-0.0 0.0 0.2 0.0 0.0 0.0-0.0 0.0 0.1 0.0-0.1-0.0 0.0-0.0-0.0 0.0-0.0 0.0 0.0 0.0
    0.0 0.0 0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.1-0.3-0.0 0.2 0.0 0.0 0.0 1.0

43  0.0-0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0 0.0-0.0-0.0 0.0
    0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0-0.0-0.0-0.0-0.0 0.0 0.0-1.0-0.0 1.0

44  0.0 0.0 0.0 0.0 0.0-0.0 0.0 0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0
    0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0-0.0 0.0-0.0-0.0 0.0 0.0 0.0-0.0 0.1 0.0 1.0

45 -0.0-0.1-0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.1 0.0 0.0-0.0-0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.1-0.0-0.0 0.1-0.1
   -0.0-0.0-0.0 0.0-0.0 0.0 0.0-0.0 0.0-0.0-0.0 0.0 0.0-0.1-0.0 0.0-0.1-0.0-0.2 1.0

46 -0.0-0.0 0.0 0.0 0.0 0.0 0.0 0.1 0.1-0.0 0.0-0.0-0.0 0.0-0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.0 0.0-0.0
   -0.0-0.0 0.0 0.0-0.0 0.0-0.0-0.0 0.0-0.0-0.1 0.0 0.0-0.0-0.0-0.0 0.2-0.0-0.1 0.3 1.0
     1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25
    26  27  28  29  30  31  32  33  34  35  36  37  38  39  40  41  42  43  44  45  46
```

TABLE I

Standard Deviations for Parameters of the Gravitational Field Resulting from Error Analyses

See pp. 7 - 10 for definition of parameters, and pp. 22 - 24 for definition of runs.

Variable	Unit	10-1A	10-1B	5-2	2.5-2	10-3A	10-3B	10-3C	10-4	10-5	10-6A	10-6B	10-7	10-8	10-9	10-10	10-11	10-12A	10-12B	10-13	10-14	10-15A	10-15B
$C_{11,0}$	10^{-9}	.002	.002	.002	.002	.002	.003	.004	.045	.002	.004	.001	.002	.001	.002	.0004	.002	.001	.002	.002	.002	.002	.002
$C_{11,3}$	"	.48	.46	.71	1.06	.48	.49	.44	7.61	.48	.49	.25	.81	.25	.49	.05	.29	.42	.53	.48	.28	.78	.45
$S_{11,3}$	"	.28	.27	.19	.14	.23	.34	.44	.98	.28	.39	.15	.37	.12	.28	.05	.22	.22	.24	.27	.22	.42	.27
$C_{11,6}$	"	.63	.61	.56	.60	.63	.63	.60	10.36	.63	.64	.29	1.14	.29	.63	.06	.41	.55	.69	.42	.25	1.10	.65
$S_{11,6}$	"	.29	.27	.14	.06	.28	.44	.59	.81	.28	.51	.16	.36	.12	.28	.06	.22	.16	.22	.20	.19	.39	.27
$C_{11,9}$	"	.39	.36	.58	.91	.39	.39	.35	7.44	.39	.39	.31	.63	.31	.39	.04	.23	.33	.42	.37	.24	.64	.37
$S_{11,9}$	"	.15	.15	.13	.09	.13	.26	.37	.68	.15	.30	.15	.18	.11	.15	.04	.13	.14	.17	.23	.18	.20	.15
$C_{12,0}$	"	.06	.05	.02	.01	.04	.09	.52	.92	.06	.19	.04	.06	.33	.06	.03	.05	.04	.07	.07	.08	.07	.06
$C_{12,3}$	"	.92	.94	1.04	1.03	.94	.98	.92	8.68	.92	.98	.70	1.65	.67	.94	.10	.70	.76	.97	.77	.51	1.69	.93
$S_{12,3}$	"	.62	.63	.46	.20	.49	.82	.93	1.32	.62	.89	.42	.79	.37	.62	.10	.51	.36	.70	.58	.47	.83	.60
$C_{12,6}$	"	.67	.63	.72	.84	.67	.69	.63	17.24	.67	.69	.48	1.19	.47	.67	.07	.37	.54	.69	.51	.29	1.10	.65
$S_{12,6}$	"	.44	.39	.23	.10	.30	.59	.65	1.08	.43	.67	.25	.48	.20	.43	.07	.29	.31	.53	.37	.25	.46	.41
$C_{12,9}$	"	.59	.51	.68	.89	.59	.59	.54	15.70	.59	.59	.25	.96	.25	.59	.06	.30	.48	.59	.58	.32	.88	.52
$S_{12,9}$	"	.22	.20	.14	.10	.20	.35	.54	.42	.22	.51	.15	.26	.10	.22	.06	.17	.21	.30	.28	.24	.24	.21
$C_{12,12}$	"	.10	.10	.16	.26	.10	.10	.10	5.94	.10	.11	.10	.18	.10	.10	.01	.06	.09	.11	.11	.06	.17	.10
$S_{12,12}$	"	.05	.05	.03	.03	.04	.06	.10	.14	.05	.08	.05	.06	.04	.05	.01	.04	.03	.04	.05	.04	.06	.05
m-NW	$10^{-9}M_E$	.55	.71	.58	.64	.54	.55	.52	25.38	.55	.55	.39	.81	.39	.55	.06	.64	.41	.58	.32	.38	1.43	.56
m-NC	"	.98	.89	2.09	1.32	.98	.99	.86	34.48	.99	.99	.45	1.35	.45	.99	.10	.88	.54	1.01	.39	.25	2.89	.96
m-NE	"	.41	.35	.82	1.23	.41	.42	.46	35.63	.42	.42	.32	.76	.32	.41	.04	.40	.59	1.04			.62	.33
m-CW	"	.41	.58	.52	.64	.41	.42	.40	45.50	.41	.42	.32	.77	.32	.42	.04	.49	.27	.57	.48	.67	1.14	.42
m-CC	"	.74	.65	2.35	1.35	.74	.74	.78	47.31	.74	.74	.40	1.29	.40	.74	.07	.74	.51	1.00	.42	.48	1.11	.77
m-CE	"	.37	.28	.89	1.25	.37	.38	.44	45.69	.37	.38	.30	.55	.30	.38	.04	.47	.67	.98	.44	.42	.49	.29
m-SW	"	.47	.56	.48	.62	.47	.47	.34	34.60	.47	.47	.36	2.15	.36	.47	.05	.44	.22	.64	.84	1.16	4.29	.48
m-SC	"	.73	.70	1.86	1.27	.73	.74	.74	35.44	.73	.74	.47	1.42	.47	.75	.08	.68	.59	1.07	.79	.80	1.10	.76
m-SE	"	.40	.29	.81	1.17	.40	.40	.42	28.09	.40	.40	.32	.72	.32	.41	.04	.71	.94	1.76	.78	.65	.46	.31

over the test area, and a second which can see the close satellite when it is over the eastern edge. The IA set gives two distant satellites off to the side, subtending an angle of about 38° at the test area, and seeing the close satellite anywhere over the area. The consequence appears to be that IB is relatively better for the eastern edge and IA is better for the western edge. Since none of the sigmas differ by more than 25%, it was decided that it was not worthwhile re-running all cases with the corrected longitude (except 10-7, a single satellite over the test area).

The results from runs 5-2 and 2.5-2 indicate the resolution limit of the ±0.5 mm/sec satellite-to-satellite range—rate with the 260 km close satellite altitude is somewhere around 3° or 4°: say 350 - 400 km. The 5° output sigmas for the point masses, 0.48 to 2.35×10^{-9}, are well under the a priori, 6.0×10^{-9} (see p. 9). The 2.5° output has six sigmas which are only slightly reduced from the a priori; 1.5×10^{-9}, even though the calculation was carried twice as long as for 5-2 and four times as long as for 10-IA. This result is somewhat affected by the imperfection of the algorithm for determining the semi-major axis, described on pp. 25 - 26; for case 2.5-2, the area was not uniformly covered.

The results for 10-3A suggest that tracking stations near the pole are helpful for determining the low order (small m) harmonics. The results for 10-3C, 10-4, and 10-5 suggest that both the ±0.5 m ranging from tracking stations to the close satellite and the ±2 m altimetry have little weight in determining the gravity field variations compared to the ±0.5 mm/sec satellite-to-satellite range-rate. The large increase $\sigma(C_{12,0})$ from 10-1A to 10-3C suggests that the principal value of the ranging from ground stations is to determine the orientation of the orbit. This is corroborated by the increases in $\sigma(\Omega_C)$ and $\sigma(\omega_C)$ between the two runs, which were by factors of 3 and $2\frac{1}{2}$ respectively.

The results for 10-6A indicate that tracking station errors have some effect on determining the broad variations of the gravity field, but not the shorter wavelength variations here represented by mass points. This result is somewhat affected by the interval between satellite-to-satellite range rates being as high as 45° far from the test area.

The poorer results from 10-7, on which a single geostationary satellite over the area was used, indicate that viewing from more than one direction is helpful in resolving the shorter wavelength variations in the gravity field. The results from 10-15A, the single drifting distant satellite, are disappointing as a means of obtaining this variety of directions; possibly the eight day duration of the test is too short -- certainly the geostationary satellite locations with respect to the test area in 10-1A are better than the average expected.

The results from 10-11 indicate that tracking from the equatorial geosynchronous distant satellites can resolve variations in the gravity field at high latitudes as well as at low. The results from 10-13 and 10-14 further indicate that the four distant satellites in polar orbits are not significantly better or worse than the three equatorial geosynchronous satellites.

The second close satellite at a lower inclination, tested by runs 10-6B and 10-8, results in the expected improvement: i.e., the sigmas are reduced by a factor of about $1/\sqrt{2}$. Some correlations between gravity coefficients are reduced from about 0.6 to 0.3 or 0.4 (e.g., $S_{11,3}$ and $S_{11,6}$; $C_{12,0}$ and $C_{12,9}$).

The distant satellite tracking interval, tested by run 10-9, has a negligible effect on the gravity field determination, but it did have some effect on determination of distant satellite orbit elements, the largest increase being in $\sigma(e)$, by a factor of 2. It would also have some effect on the determination of station location.

Significant determination of η_{oj}, the mean sea level departure from the geoid, is obtained only when much better altimetry is assumed, run 10-10. From the ±10 cm altimetry over eight days sigmas of ±4 to 6 cm were obtained.

Variation of the close satellite orbit altitude by ±23% in runs 10-12A and 10-12B produced appreciable changes in the sigmas for the point masses, on the order of ±50%. Some of the changes for the lower altitude satellite were not in the expected direction, apparently because the orbital node and semi-major axis could not be optimized. But it seems clear the message is to keep the close satellite as low as practicable.

Comparison with Other Results

Error analyses of satellite-to-satellite tracking have also been done by Martin et al. [1972] and Schwarz [1972].

That by Martin et al. [1972] assumed range, rather than range-rate, between the satellites, and analyzed only the determination of low degree spherical harmonic coefficients. Hence a meaningful comparison is difficult.

The analysis by Schwarz [1972] assumed range-rate tracking. Values of the surface density coating were assumed for $5^\circ \times 5^\circ$ and $2^\circ \times 2^\circ$ rectangles and solutions were made from simulations of orbital passes and associated tracking. Both distant-close and close-close satellite pairs were tested. It was concluded that the technique could resolve elements about the same size as the satellite altitude, the definition of resolution being that the correlation coefficients between adjacent blocks are on the order of 0.8. In the present study, the correlation coefficients between adjacent blocks were about 0.5; however, since the output sigmas approached the a priori sigmas, this value is probably lower than would have been attained without a priori sigmas. To the extent that the two analyses can be compared, the agreement seems satisfactory.

VI CONCLUSIONS

Principal Recommendations

The most useful tool for determination of the variations in the gravity field appears to be definitely the satellite-to-satellite tracking (unless and until some sort of gradiometer proves feasible); ± 0.5 mm accuracy appears capable of resolving features a little greater in extent than the satellite altitude. The orbit should be kept as low as practicable; some sort of orbit maintenance system should be considered. However, since satellite-to-satellite tracking requires determination of a reference orbit, the system should be operated intermittently, not frequently. The variations in accelerations due to drag should be small compared to the gravitational variations of comparable wavelength at any altitude at which an orbit can be maintained. There appears to be a wide variety of allowable geometries of distant satellites, provided that the close satellite is tracked from at least two distinctly different directions; whether this requirement can be satisfied by tracking at different times is not yet clear.

Satellite-to-satellite tracking would definitely allow economies of tracking from ground stations. How much is not clear from the present study, because the intervals of satellite-to-satellite tracking were deliberately varied with distance from an assumed test area. Tracking of distant satellites from ground stations remains necessary, of course.

The altimeter seems inherently less capable of resolving details in the gravity field, because it measures an integral of what is measured by the satellite-to-satellite range-rate. However, since the present error of the geoid is about ± 10 m rms, a ± 2 m system would manifestly gain considerable information. To resolve differences between geoid and sea level accuracies on the order of ± 20 cm are needed.

Limitations and Possible Future Work

In modeling the satellite and tracking systems, the present program needs to be extended to include range-rate between two close satellites, and VLBI to distant satellites. Instrumental biases need to be examined with a view to more elaborate modelling, such as has been applied in error analyses of lunar ranging [Kaula, 1972]. More investigation of alternative geometries of orbits than was done in the present study should be undertaken.

In the modeling of natural parameters, there still remain several deficiencies. Time varying effects on the sea level, both regular (tides) and stochastic (storms) should be included. Because of the broad spectrum entailed, it would take some thought how to do this economically. The short wavelength variations in the gravity field should be represented by surface coatings for area elements, rather than by point masses. The perturbations of the radial coordinates of the close satellite by short wavelength variations should be included; how to do this with an orbital representation dependent on a mean intermediary is not yet clear.

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