NASA TECHNICAL NOTE

N73-16654
NASA TN D-7158

NASA TN D-7158

NEW ASPECTS OF SUBSONIC AERODYNAMIC NOISE THEORY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1973

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-7158		

4. Title and Subtitle	5. Report Date
NEW ASPECTS OF SUBSONIC AERODYNAMIC NOISE THEORY	February 1973
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Marvin E. Goldstein and Walton L. Howes	E-7047
9. Performing Organization Name and Address	10. Work Unit No.
Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135	501-04
	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Note
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Aerodynamic sound Jet noise Turbulence	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	39	$3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151

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SUMMARY

A theory of aerodynamic noise is presented which differs from Lighthill's theory primarily in the way in which convection of the noise sources is treated. The sound directivity pattern obtained from the present theory agrees better with jet-noise directivity data than does that obtained from Lighthill's theory. The results imply that the shear-noise contribution to jet noise is smaller than previously expected.

INTRODUCTION

The theory of aerodynamic noise was developed by Lighthill in two classic papers (refs. 1 and 2). Lighthill showed that, in the absence of solid boundaries, aerodynamic noise is produced in a fluid by turbulent stress fluctuations acting as acoustic quadrupoles. In a moving fluid these quadrupoles are carried along by the mean flow at some convection Mach number M_c. For nearly parallel flows (as in jets), convection of the quadrupole sources introduces a factor $(1 - M_c \cos \theta)^{-5}$ into the noise directivity pattern, where θ is the angle measured from the mean flow direction.[1] In Lighthill's theory this convective directivity factor dominates the directivity pattern of jet noise. However, it has been found that, when experimentally measured values of M_c are used in the theory, the predicted directivity patterns do not give good agreement with experiment.

In the present report a possible alternative theory of jet noise is proposed. This theory leads to a formula for predicting noise which differs from the formulas obtained from Lighthill's theory in that the exponent -5 in the convective factor is replaced by the exponent -3. This gives better agreement with existing experimental directivity data.

Since the present theory begins with a convected wave equation, it starts from a more general point of view than Lighthill's theory. The effects of convection and refraction of the emitted sound waves by the mean flow are neglected, and the resulting wave

[1]The exponent -5 was first obtained by Ffowcs Williams (ref. 3). Lighthill erroneously obtained an exponent -6.

equation is then solved for the far-field sound pressure by using the free-space Green's function. Next, the source term (which involves the fluctuating Reynolds stress in essentially the same way as Lighthill's source term) is evaluated by using the isotropic turbulence model adopted by Lilley (ref. 4) and by Ribner (ref. 5). Finally, it is shown that the directivity patterns of the far-field sound intensity obtained from this theory are in better agreement with experimental data than those obtained from Lighthill's theory.

DERIVATION OF BASIC EQUATION

We begin by deriving an approximate wave equation very similar to Phillips exact wave equation (ref. 6). However, it is convenient to derive this equation from first principles, instead of starting with Phillips equation, since certain approximations which we shall make are most conveniently introduced during the derivation. As in reference 6, we start from the Navier-Stokes equation

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \cdot \vec{\vec{e}} \tag{1}$$

and the continuity equation

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \vec{V} = 0 \tag{2}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla$$

p is the pressure, ρ is the density, V is the velocity, and $\vec{\vec{e}}$ is the viscous stress tensor. (All symbols are defined in appendix B.) We also assume that the ideal-gas relation

$$p = R\rho\Theta \tag{3}$$

holds. The velocity $\vec{V}$ can be expressed as the sum of a time-averaged part $\vec{U}$ and a fluctuating part $\vec{V}'$. Thus,

$$\vec{V} = \vec{U} + \vec{V}' \tag{4}$$

Furthermore, suppose that the mean velocity is parallel to the x_1-coordinate axis and varies only in the x_2-direction in a locally Cartesian coordinate system (x_1, x_2, x_3). Since refraction effects will be neglected, this is a reasonable assumption for a jet. Thus,

$$\vec{U} = \hat{i}U(x_2) \tag{5}$$

The density can be expressed as

$$\rho = \rho_0 + \rho' \tag{6}$$

where ρ_0 is the time-averaged density and ρ' is the fluctuating part of the density. Although we shall allow the maximum mean-flow Mach number to be any value less than unity, the Mach number of the turbulence will always be much less than unity. In fact, for jet flows, the Mach number of the turbulence will be less than 0.1 or 0.2 (ref. 7). Hence, the turbulent flow is essentially incompressible. And the fluctuating part of the density is essentially associated with acoustic waves and not with the turbulence.

As pointed out in reference 8, the interaction of the emitted sound with the turbulence is negligible. This interaction is accounted for in the equations by the terms which involve products of the turbulent quantities with acoustic quantities. Such terms will, therefore, be neglected. We shall also neglect any terms which involve squares of acoustics quantities since these terms are assumed to be small.

It follows from equations (4) to (6) that the exact time-averaged form of the continuity equation (2) is

$$U \frac{\partial \rho_0}{\partial x_1} = - \nabla \cdot \overline{\rho' \vec{V}'}$$

Since $\vec{V}'$ is the sum of turbulent and acoustic velocities, the assumptions discussed in the preceding paragraphs imply that the right side of this equation can be neglected. Therefore, assuming an initial uniform density upstream, the time-averaged density ρ_0 will be independent of position.

It also follows from equations (4) to (6) that

$$\frac{D\rho}{Dt} = \frac{\partial \rho'}{\partial t} + U \frac{\partial \rho'}{\partial x_1} + \vec{V}' \cdot \nabla \rho'$$

The assumptions discussed in the preceding paragraphs again imply that the last term on the right side of this equation can be neglected. We can therefore write

$$\frac{D\rho}{Dt} \approx \frac{\partial \rho'}{\partial t} + U \frac{\partial \rho'}{\partial x_1}$$

so that the continuity equation becomes

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t} + U \frac{\partial \rho}{\partial x_1}\right) + \nabla \cdot \vec{V} = 0$$

Now it can easily be shown from the formula for entropy change in an ideal gas (ref. 9) that

$$\frac{d\rho}{\rho} = \frac{1}{\gamma p} dp - \frac{1}{C_p} dS$$

where S is the entropy, C_p is the specific heat at constant pressure, and γ is the ratio of specific heats. Hence,

$$\nabla \cdot \vec{V} = -\left(\frac{\partial \Pi}{\partial t} + U \frac{\partial \Pi}{\partial x_1}\right) + \frac{1}{C_p}\left(\frac{\partial S}{\partial t} + U \frac{\partial S}{\partial x_1}\right) \tag{7}$$

where we have put

$$\Pi \equiv \frac{1}{\gamma} \ln \frac{p}{p_o} \tag{8}$$

and have, for convenience, chosen p_o to be the average far-field pressure. The fluctuating velocity $\vec{V}'$ can be decomposed into a turbulent part $\vec{w}$ plus an acoustic part[2] $\vec{v}$, where $\vec{v}$ is called the acoustic particle velocity. Thus,

$$\vec{V}' = \vec{w} + \vec{v} \tag{9}$$

Since the turbulence is essentially incompressible, we can suppose that

$$\nabla \cdot \vec{w} = 0 \tag{10}$$

Hence,

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[2]Notice that we are ignoring acoustic streaming.

$$\nabla \cdot \vec{V} = \nabla \cdot \vec{V}' = \nabla \cdot \vec{v}$$

Since $\vec{v}$ is an acoustic quantity, this shows that $\nabla \cdot \vec{V}$ is essentially an acoustic quantity. In view of the assumptions previously described, we can, therefore, write

$$\frac{D}{Dt} \nabla \cdot \vec{V} \approx \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right) \nabla \cdot \vec{V}$$

Hence, it follows from equation (7) that

$$\frac{D}{Dt} \nabla \cdot \vec{V} = -\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi + \frac{1}{C_p}\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 S$$

Upon using the vector identity

$$\frac{D}{Dt} \nabla \cdot \vec{V} = \frac{\partial}{\partial x_i} \frac{DV_i}{Dt} - \frac{\partial V_j}{\partial x_i} \frac{\partial V_i}{\partial x_j}$$

given in reference 6, this becomes

$$\nabla \cdot \frac{D\vec{V}}{Dt} = -\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi + \frac{1}{C_p}\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 S + \frac{\partial V_j}{\partial x_i} \frac{\partial V_i}{\partial x_j}$$

where the summation convention has been adopted. After substituting equations (4), (5), and (9) into this equation and neglecting the squares of acoustic quantities and the products of acoustic and turbulent quantities, we obtain

$$\frac{\partial}{\partial x_i} \frac{DV_i}{Dt} = -\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi + \frac{1}{C_p}\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 S + 2 \frac{dU}{dx_2} \frac{\partial v_2}{\partial x_1} + \frac{\partial W_j}{\partial x_i} \frac{\partial W_i}{\partial x_j} \qquad (11)$$

where we have put

$$W_i = \delta_{1i} U + w_i \qquad (12)$$

with δ_{ij} the Kronecker delta. Furthermore, using the fact that, for an ideal gas, the speed of sound c is given by

$$c^2 = \frac{\gamma p}{\rho}$$

the Navier-Stokes equation becomes

$$\frac{D\vec{V}}{Dt} = - c^2 \nabla \Pi + \frac{1}{\rho} \nabla \cdot \vec{\vec{e}} \tag{13}$$

Hence, upon taking the divergence of equation (13) and using the result to eliminate the left side of equation (11), we obtain

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi - \nabla \cdot c^2 \nabla \Pi - 2 \frac{dU}{dx_2} \frac{\partial v_2}{\partial x_1} = \frac{\partial W_j}{\partial x_i} \frac{\partial W_i}{\partial x_j} - \nabla \cdot \left(\frac{1}{\rho} \nabla \cdot \vec{\vec{e}}\right) + \frac{1}{C_p} \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 S \tag{14}$$

This equation differs from Phillips' equation, (eq. (2.8) in ref. 6), which is exact, in two important respects, namely the operator D/Dt in Phillips' equation is replaced by the operator

$$\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}$$

and the terms

$$\frac{\partial V_j}{\partial x_i} \frac{\partial V_i}{\partial x_j}$$

in Phillips' equation are replaced by the terms

$$2 \frac{dU}{dx_2} \frac{\partial v_2}{\partial x_1} + \frac{\partial W_j}{\partial x_i} \frac{\partial W_i}{\partial x_j}$$

The derivation carried out above shows that these approximations amount to neglecting squares of acoustic quantities and the interaction between acoustic waves and turbulence. As pointed out in reference 8, for subsonic flow, the neglect of the interaction of the sound with the turbulence is justified on the basis that the acoustic waves emitted by the turbulence are of such long wavelengths compared with the size of the turbulent eddies that they pass right through the turbulence without being significantly affected.

The changes in entropy are related to the heat conduction through the energy equation. However, for the high Reynolds numbers which are of interest in jet flows, the direct effects of heat conduction and fluid viscosity are likely to be unimportant for cold jets. Hence, we shall assume that the entropy is constant and that the speed of sound has the constant value c_o. In addition, we shall neglect the viscous stress tensor $\overset{\Rightarrow}{e}$. Then, equation (14) becomes

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi - c_o^2 \nabla^2 \Pi - 2 \frac{dU}{dx_2} \frac{\partial v_2}{\partial x_1} = \frac{\partial W_j}{\partial x_i} \frac{\partial W_i}{\partial x_j} \tag{15}$$

Upon substituting equations (4), (9), and (12) into equation (13), neglecting terms involving squares of acoustic quantities and interaction of the sound with the turbulence, the x_2-component of equation (13) becomes

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right) v_2 = - c_o^2 \frac{\partial \Pi}{\partial x_2} - \left(\frac{\partial}{\partial t} + W_i \frac{\partial}{\partial x_i}\right) W_2 \tag{16}$$

Then, by eliminating v_2 between equations (15) and (16), we obtain

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\left[\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi - c_o^2 \nabla^2 \Pi\right] + 2c_o^2 \frac{dU}{dx_2} \frac{\partial^2 \Pi}{\partial x_1 \partial x_2}$$

$$= \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\left(\frac{\partial W_j}{\partial x_i} \frac{\partial W_i}{\partial x_j}\right) - 2 \frac{dU}{dx_2} \frac{\partial}{\partial x_1}\left(\frac{\partial W_2}{\partial t} + W_i \frac{\partial W_2}{\partial x_i}\right) \tag{17}$$

Now, it follows, from equation (12) and the fact that U is a function of x_2 only, that the right side of equation (17) can be written as

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\left(\frac{\partial w_i}{\partial x_j} \frac{\partial w_j}{\partial x_i} + 2 \frac{dU}{dx_2} \frac{\partial w_2}{\partial x_1}\right) - 2 \frac{\partial}{\partial x_1} \frac{dU}{\partial x_2}\left(\frac{\partial w_2}{\partial t} + U \frac{\partial w_2}{\partial x_1} + w_i \frac{\partial w_2}{\partial x_i}\right)$$

$$= \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\left(\frac{\partial w_i}{\partial x_j} \frac{\partial w_j}{\partial x_i}\right) - 2 \frac{dU}{dx_2} \frac{\partial}{\partial x_1}\left(w_i \frac{\partial w_2}{\partial x_i}\right).$$

However, it follows from equation (10) that

$$w_i \frac{\partial w_2}{\partial x_i} = \frac{\partial (w_i w_2)}{\partial x_i}$$

and

$$\frac{\partial w_i}{\partial x_j} \frac{\partial w_j}{\partial x_i} = \frac{\partial^2 w_i w_j}{\partial x_i \partial x_j}$$

Hence, equation (17) becomes

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\left[\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Pi - c_o^2 \nabla^2 \Pi\right] + 2c_o^2 \frac{dU}{dx_2} \frac{\partial^2 \Pi}{\partial x_1 \partial x_2}$$

$$= \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\left[\frac{\partial^2 (w_i w_j)}{\partial x_i \partial x_j}\right] - 2 \frac{dU}{dx_2} \frac{\partial^2 (w_2 w_i)}{\partial x_1 \partial x_i}$$

Since

$$\frac{\partial}{\partial x_1} U \frac{\partial^2 (w_i w_j)}{\partial x_i \partial x_j} = \frac{\partial}{\partial x_1}\left\{\frac{\partial}{\partial x_i}\left[U \frac{\partial (w_i w_j)}{\partial x_j}\right] - \frac{dU}{dx_2} \frac{\partial (w_2 w_i)}{\partial x_i}\right\}$$

$$= \frac{\partial}{\partial x_1}\left[\frac{\partial^2}{\partial x_i \partial x_j}(U w_i w_j) - 2 \frac{dU}{dx_2} \frac{\partial}{\partial x_i}(w_i w_2) - w_2^2 \frac{d^2 U}{dx_2^2}\right]$$

and

$$\frac{\partial}{\partial x_1} U \nabla^2 \Pi = \frac{\partial}{\partial x_1}\left[\nabla U \Pi - 2 \frac{dU}{dx_2} \frac{\partial \Pi}{\partial x_2} - \frac{d^2 U}{dx_2^2} \Pi\right]$$

this equation can be written as

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)^2 \Gamma - c_o^2\nabla^2\Gamma = \left[\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)(w_i w_j)\right]$$

$$- 4 \frac{\partial}{\partial x_1}\left[\frac{dU}{dx_2} \frac{\partial}{\partial x_i}\left(w_2 w_i + c_o^2 \delta_{2i}\Pi\right)\right] - \frac{\partial}{\partial x_1}\left[\frac{d^2U}{dx_2^2}\left(w_2^2 + c_o^2\Pi\right)\right] \tag{18}$$

where we have put

$$\Gamma \equiv \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1}\right)\Pi \tag{19}$$

INTERPRETATION OF BASIC EQUATION

Equation (18) is the starting point of the present analysis. We shall suppose that the turbulent and mean flows are given. Then, since the terms on the right vanish outside of the jet, we can treat the right side of equation (18) as a source term. Since no interaction between the sound wave and the turbulence is allowed once the sound is emitted, from the acoustical viewpoint each turbulent eddy acts independently. The sound is simply convected and refracted by the mean flow. In fact, the left side of equation (18) is a wave equation for the quantity Γ in a parallel flow with velocity U varying only in the x_2-direction and with the term accounting for the direct refraction of the sound by the velocity gradients neglected (see ref. 10). The source terms on the right side represent the production of sound by the turbulence. The consequences of equation (18) coincide with those of Lighthill's theory if the mean flow is negligible, but otherwise (as we shall see) equation (18) can lead to different results.

NEGLECT OF CONVECTION AND REFRACTION EFFECTS

The operator $\left(\partial/\partial t + U\, \partial/\partial x_1\right)^2$ on the left side of equation (18) is the time derivative in a coordinate system moving with the local mean velocity U, which we shall see is roughly equal to the velocity at which the individual sound sources move. This time derivative therefore roughly has the effect of multiplying the term on which it operates by the average angular frequency $\bar{\Omega}$ of the sound in a coordinate system moving with the source. Hence, if the wavelength $2\pi c_o/\bar{\Omega}$ is large compared with the jet diameter, the first term on the left side of equation (18) can be neglected by comparison with the second term at all points within the jet. On the other hand, since U is only nonzero within the jet, the operator in the first term reduces to $\partial^2/\partial t^2$ outside the jet. However, the operator $\partial/\partial t$ roughly has the effect of multiplying the term on which it operates by the average frequency $\bar{\omega}$ of the sound in a fixed frame. Hence, if the typical wavelengths $2\pi c_o/\bar{\omega}$ and $2\pi c_o/\bar{\Omega}$ are large compared to the jet diameter, we can replace the operator on the left side of equation (18) by the free-space wave operator.

$$\frac{\partial^2}{\partial t^2} - c_o^2\nabla^2$$

Now the data in references 11 and 12 show that these typical wavelengths vary from around 6 to 10 jet diameters. Hence, we shall make the assumption described in the preceding paragraph and replace equation (18) by

$$\left(\frac{\partial^2}{\partial t^2} - c_o^2\nabla^2\right)\Gamma = \left[\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)(w_i w_j)\right]$$
$$- 4\frac{\partial}{\partial x_1}\left[\frac{dU}{dx_2}\frac{\partial}{\partial x_i}\left(w_2 w_i + \delta_{2i}c_o^2\Pi\right)\right] - \frac{\partial}{\partial x_1}\left[\frac{d^2U}{dx_2^2}\left(w_2^2 + c_o^2\Pi\right)\right] \tag{20}$$

This approximation amounts to neglecting the convection and refraction of the sound by the mean flow; an approximation which is also made in Lighthill's theory.

SIMPLIFICATION OF SOURCE TERM

Within the jet the acoustic velocity is certainly negligible compared with the flow velocity

$$W_i = \delta_{1i}U + w_i \tag{12}$$

Thus, within the jet the x_2-component of the momentum equation (16) can be written as

$$-\frac{\partial}{\partial x_i}\left(w_2 w_i + \delta_{2i}c_o^2\Pi\right) = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)w_2$$

And since the source term in equation (20) vanishes outside the jet, we can substitute this expression into the source term to obtain

$$\left(\frac{\partial^2}{\partial t^2} - c_o^2\nabla^2\right)\Gamma = \left[\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)(w_i w_j)\right]$$

$$+ 4\frac{\partial}{\partial x_1}\frac{dU}{dx_2}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)w_2 - \frac{\partial}{\partial x_1}\left[\frac{d^2U}{dx_2^2}\left(w_2^2 + c_o^2\Pi\right)\right] \tag{21}$$

We now introduce the vector potential $\vec{A}$ by

$$\vec{A}(\vec{x},t) = \frac{1}{4\pi}\nabla \times \int \frac{\vec{w}(\vec{y},t)}{|\vec{x} - \vec{y}|}d\vec{y} \tag{22}$$

Then, as shown in books on electrodynamics (e.g., ref. 13, sec. 5.3), since $\nabla \cdot \vec{w} = 0$, this equation implies that

$$\vec{w} = \nabla \times \vec{A} \tag{23}$$

and

$$\nabla \cdot \vec{A} = 0 \tag{24}$$

Thus, in particular,

$$w_2 = \epsilon_{2jk}\frac{\partial}{\partial x_j}A_k = \frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1}$$

where ϵ_{2jk} is the permutation symbol. Upon substituting this into equation (21) and recalling that U is a function of x_2 only, we obtain

$$\left(\frac{\partial^2}{\partial t^2} - c_o^2\nabla^2\right)\Gamma = \left[\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)\left(w_i w_j + 4\delta_{1i}\epsilon_{2jk}\frac{dU}{dx_2}A_k\right)\right] - \frac{\partial}{\partial x_1}\left[\frac{d^2U}{dx_2^2}\left(w_2^2 + c_o^2\Pi\right)\right] \tag{25}$$

Now, as suggested by Lilley (ref. 4) and demonstrated by others (refs. 14 and 15), in a jet shear layer the mean velocity gradient dU/dx_2 (mean shear) is slowly varying over the narrow strip along the center of the mixing region where most of the turbulent energy occurs. Hence, terms corresponding to the variation in dU/dx_2 should only make a small contribution to the source term and, therefore, will be neglected. Thus, we will assume that

$$\frac{dU}{dx_2} = \text{constant}$$

Then equation (25) becomes

$$\left(\frac{\partial^2}{\partial t^2} - c_o^2\nabla^2\right)\Gamma = \frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)\tau_{ij} \tag{26}$$

where we have put

$$\tau_{ij} \equiv w_i w_j + 4\,\delta_{1i}\epsilon_{2jk}\frac{dU}{dx_2}A_k \tag{27}$$

This equation is similar to Lighthill's equation in that it is an ordinary inhomogeneous wave equation for a uniform medium containing a quadrupole source. However, it differs from Lighthill's equation in the important respects that the dependent variable is Γ instead of the condensation (i.e., $(\rho - \rho_o)/\rho_o$) and the quadrupole source term is considerably different from Lighthill's.

RETARDED POTENTIAL SOLUTION OF BASIC EQUATION

The retarded potential solution of equation (26) for Γ which vanishes at infinity is

$$\Gamma(\vec{x}) = \frac{1}{4\pi c_o^2}\int \frac{\delta^2}{\delta y_i \delta y_j}\left[\frac{\partial}{\partial t} + U(y_2)\frac{\delta}{\delta y_1}\right]\frac{\tau_{ij}\left(y, t - \frac{r}{c_o}\right)}{r}\,d\vec{y} \tag{28}$$

where $r \equiv |\vec{x} - \vec{y}|$ and the vectors $\vec{y}$ and $\vec{x}$ now denote source and receiver points, respectively. In addition, $\delta/\delta y_i$ indicates partial differentiation with respect to y_i with r held fixed. Now, for any function $F(\vec{y}, r, t)$,

$$\frac{\delta F}{\delta y_i} = \frac{\partial F}{\partial y_i} - \frac{\partial F}{\partial r}\frac{\partial r}{\partial y_i} = \frac{\partial F}{\partial y_i} + \frac{\partial F}{\partial x_i}$$

where $\partial/\partial y_i$ denotes partial differentiation with $\vec{x}$ fixed and $\partial/\partial x_i$ denotes partial differentiation with $\vec{y}$ fixed. Then,

$$\frac{\delta^2 F}{\delta y_i \delta y_j} = \frac{\partial^2 F}{\partial y_i \partial y_j} + \frac{\partial^2 F}{\partial y_i \partial x_j} + \frac{\partial^2 F}{\partial y_j \partial x_i} + \frac{\partial^2 F}{\partial x_i \partial x_j}$$

Upon inserting these results in equation (22), using the divergence theorem and the fact that

$$\tau_{ik}\left(\vec{y}, t - \frac{r}{c_o}\right) \rightarrow 0 \qquad \text{as } |\vec{y}| \rightarrow \infty$$

we obtain

$$\Gamma = \frac{1}{4\pi c_o^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \left[\frac{\partial}{\partial t} + U(y_2) \frac{\partial}{\partial x_1}\right] \frac{\tau_{ij}\left(\vec{y}, t - \frac{r}{c_o}\right)}{r} d\vec{y}$$

Since

$$\frac{\partial \tau_{ij}}{\partial x_l}\left(\vec{y}, t - \frac{r}{c_o}\right) = - \frac{(x_l - y_l)}{c_o r} \frac{\partial \tau_{ij}}{\partial t}\left(\vec{y}, t - \frac{r}{c_o}\right)$$

it follows from equation (19) (since U vanishes outside the jet) that, when r is large enough so that $\vec{x}$ is in the radiation field of each turbulent eddy (so that terms of higher order in $1/r$ can be neglected, see ref. 1), the preceding equation for Γ reduces to

$$\frac{\partial \Pi}{\partial t} \sim \frac{1}{4\pi c_o^4} \int \frac{(x_i - y_i)(x_j - y_j)}{r^3} \left[1 - \frac{\vec{U}(y_2) \cdot (\vec{x} - \vec{y})}{c_o r}\right] \frac{\partial^3}{\partial t^3} \tau_{ij}\left(\vec{y}, t - \frac{r}{c_o}\right) d\vec{y} \tag{29}$$

Since $(p - p_o)/p_o$ is certainly very small in the radiation field, it follows from equation (8) that

$$\Pi \sim \frac{p - p_o}{\rho_o c_o^2} \tag{30}$$

which is equal to the condensation in the radiation field.

SOLUTION IN TERMS OF SPECTRA

We shall assume that the turbulence is statistically stationary in time. Then we shall suppose (as usual, in order to ensure the convergence of the Fourier integrals which will be introduced below) that $\tau_{ik} = 0$ for $|t| > T$, where T is some large time which will be put equal to infinity at the end of the analysis. We now introduce the Fourier transforms

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} \frac{(p - p_o)}{\rho_o c_o^2} dt$$

$$T_{ij} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} \tau_{ij} \, dt$$

Then, upon using the asymptotic approximation (30), the Fourier transform of equation (29) becomes

$$-i\omega P = \frac{i\omega^3}{4\pi c_o^4} \int \frac{(x_i - y_i)(x_j - y_j)}{r^3} e^{i(\omega/c_o)r} \left[1 - \frac{\vec{U}(y_2) \cdot (\vec{x} - \vec{y})}{c_o r}\right] T_{ij} \, d\vec{y} \qquad (31)$$

In the far field of the entire flow, the spectral density of the intensity is given by (ref. 16)

$$I_\omega(\vec{x}) = \frac{\rho_o c_o^3 |P|^2}{2T}$$

Substituting equation (31) into this relation and using the convolution theorem gives, in view of equation (27),

$$I_\omega(\vec{x}) = \frac{\omega^4 \rho_o}{32\pi^3 c_o^5} \mathcal{R}e \iiint \frac{(x_i - y_i)(x_j - y_j)(x_k - z_k)(x_l - z_l)}{|\vec{x} - \vec{y}|^3 |\vec{x} - \vec{z}|^3} e^{i\omega[(1/c_o)(|\vec{x}-\vec{y}| - |\vec{x}-\vec{z}|) - \tau]} \left[1 - \frac{\vec{U}(y_2) \cdot (\vec{x} - \vec{y})}{c_o |\vec{x} - \vec{y}|}\right] \left[1 - \frac{\vec{U}(z_2) \cdot (\vec{x} - \vec{z})}{c_o |\vec{x} - \vec{z}|}\right] \mathcal{R}_{ijkl}(\vec{y}, \vec{\eta}, \tau) d\vec{y} \, d\vec{z} \, d\tau \qquad (31a)$$

where the operation of taking the real part is certainly justified since I_ω is real. (This operation is performed for convenience in subsequent manipulations.) Also,

$$\mathcal{R}_{ijkl}(\vec{y},\vec{\eta},\tau) = \overline{w_i w_j w_k' w_l'} - \overline{w_i w_j}\;\overline{w_k' w_l'}$$

$$+ 16\delta_{1i}\delta_{1k}\epsilon_{2jm}\epsilon_{2ln}\left(\frac{dU}{dy_2}\right)^2 \overline{A_m A_n'} + 8\delta_{1i}\epsilon_{2jm}\frac{dU}{dy_2}\overline{A_m w_k' w_l'} \tag{32}$$

is a fourth-order, two-point, time-delayed turbulence correlation tensor where $\vec{\eta} \equiv \vec{z} - \vec{y}$. The primes indicate that the quantities are to be evaluated at $\vec{z} = \vec{y} + \vec{\eta}$ and $t + \tau$, while the unprimed quantities are evaluated at $\vec{y}$ and t. The overbar denotes the average

$$\overline{f(t,t+\tau)} = \frac{1}{2T}\int_{-T}^{T} f(t,t+\tau)dt$$

The term $\overline{w_i w_j}\;\overline{w_k' w_l'}$, which does not contribute to $I_\omega(\vec{x})$, has been introduced for convenience. Upon changing the variables of integration from $\vec{y}$ and $\vec{z}$ to $\vec{y}$ and $\vec{\eta}$ and collecting terms, the equation for $I_\omega(\vec{x})$ becomes

$$I_\omega(\vec{x}) = \frac{\omega^4\rho_o}{32\pi^3 c_o^5}\,\mathcal{R}e\iiint \frac{(x_i-y_i)(x_j-y_j)(x_k-y_k-\eta_k)(x_l-y_l-\eta_l)}{|\vec{x}-\vec{y}|^3|\vec{x}-\vec{y}-\vec{\eta}|^3}\, e^{i\omega[(1/c_o)(|\vec{x}-\vec{y}|-|\vec{x}-\vec{y}-\vec{\eta}|)-\tau]}\left[1-\frac{\vec{U}(y_2)\cdot(\vec{x}-\vec{y})}{c_o|\vec{x}-\vec{y}|}\right]\left[1-\frac{\vec{U}(y_2+\eta_2)\cdot(\vec{x}-\vec{y}-\vec{\eta})}{c_o|\vec{x}-\vec{y}-\vec{\eta}|}\right]\mathcal{R}_{ijkl}(\vec{y},\vec{\eta},\tau)d\vec{y}\,d\vec{\eta}\,d\tau \tag{33}$$

FAR-FIELD EXPANSION OF SOLUTION

Because the distance $|\vec{\eta}|$ over which the correlation $\mathcal{R}_{imkn}$ is nonzero is certainly smaller than any overall dimension of the region of turbulence, we can always suppose that the observation point $\vec{x}$ is sufficiently far away from the flow so that

$$|\vec{x} - \vec{y}| >> |\vec{\eta}|$$

Therefore, upon expanding the integrand in equation (33) and neglecting terms of order $|\vec{\eta}|/r$, we obtain

$$I_\omega(\vec{x}) = \frac{\omega^4\rho_o}{32\pi^3 c_o^5}\,\mathcal{R}e\iiint \frac{r_i r_j r_k r_l}{r^6}\, e^{i\omega[(\vec{r}\cdot\vec{\eta}/c_o r)-\tau]}\left[1-\frac{U(y_2)}{c_o}\cos\theta\right]\left[1-\frac{U(y_2+\eta_2)}{c_o}\cos\theta\right]\mathcal{R}_{ijkl}(\vec{y},\vec{\eta},\tau)d\vec{y}\,d\vec{\eta}\,d\tau \tag{34}$$

where we have put

$$\vec{r} \equiv \vec{x} - \vec{y}$$

$$r_i = x_i - y_i$$

and

$$\cos\theta = \frac{r_1}{r} = \frac{\vec{r} \cdot \vec{U}}{rU}$$

is the cosine of the angle θ between the direction of mean flow and the direction of observation.

Now,

$$U(y_2 + \eta_2) = U(y_2) + \frac{dU}{dy_2}\eta_2$$

since dU/dy_2 is assumed to be constant. For the range of $|\vec{\eta}|$ over which the correlation $\mathscr{R}_{imkn}$ is nonzero, the ratio of the second term to the first term on the right side of this expression is of the order of the ratio of the transverse correlation length to the thickness of the mixing region, since for jets the change in $U(y_2)$ occurs over the width of the mixing region. It can be seen from the results presented in references 14, 15, and 17 (also see discussion in ref. 7) that this ratio is of the order of 1/6 to 1/10. Hence, we shall neglect the second term compared with the first and replace $U(y_2 + \eta_2)$ by $U(y_2)$ in equation (27). Since this U now represents a sort of average velocity over the eddy, we shall set it equal to the "eddy convection velocity" $U_c(y_2)$. In fact, as pointed out by Lilley (ref. 4) and verified experimentally in reference 15, the chief noise-emitting eddies are confined to a narrow strip in the center of the jet mixing region; and in this strip (ref. 15) the eddy convection velocity is very nearly equal to the local mean speed U. (See refs. 15 and 17 for a discussion of the convection velocity.) Upon collecting terms, equation (34) now becomes

$$I_\omega(\vec{x}) = \frac{\omega^4 \rho_o}{32\pi^3 c_o^5} \mathcal{R}e \iiint \frac{r_i r_j r_k r_l}{r^6} e^{i\omega[(\vec{r}\cdot\vec{\eta}/rc_o)-\tau]}\left(1 - M_c \cos\theta\right)^2 \mathscr{R}_{ijkl}(\vec{y}, \vec{\eta}, \tau)d\vec{y}\, d\vec{\eta}\, d\tau \tag{35}$$

where

$$M_c(y_2) \equiv \frac{U_c(y_2)}{c_o}$$

is the eddy convection Mach number.

INTRODUCTION OF MOVING COORDINATES AND NEGLECT OF RETARDED TIME

When $\vec{\eta}$ is transverse to the flow, the changes in $\vec{r} \cdot \vec{\eta}/rc_o$ over the "eddy" correlation distance are small compared with the change in τ over the eddy decay time. Hence, we can neglect the term $\vec{r} \cdot \vec{\eta}/rc_o$ in

$$\exp\left[\left(i\omega \frac{\vec{r} \cdot \vec{\eta}}{rc_o} - \tau\right)\right]$$

However, when $\vec{\eta}$ is in the direction of the mean flow (see ref. 16), the changes of the correlation with respect to time are much more rapid since the eddies are moving with the flow. In order to compensate for this, we introduce a set of coordinates which move with the convection velocity $\vec{U}_c$. Thus, we introduce the new variable

$$\vec{\xi} = \vec{\eta} - \vec{U}_c(y_2)\tau \tag{36}$$

where $\vec{U}_c$ is in the y_1-direction. Following Ffowcs Williams, we define the moving-axis correlation function by

$$R^{+}_{ijkl}(\vec{y}, \vec{\xi}, \tau) \equiv \mathscr{R}_{ijkl}(\vec{y}, \vec{\eta}, \tau) \tag{37}$$

Upon introducing this change of variable into the integral in equation (35) and noting that the Jacobian of the transformation is unity, we find that

$$I_\omega(\vec{x}) = \frac{\omega^4 \rho_o}{32\pi^3 c_o^5} \mathcal{R}e \int \frac{r_i r_j r_k r_l}{r^6} \left(1 - M_c \cos\theta\right)^2 \int e^{-i\omega(1-M_c \cos\theta)\tau} \int e^{(i\omega/c_o r)\vec{\xi}\cdot\vec{r}} R^{+}_{ijkl}(\vec{y}, \vec{\xi}, \tau) d\vec{\xi}\, d\tau\, d\vec{y} \tag{38}$$

Now let l denote a typical correlation length of the turbulence, let M_e be the characteristic Mach number of the turbulence, and let τ_f denote a typical decay time of the moving-axis correlation. It is shown in reference 7 (see sec. 3, eq. (17)) that

$$\frac{l}{\tau_f c_o} \approx M_e$$

However, for jet flows the maximum turbulent velocity is about one-tenth, and certainly not more than two-tenths, of the maximum mean-flow velocity. Consequently, for subsonic flows

$$\frac{l}{c_o} << \tau_f$$

This shows that $\vec{\xi} \cdot \vec{r}/c_o r$ is negligible in comparison with $(1 - M_c \cos \theta)\tau$. Hence, we can set $\vec{\xi} \cdot \vec{r}/c_o r$ equal to zero to obtain

$$I_\omega(\vec{x}) = \frac{\omega^4 \rho_o}{32\pi^3 c_o^5} \mathcal{R}e \int \frac{r_i r_j r_k r_l}{r^6} \left(1 - M_c \cos \theta\right)^2 \int e^{-i\omega(1-M_c \cos \theta)\tau} \int R^+_{ijkl}(\vec{y}, \vec{\xi}, \tau) d\vec{\xi}\, d\tau\, d\vec{y} \tag{39}$$

Following Ribner (ref. 5) we let $I_\omega(\vec{x}|\vec{y})$ denote the intensity at the point $\vec{x}$ caused by the sound emitted from a unit volume at $\vec{y}$. Then,

$$I_\omega(\vec{x}) = \int I_\omega(\vec{x}|\vec{y}) d\vec{y}$$

and it follows from equation (39) that

$$I_\omega(\vec{x}|\vec{y}) = \frac{\Omega^2 \omega^2 \rho_o}{32\pi^3 c_o^5} \frac{r_i r_j r_k r_l}{r^6} \mathcal{R}e \int e^{-i\Omega\tau} \int R^+_{ijkl}(\vec{y}, \vec{\xi}, \tau) d\vec{\xi}\, d\tau \tag{40}$$

where

$$\Omega \equiv \omega(1 - M_c \cos \theta)$$

is the frequency which would be observed in a frame of reference moving with the eddy convection Mach number M_c.

We can now write, in view of equations (32) and (37),

$$R^{+}_{ijkl}(\vec{y}, \vec{\xi}, \tau) = R_{ijkl}(\vec{y}, \vec{\xi}, \tau) + 16\delta_{1i}\delta_{1k}\epsilon_{2jm}\epsilon_{2ln}\left(\frac{dU}{dy_2}\right)^2 Q_{mn}(\vec{y}, \vec{\xi}, \tau)$$

$$+ 8\delta_{1i}\epsilon_{2jm}\frac{dU}{dy_2} Q_{m,kl}(\vec{y}, \vec{\xi}, \tau) \tag{41}$$

where

$$R_{ijkl}(\vec{y}, \vec{\xi}, \tau) \equiv \overline{w_i w_j w_k' w_l'} - \overline{w_i w_j}\;\overline{w_k' w_l'} \tag{42}$$

$$Q_{m,kl}(\vec{y}, \vec{\xi}, \tau) \equiv \overline{A_m w_k' w_l'} \tag{43}$$

$$Q_{m,n}(\vec{y}, \vec{\xi}, \tau) \equiv \overline{A_m A_n'} \tag{44}$$

We shall now show, under the relatively mild restriction that the turbulence is locally homogeneous and incompressible, that the volume integrals of the correlations $Q_{m,kl}$ and Q_{mn} can be expressed in a simple way entirely in terms of two-point velocity correlations.

LOCALLY HOMOGENEOUS TURBULENCE; REDUCTION OF CORRELATION VOLUMES

The theory of incompressible, locally homogeneous turbulence is developed in reference 18 and in chapter 4 of reference 19. The assumption of local homogeneity implies that any two-point, two-time correlation function, say $Q(\vec{y}, \vec{z}, \tau)$, is a function only of $\vec{\eta} = \vec{z} - \vec{y}$ and τ.

It follows from equation (22) that

$$\overline{A_m w_k' w_l'} = -\frac{1}{4\pi}\epsilon_{mrp}\int \overline{w_p'' w_k' w_l'}\frac{\partial}{\partial y_r''}\frac{1}{|\vec{y}-\vec{y}''|}d\vec{y}''$$

$$= -\frac{1}{4\pi}\epsilon_{mrp}\int \overline{w_p'' w_k' w_l'}\frac{\partial}{\partial \eta_r^{(1)}}\frac{1}{|\vec{\eta}-\vec{\eta}^{(1)}|}d\vec{\eta}^{(1)}$$

where the double primes denote quantities evaluated at $\vec{y}''$ and t and where $\vec{\eta}^{(1)} = \vec{z} - \vec{y}''$. Or, since the Jacobian of the transformation $\vec{\xi}^{(1)} \to \vec{\eta}^{(1)}$ [where $\vec{\xi}^{(1)} = \vec{\eta}^{(1)} - \vec{U}_c(y_2)\tau$] is unity,

$$Q_{m,kl} = -\frac{1}{4\pi}\epsilon_{mrp}\int R_{p,kl}(\xi^{(1)},\tau)\frac{\partial}{\partial \xi_r^{(1)}}\frac{1}{|\vec{\xi}-\vec{\xi}^{(1)}|}d\xi^{(1)}$$

where

$$R_{p,kl}(\vec{\xi},\tau) \equiv \overline{w_p w_k' w_l'} \tag{45}$$

is the two-point, time-delayed, triple-velocity correlation. Thus,

$$\int Q_{m,kl}\,d\vec{\xi} = -\frac{1}{4\pi}\epsilon_{mrp}\int R_{p,kl}(\vec{\xi}^{(1)},\tau)\left[\int \frac{\partial}{\partial \xi_r^{(1)}}\frac{1}{|\vec{\xi}-\vec{\xi}^{(1)}|}d\vec{\xi}\right]d\vec{\xi}^{(1)}$$

Hence, upon using equation (A1) derived in appendix A, this becomes

$$\int Q_{m,kl}\,d\vec{\xi} = \frac{1}{3}\epsilon_{mrp}\int \xi_r^{(1)} R_{p,kl}(\vec{\xi}^{(1)},\tau)d\vec{\xi}^{(1)} = \frac{1}{3}\epsilon_{mrp}\int \xi_r R_{p,kl}(\vec{\xi},\tau)d\vec{\xi} \tag{46}$$

It is shown in reference 20 that this integral is indeed convergent (see remarks immediately proceeding equation (6.17) of ref. 20).

In order to evaluate the volume integral of Q_{mn}, notice that it follows from equation (22) that

$$\overline{A_m A_n'} = \frac{\epsilon_{mpq}\epsilon_{nrs}}{16\pi^2} \iint \overline{w_q'' w_s'''} \frac{\partial}{\partial y_p''} |\vec{y} - \vec{y}''|^{-1} \frac{\partial}{\partial y_r'''} |\vec{z} - \vec{y}'''|^{-1} d\vec{y}'' d\vec{y}'''$$

$$= \frac{\epsilon_{mpq}\epsilon_{nrs}}{16\pi^2} \int \overline{w_q'' w_s'''} \left[\int \frac{\partial}{\partial y_p''} |\vec{y} - \vec{y}''|^{-1} \frac{\partial}{\partial \eta_r^{(2)}} |\vec{z} - \vec{y}'' - \vec{\eta}^{(2)}|^{-1} d\vec{y}'' \right] d\vec{\eta}^{(2)}$$

where the triple-primed quantities are to be evaluated at $\vec{y}'''$ and $t + \tau$ and where $\vec{\eta}^{(2)} = \vec{y}''' - \vec{y}''$. The integral in square brackets was evaluated in the appendix of reference 21 and found to be

$$2\pi \frac{\partial^2 |\vec{\eta}^{(2)} - \vec{\eta}|}{\partial \eta_p^{(2)} \partial \eta_r^{(2)}}$$

Hence,

$$\overline{A_m A_n'} = \frac{\epsilon_{mpq}\epsilon_{nrs}}{8\pi} \int \overline{w_q'' w_s'''} \frac{\partial^2 |\vec{\eta} - \vec{\eta}^{(2)}|}{\partial \eta_p^{(2)} \partial \eta_r^{(2)}} d\vec{\eta}^{(2)}$$

Or since the Jacobian of the transformation $\vec{\xi}^{(2)} \rightarrow \vec{\eta}^{(2)}$ $\left[\text{where } \vec{\xi}^{(2)} \equiv \vec{\eta}^{(2)} - \vec{U}_c(y_2)\tau\right]$ is unity,

$$Q_{mn} = \frac{\epsilon_{mpq}\epsilon_{nrs}}{8\pi} \int R_{qs}(\vec{\xi}^{(2)}, \tau) \frac{\partial^2 |\vec{\xi} - \vec{\xi}^{(2)}|}{\partial \xi_p^{(2)} \partial \xi_r^{(2)}} d\vec{\xi}^{(2)}$$

where

$$R_{ij}(\vec{\xi}, \tau) \equiv \overline{w_i w_j'} \tag{47}$$

is the two-point, time-delayed, second-order velocity correlation tensor. Hence,

$$\int Q_{mn}(\vec{\xi}, \tau)d\vec{\xi} = \frac{\epsilon_{mpq}\epsilon_{nrs}}{8\pi} \int R_{qs}(\vec{\xi}^{(2)}, \tau) \int \frac{\partial^2|\vec{\xi} - \vec{\xi}^{(2)}|}{\partial\xi_p^{(2)}\partial\xi_r^{(2)}} d\vec{\xi}\, d\vec{\xi}^{(2)}$$

The double integral was evaluated in appendix I of reference 4 by using the methods of reference 21 and found to be

$$-\frac{8\pi}{30} \int \left\{2\xi_p^{(2)}\xi_r^{(2)} + \left[\xi^{(2)}\right]^2 \delta_{pr}\right\} R_{qs}\left[\vec{\xi}^{(2)}, \tau\right] d\vec{\xi}^{(2)} = -\frac{8\pi}{30} \int \left(2\xi_p\xi_r + \delta_{pr}\xi^2\right) R_{qs}(\vec{\xi}, \tau)d\vec{\xi}$$

Hence,

$$\int Q_{mn}(\vec{\xi}, \tau)d\vec{\xi} = -\frac{\epsilon_{mpq}\epsilon_{nrs}}{30} \int \left(2\xi_p\xi_r + \delta_{pr}\xi^2\right) R_{qs}(\vec{\xi}, \tau)d\vec{\xi} \tag{48}$$

It is shown in reference 20 that this integral is indeed convergent (see remarks on top of p. 386, ref. 20).

Upon using the identity $\epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$ and the fact that $\epsilon_{ijk} = -\epsilon_{ikj}$, it follows from equations (41), (46), and (48) that

$$\int R_{ijkl}^{+}\, d\vec{\xi} = \int R_{ijkl}\, d\vec{\xi} - \frac{8}{15}\, \delta_{1i}\delta_{1k}(\delta_{2p}\delta_{jq} - \delta_{2q}\delta_{jp})(\delta_{2r}\delta_{ls}$$

$$- \delta_{2s}\delta_{lr})\left(\frac{dU}{dy_2}\right)^2 \int \left(2\xi_p\xi_r + \delta_{pr}\xi^2\right) R_{qs}\, d\vec{\xi} + \frac{8}{3}\, \delta_{1i} \frac{dU}{dy_2} \int (\xi_2 R_{j,kl} - \xi_j R_{2,kl})d\vec{\xi} \tag{49}$$

The first two integrals on the right side of equation (49) represent the self noise and the shear noise, respectively (refs. 22 and 4). This terminology was introduced by Lilley in reference 4 to indicate that the former term represents noise generated by turbulent-turbulent interactions, whereas the latter term represents noise generated by turbulent - mean-shear interactions. The last integral represents a coupling between the shear noise and the self noise.

Equation (49) cannot be further simplified without introducing some additional assumptions about the turbulence. We first introduce the joint normality approximation.

REPRESENTATION OF FOURTH-ORDER CORRELATIONS IN TERMS OF SECOND-ORDER CORRELATIONS

It is argued by Batchelor (ref. 18) that the part of the joint probability distribution of the velocities at a fixed time associated with the energy-bearing eddies is approximately normal, at least insofar as the velocities at two points are concerned. This approximation is better for some purposes than others. Thus, in reference 18 (p. 176) it gives reasonably accurate predictions about the relation between the second- and fourth-order correlations. This relation (ref. 18, eq. (8. 3. 11) is found to be (see ref. 23, sec. 2. 1. 7 for derivation)

$$\overline{w_i w_j w_k' w_l'} = \overline{w_i w_j}\ \overline{w_k' w_l'} + \overline{w_i w_k'}\ \overline{w_j w_l'} + \overline{w_i w_l'}\ \overline{w_j w_k'} \qquad \text{at} \quad \tau = 0 \tag{50}$$

But, by extending the reasoning used by Batchelor in section 8. 2, we can argue that, when the velocity correlations in the moving frame are separated in time as well as in space, their correlation will be subject to even more random influences from the neighboring flow than when they occur at the same time. In accordance with the central limit theorem, these influences will tend to ensure further the normality of the joint probability distribution. Hence, we have even more reason to expect equation (50) to be valid with $\tau \neq 0$, and we can now write from equation (42)

$$R_{ijkl}(\vec{y}, \vec{\xi}, \tau) = R_{ik}(\vec{y}, \vec{\xi}, \tau) R_{jl}(\vec{y}, \vec{\xi}, \tau) + R_{il}(\vec{y}, \vec{\xi}, \tau) R_{jk}(\vec{y}, \vec{\xi}, \tau)$$

so that equation (49) now becomes

$$\int R_{ijkl}^{+}\, d\vec{\xi} = \int (R_{ik}R_{jl} + R_{il}R_{jk}) d\vec{\xi} - \frac{8}{15}\, \delta_{1i}\delta_{1k}(\delta_{2p}\delta_{jq} - \delta_{2q}\delta_{jp})(\delta_{2r}\delta_{ls} - \delta_{2s}\delta_{lr}) \left(\frac{dU}{dy_2}\right)^2 \int \left(2\xi_p\xi_r + \delta_{pr}\xi^2\right) R_{qs}\, d\vec{\xi} + \frac{8}{3}\, \delta_{1i} \frac{dU}{dy_2} \int (\xi_2 R_{j,kl} - \xi_j R_{2,kl}) d\vec{\xi} \tag{51}$$

RESULTS FOR ISOTROPIC TURBULENCE MODEL

In order to deduce the jet-noise directivity pattern from equations (40) and (51), it is necessary to make some additional assumption about the turbulence. Perhaps the simplest plausible assumption is that made by Ribner and Lilley (refs. 4, 5, and 22) namely that the turbulence is locally homogeneous and isotropic. A more refined model for the turbulence (assumption of axisymmetric turbulence) is given in reference 24. However, since (as we shall see) the convective factor dominates the overall directivity pattern (i.e., at least insofar as the overall intensity itself, but not its spectral density, is concerned), it is felt that the assumption of isotropic turbulence will suffice for predicting the directivity of the intensity. We shall, therefore, assume that the moving-axis correlation tensors are isotropic tensors. Thus, it is shown in reference 18 (p. 42) that the general third-order isotropic tensor which is symmetric in its second two indices is

$$C\xi_j\xi_k\xi_l + D(\delta_{jl}\xi_k + \delta_{kj}\xi_l) + E\xi_j\delta_{kl}$$

where C, D, and E are functions of $\vec{y}$ and τ and are functions only of the magnitude ξ of $\vec{\xi}$ and not its direction. Hence, we put

$$R_{j,kl} = C\xi_j\xi_k\xi_l + D(\delta_{jl}\xi_k + \delta_{kj}\xi_l) + E\xi_j\delta_{kl}$$

Then

$$\xi_2 R_{j,kl} - \xi_j R_{2,kl} = D(\xi_2\xi_k\delta_{lj} + \xi_2\xi_l\delta_{kj} - \xi_j\xi_k\delta_{l2} - \xi_j\xi_l\delta_{k2})$$

And, therefore, upon noting that only even functions of ξ_i can contribute to the integral, we find that

$$\int(\xi_2 R_{j,kl} - \xi_j R_{2,kl})d\vec{\xi} = (\delta_{2k}\delta_{lj} + \delta_{l2}\delta_{jk})\int D\left(\xi_2^2 - \xi_j^2\right)d\vec{\xi} \qquad \text{(no sum on j)}$$

$$= 0 \tag{52}$$

since $\int D\xi_2^2\, d\vec{\xi} = \int D\xi_j^2\, d\vec{\xi}$. This shows that the coupling term in equation (51) vanishes.

It is shown in reference 24 that the second-order isotropic correlation tensor is

$$R_{ij}(\vec{y}, \vec{\xi}, \tau) = A\xi_i\xi_j + B\delta_{ij} \tag{53}$$

where A and B are functions of $\vec{y}$ and τ and are functions only of the magnitude ξ of $\vec{\xi}$ and not of its direction. It is also shown in reference 24 that the requirements of continuity dictate that the functions A and B are related to a single function F by

$$\left.\begin{aligned} A &= -\frac{1}{2}\frac{1}{\xi}\frac{\partial F}{\partial \xi} \\ B &= F + \frac{\xi}{2}\frac{\partial F}{\partial \xi} \end{aligned}\right\} \tag{54}$$

Hence, upon noting that only even functions of ξ_i contribute to the integral, we find that

$$(\delta_{2p}\delta_{jq} - \delta_{2q}\delta_{jp})(\delta_{2r}\delta_{l s} - \delta_{2s}\delta_{l r})\int\left(2\xi_p\xi_r + \delta_{pr}\xi^2\right)R_{qs}\,d\vec{\xi}$$

$$= 2\delta_{jl}\left[\int\left(2\xi_j^2 + \xi^2\right)R_{22}\,d\vec{\xi} - \int\left(2\xi_2\xi_j + \delta_{2j}\xi^2\right)R_{2l}\,d\vec{\xi}\right] \quad \text{(no sum on } j, l)$$

$$= 2\delta_{jl}(1 - \delta_{2j})\int\left[\left(2\xi_1^2 + \xi^2\right)R_{22} - 2\xi_1\xi_2R_{12}\right]d\vec{\xi} \quad \text{(no sum on } j)$$

$$= \frac{2}{3}\delta_{jl}(1 - \delta_{2j})\int\left(5F + 2\xi\frac{dF}{d\xi}\right)\xi^2\,d\vec{\xi} \quad \text{(no sum on } j)$$

where we have used the fact that for any function $f(\xi)$,

$$\int f(\xi)\xi_j^2\,d\vec{\xi} = \frac{1}{3}\int f(\xi)\xi^2\,d\vec{\xi}$$

Hence, upon integrating by parts, we find that

$$(\delta_{2p}\delta_{jq} - \delta_{2q}\delta_{jp})(\delta_{2r}\delta_{l s} - \delta_{2s}\delta_{l r})\int\left(2\xi_p\xi_r + \delta_{pr}\xi^2\right)R_{qs}\,d\vec{\xi}$$

$$= -\frac{10}{3}\delta_{jl}(1 - \delta_{2j})\int F\xi^2\,d\vec{\xi} = 10\delta_{jl}(1 - \delta_{2j})\int \xi_2^2R_{11}\,d\vec{\xi} \quad \text{(no sum on } j) \tag{55}$$

Finally, it follows from equations (53) and (54), after applying some routine manipulations given in appendix C of reference 25, that

$$\int R_{ik}R_{jl}\, d\vec{\xi} = \frac{1}{8}\left(6\delta_{ik}\delta_{jl} + \delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}\right)\int R_{11}^2\, d\vec{\xi} \tag{56}$$

Hence, upon substituting equations (52), (55), and (56) into equation (51) and using the result in equation (40), we obtain

$$I_\omega(\vec{x}|\vec{y}) = \frac{\Omega^2\omega^2\rho_o}{32\pi^3 c_o^5 r^2}\left[2\,\mathcal{Re}\int e^{-i\Omega\tau}\int R_{11}^2\, d\vec{\xi}\, d\tau - \frac{16}{3}\cos^2\theta\left(\cos^2\theta + \sin^2\theta\sin^2\varphi\right)\left(\frac{dU}{dy_2}\right)^2 \mathcal{Re}\int e^{-i\Omega\tau}\int \xi_2^2 R_{11}\, d\vec{\xi}\, d\tau\right] \tag{57}$$

where the azimuthal angle φ is given by $\sin\theta\sin\varphi = r_3/r$.

EQUATIONS FOR AXISYMMETRIC FLOWS AND TOTAL INTENSITY

The total intensity $I(\vec{x}|\vec{y})$ at each point is obtained by integrating the spectral density of the intensity over all frequencies. Thus,

$$I(\vec{x}|\vec{y}) = \int_{-\infty}^{\infty} I_\omega(\vec{x}|\vec{y})\, d\omega$$

Substituting equation (57) into this expression shows that

$$I(\vec{x}|\vec{y}) = \frac{\rho_o}{16\pi^2 c_o^5 r^2 (1 - M_c\cos\theta)^3}$$

$$\times\left[2\left(\frac{\partial^4}{\partial\tau^4}\int R_{11}^2\, d\vec{\xi}\right)_{\tau=0} - \frac{16}{3}\cos^2\theta\left(\cos^2\theta + \sin^2\theta\sin^2\varphi\right)\left(\frac{dU}{dy_2}\right)^2\left(\frac{\partial^4}{\partial\tau^4}\int \xi_2^2 R_{11}\, d\vec{\xi}\right)_{\tau=0}\right] \tag{58}$$

As indicated by Ribner (refs. 5 and 22), for round jets it is appropriate to average the intensity over the azimuthal angle φ. When this is done, equations (57) and (58) become, respectively,

$$I_\omega(\vec{x}|\vec{y})_{av} = \frac{\Omega^4 \rho_o}{32\pi^3 c_o^5 r^2 \left(1 - M_c \cos\theta\right)^2}$$

$$\times \left[2 \int \cos\Omega\tau \int R_{11}^2 \, d\vec{\xi}\, d\tau - \frac{16}{3}\left(\frac{\cos^4\theta + \cos^2\theta}{2}\right)\left(\frac{dU}{dy_2}\right)^2 \int \cos\Omega\tau \int \xi_2^2 R_{11}\, d\vec{\xi}\, d\tau\right] \tag{59}$$

$$I(\vec{x}|\vec{y})_{av} = \frac{\rho_o}{16\pi^2 c_o^5 r^2 \left(1 - M_c \cos\theta\right)^3}$$

$$\times \left[2\left(\frac{\partial^4}{\partial\tau^4}\int R_{11}^2\, d\vec{\xi}\right)_{\tau=0} - \frac{16}{3}\left(\frac{\cos^4\theta + \cos^2\theta}{2}\right)\left(\frac{dU}{dy_2}\right)^2\left(\frac{\partial^4}{\partial\tau^4}\int \xi_2^2 R_{11}\, d\vec{\xi}\right)_{\tau=0}\right] \tag{60}$$

DISCUSSION OF EQUATIONS FOR FAR-FIELD INTENSITY; COMPARISON WITH LIGHTHILL'S THEORY AND RIBNER'S QUADRUPOLE MODEL

The first term on the right side in either equations (59) or (60) represents the self noise, and the other terms represent the shear noise (refs. 22 and 4). The factor $\left(1 - M_c \cos\theta\right)^{-3}$ in equation (60) represents the effect on the directivity pattern of the convection of the noise sources by the mean flow. The results of the present theory differ from those of Lighthill's theory (as corrected by Ffowcs Williams in ref. 3) principally in that Lighthill obtains an exponent of -5 instead of -3 for this factor (refs. 2 and 7). Ribner (ref. 5) refers to this term as the "convection factor." Because of the large exponent, it is believed to be the most important factor in determining the directivity pattern of the jet noise.[3]

The term in brackets in equation (60) represent the actual generation term for the jet noise. It is composed of a self-noise part and a shear-noise part. The directivity pattern associated with this term is called by Ribner in reference 5 the "basic" direc-

[3]This may not be true near the jet axis where refraction effects may be dominant.

tivity pattern of the jet noise. This pattern is a result of the quadrupole nature of the noise source.

In order to represent the "basic" directivity pattern given in equation (60), it was necessary to make some assumptions about the turbulence. These assumptions are the same as those made by Ribner (ref. 5) to deduce the basic directivity pattern implied by Lighthill's theory. Hence, it is appropriate to compare the basic directivity pattern obtained herein with that obtained by Ribner. The two results are almost identical. Thus, the self-noise term is the same in both cases and the directional dependence

$$\frac{\cos^4 \theta + \cos^2 \theta}{2}$$

of the shear-noise term is also the same in both cases. The time derivative of the correlation volume multiplying $(\cos^4 \theta + \cos^2 \theta)/2$ in Ribner's model will reduce to the time derivative of the correlation volume

$$\left(\frac{dU}{dy_2}\right)^2 \left(\frac{\partial^4}{\partial \tau^4} \int \xi_2^2 R_{11}\, d\vec{\xi}\right)_{\tau=0}$$

obtained herein if the mean shear dU/dy_2 is assumed to be constant over this volume. Thus, the two shear-noise terms differ only by a small numerical factor.

Ribner (ref. 5) attempted to carry his model further and estimated the ratio of the maximum shear noise to the self noise by making an assumption about the scalar F appearing in the turbulence correlation R_{ij}. Thus, (in the present notation) he assumed

$$F = \overline{w_1^2} e^{-\pi^2 \xi^2 / L^2} g(\tau)$$

where L is a longitudinal correlation length and $g(\tau)$ is a function of the time delay. With this assumption he shows that the ratio of the maximum shear noise to the self noise is proportional to

$$\frac{\left[\dfrac{\partial^4 g(\tau)}{\partial \tau^4}\right]_{\tau=0}}{\left[\dfrac{\partial^4 g^2(\tau)}{\partial \tau^4}\right]_{\tau=0}} \tag{61}$$

And by assuming $g(\tau) = e^{-\omega_f^2 \tau^2}$, he determines this ratio to be equal to 1/4. However, it is easy to show that upon choosing $g(\tau)$ to be a decaying exponential times a low-degree polynomial, the coefficients of the polynomial can be adjusted to make the ratio (61) equal to zero or to any other value. Hence, it is not possible to use Ribner's argument with any confidence to estimate the ratio of the shear noise to the self noise. However, if it is assumed that the maximum shear noise is less than or equal to the self noise, the "basic" directivity associated with the noise sources will be small compared with the directivity due to the convection factor $\left(1 - M_c \cos \theta\right)^{-3}$. It will be seen subsequently that this assumption leads to excellent agreement with the experimental data.

COMPARISON WITH EXPERIMENT

In this section we compare the predictions of the present theory with experimentally observed results on jet noise. The most striking comparison occurs when the theory is compared with the observed directivity data.

The directivity patterns of the far-field intensity of noise from round jets have been measured by a number of investigators. We shall compare the present theoretical directivity patterns with those found in the most recent experiments. In order to make this comparison, it is necessary to know the relation between the convection Mach number of the eddies M_c and the jet Mach number M_J. Measurements of these Mach numbers are presented in references 15 and 26 to 28. There is general agreement among the various investigators. Their results show that the convection velocity is fairly constant across the mixing region and that in the center of the mixing region, where the most intense turbulence occurs, the relation $M_c = 0.63\ M_J$ holds. (As indicated previously, most of the turbulent energy lies in a narrow region centered along the center of the mixing region.) We shall use this relation in comparing the theoretical results with experiment.

As explained in the preceding section we shall neglect the effect of the shear noise on the directivity pattern, and therefore this pattern will be completely determined by the convection factor $\left(1 - M_c \cos \theta\right)^{-3}$. This quantity is compared in figures 1 to 4 with the recent experimental results of Olsen[4] (ref. 29), Lush (ref. 11), and Krishnappa and Csanady (ref. 30) and with the older data of Howes (ref. 12). In all cases the level of the theoretical directivity curve is determined by putting it through the experimental data at 90° to the jet axis, where the convection effect is zero. The curve corresponding to the

[4]These data are an improved version of the data in ref. 29 and will be presented as an AIAA paper.

present theory is shown as a solid line. For comparison purposes, the directivity factor predicted by Lighthill's theory is shown as a dashed line. In order to be as fair as possible, the effects on the directivity of the shear noise are also neglected in the representation of Lighthill's theory. Including the shear noise would make the disagreement between the predictions of Lighthill's theory and the data greater in all cases shown. It can be seen from the figures that the present theory always agrees well with the data, and at least as well as Lighthill's theory at all jet Mach numbers. The agreement between Lighthill's theory and the data is poor at the higher Mach numbers. Both theories and the data tend toward better agreement as the jet Mach number is reduced, primarily because the convection factor tends to unity as M_c approaches 0.

CONCLUDING REMARKS

A theory of aerodynamic noise has been developed which differs from Lighthill's theory primarily in the way the convection of the noise sources is treated.

For high subsonic convection velocities, the present theory provides much better agreement with directivity measurements than does Lighthill's theory. For moderate and low subsonic velocities, both theories and the experiments tend toward agreement, as expected.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 3, 1972,
501-04.

APPENDIX A

EVALUATION OF $\int \frac{\partial}{\partial \xi_r^{(1)}} \frac{1}{|\vec{\xi} - \vec{\xi}^{(1)}|} d\vec{\xi}$

In order to evaluate this integral, notice that

$$\int \frac{1}{|\vec{\xi}^{(1)} - \vec{\xi}|} d\Gamma = \begin{cases} \dfrac{4\pi}{\xi^{(1)}} & \text{for } \xi^{(1)} > \xi \\[2ex] \dfrac{4\pi}{\xi} & \text{for } \xi^{(1)} < \xi \end{cases}$$

where $d\Gamma$ is the element of solid angle centered about $\xi = 0$ and $\xi = |\vec{\xi}|$, $\xi^{(1)} = |\vec{\xi}^{(1)}|$. Hence, if S_R is any sphere of radius R about $\vec{\xi} = 0$,

$$\int_{S_R} \frac{1}{|\vec{\xi}^{(1)} - \vec{\xi}|} d\vec{\xi} = 4\pi \left(\frac{R^2}{2} - \frac{{\xi^{(1)}}^2}{6} \right)$$

and

$$\int \frac{\partial}{\partial \xi^{(1)}} \frac{1}{|\vec{\xi} - \vec{\xi}^{(1)}|} d\vec{\xi} = - \frac{4\pi}{3} \xi_r^{(1)}. \tag{A1}$$

APPENDIX B

SYMBOLS

A	function in isotropic tensor (eq. (53))
$\vec{A}$	vector potential
B	function in isotropic tensor (eq. (53))
C	function in isotropic tensor
C_p	specific heat at constant pressure
c	speed of sound
c_o	average sound speed
D	jet nozzle diameter, function in isotropic tensor
D/Dt	$\partial/\partial t + \vec{V} \cdot \nabla$
E	function in isotropic tensor
$\vec{\vec{e}}$	viscous stress tensor
F	arbitrary function of $\vec{y}$, r, and t; function of ξ, defined in eq. (54))
f	arbitrary function
$g(\tau)$	time dependence of correlation tensor
$I(\vec{x})$	far-field intensity
$I(\vec{x}\|\vec{y})$	intensity at $\vec{x}$ due to unit volume of turbulence at $\vec{y}$
$I_\omega(\vec{x})$	spectral density of far-field intensity
$I_\omega(\vec{x}\|\vec{y})$	spectral density of far-field intensity of sound emitted from point $\vec{y}$
$\hat{i}$	unit vector in x_1-direction
L	correlation length
l	eddy correlation length
M_c	convection Mach number
M_J	jet-exit Mach number
P	Fourier transform of $(p - p_o)/\rho_o c_o^2$
p	pressure
p_o	average pressure in far field
$Q_{m,kl}$; Q_{mn}	moving-axis, vector-potential correlation tensors

R	gas constant; radius of sphere about $\vec{\xi} = 0$
$R_{ij}(\vec{y}, \vec{\xi}, \tau)$, $R_{i,jk}(\vec{y}, \vec{\xi}, \tau)$	moving-axis velocity correlation tensors
$R_{imkn}(\vec{y}, \vec{\xi}, \tau)$	moving-axis, turbulence velocity correlation tensor
$\mathcal{Re}$	real part
$\mathcal{R}_{imkn}(\vec{y}, \vec{\eta}, \tau)$	fixed-axis turbulent correlation tensor
r	distance from source point to field point, $\lvert\vec{x} - \vec{y}\rvert$
r_i	component of r; $x_i - y_i$
S	entropy
T	large time duration
T_{ij}	Fourier transform of τ_{ij}
t	time
$U, \vec{U}$	mean velocity
$U_c, \vec{U}_c$	convection velocity
$\vec{V}, V_i$	total fluid velocity
$\vec{V}'$	fluctuating part of velocity
$\vec{v}, v_i$	acoustic part of fluctuating velocity
$\vec{W}, W_i$	incompressible velocity
$\vec{w}$	turbulent part of fluctuating velocity
$\vec{x}, x_i$	Cartesian coordinates; location of observation point
$\vec{y}, \vec{y}', \vec{y}'', \vec{y}''', \vec{z}$	location of source point
Γ	$(\partial/\partial t + U\, \partial/\partial x_1)\Pi$
γ	ratio of specific heats
δ_{ij}	Kronecker delta
$\delta/\delta y_i$	partial derivative with r fixed
ϵ_{ijk}	permutation tensor
$\vec{\eta}$	$\vec{z} - \vec{y}$
$\vec{\eta}^{(1)}$	$\vec{z} - \vec{y}''$
$\vec{\eta}^{(2)}$	$\vec{y}''' - \vec{y}''$
η_i	component of $\vec{\eta}$

Θ temperature

θ angle between mean flow direction and observation direction

$\vec{\xi}$ $\vec{\eta} - \vec{U}_c \tau$

$\vec{\xi}^{(1)}$ $\vec{\eta}^{(1)} - \vec{U}_c(\vec{y})\tau$

$\vec{\xi}^{(2)}$ $\vec{\eta}^{(2)} - \vec{U}_c(\vec{y})\tau$

ξ_i component of $\vec{\xi}$

Π $\gamma^{-1} \ln (p/p_o)$

ρ density

ρ' fluctuating part of density

ρ_o average density

τ time delay

τ_f characteristic decay time of moving-axis correlation

τ_{ij} turbulent stress tensor

φ azimuthal angle

Ω $\omega(1 - M_c \cos \theta)$

ω circular frequency

ω_f characteristic frequency of second-order correlation

Superscripts:

' quantity evaluated at $\vec{y}'$ and $t + \tau$

'' quantity evaluated at $\vec{y}''$ and t

''' quantity evaluated at $\vec{y}'''$ and $t + \tau$

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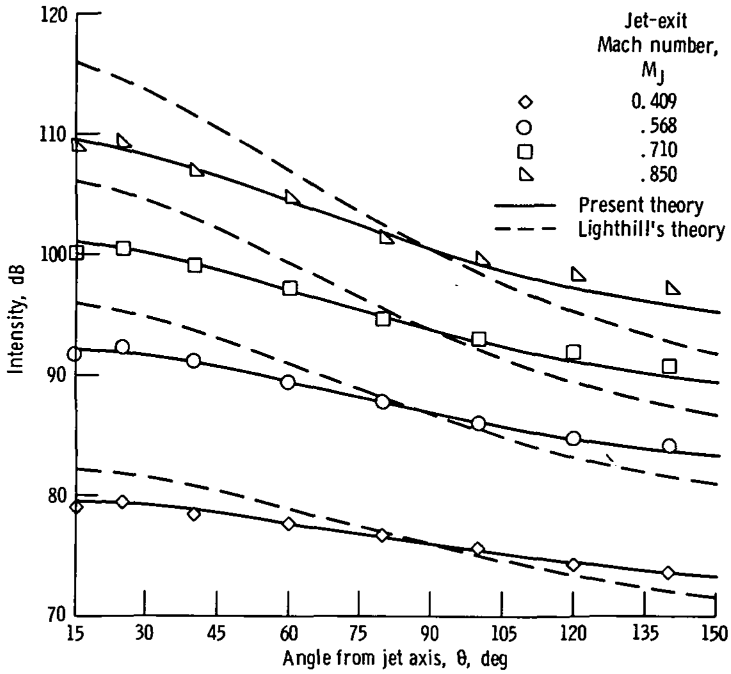


Figure 1. - Experimental directivity data of reference 29. Jet nozzle diameter, 5.08 centimeters (2 in.).

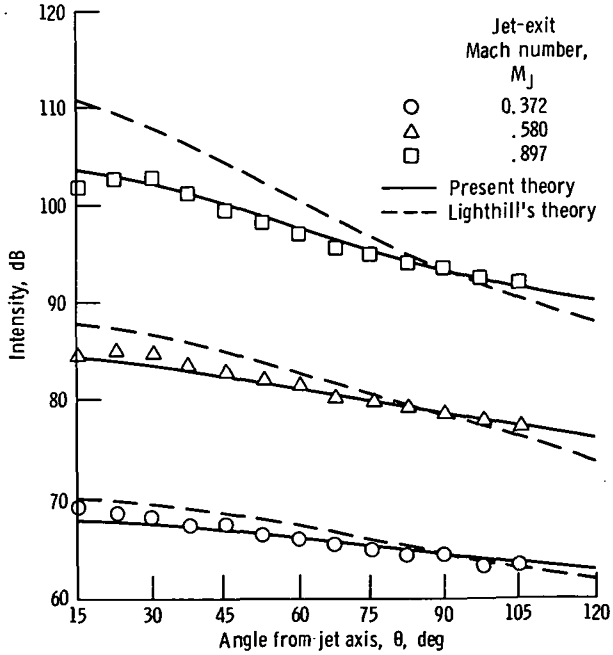


Figure 2. - Experimental directivity data of reference 11. Jet nozzle diameter, 2.54 centimeters (1 in.).

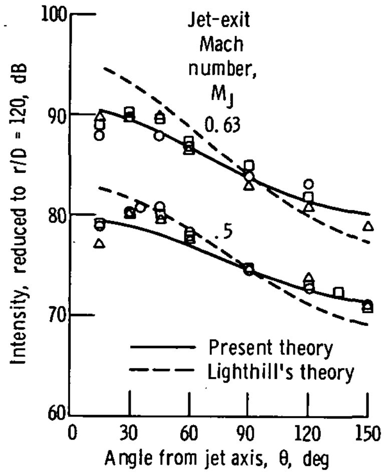
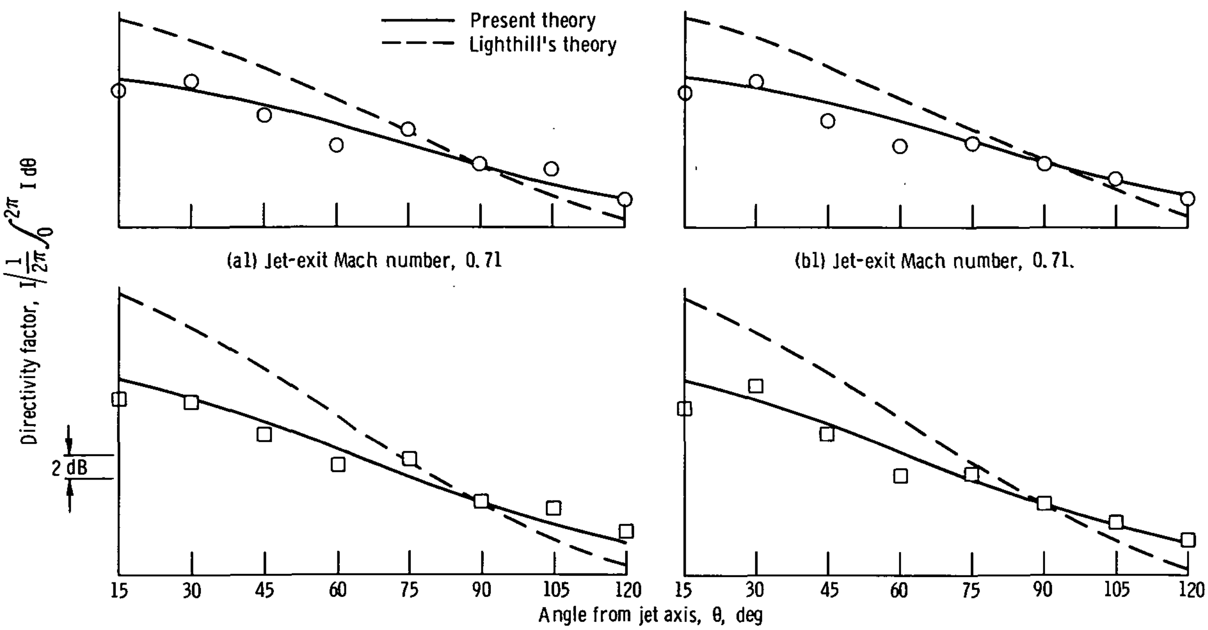


Figure 3. - Experimental directivity data of reference 30. Jet nozzle diameter, 2.54 centimeters (1 in.).



(a1) Jet-exit Mach number, 0.71

(b1) Jet-exit Mach number, 0.71.

(a2) Jet-exit Mach number, 0.90.

(b2) Jet-exit Mach number, 0.90

(a) Jet nozzle diameter, 12.7 centimeters (5 in.).

(b) Jet nozzle diameter, 7.62 centimeters (3 in.).

Figure 4. - Experimental directivity data of reference 12.

NASA-Langley, 1973 —— 23 E-7047

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