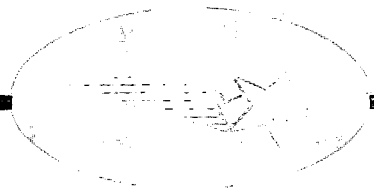

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HREC-7012-1
LMSC-HREC D225823

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STUDY OF PROPELLANT DYNAMICS IN A SHUTTLE-TYPE LAUNCH VEHICLE

INTERIM REPORT

April 1972

Contract NAS8-27012

by

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FOREWORD

This report summarizes work performed during the first ten months of Contract NAS8-27012, "Study of Propellant Dynamics in a Shuttle-Type Launch Vehicle." The work was conducted by Lockheed Missiles & Space Company, Inc., Huntsville Research & Engineering Center, for George C. Marshall Space Flight Center of the National Aeronautics and Space Administration. The contract is administered under the direction of the Aero-Astrodynamics Laboratory, NASA-MSFC, with Mr. Larry Kiefling as Contracting Officer Representative.

SUMMARY

This report describes a method and an associated digital computer program for evaluating the vibrational characteristics of large liquid-filled rigid wall tanks of general shape. A solution procedure was developed in which slosh modes and frequencies are computed for systems mathematically modeled as assemblages of liquid finite elements. To retain sparsity in the assembled system mass and stiffness matrices, a compressible liquid element formulation was incorporated in the program.

The approach taken in the liquid finite element formulation is compatible with triangular and quadrilateral structural finite elements so that the analysis of liquid motion can be coupled with flexible tank wall motion at some future time. The liquid element repertoire developed during the course of this study consists of a two-dimensional triangular element and a three-dimensional tetrahedral element.

A number of example problems were analyzed with the propellant dynamics computer program. The results of these analyses compared favorably with known closed form solutions. Solutions computed for several finite element models of liquid sloshing in a rigid rectangular container are presented.

CONTENTS

Section 1
INTRODUCTION

During this study, a method and an associated digital computer program were developed for computing the free undamped vibrational characteristics of large liquid-filled tanks of general shape. Solution procedures were formulated in which systems are mathematically modeled as assemblages of liquid finite elements.

Unlike previous investigations (Refs. 1, 2, and 3), the methods developed during this study satisfy two fundamental requirements: (1) the procedure is suitable for analyzing large systems such as Space Shuttle vehicles, which typically involve thousands of degrees of freedom; and (2) the approach to liquid representation is compatible with triangular and quadrilateral structural finite elements, so that the analysis of liquid motion can be coupled with flexible tank wall motion at some future time.

Solutions to eigenvalue problems resulting from large finite element networks can be obtained economically only if the assembled system mass and stiffness matrices exhibit sparse and/or banded characteristics. Most past investigations of finite element liquid sloshing problems that are reported in the literature are based on the assumption that fluid is incompressible. This assumption invariably yields system matrices that are completely full; consequently, the resulting computer programs are restricted to problems of moderate size (usually 200 degrees of freedom or less). In this study the effects of liquid compressibility are included in the finite element formulation, and the resulting system matrices exhibit the same sparse characteristics as an analogous three-dimensional solid finite element formulation. As a result, sparse and/or bandmatrix solution procedures (such as the matrix iteration method incorporated in the Lockheed-Huntsville SNAP/Dynamics program, Ref. 4 can be used to solve the ω^2 MX - KX = 0 eigenvalue problem.

The mathematical model of a fluid system consists of liquid finite element representations that can easily be merged with solid shell finite element representations of tank walls. The element repertoire developed during the course of the study consists of a two-dimensional triangular element and a three-dimensional tetrahedral element. Particular care was taken to ensure complete compatibility between the liquid element formulations and the solid plate and membrane formulations incorporated in the SNAP/Dynamics program. As in SNAP, individual element energy matrices are formulated relative to imbedded intrinsic reference frames. This apparatus makes it easy to add new element formulations to the program, since it is necessary only to construct subroutines for computing the corresponding intrinsic mass and stiffness matrices, with element dimensions and fluid properties supplied through the calling sequence from general routines.

The generalized coordinates used to characterize individual element energies are three-component displacement vectors associated with each element node. System mass and stiffness matrices are constructed by expressing total system energies as the sum of individual element energies and invoking displacement compatibility among connecting elements through their common nodes.

Section 2
FORMULATION OF THE EIGENVALUE PROBLEM

2.1 SYSTEM EQUATIONS OF MOTION

The kinetic and potential energies of a system may be expressed in matrix form as:

$$T = \frac{1}{2} \dot{\Phi}^* M \dot{\Phi}$$

and (1)

$$V = \frac{1}{2} \Phi^* K \Phi \ ,$$

where Φ is the generalized coordinate vector, and M and K are the mass and stiffness matrices of the system. In the absence of dissipative effects and external forces, the Lagrange equations yield

$$M \ddot{\Phi} + K \Phi = 0 \ . \tag{2}$$

Assuming solutions are in the form of $\Phi = X \cos\omega t$, Eq. (2) is reduced to the usual linear vibrational eigenproblem

$$\omega^2 M X - K X = 0 \ . \tag{3}$$

In this study, node point displacement components are used explicitly as system generalized coordinates, so that for a network composed of a total of n system nodes

$$\Phi = \begin{Bmatrix} v^1 \\ v^2 \\ \cdot \\ \cdot \\ \cdot \\ v^n \end{Bmatrix} , \tag{4}$$

where v^i is a vector containing the three motion components of node i.

The total system kinetic and potential energies may be expressed as

$$T = \sum_{j=1}^{N} \overline{T}_j, \quad \text{and}$$

$$V = \sum_{j=1}^{N} \overline{V}_j \quad , \tag{5}$$

where $\overline{T}_j$ and $\overline{V}_j$ represent the kinetic and potential energies of the j^{th} liquid element in terms of the system generalized coordinates, and N is the total number of finite elements in the network.

The kinetic and potential energies of liquid element j are expressed as

$$T_j = \frac{1}{2} \dot{\varphi}_j^* M_j \dot{\varphi}_j, \quad \text{and}$$

$$V_j = \frac{1}{2} \varphi_j^* K_j \varphi_j \quad , \tag{6}$$

where M_j and K_j are the mass and stiffness matrices of the element in terms of element generalized coordinates. In the propellant dynamics computer program, element energies are expressed in terms of explicit nodal displacements; so that, where u^i is a vector containing the three motion components of node i in directions parallel to the intrinsic reference frame axes of element j,

$$\varphi_j = \begin{Bmatrix} u^1 \\ u^2 \\ \cdot \\ \cdot \\ \cdot \\ u^m \end{Bmatrix} . \tag{7}$$

In Eq. (6), m is number of nodes interconnected by element j.

To transform element energy matrices M_j and K_j into system coordinates, a coordinate transformation is performed such that

$$\varphi_j = R_j \Phi \tag{8}$$

Substituting Eq. (8) into Eqs. (6) yields

$$\begin{aligned} \overline{T}_j &= \frac{1}{2} \dot{\Phi}^* \overline{M}_j \dot{\Phi} , \text{ and} \\ \overline{V}_j &= \frac{1}{2} \Phi^* \overline{K}_j \Phi , \end{aligned} \tag{9}$$

where

$$\begin{aligned} \overline{M}_j &= R_j^* M_j R_j, \text{ and} \\ \overline{K}_j &= R_j^* K_j R_j . \end{aligned} \tag{10}$$

Substitution of Eqs. (9) into Eqs. (5) yields the total system kinetic and potential energies in terms of the desired coordinates.

For purposes of specifying node restraints and system boundary conditions, unique reference frames are associated with each system node, and the directions of node displacements are defined relative to these nodal reference frames. Consequently, the transformation matrix R_j includes the effects of: (1) the orientation of the intrinsic reference frame of element j; and (2) the orientations of the nodal reference frames associated with nodes 1 through m.

The procedure described above is identical to that incorporated in SNAP/Dynamics, and the sparsity of the resulting system matrices is the same as would be obtained in a solid finite element formulation. However, if the element matrices, M_j and K_j, were formulated assuming liquid incompressibility, modification of the system matrices produced by Eqs. (5) would be required before solution to the eigenproblem could be attempted. Expressions would be introduced to ensure that the entire liquid volume would undergo zero volume change. These expressions would necessarily relate the relative motions of all network nodes that appear on the surfaces of the finite element model. Consequently, sparsity in the assembled system matrices would be lost, and those rows and columns associated with surface nodes would be completely full. This procedure would drastically restrict the size capability of the resulting computer program. By incorporating the effects of compressibility in the element formulations, the system matrices produced by Eqs. (5) can be used directly, and sparse and/or bandmatrix procedures can be used to solve the resulting eigenvalue problem.

2.2 LIQUID FINITE ELEMENTS

The liquid is assumed to be inviscid and compressible. The effect of small variation in mass density caused by the compressible assumption is neglected in formulating element kinetic energies.

The intrinsic reference frame associated with a tetrahedral liquid element is illustrated on Fig. 1. The origin of the reference frame coincides with node 1; intrinsic axis 1 extends from node 1 to node 2; axis 2 lies in the plane of nodes 1, 2, and 3 and is directed so that node 3 has a positive 2 coordinate. The relative displacements q_1, q_2, ..., and q_6 as shown on Fig. 1 completely define the deformation of the element. As the element moves with the liquid network, the intrinsic reference frame travels with node 1. The motion of the reference frame is represented by $\bar{q}_1$, $\bar{q}_2$, ..., and $\bar{q}_6$, as illustrated on Fig. 1. Liquid element mass and stiffness matrices are derived in terms of the twelve coordinates q_1 through q_6 and $\bar{q}_1$ through $\bar{q}_6$.

A linear deformation field is assumed for the element; it is expressed as

$$\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & x_2 & x_3 \\ 1 & x_1 & x_2 & x_3 \\ 1 & x_1 & x_2 & x_3 \end{bmatrix} A\,Q, \quad \text{or} \tag{11}$$

$$w = X\,A\,Q, \tag{12}$$

where w defines the motion of a particle on the interior of the element with intrinsic coordinates (x_1, x_2, x_3) and Q is a vector of the twelve coordinates q_1 through q_6 and $\bar{q}_1$ through $\bar{q}_6$. The terms of the coefficient matrix A are determined according to the physical dimensions of the element.

The kinetic energy of a liquid element is expressed as the following volume integral

$$\tilde{T} = \frac{1}{2}\rho \int_{\text{volume}} (\dot{w}_1^2 + \dot{w}_2^2 + \dot{w}_3^2)\, dV, \tag{13}$$

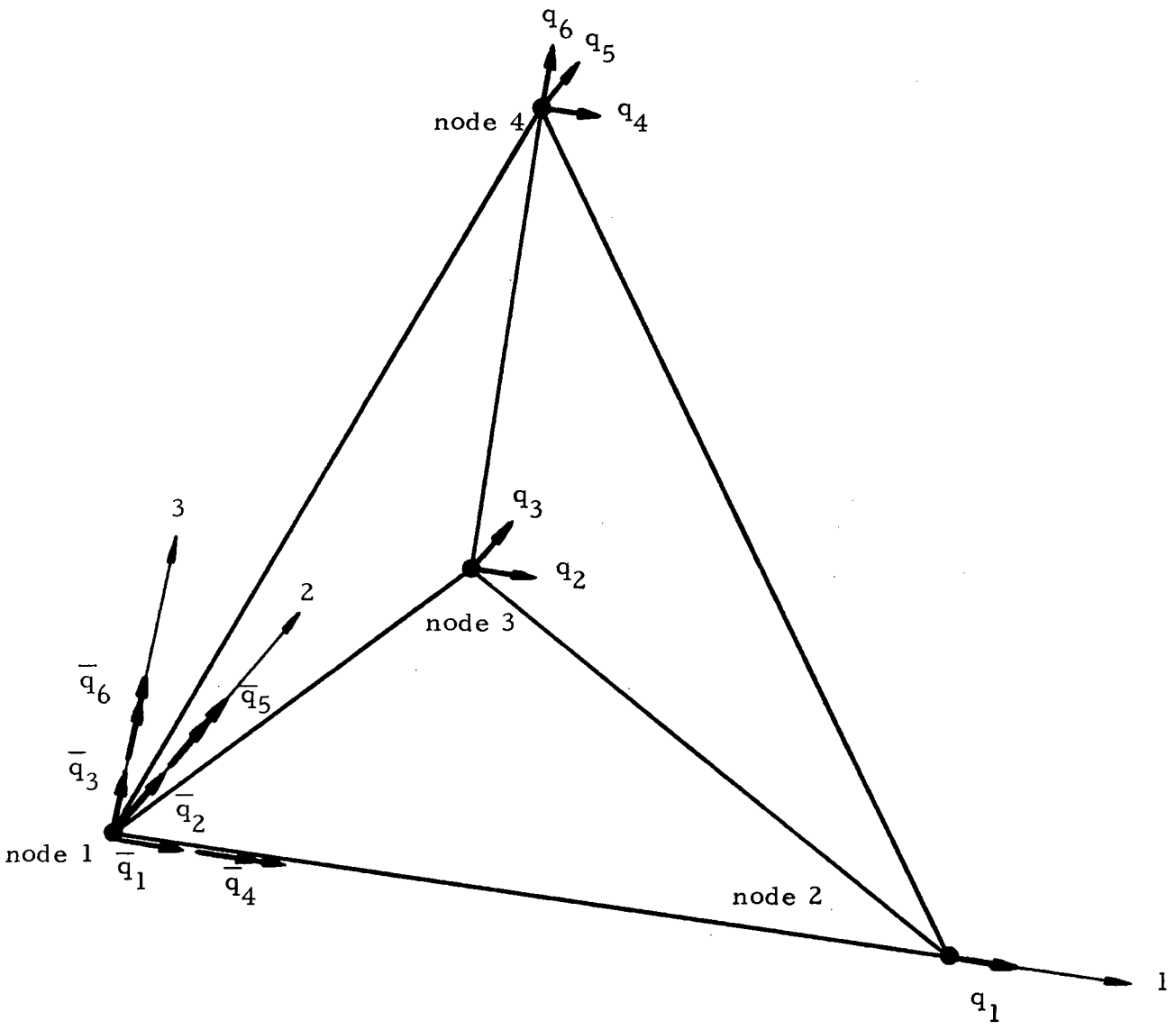


Fig. 1 - Intrinsic Reference Frame and Generalized Coordinates

where ρ is the mass density of the liquid. Substituting Eq. (12) into Eq. (13) yields the element kinetic energy as a quadratic form in terms of the intrinsic generalized coordinates, so that

$$\tilde{T} = \frac{1}{2} \Omega^* \tilde{M} \Omega \ , \tag{14}$$

where $\tilde{M}$ represents the intrinsic element mass matrix.

The total potential energy of a liquid element is expressed as the summation of two energies: (1) gravitational potential; and (2) dilatational potential. The potential energy incurred by gravity considerations is expressed by the following surface integral

$$\tilde{V}_G = \frac{1}{2} \rho \int_{\text{surface}} g_n \zeta^2 \, dS \ , \tag{15}$$

where g_n is the component of the gravitational acceleration vector normal to the surface differential, dS, and ζ is the component of surface motion normal to the surface differential. Dilatational energy is expressed as

$$\tilde{V}_D = \frac{1}{2} \kappa \int_{\text{volume}} \theta^2 \, dV \ , \tag{16}$$

where κ is the liquid bulk modulus and θ represents the volume change of the element. Substituting into Eqs. (15) and (16) the normal component of surface motion, ζ, and the element dilatation, θ, in terms of the assumed displacement field represented by Eqs. (12) yields the element potential energies as quadratic forms in terms of the intrinsic generalized coordinates, so that

$$\tilde{V}_G = \frac{1}{2} Q^* \tilde{K}_G Q \ , \quad \text{and}$$

$$\tilde{V}_D = \frac{1}{2} Q^* \tilde{K}_D Q \ . \tag{17}$$

The total element potential energy is

$$\tilde{V} = \tilde{V}_G + \tilde{V}_D \ , \quad \text{or}$$

$$\tilde{V} = \frac{1}{2} Q^* \tilde{K} Q \ , \tag{18}$$

where $\tilde{K}$ represents the intrinsic element stiffness matrix and is equal to the summation of $\tilde{K}_G$ and $\tilde{K}_D$.

A transformation of coordinates is performed so that the element energy matrices appearing in Eqs. (14) and (18) are expressed in terms of explicit nodal displacements. The resulting matrices are then used to form the assembled system mass and stiffness matrices as described in the previous section.

Energy expressions for the two-dimensional triangular element included in the program are derived in a manner similar to the tetrahedral element.

Section 3
COMPUTER PROGRAM

During this study a general purpose digital computer program was developed for computing the slosh modes and frequencies of finite element representations of liquid filled rigid wall tanks of general shape. Some of the features of the program are as follows:

- The element repertoire consists of a two-dimensional triangular element and a three-dimensional tetrahedral element.
- All operations involved in forming element and system matrices are performed in double precision.
- A moderate-size eigensolution routine is incorporated that will accommodate ω^2 M X - K X = 0 eigenproblems of up to approximately 75 degrees of freedom in double precision and 100 degrees of freedom in single precision.
- System nodes may be resequenced, giving the user maximum control over the bandwidth of system matrices.
- Individual reference frames may be assigned to the system nodes. Nodal motion components are given relative to these reference frames.
- Automatic network generators are incorporated to facilitate the input of node reference frame assignments, node restraint conditions, node position coordinates, and element definitions.
- SC 4020 plotting of undeformed and deformed element networks.
- Intrinsic local coordinate systems are used for individual elements.

Like SNAP/Dynamics, the program is arranged in a modular fashion to provide a convenient framework for modifying and expanding various parts of the program. New element formulations can be included in the element repertoire without altering any of the existing code. The module for computing eigensolutions can also be easily replaced.

The program is coded in Fortran V and designed for use on the NASA-MSFC Univac 1108 Exec VIII system.

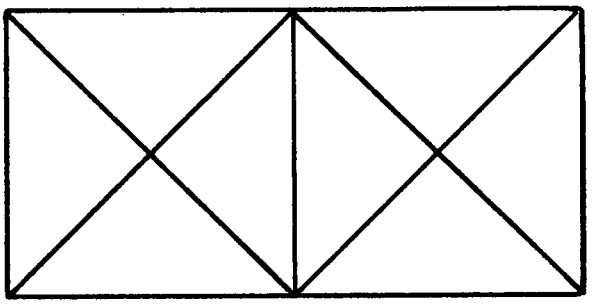
Section 4
RESULTS

Solutions computed for liquid sloshing in a rigid rectangular container are presented. The solutions are compared with known closed-form solutions. The geometric dimensions of the liquid volume are 1 meter wide, 2 meters long and 1 meter deep. The mass density, bulk modulus and equivalent gravitationsl acceleration of the liquid are 1000 kg/m^3, 2 x 10^8 $kg/m\text{-}sec^2$ and 9.807 m/sec^2, respectively.

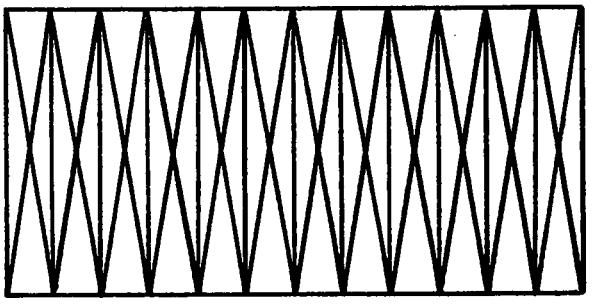
Results for six finite element models of the same liquid volume are presented. Three of the networks are two-dimensional and are composed of triangular elements. The two-dimensional models, which are referred to as 2D-a, 2D-b, and 2D-c, are illustrated on Fig. 2. The other three models are three-dimensional and are made up of tetrahedral elements. The three-dimensional models, which are referred to as 3D-a, 3D-b, and 3D-c, are shown on Fig. 3. The number of elements contained in each model and frequency comparisons between closed-form solutions are given in Table 1. Plots illustrating the slosh modes computed with the two-dimensional models are given in Fig. 4. Plots illustrating the modes computed with the three-dimensional models are given in Fig. 5.

Table 1
COMPARISON OF FREQUENCIES

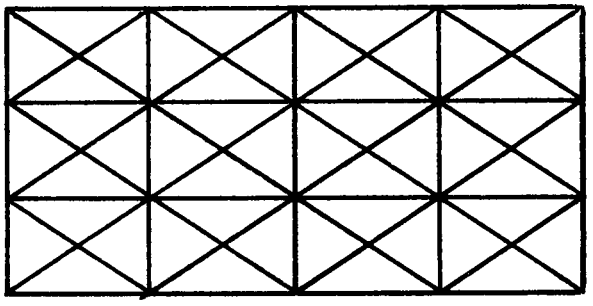
Cases Studied	Number of Elements	Frequencies (cps)			
		ω_1	ω_2	ω_3	ω_4
Closed Form		0.59823	0.88124	1.082	1.249
2D-a	8	0.587866	0.704812		
2D-b	48	0.595833	0.848202	0.970054	1.02322
2D-c	48	0.598934	0.889266	1.05944	1.08128
3D-a	48	0.588683	0.709116		
3D-b	96	0.589386	0.711981		
3D-c	96	0.589449	0.815962		



2D-a

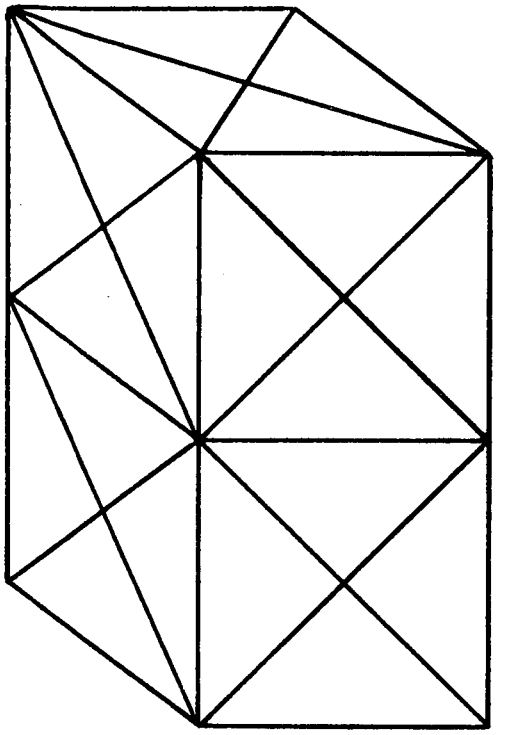


2D-b

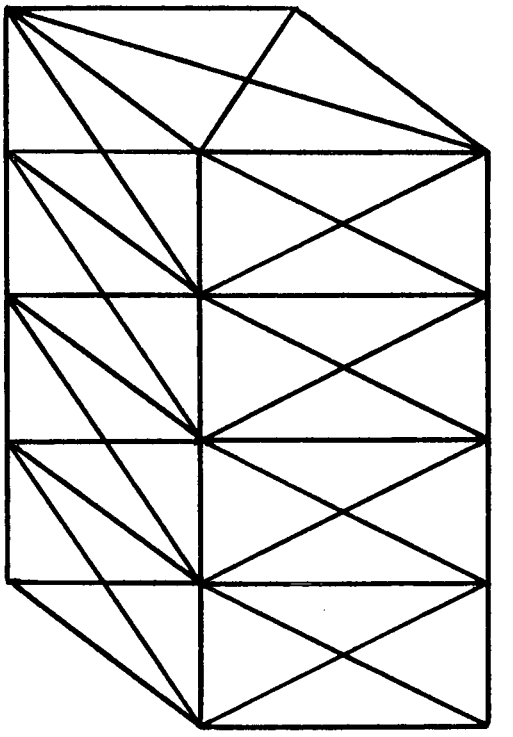


2D-c

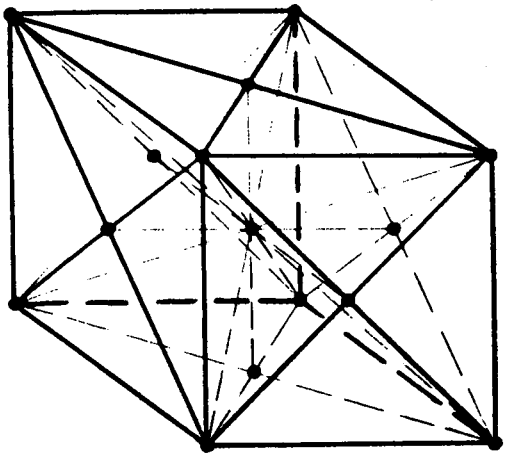
Fig. 2 - Arrangements of Two-Dimensional Elements



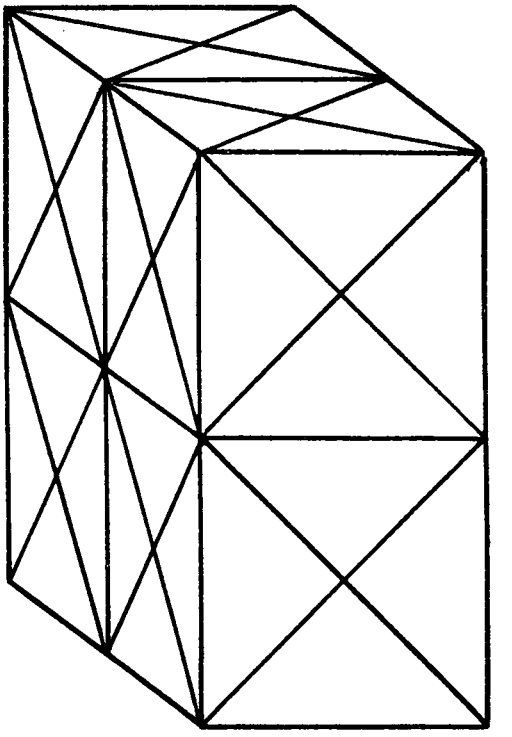
3D-a



3D-c

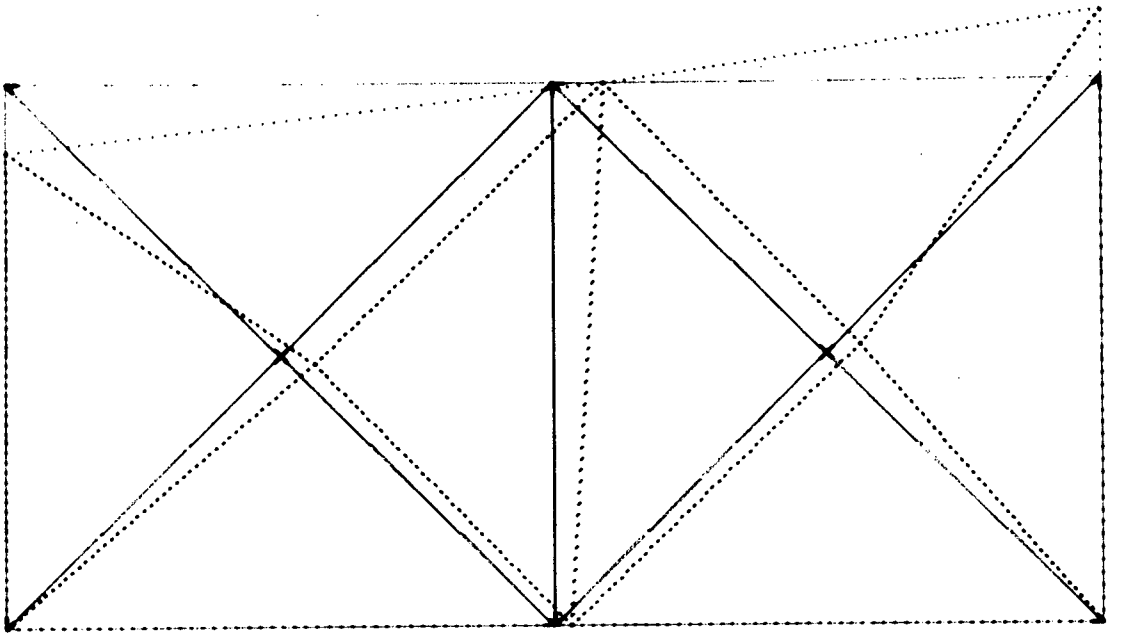


Typical Tetrahedron Pattern



3D-b

Fig. 3 - Arrangements of Three-Dimensional Elements



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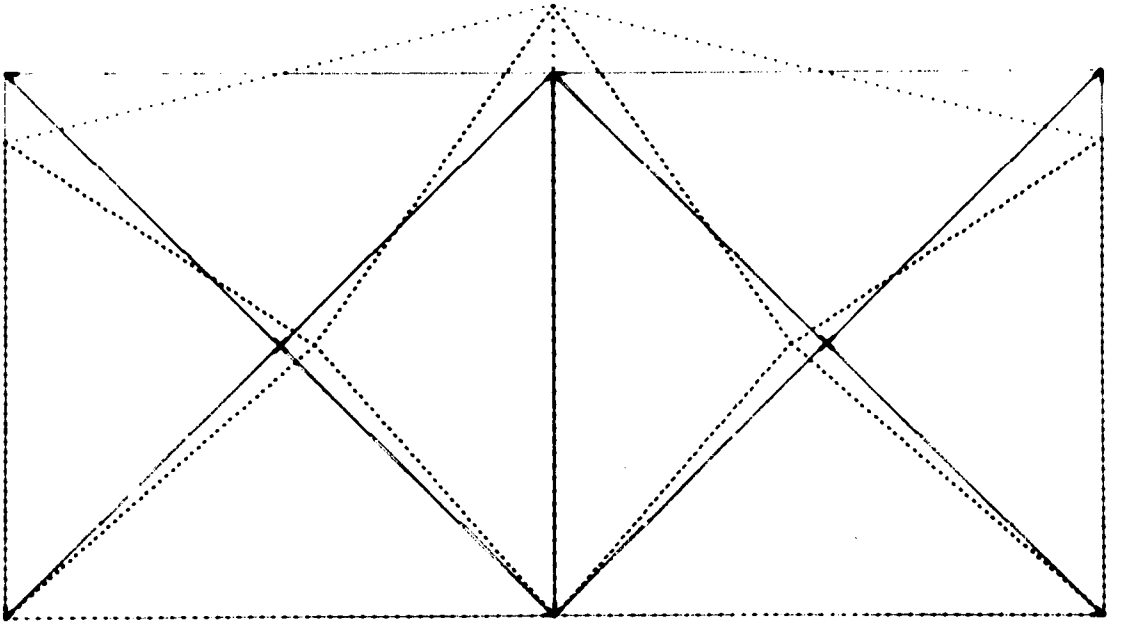

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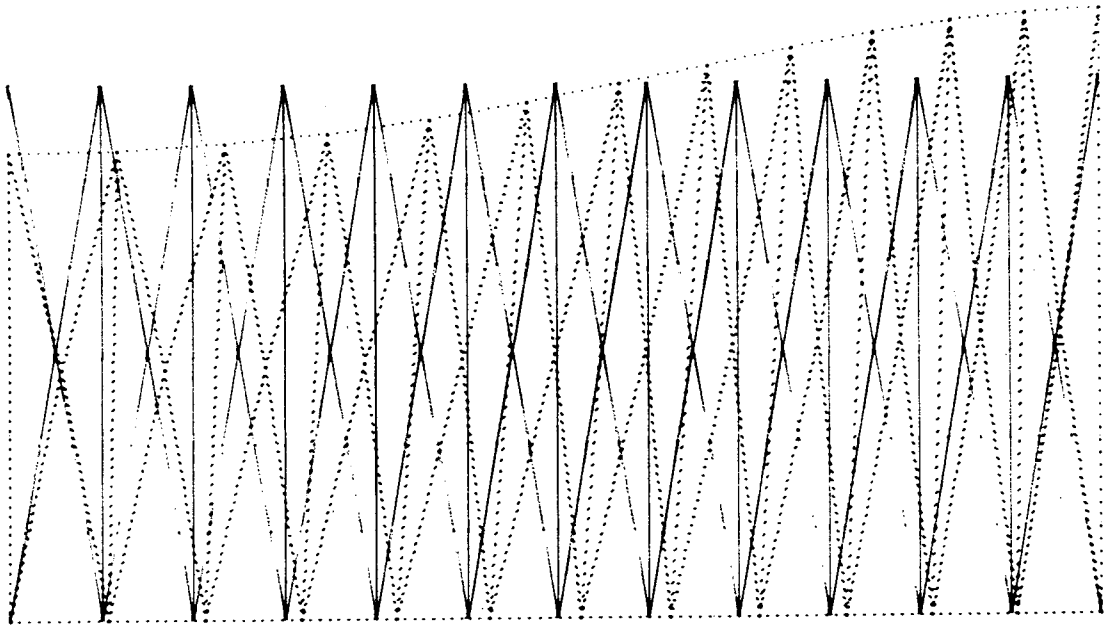
Fig. 4 - Mode Shapes and Frequencies — Case 2D-a



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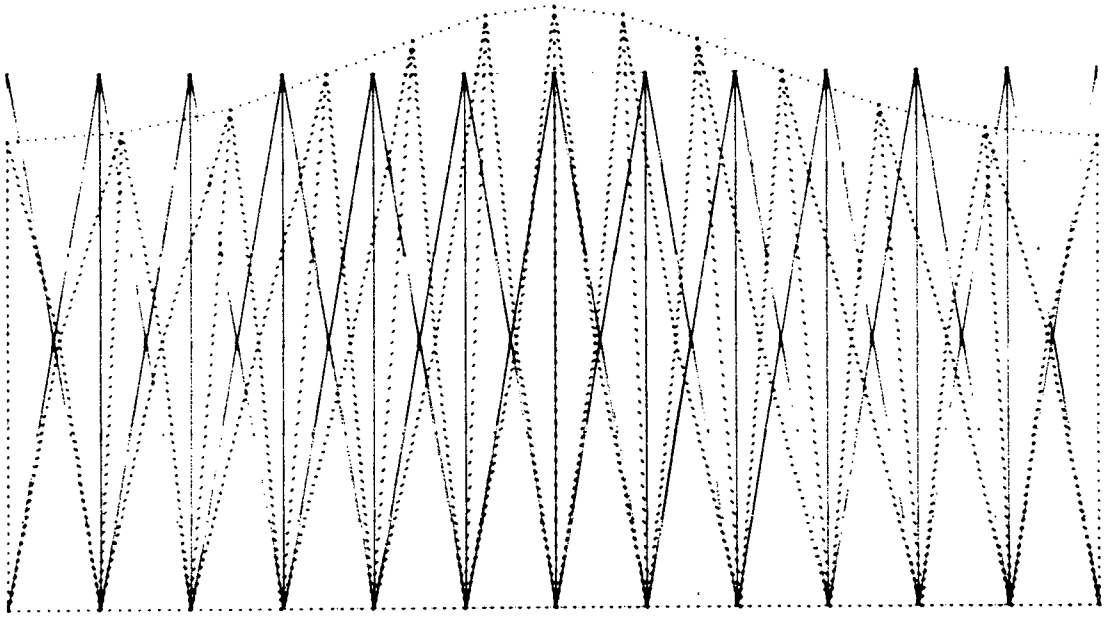

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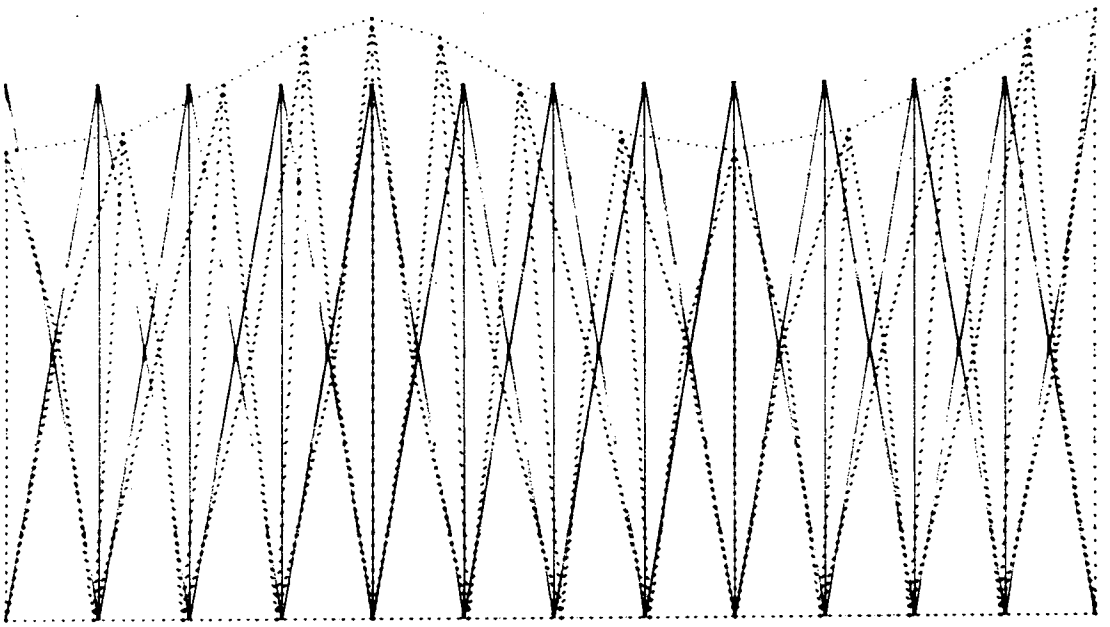
Fig. 4 - (Continued) – Case 2D-b



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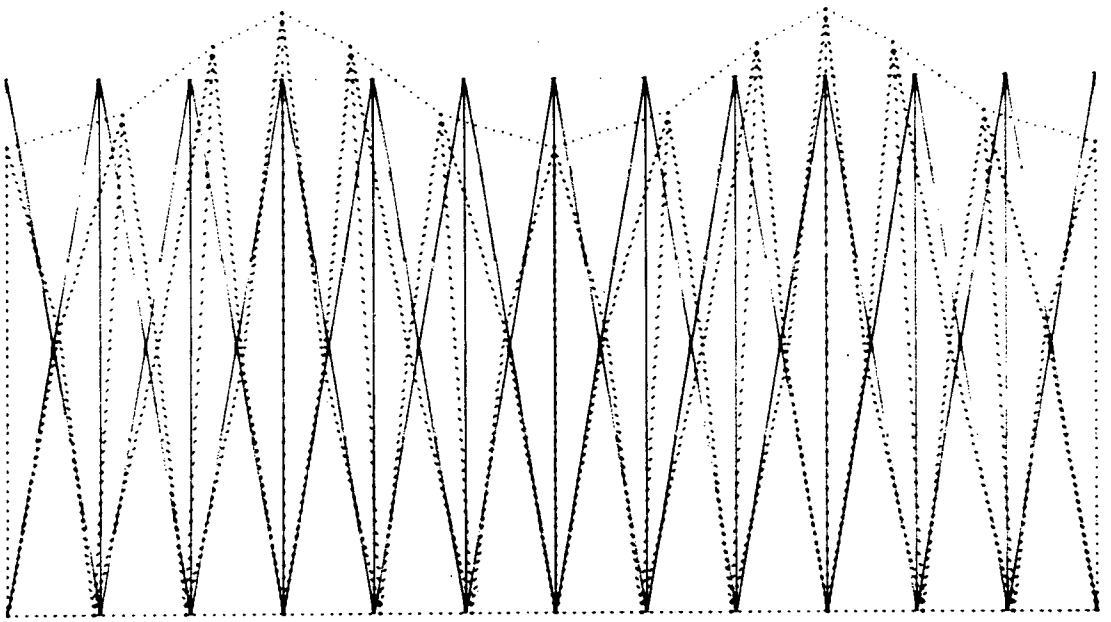

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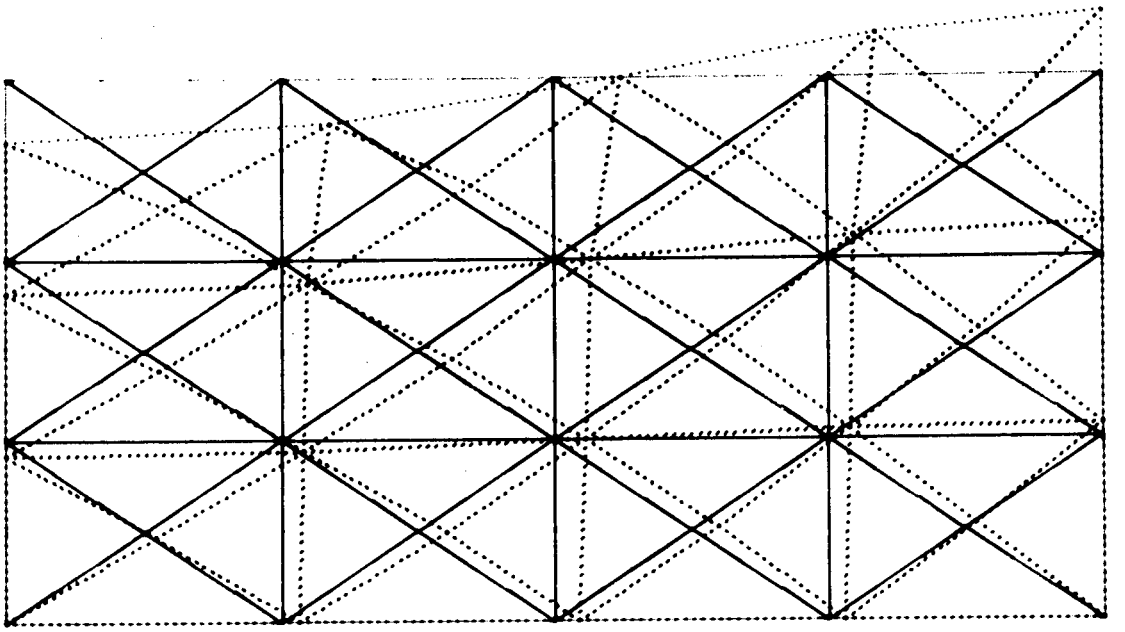

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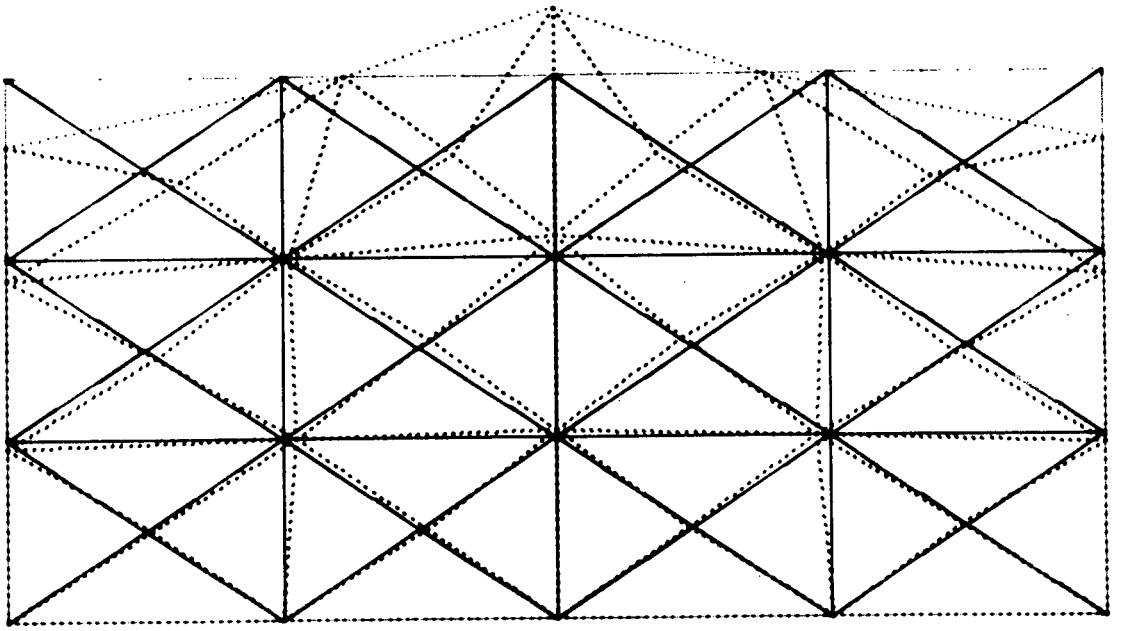

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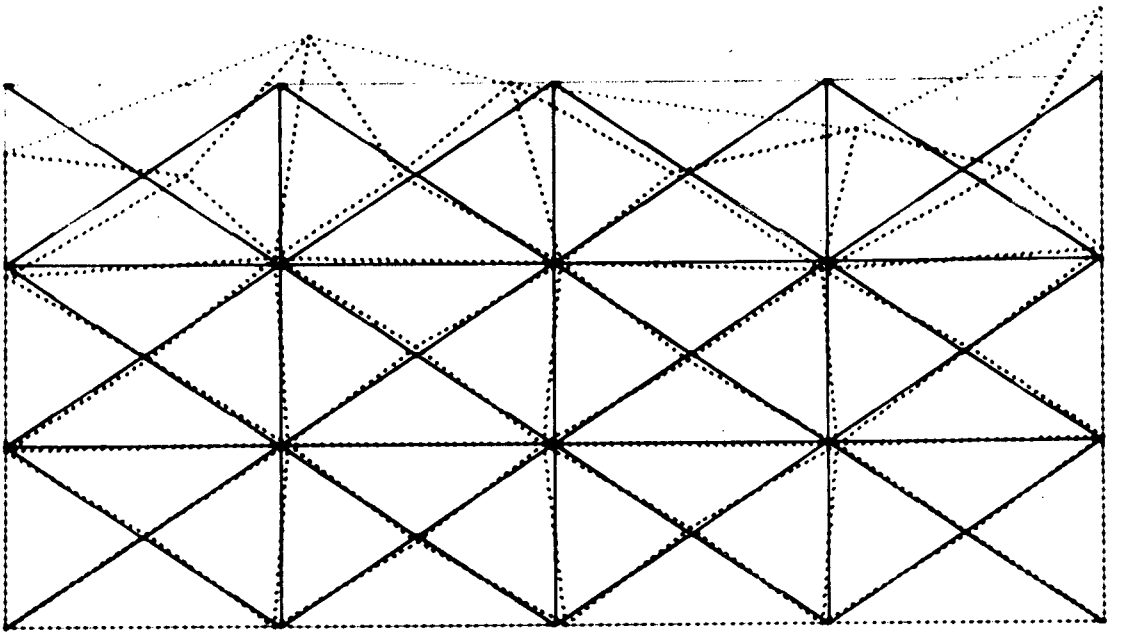
Fig. 4 - (Continued) — Case 2D-c



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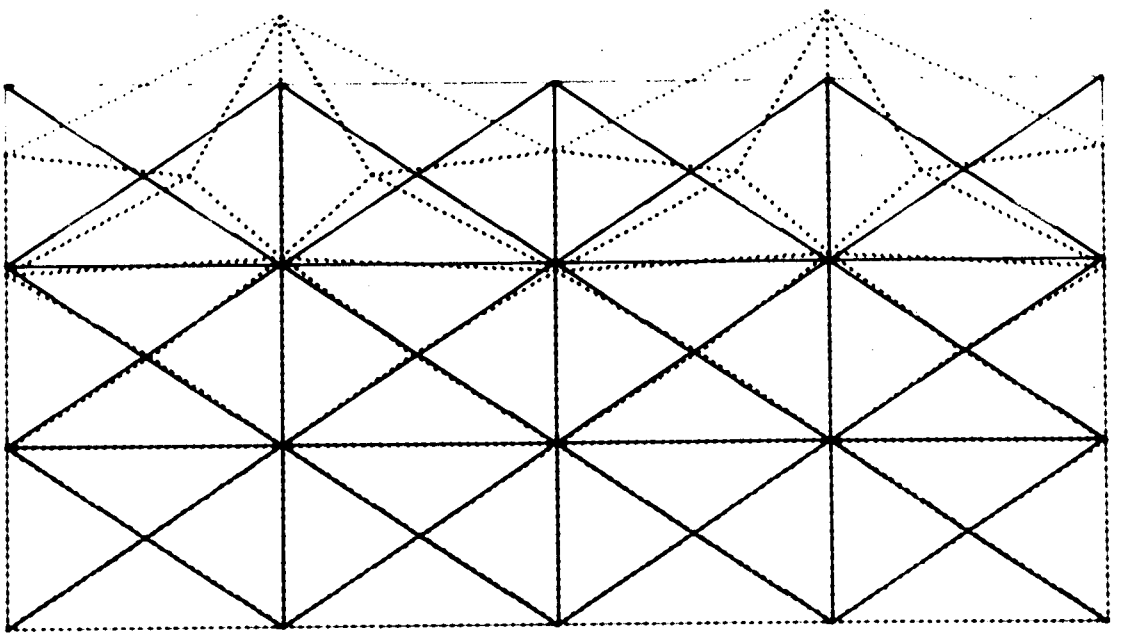

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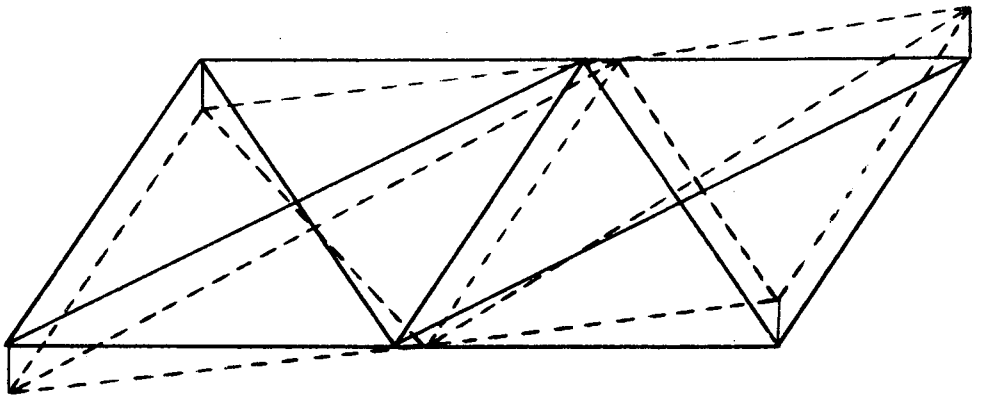
Fig. 4 - (Continued) — Case 2D-c



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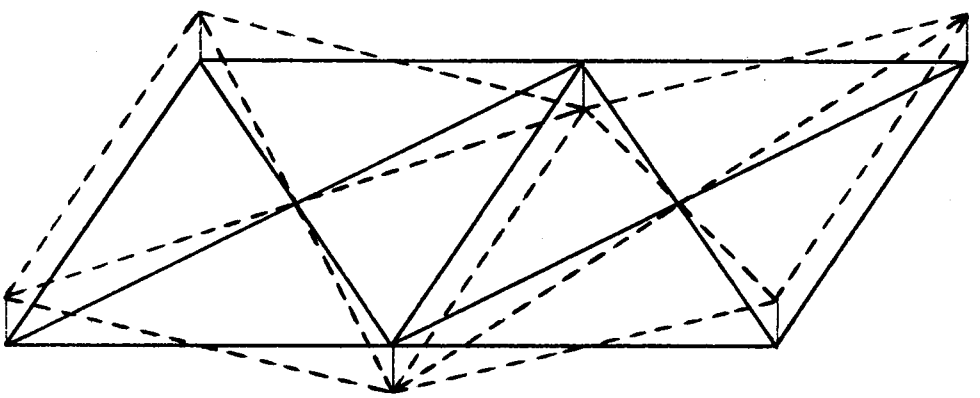

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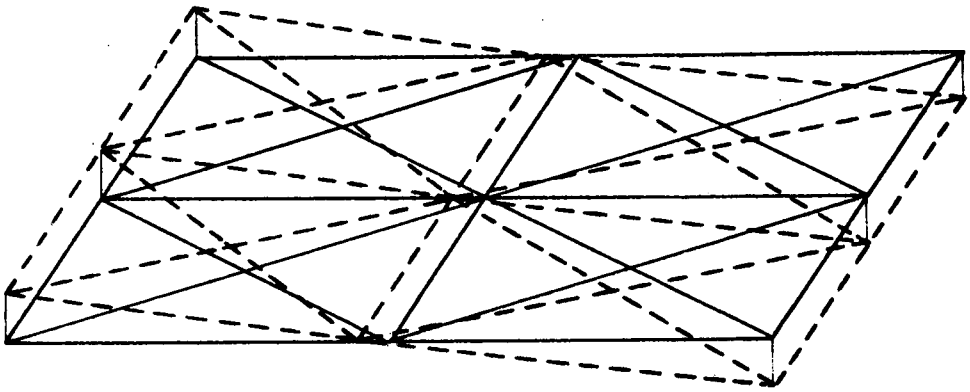
Fig. 5 - Free Surface Mode Shapes — Case 3D-a



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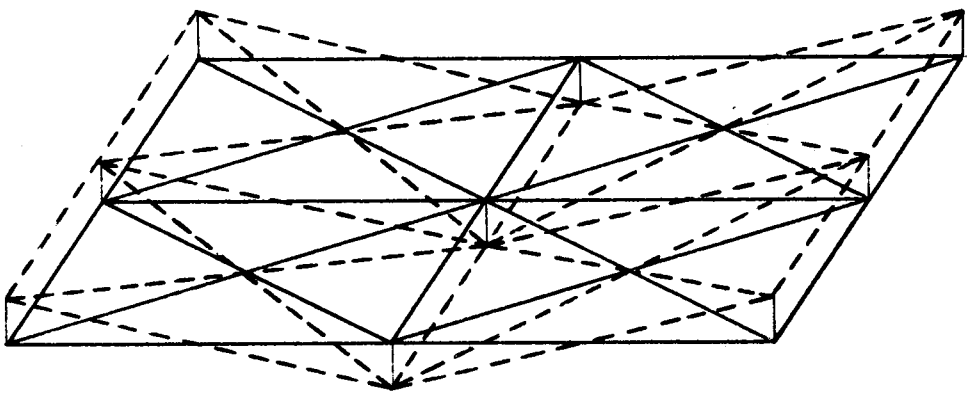

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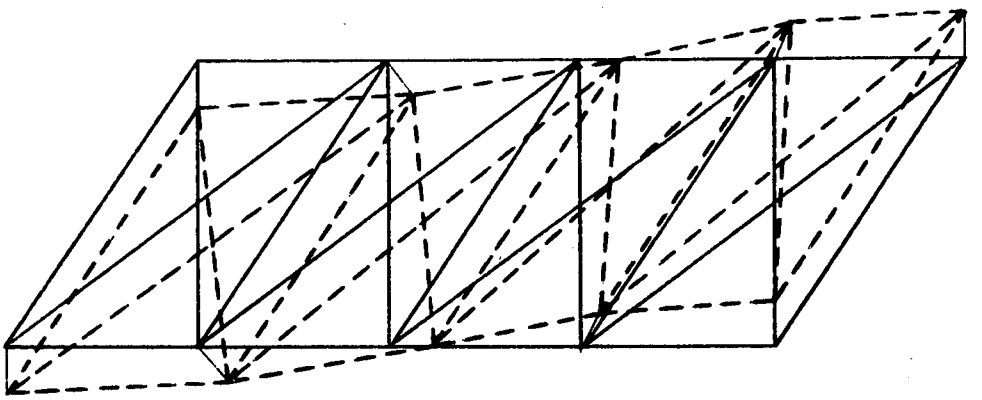
Fig. 5 - (Continued) — Case 3D-b



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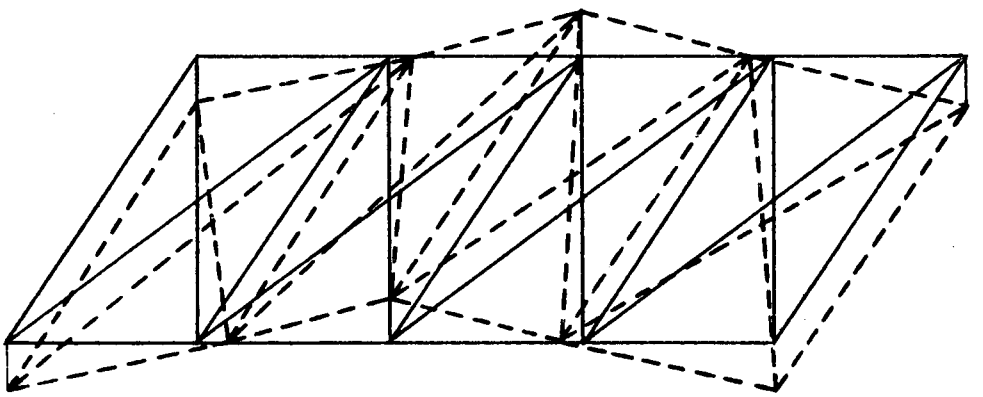

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Fig. 5 - (Continued) — Case 3D-c

Section 5
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