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THEORY OF ZONE
RADIOMETRY

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THEORY OF ZONE RADIOMETRY

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FOREWORD

This document is one of two documents that constitute the final report for Contract NAS8-28089, "Study of Viscous Mixing Plume Flow Field." This study was performed by the Lockheed-Huntsville Research & Engineering Center, Inc., for the National Aeronautics & Space Administration, George C. Marshall Space Flight Center, Alabama.

The other document is "Mathematical Representations of Turbulent Mixing," LMSC-HREC D306102, January 1973.

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Section 1
INTRODUCTION AND SUMMARY

A spectroscopic instrumentation system was developed by Rocketdyne Division of North American Rockwell which was used to measure temperature and concentration distributions in (hopefully) axisymmetric and two-dimensional combusting flows (Ref. 1). This measurement technique has become known as zone radiometry.

The success of the method depends both on how accurately the detected radiation can be converted by analysis into the desired temperatures and concentrations and on how closely the flow meets the dimensional limitations of this measurement scheme. Since the technology of radiative transfer was being very actively researched during the same time that the zone radiometry experiments were being performed, a critique of the Rocketdyne data redcution procedures is in order to determine whether or not application of the present state-of-the-art radiation analyses can yield more accurate flowfield information.

Theoretically, a temperature and a partial pressure distribution for a given species can be determined from a set of measurements made at one particular spectral level. If sets of measurements at more than one spectral level are made, partial pressures of several species and some average temperature can be determined. Practically, such multiple determinations have not yet been possible. At best, one set of measurements has been used to establish temperature and the partial pressure of one species, and another set used to determine the partial pressure of a second species. Therefore, an analysis at one particular wavelength is all that is required of available zone radiometry data. Such an analysis is described in this report.

The final goal of this report is to present a recommended data reduction scheme for the zone radiometry system. The limitations in this scheme will be clearly stated and quantitatively evaluated when possible.

To appreciate the utility of zone radiometry methods, one should realize that the technique was developed and used extensively to measure "axisymmetric" rocket motor plumes. All propellant systems cannot be measured with satisfactory accuracy by this method. Plumes with carbon particles may become too "optically thick" for the transmission part of the measurement to be made, and plumes with only water vapor as the optically active species may be too "thin" for accurate measurements. Furthermore, all motors lack axial symmetry to some degree; no good measure of this feature has yet been devised.

Despite the fact that much experimental data are available, no definitive comparison with calculated flows exists. Experiments with carefully designed burners are the primary source of radiation property data, but when a real rocket motor is studied both analysis and experiment become more difficult.

Zone radiometry has also been used on two-dimensional mixing studies. These studies have not been as extensive as those on motors. Again definitive comparisons with calculations have not been made, nor have error analysis for reducing data been previously reported.

This report serves as a prelude to a more complete data comparison study which will be forthcoming. A detailed treatment of the radiation analysis and synopsis of the zone radiometry method is reported herein, so that questions regarding the accuracy of reported data and experiments can be determined.

Section 2
RADIATION ANALYSIS

The zone radiometry system is an instrument used to determine temperature and composition of optically active species in either an axisymmetric or planar gas flow field. The radiative energy transfer analysis which must be used to relate the radiation measurements to the flow properties is described in this section.

The basic radiative exchange process is sketched in Fig. 1. Radiation from the hot zones is seen by the detector at all times. When the chopper is open, radiation from the source which is even hotter than the zones plus radiation from the hot zones is seen. Particular frequency (or wave length) intervals are measured with the detector by selectively filtering away the unwanted radiation. The radiative exchange process is essentially one-dimensional, as the angle β is quite small; hence, one line of sight is viewed. The dimensions of the gases on the immediate sides of β are assumed to be such that radiative equilibrium with the adjacent, lateral-gas zones is maintained, so there is no net radiative exchange in the lateral direction.

To quantitatively describe radiative exchange, the concept of intensity must be used. At a point, P, consider the monochromatic intensity,

$$I_\nu = \underset{\substack{d\sigma, d\Omega, d\nu \\ dt \longrightarrow 0}}{\text{Limit}} \left(\frac{dE_\nu}{d\sigma \cos\theta \, d\Omega \, d\nu \, dt} \right) \tag{2.1}$$

The term E_ν represents the radiant energy in $(\nu, \nu + d\nu)$, where ν is the frequency of the radiation. The term t is time; the geometric factors σ, θ, Ω are defined by Fig. 2. The fact that the above limit exists is an

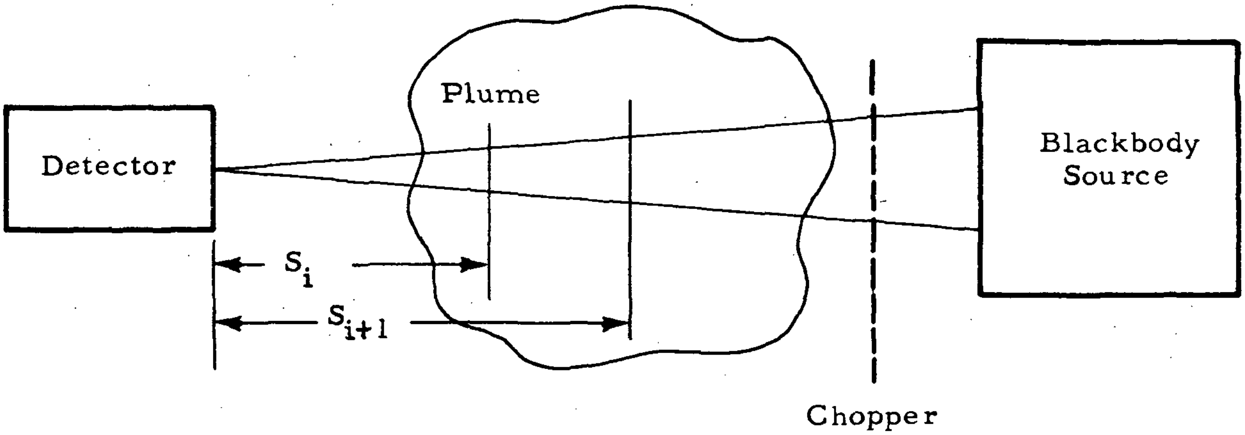
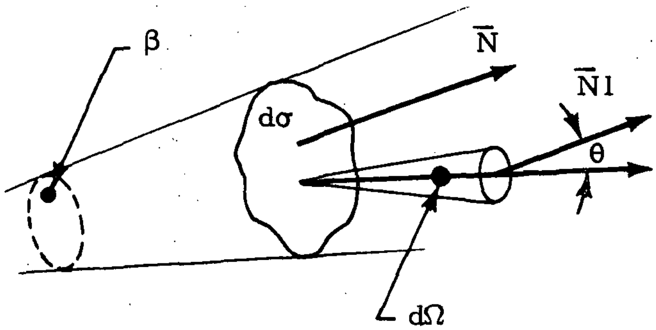


Fig. 1 - Zone Radiometer System



dσ = incremental surface

$\overline{N}$ = surface normal

$\overline{N}1$ = parallel to surface normal

dΩ = increment of solid angle

θ = angle between solid angle direction and surface normal

β = solid angle of all dΩ over dσ

(dσ is located at point R)

Fig. 2 - Definition of Geometric Terms Used to Define Intensity

experimentally observed fact (Milne, Ref. 2, p. 84). Radiation is emitted from each point on $d\sigma$; therefore, an integration in Ω is required to calculate the radiation flux through $d\sigma$. In optics, this is not the case as intensity is defined at a point with $d\sigma$ missing in the limit expression.

I_ν is independent of S unless it is modified by the transmitting medium, whereas E_ν is not. There are other intensities which could have been defined; they are I_ω and I_λ. These are defined on the basis of a unit of wave number, ω, or wave length, λ, in the limiting expression. If the transporting medium has a unit or known index of refraction, conversions between these intensities can be easily made. I_ν is somewhat more basic because it is independent of the index of refraction. However, the overriding criteria to use in selecting the intensity to use is the availability of property data. These data are available in select wave number increments; therefore, I_ω will be used.

Radiation in the absence of emission is attenuated according to

$$I_\omega \{S_i\} = I_\omega \{S_{i+1}\} \exp\left(-\int_{S_i}^{S_{i+1}} K_\omega \rho \, dS\right) \tag{2.2}$$

where here and henceforth brackets indicate functionality and where K_ω is mass absorption coefficient and ρ is the density of the absorbing medium. If K_ω and ρ are independent of S this relationship is called the Beer-Lambert law. In general, optical thickness = $\int_{S_1}^{S_2} K_\omega \rho \, dS$. A spectral absorption coefficient may be defined by the equation shown on the following page.

$$\alpha_\omega = \frac{I_\omega(S_{i+1}) - I_\omega(S_i)}{I_\omega(S_{i+1})} = \frac{I_{\omega\,(\text{absorbed})}}{I_{\omega\,(\text{incident})}} \tag{2.3}$$

In general, radiation may be absorbed, reflected or transmitted; or, fractionally,

$$\alpha_\omega + \rho_\omega + \tau_\omega = 1 \tag{2.4}$$

Thus

$$\tau_\omega = 1 - \alpha_\omega = \cancel{1} - \cancel{1} + \frac{I_\omega(S_i)}{I_\omega(S_{i+1})}\,, \quad \text{if } \rho_\omega = 0 \tag{2.5}$$

or

$$\tau_\omega = \exp\left(\int - K_\omega \rho \, dS\right) \tag{2.6}$$

The two absorption coefficients are related by:

$$\alpha_\omega = 1 - \exp\left(\int - K_\omega \rho \, dS\right) \tag{2.7}$$

Elements along the solid angle will not only absorb radiation but will emit at a rate of

$$\frac{dE_\omega}{dt} = J_\omega \, (\rho \, d\sigma \, dS) \; d\omega \; d\Omega \tag{2.8}$$

where J_ω is the emission coefficient. J_ω will be isotropic.

The geometry is such that the detector is normal to the view angle through the plume; hence, cos $\theta = 1$.

2.1 THE EQUATION OF TRANSFER

Now a radiation heat balance on a control volume consisting of the solid angle β between S_i and S_{i+1} can be made. Consider three cross sections of β, those at S_i, S_{i+1}, $S_{i+1/2}$; call them A_i, A_{i+1}, $A_{i+1/2}$. Since we wish to calculate the radiation to the detector, let the radiation at S_{i+1} be I_ω and that at S_i be $I_\omega + d\,I_\omega$, i.e. I_ω is positive in the negative S direction. The heat balance becomes:

$$\int_\Omega I_\omega \, d\Omega \; A_i \, dt - \int_\Omega \left(I_\omega + dI_\omega\right) d\Omega \; A_{i+1} \, dt =$$
$$- \int_\Omega K_\omega \, \rho \left(S_{i+1} - S_i\right) I_\omega \, d\Omega \, dt \, A_{i+1/2}$$
$$+ \int_\Omega J_\omega \, \rho \, A_{i+1/2} \left(S_{i+1} - S_i\right) d\Omega \, dt \qquad (2.9)$$

The limits on the Ω integration are over the solid angle β; all variables are constant with respect to this integration. The integration converts intensity to flux (Milne, Ref. 2, p. 85). Since β is small and $S_i \longrightarrow S_{i+1}$, Eq. (2.9) becomes,

$$I_\omega A_i \, dt \int_o^\beta d\Omega - \left(I_\omega + dI_\omega\right) A_{i+1} \, dt \int_o^\beta d\Omega =$$
$$- K_\omega \, \rho \, dS \, I_\omega \, dt \, A_{i+1/2} \int_o^\beta d\Omega$$
$$+ J_\omega \, \rho \, A_{i+1/2} \, dS \, dt \int_o^\beta d\Omega \qquad (2.10)$$

Let

$$A_{i+1/2} = A_i + \frac{dA}{2} = A_{i+1} - \frac{dA}{2} \tag{2.11}$$

$$I_\omega \left(A_{i+1/2} - \frac{dA}{2}\right) dt\,\beta - \left(I_\omega + dI_\omega\right)\left(A_{i+1/2} + \frac{dA}{2}\right) dt\,\beta$$

$$= - K_\omega\, \rho\, dS\, I_\omega\, dt\; A_{i+1/2}\, \beta$$

$$+ J_\omega \rho\, A_{i+1/2}\, dS\; dt\, \beta \tag{2.12}$$

Dividing by $\left(A_{i+1/2}\right)$ dt β dS and neglecting products of differentials.

$$- \frac{dI_\omega}{dS} = - K_\omega \rho\, I_\omega + \rho\, J_\omega \tag{2.13}$$

Assume the flow to be in local thermodynamic equilibrium, such that Kirchoff's theory can be used to give

$$J_\omega = K_\omega\, I_{\omega b} \tag{2.14}$$

where $I_{\omega b}$ is Planck's blackbody intensity

$$I_{\omega b} = \frac{2\, C^2 \hbar \omega^3}{\left[\exp\left(\frac{\hbar\, C\omega}{k t}\right) - 1\right]} \tag{2.15}$$

Similar intensities based on other measures of spectral interval may also be defined. For convenience, several of these are tabulated in Table 1. Let

$$dx = \rho\, dS \tag{2.16}$$

Table 1

RADIATION RELATIONSHIPS WITH RESPECT TO FREQUENCY, WAVELENGTH AND WAVE NUMBER

I. INTENSITIES (SIEGEL AND HOWELL, REF. 3, PP. 20 AND 31 – FOR THE INDEX OF REFRACTION EQUAL TO ONE)

$$I_{\nu b} = \frac{2 \hbar \nu^3}{C^2 \left(\left[\exp \left(\frac{\hbar \nu}{k T} \right) \right] - 1 \right)} = \frac{N_{\nu b}}{\pi}$$

$$I_{\lambda b} = \frac{2 \hbar C^2}{\lambda^5 \left(\left[\exp \left(\frac{\hbar C}{k \lambda T} \right) \right] - 1 \right)} = \frac{N_{\lambda b}}{\pi}$$

$$I_{\omega b} = \frac{2 h C^2 \omega^3}{\left(\left[\exp \left(\frac{\hbar C \omega}{k T} \right) \right] - 1 \right)} = \frac{N_{\omega b}}{\pi}$$

where ν is frequency in (time^{-1}), λ is wavelength in (length), and ω is wave number in (length^{-1}). C is the speed of light in a vacuum, $\hbar$ is Planck's constant, and k is Boltzmann's constant.

(Continued)

Table 1 - (Continued)

II. ILLUSTRATION OF ν, λ AND ω RELATIONSHIPS (SIEGEL AND HOWELL, REF. 3, p. 20)

For Carbon Dioxide (CO_2)

$\lambda\nu = C$ (Speed of light in a vacuum)

for $\lambda_o = 4.45\mu$ for CO_2

$$\nu_o = \frac{C}{\lambda_o} = 6.736 \times 10^{13}\ \text{sec}^{-1}$$

$$\lambda_o\omega_o = 1$$

$$\omega_o = \frac{1}{\lambda_o} = 2247\ \text{cm}^{-1}$$

for $\underline{d\omega = +25\ \text{cm}^{-1}}$

$$d\lambda = -\frac{1}{\eta_o^2}\, d\omega = -1.98 \times 10^{-7} d\omega$$

$$\underline{d\lambda = -0.465 \times 10^{-5}\mu}$$

$$d\nu = -\frac{\nu_o^2}{C}\, d\lambda = -1.514 \times 10^{-16} d\lambda$$

$$\underline{d\nu = +0.705 \times 10^{+11}\ \text{sec}^{-1}}$$

For Water (H_2O)

$\lambda\nu = C$ (Speed of light in a vacuum)

for $\lambda_o = 2.49\mu$ for H_2O

$$\nu_o = \frac{C}{\lambda_o} = 1.2039 \times 10^{14}\ \text{sec}^{-1}$$

$$\lambda_o\omega_o = 1$$

$$\omega_o = \frac{1}{\lambda_o} = 4016\ \text{cm}^{-1}$$

for $\underline{d\omega = +25\ \text{cm}^{-1}}$

$$d\lambda = -\frac{1}{\omega_o^2}\, d\omega = -6.20 \times 10^{-8} d\omega$$

$$\underline{d\lambda = -1.550 \times 10^{-6}\mu}$$

$$d\nu = -\frac{\nu_o^2}{C}\, d\lambda = -4.834 \times 10^{17} d\lambda$$

$$\underline{d\nu = +0.748 \times 10^{+12}\ \text{sec}^{-1}}$$

Then the energy balance becomes

$$-\left(\frac{d\,I_\omega}{K_\omega\,dx}\right) + I_\omega = I_{\omega b} \tag{2.17}$$

Equation (2.17) is called the <u>equation</u> <u>of</u> <u>transfer</u> and is of fundamental importance in radiative transfer. The derivation given is consistent with Milne (Ref. 2). Other discussions of this equation are given by Viskanta (Ref. 4) and Kourganoff (Ref. 5).

The detector in the zone radiometer system does not indicate I_ω, but rather the product ($\beta\ I_\omega$). Since β is a constant, the signal is proportional to I_ω. β is a definite number, the view angle of the radiometer. The important point is that Eq. (2.17) is valid for any constant value of β. More will be said of these choices in subsequent pages.

To solve the equation of transfer, an integrating factor is introduced so that the two terms on the LHS of Eq. (2.17) may be combined.

$$\frac{d}{K_\omega\,dx}\left[-I_\omega \exp\left(-\int_o^x K_\omega\,dx'\right)\right] = I_{\omega b} \exp\int_o^x - K_\omega\,dx' \tag{2.18}$$

Primes denote dummy variables.

$$I_\omega \exp\left(\int_{x_1}^{x_2} - K_\omega\,dx'\right)\Bigg|_o^x = \int_o^x - I_{\omega b} \exp\left(\int_o^{x'} - K_\omega\,dx''\right) K_\omega dx' \tag{2.19}$$

$$I_\omega\{0\} = I_\omega\{x\} \exp\left(\int_o^x - K_\omega\,dx'\right) + \int_o^x I_{\omega b} \exp\left(\int_o^{x'} - K_\omega\,dx''\right) K_\omega dx' \tag{2.20}$$

This equation affirms that the intensity which arrives at 0 comes from x and is attenuated between x and 0 or from emission plus self-absorption along x to 0, thus the two terms on the RHS of Eq. (2.20).

Note the first term on the RHS of Eq. (2.20) is often omitted with the understanding that x becomes so large that the path becomes "optically thick." This point is discussed in Goody (Ref. 6) p. 456. When this omission is used, the term is recovered by using a boundary condition on the RHS, when the integration is performed, that equals the omitted term. Such a ploy will be used here.

I_ω is a monochromatic radiation intensity. Experimentally, a specific wave number cannot be isolated for study, so a wave number interval is used. An appropriately averaged intensity, called radiance, is obtained by

$$\bar{I}_\omega = \frac{1}{\Delta\omega}\int_{\omega 1}^{\omega 2} I_\omega\{0\}\, d\omega = + \frac{1}{\Delta\omega}\int_{\omega 1}^{\omega 2}\int_{o}^{x} I_{\omega b}\left[\exp\left(-\int_{o}^{x'} K_\omega\, dx''\right)\right] K_\omega\, dx'\, d\omega \qquad (2.21)$$

Recall

$$\tau_\omega = \exp\left(-\int_{o}^{x} K_\omega\, dx'\right) \qquad (2.22)$$

Therefore

$$\frac{d\tau_\omega}{dx} = \left[\exp\left(-\int_{o}^{x} K_\omega\, dx'\right)\right](-K_\omega) \qquad (2.23)$$

$$\bar{I}_\omega = \frac{1}{\Delta\omega}\int_{\omega 1}^{\omega 2}\int_{o}^{x} - I_{\omega b}\left(\frac{d\tau_\omega}{dx}\right) dx'\, d\omega \qquad (2.24)$$

To exchange the order of integration, define

$$\overline{\tau} = \frac{1}{\Delta\omega} \int_{\omega 1}^{\omega 2} \tau_\omega \, d\omega \tag{2.25}$$

and

$$\overline{I}_{\omega b} = \frac{1}{\Delta\omega} \int_{\omega 1}^{\omega 2} I_{\omega b} \, d\omega \tag{2.26}$$

Then

$$\overline{I}_\omega = - \int_{o}^{\overline{\tau}} \overline{I}_{\omega b} \, d\overline{\tau} \tag{2.27}$$

To be consistent $\overline{\tau}$ should have a subscript ω, but this is not convenient.

Golden (Ref. 7) strongly contested the possibility of this inversion of order. Simmons (Ref. 8) presented several other ways of accomplishing the averaging and an alternate derivation.

Equation (2.27) may be approximated with finite-differences as:

$$\overline{I}_\omega = \sum_{i=1}^{N} \overline{I}_{\omega b} \left\{ \overline{\tau}_i \right\} \left[\overline{\tau}_{i-1} - \overline{\tau}_i \right] \tag{2.28}$$

or

$$\overline{I}_\omega = \overline{I}_{\omega b1} \left(1 - \overline{\tau}_1\right) + \overline{I}_{\omega b2} \left(\overline{\tau}_1 - \overline{\tau}_2\right) + \ldots \overline{I}_{\omega bN} \left(\overline{\tau}_{N-1} - \overline{\tau}_N\right) \tag{2.29}$$

since

$$\overline{\alpha}_o = 0, \quad \overline{\tau}_o = 1.$$

It is crucial to the understanding of non-grey, radiation analysis to appreciate that the quantities in Eqs. (2.27), (2.28) and (2.29) are spectrally averaged; however, from this point on the overbars will not be shown because only averaged quantities are of interest.

2.2 THE EVALUATION OF τ AVERAGE

Equation (2.28) represents the intensity which is proportional to that detected in the zone radiometery experiments. The summation can be directly evaluated, but even though the blackbody intensities are well behaved the transmittances are not. This is the reason that Krakow et al. (Ref. 9) developed the Curtis-Godson approximation to properly average the τ's. The development of this approximation is given below.

Two problems must be simultaneously solved. First, an average over a certain wave number interval must be established, because radiative transitions occur in a discontinuous manner even with respect to the narrow acceptance interval of the detecting device. For isothermal, homogenous gases, this wave number averaging has been accomplished with "band models." The Random Band model with constant line widths was chosen for this purpose. It is stated:

$$- \ln \tau = 2\pi \left(\frac{\gamma}{d}\right) f\{X\} \tag{2.30}$$

where $\{\ \}$ denotes functionality and

$$X = \frac{\left(\frac{s}{d}\right) S}{2\pi \left(\frac{\gamma}{d}\right)} \tag{2.31}$$

$\left(\frac{s}{d}\right)$ and $\left(\frac{\gamma}{d}\right)$ are band model parameters and f is the probability distribution for line strengths. S is still distance. Specific values of the band model parameters will be subsequently quoted.

The second problem which must be overcome is to devise a way that the band model representation of the transmittance may be used for a non-homogeneous, nonisothermal path. The Curtis-Godson approximation may be used to calculate an average for such paths. Let

$$\left(\frac{s}{d}\right) S = \sum_h \left(\frac{s}{d}\right)_h S_h \tag{2.32}$$

and

$$\left(\frac{s}{d}\right) S \left(\frac{\gamma}{d}\right) = \sum_h \left(\frac{s}{d}\right)_h S_h \left(\frac{\gamma}{d}\right)_h \tag{2.33}$$

where h represents a zonal increment of constant temperature and composition. By combining Eqs. (2.31) and (2.33)

$$X = \left[\sum_h (s/d)_h S_h\right]^2 \Big/ 2\pi \sum_h (s/d)_h S_h (\gamma/d)_h \tag{2.34}$$

Within each zone

$$X_h^* = (s/d)_h S_h / 2\pi (\gamma/d)_h \tag{2.35}$$

$$-\ln \tau_h^* = 2\pi (\gamma/d)_h f\left\{X_h^*\right\} \tag{2.36}$$

where stars emphasize zonal properties.

Therefore, Eq. (2.30) becomes

$$-\ln \tau = \frac{\left[\sum_h X_h^* \left(-\ln \tau_h^* / f\left\{X_h^*\right\}\right)\right]^2}{\sum_h X_h^* \left(-\ln \tau_h^* / f\left\{X_h^*\right\}\right)} \; f \left\{ \frac{\left[\sum_h X_h^* \left(-\ln \tau_h^* / f\left\{X_h^*\right\}\right)\right]^2}{\sum_h X_h^* \left(-\ln \tau_h^* / f\left\{X_h^*\right\}\right)^2} \right\} \qquad (2.37)$$

Equation (2.37) was derived and substantiated by experiments in Krakow et al., (Ref. 9). Two limits for this expression exist.

If $X \rightarrow 0$, i.e., is less than 0.1,

$$-\ln \tau \cong \sum_h \left(-\ln \tau_h^*\right). \qquad (2.38)$$

If $X \rightarrow \infty$, i.e., is greater than 3,

$$-\ln \tau \cong \left[\sum_h \left(-\ln \tau_h^*\right)^2\right]^{1/2}. \qquad (2.39)$$

Now the equations developed in the two previous paragraphs may be used, if appropriate band model data are available. The General Dynamics experiments (Refs. 10, 11 and 12) provide such data. An exponential probability distribution was used and values of (s/d) and (γ/d) were determined.

$$f\{X\} = X \left[1 + \pi X/2\right]^{-1/2} \qquad (2.40)$$

$$-\ln\tau = \frac{(s/d)\,S}{\left[1 + \frac{(s/d)}{4}\,\frac{S}{(\gamma/d)}\right]^{1/2}} \tag{2.41}$$

Furthermore, let $(s/d) = k$ and $(\gamma/d) = a$

$$-\ln\tau = \frac{kS}{\left[1 + \frac{kS}{4a}\right]^{1/2}} = \frac{\sum_h k_h S_h}{\left[1 + \frac{\left(\sum_h k_h S_h\right)^2}{4 \sum_h k_h S_h a_h}\right]^{1/2}} \tag{2.42}$$

This is the same relationship that is used by Reardon and Huffaker (Ref. 13) to calculate radiation from a line of sight. For a single isothermal, constant-composition zone:

$$-\ln\tau^* = \frac{k S^*}{\left[1 + \frac{kS^*}{4a}\right]^{1/2}} \tag{2.43}$$

$$k = \left(\frac{k_o}{P_o}\right)\left(\frac{273}{T\,^oK}\right) P_i \tag{2.44}$$

P_o = reference state of 1 atmosphere

k_o is in (cm^{-1}) and is tabulated for H_2O, CO and CO_2 in the General Dynamics reports (Refs. 10, 11 and 12). The term P_i represents the partial pressure of the radiating species in atmospheres.

Unless the pressure is much lower than one atmosphere, Doppler broadening is negligible with respect to collision broadening. Assuming such a case, "a" can be calculated from the tabulated data in Ref. 12, pp. 22-23 or from Reardon and Huffaker (Ref. 13) pp. 141-144.

For continuous radiators, grey gases (throughout the spectral range of interest), "a" $\longrightarrow \infty$ and

$$-\ln \tau^* = k\, S^* \tag{2.45}$$

This correctly implies that the optically thin limit Eq. (2.38), can be used to calculate integrated values of τ. Thus non-grey gases, which are optically thin because of geometry and density distributions in addition to grey gases obey Eq. (2.38).

In general, "a" is the fine structure parameter which is the ratio of the line width, γ, to line spacing, d. Width of a radiating line is broadened by collisions between the atoms and molecules of the gas. The general form of the line width with collisional broadening terms included according to Reardon and Huffaker (Ref. 13) and Reardon et al. (Ref. 14) is

$$\gamma_{c_i} = \sum_j \left(\gamma_{i,j}\right)_{\substack{\text{at} \\ T=273^\circ K}} P_j \left(\frac{273}{T}\right)^{n_{i,j}} + \left(\gamma_{i,i}\right)_{\substack{\text{at} \\ T=273^\circ K}} P_i \left(\frac{273}{T}\right)^{n_{i,i}} \tag{2.46}$$

The term i is the species being considered, P_j are the species partial pressures in atmospheres. The exponent values of $n_{i,j} = 1/2$ and $n_{i,i} = 1$ were recommended by General Dynamics/Convair (Ref. 12). The summation, j, runs through the number of species in the gas. A representative set of the constants needed to calculate the line width with collisional broadening for, in this example, the water molecule is presented in Table 2.

Table 2

LINE WIDTH WITH COLLISIONAL BROADENING FOR WATER

$$\gamma_{c_i} = \sum_{j}^{N} \left(\gamma_{i,j}\right)_{\substack{at\\T=273^{\circ}K}} P_j \left(\frac{273}{T}\right)^{n_{i,j}} + \left(\gamma_{i,i}\right)_{\substack{at\\T=273^{\circ}K}} P_i \left(\frac{273}{T}\right)^{n_{i,i}}$$

$n_{i,j} = 1/2$ and $n_{i,i} = 1$

for $i = H_2O$		for j ↓	γ_{ij} becomes ↓
	(nonresonating)	$j = H_2O$	$\gamma_{ij} = 0.09$
		$= N_2$	$= 0.09$
		$= O_2$	$= 0.04$
		$= H_2$	$= 0.05$
		$= CO_2$	$= 0.12$
		$= CO$	$= 0.10$
	$\gamma_{i,i}$ for water (resonating)		$= 0.44$

The evaluation of the $1/d$ value to complete the calculation of a, is defined by Reardon and Huffaker (Ref. 13) as

$$\frac{1}{d} = \frac{a^o}{\gamma^o} \tag{2.47}$$

where

$$a^o = 10^{(b_\nu + c_\nu T^2)} \tag{2.48}$$

The b_ν and c_ν are constant over spectral regions and for water are listed as a function of temperature in Ref. 13.

While

$$\gamma^o = \left[0.44 \left(\frac{273}{T}\right) + 0.09 \left(\frac{273}{T}\right)^{1/2}\right] C^o + 0.044 \left(\frac{273}{T}\right)^{1/2} (1 - C^o) \tag{2.49}$$

where

$$C^o = -0.1002 + 0.2802 \times 10^{-3}\, T - 0.1089 \times 10^{-6}\, T^2 + 0.0291 \times 10^{-9} T^3 \tag{2.50}$$

Values of $1/d$ shall be tabulated as a function of λ and T in Ref. 14. These tabulated values of $1/d$ provide an alternate method of obtaining the fine structure parameter, a.

If more than one species is optically active in a given spectral interval, Eq. (2.42) is modified and used thusly:

$$-\ell n(\tau_{MS}) = \sum_i \left(\frac{\sum_h k_h S_h}{\left[1 + \frac{\sum_h k_h S_h{}^2}{4\sum_h k_h S_h a_h}\right]^{1/2}} \right)_i \qquad (2.51)$$

where the i summation is on all active species. Remember $\left[-\ell n(\tau_{MS})\right]$ is called optical depth and the MS indicates multi-species. The type of summation indicated in Eq. (2.51) is not obvious, but is what is used.

The origin of the relationships necessary to determine temperature and concentrantions from zone radiometry experiments has now been established. These relationships will now be applied to the specific experiments which have been performed by Rocketdyne.

Section 3
DATA REDUCTION PROCEDURE

The solution to the equation of transfer – Eq. (2.29), the Curtis-Godson approximation as stated in Eq. (2.37), and specified band model parameters may now be used to reduce zone radiometry data. Brewer (Ref. 16) describes a computer program to perform such a calculation; unfortunately, he uses a distribution function, f, which is not compatible with the reported General Dynamics/Convair k's and a's. This may not introduce a significant error, but it prevents one from using the reported program directly.

Rocketdyne chose not to reduce their data in this manner. They approximated Eq. (2.37) with its grey gas limit Eq. (2.38) and then reduced the radiometry data, arguing that since $a \longrightarrow \infty$, $X \longrightarrow 0$ for CO_2 and that experiments with water first were optically thin and second that they used water vapor radiation data which were taken with the same resolution as their spectrometer (Ref. 15). The first two of these arguments may well be valid, and their validity can be determined with analysis. The third is highly improbable because not only isothermal property data must be available (which may be) but data for the same temperature and compositions as those in the measured plumes must be also. However, if the first two arguments are valid, the third is unnecessary. Herget (Ref. 15) contends because of these arguments that his studies are not limited to optically thin cases. Let us reserve judgment on this contention until some subsequent calculations and experimental observations are made.

Specifically, Eq. (2.38) may be written as

$$\tau_k = (\tau_1^*)(\tau_2^*) \ldots (\tau_h^*) \qquad (3.1)$$

where the subscript k represents the k^{th} row of zones and h represents the number of zones. Eq. (2.29) may be multiplied by π, so that $N_{\omega bi}$'s appear on the RHS where N_{ω_b} is emissive power. In fact, a summation of any of the terms listed in Table 1 could have been used. Actually neither intensity nor emissive power is measured but the radiation from the solid angle β which intersects the detector surface. This angle is not measured, but the radiation from an internal blackbody source along β is. Since any of the terms in Table 1 may be calculated from the internal blackbody temperature, the LHS of Eq. (2.29) is simply calibrated. Eq. (2.29) is used as stated below.

$$
\begin{aligned}
I_\lambda &= I_{\lambda b1}(1 - \tau_1^*) + I_{\lambda b2}(\tau_1^* - \tau_1^* \tau_2^*) + \dots \\
&= I_{\lambda b1}(\epsilon_1^*) + I_{\lambda b2}(\tau_1^* [1 - \tau_2^*]) + \dots \\
&= I_{\lambda b1}(\epsilon_1^*) + I_{\lambda b2}(\tau_1^* \epsilon_2^*) + I_{\lambda b3} \underbrace{(\tau_1^* \tau_2^* - \tau_1^* \tau_2^* \tau_3^*)}_{\underbrace{\tau_1^* \tau_2^*}_{\tau_2} \epsilon_3^*} \qquad (3.2) \\
&= \sum_i I_{\lambda bi}\, \epsilon_i^*\, \tau_{i-1}
\end{aligned}
$$

ϵ^* is defined as $1 - \tau^*$, and τ^* is defined by Eq. (2.43). Since I_λ is an averaged intensity over some small spectral interval and is measurable, it will be called <u>radiance.</u>

Before considering the Rocketdyne experiments in more detail, consider the following definition of a new τ^*, namely,

$$
-\ln(\tau_h^{**}) = \frac{k_h S_h}{\left[1 + \dfrac{\left(\sum_{n=1}^{h} k_n S_n\right)^2}{4 \sum_{n=1}^{h} k_n S_n a_n}\right]^{1/2}} \qquad (3.3)
$$

This τ_h^{**} has the property that if it is used in Eqs. (3.1) and (3.2), Eq. (2.42) will result. This means that the assumption on optical depth which was used by Rocketdyne will be removed. Other means could be used to eliminate this assumption, but, as will be confirmed in subsequent discussion, this means would require the least amount of revision to the existing Rocketdyne data reduction program. An additional benefit is that a convenient check of the deviation from optical thinness can also be made with this parameter.

In summary, the following equation must be solved, either exactly or approximately, to reduce zone radiometry data for one component and wavelength. Functionality is emphasized for clarity.

$$I_\lambda \{\lambda, \Delta\omega\} = \sum_i I_{\lambda b} \{\lambda, T_i\} \left[\tau_{i-1} \{\lambda, \Delta\omega, T_{i-1}\} - \tau_i \{\lambda, \Delta\omega, T_i\}\right] \tag{3.4}$$

Now the specific geometry of the zone radiometry experiments can be considered.

3.1 ONE-DIMENSIONAL ISOTHERMAL FLOWS

The one-dimensional test situation (Fig. 3) such as the Rocketdyne Composite Engine Study (Ref. 17) will be used to demonstrate the procedure used to convert measured values of emissive power into temperature and composition values. Recalling Fig. 1, the basic geometry of this one-dimensional flow contains all of the features shown except that there is a single zone. Two radiance readings are made using the zone radiometer. One radiance reading is made with the chopper closed giving the radiance of only the zone while the other reading made with the chopper open provides a radiance value containing the zone radiation and the grey body source radiation. Using the finite difference form of Eq. (3.2) with these measured

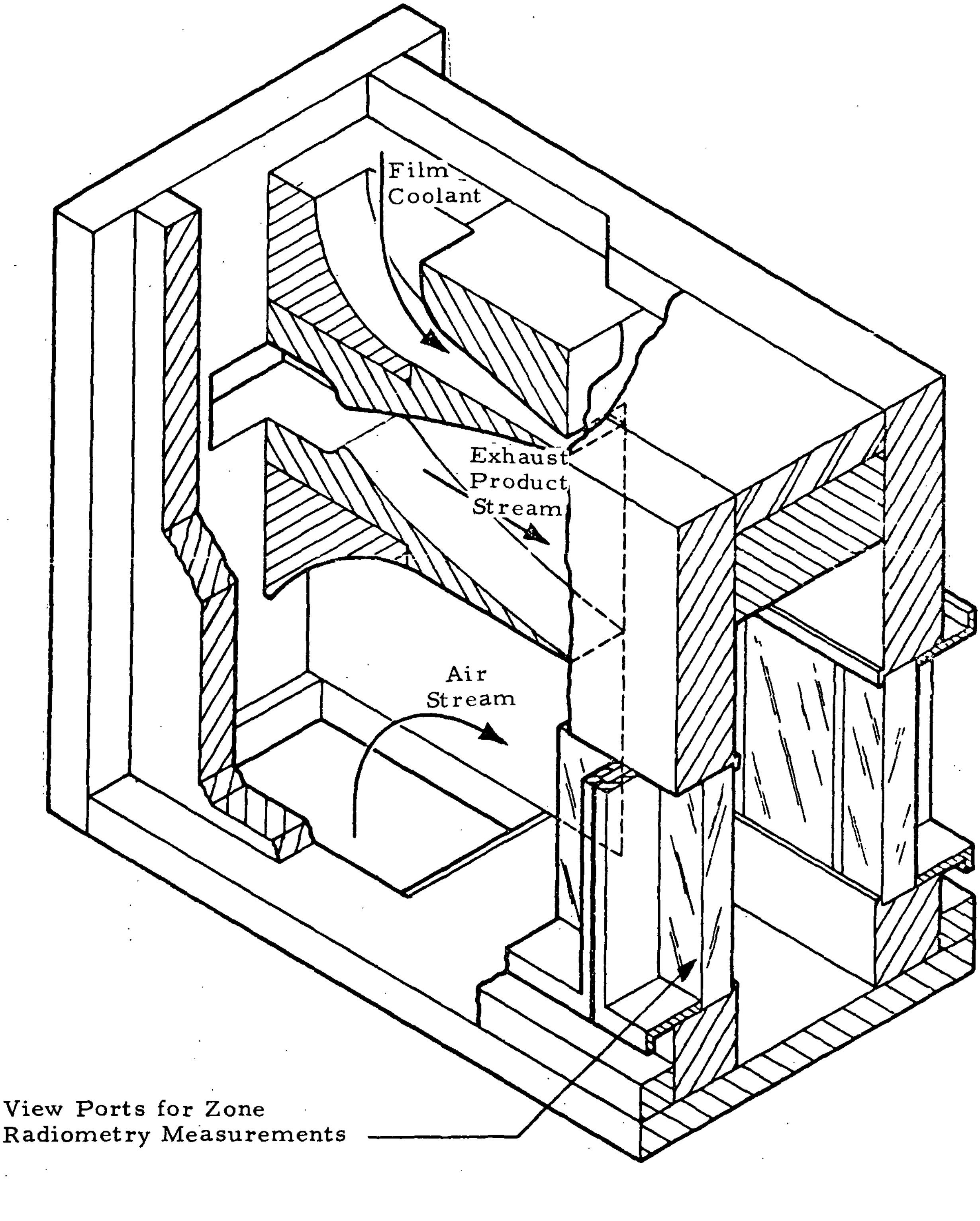


Fig. 3 - Rocketdyne Composite Engine Study (Ref. 17)

radiances, $I_{\lambda A}$ (with the chopper closed) and $I_{\lambda B}$ (with the chopper open), the transmittance and blackbody radiance of the zone can be calculated from:

$$I_{\lambda A} = I_{\lambda b}(1 - \tau) \tag{3.5}$$

$$I_{\lambda B} = I_{\lambda b}(1 - \tau) + I_{\lambda b_{source}} \tau \tag{3.6}$$

Note that the temperature of the radiating gas is assumed not be change when it is impinged upon by the radiating source.

The zone radiometry measurement data presented by Rocketdyne are in terms of radiance with units of $W\,cm^{-2}\,sr^{-1}\,\mu^{-1}$. The form of Planck's law applicable for relating the blackbody radiance at a particular wavelength to the temperature was used to evaluate the temperature of the zone (see Table 1).

$$I_{\lambda b} = \frac{2C^2}{\lambda^5\left(\left[\exp\left(\frac{\hbar C}{k\lambda T}\right)\right] - 1\right)} \tag{3.7}$$

The evaluation of the partial pressure of the radiating species makes use of the representation of transmittance. When the species can be treated as a continuum radiator the transmittance can be calculated using Eq. (2.45)

$$-\ell n\, \tau^* = k\, S$$

where the S is distance and the k value contains the partial pressure of the species as a correction factor to the absorption coefficient at standard conditions, k_o. Repeating Eq. (2.44)

$$k = \left(\frac{k_o}{P_o}\right)\left(\frac{273}{T}\right) P_i$$

The tabulated k_o value corresponding to the calculated zone temperature makes the solution for the partial pressure straightforward.

Whenever band models must be used to represent the radiation process, the transmittance is given by Eq. (2.42). The solution for the partial pressure is no longer a simple process. The line width parameter, a, is dependent on temperature, the local pressure and partial pressure of all the constituent species as described previously. An iterative method is used in which an estimate of the partial pressure of the radiating spacies is made using the simpler continuum radiation transmittance (Eq. (2.45)). With this estimated pressure and the calculated temperature, a corresponding "a" value is evaluated using Eqs. (2.46-2.50). The first iteration on the partial pressure can then be made using Eq. (2.43). The iteration procedure is continued until the desired degree of agreement is obtained between succeeding pressure values.

Summarizing the procedure for evaluating the temperature and partial pressure from zone radiometry measurements of a one-dimensional flow:

1. Obtain the zone radiance and transmittance using Eqs. (3.5) and (3.6).
2. Solve for the zone temperature using Eq. (3.7).
3. Use Eq. (2.45) for continuum radiation to calculate the partial pressure to complete the solution, or
4. Use an iterative procedure to calculate the partial pressures for non-continuum radiation requiring a band model representation of transmittances as follows:
    a. Estimate a partial pressure value using the continuum radiation representation for the transmittance, Eq. (2.45).

b. Evaluate the fine structure parameter, a, with Eqs.(2.46) through (2.50) using the temperature and the estimated pressure value.

c. Calculate the partial pressure using Eq. (2.43).

d. Compare the newly calculated partial pressure with that used in Step 4b.

- "Poor" agreement: Repeat from Step 4b using new partial pressure.
- "Good" agreement: Consider solution complete.

3.2 AXISYMMETRIC NONISOTHERMAL FLOWS

Application of zone radiometry measurement techniques to axisymmetric flows uses the same principles as for the one-dimensional situation but the reduction of the measured intensities to temperature and partial pressure becomes more complex.

A schematic of the axisymmetric zone layout is given in Fig. 4.

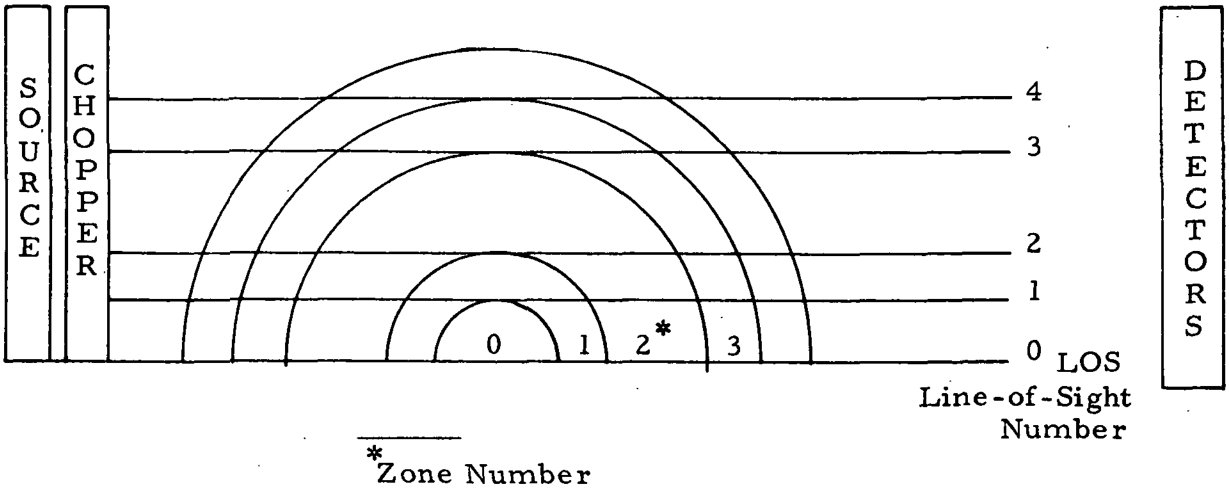


Fig. 4 - Definition of Zones and Lines of Sight in Axisymmetric Zone Radiometry

The zones consist of concentric circular regions in which the physical properties are assumed uniform. The line of sight (LOS) is defined such that the n^{th} LOS passes through the n^{th} zone and all zones outside of it. The zones are not necessarily of the same size. (In the Rocketdyne zone radiometry data reduction program (see Appendix and Ref. 18) the zones are nevertheless assumed to all be of the same size.) Whether or not the same size zones are used the path lengths within the zones are variable and dependent upon the location of the zone within the axisymmetric array. The path length, ℓ, in the fourth zone on the third line of sight is (see Fig. 5) calculated as

$$\ell = \sqrt{R_3^2 - R_1^2} - \sqrt{R_2^2 - R_1^2} \tag{3.8}$$

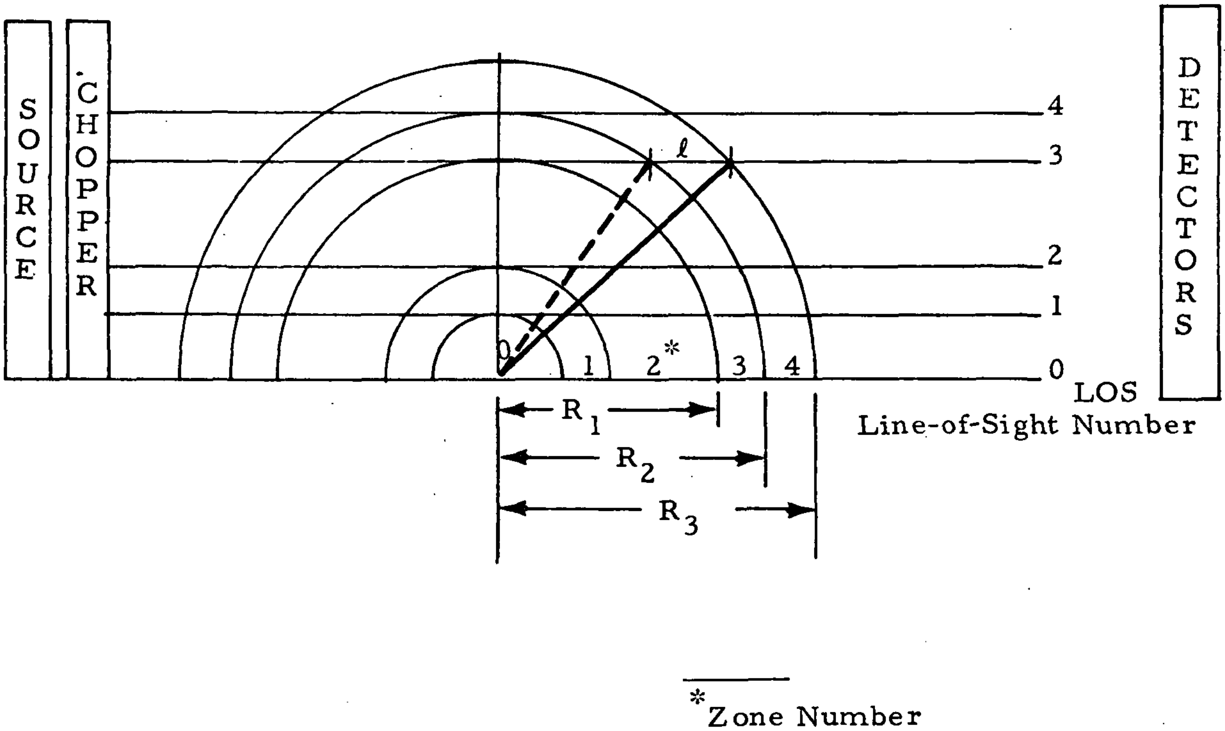


Fig. 5 - Path Length Definition in an Axisymmetric Zone Radiometry Array

From Fig. 4 it can be seen that the signal received by the detectors along a respective LOS has in general passed through an inhomogeneous region. The radiances measured for the LOS can be used to calculate the temperatures and partial pressure within the zones using the following procedure.

Again the two radiance readings (one with the chopper and one without it) are made along each line of sight. The zones are maintained at sufficiently small sizes that the line of sight through the concentric zones can be approximated by one-dimensional slabs as in Fig. 6.

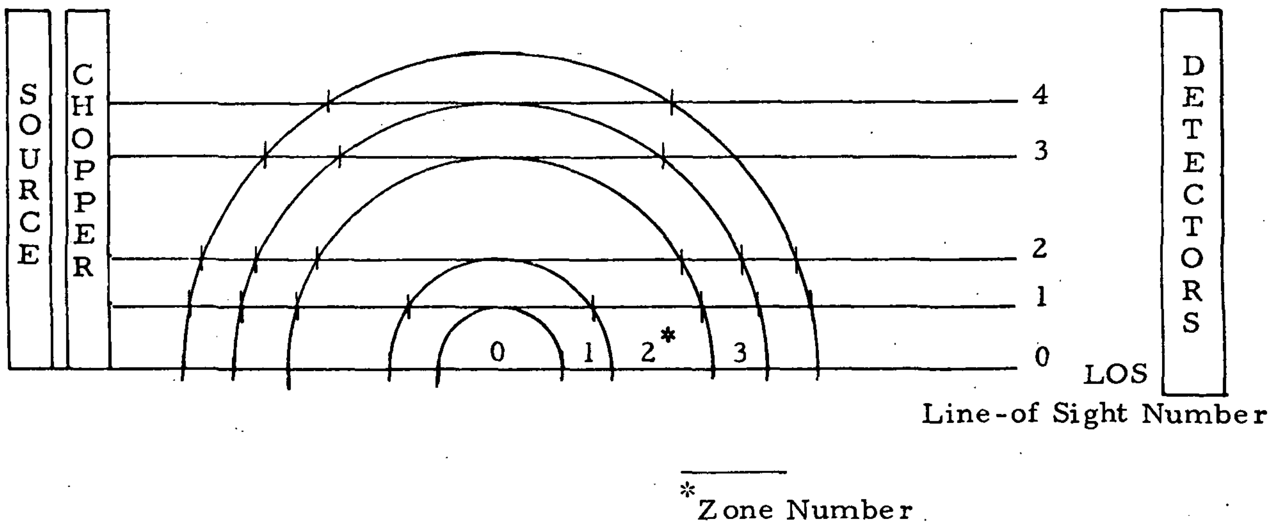


Fig. 6 - One-Dimensional Approximation of the Lines of Sight for Axisymmetric Zone Radiometry

The radiance values measured for these lines of sight can be mathematically represented as they were in Eqs. (3.5) and (3.6) for the single one-dimensional case.

$$I_{\lambda Aj} = \sum_{i=1}^{n} I_{\lambda bi,j}\, \epsilon^{*}_{i,j} \quad \tau_{i-1,j} \tag{3.9}$$

$$I_{\lambda \beta j} = \sum_{i=1}^{n} I_{\lambda bi,j}\, \epsilon^{*}_{i,j} \quad \tau_{i-1,j} + I_{\lambda b\,\mathrm{Source}}\, \tau_j \tag{3.10}$$

where j is the line of sight under study and the i is summed over all the zones to n. Subtracting Eq. (3.9) from Eq. (3.10) provides n relationships for τ_j which is the mean transmittance of the entire j^{th} line of sight. The other n equations needed to solve for the 2n unknowns, T_i and P_i come from Eq. (3.9). The representation of the transmittance now becomes the prime question. Rocketdyne uses the grey gas limit for the transmittance, Eq. (3.1). The n values of τ_j can then be expanded as

$$\tau_j = \prod_{i=1}^{n} \tau^{*}_{i,j} = (\tau^{*}_{1,j})(\tau^{*}_{2,j}) \ldots (\tau^{*}_{n,j}) \tag{3.11}$$

The 2n equations consisting of Eqs. (3.9) and (3.11) are solved using matrix algebra. Since Rocketdyne has automated the solution procedure in a data reduction program, an iterative process is used to evaluate the unknowns.

Summarizing the procedure for evaluating the temperatures and partial pressures in the n zones of an axisymmetric flow using the zone radiometry measurements is:

1. Construct equations for the measured line of sight radiance using Eq. (3.9).
2. Construct equations for the mean transmittance through an entire line of sight using Eq. (3.11).
3. Place the transmittance represented by Eq. (3.1) in matrix form. (Using the measured mean line-of-sight transmittance values, τ_j, the matrix can be solved for the zone transmittances $\tau^{*}_{i,j}$.)

4. Replace the transmittances in the equations constructed in Step 1 with the calculated zone transmittances. (The matrix representing the mean measured radiance is then ready for solution for the zonal blackbody radiance functions.)

5. Solve for the temperatures in the zones using Eq. (3.7). (For continuum radiators this completes the solution procedure since the zone partial pressures can be evaluated using the calculated k_i's, tables of k_o versus temperature and Eq. (2.44).)

6. Use an iteration procedure (for noncontinuum radiation, requiring a band model representation of the transmittance) to solve for the temperatures and partial pressures in the zones as follows:

   a. Use Eqs. (2.46) and (2.50) to evaluate the fine structure parameter, a, using the zone temperatures and the zone partial pressures from Step 5.

   b. Reevaluate the zone transmittance values using Eq. (2.43) The modification to the Rocketdyne program to define the transmittance of the zone with Eq. (3.3) would eliminate the grey gas assumption inherent in Eq. (2.43) and make the solution valid for all optical thicknesses.

   c. Return to Step 4 and repeat Steps 4 and 5.

   d. Compare the newly calculated partial pressures and temperatures of the zones with those obtained previously in Step 5.

      - "Poor" agreement: Repeat from Step 6 using new partial pressures and temperatures for the zones.

      - "Good" agreement: Consider solution complete.

3.3 ROCKETDYNE ZONE RADIOMETER DATA REDUCTION PROGRAM

The Rocketdyne automated data reduction program is listed in the Appendix. An input guide and flow chart of the program are also presented. To aid potential users of the data reduction program, a sample case is given along with sample input and output.

Section 4
EXAMPLE PROBLEMS

To illustrate the calculation techniques discussed in this report, several example problems will be solved. The first is one typical of an axisymmetric alcohol-burning Atlas vernier engine; the second represents a planar, hydrogen-oxygen diffusion flame, i.e., the composite engine experiment.

- Problem 1 — LOX-Alcohol Engine

Due to the behavior of gaseous radiation properties, it is desirable to choose example problems in which the temperature and composition are specified. Consider first the following thermal path:

T (^{o}K)

Detector	----	1500	2000	2500	2000	1500	---- 3500^{o} source
	Zone:	1	2	3	4	5	

Zones 1, 2, 4 and 5 are 2 cm long; zone 3 is 4 cm long.

Partial pressure, CO_2: 0.27 atm

H_2O: 0.58 atm

Wave lengths of measurement: 4.45 (μ) or 2247 (1/cm)

2.49 (μ) or 4016 (1/cm)

<u>Solution Procedure</u>

1. Evaluate $$k = k_o \left(\frac{273^{o}K}{T^{o}K}\right)\left(\frac{P_i \text{ atm}}{1 \text{ atm}}\right)$$

k_o (1/cm)	T (^oK)			ω (1/cm)	Ref.
Species:	1500	2000	2500		
H_2O	1.22×10^{-2}	2.33×10^{-2}	3.05×10^{-2}	2247	(12), p. 70
	1.47×10^{-1}	1.43×10^{-1}	1.50×10^{-1}	4016	(12), p. 73
CO_2	10.99	13.30	13.55	2247	(10), p. 104
	(Not measured but assumed zero)			4016	(12), p. 33

T (^oK)	T/273 (^oK)	$\left[T/2.73\ (^oK)\right]^{1/2}$	T^2	T^3
1500	5.50	2.35	2.25×10^6	3.38×10^9
2000	7.34	2.73	4.0×10^6	8.0×10^9
2500	9.15	3.03	6.25×10^6	15.63×10^9

k (1/cm)	T (^oK)			ω (1/cm)
Species:	1500	2000	2500	
H_2O	1.29×10^{-3}	1.84×10^{-3}	1.94×10^{-3}	2247
	1.55×10^{-2}	1.12×10^{-2}	0.95×10^{-2}	4016
CO_2	0.540	0.485	0.400	2247

2. Evaluate fine structure parameters.

a. line density, (1/d)

(i) CO_2, (1/d) = (1/DLR) (cm) at ω = 2247 (1/cm)

(1/DLR)	T (^oK)
181.1	1500
356	2000
510	2500

Ref. 11, p. 59

(ii) H_2O, $(1/d) = a^*/\gamma^*$ Ref. 13, p. 194

Evaluation of a^* is a rather nebulous operation, but Rocketdyne's programmed values will be presented. Reference 14 is supposed to have tabulated values of (1/d) when it is published.

$$a^* = 10^{(b_i + C_i T^2)}$$

b_i	c_i	i	Ref. 12
-1.366	0.165×10^{-6}	2247 (1/cm)	p. 134
-1.62	0.180×10^{-6}	4016 (1/cm)	

$T(^oK)$	$C_i T^2$	$b_i + C_i T^2$	i	$1/a^*$	a^*
1500	0.370	-0.996	2247 (1/cm)	9.9	0.101
2000	0.660	-0.706		5.09	0.196
2500	1.030	-0.336		2.166	0.461
1500	0.405	-1.215	4016 (1/cm)	16.4	0.061
2000	0.720	-0.900		7.92	0.126
2500	1.125	-0.495		3.13	0.319

To complete the (1/d) calculation for H_2O, define $\gamma^* = \left[0.44\,(T_o/T) + 0.09\,(T_o/T)^{1/2}\right] C^* + 0.044\,(T_o/T)^{1/2}(1 - C^*)$ Ref. 13, p. 194

where

$$C^* = -0.1002 + 0.2802 \times 10^{-3}\, T - 0.1089 \times 10^{-6}\, T^2 + 0.0291 \times 10^{-9}\, T^3.$$

Evaluate C^* and γ^* for the temperatures in the zones.

For T = 1500°K

$$C^* = \frac{-0.1002 \quad +0.423}{-0.245 \quad +0.098} \qquad \frac{-0.345}{} \quad \frac{+0.521}{} = +0.176$$

T = 2000°K

$$C^* = \frac{-0.100 \quad +0.560}{-0.436 \quad +0.233} \qquad -0.536 \quad +0.793 = +0.257$$

T = 2500°K

$$C^* = \frac{-0.100 \quad +0.701}{-0.682 \quad +0.456} \qquad -0.782 \quad +1.157 = +0.377$$

$$\gamma^* = \left[0.44\,(T_o/T) + 0.09\,(T_o/T)^{1/2}\right] C^* + 0.044\,(T_o/T)^{1/2}\,(1 - C^*)$$

Use the C^* values to calculate the γ^* values.

$$\begin{aligned}
\gamma^*_{1500} &= [0.080 + 0.0383]\,(0.176) + (0.0187)(0.824) \\
&= (0.1183)(0.176) + 0.0154 \\
&= 0.0209 + 0.0154 = 0.0363
\end{aligned}$$

$$\begin{aligned}
\gamma^*_{2000} &= [0.0593 + 0.033]\,(0.257) + (0.0161)(0.743) \\
&= 0.0237 + 0.0120 = 0.0357
\end{aligned}$$

$$\begin{aligned}
\gamma^*_{2500} &= [0.0481 + 0.0297\,[0.377] + (0.0145)(0.623) \\
&= 0.0292 + 0.0091 = 0.0393
\end{aligned}$$

The (1/d) values for H_2O at the temperatures and in the wave lengths of interest are:

(1/d) (cm)	T (^oK)	i (1/cm)
2.78	1500	2247
5.49	2000	
11.78	2500	
1.68	1500	4016
3.52	2000	
8.15	2500	

b. Obtain the collision half widths, γ_c's, to complete the calculation of the fine structure parameters, a's.

$$\gamma_{c_i} = \sum_j \gamma_{i,j} P_j \left(\frac{273}{T}\right)^{1/2} + \gamma_{i,i} P_i \left(\frac{273}{T}\right) \tag{2.46}$$

j = all species.

For CO_2 at 1500^{o}K, Ref. 14

$$\begin{aligned} \gamma_{c_{CO_2}} &= (0.07)(0.58)/2.35 + (0.09)(0.27)/2.35 + (0.01)(0.27)/5.50 \\ &= 0.0173 + 0.0103 + 0.0005 = 0.0281 \text{ (1/cm)} \end{aligned}$$

At 2000^{o}K,

$$\begin{aligned} \gamma_{c_{CO_2}} &= (0.07)(0.58)/2.73 + (0.09)(0.27)/2.73 + (0.01)(0.27)/7.34 \\ &= 0.0149 + 0.0089 + 0.0004 = 0.0242 \end{aligned}$$

At 2500°K

$$\gamma_{c_{CO_2}} = \frac{(0.07)(0.58) + (0.09)(0.27)}{3.03} + (0.01)(0.27)/9.15$$

$$= \frac{(0.0406 + 0.0243)}{3.03} + 0.0003 = 0.0217$$

For H_2O: Ref. 14

$$\gamma_{c_{H_2O}} = \frac{(0.09)(0.58) + (0.12)(0.27)}{(T/273)^{1/2}} + (0.44)(0.58)/(T/273)$$

$$\frac{0.0845}{(T/273)^{1/2}} + \frac{0.255}{(T/273)}$$

At 1500°K

$$\gamma_{c_{H_2O}} = 0.0360 + 0.0465 = 0.0825$$

At 2000°K

$$\gamma_{c_{H_2O}} = 0.0310 + 0.0347 = 0.0657$$

At 2500°K

$$\gamma_{c_{H_2O}} = 0.0279 + 0.0279 = 0.0558$$

The radiation parameters for this sample problem are summarized on the following page.

$a = (\gamma_c)(l/d)$	$T(^oK)$	Species	ω (1/cm)	k (1/cm)
5.09 8.61 11.10	1500 2000 2500	CO_2	2247	0.540 0.485 0.400
0.229 0.358 0.658	1500 2000 2500	H_2O	2247	1.29×10^{-3} 1.84×10^{-3} 1.94×10^{-3}
0.139 0.230 0.455	1500 2000 2500	H_2O	4016	1.55×10^{-2} 1.12×10^{-2} 0.95×10^{-2}

The intensities measured by the detector are the blackbody radiation of the species within the zones attenuated by the zones between that particular zone and the detector.

The blackbody intensity for the zonal temperatures and the wave numbers of interest are

$I_{\lambda b}$ (Ref. 19)	T	i
0.8942 ($W/cm^2 - \mu$-sr)	1500 (^oK)	2247 (1/cm)
1.691	2000	
2.580	2500	
4.493	3500	
2.687	1500	4016
7.271	2000	
13.56	2500	
29.19	3500	

The attenuation (or in the converse sense, the transmittance, τ) of the radiation of the zones to the detector is calculated for each of the optical paths. Starting from the detector, the first transmittance, τ_1, involves only zone one. The calculation of the second transmittance, τ_2, includes the first and second zones. These calculations continue until all the transmittances are determined.

For CO_2 radiation:

$$K_1 L_1 = (0.540 \text{ cm}^{-1})(2 \text{ cm}) = 1.080$$

$$-\ell n\, \tau_1 = \frac{1.080}{\left(1 + \frac{1.080}{4(5.09)}\right)^{1/2}} = \frac{1.080}{(1.053)^{1/2}} = 1.050$$

$\underline{\tau_1 = 0.350}$

$$K_2 L_2 = (0.485)(2) = 0.970$$

$$-\ell n\, \tau_2 = \frac{\sum K_h L_h}{\left(1 + \frac{\left(\sum K_h L_h\right)^2}{4 \sum (\gamma_c/d)_h K_h L_h}\right)^{1/2}}$$

$$= \frac{2.05}{\left(1 + \frac{4.20}{4\left[(5.09)(1.08) + (8.61)(0.97)\right]}\right)^{1/2}}$$

$$= \frac{2.05}{\left(1 + \frac{1.05}{5.50 + 8.35}\right)^{1/2}} = \frac{2.05}{1.04} = 1.94$$

$\underline{\tau_2 = 0.144}$

$$K_3 L_3 = (0.400)(4) = 1.60$$

$$-\ln \tau_3 = \frac{2.05 + 1.60}{\left(1 + \frac{(3.65)^2}{[4(13.85 + (11.1)(1.6)]}\right)^{1/2}} = \frac{3.65}{(1.052)} = 3.48$$

$\underline{\tau_3 = 0.0310}$

$K_4L_4 = K_2L_2 = 0.970$

$$-\ln \tau_4 = \frac{3.65 + 0.97}{\left(1 + \frac{(4.62)^2}{4\left[31.65 + (8.61)(0.97)\right]}\right)^{1/2}} = \frac{4.62}{1.06} = 4.35$$

$\underline{\tau_4 = 0.0130}$

$K_5L_5 \doteq K_1L_1 = 1.080$

$$-\ln \tau_5 \approx \frac{4.62 + 1.08}{\left(1 + \frac{(5.70)^2}{4\left[40.00 + (5.09)(1.08)\right]}\right)^{1/2}} = \frac{5.70}{1.09} = 5.25$$

$\underline{\tau_5 = 0.0053}$

Use the zonal blackbody radiation and transmittance values to evaluate the measured intensity.

$$I_\lambda = I_{\lambda b1}(1-\tau_1) + I_{\lambda b2}(\tau_1-\tau_2) + I_{\lambda b3}(\tau_2-\tau_3) + I_{\lambda b4}(\tau_3-\tau_4) + I_{\lambda b5}(\tau_5-\tau_4)$$

$$I_\lambda = \begin{pmatrix} 0.894\,(1 - 0.350) & = (0.894)\,(0.650) = 0.580 \\ 1.691\,(0.350 - 0.144) & = (1.691)\,(0.206) = 0.348 \\ 2.580\,(0.144 - 0.0310) & = (2.580)\,(0.113) = 0.292 \\ 1.691\,(0.0310 - 0.0130) & = (1.691)\,(0.018) = 0.031 \\ 0.894\,(0.0130 - 0.0053) & = (0.894)\,(0.008) = \underline{0.007} \end{pmatrix}$$

$$I_\lambda = 1.258 \quad \mathrm{W/cm^2\text{-}\mu\text{-}sr}$$

If a 3500°K blackbody source has also been transmitting,

$$\begin{aligned} I_{\lambda WS} &= 1.258 + (0.0053)(4.493) \\ &\doteq 1.272 \quad \mathrm{W/cm^2\text{-}\mu\text{-}sr} \end{aligned}$$

Neglecting non-grey effects introduces errors of 5 to 10% in intensity; these become a smaller precentage when they are converted to temperature. Such errors do not become unreasonably increased when using the data reduction programs — as evidenced by Appendix D of Ref. 18.

For H_2O radiation at 2247 (1/cm), the transmittances are

$$K_1L_1 = 2.58 \times 10^{-3}$$

$$-\ln \tau_1 = \frac{2.58 \times 10^{-3}}{\left(1 + \dfrac{2.58 \times 10^{-3}}{4\,(0.229)}\right)^{1/2}} = \frac{2.58 \times 10^{-3}}{(1.0028)^{1/2}} = 2.58 \times 10^{-3}$$

$\underline{\tau_1 \cong 1}$

$$K_2L_2 = 3.68 \times 10^{-3}$$

$$-\ln \tau_2 = \frac{6.26 \times 10^{-3}}{\left(1 + \dfrac{(6.26 \times 10^{-3})^2}{4\left[0.59 \times 10^{-3} + 1.32 \times 10^{-3}\right]}\right)^{1/2}} = \frac{6.26 \times 10^{-3}}{1.0026} = 6.26 \times 10^{-3}$$

$\underline{\tau_2 \cong 1}$

$K_3L_3 = 7.76 \times 10^{-3}$

$$-\ln\tau_3 = \frac{14.02 \times 10^{-3}}{\left(1 + \dfrac{(1.40 \times 10^{-2})^2}{4\left[1.91 \times 10^{-3} + 5.11 \times 10^{-3}\right]}\right)^{1/2}} = \frac{1.40 \times 10^{-2}}{1.005} = 1.40 \times 10^{-2}$$

$\tau_3 = 0.9861$

$K_4L_4 = K_2L_2 = 3.68 \times 10^{-3}$

$$-\ln\tau_4 = \frac{17.70 \times 10^{-3}}{\left(1 + \dfrac{(1.770 \times 10^{-2})^2}{4\left[7.02 \times 10^{-3} + 1.32 \times 10^{-3}\right]}\right)^{1/2}} = \frac{1.77 \times 10^{-2}}{1.005} = 1.77 \times 10^{-2}$$

$\underline{\tau_4 = 0.9825}$

$K_5L_5 = K_1L_1 = 2.58 \times 10^{-3}$

$$-\ln\tau_5 = \frac{20.28 \times 10^{-3}}{\left(1 + \dfrac{400 \times 10^{-6}}{4\left[8.34 \times 10^{-3} + 0.59 \times 10^{-3}\right]}\right)^{1/2}} = \frac{2.03 \times 10^{-2}}{1.005} = 2.03 \times 10^{-2}$$

$\underline{\tau_5 = 0.9799}$

$$I_{\lambda b1}(1 - \tau_1) = 0$$

$$I_{\lambda b2}(\tau_1 - \tau_2) = 0$$

$$I_{\lambda b3}(\tau_2 - \tau_3) = (1 - 0.9861) = 0.0139$$

$$I_{\lambda b4}(\tau_3 - \tau_4) = (0.9861 - 0.9825) = 0.0036$$

$$I_{\lambda b5}(\tau_4 - \tau_5) = (0.9825 - 0.9799) = 0.0026$$

$$
\begin{aligned}
I_\lambda = \; & 0.892\,(0) + && = 0 \\
& 1.691\,(0) + && = 0 \\
& 2.580\,(0.0139) + && = 0.0358 \\
& 1.691\,(0.0036) + && = 0.0061 \\
& 0.8921\,(0.0026) && = \underline{0.0023}
\end{aligned}
$$

$$I_\lambda = 0.0442 \quad W/cm^2\text{-}\mu\text{-sr}$$

The intensity when the 3500°K blackbody source is also transmitting is

$$
\begin{aligned}
I_{\lambda WS} &= 0.0442 + (0.9799)(4.493) \\
&= 4.434 \;\; W/cm^2\text{-}\mu\text{-sr}
\end{aligned}
$$

In this example, water vapor radiation is very optically thin.

Total Radiation at 2247 (1/cm)

Since both CO_2 and H_2O are optically active at 2247 (1/cm), both contribute to the total radiation flux. To account for multi-species emission, the natural log of the transmittance of each is calculated and then all such logs are summed.

$$-\ln(\tau_1)_{MS} = 1.050 + 0.0026 = 1.053$$

$$-\ln(\tau_2)_{MS} = 1.940 + 0.0062 = 1.946$$

$$-\ln(\tau_3)_{MS} = 3.48 + 0.014 = 3.49$$

$$-\ln(\tau_4)_{MS} = 4.35 + 0.018 = 4.37$$

$$-\ln(\tau_5)_{MS} = 5.25 + 0.020 = 5.27$$

$(\tau_h)_{MS}$	$(\tau_{h-1} - \tau_h)_{MS}$	$I_{\lambda b}(\tau_{h-1} - \tau_h)_{MS}$
0.348 0.143 0.0305 0.0126 0.0051	0.652 0.205 0.112 0.0179 0.0075	In this case the results are identical to those for CO_2 alone.

Since the radiance from the water vapor is only 4% of that from CO_2, Rocketdyne assumed it negligible (consistent with the exact calculation).

Repeat the transmittance and intensity calculations <u>for H_2O radiation at 4016 (1/cm)</u>

$$K_1L_1 = (1.55 \times 10^{-2})(2) = 3.10 \times 10^{-2}$$

$$a_1 = 0.139$$

$$-\ln\tau_1 = \frac{3.10 \times 10^{-2}}{\left(1 + \frac{3.10 \times 10^{-2}}{4(0.139)}\right)^{1/2}} = \frac{3.10 \times 10^{-2}}{1.025} = 3.02 \times 10^{-2}$$

$$\underline{\tau_1} = \exp(-0.031) = \underline{0.9704}$$

$$K_2L_2 = 2.24 \times 10^{-2}$$

$$-\ln\tau_2 = \frac{5.34 \times 10^{-2}}{\left(1 + \frac{28.4 \times 10^{-4}}{4\left[4.31 \times 10^{-3} + 5.15 \times 10^{-3}\right]}\right)^{1/2}} = \frac{5.34 \times 10^{-2}}{1.04} = 5.13 \times 10^{-2}$$

$$\underline{\tau_2 = 0.950}$$

$$K_3L_3 = 3.80 \times 10^{-2}$$

$$-\ln\tau_3 = \frac{9.14 \times 10^{-2}}{\left(1 + \frac{83 \times 10^{-4}}{4\left[9.46 \times 10^{-3} + 1.73 \times 10^{-2}\right]}\right)^{1/2}} = \frac{9.14 \times 10^{-2}}{1.04} = 8.80 \times 10^{-2}$$

$\underline{\tau_3 = 0.916}$

$K_4L_4 = 2.24 \times 10^{-2}$

$$-\ln \tau_4 = \frac{11.38 \times 10^{-2}}{\left(1 + \frac{130 \times 10^{-4}}{4[26.76 \times 10^{-3} + 5.15 \times 10^{-3}]}\right)^{1/2}} = \frac{1.138 \times 10^{-1}}{1.05} = 1.08 \times 10^{-1}$$

$\underline{\tau_4 = 0.898}$

$K_5L_5 = 3.10 \times 10^{-2}$

$$-\ln \tau_5 = \frac{14.48 \times 10^{-2}}{\left(1 + \frac{210 \times 10^{-4}}{4[31.91 \times 10^{-3} + 4.31 \times 10^{-3}]}\right)^{1/2}} = \frac{1.448 \times 10^{-1}}{1.07} = 1.35 \times 10^{-1}$$

$\underline{\tau_5 = 0.874}$

$$\begin{aligned} I_{\lambda b} = I_{\lambda b1}(1-\tau_1) &= (2.687)\ (0.0296) = 0.080 \\ I_{\lambda b2}(\tau_1-\tau_2) &= (7.271)\ (0.0204) = 0.150 \\ I_{\lambda b3}(\tau_2-\tau_3) &= (13.56)\ (0.034) = 0.460 \\ I_{\lambda b4}(\tau_3-\tau_4) &= (7.271)\ (0.018) = 0.131 \\ I_{\lambda b5}(\tau_4-\tau_5) &= (2.687)\ (0.024) = 0.065 \end{aligned}$$

$I_\lambda = 0.886 \quad W/cm^2\text{-}\mu\text{-}sr$

$I_{\lambda_{WS}} = 0.886 + (0.874)(29.19) = 0.886 + 25.50 = 26.39 \quad W/cm^2\text{-}\mu\text{-}sr$

- Problem 2 — Composite Engine

This is an example of single-zone radiation. Let the path length be the distance between the side walls less the initial jet widths of coolant gases, 4.46 — 0.80 = 3.66 in. or 9.30 cm. Static pressure 15.3 psia. Mass fractions: 0.9438, water and 0.0545, hydrogen.

The mole fraction of water is 0.656, giving a partial pressure of 0.683 atm. The wave number of interest is 4016 (1/cm).

k (1/cm)	T ($^{\circ}$K)	kL	$(1 + kL/4a)^{1/2}$	τ
1.825×10^{-2}	1500	0.1695	1.145	0.8630
1.33×10^{-2}	2000	0.1235	1.065	0.8910
1.12×10^{-2}	2500	0.1040	1.030	0.9040

The measured intensity for the zone can be represented by $I_\lambda = (1-\tau)(I_{\lambda b})$. In particular for each zone

$$(0.137)(2.687) = 0.368 \quad \text{at} \quad 1500^{\circ}\text{K}$$
$$(0.109)(7.271) = 0.792 \quad \text{at} \quad 2000^{\circ}\text{K}$$
$$(0.096)(13.56) = 1.310 \quad \text{at} \quad 2500^{\circ}\text{K}$$

When a 3500°K blackbody source is also radiating, the measured intensity for each zone is calculated as

$$I_{\lambda WS} = 0.368 + (0.863)(29.19) = 0.368 + 25.05 = 25.418 \quad \text{at} \quad 1500^{\circ}\text{K}$$
$$= 0.792 + (0.891)(29.19) = 0.792 + 25.95 = 26.74 \quad \text{at} \quad 2000^{\circ}\text{K}$$
$$= 1.310 + (0.904)(29.19) = 1.310 + 26.20 = 27.51 \quad \text{at} \quad 2500^{\circ}\text{K} .$$

The experiment corresponding to this calculation would indicate:

$$I_{\lambda WS} = 27.51$$

$$I_{\lambda} = 1.310$$

therefore,

$$\tau = \frac{27.51 - 1.310}{29.19} = \frac{26.20}{29.19} = 0.900$$

$$1 - \tau \doteq 0.100$$

$$I_{\lambda b} = \frac{1.310}{0.100} = 13.10, \quad \therefore T = 2470^{\circ}K \cong 2500^{\circ}K \cdot$$

Not using band models would introduce intensity errors of up to 15% in the variable range presented here; temperature errors would be somewhat less. If ϵ is much less than 0.1, serious errors would be introduced into the temperature determination.

The assumption of a grey gas may be used for CO_2 and of an optically thin gas may be used for H_2O, in the examples presented, without introducing excessive errors. Such errors could be removed by using a more complete data analysis program. The theoretical radiation analysis presented in Section 2 should provide a very adequate basis for an accurate data analysis calculation in ranges of experiments for which the illustrative examples are typical, i.e., no improvement to the Curtis-Godson approximation is necessary.

Section 5
CONCLUSIONS AND RECOMMENDATIONS

This report has demonstrated that sufficient radiation property data exist to study zone radiometry of CO_2 and H_2O. Such data also exist for soot and CO.

Data reduction procedures currently used are adequate for the studies which Rocketdyne has performed. These work because CO_2 is a grey gas and H_2O is optically thin in their experiments. Sample problems show this behavior. More accurate data reduction schemes could be devised, but this would not substantially improve existing data. However, to remove that criticism such procedures should be developed.

The only apparent reason for experiments of the alcohol-LOX vernier engine type not yielding accurate temperature and partial pressure data is lack of axial symmetry. The two-dimensional mixing study is probably so optically thin, that accurate transmittances would be very difficult to determine. In any event, all future studies should be preceded by an error analysis of expected data.

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Appendix
ZONE RADIOMETER DATA REDUCTION PROGRAM

Appendix

The data reduction procedure for axisymmetric zone radiometry readings was automated by Rocketdyne in a computer program* called the Axisymmetric Zone Radiometer Data Reduction Program. A description of the program is given here by outlining the operations occurring in the subroutines, a detailed flow chart of the procedure and a listing of the program. An input guide is given, and also a sample input and output are included to aid potential users of the program.

PROGRAM SUBROUTINES

MAIN Subroutine

The Axisymmetric Zone Radiometer Data Reduction Program main driver is responsible for reading the program input and calculating the local temperature and species concentrations of water or carbon dioxide.

The main program does the following (in sequence):

1. Evaluates the path lengths
2. Reads data
3. Solves for the product of zonal absorption coefficient and zonal partial pressure of radiating species, $(kp)_i$
4. Constructs attenuation matrix to solve for zonal blackbody radiance
5. Solves for the zonal blackbody radiance
6. Uses Planck's distribution law to solve for zonal temperatures
7. If the radiating species are considered to be continuous radiators, the solution is complete. The local temperatures along with the product of the absorption coefficient and partial pressures for each zone is output.

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*North American Rockwell Corp., "A Compendium of Zone Radiometry Measurements of Exhaust Plumes," R-8140, Rocketdyne Div., Canoga Park, Calif., 25 February 1970.

8. If the radiating species require the use of spectral averaged data (band models), the zonal temperatures and partial pressures are iterated.
9. Evaluate the fine structure parameter, a, for each zone using the zonal temperatures and partial pressures.
10. Calculate new absorption coefficients for each zone.
11. Repeat calculation steps from Step 4 until successive values of the temperatures and partial pressures are within a preset limit.

QUAD Subroutine

This subroutine contains three entries: QUAD0, QUAD1 and QUAD2. The QUAD0 entry generates the appropriate weighting function for quadratic distribution of properties. The QUAD1 entry sums the product of path lengths and weighting factors to obtain the coefficient for the average (kp) values. The QUAD2 entry sets up the average kp values using the path lengths and the weighting factors.

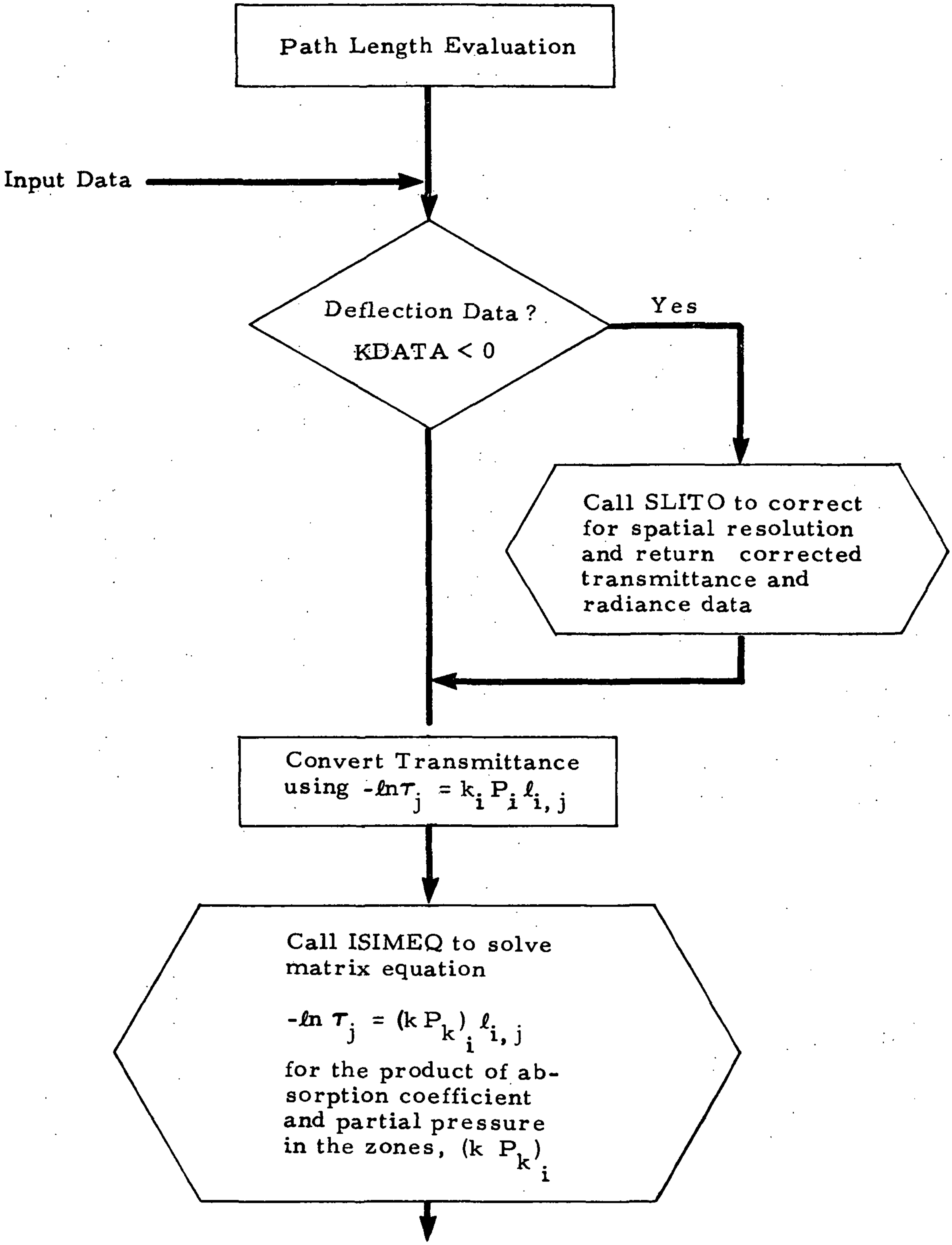
SLIT Subroutine

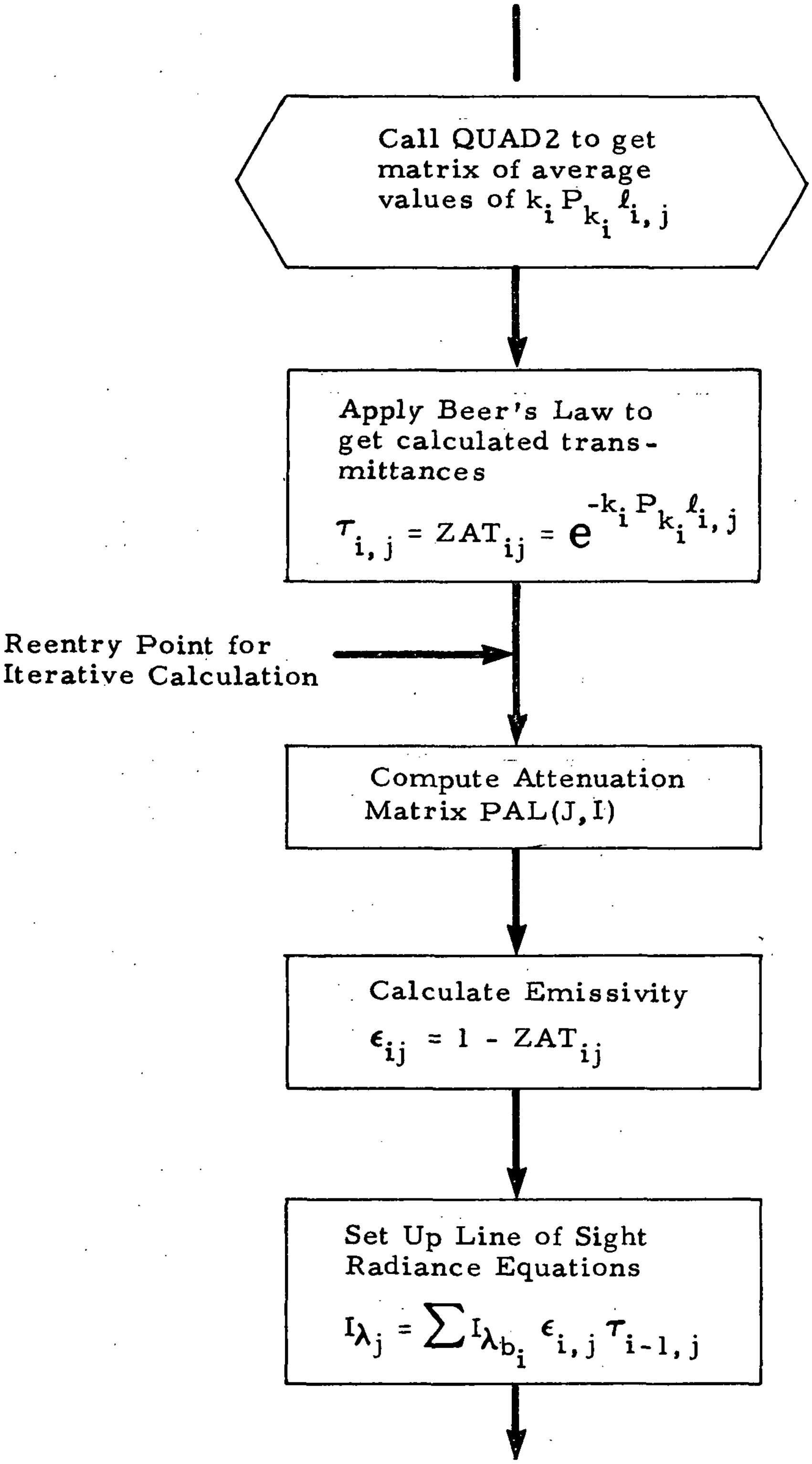
A routine to correct for spatial resolution in deflection data. The correction technique is from the University of Tennessee described in AF CRL 465, pages 59-62. These corrections are usually small and occur at the edge of the plume. Rocketdyne has modified the experimental procedure such that radiance and transmission (smoothed) data are available rather than deflection data making it unnecessary to apply the slit corrections in this subroutine. For completeness the capability to read in deflection data and correct it for spatial resolution has been left in the program.

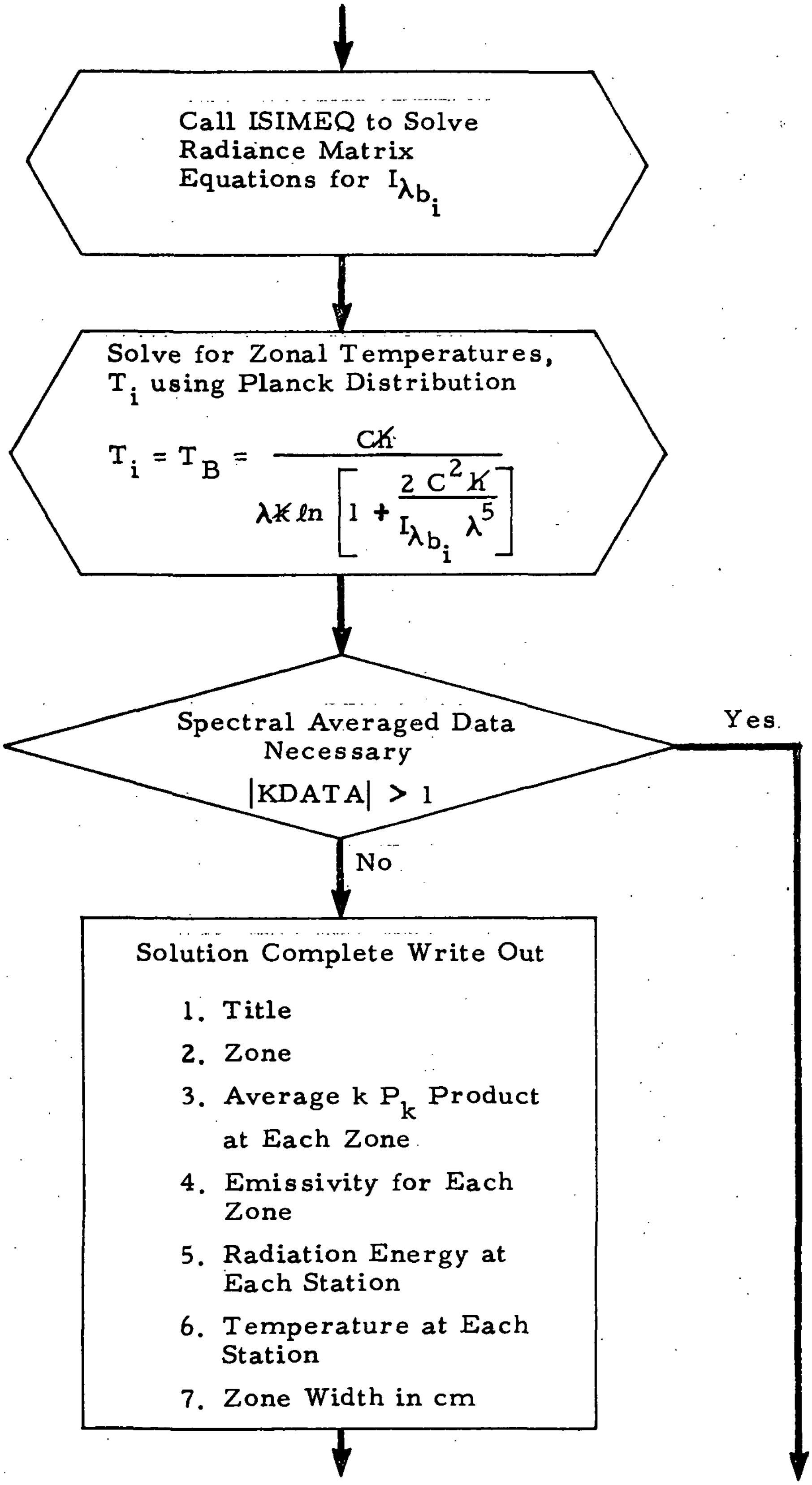
ISIMEQ Function Subroutine

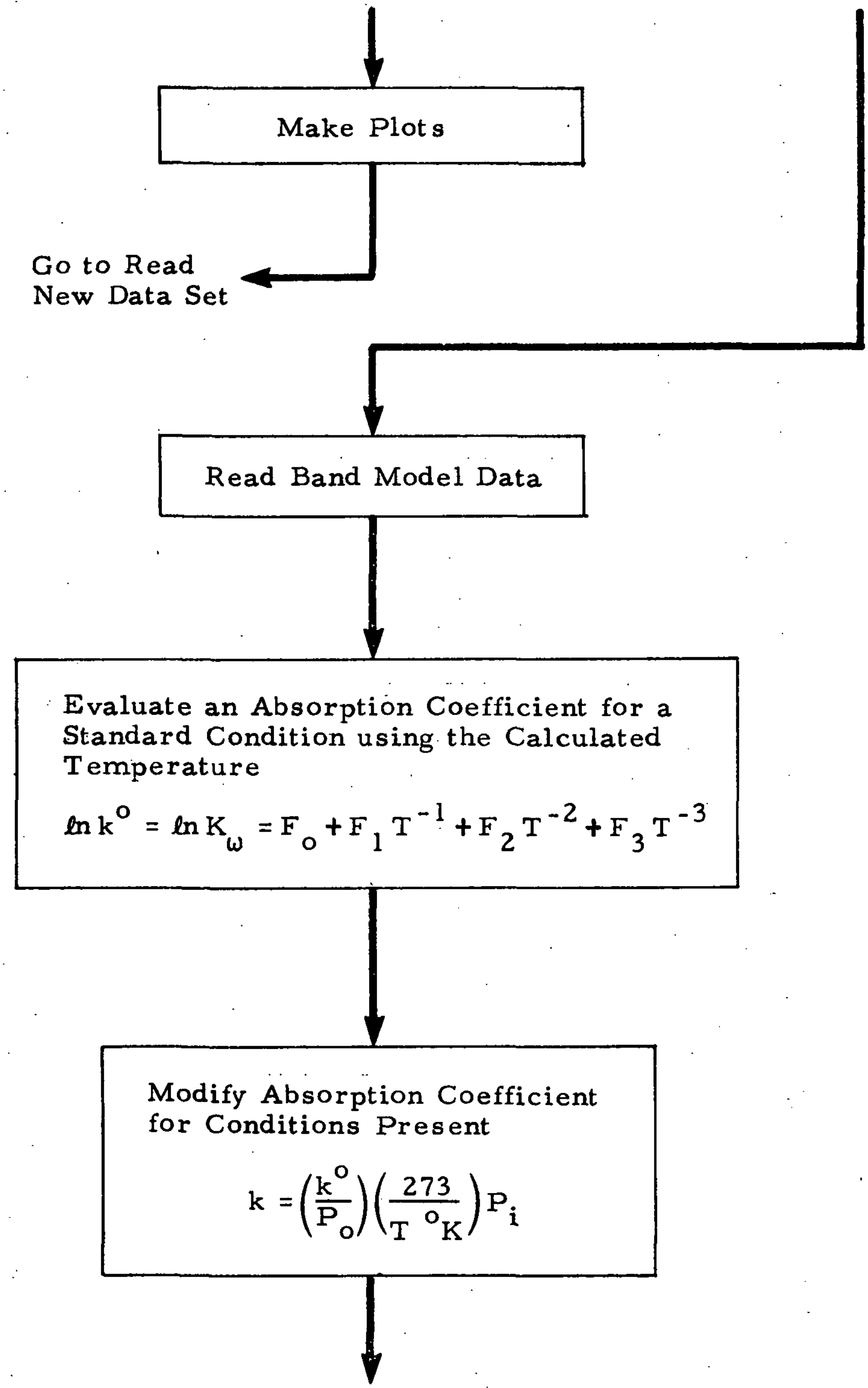
This subprogram solves a set of simultaneous linear equations with up to 30 variables. This is a standard matrix solution subprogram. Throughout the zone radiometry data reduction program, this subroutine is used to solve for the variable of interest in each zone and then returns the answers as a column matrix in column one.

PROGRAM FLOW CHART


Path Length Evaluation
Input Data
Deflection Data ?
KDATA < 0
Yes
Call SLITO to correct for spatial resolution and return corrected transmittance and radiance data
Convert Transmittance using $-\ln \tau_j = k_i P_i \ell_{i,j}$
Call ISIMEQ to solve matrix equation
$-\ln \tau_j = (k P_k)_i \ell_{i,j}$
for the product of absorption coefficient and partial pressure in the zones, $(k\ P_k)_i$


Call QUAD2 to get matrix of average values of $k_i P_{k_i} \ell_{i,j}$
Apply Beer's Law to get calculated transmittances
$\tau_{i,j} = ZAT_{ij} = e^{-k_i P_{k_i} \ell_{i,j}}$
Reentry Point for Iterative Calculation
Compute Attenuation Matrix PAL(J, I)
Calculate Emissivity
$\epsilon_{ij} = 1 - ZAT_{ij}$
Set Up Line of Sight Radiance Equations
$I_{\lambda_j} = \sum I_{\lambda_{b_i}} \epsilon_{i,j} \tau_{i-1,j}$


Call ISIMEQ to Solve Radiance Matrix Equations for $I_{\lambda b_i}$
Solve for Zonal Temperatures, T_i using Planck Distribution
$T_i = T_B = \dfrac{CK}{\lambda K \ln\left[1 + \dfrac{2C^2 K}{I_{\lambda b_i} \lambda^5}\right]}$
Spectral Averaged Data Necessary
$|KDATA| > 1$
Yes
No
Solution Complete Write Out
1. Title
2. Zone
3. Average k P_k Product at Each Zone
4. Emissivity for Each Zone
5. Radiation Energy at Each Station
6. Temperature at Each Station
7. Zone Width in cm


Make Plots
Go to Read New Data Set
Read Band Model Data
Evaluate an Absorption Coefficient for a Standard Condition using the Calculated Temperature
$\ln k^o = \ln K_\omega = F_o + F_1 T^{-1} + F_2 T^{-2} + F_3 T^{-3}$
Modify Absorption Coefficient for Conditions Present
$k = \left(\frac{k^o}{P_o}\right)\left(\frac{273}{T\ {}^oK}\right) P_i$

Evaluate Fine Structure Parameter

$$\log_{10} \bar{a}_i(T) = A_{o_i} + A_{2_i} T^2$$

Modify Previously Calculated Partial Pressures Using Band Model Absorption Coefficients

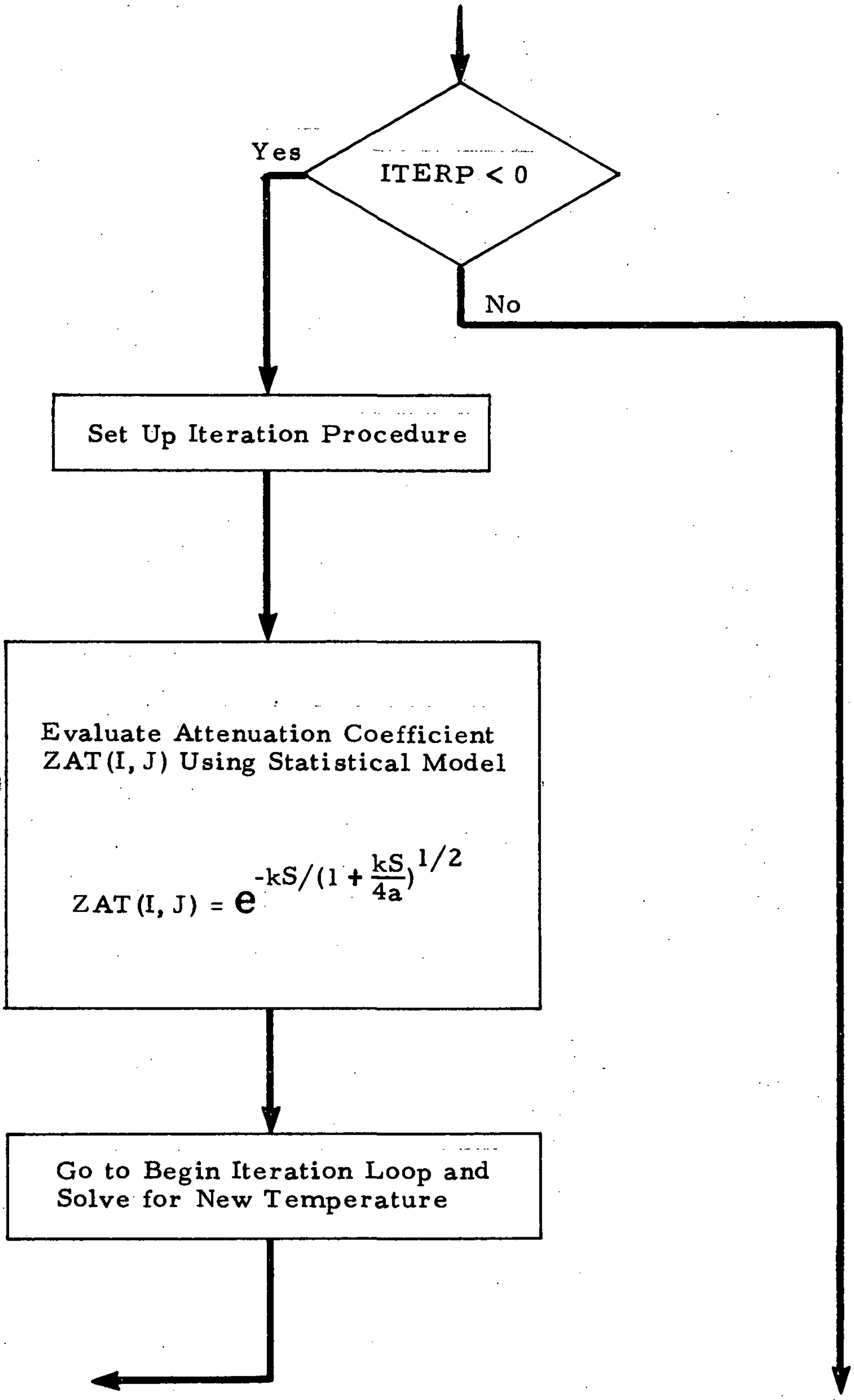
$$P_{k_i} = \frac{\text{average } (k_i P_{k_i} \ell_i}{(\text{zone width}) \; (\text{band model } k_i)}$$

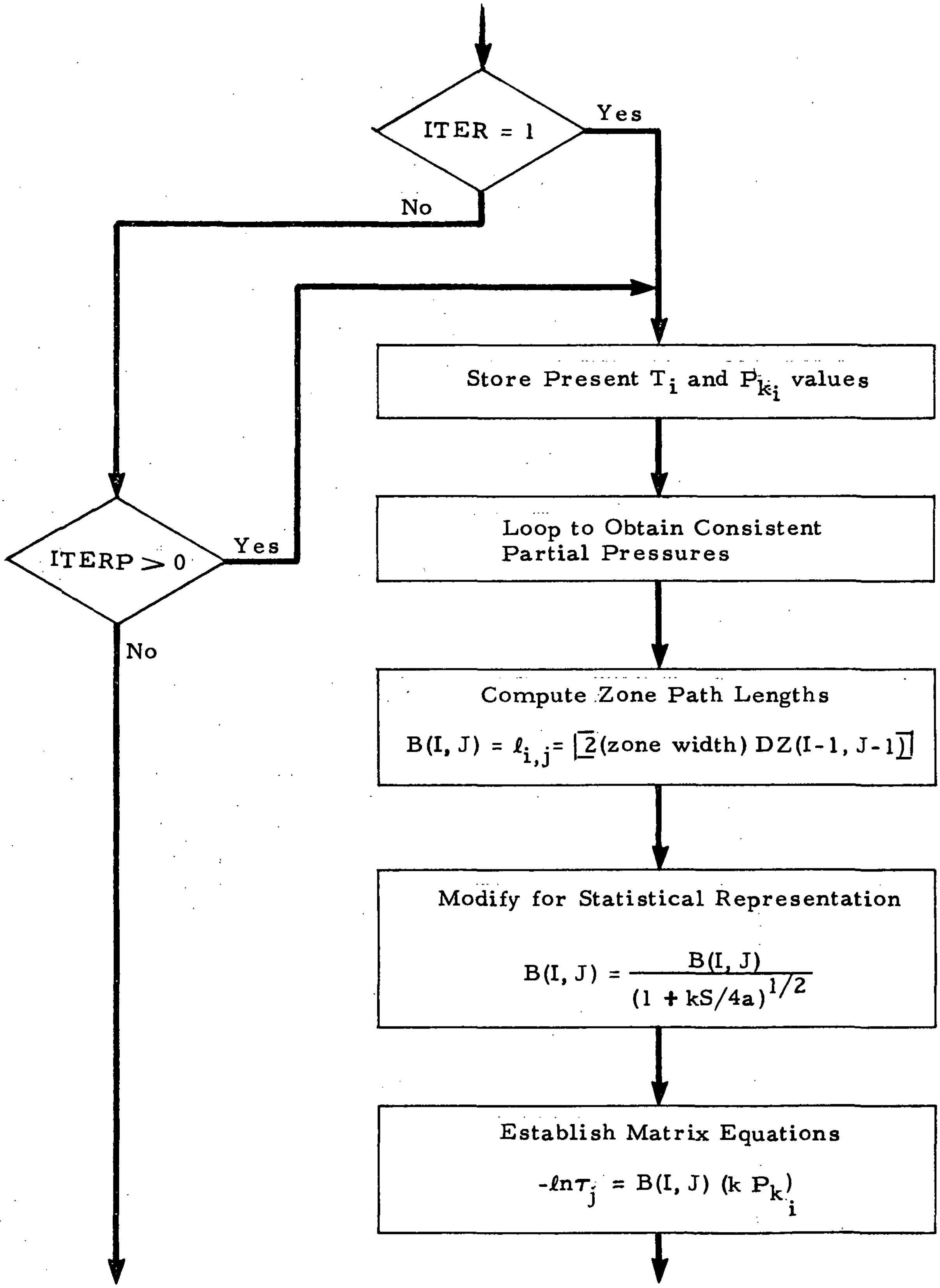
Evaluate ASTAR
The Average Fine Structure Parameter, a

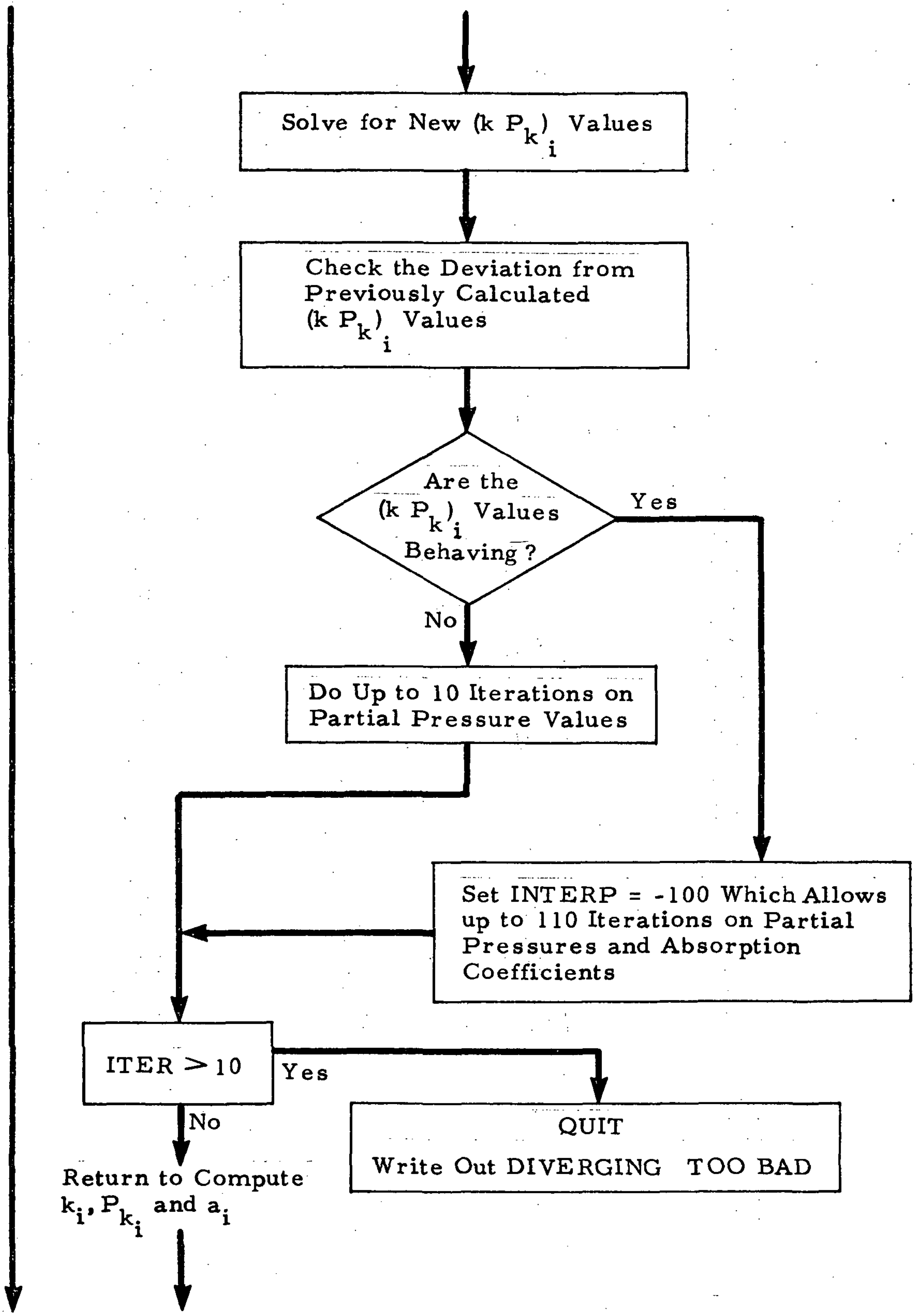
$$a = \text{ASTAR} = \frac{a^* \gamma_o}{\gamma_o^*}$$

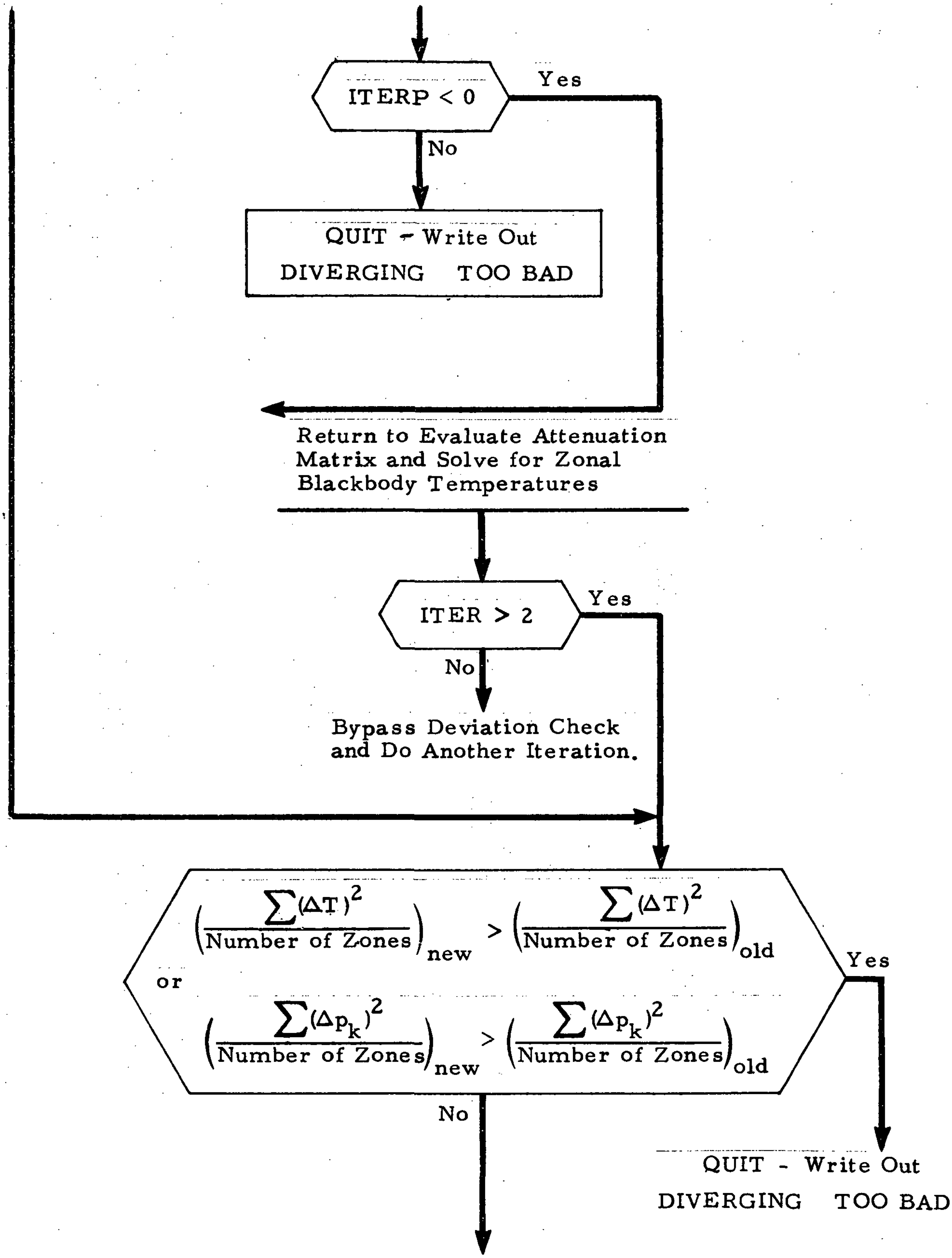
Write Out Calculated Values for Each Zone

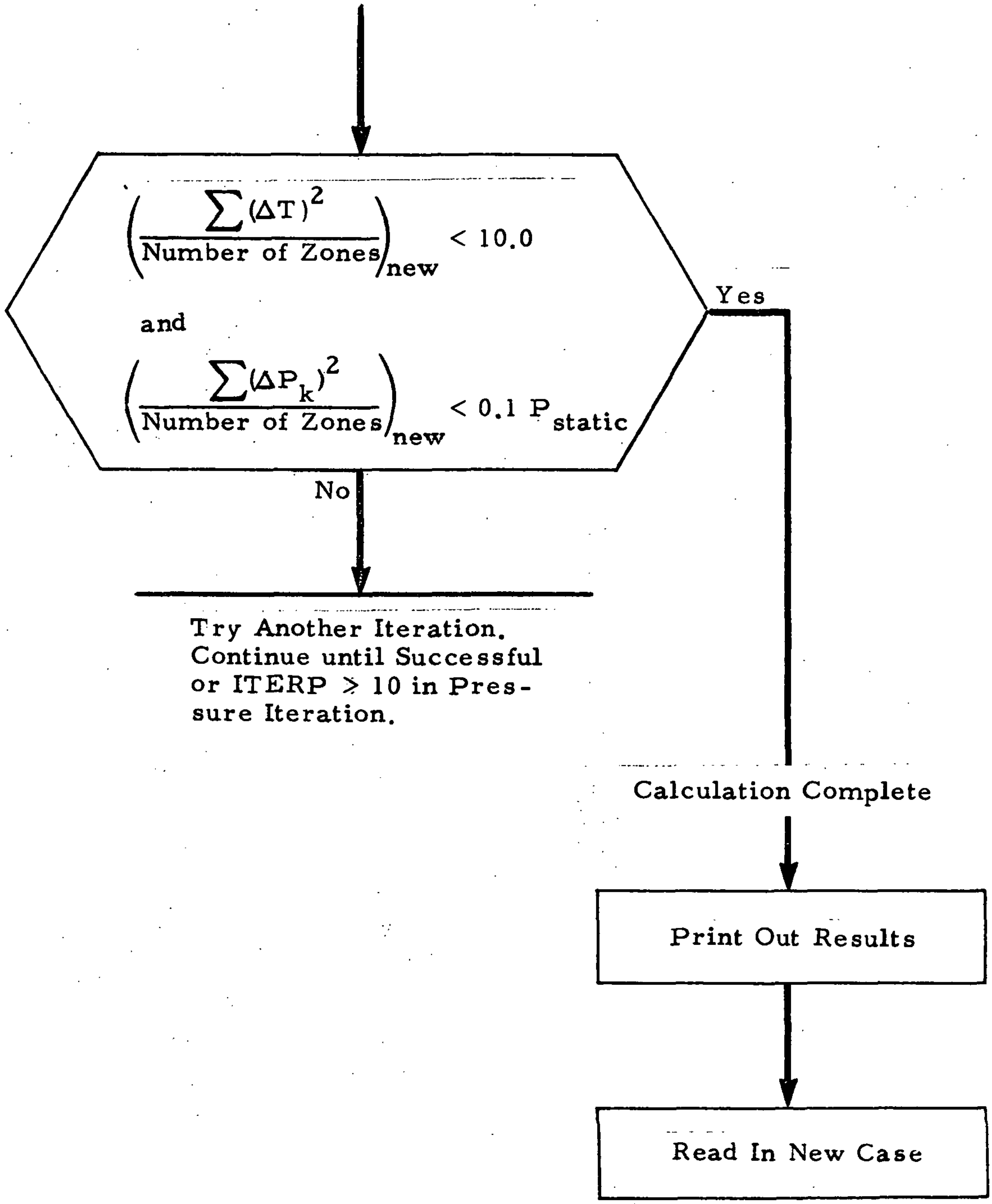
1. Number of Iteration
2. Number of Zone
3. Band Model Absorption Coefficient
4. Partial Pressure of Radiating Species
5. Temperature
6. Fine Structure Parameter


Yes
ITERP < 0
No
Set Up Iteration Procedure
Evaluate Attenuation Coefficient ZAT(I, J) Using Statistical Model
$ZAT(I, J) = e^{-kS/(1+\frac{kS}{4a})^{1/2}}$
Go to Begin Iteration Loop and Solve for New Temperature


ITER = 1
Yes
No
ITERP > 0
Yes
No
Store Present T_i and P_{k_i} values
Loop to Obtain Consistent Partial Pressures
Compute Zone Path Lengths
$B(I, J) = \ell_{i,j} = [2(\text{zone width})\,DZ(I-1, J-1)]$
Modify for Statistical Representation
$B(I, J) = \frac{B(I, J)}{(1 + kS/4a)^{1/2}}$
Establish Matrix Equations
$-\ell n \tau_j = B(I, J)\,(k\,P_{k_i})$


Solve for New $(k\ P_k)_i$ Values
Check the Deviation from Previously Calculated $(k\ P_k)_i$ Values
Are the $(k\ P_k)_i$ Values Behaving ?
Yes
No
Do Up to 10 Iterations on Partial Pressure Values
Set INTERP = -100 Which Allows up to 110 Iterations on Partial Pressures and Absorption Coefficients
ITER > 10
Yes
No
QUIT
Write Out DIVERGING TOO BAD
Return to Compute k_i, P_{k_i} and a_i


ITERP < 0
Yes
No
QUIT - Write Out
DIVERGING TOO BAD
Return to Evaluate Attenuation Matrix and Solve for Zonal Blackbody Temperatures
ITER > 2
Yes
No
Bypass Deviation Check and Do Another Iteration.
$\left(\frac{\sum(\Delta T)^2}{\text{Number of Zones}}\right)_{new} > \left(\frac{\sum(\Delta T)^2}{\text{Number of Zones}}\right)_{old}$
or
$\left(\frac{\sum(\Delta p_k)^2}{\text{Number of Zones}}\right)_{new} > \left(\frac{\sum(\Delta p_k)^2}{\text{Number of Zones}}\right)_{old}$
Yes
No
QUIT - Write Out
DIVERGING TOO BAD


$\left(\dfrac{\sum(\Delta T)^2}{\text{Number of Zones}}\right)_{new} < 10.0$
and
$\left(\dfrac{\sum(\Delta P_k)^2}{\text{Number of Zones}}\right)_{new} < 0.1\ P_{static}$
Yes
No
Try Another Iteration. Continue until Successful or ITERP > 10 in Pressure Iteration.
Calculation Complete
Print Out Results
Read In New Case

LISTING OF THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

```
C     SUBROUTINE MAIN
C     ZONE RADIOMETRY DATA REDUCTION                                            30
C                                                                               60
C     DESIGNED TO PROCESS DATA ON CYLINDRICALLY SYMMETRIC PLUMES           00000070
C     USES SIMULTANEOUS EQUATION APPROACH TO OBTAIN TEMPERATURES.         00000080
C     CAN USE NON-LINEAR ABSORPTION AND ITERATIVE TECHNIQUE FOR           00000090
C          REDUCING SPECTRALLY AVERAGED DATA                              00   100
C                                                                              110
C     MODIFIED TO INCLUDE SLIT FUNCTION CORRECTION USING TECHNIQUE FROM 00000111
C         U. OF TENNESSEE DESCRIBED IN AFCRL 465, PAGES 59-62.            00000112
C     DEFAULT OPTION NOW USES A QUADRATICALLY VARYING PRESSURE DISTRIBU-00000113
C         TION ALTHOUGH A DISCRETE STEP VARIATION CAN BE SELECTED         00000114
C                                                                              115
C     THE RADIANCE AND TEMPERATURE ARE STILL ASSUMED CONSTANT IN A ZONE 00000116
C     INPUT DATA                                                               120
C        CARD     CONTENTS                                                     130
C          1      TITLE                                                        140
C          2      NO. OF ZONES                                                 150
C                 ZONE WIDTH IN CM.                                            151
C                 CORRECTION TO PLUME RADIANCE DUE TO WINDOW ABSORPTION   00000152
C                 BLACKBODY RADIANCE IN WATTS /(CM**2 MICRON STER)        00000153
C                 QUADRATIC PROPERTY VARIATION (0) OR STEP FUNCTION (1)   00000154
C                 SLIT FUNCTION CORRECTION CONTROL PARAMETER              00000160
C                     = 1 TO APPLY A NEW FUNCTION                         00* *161
C                     = 0 TO APPLY PREVIOUSLY USED FUNCTION               00000162
C                     =-1 TO NOT CORRECT FOR SLIT FUNCTION                00000163
C                 DATA TYPE = 0 OR 1 FOR SPECTRAL DATA                    00000170
C                             2 FOR SPECTRALLY AVERAGED DATA              00000180
C                             POSITIVE FOR SMOOTHED INPUT                 00000190
C                     ZERO OR NEGATIVE FOR DEFLECTION VALUES AS INPUT     00000200
C                 WAVELENGTH IN MICRONS                                   0    205
C        DEFLECTION DATA     BLACKBODY                                    0    210
C                            FLAME                                             220
C                            BLACKBODY PRIME                              00   230
C                            FLAME PRIME                                  00   240
C                            GREYBODY ALONE                               00   250
C                            GREYBODY THRU FLAME                          000  260
C                            GREYBODY ALONE PRIME                         00000270
C                            GREYBODY THRU FLAME PRIME                    00000280
C            OR                                                                290
```

```
C        SMOOTHED DATA       RADIANCE                                    *    300
C                            TRANSMISSION                                *0   310
C              6 POINTS / CARD                                                320
C                                                                             330
C        ABSORPTION COEFFICIENT PARAMETERS FOR SPECT. AVG. DATA         00000340
C        TO USE PREVIOUS PARAMETERS SET DATA TYPE = 3                   00000350
C                                                                             360
      REAL * 4   SMN(30,30), DZ(30,30), ZAT(30,30), PAL(30,30), AM(18), 00000370
     1     B(30,30), T(30), TN(30), TP(30), ENF(30), EFN(30), ENNF(30),00000380
     2     ENPF(30), EPS(30), BF(30), AKP(30), RAD(30), TB(30), X(30), 00000390
     3     DF(30), DGB(30), DGBF(30), DGBP(30), DGBFP(30), DFP(30),    00000400
     4     DBBP(30), DBB(30), AK(30), P(30), POLD(30), TOLD(30),       00000410
     5     ASTAR(30)                                                          420
C                                                                             430
      INTEGER * 4  IA(30)                                                     440
C                                                                             450
      COMMON  N, A, SMN, DZ, B, AKP, AVKP(30,30)                        00   451
C                                                                             460
C                                                                             470
    1 FORMAT ( 18A4)                                                          480
    2 FORMAT (I12, 3F12.8, 2I3, I6, F12.8)                              00   490
    3 FORMAT (6F12.8)                                                         500
    4 FORMAT ('1', 18X, 18A4 / '-', 7X, 'K', 9X, 'TN(K)', 6X, 'TP(K)', 00000510
     1    9X, 'T(K)', 7X, 'NFN(K)', 6X, 'NFP(K)', 6X, 'NF(K)' / '0')   00000520
    5 FORMAT ('1', 18X, 18A4 / '-', 7X, 'K', 7X, 'T(K)', 8X, 'NF(K)'/) 00000530
    6 FORMAT (' ', I8, 2F12.4)                                                540
    7 FORMAT (' ', I8, 6F12.4)                                                550
    8 FORMAT ('1', 18X, 18A4 / '0', 18X, 'RADIAL PROPERTIES' / '0', 9X, 00000560
     1    'K', 6X, ' KP(K)', 6X, 'EPS(K)', 6X, 'RAD(K)', 7X, 'T(K)' /  00000570
     2    30(' ', I10, 3F12.4, 6X, F6.0 / ))                            00   580
    9 FORMAT ('1', I8 / 150(' ', 6F12.8 / ))                            00   590
   10 FORMAT ('1 ITERATION NO.',I3 / '0', 5X,'ZONE', 8X, 'K', 11X, 'P', 00000600
     1    11X, 'T', 11X, 'A*' / 30(' ', I8, 2F12.4, F12.1, F12.4/))    00000610
   11 FORMAT ('1',18X,18A4/ '0INPUT DATA',I12,3F12.4,2I3,I6,F12.4)      00000620
   12 FORMAT ('0 ITERATION NO.', I3, '  WAS CONSISTENT AT ITERP =' I3)  00000630
   13 FORMAT ('0', 18X, I3, ' ITERATIONS WERE REQUIRED' )               00000640
   14 FORMAT (19X, 'SLIT CORRECTION INCLUDED')                          00   642
   15 FORMAT (19X, 'STEP FUNCTION PROPERTY DISTRIBUTION ASSUMED')       00000644
   16 FORMAT ('0', 18X, 'ZONE WIDTH =', F6.3, ' CM USED TO COMPUTE EPS')00000646
C     GENERATE SMN AND X                                                      650
```

```
      DO 104  I = 1,30
      X(I) = I
      DO 104  J = 1,30
  104 SMN(I,J) = SQRT(FLOAT(J*J - I*I))
C
C     REDUCED PATH LENGTH MATRIX, 0TH LOS OMITTED - DZ(LOS,ZONE)=(Y2-Y1)
      DO 110  I = 1,29
      L = I - 1
      DO 108  J = 1,L
  108 DZ(I,J) = 0.0
      DO 110  J = I,29
  110 DZ(I,J) = SMN(I,J+1) - SMN(I,J)
C
C     GENERATE QUADRATIC FUNCTION WEIGHTING FACTORS
      CALL QUADO
C
C     READ TITLE AND PARAMETER CARDS
C     LOOP TO HERE FOR MULTIPLE PROBLEMS
C
  120 READ (5,1) AM
      READ (5,2) N,A,WC,ENBB,IQA,ISF,KDATA,WL
      WRITE (6,11) AM, N, A, WC, ENBB, IQA, ISF, KDATA, WL
      IF (ISF.EQ.1)  CALL SLIT1
      IF (N.EQ.0)  GO TO 120
      IF (KDATA.GT.0)  GO TO 150
C     READ DEFLECTION DATA
      READ (5,3) (DBB(K),K=1,N)
      READ (5,3) (DF(K), K=1,N)
      READ (5,3) (DBBP(K),K=1,N)
      READ (5,3) (DFP(K), K=1,N)
      READ (5,3) (DGB(K), K=1,N)
      READ (5,3) (DGBF(K),K=1,N)
      READ (5,3) (DGBP(K),K=1,N)
      READ (5,3) (DGBFP(K),K=1,N)
C
C     GET TRANSMISSION AND RADIANCE FROM DEFLECTIONS
C
      CAL = WC * ENBB
      DO 145  K = 1,N
      TN(K) = DGBF(K) / DGB(K)
```

```
      TP(K) = DGBFP(K) / DGBP(K)
      IF (TN(K).GT.1.0)  TN(K) = 1.0
      IF (TP(K).GT.1.0)  TP(K) = 1.0
      T(K) = 0.5 * (TN(K) + TP(K))
      ENNF(K) = CAL * DF(K) / DBB(K)
      FNPF(K) = CAL * DFP(K) / DBBP(K)
  145 ENF(K)  = 0.5 * (ENNF(K) + ENPF(K))
      WRITE (6,4) AM
      WRITE (6,7) (K,TN(K),TP(K),T(K),ENNF(K),ENPF(K),ENF(K),K=1,N)
      GO TO 160
C
C     READ SMOOTHED DATA
C
  150 READ (5,3) (ENF(K), K=1,N)
      READ (5,3) (T(K), K=1,N)
C
C     LIST LINE OF SIGHT TRANSMISSION AND RADIANCE
  155 WRITE (6,5) AM
      WRITE (6,6) (K,T(K),ENF(K), K=1,N)
C
C
C
C     CHECK FOR SLIT FUNCTION
  160 IF (ISF.LT.0)  GO TO 165
      CALL SLITO(T)
      CALL SLITO(ENF)
C
C     DELETE EXTRA POINTS
  161 IF (T(N).LT.1.0)  GO TO 162
      N = N - 1
      GO TO 161
C     LIST CORRECTED PROPERTIES
  162 ISF = - 1
      WRITE (6,163) N
  163 FORMAT ('0MODIFIED BY APPLYING SLIT FUNCTION', I6, ' POINTS LEFT')
      GO TO 155
C     CONVERT TRANSMITTANCE
  165 DO 170  K = 1,N
  170 FEN(K) = -ALOG(T(K))
C
```

```
C     GET WEIGHTED LOS MATRIX IN B                                     *  1270
      CALL QUAD1(IQA)                                                     1280
C                                                                         1290
C     SOLVE MATRIX EQUATION  LN T = DZ * KP                           00  1300
      DO 175   I = 1,N                                                    1310
  175 TP(I) = EFN(I)                                                      1320
      F = 1.0                                                             1380
      K = 1SIMEQ (30,N,1,B,TP,F,IA)                                    0  1390
      IF (K.GT.1) WRITE (6,9) K, B                                     0  1400
C                                                                         1410
      DO 1798 I = 1,N                                                     1420
 1798 AKP(I) = B(I,1)                                                     1430
C                                                                         1440
C     GET MATRIX OF AVERAGE VALUES -  AVKP(LOS+1, ZONE+1) - INCLUDES A 00001442
      CALL QUAD2(IQA)                                                     1444
C                                                                         1446
C     COMPUTE BEER'S LAW ATTENUATION FOR EACH ZONE ALONG EACH LOS      00001450
C     ZAT(LOS+1,ZONE+1)                                                   1460
      DO 185   I = 1,N                                                    1470
      DO 185   J = I,N                                                    1480
  185 ZAT(I,J) = EXP(-AVKP(I,J))                                          1490
C                                                                         1520
C     RE-ENTRY POINT FOR ITERATIVE CALCULATION                        000 1530
      ITER = 0                                                            1540
C                                                                         1550
C     COMPUTE ATTENUATION MATRIX  PAL(LOS,ZONE)                        00001560
  200 DO 218   I = 1,N                                                    1570
      PAL(I,N) = 1.0                                                      1580
C     WORK IN FROM ZONE N-1 TO CENTER                                 00  1590
      DO 209   J = 2,N                                                    1600
      NJ = N + 1 - J                                                      1610
C     NJ = N-1,1,-1                                                       1620
      PAL(I,NJ) = 0.0                                                     1630
      IF (NJ.LT.I) GO TO 209                                              1640
      PAL(I,NJ) = PAL(I,NJ+1) * ZAT(I,NJ+1)                           000 1650
  209 CONTINUE                                                            1660
C                                                                         1670
C     WORK BACK OUT FROM SYMMETRY PLANE                               00  1680
C     ADDITIVE CONTRIBUTION FROM OTHER HALF                           000*1690
C     KEEP MULTIPLYING TO GET ATTENUATION                             00* 1700
```

```
C     MULTIPLY BY EMISSIVITY = 1 - ZAT FOR EACH ZONE AFTER ADDITION
C
      ATT = PAL(I,I) * ZAT(I,I)
      DO 218  J = I,N
      PAL(I,J) =(PAL(I,J) + ATT) * (1 - ZAT(I,J))
  218 ATT = ZAT(I,J) * ATT
C
C     SAVE LOS RADIANCE
      DO 222  I = 1,N
  222 RAD(I) = FNF(I)
C     SOLVE FOR ZONAL BLACKBODY FUNCTIONS
      F = 1.0
      K = ISIMEQ(30,N,1,PAL,RAD,F,IA)
      IF (K.GT.1)  WRITE (6,9) K,PAL
C
      DO 229  I = 1,N
      EPS(I) = 1.0 - ZAT(I,I)
  229 RAD(I) = PAL(I,1)
C
C     FIND BLACKBODY TEMP
      IF (WL.LE.0.0)  GO TO 270
      DO 235  I = 1,N
      IF (RAD(I) .LT. 0.001)  RAD(I) = 0.001
      TS = ALOG (1.0 + 11909.0 / (RAD(I) * WL**5))
      TB(I) = 14388.0 / (WL *TS)
  235 CONTINUE
C
C     CHECK FOR ITERATION
      IF (IABS(KDATA).GT.1)  GO TO 300
C
C     DONE  -  OUTPUT
  250 WRITE (6,8) AM, (K, AKP(K), EPS(K), RAD(K), TB(K), K=1,N)
      WRITE (6,16) A
      IF (ISF .GE. 0)  WRITE(6,14)
      IF (IQA .NE. 0)  WRITE(6,15)
      IF (IABS(KDATA).GT.1)  WRITE (6,13) ITER
C
      DO 252  I = 1,N
      IF (TB(I).LT.4000.0)  GO TO 252
      TB(I) = 0.0
```

```
      RAD(I) = 0.0
  252 CONTINUE
C     MAKE CHARTS
      N = N + 1
      TB(N) = 0.0
      RAD(N) = 0.0
      EPS(N) = 0.0
C
C     GO GET NEXT DATA SET
      GO TO 120
C
C     DATA ERROR
  270 WRITE (6,271) WL
  271 FORMAT ('0 BAD VALUE FOR WL=', E12.2)
      IF (IABS(KDATA).LE.1)  GO TO 120
      READ (5,1) DUMMY
      READ (5,1) DUMMY
      GO TO 120
C
C
C     ITERATE -  IS THIS FIRST TIME
  300 IF (ITER.GT.0) GO TO 350
      IF (IABS(KDATA).EQ.3)  GO TO 340
C     READ DATA ON K AND A
C
C     LN K = F0 + F1/T + F2/T**2 + F3/T**3
C     LOG ABAR  = A0 + A2*T**2
C     DATA AND ANALYSIS ARE FROM GD/C REPORT OF DEC. 1966
      READ (5,307) F0, F1, F2, F3, A0, A2
      READ (5,307) PT
  307 FORMAT (6E12.7)
C
  340 WRITE (6,341) F0,F1,F2,F3,A0,A2,PT
  341 FORMAT (19X, 'BAND MODEL DATA USED' / 19X, 6E12.4/ 19X,'PT =',
     1          F12.4)
C
  350 ITER = ITER + 1
      ITERP = 0
C
C     COMPUTE K FOR EACH ZONE - ALSO P AND ASTAR
```

```
  353 DO 355  I = 1,N
      T(I) = TB(I)
      IF (T(I).GT.4000.0)  T(I) = 4000.0
      AK(I) = EXP (FO + F1/T(I) + F2/T(I)**2 + F3/T(I)**3)
      AK(I)=(273.0/T(I)).AK(I)
      P(I) = (AVKP(1,I) / A) / AK(I)
      TS = T(I) / 273.0
      THETA = 0.44 / TS  + 0.09 / SQRT(TS)
      CSTAR = -0.1002 + TS*(0.076495 - TS*(0.008116 - TS * 0.000592))
      G = THETA * P(I)  +(PT -P(I)) * 0.044 / SQRT(TS)
      GS = THETA * CSTAR + (1.0 - CSTAR) * 0.044 / SQRT(TS)
  355 ASTAR(I) = 10.0 ** (A0 + A2*T(I)**2)  * G / GS
C
      WRITE (6,10)  ITER, (K, AK(K), P(K), T(K), ASTAR(K), K=1,N)
      IF (ITERP.LT.0)  GO TO 406
      IF (ITER.EQ.1)  GO TO 377
      IF (ITERP.GT.0)  GO TO 377
      STO = ST
      SPO = SP
      ST = 0.0
      SP = 0.0
C     CALCULATE DEVIATION
      DO 365  I = 1,N
      ST = ST + (T(I)-TOLD(I))**2
  365 SP = SP + (P(I)-POLD(I))**2
      ST = SQRT (ST / N)
      SP = SQRT (SP / N)
      WRITE (6,368) ITER, ST, SP
      IF(ITER.EQ.2)  GO TO 377
  368 FORMAT ('0 ITER =',I3, '  ST =', F12.2, '  SP =' F12.4)
C
C     ARE WE DOING BETTER
      IF (ST.GT.STO.OR.SP.GT.SPO)  GO TO 425
      IF (ST.LT.10.0.AND.SP.LT.0.01*PT) GO TO 250
C
C     TRY AGAIN
C
C     STORE PRESENT VALUES
  377 DO 379  I = 1,N
      TOLD(I) = T(I)
```

```
  379 POLD(I) = P(I)                                                    2940
C                                                                       2950
C                                                                       2960
C     LOOP TO OBTAIN CONSISTENT PRESSURES                            00 2970
C     B = 2*L /(1+KPL/4A)**0.5                                          2980
C     B * KP = -LN T                                                    2990
      CALL QUAD1(IQA)                                                   2995
      DO 386  I = 1,N                                                   3000
      TP(I) = EFN(I)                                                    3010
      DO 386  J = 1,N                                                   3020
C                CONVERT FOR STATISTICAL REPRESENTATION                 00003030
  386 B(I,J) = B(I,J)  /  SQRT(1.0 + AVKP(I,J) / (4.0 * ASTAR(J)))      00003070
      F = 1.0                                                           3080
      K = ISIMEQ (30,N,1,B,TP,F,IA)                                   0 3090
      IF (K.GT.1) WRITE (6,9) K,B                                     0 3100
C     B(I,1) CONTAINS NEW KP                                            3110
      DO 398  I = 1,N                                                   3120
      DELTA =(B(I,1) - AKP(I)) / AKP(I)                              00 3130
  398 IF (ABS(DELTA).GT.0.01)  GO TO 400                             00 3140
      WRITE (6,12) ITER, ITERP                                          3150
      ITERP = - 100                                                     3160
  400 DO 401  I = 1,N                                                   3170
  401 AKP(I) = B(I,1)                                                   3180
      CALL QUAD2(IQA)                                                   3185
      IF (ITERP.LT.0)  GO TO 353                                      0 3190
C     NEED TO ITERATE ON P                                              3200
      WRITE (6,403) ITERP                                               3210
      ITERP = ITERP + 1                                                 3220
  403 FORMAT ('0 ITERP = ',I3)                                          3230
      IF (ITERP.GT.10)  GO TO 425                                     0 3240
      GO TO 353                                                         3250
C                                                                       3260
C     SET UP ZAT USING STATISTICAL MODEL                             00 3270
  406 DO 411  I = 1,N                                                   3280
      DO 411  J = I,N                                                   3310
  411 ZAT(I,J) = EXP(-AVKP(I,J)/SQRT(1.0 + AVKP(I,J)/(4.0 * ASTAR(J)))) 00003330
C     SOLVE FOR NEW TEMP                                                3340
      GO TO 200                                                         3350
C                                                                       3360
C     DIVERGING                                                         3370
```

```
425 WRITE (6,426) ITER, ST, STO, SP, SPO
426 FORMAT ('1 DIVERGING - TOO BAD - ITER =' I4, 'ST =' F12.2,
   1      'STO =', F12.2, 'SP =', F12.2, 'SPO =', F12.2)
C      GIVE UP
       GO TO 250
       END
```

```
      SUBROUTINE QUAD
C
C     USED WITH ZR-5 TO REPLACE STEP FUNCTION WITH QUADRATIC
C     IQA .NE. 0 REVERTS TO STEP FUNCTION
C     3 ENTRIES9 QUAD0       GENERATES WEIGHT MATRICES  WIN(LOS+1,ZONE+1)
C                QUAD1       SUMS WIN TO GET COEFFICIENT OF AKP(ZONE+1)
C                QUAD2       DOES AVKP = WIN * AKP
C     WIN DOES NOT CONTAIN ZONE LENGTH = A  SO NEEDS ONLY TO BE SET ONCE
C
C     BOTH QUAD1 AND QUAD2 APPLY LENGTH FACTOR
C
      COMMON  N, A, SMN(30,30), DZ(30,30), B(30,30), AKP(30),AVKP(30,30)
C
      REAL *8  WIN(30,30,3), C0, C1, C2, DY, Y1, Y2
C
C                SETUP WEIGHT MATRIX
      ENTRY QUAD0
      DO 3    I = 2,30
C     ZERO EXTRA ELEMENTS FOR THIS LOS+1
      I1 = I - 1
      DO 1     J = 1,I1
      DO 1     K = 1,3
    1 WIN(I,J,K) = 0.0
C
C     GET 'EM FOR EACH ZONE+1
      C0 = (I-1) * (I-1)
      DO 3     J = 1,30
      Y1 = DSQRT((J-1)*(J-1) - C0)
      Y2 = DSQRT(J * J - C0)
      DY = Y2 - Y1
      C1 = C0 + Y1 * Y2
      C2 = 12.0 * (Y2 + C0 * DLOG((Y2 + J) / (Y1 + J - 1))) / DY
      DY = DY / 24.0
      WIN(I,J,1) = DY *(1.0 + 4.0 *(C1 + J * (1. + 2 * J)) - J * C2)
      WIN(I,J,2) = DY *(22.0 + 4*J*(1-4*J) - 8.0 * C1 + (2*J-1) * C2)
    3 WIN(I,J,3) = DY *(1.0 + (J-1) * (8.*J - C2) + 4.0 * C1)
C
C     DO FIRST ROW = 0TH LOS
      DO 4     J = 1,30
      WIN(1,J,1) = 0.04167
```

```
      WIN(1,J,3) = 0.04167
    4 WIN(1,J,2) = 0.91666
      GO TO 20
C
C     SUM APPROPRIATE WIN ELEMENTS                                       0
      ENTRY QUAD1(IQA)
      DY = 2.0 * A
      IF (IQA.NE.0)  GO TO 7
      K = N - 1
      DO 6   I = 1,N
      B(I,1) = 0.0
      DO 5   J = 2,K
    5 B(I,J) = DY * (WIN(I,J-1,3) + WIN(I,J,2) + WIN(I,J+1,1))           0000
    6 B(I,N) = DY * (WIN(I,J-1,3) + WIN(I,J,2))                          000
      B(1,1) = DY
      B(2,1) = DY * WIN(2,2,1)
      GO TO 20
C
C     STEP FUNCTION OPTION
    7 DO 8   I = 1,N
      B(I,1) = 0.0
    8 B(1,I) = DY
      DO 9   I = 2,N
      DO 9   J = 2,N
    9 B(I,J) = DY * DZ(I-1,J-1)
      GO TO 20
C
C     CALCULATE AVKP
      ENTRY QUAD2(IQA)
      DY = A
      IF(IQA.NE.0)  GO TO 11
      AKP(N+1) = 0.0
      AVKP(1,1) = DY*((WIN(1,1,1)+WIN(1,1,2))*AKP(1)+ WIN(1,1,3)*AKP(2))0000
      K = N - 1
      DO 10  I = 2,N
      AVKP(1,I) = DY*(WIN(1,I,1)*AKP(I-1) + WIN(1,I,2)*AKP(I)           0000
     1               + WIN(1,I,3)*AKP(I+1))                              00
      AVKP(I,N) = DY * (WIN(I,N,1) * AKP(N-1) + WIN(I,N,2) * AKP(N))     0000
      AVKP(I,1) = 0.0
      DO 10  J = 2,K
```

```
10 AVKP(I,J) = DY * (WIN(I,J,1) * AKP(J-1) + WIN(I,J,2) * AKP(J)      00004730
  1                  +  WIN(I,J,3) * AKP(J+1))                         000 4740
   GO TO 20                                                                4760
C                                                                          4770
C     NON-QUAD AVKP                                                        4780
11 AVKP(1,1) = DY * AKP(1)                                                 4790
   DO 12  I = 2,N                                                          4800
   AVKP(1,I) = DY * AKP(I)                                                 4810
   AVKP(I,1) = 0.0                                                         4815
   DO 12  J = 2,N                                                          4820
12 AVKP(I,J) = DY * DZ(I-1,J-1) * AKP(J)                               00  4830
20 RETURN                                                                  4840
   END                                                                     4850
```

```
      SUBROUTINE SLIT
C     ROUTINE TO CORRECT FOR SPATIAL RESOLUTION ELEMENT
C     TWO ENTRIES9 SLIT1     READ A NEW SLIT FUNCTION
C                  SLITO     APPLY SLIT FUNCTION
C
      COMMON  NZONE, A, DUMMY(3630)
      REAL * 4  SFO(50), SF1(50), X(400), Y(400), Z(400), T(30)
      INTEGER   JCHAR(2)/55,16/
C
C     READ SLIT FUNCTION
      ENTRY SLIT1
      READ (5,1) NSO, ASO, (SFO(I+1), I=1,NSO)
    1 FORMAT (I12, 5F12.6 / (6F12.6))
C                NSO = NO. OF ELEMENTS TO BE READ IN
C                ASO = SPACING BETWEEN ELEMENTS IN CM.
      WRITE (6,10) NSO, ASO, (I,SFO(I+1), I=1,NSO)
   10 FORMAT ('OSLIT FUNCTION INPUT' / '0', I3, ' POINTS WITH', F7.4,
     1        ' CM SPACING' // (' ', I6, F12.4))
      SFO(1) = 0.0
      SFO(NSO+2) = 0.0
      AS1 = 0.
C
      GO TO 99
C
C     APPLY SLIT FUNCTION
      ENTRY SLITO(T)
C     EXPAND T BY FACTOR OF 10 USING CUBIC FIT TO 4 POINTS
C     USE LINEAR WEIGHTED AVERAGE TO REDUCE DISCONTINUITIES
C     USE SYMMETRY TO GET EXTRA POINTS IN CENTER
C     0TH LOS GOES TO X(31)
      X(1) = T(4)
      X(11) = T(3)
      X(21) = T(2)
      DO 2   I = 1,NZONE
    2 X(10*I + 21) = T(I)
C     EXTEND OUTER EDGE
      K = 10 * NZONE  + 31
      X0 = 0.0
      IF (T(NZONE) .GT. T(NZONE-1))     X0 = 1.0
      IF (T(NZONE) .EQ. T(NZONE-1))     X0 = T(NZONE)
```

```
      X(K) = XO
      X(K+10) = X(K-10)
C     FILL IN GAPS
      DO 20  I = 31,K,10
      XO = X(I)
      X2 = (X(I+10) + X(I-10) - 2.0 * XO) / 400.0
      X3 = XO - X(I-10) - (X(I+10) - X(I-20)) / 3.0
      X1 = (X(I-10) - X(I+10) - X3) / 40.0
      XO = XO / 2.0
      X3 = X3 / 4000.0
      DO 20  J = 1,9
      X(I-J) = XO + J * (X1 + J * (X2 + J * X3))  + X(I-J)
      C = (10. - J) / 10.0
      X(I+J) =(XO - J * (X1 - J * (X2 - J * X3))) * C
      L = J + 10
   20 X(I-L) =(XO + L * (X1 + L * (X2 + L * X3))) * C  + X(I-L)
C
C     ADJUST LAST ZONE USING QUADRATIC CONTRIBUTION
      C = (X(K-10) - X(K)) / 2000.0
      X1 = X(K) / 20.0
      DO 22  I = 1,9
      J = 10 - I
      XO = X(K-J) +  I * (X1 + C * J * J)
      IF (XO .LT. 0.0) XO = 0.0
      IF (XO .GT. 1.0) XO = 1.0
      IF (ABS(XO-X(K)) .GE. ABS(X(K-J-1)-X(K)))XO= X(K) + (J/(J+1.0))
     1  ** 2 * (X(K-J-1) - X(K))
   22 X(K-J) = XO
C
C     FIND FIRST CROSSING POINT
      XO = X(K)
      IF(XO .EQ. 1.0)  GO TO 24
      DO 23  I = 62,K
   23 IF(X(I).LE. 0.0) GO TO 26
   24 DO 25  I = 62,K
   25 IF(X(I).GE. 1.0) GO TO 26
   26 DO 27  J = I,K
   27 X(J) = XO
      DO 28  I = 1,30
      X(K+I) = XO
```

```
   28 X(I) = X(62 -I)
C
C     ADJUST SLIT FUNCTION SPACING TO MATCH DATA
C
      IF (AS1 .EQ. A/10.)  GO TO 55
      AS1 = A/10.
      AS2 = AS1 / AS0
C     EQUALLY SPACED - SYMMETRY NOT ASSUMED - GIVEN CENTER TO EDGE
C     NO. OF POINTS TO MATCH DATA SPACING
      NS1 =((NS0 - 1) / AS2 + 1.75)
C     MAKE NEW SLIT FUNCTION ODD
      IF (MOD(NS1,2).EQ.0)  NS1 = NS1 + 1
C
      IC0 = NS0 / 2 + 2
      IC1 = NS1 / 2 + 1
      J = NS1 - IC1
      IF (MOD(NS0,2).EQ.0)  GO TO 45
C     ODD NO. IN ORIGINAL
C     ODD - ODD
      SF1(IC1) = SF0(IC0)
      DO 30  I = 1,J
      X0 = I * AS2
      N = X0
      X0 = X0 - N
      M = N + IC0
      SF1(IC1 + I) = SF0(M) + X0 *(SF0(M+1) -  SF0(M))
      M = IC0 - N
   30 SF1(IC1 - I) = SF0(M) + X0 * (SF0(M-1) - SF0(M))
      GO TO 52
C

C     EVEN IN ORIGINAL
   45 SF1(IC1) = 0.5 * (SF0(IC0-1) + SF0(IC0))
      DO 50  I = 1,J
      X0 = I * AS2 - 0.5
      N = X0
      X0 = X0 - N
      M = IC0 + N
      SF1(IC1 + I) = SF0(M) + X0 * (SF0(M+1) - SF0(M))
      M = IC0 - 1 - N
```

```
50 SF1(IC1 - I) = SFO(M) + XO * (SFO(M-1) - SFO(M))
C
C
C     NORMALIZE
52 C = 0.0
   DO 53  I = 1,NS1
   IF (SF1(I).LT.0)  SF1(I) = 0.0
53 C = C + SF1(I)
   DO 54  I = 1,NS1
54 SF1(I) = SF1(I) / C
C
   WRITE (6,61)  NS1, AS1, (I, SF1(I), I = 1,NS1)
61 FORMAT ('1SLIT FUNCTION USED' / '0', I3, ' POINTS WITH', F7.4,
  1        ' CM SPACING' / 50(' ', I6, F7.4/))
C
C     SET LOOP PARAMETERS
55 J = NS1 / 2
   JO = J + 1
C     FIND LAST OBSERVED DEVIATION
   XO = X(K)
   I = K - 1
56 IF (X(I).NE.XO)  GO TO 57
   I = I - 1
   GO TO 56
57 KMAX = I - J
C     KMAX IS LAST ALLOWED DEVIATION IN TRUE FUNCTION - 1ST TRY Z = X
   DO 58  I = 1,KMAX
58 Z(I) = X(I)
C     CLAMP END OF TRUE FUNCTION
   DO 59  I = KMAX,K
59 Z(I) = XO
C
   NCY = 0
C                LOOP FOR SLIT CORRECTION
69 DO 68  I = JO,KMAX
   C = SF1(JO) * Z(I)
   DO 67  L = 1,J
67 C = C + SF1(JO - L) * Z(I - L)  +  SF1(JO + L) * Z(I + L)
68 Y(I) = C
C
```

```
C     CORRECT AND COMPARE                                                    6290
      C3 = 0.0                                                               6300
      C1 = 0.0                                                               6310
      DO 70  I = J0,KMAX                                                     6320
      C2 = X(I) - Y(I)                                                       6330
      Z(I) = Z(I) + C2                                                       6340
      C2 = ABS(C2)                                                           6360
      IF (C2 .GT. C1)  C1 = C2                                               6370
      C3 = C3 + C2 * C2                                                      6375
   70 CONTINUE                                                               6380
      DO 71  I = 1,J                                                         6382
   71 Z(I) = Z(62 - I)                                                       6385
      C3 = SQRT (C3 / (KMAX -J))                                           * 6387
      NCY = NCY + 1                                                          6390
      WRITE (6,72) NCY, C1, C3                                               6392
   72 FORMAT(' SLIT FUNCTION CYCLE', I4, 6X, 'MAX. DEV. =', E12.3, 6X,  00006394
     1        'STD. DEV. =', F12.3)                                        * 6395
      IF(NCY.GT.10)  GO TO 75                                                6400
      IF (C1 .GT. 0.005)  GO TO 69                                         0 6410
C                                                                            6420
C                                                                            6430
   75 DO 76  I = 1,K                                                         6440
   76 Y(I) = I-1                                                             6450
C     PLOT RESULTS                                                           6460
C                                                                            6480
C     REPLACE ELEMENTS                                                       6490
      DO 80  I = 1,NZONE                                                     6500
   80 T(I) = Z(21 + 10 * I)                                                  6510
   99 RETURN                                                                 6520
      END                                                                    6530
```

```
      FUNCTION ISIMEQ( DSM , NE , NC , A , B , DET , C )                AWCT0190
C     SUBPROGRAM TO SOLVE SIMULTANEOUS LINEAR EQUATIONS                 AWCT0030
C       ARGUMENTS-                                                      AWCT0040
C                                                                       AWCT0050
C        DATE- 1/13/67   MODIFIED  FOR COMPILATION IN RELEASE 14        AWCT0060
C                                                                       AWCT0070
C                 DSM        DIMENSIONED SIZE OF COEFFICIENT MATRIX     AWCT0080
C                 NE         ACTUAL NUMBER OF EQUATIONS FOR THIS CALL   AWCT0090
C                 NC         NUMBER OF COLUMNS IN CONSTANT MATRIX       AWCT0100
C                 A          COEFFICIENT MATRIX                         AWCT0110
C                 B          CONSTANT MATRIX                            AWCT0120
C                 DET        INPUT - SCALE FACTOR, OUTPUT - FACTOR TIMES AWCT0130
C                            DETERMINANT VALUE OF COEFFICIENT MATRIX    AWCT0140
C                 C          TEMPORARY STORAGE FOR SUBROUTINE           AWCT0150
C                 ISIMEQ     RETURNS 1 IF OK, 2 IF OVFLO, 3 IF SINGULAR AWCT0160
C                 IF NC IS NEGATIVE, THE INVERSE OF THE COEFFICIENT     AWCT0170
C                 MATRIX IS REQUIRED, MATRIX B IS SET UP AS IDENTITY.   AWCT0180
      LOGICAL DVO                                                       AWCT0200
      INTEGER DSM, C, T, SUB1, SUB2,     R, D                           AWCT0210
      DIMENSION  B(1), C(1)                                             AWCT0220
C     INITIALIZE                                                        AWCT0230
      N            =  NE                                                AWCT0240
      D = DSM                                                           AWCT0250
      M = IABS(NC)                                                      AWCT0260
      ISIMEQ = 1                                                        AWCT0270
      DVO          =  .FALSE.                                           AWCT0280
      DO 1 I = 1,N                                                      AWCT0290
    1 C(I) = I                                                          AWCT0300
      IF(NC) 5, 15, 15                                                  AWCT0310
C     INVERSE REQUIRED                                                  AWCT0320
    5 SUB2 = 0                                                          AWCT0330
      DO 10 J = 1,N                                                     AWCT0340
        SUB1 = SUB2                                                     AWCT0350
        DO 6 I = 1,N                                                    AWCT0360
        SUB1 = SUB1 + 1                                                 AWCT0370
    6   B(SUB1) = 0.0                                                   AWCT0380
       SUB1  = SUB2 + J                                                 AWCT0390
       B(SUB1) = 1.0                                                    AWCT0400
   10 SUB2 = SUB2 + D                                                   AWCT0410
      GO TO 15                                                          AWCT0420
```

```
      ENTRY IDETRM(DSM, NE, A, DET)                                     AWCT0430
      DIMENSION  A(1)                                                   AWCT0440
      N             =  NE                                               AWCT0450
      D             =  DSM                                              AWCT0460
      DVO           =   .TRUE.                                          AWCT0480
C     START MAIN LOOP                                                   AWCT0490
   15 DO 1000 L = 1,N                                                   AWCT0500
         LP1  = L+ 1                                                    AWCT0510
        DO 40  I = L,N                                                  AWCT0520
         PIVOT  = 0.0                                                   AWCT0530
          SUB1  =  (L-1)  *  D  +  I                                    AWCT0540
          SUB2  =  SUB1                                                 AWCT0550
            DO 20  J  = L,N                                             AWCT0560
             IF(ABS(PIVOT)  .GE.  ABS(A(SUB1)))  GO  TO  20             AWCT0570
             PIVOT  =  A(SUB1)                                          AWCT0580
             JB  =  J                                                   AWCT0590
   20       SUB1  =  SUB1  +  D                                         AWCT0600
C     COMPUTE  DETERMINANT                                              AWCT0610
        DET  =  DET  *  PIVOT                                           AWCT0630
      IF(.NOT. DVO )   GO  TO  24                                       AWCT0640
C     TEST  FOR  SINGULAR  MATRIX                                       AWCT0670
   24 IF(PIVOT  .EQ.  0.0)  GO  TO  2000                                AWCT0680
            DO 25  J  =  L,N                                            AWCT0690
            A(SUB2)  =  A(SUB2)  /  PIVOT                               AWCT0700
   25       SUB2  =  SUB2  +  D                                         AWCT0710
         IF  (DVO)  GO  TO  35                                          AWCT0720
           SUB1  =  I                                                   AWCT0730
            DO 30  J  =  1,M                                            AWCT0740
            B(SUB1)  =  B(SUB1)  /  PIVOT                               AWCT0750
   30       SUB1  =  SUB1  +  D                                         AWCT0760
   35    IF  (I  .EQ.  L)  JP  =  JB                                    AWCT0770
   40   CONTINUE                                                        AWCT0780
C     INTERCHANGE  COLUMNS                                              AWCT0790
  100   IF  (JP  .EQ.  L)  GO  TO  260                                  AWCT0800
      IF  (DVO)  GO  TO  110                                            AWCT0810
        T  =  C(L)                                                      AWCT0820
        C(L)  =  C(JP)                                                  AWCT0830
        C(JP)  =  T                                                     AWCT0840
  110     P  =  D  *  L  -  D                                           AWCT0850
          T  =  D  *  JP  -  D                                          AWCT0860
```

```
      DO 120 I = 1.N                                                    AWCT0870
        SUB1 = R + I                                                    AWCT0880
        SUB2 =  T + I                                                   AWCT0890
        S = A(SUB1)                                                     AWCT0900
        A(SUB1) = A(SUB2)                                               AWCT0910
  120   A(SUB2) = S                                                     AWCT0920
      DET = -DET                                                        AWCT0930
C     REDUCE PIVOT COLUMN                                               AWCT0940
  260 R = D* L - D                                                      AWCT0950
      DO 400 I = 1.N                                                    AWCT0960
       IP = R + I                                                       AWCT0970
       PIVOT = A(IP)                                                    AWCT0980
      IF (I .EQ. L .OR. PIVOT .EQ. 0.0) GO TO 400                       AWCT0990
       SUB1 = L                                                         AWCT1000
       SUB2 = I                                                         AWCT1010
        DO 360 J = 1.N                                                  AWCT1020
         IF (J .LT. LP1) GO TO 300                                      AWCT1030
          S = PIVOT * A(SUB1)                                           AWCT1040
          A(SUB2) = A(SUB2) - S                                         AWCT1050
         IF (ABS(A(SUB2)) .LT. ABS(2.0E-6 * S)) A(SUB2) = 0.0           AWCT1060
  300   IF (DVO .OR. J .GT. M) GO TO 350                                AWCT1070
          B(SUB2) = B(SUB2) - PIVOT * B(SUB1)                           AWCT1080
  350    SUB1 = SUB1 + D                                                AWCT1090
  360   SUB2 = SUB2 + D                                                 AWCT1100
  400 CONTINUE                                                          AWCT1110
 1000 CONTINUE                                                          AWCT1120
      IF (DVO) GO TO 1500                                               AWCT1130
C     REARRANGE VARIABLES                                               AWCT1140
 1100 DO  1201   L=1.N                                                  AWCT1150
       SUB1 = C(L)                                                      AWCT1160
       SUB2 = L                                                         AWCT1170
        DO 1200 J = 1.M                                                 AWCT1180
         A(SUB1) = B(SUB2)                                              AWCT1190
         SUB1 = SUB1 + D                                                AWCT1200
 1200    SUB2 = SUB2 + D                                                AWCT1210
 1201 CONTINUE                                                          AWCT1220
 1500 RETURN                                                            AWCT1230
C     SINGULAR COEFFICIENT MATRIX                                       AWCT1240
 2000 IF(DVO)     GO TO 3000                                            AWCT1250
      ISIMEQ      =  3                                                  AWCT1260
```

```
      GO TO 1500                                                        AWCT1270
 3000 CONTINUE
      GO TO 1500                                                        AWCT1290
      END                                                               AWCT1300
```

LMSC-HREC D306101

INPUT GUIDE TO THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

LOCKHEED - HUNTSVILLE RESEARCH & ENGINEERING CENTER

INPUT GUIDE TO THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

Card	Col.	Format	Description
1	1-72	18A4	Title card
2	1-12	I12	Number of zones
	13-24	F12.8	Zone width, cm
	25-36	F12.8	Correction to plume radiance due to window absorption
	37-48	F12.8	Blackbody radiance, W/cm^2-μ-sr
	49-51	I3	Property variation
			= 0 Quadratic property variation
			= 1 Step function; constant zonal properties
	52-54	I3	Slit function correction control parameter
			= 1 Apply a new function
			= 0 Apply previously used function
			=-1 Do not correct for slit function
	55-60	I6	Data type
			= 0 or 1 For spectral data
			= 2 For spectrally averaged data
			(Positive values imply smoothed input — zero or negative values imply deflection values input)
2	61-72	F12.8	Wavelength in microns
3a			Deflection Data (or) Smoothed Data For Deflection Data
	1-12	F12.8	Blackbody Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	— —	F12.8	
	61-72	F12.8	
3b	1-12	F12.8	Flame Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	— —	F12.8	
	61-72	F12.8	

Card	Col.	Format	Description
3c	1-12	F12.8	Blackbody Prime Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3d	1-12	F12.8	Flame Prime Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3e	1-12	F12.8	Greybody Alone Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3f	1-12	F12.8	Greybody through Flame Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3g	1-12	F12.8	Greybody Alone Prime Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3h	1-12	F12.8	Greybody through Flame Prime Data; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3'a	1-12	F12.8	For Smoothed Data Line of Sight Spectral Radiance Data in W/cm^2-micron-ster; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	
3'b	1-12	F12.8	Line of Sight Transmittance Data which is dimensionless; 6 values to a card until all zone data are input
	13-24	F12.8	
	--	F12.8	
	61-72	F12.8	

SAMPLE INPUT TO THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

```
      SAMPLE CASE FOR ZONE RADIOMETRY   CO2 RADIATING
        16  0.45                                1 -1      1  4.45
1.229        1.219       1.196       1.163      1.114        1.05
 .953         .823        .647        .442       .28          .167
 .09          .033        .004        .001
 .18          .182        .184        .19        .203         .227
 .263         .31         .372        .453       .549         .661
 .784         .92         .99         .999
```

OUTPUT OF THE DATA REDUCTION PROGRAM FOR AXISYMMETRIC ZONE RADIOMETRY

```
                    SAMPLE CASE FOR ZONE RADIOMETRY   CO2 RADIATING
INPUT DATA      16      .4500      .0000      .0000  1 -1      1      4.4500
```

SAMPLE CASE FOR ZONE RADIOMETRY   CO2 RADIATING

K	T(K)	NF(K)
1	.1800	1.2290
2	.1820	1.2190
3	.1840	1.1960
4	.1900	1.1630
5	.2030	1.1140
6	.2270	1.0500
7	.2630	.9530
8	.3100	.8230
9	.3720	.6470
10	.4530	.4420
11	.5490	.2800
12	.6610	.1670
13	.7840	.0900
14	.9200	.0330
15	.9900	.0040
16	.9990	.0010

SAMPLE CASE FOR ZONE RADIOMETRY   CO2 RADIATING

RADIAL PROPERTIES

K	KP(K)	EPS(K)	RAD(K)	T(K)
1	.1800	.0778	1.9660	2159.
2	.1743	.0754	1.9430	2146.
3	.1823	.0787	1.8430	2088.
4	.1895	.0818	1.7679	2045.
5	.1922	.0829	1.6944	2002.
6	.1851	.0799	1.6733	1990.
7	.1707	.0740	1.6249	1961.
8	.1551	.0674	1.5330	1906.
9	.1366	.0596	1.3270	1781.
10	.1140	.0500	.9889	1564.
11	.0917	.0404	.7069	1367.
12	.0687	.0305	.5231	1224.
13	.0458	.0204	.4173	1133.
14	.0169	.0076	.4121	1128.
15	.0020	.0009	.3736	1093.
16	.0002	.0001	1.0000	1572.

ZONE WIDTH = .450 CM USED TO COMPUTE EPS
STEP FUNCTION PROPERTY DISTRIBUTION ASSUMED