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APPLICATION OF THE FLOATING-POTENTIAL PROBE FOR STUDIES OF LOW-FREQUENCY OSCILLATIONS IN A PLASMA

B. Ye. Dzhakov

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LIST OF SYMBOLS

C - Input capacitance of measuring device
C_1 - Capacitance between probe and unit length of column
C_s - Capacitance of layer next to probe

e - Charge of an electron
f, g - Bernoulli equation coefficients

$G = \ln(n^-/n^+)$

$H = \ln(v^-/v^+)$

i - Total current on probe
I^-, I^+ - Saturation current on probe (electrons and ions respectively)
j^-, j^+ - Electron and ion current on probe
k - Wave number of ion-sonic wave
m, M - Mass of electrons and ions
n - Amplitude of ion density oscillations
n^-, n^+ - Electron and ion densities
Δn^-, Δn^+ - Changes in electron and ion densities from equilibrium
S - Surface area of probe
t - Time
u - Floating probe potential
U - Plasma potential
U^- - Electron Temperature in volts
v^-, v^+ - see Formula (2)
z - Coordinate in direction of discharge axis
z_0 - Probe coordinate
α, β - Constants on the order of one
ε_0 - Electrical constant
λ, $1/\nu$ - Length and time of plasma heterogeneity (with periodic wave, wavelength and period)
λ_D - Debye length
ω - Frequency of ion-sonic wave
ω_i - Angular ionic plasma frequency
U_0, G_0,... - Oscillating amplitudes
$\overline{\quad}$ - Average with respect to time, for example $\overline{g(\nu t)}$

$N = \overline{n^-} = \overline{n^+}$

APPLICATION OF THE FLOATING-POTENTIAL PROBE
FOR STUDIES OF LOW-FREQUENCY OSCILLATIONS IN A PLASMA

B. Ye. Dzhakov

1. Introduction

An electric probe connected to a high-impedence input of a recording device is frequently used as a low-frequency oscillation detector. Thus, the constant component of the probe current is sharply limited. Most floating potential measurements are used for determination of the frequency and phase of oscillations, in order to confirm the dispersion equation of some type of oscillation. In the case of ionic oscillations, for example, a small electrode capacitively coupled with a plasma [1], a Langmuir probe with floating potential [2] and a metal ring surrounding a discharge tube [3] have been used. The use of a floating potential method in the case of moving striae [4, 5] has allowed determination of the instantaneous values of the main parameters of a plasma. Actually, direct recording of the dependence of electron temperature (or plasma potential) on time is possible if we use the following expression for floating probe potential /63*

$$u=U-HU^{-}, \tag{1}$$

where H is a quantity on the order of 1 [5].

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* Numbers in the margin indicate pagination in the foreign text.

The present work studies the more general cases of utilization of these methods, in particular the possibility for production of information on slight oscillations in ion density in the case of ion-sonic oscillations using the floating potential method. Little attention is given to capacitive probes, since they were covered by [6], which was recently published.

2. Limitations of Floating Probe Method in Dynamic Mode

2.1. Consideration of Charge Separation

Charge separation should, generally speaking, limit the applicability of the theories of the electron and ion current to a Langmuir probe [7]. The present work studies only those /64
slight deviations from electrical neutrality which do not distort the expressions for currents on probe significantly. Actually, in almost all cases of striae and ionic sound, the separation of charges is so slight that the theory of Langmuir probes remains correct, as was demonstrated in [8].

Let us clarify the question of the floating potential of a probe immersed in a medium with variable density of positive and negative charges. The expressions for the components of current on an ideal, flat probe are

$$j^- = I^- \exp\frac{u-U}{U^-}, \quad I^- = \frac{1}{4} en^- v^-, \quad v^- = \left(\frac{aeU^-}{m}\right)^{1/2},$$
$$j^+ = I^+, \qquad I^+ = \frac{1}{4} en^+ v^+, \quad v^+ = \left(\frac{aeU^-}{M}\right)^{1/2} \exp\left(-\frac{1}{2}\right), \qquad (2)$$

where v^- and v^+ are the normal components of particle velocities at the boundary of the probe layer.

Equality of electron and ion current corresponds to a floating potential

$$u = U - HU^- - GU^-, \tag{3}$$

where the quantity G is a measure of the charge separation. All potentials are read off relative to either electrode with fixed potential, for example the relative anode.

Where $n^- = n^+ = N$, G vanishes and relationship (3) is converted to (1), as would be expected. For slight charge separation

$$n^- = N + \Delta n^-, \quad n^+ = N + \Delta n^+,$$

we have

$$G \approx \frac{\Delta n^- - \Delta n^+}{N}, \tag{4}$$

or, considering Poisson's equation

$$G \approx \frac{\varepsilon_0 \nabla^2 U}{Ne}. \tag{4a}$$

The use of the theory of probes assumes that the length of heterogeneities in the plasma λ is significantly greater than the dimensions of the area perturbed by the probe. Suppose now the perturbation of a space charge propagates as a wave. If the change in potential caused by this perturbation can be described (at least formally) by the wave equation, relationship (4a) can be written as

$$G=\frac{\varepsilon_0}{Ne(\lambda\nu)^2}\frac{\partial^2 U}{\partial t^2}. \qquad (5)$$

2.2. Dynamic Floating Potential

Connection of the input of the recording device to a probe is equivalent to connection of capacitance C. It has been assumed up to now that the characteristic time of change of parameters of the plasma $1/\nu$ is more than the charging time of the capacitance. In practice, probe measurements must be studied in which the capacitance of the measuring circuit is significant, but the frequency spectrum of oscillations is limited at the high frequency end, so that transient processes in the probe layer can be ignored.

Figure 1 shows an equivalent circuit for this case of measurements. The plasma is represented by a voltage generator with low internal impedence. The layer is described by a nonlinear ohmic resistance (U-u)/i. This example of an equivalent circuit is rather general and encompasses many important cases of electrical probes with high dc resistance. /65

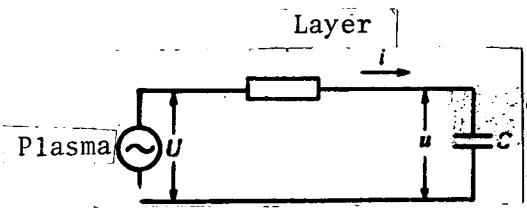

Figure 1.

The equivalent circuit (Figure 1) can be studied using the equation system

$$\begin{aligned} i &= -I^- \exp\frac{u-U}{U^-} + I^+, \\ u &= \frac{1}{C}\int i\,dt, \end{aligned} \qquad (6)$$

which can be written as a single equation

$$\frac{du}{dt}=\frac{I^{+}}{C}\left[1-\exp\left(H+G-\frac{U}{U^{-}}+\frac{u}{U^{-}}\right)\right]. \tag{6a}$$

Let us introduce the dimensionless variables $y=\exp(u/U^{-})$, $x=\nu t$. If

$$\left|\frac{1}{U^{-}}\frac{\partial U^{-}}{\partial t}\right|\ll\nu\left|y\frac{dy}{dx}\right|, \tag{7}$$

then from (6a) we produce the Bernoulli equation

$$y'+f(x)y^{2}+g(x)y=0 \tag{8}$$

with coefficients

$$\begin{aligned} f(x)&=-g(x)\exp\left[H+G(x)-\frac{U(x)}{U^{-}(x)}\right], \\ g(x)&=-\frac{I^{+}(x)}{U^{-}(x)}\frac{1}{\nu C}. \end{aligned} \tag{9}$$

The solution of equation (8) gives us the relationship between the instantaneous values of floating potential and the plasma parameters U^{-}, U, I^{+} and G. The measure of separation of charges G can be determined unambiguously by the derivatives of the potential, for example, from formula (5). It is more convenient to take (8) as the relationship between floating potential $u(t)$ and the three unknown variables $U^{-}(t)$, $U(t)$ and $n^{+}(t)$, referring to formulas (2) which relate $I^{+}(t)$ with $n^{+}(t)$ and $U^{-}(t)$.

2.3. Influence of Capacitance of Layer by Probe on Dynamic Floating Potential /66

The equivalent circuit, considering the capacitance of the layer by the probe, becomes as shown on Figure 2. In this case, the system of equations for the floating potential is

$$\begin{cases} i=-I^{-}\exp\frac{u-U}{U^{-}}+I^{+}+\left[C_s+\frac{dC_s}{d(U-u)}(U-u)\right]\left(\frac{dU}{dt}-\frac{du}{dt}\right), \\ u=\frac{1}{C}\int i\,dt. \end{cases} \quad (10)$$

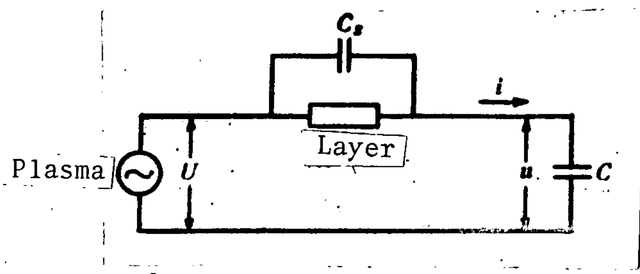

Figure 2

Equation system (10) is reduced to Bernoulli equation (8) only for the case when we can consider C_s independent of U-u near the floating potential, which is true only for slight oscillations. Near the floating potential, the capacitance of the layer, according to [9], is given by the expression

$$C_s=\beta\frac{\varepsilon_0 S}{\lambda_D}. \quad (11)$$

This time, the coefficient of the Bernoulli equation should be written as

$$\begin{aligned} &f(x)=-g_1(x)\exp\left[H+G(x)-\frac{U(x)}{U^{-}(x)}\right], \\ &g(x)=g_1(x)+g_2(x), \\ &g_1(x)=-\frac{I^{+}(x)}{U^{-}(x)}\frac{1}{(C+C_s)}, \\ &g_2(x)=-\frac{C_s}{C+C_s}. \end{aligned} \quad (12)$$

Here also we assume slight oscillations of the electron temperature, satisfying inequality (7).

Recording of plasma oscillations by an electrode with no contact with the plasma (for example, a metal ring seated on a discharge tube) occurs only due to the capative coupling with the plasma. Due to the difference in the structure of the discharge layer near the wall, the value of C_s for an external electrode is generally different from the value of C_s for a Langmuir probe.

2.4. Influence of Capacitive Currents from Remote Areas of the Plasma /67

Capacitive coupling with more distant areas of the discharge may play a significant role for the variable potential of an unshielded probe. The degree of influence of this capacitive coupling depends, obviously, on the shape and size of the probe and its placement. Any quantitative estimates of the distortion of the probe signal by parasitic capacitive currents is quite difficult; therefore, we must greatly simplify the task.

Let us seek the probe potential defined by bias currents alone, in order to compare it with the potential resulting from conductivity current. The probe is located at distance z_0 from one end of the plasma column of length l (Figure 3) and is connected to a recording device with input capacitance C. Let us assume that the cross section of the plasma column is negligible in comparison to its length. Each element of the column of length dz has a certain capacitance $C_1(z-z_0)dz$ in relation to the probe. C_1 represents the capacitance between the probe and a unit length of the column. The full bias current is equal to the sum of the currents of all such elements. In order to estimate the probe potential determined by this current, we can use the equation system

$$\left.\begin{aligned} i(z_0, t) &= \int\limits_{z_0-\zeta_1}^{z_0+\zeta_2} \left[\frac{\partial U(z, t)}{\partial t} - \frac{\partial u(z_0, t)}{\partial t}\right] C_1(z-z_0)\, dz, \\ \frac{\partial u(z_0, t)}{\partial t} &= \frac{1}{C}\, i(z_0, t). \end{aligned}\right\} \tag{13}$$

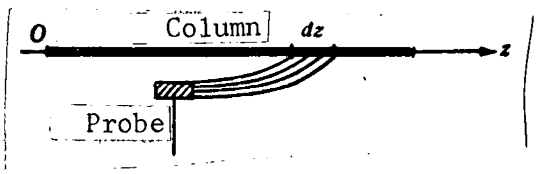

Figure 3

The limits of integration of (13), determined by the values of ζ_1 and ζ_2, may differ, depending on the specific problem studied. U(z, t) is the potential along the plasma column, which we will consider fixed. In other words, we will look upon the plasma as a voltage generator. We can with equal success fix the distribution of charge density in the column, which is equivalent to a current generator. Actually, these two methods of analysis are not independent, since the distributions of plasma parameters along the column are not arbitrary, but rather are interrelated by Poisson's equation. This factor has a significant influence on the course of the solution, as can be seen from the example studied in 2.4.

3. Foundation and Discussion of Floating Probe Method for a Plasma with Low-Frequency Oscillations

3.1. Foundation of the Floating Probe Method for Moving Striae

The static method of the floating probe [4, 5] was used to study traveling layers in inert gasses; the oscillating frequency was not over 10 kHz, the dynamic input capacitance of the electrometric apparatus used was on the order of 100 pF. Based /68
on an ion current value on the probe of 10^{-5} A and a floating potential of about 0.5 V, it is not difficult to show that under the conditions of [4] we can ignore the term in equation (6a) proportional to du/dt[1]. Therefore, the probe potential is determined by (3). Recording of currents $I^{-}(t)$ and $I^{+}(t)$ by an oscillograph showed that there is a definite, slight phase shift between them. Consequently, G may differ significantly from zero only at the head of a stria. Significant deviations in the distribution of velocities of electrons from the equilibrium distribution have been found in this area, so that near the area of the space charge, probe measurements are unreliable for another reason.

We must note that the temperature "profiles" in a stria, both those indicated by the volt-ampere probe characteristics measured at various points in the stria and those taken from the floating potential oscillograms, coincide within the limits of error of the experiments [4].

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1 And, apparently, the effect related to capacitance.

3.2. Dynamic Floating Potential in the Case of Ion-Sonic Oscillations

Let us study unidimensional ion density waves of low amplitude

$$\Delta n^{+}(t)=n\sin(\omega t-kz),\quad \frac{n}{N}\ll 1. \tag{14}$$

As we know, in the case of ion-acoustical waves, the remaining parameters of the plasma can be written as

$$\begin{aligned} &U^{-}=\text{const},\\ &\Delta n^{-}(t)=N\frac{U(t)}{U^{-}},\\ &\nabla^{2}U(t)=-\frac{e}{\varepsilon_0}[\Delta n^{+}(t)-\Delta n^{-}(t)]. \end{aligned} \tag{15}$$

It follows from the first relationship of (15) that inequality (7) is fulfilled. Keeping in mind (5), (15) is reduced to the expressions

$$\begin{aligned} &U(t)=U_0\sin(\omega t-kz),\\ &G(t)=-G_0\sin(\omega t-kz),\\ &G_0=\frac{\omega^2}{\omega^2+\omega_i^2}\frac{n}{N},\\ &G_0+\frac{U_0}{U^{-}}=\frac{n}{N}. \end{aligned} \tag{16}$$

Let us study a Langmuir probe placed at the coordinate origin z=0. We substitute (15) into (9), using (2) for determination of I^{+}. The Bernoulli equation is solved by the standard method. During the course of the solution, /69

$\exp[n/N(\sin 2\pi x - P\cos 2\pi x)]$ is replaced by $1+[n/N(\sin 2\pi x - P\cos 2\pi x)$, which introduces an error of about 0.5% for $n/N=1\%$[2]. If we limit our calculations to the case of periodic solutions (which is equivalent to the boundary condition), the solution becomes simple. Ignoring the "overtones," the amplitudes of which are proportional to $(n/N)^2$, $(n/N)^3$, etc., we find the following final expression for the dynamic floating potential:

$$u(t)=\frac{n}{N}U^{-}(A\sin\omega t - B\cos\omega t), \tag{17}$$

where

$$A=\frac{P^2}{1+P^2},\quad B=\frac{P}{1+P^2}, \tag{18}$$

$$P=\frac{\overline{I^{+}}}{\omega C U^{-}}=\overline{g(2\pi x)}. \tag{19}$$

We note that in concluding these equations it was assumed that the probe does not change the structure of the wave, does not excite standing waves in the column, etc. The values of N, U^{-} and $I^{+}=1/4eN(aeU^{-}/M)^{1/2}\exp(-1/2)$ can be easily determined from the static volt-ampere probe characteristics or estimated using tabular data on a positive column. Thus, based on formulas

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2 We note that the value of P(19) is on the order of 1 even in the most unfavorable cases, for example $I^{+}=10^{-5}$, $\omega/2\pi=10^{4}$ Hz, $C=10^{-10}$F.

(17), (18) and (19), we can determine the instantaneous values of ion density if the dynamic floating potential is measured using a high input impedence.

It is not difficult to see that where $\omega C \to 0$, formula (17) becomes

$$u(t)=G(t)U^{-}+U(t),$$

as would be expected. We can conclude from this that the input capacitance of the device introduces a certain phase shift to the observed signal, depending on the input impedence of the device.

3.3. Influence of Capacitance of Layer in the Case of Ion Sound

Calculation of the floating potential considering the capacitance of the layer next to the probe in the case of a plasma with ion-acoustic oscillations is performed just as above (see 3.2), but formulas (12) should be used to find f(x) and g(x) in this case. As a result, the floating pontential U(t) is determined by the same expression (17), but coefficients A and B are changed:

$$\begin{gathered} A=\frac{P^2}{1+P^2}\left(1+Q\,\frac{1+P^2+P^3}{P^2}\right), \quad B=\frac{P}{1+P^2}(1+Q), \\ P=\frac{\overline{I^+}}{\omega(C+C_s)U^-}, \quad Q=\frac{C_s}{C+C_s}\,\frac{\omega_1^2}{\omega^2+\omega_1^2}. \end{gathered} \tag{20}$$

The effect of layer capacitance causes a change both of the phase and of the amplitude of the floating potential oscillations.

3.4. Bias Currents on the Probe in the Case of Periodic Structure of the Space Charge /70

Let us return to analysis of the influence of capacitive currents from remote areas of the plasma (see 2.4), assuming periodic distribution of charges moving along the column. The force lines leaving positive charges in the column are closed to the negative charges in the column without moving far from the column. At any moment in time the lines closing through the probe cannot begin at a distance from the probe greater than a half wave length of the periodic structure. This determines the integration limits in (13), assuming $\zeta_1=\zeta_2=\lambda/2$.

Due to electrostatic induction, positive and negative charges appear on the probe, but the summary charge on the probe does not change with time. With the model studied, only in the case of some stable asymmetry of charges can a non-zero signal be expected, resulting from the bias currents from neighboring charge areas (near the probe). Asymmetry will develop in the distribution of the space charge if the probe is near the end of a periodic structure, for example $z_0<\lambda/2$.

Another effect always present to some extent during probe measurements in striae consists of distortions of amplitude, form and phase of the striae passing near the probe. Observations using a rotating mirror (stroboscope) or photomultiplier have shown that each stria "adheres" to the probe for a certain time, yielding its place suddenly to the next stria. The intensity of oscillations of radiated light also changes near the probe.

Suppose the probe distorts the oscillations of plasma parameters quite strongly--by a value on the order of the oscillations themselves. This is equivalent to the assumption that the oscillations exist only on one side of the probe, i.e., all the lines of force close at the probe. This estimate of the value of $u(z_0, t)$ will be quite high, but still may yield certain useful information.

The use of this rough model of the phenomena at the probe allows us to note a path to the solution of equation system (13). In order to estimate the value of $C_1(z-z_0)dz$, a column sector of length dz should be replaced by a point charge or a distribution of charges of more complex configuration. The relationships produced for $U(z_0, t)$ depend strongly on the method of approximation of column element of length dz. Apparently, this is true only for very rough and unreliable methods of estimation; therefore, due to their roughness, it does not seem useful to write out all of the expressions produced.

If the distribution of potential along the column is fixed as (14), the various approximations for calculation of the capacitance between the probe and a plasma element (Figure 3) yield for the floating potential

$$u(z_0, t) = u_0 F(kz_0, \omega t - kz_0), \tag{21}$$

where u_0 is the amplitude of oscillations of the floating potential. The phase of the floating potential oscillations, as we can see from (21), depends on the position of the probe in relationship to the column, which is considered by function F, which is always on the order of one. In the simplest case (approximation of point charges) it is cos $(\omega t-kz_0)$. In other /71 cases, it is expressed by a combination of sines and cosines, and of integral sines and integral cosines of the quantities kz_0 and $\omega t-kz_0$.

It is also important to note that the various approximations for $C_1(z-z_0)dz$ yield approximately the same result for the oscillating amplitudes of the floating potential. In the point charge approximation

$$u_0=\frac{U_0}{1+\frac{2C}{\varepsilon_0\lambda}}. \tag{22}$$

The frequency of oscillations ω is not included in the expression for u_0. This is essentially a reflection of the fact that the probe is a capacitive divider in the model used.

Estimates were performed for the same electrode placed in a plasma column with periodic structure, which served as a model of a discharge with striae or ion sound. The values produced for the floating potential depend on the values of C and ω. The signal on the probe resulting from bias currents alone amounts to 0.1--10% of the signal resulting from conductivity currents alone for most cases encountered in practice ($C=5.10^{-11}-10^{-9}$F, $\omega/2\pi=10^3-10^6$Hz).

4. Conclusions

We can see from the preceding analysis that when a single floating probe is used to study low frequency oscillations, we must consider deviations of the plasma from electical neutrality, and in many cases the capacitive currents as well. However, it has been shown that the method of [5] is applicable for the study of fluctuations in electron temperature and plasma potential in moving striae. The difficulties of the method are clearly seen in the case of ionic oscillations of the plasma. Artificially increasing the impėdence of the probe helps to overcome the harmful influence of capacitive currents. Therefore, probes with more complex design [6, 10] are free of these shortcomings. We find that an ordinary Langmuir probe in combination with a special electrometric apparatus (floating grid amplifier, see [5]) can be used to study ion-sonic oscillations. The dynamic floating potential recorded is related by simple formula (16) to the instantaneous value of ion density in the plasma. By introducing a suitable variable condensor with known capacitance to the circuit (in parallel to the input capacitance of the recording device), we can estimate not only the amplitude but also the phase of oscillations of ion density. The dynamic input capacitance of the electrometric tube must be known in advance. The phase of oscillations can also be determined by comparison of the signals of the the probes.

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