A TWO DIMENSIONAL, ITERATIVE SOLUTION FOR THE JET FLAP

by Alan Charles Herold

Prepared by
UNIVERSITY OF WASHINGTON
Seattle. Wash. 98195
for Langley Research Center

1. Report No. NASA CR-2190	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A TWO-DIMENSIONAL, ITERATIVE SOLUTION FOR THE JET FLAP		5. Report Date February 1973
		6. Performing Organization Code
7. Author(s) Alan Charles Herold		8. Performing Organization Report No.
9. Performing Organization Name and Address University of Washington Seattle, Washington 98195		10. Work Unit No. 760-61-02-81-23
		11. Contract or Grant No. NGL 48-002-010
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Contractor Report
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A solution is presented for the jet-flapped wing in two dimensions. The main flow is assumed to be inviscid and incompressible. The flow inside the jet is considered irrotational and the upper and lower boundaries between the jet and free stream are assumed to behave as vortex sheets which allow no mixing.

The solution is found to be in satisfactory agreement with two dimensional experimental results and other theoretical work for intermediate values of momentum coefficient $(1.0 < C_J < 5.0)$, but the regions of agreement vary with jet exit angle. At small values of momentum coefficient, the trajectory for the jet, as computed by this method, has more penetration than that of other available data, while at high C_J's this solution results in less penetration of the jet into the main flow.

17. Key Words (Suggested by Author(s)) Jet flap Two dimensional Numerical analysis	18. Distribution Statement Unclassified – Unlimited		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 47	22. Price* $3.00

PREFACE

This report was prepared with the support of the NASA through

Grant No. NGL-48-002-010. The work herein was presented by Mr.

Herold as a thesis for the degree of Master of Science in

Aeronautics and Astronautics at the University of Washington,

and was carried out under the supervision of Professor Robert G.

Joppa.

TABLE OF CONTENTS

PAGE

LIST OF FIGURES

LIST OF SYMBOLS

α Airfoil angle of attack

a The distance between vortices

c Airfoil chord length

C_L Lift Coefficient

C_J Momentum coefficient of jet

ds Arc length at centerline of jet element

ds_1, ds_2 Arc lengths of upper and lower boundaries of jet element

$d\psi$ Angle subtended by element of jet

δ Jet thickness

δ_0 Jet thickness at infinity

γ_a Vorticity per unit length in airfoil

γ_J Vorticity per unit length in jet

$\gamma_{1,2}$ Vorticity per unit length on upper and lower boundaries of jet element

Γ_T The total circulation about a contour which enclosed the entire lifting system

$\Gamma_{i...n}$ Point vortices used to represent chord and jet

J Momentum flux of jet

J_0 Momentum flux of jet at infinity

L Aerodynamic lift

p_0 Mean pressure at infinity

Φ_i The angle between a normal and the velocity induced by the ith vortex

P_1, P_2 Static pressures at upper and lower jet boundaries

q Dynamic pressure

Q Mass flow of jet

r_i	The distance from a control point to the ith point vortex
R	Radius of curvature of elemental section of jet
ρ	Jet density
ρ_0	Main stream density
τ	Exit angle of jet sheet relative to airfoil chord
θ	The angle made by a jet segment with the horizontal
u_1, u_2	Main stream velocities just outside jet boundaries
U	Average free stream velocity across jet
U_0	Average free stream velocity across jet at infinity
U_n	The component of the free stream velocity along a normal
v_1, v_2	Jet velocities inside wake boundaries
v	The velocity induced by a vortex
V	Average velocity across jet section
V_0	Average velocity across jet section at infinity

SUMMARY

A solution is presented for the jet-flapped wing in two dimensions. The main flow is assumed to be inviscid and incompressible. The flow inside the jet is considered irrotational and the upper and lower boundaries between the jet and free stream are assumed to behave as vortex sheets which allow no mixing.

In the case of an infinitely thin jet, as is explored in this paper, the jet behaves as would a single vortex sheet which has a vortex distribution in which the vorticity, at any point, is proportional to the momentum coefficient and the free stream velocity squared and is inversely proportional to the radius of curvature and the tangential velocity at that point.

The above-mentioned relation is derived and used along with the condition that the vorticity in the airfoil and in the jet be such that they are streamlines of the main flow to create a numerical iterative solution for the jet flap. The method was programmed in FORTRAN IV for digital computation. Products of the solution are the vorticity distribution and the jet shape for any attack angle, jet exit angle and momentum coefficient.

Satisfactory solutions have been computed for a wide range of momentum coefficients, jet and attack angles and jet lengths.

The solution is found to be in satisfactory agreement with two dimensional experimental results and other theoretical work for intermediate values of momentum coefficient $(1.0 < C_J < 5.0)$, but the regions of agreement vary with jet exit angle. At small values of momentum coefficient, the trajectory for the jet, as computed by this method, has more penetration than that of other available data, while at high C_J's this solution results in less penetration of the jet into the main flow.

INTRODUCTION

The jet-flapped wing is an airfoil augmented with a jet of high velocity air which issues from a span-wise slot near the trailing edge.
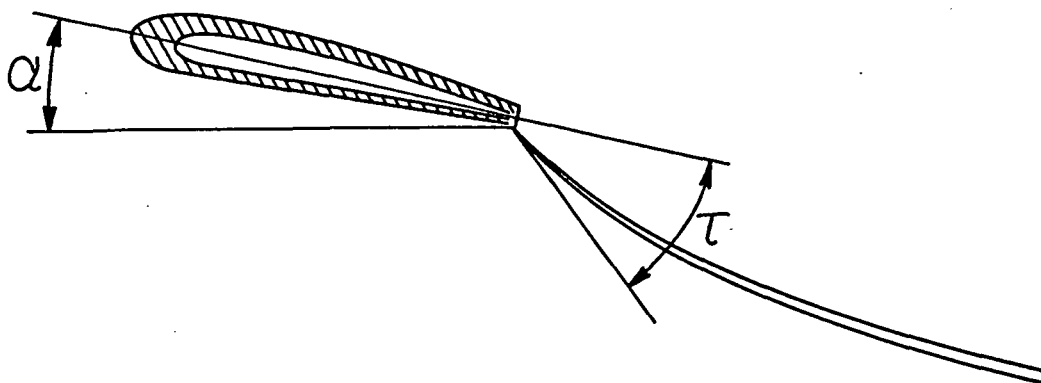
Figure 1. The Jet Flap Lifting System

The jet, as considered in this two-dimensional treatment, exists at an angle τ with the airfoil which is inclined at an angle α with the free stream, remains in its thin-sheet form and eventually aligns itself with the main flow.

The aerodynamic advantages of such a lifting system are significant. Since the jet ultimately aligns with the free stream direction, theoretically all of the jet momentum is recovered as usable thrust. In addition, high lift is achieved from the vertical component of jet momentum as well as the pressure lift on the airfoil which arises from the asymmetry created in the main stream by the presence of the jet. These advantages suggest the diverting of conventional jet thrust through such a slotted wing resulting in shorter runway distances and lower stall speeds.

The jet flap was investigated as early as 1933 by Shubauer[1] but the most significant theoretical investigations were carried out by the British in the mid 1950's[2,3,4]. Spence[4] has presented a solution for the jet flap in two dimensions but is limited by the assumption of small deflections of the jet and small attack angles.

The purpose of this approach is to avoid the small deflection assumptions to arrive at a non-linearized solution suitable for any

2

attack or jet angle.

ANALYSIS OF POLAR ELEMENT OF JET WAKE
IN UNIFORM FLOW FIELD

A polar element of the jet will be analyzed as was done by Preston[2] in order to arrive at a relation between the jet vorticity, its shape, its momentum coefficient and the velocity field of the main stream.

It is assumed in the following that the flow both inside and outside the jet is irrotational and that the total pressure within the jet is greater than in the free stream.

Consider an element of the wake of thickness δ , which sub-tends an angle $d\psi$ at its center of curvature, the radius of which is R .
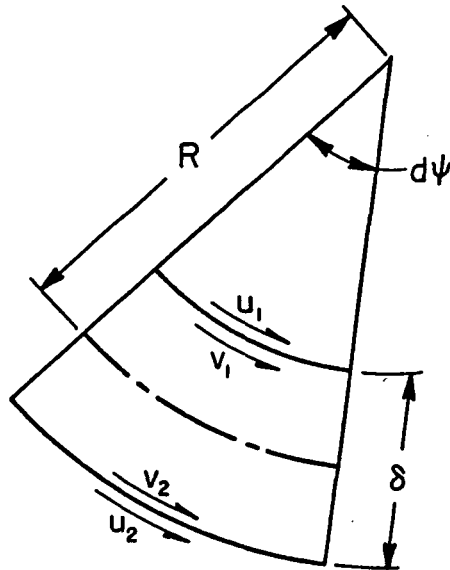
Figure 2. A Polar Element of Jet Wake

The main stream velocities just outside the wake boundaries are u_1 and u_2 . The jet velocities inside the boundaries are v_1 and v_2 .

The total heads are constant in both the main flow and jet which are incompressible, resulting in the relations,

3

$$p_1 + \frac{1}{2}\rho_0 u_1^2 = p_2 + \frac{1}{2}\rho_0 u_2^2 \tag{1}$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \tag{2}$$

Since the pressures on the boundaries, p_1 and p_2, must be continuous, (1) and (2) may be combined in the form

$$u_1^2 - u_2^2 = \frac{\rho}{\rho_0}(v_1^2 - v_2^2) \tag{3}$$

It is assumed that the jet flow is irrotational, or

$$v_1(R - \frac{\delta}{2}) = v_2(R + \frac{\delta}{2}) \tag{4}$$

Average velocities for the jet and main flow may be introduced

$$V = \frac{1}{2}(v_1 + v_2) \tag{5}$$

$$U = \frac{1}{2}(u_1 + u_2) \tag{6}$$

Expanding (4),

$$v_1 R - v_1\frac{\delta}{2} = v_2 R + v_2\frac{\delta}{2}$$

$$R[v_1 - v_2] = \frac{\delta}{2}[v_2 + v_1]$$

$$v_1 - v_2 = \frac{\delta V}{R} \tag{7}$$

4

factoring (3) and using (7)

$$(u_1 - u_2)(u_1 + u_2) = \frac{\rho}{\rho_0}(v_1 - v_2)(v_1 + v_2)$$

$$u_1 - u_2 = \frac{\rho}{\rho_0}\frac{\delta V}{R}\frac{V}{U} = \frac{\rho \delta V^2}{\rho_0 RU} \tag{8}$$

Now from (1),

$$p_1 - p_2 = \frac{1}{2}\rho_0 (u_2{}^2 - u_1{}^2)$$

$$= \frac{1}{2}\rho_0 (u_2 - u_1)(u_2 + u_1)$$

$$= -\rho_0 \frac{\rho \delta V^2}{\rho_0 RU} U = -\frac{\rho \delta V^2}{R} \tag{9}$$

It is now necessary to investigate the effect of the vortex sheet on the external flow. The value of circulation or strength of a vortex sheet of unit width is equal to the integral of the curl of the velocity vector V over the enclosed area. Referring to the polar element in Fig. 2,

$$u_1 - u_2 = \iint_s \text{curl } \overline{V} \, ds = \gamma_J$$

or

$$\gamma_J \, ds = u_1 \, ds_1 - u_2 \, ds_2$$

$$\gamma_J \, ds = u_1 (R - \tfrac{\delta}{2}) \, d\psi - u_2 (R + \tfrac{\delta}{2}) \, d\psi$$

The values of the elemental vorticities,

$$\gamma_1 \, ds_1 = u_1 (R - \tfrac{\delta}{2}) \, d\psi \tag{10}$$

$$\gamma_2 \, ds_2 = -u_2 (R + \tfrac{\delta}{2}) \, d\psi \tag{11}$$

on the upper and lower boundaries are equivalent to a single vortex of magnitude

5

$$\gamma_J \, ds = \gamma_1 \, ds_1 + \gamma_2 \, ds_2 \tag{12}$$

and a doublet of magnitude

$$\frac{1}{2} \delta (\gamma_1 \, ds_1 - \gamma_2 \, ds_2) \tag{13}$$

The single vortex may be expressed as

$$\gamma_J \, ds = u_1 \left(R - \frac{\delta}{2}\right) d\psi - u_2 \left(R + \frac{\delta}{2}\right) d\psi$$

$$\gamma_J \, ds = u_1 R d\psi - u_1 \frac{\delta}{2} d\psi - u_2 R d\psi - u_2 \frac{\delta}{2} d\psi$$

$$\gamma_J \, ds = u_1 \, ds - u_2 \, ds - \frac{\delta}{2} (u_1 + u_2) d\psi$$

$$\gamma_J = (u_1 - u_2) - \frac{\delta U}{R}$$

$$\gamma_J = \frac{\rho \delta V^2}{\rho_o R U} - \frac{\delta U}{R}$$

$$\gamma_J = \frac{U}{R} \left[\frac{\rho \delta V^2}{\rho_o U^2} - \delta \right] \tag{14}$$

The doublet may be expressed

$$\frac{1}{2} \delta (\gamma_1 \, ds_1 - \gamma_2 \, ds_2) = \frac{\delta}{2} \left(u_1 R d\psi - u_1 \frac{\delta}{2} d\psi + u_2 R d\psi + u_2 \frac{\delta}{2} d\psi \right)$$

$$= \frac{\delta}{2} \left((u_1 + u_2) ds - \frac{\delta}{2} (u_1 - u_2) d\psi \right)$$

$$= \delta \left[U ds - \frac{\delta}{4} d\psi \, \frac{\rho \delta V^2}{\rho_o R U} \right]$$

6

$$= \delta U \left[ds - \frac{\rho \delta^2 v^2 d\psi}{4 \rho_0 R U^2} \right]$$

or $\delta U \left[1 - \frac{\rho \delta^2 v^2}{4 \rho_0 R^2 U^2} \right]$ per unit length of jet centerline. (15)

The momentum of the jet per unit span is given by

$$J = \rho \int_{R - \frac{\delta}{2}}^{R + \frac{\delta}{2}} v^2 \, dr = \rho v^2 \delta$$

in terms of which the vortex and doublet become

$$\gamma_J = \frac{U}{R} \left[\frac{J}{\rho_0 U^2} - \delta \right] \tag{16}$$

and

$$\delta U \left[1 - \frac{\delta J}{4 \rho_0 R^2 U^2} \right] \tag{17}$$

per unit span respectively.

It is now desired to explore the limit as jet momentum remains finite and velocity goes to infinity. Let J_0 , V_0 and p_0 be momentum flux, mean jet velocity and mean pressure far downstream and, as before, J , V and p at any other point.

The mass flow is defined as

$$Q = \rho \int_{R - \frac{\delta}{2}}^{R + \frac{\delta}{2}} V dr = \rho V \delta \quad \text{and is here constant.}$$

Along the jet

$$Q = \rho V \delta = \rho \delta_0 V_0 \tag{18}$$

7

The momentum flux in terms of mass flow is

$$J = QV \quad ; \quad J_0 = QV_0 \tag{19}$$

from which

$$\frac{J - J_0}{J_0} = \frac{QV - QV_0}{QV_0} = \frac{V - V_0}{V_0} \tag{20}$$

Now the head in the jet is constant

$$p + \frac{1}{2}\rho V^2 = p_0 + \frac{1}{2}\rho V_0^2$$

or

$$p_0 - p = \frac{1}{2}\rho(V^2 - V_0^2) = \frac{1}{2}\rho(V - V_0)(V + V_0)$$

or

$$V - V_0 = \frac{2(p_0 - p)}{\rho(V + V_0)} \tag{21}$$

Combining (21) with (20)

$$\frac{J}{J_0} = 1 + \frac{2(p_0 - p)}{\rho V_0(V + V_0)} \tag{22}$$

But $p_0 - p$ is also the pressure difference in the exterior flow and hence proportional to $1/2 \, \rho_0 U_0^2$ or

$$\frac{J}{J_0} = 1 + 0 \frac{U_0^2}{V_0^2}$$

Also from (18) and (19)

$$\frac{V}{V_0} = \frac{\rho \delta_0}{\rho \delta} = \frac{\delta_0}{\delta} = \frac{J}{J_0} = 1 + 0 \frac{U_0^2}{V_0^2} \tag{23}$$

Now consider the limit as $V_0 \to \infty$ and the momentum remains finite,

from (23),

$$\frac{J}{J_0} \rightarrow 1 \quad \text{or} \quad J = \text{constant}$$

and from (19)
$$Q = \frac{J_0}{V_0} \rightarrow 0$$

and from (18)
$$\delta = \frac{Q}{\rho V} \rightarrow 0$$

Under these limiting conditions, the vortex distribution on the centerline becomes, from (16)

$$\gamma_J = \frac{U}{R} \frac{J}{\rho_0 U^2} = \frac{J}{R\rho_0 U} \tag{24}$$

and the doublet (17) vanishes.

Introducing the momentum coefficient C_J by $J = C_J \frac{1}{2} \rho_0 U_0^2 c$, the relation for vortex distribution becomes

$$\gamma_J = \frac{C_J \frac{1}{2} \rho_0 U_0^2 c}{R\rho_0 U} = \frac{C_J U_0^2 c}{2RU} \tag{25}$$

(24) may be combined with (9)

$$P_1 - P_2 = -\frac{\rho \delta V^2}{R} = -\frac{J}{R}$$

$$P_2 - P_1 = \frac{\gamma_J R\rho_0 U}{R} = \gamma_J \rho_0 U$$

$$P_2 - P_1 = \rho_0 U \gamma_J \tag{26}$$

What has been accomplished, then, is the derivation of an expression for the vorticity in the jet (Eq. 25). It is seen to be directly proportional to the momentum coefficient, the airfoil chord and the square of the remote wind. It varies inversely with the jet radius

9

of curvature and the tangential velocity. An expression has also been found for the pressure difference across the jet at a point (Eq. 26).

These results have been derived by considering a polar element of the jet and considering the effects of letting the jet momentum remain finite while allowing the velocity to become infinite.

THE AIRFOIL AND JET AS STREAMLINES
OF THE MAIN FLOW

The condition that the airfoil and jet be streamlines of the main flow field, or alternatively, that no fluid flow across these boundaries must be met and allows a relation between jet shape and circulation which, when combined with Eq. (25), makes the solution of the jet flap problem possible.

It is assumed in the paper that the chord and jet may be replaced, as far as the external flow is concerned, with a series of two-dimensional vortices located at equal spacing throughout the length of the system.
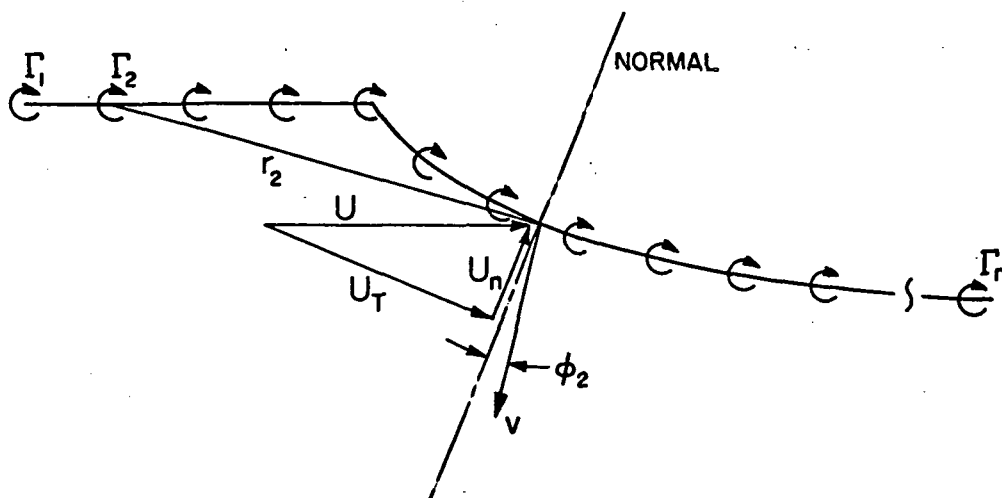
Figure 3. Representation of Jet Flap with
Two-Dimensional Vortices.

The vortices may assume strengths such that the main flow has a velocity field in which the airfoil and jet are exactly stream-lines. The vortex strength distribution required to do this may be found by first establishing at control points between vortices, normals to the jet or chord direction. Now, no flow may take place along these normals and the negative of the main stream velocity in the normal direction may be equated, at each control point, to the

10

sum of all the velocities induced in this direction by the vortices. The velocity induced at a point by a two-dimensional vortex is given by

$$v = \frac{\Gamma}{2\pi r}$$
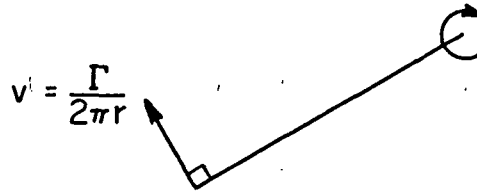
Figure 4. Velocity Induced by Two-Dimensional Vortex.

where v is the velocity induced perpendicular to the radius from the vortex to the point and Γ is the vortex strength with units ft^2/sec.

In order to make a determinant system, there are established as many control points as vortices and a system of equations results by equating the free stream component and the vortex-induced velocities along each control point normal. That is, for each control point, there exists an equation of the form

$$\frac{\Gamma_1 \cos \Phi_1}{2\pi r_1} + \frac{\Gamma_2 \cos \Phi_2}{2\pi r_2} + \ldots + \frac{\Gamma_n \cos \Phi_n}{2\pi r_u} = -U_n$$

where Φ is the angle between the vortex induced velocity and the normal and U_n indicates the normal component of the main flow (see Fig. 3). The solution of this system of equations then for the vortex strengths $\Gamma_1 \ldots \Gamma_n$ gives the values which make the airfoil and jet streamlines of the main flow field.

AN ITERATIVE METHOD OF SOLUTION

A numerical solution for the jet flap may be accomplished by an iterative procedure which first computes the coordinates of the vortices and normals starting with an initial guess at the jet trajectory. With these points established, the vortex strengths are found which make the chord and jet streamlines of the main flow with this jet configuration.

Then knowing these strengths, it is possible to readjust the shape of the jet by finding the tangential velocity at each vortex and balancing the lift produced by the jet vorticity to the wake

11

momentum.   This is expressed by (25) which may be rewritten in a
form which gives jet shape as a function of vortex strength, tan-
gential velocity and momentum coefficient.   If this new shape is
sufficiently close to the shape before improvement, the solution
has been found.   If the improved shape differs significantly from
the previous curve, the new trajectory is used to compute new vor-
tex strengths and the readjustment continues until convergence
occurs.

DESCRIPTION OF PROCEDURE AND SETUP
OF COMPUTER PROGRAM SOLUTION

The computerized solution begins by reading in, as input, the
number of subdivisions to be made in the airfoil chord (taken to be
of unit length) and in the jet itself.   The jet sheet is to be
approximated in shape by straight line segments of the same length
as in the airfoil.
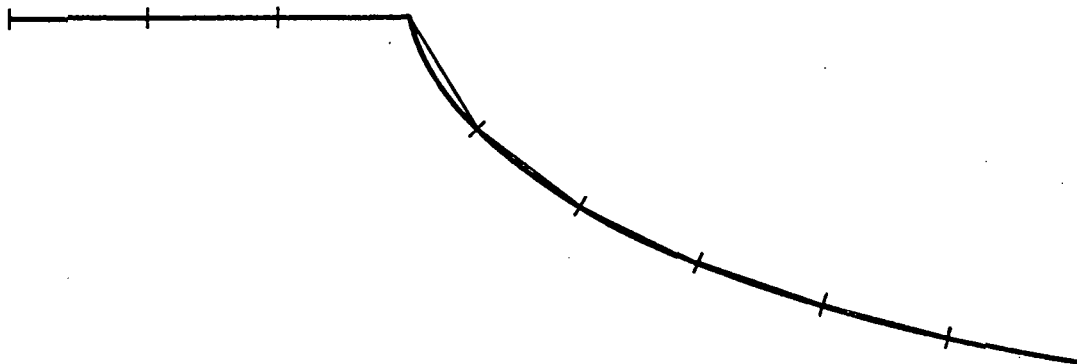
Figure 5.   Approximation of Jet Wake with
Straight Line Segments.

The shape of the jet is determined by reading in as input the
angles which the jet segments make with the horizontal (positive
for the usual downward deflection).   The coordinates of all the
joints of the segments in the airfoil and chord are computed and
at each is located a two-dimensional vortex.   A vortex is also
located at the leading edge.   Normals are located midway between
each vortex, crossing the segments perpendicularly and, in order to
satisfy the Kutta condition, a normal is located on the final seg-
ment downstream of the last vortex.   The last segment, in general,
need not align with the free stream.

Next, a matrix of coefficients is computed which, when multi-
plied by the vortex strength vector, gives the velocity components

12

along the normals. In this calculation, the distance between the
vortex and normal is computed, followed by the angle which the
normal makes with the vortex induced velocity vector. Each element
of the array then is the value of

$$\frac{1}{2\pi r} \cos \Phi$$

where r and Φ are defined as before (Fig. 3). All vortices are
assumed from the onset to be positive if they induce a clockwise
velocity field. The signs of the matrix elements then are positive
if the velocity induced by the vortex under consideration is direc-
ted "upward" along the normal.

To complete the matrix equation, the right hand side or vector
of free stream normal components is next computed.

The angle of attack and the jet angles are considered in com-
puting the components of the free stream velocity along the normals
that have been established. The velocity component is taken to be
positive if it is directed "upward" along the normal.

The matrix equation then, equates the vortex induced velocity
components to the negative of the free stream components in the
same direction in order that the sum of the two sources of velocity
gives zero net flow across the airfoil and jet.

The solution of the matrix equation is accomplished by calling
the subroutine INVR which uses the Jordan method of solution. It
returns to the main program, in place of the free stream component
vector, the vector of vortex strengths, the first element of which
is the vortex strength at the leading edge.

Subroutine TV is next called, which computes the tangential
velocity felt at each jet vortex location. The average of the
angles of the two segments which adjoin each vortex is computed
which represents the angle of the tangent to the jet trajectory
at the vortex. Next, the components along this tangent of the
velocities induced by all the vortices in the chord and jet except
the vortex at the location being considered are computed by a method
similar to the one used to determine the normal components. To the
sum of these velocities is added the component of the free stream
velocity along the tangent and control is returned to the main
program.

Before the jet trajectory improvement takes place, the jet
segment angles are stored for comparison with the corrected values.

The proper trajectory for the above computed values of tan-
gential velocity, vortex strength and momentum coefficient is
obtained by application of a modified form of Eq. (25). The radius

13

of curvature, R , may be found in terms of jet segment angles by
considering the arc subtended by two jet segments as being circular
and using the law of cosines. The length of the segments in the
jet is taken here to be a .
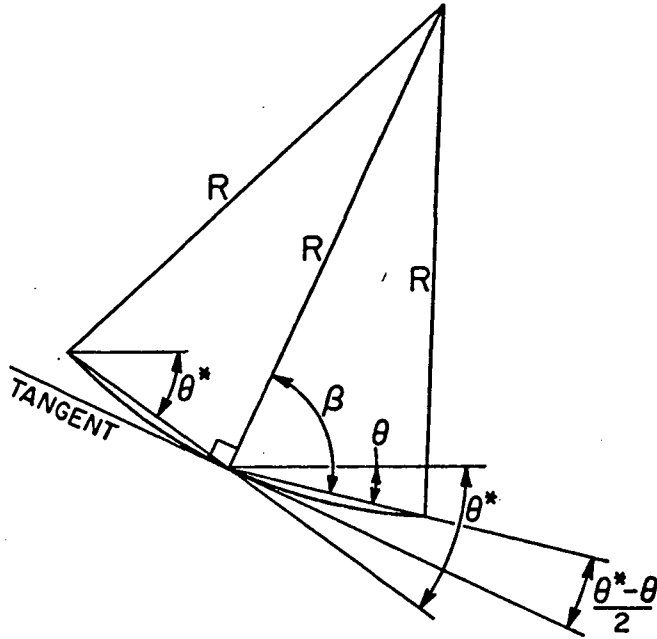
Figure 6. Geometry for Relation between Radius
of Curvature and Jet Angles.

$$R^2 = a^2 + R^2 - 2aR \cos \beta$$

or
$$a = 2R \cos \beta$$

and
$$R = a/2 \cos \beta$$

but
$$\beta = \frac{\pi}{2} - \frac{\theta^* - \theta}{2} = \frac{\pi}{2} + \frac{\theta - \theta^*}{2}$$

so
$$R = a/2 \cos \left[\frac{\pi}{2} + \frac{\theta - \theta^*}{2} \right]$$

$$= a/2 \sin \left[\frac{1}{2} (\theta - \theta^*) \right]$$

14

With this expression for R, (25) becomes.

$$\gamma_J = \frac{C_J U_0{}^2 c \sin\left[\frac{1}{2}(\theta - \theta^*)\right]}{aU}$$

Now, it is assumed that the point vortex Γ_J anywhere in the jet is the result of integrating the vorticity distribution γ_J over the interval of length a which has the point vortex at its midpoint, i.e.,

$$\gamma_J a = \Gamma_J$$

Then the above becomes

$$\Gamma_J U = C_J U_0{}^2 c \sin\left[\frac{1}{2}(\theta - \theta^*)\right]$$

$$\theta - \theta^* = 2 \sin^{-1}\left[\frac{\Gamma_J U}{C_J U_0{}^2 c}\right]$$

The free stream velocity U_0 and the chord c are taken to be unity and the jet angles may be found from the simpler expression

$$\theta - \theta^* = 2 \sin^{-1}\left[\frac{\Gamma_J U}{C_J}\right]$$

Subroutine WANG accomplishes the computation of the improved trajectory.

It should be remarked that U, the tangential velocity, is a function of θ and θ^* as well as the entire jet angle array. Rather than include its dependence upon the undetermined angles, the solution is simplified by using the value of vorticity and the vortex locations before the trajectory improvement takes place. This is justified since the tangential velocities after shape correction are only slightly different from the values before iteration, becoming more nearly identical as convergence is achieved.

To determine whether the improved values differ significantly from the values before correction, the absolute value of the difference between the two is computed and compared with an input variable

15

called EPS. If the error in any one angle is greater than or equal to EPS, a new guess at the jet segment angles is computed by adding to the uncorrected angles, a fraction (COEF in decimal form, an input variable) of the difference between the improved and unimproved values.

After a new shape for the jet has been determined, control is returned to that section of the program which computes the normal and vortex coordinates with the new jet segment angles and another iteration continues.

When convergence is achieved, the lift coefficient is computed and the vortex coordinates and strengths are printed. Lift coefficients are computed as twice the sum of the vortex strengths,

$$C_L = \frac{L}{qc} = \frac{2\rho_0 U_0 \Gamma_T}{\rho_0 U_0{}^2 c} = \frac{2\Gamma_T}{U_0 c}$$

where Γ_T is the total circulation about a contour which encloses the entire lifting system.

$$\Gamma_T = \Gamma_{airfoil} + \Gamma_{jet}$$

$$\Gamma_T = \int_{chord} \gamma_A \, ds_A + \int_{jet} \gamma_J \, ds_J$$

$$\Gamma_T = \sum_i \Gamma_{A_i} + \sum_i \Gamma_{J_i}$$

Therefore

$$C_L = \frac{2\sum_i \Gamma_i}{U_0 c}$$

where the summation includes all point vortices in the airfoil and jet. Since the remote wind and chord length are taken as unity,

$$C_L = 2 \sum_i \Gamma_i$$

16

ANALYSIS OF ACCURACY DEPENDENCE
UPON INPUT VARIABLES

Several input variables exist in the program which determine such things as the number of unknowns, how long iteration will continue, etc. The effect of the choice of these variables on the ultimate accuracy of the output has been investigated.

Effect of Number of Unknowns in Chord

The program was run for a lifting system with a momentum coefficient of 2.0 and a jet exit angle of 45.0 degrees. The number of airfoil segments was taken to be 3, 5, 7, 9 and 11 with the number of jet segments chosen to make the jet a length of four chords behind the trailing edge in each case. The results are shown in Fig. (7) and (8). The lift coefficient (the most sensitive index of accuracy) is sufficiently convergent at 10 or 11 segments in the airfoil. Too few unknowns results in a trajectory which is too low and corresponds to a vorticity distribution which is too large in magnitude. This effect is independent of the initial guess for the jet shape.

The Effect of the Jet Length

A test of the program was made to determine how the solution was affected by an increase or decrease in jet length with the number of unknowns in the airfoil remaining fixed at six. The jet was allowed lengths of two, three and four chords by adjusting the number of segments (vortices) in the jet. The momentum coefficient was 2.0 and the jet angle was 45.0 degrees. The results are found in Figures (9) and (10). The trajectories and vortex distributions are very similar while the total lift may be seen to increase with increased jet length. As the jet length changes, variations in vorticity are found to be distributed over the jet and airfoil. The error in the value of lift coefficient at 3 chords, when compared with that at 4 chords is 1.3 per cent.

Effect of Varying the Convergence Value, Epsilon

An indication of the importance of the choice for the value of epsilon, which determines how much error may exist in jet angles after iteration, was obtained by running the program for a fixed configuration. Epsilon was varied between 0.01 and 0.60 degrees.

The plot of Fig. (11) shows values of lift coefficient which resulted for various values of ϵ. The points, however, do not represent the maximum possible error in lift coefficnet which may result. The values were computed by requiring that all angle

17

errors (the difference between angles before and after iteration) be less than the value, epsilon. If all angle errors were a maximum while still being less than ϵ, which they are not for the usual solution, slightly greater errors in the solution may be found. Further, the solution converges not from one direction but alternates between a solution which is too high and then too low. The result is that the final trajectory may be either above or below the asymptotic solution, each case having an equal probability.

Assuming that the maximum possible error in the solution has a small probability of occurring and considering that the solution may converge to a trajectory above or below the true shape, the estimated convergence limits were constructed, which represent a good approximation at the boundaries for error in the lift coefficient. In particular, for epsilon equal to 0.3, the error in C_L (assuming an extrapolated value at $\epsilon = 0.0$ to be without error) is only 0.23 per cent.

The variation in jet trajectory and values of individual vortices is very small. Comparing results for epsilon equal to 0.01 and 0.6 degrees, the discrepancy in trajectory was found to be only 0.00264 chords in the vertical direction at the downstream end of the jet. The error between the values of vortex strengths at the leading edge for the same two values of ϵ is 0.33 per cent.

DISCUSSION OF RANGE OF APPLICABILITY

Because the solution has not been linearized in any way, the program may be used for any lifting system for which it will converge. A solution has been obtained using the program for jet angles as high as 90 and as low as 30 degrees. (Lower jet exit angles will produce no difficulty; convergence is more easily achieved at small jet angles because of the lower values of vorticity).

The value of C_J has been allowed to range from 0.1 to 5.0 (at $\tau = 45°$) without any difficulty. Lower values may be achieved if the solution input is carefully chosen. (A discussion of convergence follows). Higher values will be easily handled by the program.

The program also is applicable for a useful range of angle of attack. Lift curve slopes were computed for a wing with jet exit angle equal to 45°. C_J values were taken to be 5, 2 and 1 and the attack angle ranged from - 10 to + 10 degrees. The results are depicted in Fig. (12). Five points were computed for $C_J = 2$ which indicate an almost exactly linear relationship between attack angle and lift. The upper and lower bounds on alpha for which the solution will converge is difficult to ascertain. Solutions for any moderate alpha will be possible.

SOME REMARKS ON THE CONVERGENCE OF THE SOLUTION

The program will not yield a solution for its input when the ratio, at any jet vortex location, of the product of the vortex strength and tangential velocity to the momentum coefficient is larger than unity. The inverse sine function in subroutine WANG encounters this value as its argument and the execution of the program is terminated.

Subroutine WANG computes the jet angles that should exist for the above-mentioned ratios. If these exact values are used by the main program for the jet shape, in some cases over-correction occurs, the shape of the trajectory diverges and the arcsine function error is encountered. To prevent this, the input variable COEF prescribes a percentage of the correction that is to be added (or subtracted) to the old value and the new trajectory remains within an "interval of convergence".

Despite this, the trajectory of the jet before iteration must be somewhat close which means that the initial guess must be reasonable. To prevent divergence at small values of C_J (large ratio values), higher values of C_J must be solved first and the corresponding trajectories will be used automatically as initial guesses for the smaller C_J's.

COMPARISON WITH EARLIER THEORY AND EXPERIMENT

Several authors have explored the jet flap problem in two dimensions using analytical methods[3,4,5,6]. D. A. Spence has published rather complete results of his solution which has been linearized by assuming small deflections of the jet. Figure (13) depicts his curves of the jet wake for values of C_J from 0.5 to 5.0 with the jet exit angle equal to 45°. A solution to three chords downstream was computed by this method for comparison. As may be seen, this solution results in a slightly more shallow trajectory (smaller lift coefficient) at momentum coefficients above 1.0 and more penetration into the main flow than Spence's plot at $C_J = 0.5$. The solutions for $C_J = 1.0$ appear to be the same.

Further comparison may be found in Fig. (14) which shows a plot of C_L versus C_J for $\tau = 31.4°$. Lift coefficients computed by Spence are lower below a C_J of about 3.5. Note that C_L values of this method become increasingly higher than Spence's with decreasing C_J which is a trend consistent with the jet plots of Fig. (13) except that the C_J at which the solutions are the same is higher (3.5) for this lower jet angle.

Extensive experimental work with a two-dimensional jet flap was conducted by the British at the National Gas Turbine Establishment during 1953 - 1954,[7,8,9]. This model was a 12.5

per cent thick elliptic cylinder with jet exit directions of 30, 60 and 90 degrees.

For $\tau = 31.4^\circ$ (nominally 30°), the jet trajectory was presented and is compared with the result of this theory in Fig. (15). The discrepancy is found to be in the same direction as indicated in a comparison with the results of Spence. That is, the jet computed by this method is again slightly low when compared at small momentum coefficients.

Figure (14) also shows the comparison of lift coefficients over a range of C_J. Since the model used in the N.G.T.E. is too thick for direct comparison, an approximate correction for thickness has been made by assuming that thickness affects lift coefficients in the jet flap case in the same way as in the case of irrotational circulatory flow. That is, the lift coefficient of an ellipse of thickness to chord ratio t/c is $(1 + t/c)$ times that of a flat plate with the same incidence angle[4].

CONCLUSIONS

An iterative numerical solution for the jet flap in two dimensions has been programmed for digital computation. The flow past the wing is considered to be inviscid and incompressible. The jet flow is assumed to be irrotational and bounded by vortex sheets. In the limiting case of an infinitely thin jet, the vortex boundaries are found to behave as would a single vortex sheet of strength proportional to momentum and inversely proportional to curvature and tangential velocity.

The condition that the chord and jet both be streamlines may be combined with the aforementioned relation to produce an iterative solution by satisfying both relations alternatively until both produce the same jet shape and vortex distributions.

Point vortices have been located along the airfoil chord and the jet trajectory to represent the circulation of the system. An error analysis, carried out for this approach, has determined that the solution is convergent for 11 or 12 vortices equally spaced in the airfoil and at the same spacing in the jet. A jet length of 3 to 4 chords behind the trailing edge will insure convergent results.

Solutions have been found by this method for jet angles as high as 90° and as low as 30°. The momentum coefficient range has been found to be 0.5 and above.

Attack angles from minus ten to plus ten degrees have been investigated. Lift curve slopes computed by this method have been found to be linear over the range of attack angles mentioned.

It has, therefore, been demonstrated that point vortices may be used to represent the wake of a jet flap just as they are assumed to represent circulation about an ordinary airfoil.

It is believed that the absence of linearization in this approach accounts for the discrepancies which exist at very low or high momentum coefficients.

REFERENCES

1. Schubauer, G. B., "Jet Propulsion with Special Reference to Thrust Augmentations," NACA TN 442, January 1933.

2. Preston, J. H., Rep. Memor. Aero. Res. Comm., London, No. 2957, 1956.

3. Maskell, E. C. and Gates, S. B., "Preliminary Analysis for a Jet Flap System in Two Dimensional Inviscid Flow," Great Britain, A.R.C., C.P. 359, 1957.

4. Spence, D. A., "The Lift Coefficient of a Thin Jet-Flapped Wing," Proc. Roy. Soc. (London), Ser. A, pp. 46-68, December 4, 1956.

5. Hirsch, R., "Establishment of Lift on an Aerofoil with a Jet Flap," Aircraft Eng., pp. 366-375, December 1957.

6. Davis, H. J. and Ross, A. J., "A Jet Deflected from the Lower Surface of an Aerofoil," Quart. J. Mech. and Appl. Math., pp. 291-301, August 1957.

7. Dimmock, N. A., "An Experimental Introduction to the Jet Flap," Great Britain, A.R.C., C.P. 344, 1957.

8. Dimmock, N. A., "Some Further Jet Flap Experiments," Great Britain, A.R.C., C.P. 345, 1957.

9. Davidson, I. M., "The Jet Flap," Journal R. Ae. Soc., January 1956.

APPENDIX I

FLOW CHART OF JET FLAP ITERATIVE SOLUTION METHOD

```
┌─────────────────────────────────┐
│ Read in model momentum coefficient,│
│ attack and jet angle, number of  │
│ unknowns and the initial guess at │
│ the jet trajectory               │
└─────────────────────────────────┘
                │
                ▼
      ┌─────────────────────┐
      │ Compute coordinates │
      │ of all vortices     │
      └─────────────────────┘
                │
                ▼
      ┌─────────────────────┐
      │ Compute coordinates │
      │ of normals          │
      └─────────────────────┘
                │
                ▼
      ┌─────────────────────┐
      │ Determine matrix    │
      │ of coefficients     │
      └─────────────────────┘
                │
                ▼                          INVR Subroutine
      ┌─────────────────────┐      ┌─────────────────────────┐
      │ Compute free stream │      │ The matrix equation is  │
      │ components along normals│   │ solved for the vortex   │
      └─────────────────────┘      │ strengths which make    │
                │                  │ the airfoil and jet     │
                ▼                  │ streamlines of the main │
      ┌─────────────────────┐─────▶│ flow                    │
      │ Call INVR Subroutine│◀─────│                         │
      └─────────────────────┘      └─────────────────────────┘
                │                          TV Subroutine
                ▼                  ┌─────────────────────────┐
      ┌─────────────────────┐─────▶│ The tangential          │
      │ Call TV Subroutine  │◀─────│ velocities at jet       │
      └─────────────────────┘      │ vortex locations        │
                │                  │ are computed            │
                ▼                  └─────────────────────────┘
```

```
                    ┌──────────────────────┐
                    │ Store jet angles for │
                    │ comparison with      │
                    │ corrected values     │
                    └──────────────────────┘
                               │
                               ▼                              WANG Subroutine
                    ┌──────────────────────┐    ┌──────────────────────────────┐
                    │ Call WANG Subroutine │───▶│ Computes corrected jet       │
                    │                      │◀───│ angles using the relation    │
                    └──────────────────────┘    │                              │
                               │                │                  ⌈ Γ_J u ⌉   │
                               │                │ θ - θ* = 2 sin⁻¹ │ ───── │   │
                               │                │                  ⌊  C_J  ⌋   │
                               │                └──────────────────────────────┘
 ┌──────────────┐              │
 │ Compute new  │              │
 │ trajectory   │              │
 └──────────────┘              │
         ▲                     ▼
         │          ┌──────────────────────┐
         │          │ Is new trajectory    │
     no  │          │ for jet sufficiently │
         └──────────│ similar to shape     │
                    │ before iteration     │
                    └──────────────────────┘
                               │
                               ▼ yes
                    ┌──────────────────────┐
                    │ Compute lift         │
                    │ coefficient          │
                    └──────────────────────┘
                               │
                               ▼
                    ┌──────────────────────┐
                    │ List results         │
                    └──────────────────────┘
                               │
                               ▼
                             STOP
```

$$\theta - \theta^* = 2\sin^{-1}\left[\frac{\Gamma_J u}{C_J}\right]$$

24

APPENDIX II

SOURCE PROGRAM IN FORTRAN IV
FOR SOLUTION OF JET FLAP

25

```
            PROGRAM FLAP (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

C     PROGRAM TO COMPUTE VORTICITY DISTRIBUTION AND WAKE
C     TRAJECTORY OF JET FLAP IN TWO DIMENSIONS
C
C
C     INPUT SEQUENCE
C
C     CARD 1   NC,NW - THE NUMBER OF SUBDIVISIONS IN THE AIRFOIL
C     AND THE JET RESPECTIVELY.NOTE-NW DOES NOT INCLUDE THE
C     SEGMENT DOWNSTREAM OF THE LAST VORTEX. (2I5)
C
C     CARD 2   EPS - IN THE CONVERGING PROCESS,ITERATION WILL
C     CONTINUE UNTIL EVERY JET ANGLE AFTER AN ITERATION DIFFERS
C     FROM ITS VALUE BEFORE ITERATION BY AN AMOUNT LESS THAN
C     EPS (IN DEGREES) (F15.8)
C
C     CARDS 3 THRU 3+NW   WA(I) - THE INITIAL GUESS AT THE ANGLES
C     (IN DEGREES) MADE BY THE JET SEGMENTS WITH THE HORIZONTAL,
C     BEING POSITIVE IF THE SEGMENT SLOPES ARE NEGATIVE.
C     NW+1 ANGLES ARE NEEDED,THE LAST BEING FOR THE SEGMENT DOWNSTREAM
C     OF THE LAST VORTEX. (F15.8)
C
C     CARD NW+4   ALPHA - THE ANGLE OF ATTACK OF THE AIRFOIL
C     (DEGREES) (F15.8)
C
C     CARD NW+5 AND ANY NUMBER OF SUBSEQUENT CARDS   CJ,COEF - THE
C     MOMENTUM AND CONVERGENCE COEFFICIENTS.(2F15.8) ONE CARD FOR
C     EVERY VALUE OF CJ IS NEEDED.
C
      DIMENSION WA(60),A(60),B(60),F(60,60),FS(60),C(60),D(60)
      DIMENSION UU(60),V(60),G(60),WAH(60)
      INTEGER CC
1     FORMAT (2I5)
2     FORMAT (F15.8)
3     FORMAT (2F15.8)
4     FORMAT (/,13X,F10.6)
305   FORMAT (/,13X,F10.6,12X,F10.6,12X,F10.6)
310   FORMAT (/,10X,16HVORTEX STRENGTHS,8X,12HX COORDINATE,10X,
     112HY COORDINATE)
701   FORMAT (/,/,/,41H THE NUMBER OF SEGMENTS IN THE AIRFOIL IS,
     1I5)
702   FORMAT (/,37H THE NUMBER OF SEGMENTS IN THE JET IS,I5)
703   FORMAT (/,43H THE INITIAL GUESSES FOR THE JET ANGLES ARE)
704   FORMAT (/,24H THE VALUE OF EPSILON IS,F7.4, 8H DEGREES)
705   FORMAT (/,37H THE VALUE OF MOMENTUM COEFFICIENT IS,F7.4)
706   FORMAT (/,31H ITS CONVERGENCE COEFFICIENT IS,F7.4)
707   FORMAT (/,23H THE ANGLE OF ATTACK IS,F7.4, 8H DEGREES)
708   FORMAT (/,22H THE JET EXIT ANGLE IS,F9.4, 8H DEGREES)
709   FORMAT (/,38H THE NUMBER OF ITERATIONS REQUIRED WAS,I5)
710   FORMAT (/,30H THE CONVERGENT JET ANGLES ARE)
```

```
711       FORMAT (/,24H THE LIFT COEFFICIENT IS,F7.4)
          ITER=0
C
C         READ IN NUMBER OF UNKNOWNS,JET ANGLES,ATTACK ANGLE,EPSILON,
C         MOMENTUM COEFFICIENT AND CONVERGENCE COEFFICIENT
C
          READ (5,1) NC,NW
          WRITE (6,701) NC
          WRITE (6,702) NW
          KK=NW+1
          READ (5,2) EPS
          WRITE (6,704) EPS
          EPS=EPS/57.2958
          WRITE (6,703)
          DO 51 I=1,KK
          READ (5,2) WA(I)
          WRITE (6,4) WA(I)
51        WA(I)=WA(I)/57.2958
          READ (5,2) ALPHA
          WRITE (6,707) ALPHA
          ALPHA=ALPHA/57.2958
606       CONTINUE
          READ (5,3) CJ,COEF
          WRITE (6,705) CJ
          WRITE (6,706) COEF
38        CONTINUE
          NCC=NC+1
C
C         COMPUTE COORDINATES OF VORTICES
C
          SEG=1.0/FLOAT(NC)
          DO 100 I=1,NCC
          AA=FLOAT(I)/FLOAT(NC)
          A(I)=AA-SEG
100       B(I)=0.0
          DO 15 I=1,NW
          NN=NC+I+1
          NNN=NC+I
          A(NN)=A(NNN)+SEG*COS(WA(I))
15        B(NN)=B(NNN)-SEG*SIN(WA(I))
          CC=NW+NC+1
          NWC=NW+NC
C
C         COMPUTE COORDINATES OF NORMALS
C
          DO 25 I=1,NWC
          C(I)=0.5*(A(I)+A(I+1))
25        D(I)=0.5*(B(I)+B(I+1))
          SEG2=SEG/2.0
          C(CC)=A(CC)+SEG2*COS(WA(KK))
          D(CC)=B(CC)-SEG2*SIN(WA(KK))
```

```
          PI=0.15915
C
C         COMPUTE MATRIX OF COEFFICIENTS
C
          DO 30 I=1,NC
          DO 30 J=1,CC
          R=((ABS(C(I)-A(J)))**2.0+(ABS(D(I)-B(J)))**2.0)**0.5
          IF (B(J).EQ.0.0) GO TO 90
          ARG=-(A(J)-C(I))/(B(J)-D(I))
          ANN=ABS(COS(-ATAN(ARG)+1.5708))
          GO TO 50
90        ANN=1.0
50        CONTINUE
          E(I,J)=PI*ANN/R
          IF (A(J).GT.C(I)) GO TO 35
          E(I,J)=-E(I,J)
35        CONTINUE
30        CONTINUE
          DO 40 I=NCC,CC
          DO 40 J=1,CC
          R=((ABS(C(I)-A(J)))**2.0+(ABS(D(I)-B(J)))**2.0)**0.5
          ARG=-(A(J)-C(I))/(B(J)-D(I))
          K=I-NC
          ANN=ABS(COS(ABS(ATAN(ARG)+WA(K)-1.5708)))
          E(I,J)=PI*ANN/R
          IF (A(J).GT.C(I)) GO TO 351
          E(I,J)=-E(I,J)
351       CONTINUE
40        CONTINUE
C
C         COMPUTE FREE STREAM COMPONENTS ALONG NORMALS
C
          DO 59 I=1,NC
59        FS(I)=-1.0*SIN(ALPHA)
          DO 60 I=1,KK
          III=I+NC
60        FS(III)=-1.0*SIN(WA(I)+ALPHA)
C
C         SOLVE MATRIX EQUATION FOR VORTEX STRENGTHS
C
          CALL INVR (E,CC,FS,1,DETERM,60,60)
C
C         COMPUTE TANGENTIAL VELOCITIES
C
          CALL TV (NW,NC,UU,WA,A,B,FS,ALPHA)
          LN=NC+2
C
C         CHANGE INDEXING FOR USE IN FOLLOWING SUBROUTINE
C
          DO 45 I=1,NW
          NE=NC+I+1
```

28

```
        G(I)=FS(NE)
45      V(I)=UU(NE)
C
C       STORE JET ANGLES FOR COMPARISON WITH VALUES OF NEXT
C       ITERATION
C
        DO 14 I=1,KK
14      WAH(I)=WA(I)
C
C       CORRECT JET ANGLES USING VORTICITY AND TANGENTIAL
C       VELOCITIES
C
        CALL WANG (CJ,V,G,WA,NW)
C
C       IF CONVERGENCE HAS BEEN ACHIEVED,CONTINUE TO 800
C
        DO 70 I=1,KK
        ERR=ABS(WAH(I)-WA(I))
        IF (ERR.GE.EPS) GO TO 500
70      CONTINUE
        GO TO 800
500     CONTINUE
C
C       IF NOT CONVERGED,COMPUTE NEW GUESS FOR JET ANGLES
C
        DO 600 I=1,KK
600     WA(I)=WAH(I)+COEF*(WA(I)-WAH(I))
        ITER=ITER+1
        GO TO 38
800     CONTINUE
        CALL TV (NW,NC,UU,WA,A,B,FS,ALPHA)
        DO 73 J=1,60
73      WRITE (6,4) UU(J)
C
C       COMPUTE LIFT COEFFICIENT
C
        CL=0.0
        DO 105 I=1,CC
105     CL=2.0*FS(I)+CL
C
C       PRINT VORTEX STRENGTHS,TRAJECTORY,CJ,JET ANGLES,ITERATIONS
C       REQUIRED AND LIFT COEFFICIENT
C
        TAU=WA(1)*57.2958
        WRITE (6,708) TAU
        WRITE (6,709) ITER
        WRITE (6,710)
        DO 903 I=1,KK
        WA(I)=WA(I)*57.2958
903     WRITE (6,4) WA(I)
        WRITE (6,711) CL
```

```
      WRITE (6,310)
      DO 505 I=1,CC
505   WRITE (6,305) FS(I),A(I),R(I)
      GO TO 606
      END
      SUBROUTINE INVR(A,N,B,M,DETERM,ISIZE,JSIZE)
C
C     SUBROUTINE TO COMPUTE THE INVERSE OF A MATRIX OF SIZE LESS
C     THAN OR EQUAL TO 100 OR,IF REQUIRED,FIND THE SOLUTION OF
C     A SET OF SIMULTANEUS EQUATIONS
C
      DIMENSION IPIVOT(100),A(ISIZE,JSIZE),B(ISIZE,M),
     1INDEX(100,2),PIVOT(100)
      EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)
C
C
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1) 80, 100, 740
   80 IF(ABS(AMAX)-ABS(A(J,K)))85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140, 260, 140
  140 CONTINUE
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205 IF(M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  310 PIVOT(I)=A(ICOLUM,ICOLUM)
```

```
C
C        DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C
C        REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
  550 CONTINUE
C
C        INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
      END
      SUBROUTINE TV (NW,NC,UU,WA,A,B,G,ALPHA)
C
C     SUBROUTINE TO COMPUTE TANGENTIAL COMPONENTS OF VELOCITY
C
      DIMENSION UU(60),WA(60),A(60)
      DIMENSION B(60),TANG(60),G(60)
      INTEGER CC
      DO 30 I=1,NW
      I2=I+1
      I1=I+NC+1
  30  TANG(I1)=0.5*(WA(I)+WA(I2))
      NNN=NC+2
      NMM=NW+NC+1
```

```
          DO 100 I=NNN,NMM
          EE=0.0
          NO=I-1
          DO 10 J=1,NO
          R=((ABS(A(J)-A(I)))**2.0+(ABS(B(J)-B(I)))**2.0)**0.5
          THETA=ATAN((B(J)-B(I))/(A(I)-A(J)))
          ANG=(1.5708+TANG(I)-THETA)
          ANN=COS(ANG)
          EE=-(G(J)*ANN/(6.2832*R))+EE
10        CONTINUE
          NP=I+1
          CC=NW+NC+1
          FF=0.0
          IF (I.EQ.CC) GO TO 99
          DO 20 J=NP,NMM
          THETA=ATAN((B(I)-B(J))/(A(J)-A(I)))
          R=((ABS(A(J)-A(I)))**2.0+(ABS(B(J)-B(I)))**2.0)**0.5
          ANG=(1.5708+THETA-TANG(I))
          ANN=COS(ANG)
          FF=-(G(J)*ANN/(6.2832*R))+FF
20        CONTINUE
99        CONTINUE
          UU(I)=FF+EE+COS(TANG(I)+ALPHA)
100       CONTINUE
          RETURN
          END
          SUBROUTINE WANG (CJ,U,G,WA,NW)
C
C         SUBROUTINE TO CORRECT JET ANGLES
C
          DIMENSION G(60),WA(60),U(60)
          DO 5 I=1,NW
          I2=I+1
          FG=G(I)*U(I)/CJ
5         WA(I2)=WA(I)-2.0*ASIN(FG)
          RETURN
          END
```
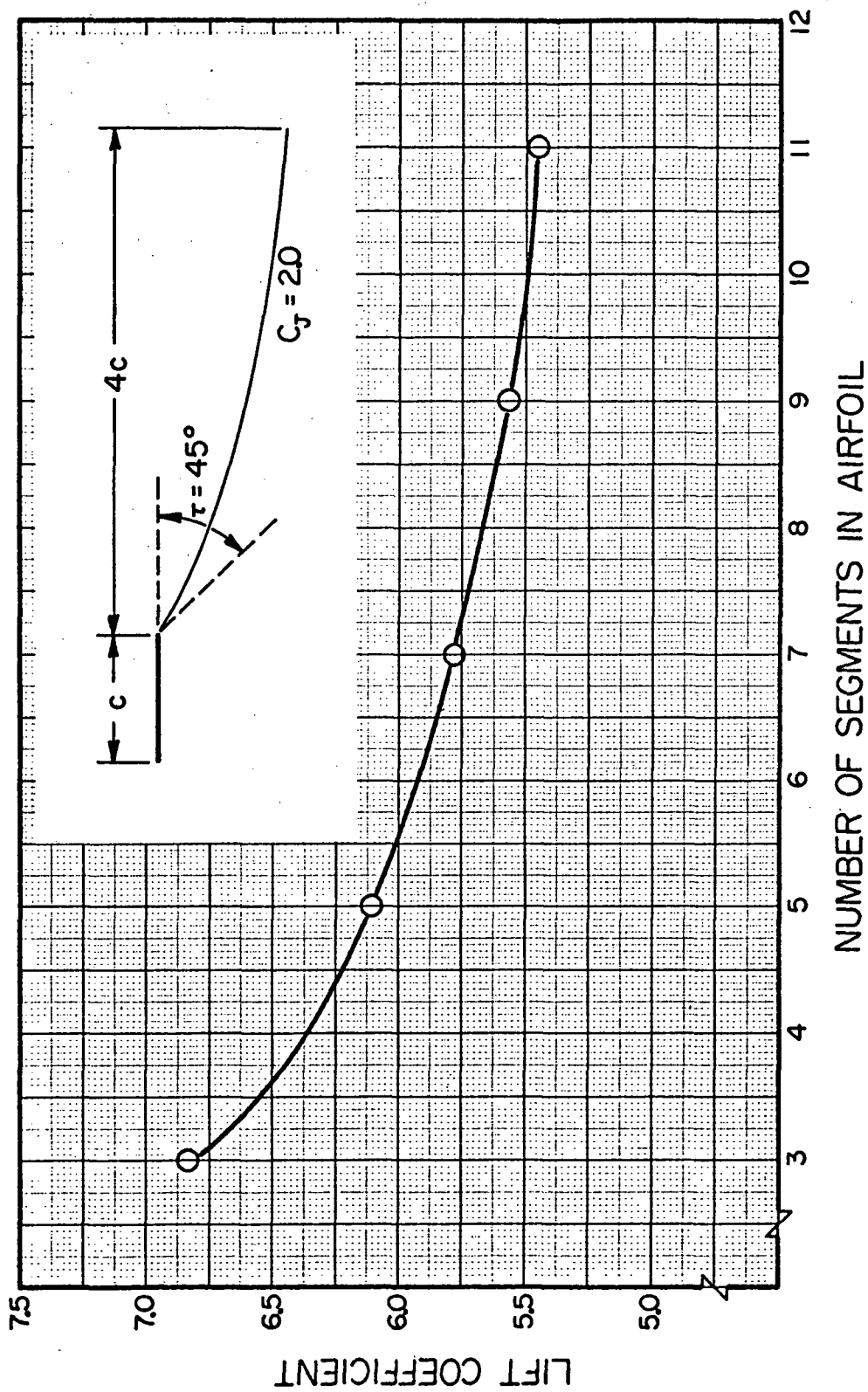
FIGURE 7. EFFECT OF NUMBER OF UNKNOWNS IN AIRFOIL ON LIFT COEFFICIENT.

33
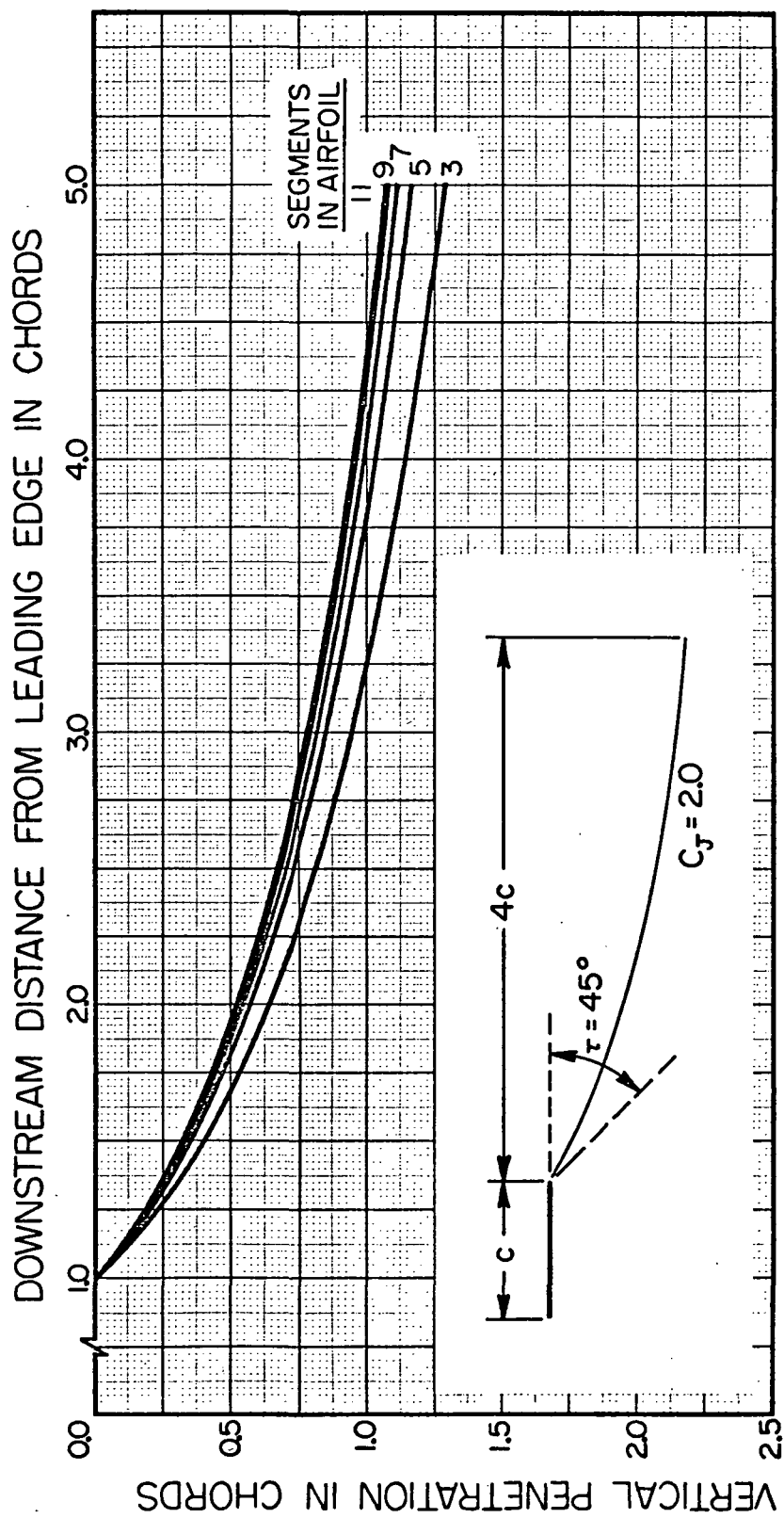
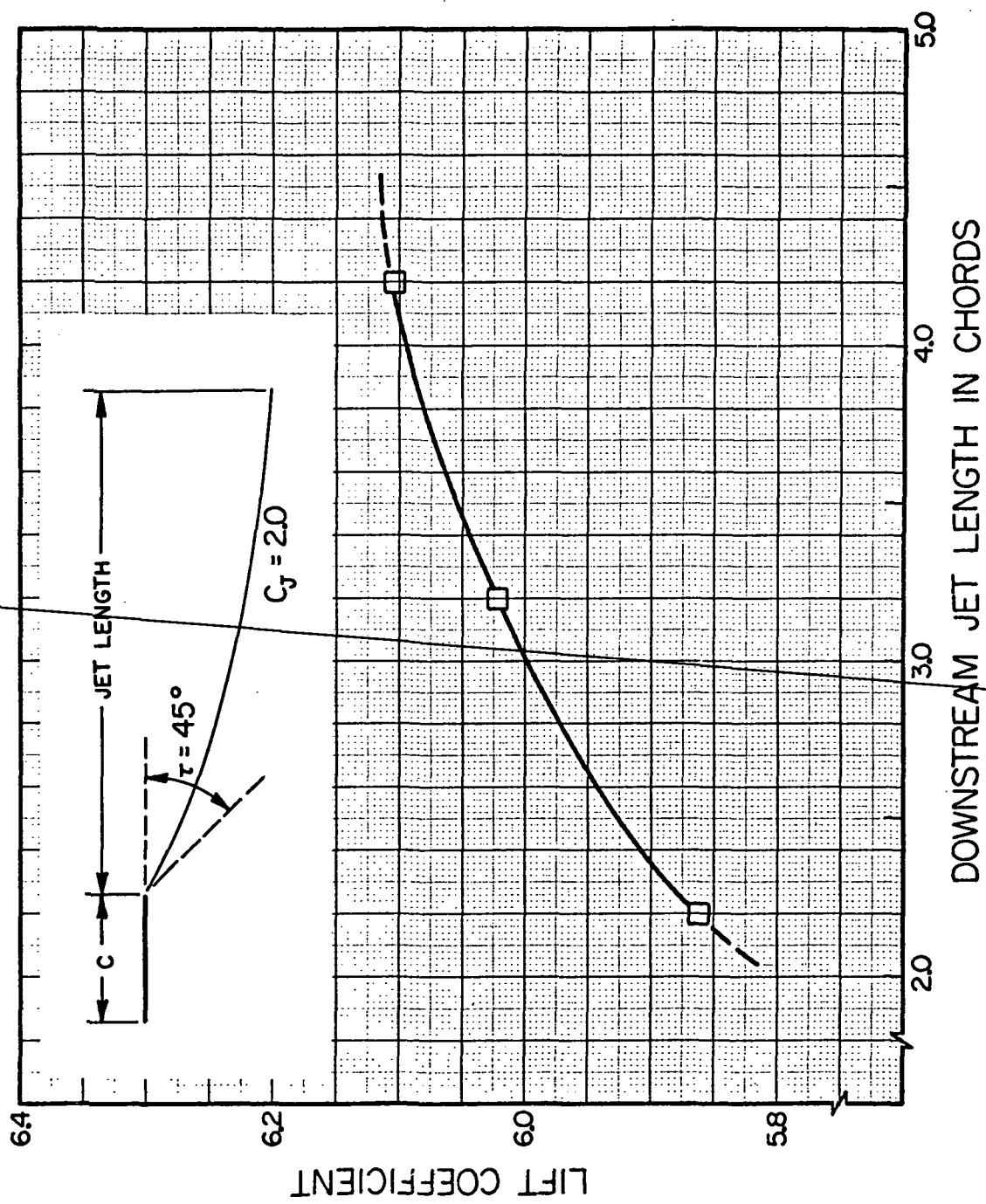FIGURE 8. DEPENDENCE OF JET TRAJECTORY ON NUMBER OF UNKNOWNS IN AIRFOIL.

34

FIGURE 9. VARIATION OF LIFT COEFFICIENT WITH LENGTH OF JET.
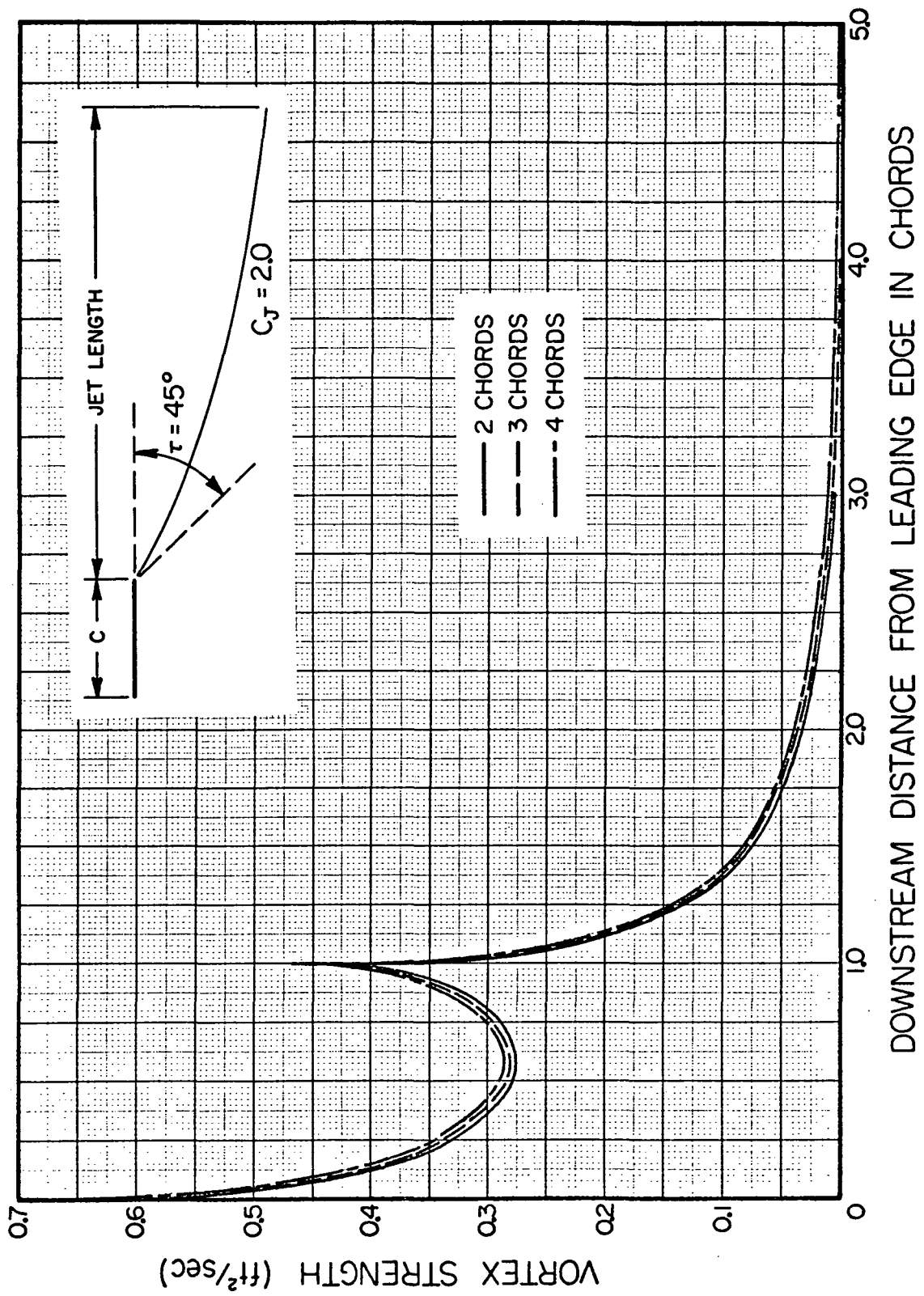
35

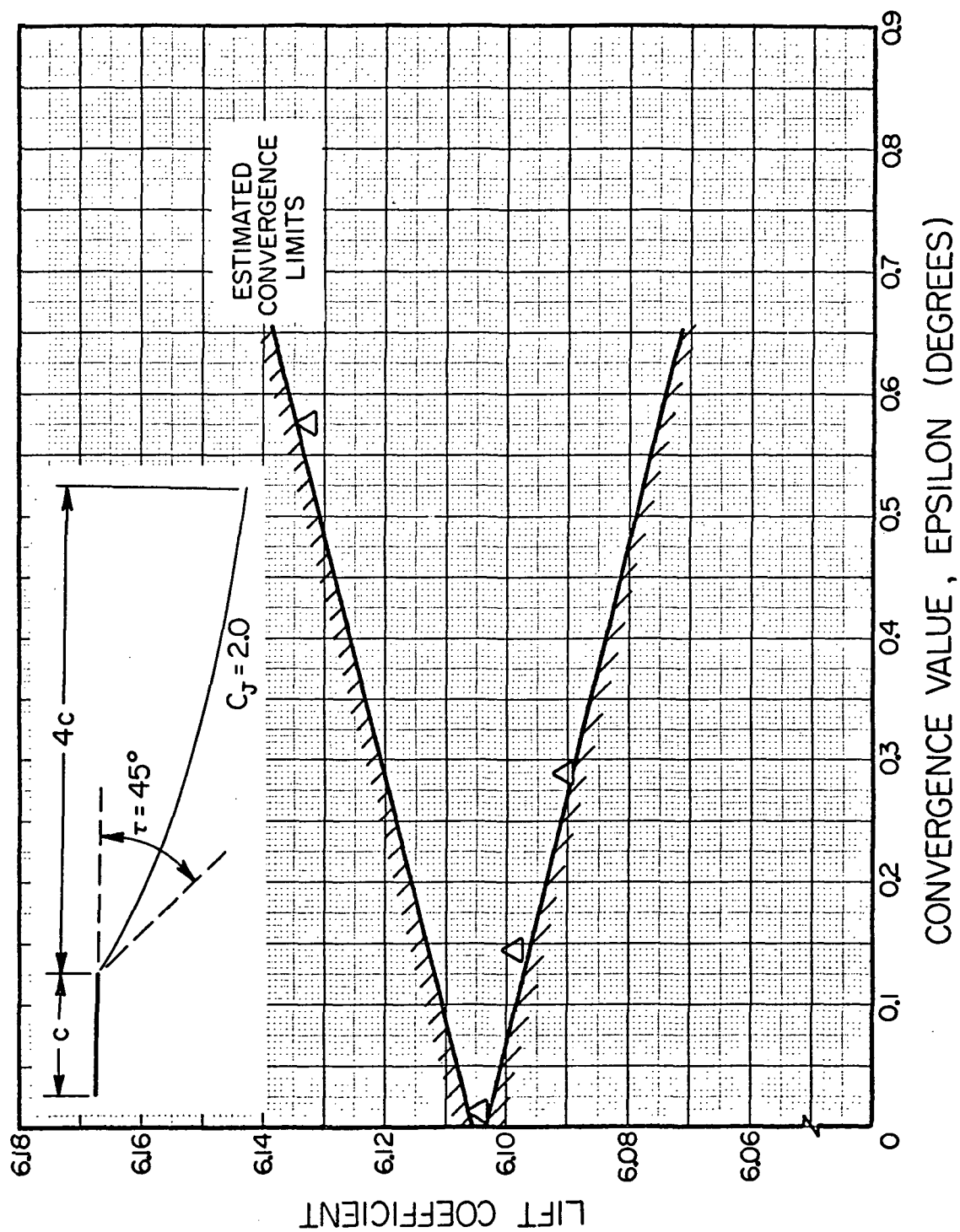FIGURE 10. VARIATION IN VORTEX STRENGTH DISTRIBUTION WITH JET LENGTH.

36

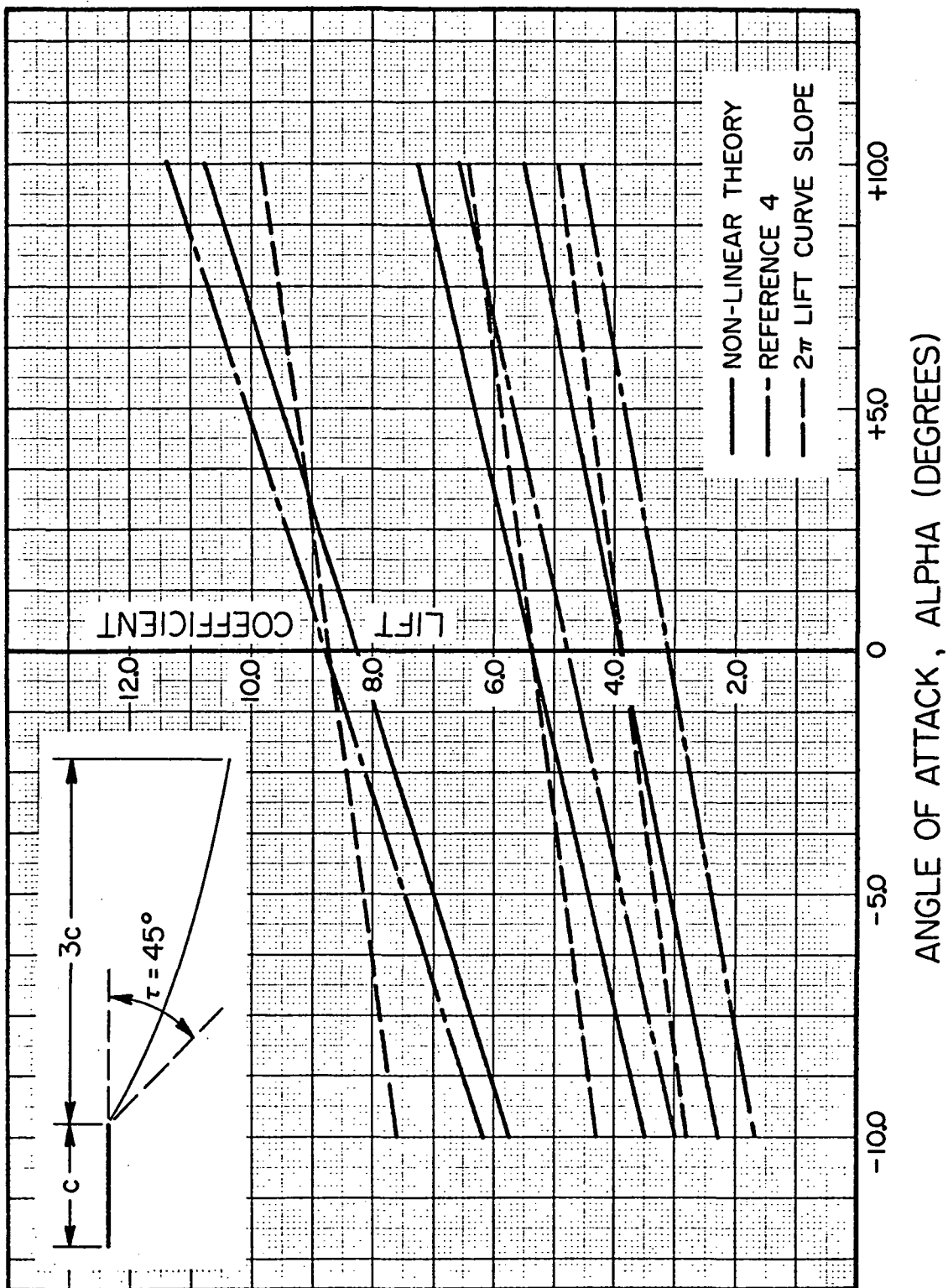FIGURE II. EFFECT OF CONVERGENCE VALUE, EPSILON, UPON LIFT COEFFICIENT.

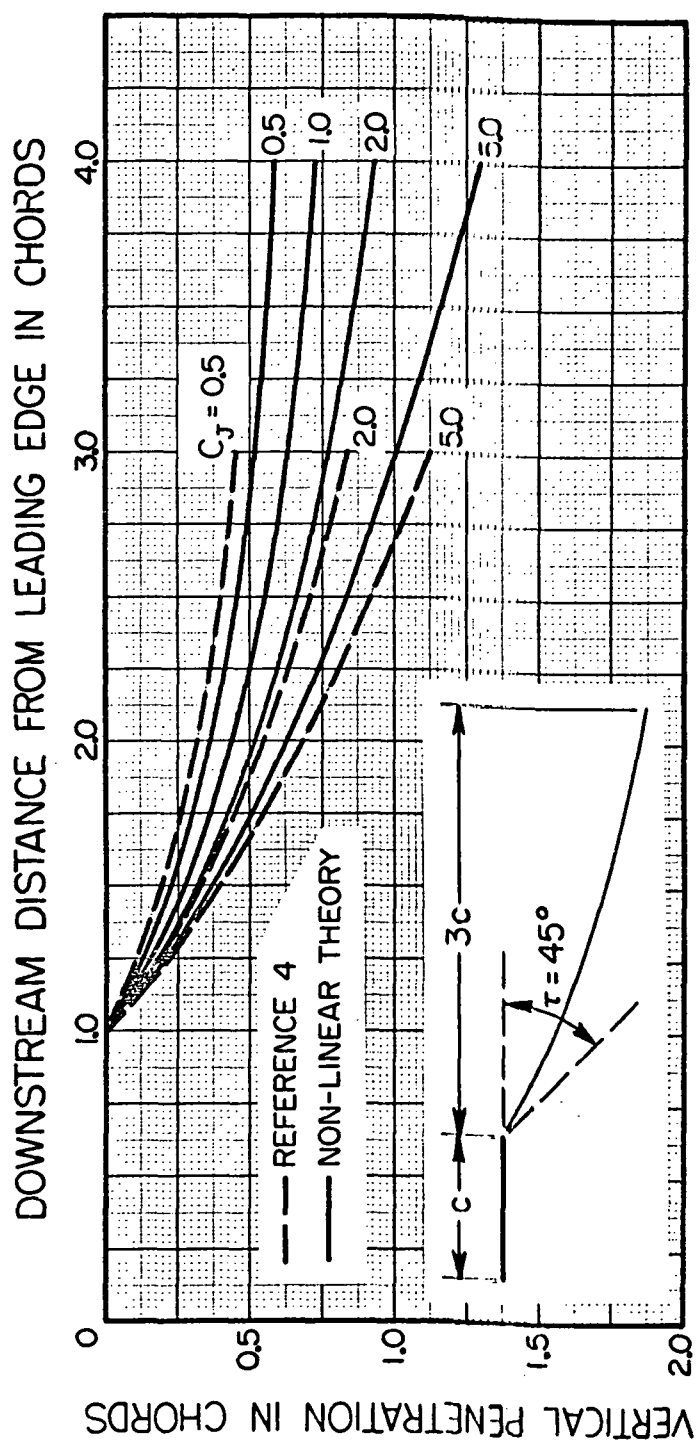FIGURE 12. LIFT COEFFICIENT AS A FUNCTION OF ATTACK ANGLE.

38

DOWNSTREAM DISTANCE FROM LEADING EDGE IN CHORDS

VERTICAL PENETRATION IN CHORDS

FIGURE 13. COMPARISON OF TRAJECTORY RESULTS WITH THOSE OF REFERENCE 4.

39
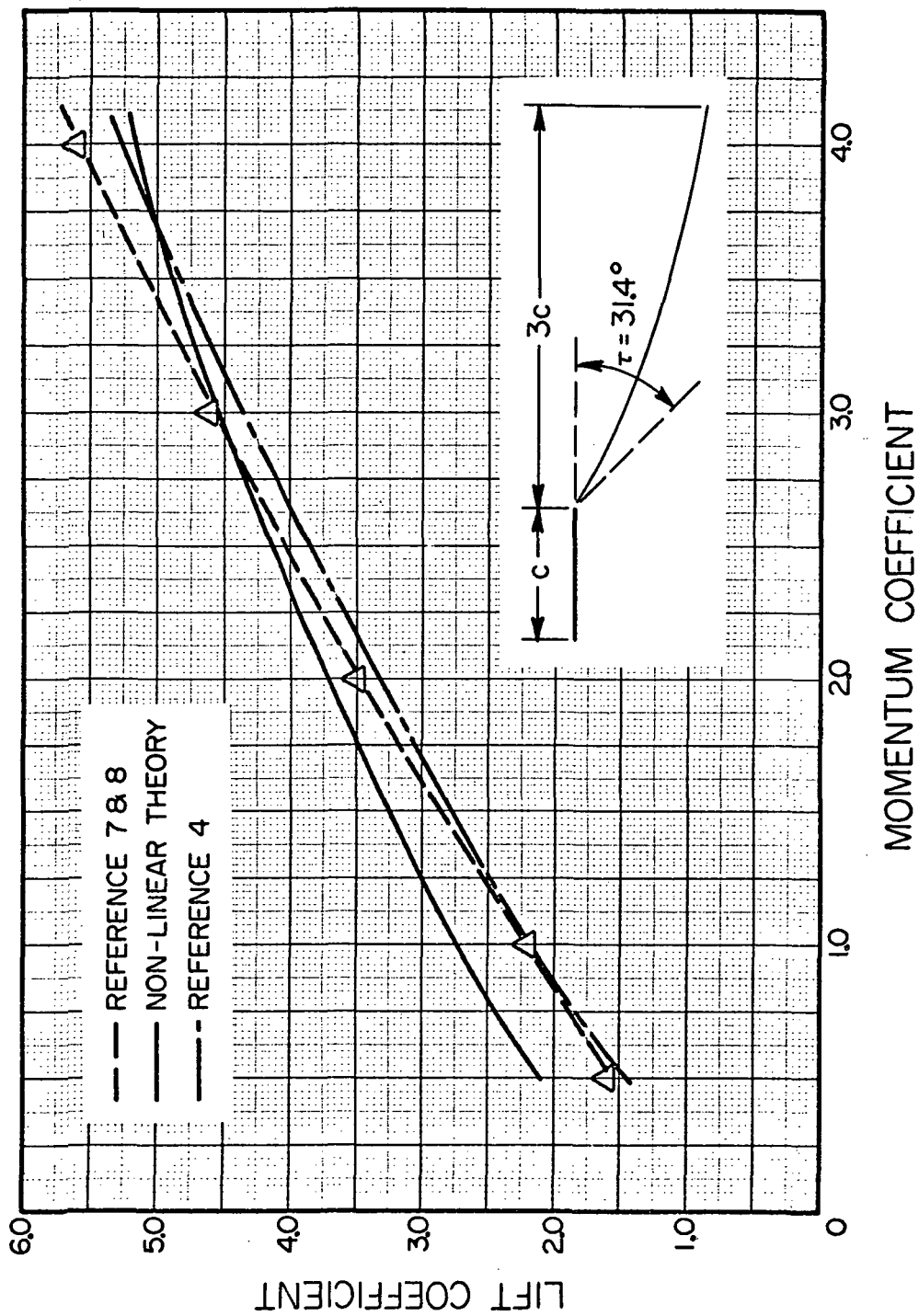
FIGURE 14. COMPARISON OF C_L VERSUS C_J CURVES WITH REFERENCES 4,7 & 8.
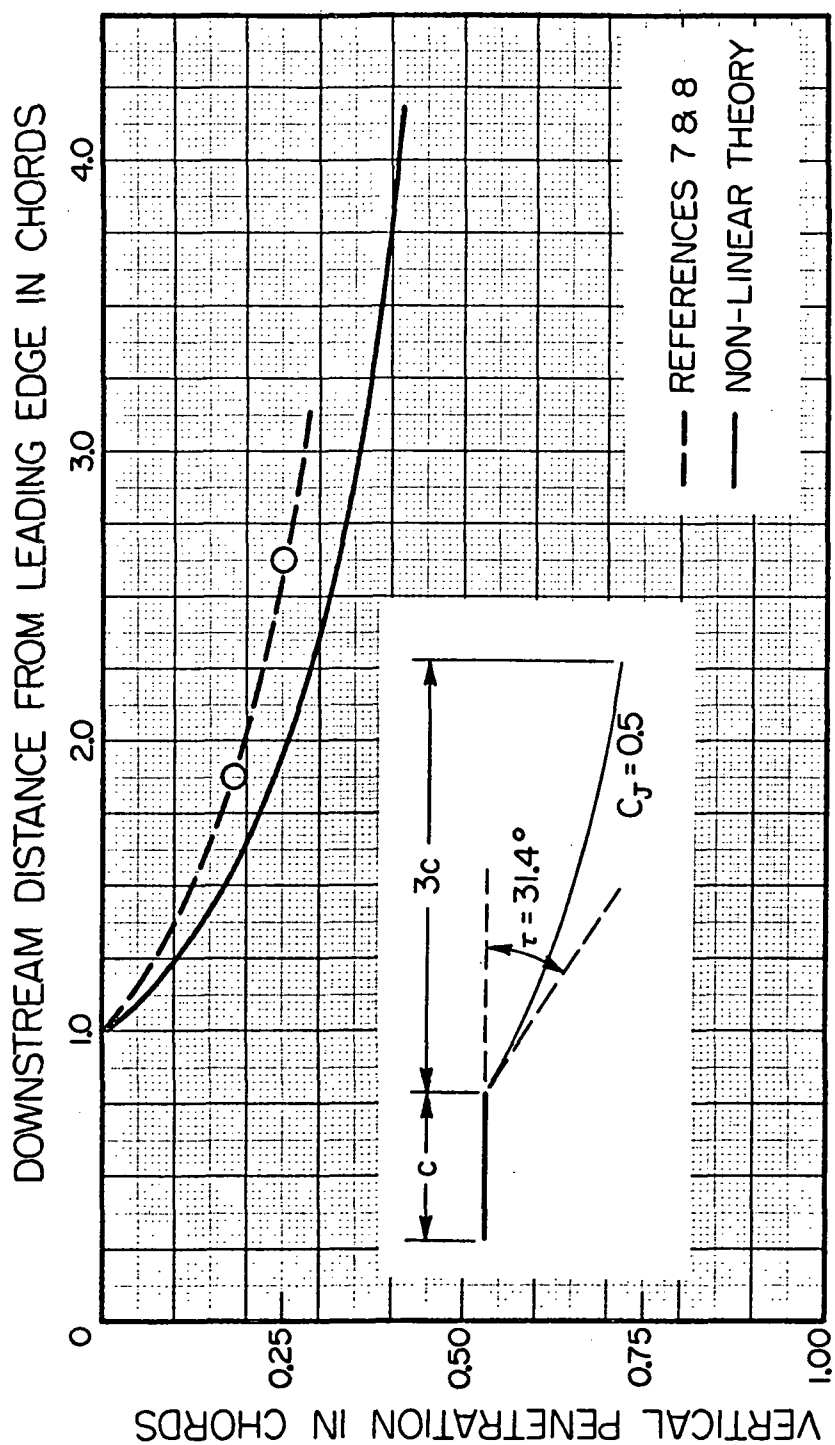
40

FIGURE 15. FURTHER COMPARISON OF RESULTS WITH EXPERIMENTAL WORK. (REFERENCES 7 & 8)

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."
—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546