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TIDAL PERTURBATIONS ON THE ORBITS OF GEOS-1 AND GEOS-2

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ABSTRACT

Analysis of the luni-solar tidal perturbations of the inclination of GEOS-1 (1965-89A) and GEOS-2 (1968-002A) has yielded the values $k_2 = 0.22$ ($\sigma = 0.02$) and 0.31 ($\sigma = 0.01$) respectively for the second degree Love number. For GEOS-1 a new, purely numerical method involving osculating elements was employed. For GEOS-2 it was necessary to analyze the variations of the mean elements because of the very long period (450^d) of the dominant solar tidal perturbation. An additional analysis of the variation of the mean elements of GEOS-1 confirmed the value of k_2 obtained from the osculating elements.

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1.0 INTRODUCTION

Recent determinations of geopotential models and satellite tracking station locations permit computation of satellite position to an accuracy of 10 meters or better along-track and to a fraction of this in the cross-track and radial directions for geodetic satellites such as the Geodetic Earth Orbiting Satellites (GEOS) 1 and 2. Since the effect of Earth tides is of the order of two seconds of arc or more out-of-plane (about 60 m) on the GEOS-1 orbit and ten seconds of arc (about 300 m) on the GEOS-2 orbit, it is reasonable to expect that a new determination of the 2nd degree Love number (k_2) with high accuracy is possible. Table 1 gives the specifications of the GEOS orbits.

The pioneering attempts at determination of tidal parameters from observations of satellites were by Newton (1968) and Kozai (1968). Both authors studied the variation of the inclination and/or node after removal of other perturbations by analytic methods. More recent determinations by Anderle (1971), Smith, et al. (1972), and the present authors employed numerical methods and have given results more precise than the earlier efforts.

The advantage of the recent numerical investigations is in the accuracy of computing the perturbations. The main disadvantage is that orbital arcs longer than about 100 days are difficult to analyze if numerical methods are employed exclusively. However, by combining numerical and analytic methods we were able to analyze a very long GEOS-2 arc (2 years).

2.0 DETERMINATION OF GEOS-1 TIDAL PARAMETERS FROM OSCULATING ELEMENTS

In the solution of Anderle the tidal parameter was treated like any other parameter and estimated directly from the data. In our pure numerical solution and the somewhat similar solution of Smith et al., a different method was used that permitted determination of k_2 from the inclination variation alone to reduce the effect of radiation pressure model error. This method is simple in concept but unfortunately requires a large amount of computer time.

The principle of this purely numerical method is to select an arc of data long enough for the tidal perturbation to reach a substantial value and if possible go through a full cycle of a dominant term. This long arc is then divided into shorter arcs and orbits determined (by Cowell's method) for both the long and short arcs with various values of k_2. If an erroneous value of k_2 is used in the determination of the long arc, the long arc orbit will be in error systematically by the amount of the error in the modeling of the Earth tide effects. In contrast, the short arc orbits, being short in length compared to the period of the tidal effects, adjust the orbit to best fit the actual inclination during the short arc. By differencing long and short arcs the method amounts to finding a value of k_2 that results in the short arc and long arc orbits being consistent (the same). We shall see that this method is sensitive to changes as small as 0.01 in k_2.

All erroneously modeled long periodic effects are revealed by this method. But in the case of GEOS-1 and 2, the frequencies of the tidal perturbations are distinct from other perturbations (except solar radiation pressure), so the possibility of error arising from other sources is reduced. Radiation pressure effects are discussed in Section 4.

This technique was used on a 65 day GEOS-1 orbital arc from March 11-May 15, 1966 (MJD 39195-39260). The SAO 1969 Standard Earth (Gaposchkin and Lambeck 1970) geopotential model was employed and about 7700 optical flash observations from a worldwide network of SAO, NASA, and International cameras at coordinates determined by Marsh, Douglas and Klosko (1971) were used to determine the orbit. In addition, BIH polar motion and UT1 time corrections were employed in the GSFC Cowell-type orbit determination program GEODYN (Martin, 1972)

We studied both the solar and lunar tidal perturbations. The effects are about equal in the 65 day arc so that our determination is based equally upon lunar and solar tidal effects. This is a very important fact, because of course the frequencies of the lunar effects are distinct from those of radiation pressure.

Figure 2.1 shows the combined luni-solar tidal variation of the inclination of GEOS-1 for the value $k_2 = 0.22$. This theoretical curve was obtained by numerical integration of the long period tidal perturbations of the inclination and the secular effects of oblateness. Note that the combined effect over the 65 day arc is about 1''.5, or about 50 m in terms of satellite position. This is very large compared to the position error out-of-plane for this geodetic satellite orbit.

As usual, resonance caused some problems in the investigation. GEOS-1 has significant resonant perturbations from 12th order geopotential terms. To smooth out errors of several tenths of an arc second in the inclination due to resonance, we divided the 65 day arc into arcs of length equal to the beat period (6.5 days) and compared inclinations from these arcs to inclinations obtained on the 65 day orbital arc. The procedure then involved finding a value of k_2 that when used in both the 65 day and 6.5 day data reductions eliminated differences in the inclination. Figure 2.2 shows these comparisons for values of $k_2 = 0.18$, 0.21, 0.24, 0.27 and 0.30. The values plotted are the inclination differences (residuals) between the 6.5 day arcs and the 65 day arc with Earth tidal effects modeled in both the long arc and the short arcs at the value indicated. We have plotted the differences at six hour intervals to show how the 6.5 day arcs respond to an error in k_2. The differences over a particular 6.5 day arc are essentially constant, showing that the 6.5 day arcs simply average over the unmodeled tidal effect. But of course since the tidal effects vary with time, this average will also vary with time. Further examination of the residual patterns in Figure 2.2 shows that the trends reverse between $k_2 = 0.21$ and $k_2 = 0.24$. Figure 2.3 shows the relation of the residual amplitudes to the value of k_2. An interpolation yields the value $k_2 = 0.22$. It is also possible to extrapolate to the residual effect to be seen with $k_2 = 0$. Such an extrapolation agrees with the observed tidal variation.

Because of the nature of the osculating element method, it is difficult to obtain the usual formal uncertainty of the result. The method is however, very sensitive. Some indication of uncertainty can be seen in Figure 2.3. Note that the residual amplitude values are not exactly on a straight line, but an (1σ) uncertainty of less than 0.01 in k_2 is probable. The analysis of the mean elements in the following section does provide a formal uncertainty of about this value. As an interesting aside, the osculating element method was sensitive enough to show that BIH polar motion values are superior to IPMS values for this (March 11-May 15, 1966) period.

A value of k_2 could not be obtained from GEOS-2 by the pure numerical method described above. The dominant solar tidal perturbation has a period of 450^d for GEOS-2. This is too long an orbital arc to be determined by numerical integration of Cartesian coordinates. Instead we evaluated the variation of the inclination of GEOS-2 by numerical integration of mean elements. The mean inclination of GEOS-1 was also analyzed to confirm the value obtained from the osculating elements.

3.0 DETERMINATION OF TIDAL PARAMETERS FROM MEAN ELEMENTS

The most common method of determining geodetic parameters from long-periodic orbital variations is to compare the changes in the mean elements of an orbit with the changes predicted by theory. In this paper we used this method for GEOS-2 exclusively because of the very long (2 year) orbital arc considered. The mean elements of GEOS-1 were also analyzed, and the results obtained from the osculating elements confirmed.

The mean elements of GEOS-1 and 2 were taken from Douglas, Marsh, and Mullins (1972). These elements were obtained by applying a combined analytical-numerical averaging technique to osculating elements. The osculating elements were based upon 2 day orbital arcs determined from optical flash data from a worldwide network of tracking stations. The precision of these elements appears to be a few tens of centimeters in the semi-major axis, and about $0\overset{''}{.}2$ in the orientation angles. The very high precision of the mean semi-major axis probably results from the lack of m-daily perturbations of that element. On the other hand, the uncertainty of the mean inclination and node of a few tenths (or about 6 m) is comparable to the satellite position error out of plane for the GEOS satellites. Nevertheless, since the tidal perturbation of the inclination exceeds 10" for GEOS-2, and 2" for GEOS-1 these mean elements are sufficient for a good determination.

In order to analyze the mean inclination variations we chose to numerically integrate simultaneously all long periodic and secular effects on the orbit. A computer program called Rapid Orbit Analysis and Determination (ROAD) program (Douglas et al., 1972; Williamson and Mullins, 1972) was prepared to estimate geodetic parameters using this idea. This program models all significant effects including the geopotential, luni-solar, drag, solar radiation pressure, precession and nutation, and Earth tides using the formulation of Kaula (1969). Geodetic parameters can be estimated from any number of arcs of many satellites using any or all of the mean Kepler elements as data. Because of the absence of high frequency variations in the equations of motion, integration step sizes of 1 - 2 days are possible with a 10th order predictor-corrector scheme. Thus a one year orbital arc requires only about 1 minute of IBM 360/95 computer time for computation.

As is well known, inaccurate modeling of radiation pressure effects poses one of the greatest potential threats to a successful determination of tidal parameters. This is especially so for GEOS-1 because of its substantial ($e = 0.07$) eccentricity. The solar radiation pressure perturbation of the inclination of GEOS-1 is a significant fraction of the tidal perturbation. However, since the semi-major axis of an eccentric orbit is also perturbed by solar radiation pressure but not by Earth tides, the solar radiation pressure effects can be evaluated separately from the tidal effects. This is done in Section 4, below. In the case of GEOS-2, solar radiation pressure effects on i are small and easily modeled and have little effect on the determination of k_2.

In order to verify the results obtained from GEOS-1 osculating elements we analyzed 3 different arc lengths of the GEOS-1 elements. The shortest of the three was the same 65 day period used in the determination from osculating elements. This arc was extended to 122 days, and a final eleven month arc covering all available data in 1965-1966 was also used. All cases gave the same value $k_2 = 0.22$ to two significant figures. The lag angle ϵ varied slightly from the a priori value, but the differences were not statistically different from each other or the a priori value of $2^\circ\!.5$.

Figure 3.1 shows the residual inclination in the 122 day arc of GEOS-1 after removal of all perturbations except Earth tides. Note that the tidal effect is plainly visible. Figure 3.2 shows the residuals in the same arc after deriving the value of $k_2 = 0.22$. The rms of fit is $0''\!.15$. Similar good results were obtained from the 65^d and 11 month arcs.

Figure 3.3 shows the residual inclination for a 2 year GEOS-2 arc before solution for k_2. Figure 3.4 shows the results after solution for k_2. The amplitude of the solar tidal perturbations is so large that the accidental errors in the data have little effect on the accuracy of the determination of k_2. Unfortunately, even for GEOS-2 the phase angle is not determinable to a precise enough value other than to confirm that it is very small.

4.0 ERROR ANALYSIS

The major error sources affecting this determination are radiation pressure and geopotential uncertainty. Drag perturbations do not affect the determination since our attention is confined to variations of the inclination.

The data reduction for the GEOS-1 65 day orbital arc gave a fit of 13.″9 in right ascension and 25.″4 in declination. In contrast, the 6.5 day arcs yielded fits of about 2.″0 in each coordinate. At first glance it seems surprising that the inclination differences between the long and short arc orbits could give a precision of better than 0.″1, but there is no paradox. This situation results from the nature of the geopotential perturbations.

The tesseral harmonics of the geopotential do not produce long period perturbations of the elements (excluding resonance). The effect of the tesseral harmonics is to produce oscillations of frequency m cycles/day, where m is the order of a tesseral harmonic. A frequency of m times daily is relatively short periodic compared to either the 6.5 day or 65 day arcs. Therefore both orbital arc lengths yield the average inclination over the arc and the inclination is well determined in an average sense. An error in the orbital energy does, however, also cause an error in the period which propagates along track rapidly because of Kepler's third law. For example, a 10 cm error in the semi-major axis of the GEOS satellites causes an along-track error of nearly 100 m after one week. Thus a small error in the orbital energy has serious consequences. Since the

GEOS-1 orbit is not polar, along track error of this type will propagate into both declination and right ascension according to the tangent of the inclination. We note that the ratio of the rms fits in declination and right ascension is very nearly the tangent of 59°, the inclination of the orbit.

The most important potential error source by far for determination of tidal parameters from GEOS-1 orbital variations is radiation pressure. Errors in the modeling of the radiation pressure cause effects resembling the solar tidal perturbations on the orbit.

Figure 4.1 shows the variation of the inclination of GEOS-1 for 1966 obtained by numerical integration of the long-periodic changes of the elements due to solar radiation pressure. (The secular effects of oblateness were also included to keep the orbit properly oriented with respect to the sun.) It is immediately apparent that the radiation pressure perturbations could corrupt the determination of the tidal parameters. Note that the total range of variations due to radiation pressure is about 0".3 during March 11-May 15. The total variation due to tidal effects is about 1.5 arc second, so errors in the radiation pressure model could degrade the accuracy of the tidal parameter determination. However, since drag is very small for GEOS-1, we are in a position to evaluate the adequacy of our radiation pressure model for that orbit by studying the variations of the semi-major axis. This element has no long periodic variations (apart from resonance) due to gravitational forces. (The effect of resonance is readily evaluated because the beat period of the orbit is accurately known.) Figure 4.2 taken from Douglas, Marsh and Mullins (op cit) shows the evolution of the mean semi-major axis of GEOS-1 during 1966. Note that the orbit experiences very large perturbations due to radiation pressure.

Figure 4.3 presents the residuals in mean semi-major axis for the 65^d period investigated. This plot reflects all unmodeled long periodic variations, in this case radiation pressure error. The total variation of the residuals in Figure 4.3 is about 25 cm. Referring to Figure 4.1, we see that the variation in a due to radiation pressure is about 5 meters during this period. Thus we conclude that we have modeled the solar radiation pressure to high accuracy for GEOS-1. Also, since the lunar tidal effects are of comparable importance to the solar effects for GEOS-1 and of distinctly different period, the effect of a solar radiation pressure model error is reduced.

The adequacy of the radiation pressure model for GEOS-2 cannot be evaluated in the same manner as GEOS-1. The orbit of GEOS-2 is nearly circular ($e = 0.02$) and there is a significant decay (10 m/year) of the semi-major axis due to atmospheric drag. However, the radiation pressure perturbations are much smaller on GEOS-2 than GEOS-1, so the effect of an error in the radiation pressure coefficient C_R will be much less than in the case of GEOS-1. To determine the effect of an error in C_R, k_2 was determined with the value obtained from the analysis of the GEOS-1 semi-major axis ($C_R = 1.51$) and with the value $C_R = 2.0$. The result was to change the value of k_2 determined by 0.02, or less than 10%. Thus we conclude that for either GEOS-1 or GEOS-2, radiation pressure error is unlikely to have influenced our determination by an amount as large as 0.01.

5.0 CONCLUSIONS

The results of this investigation show that osculating elements can be used with great effectiveness to investigate long period orbital variations. In addition the mean element technique presented by Douglas, Marsh, and Mullins (1972) yields a precision of about 0.″2 or better in the inclination making elements so obtained a very powerful means of investigating long periodic orbital changes.

The formal standard errors for k_2 obtained from the mean element data analyses were about 0.01 for GEOS-1 and less than 1/2 of this for GEOS-2. Of course in investigations like this one it is commonplace for the true uncertainty to be 2 or 3 times the uncertainty obtained from the least-squares process. Therefore we assign somewhat arbitrarily an uncertainty of 0.02 to the GEOS-1 value of 0.22 and 0.01 to the GEOS-2 value of 0.31. Based upon these low uncertainties it is concluded that the large difference between the values of k_2 derived from satellites at different orbital inclinations is real and requires explanation.

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Table 1

Orbital Elements of GEOS-1 and 2

	GEOS-1	GEOS-2
Epoch	Jan. 2, 1966	April 28, 1968
Apogee Height	2273 km	1569 km
Perigee Height	1116 km	1077 km
Eccentricity	0.07	0.03
Inclination	59°.4	105°.8
Anomalistic Period	120.3 min	112.1 min

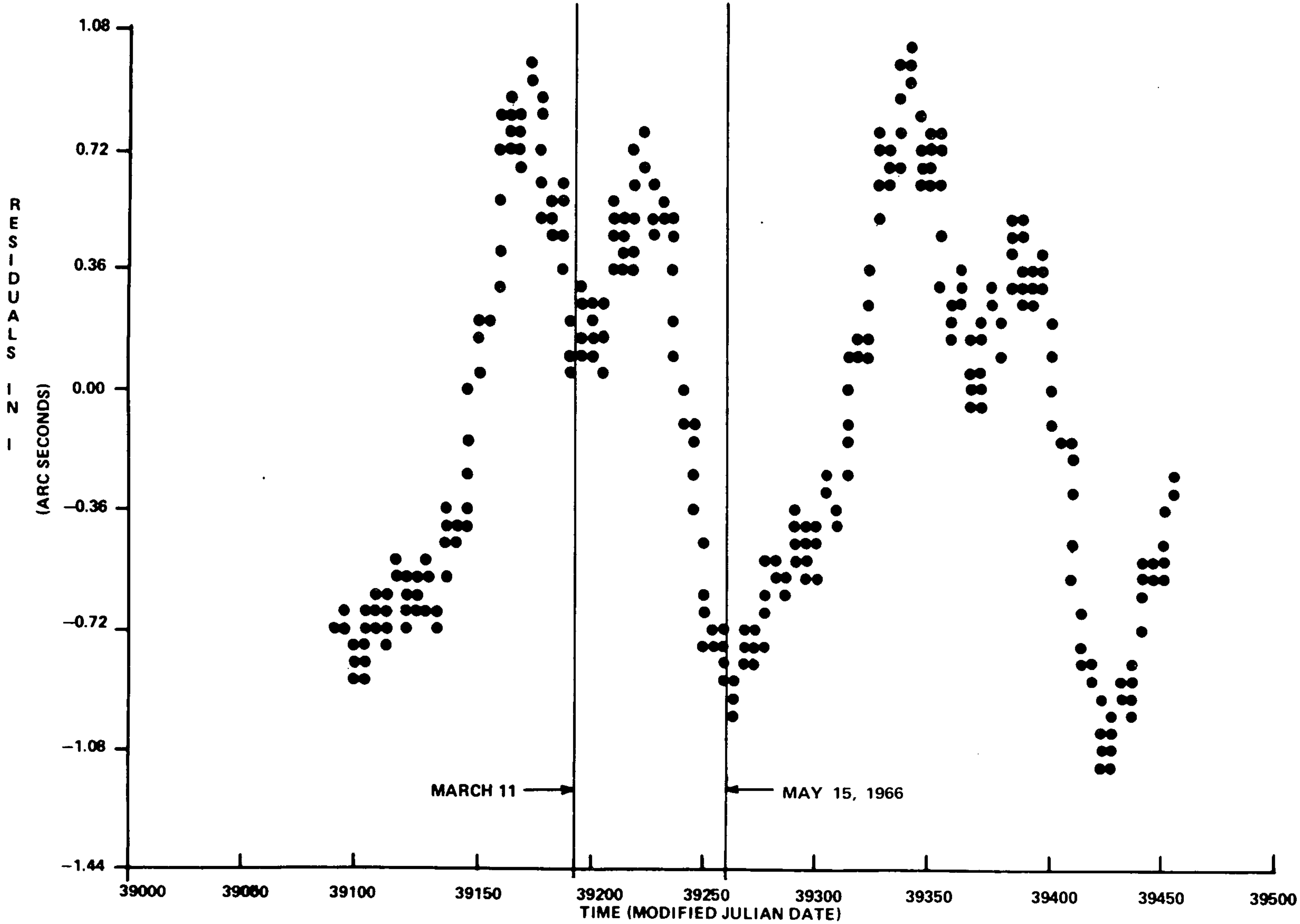

Figure 2.1. Combined Effects of Lunar and Solar Tides in the Inclination of GEOS-1

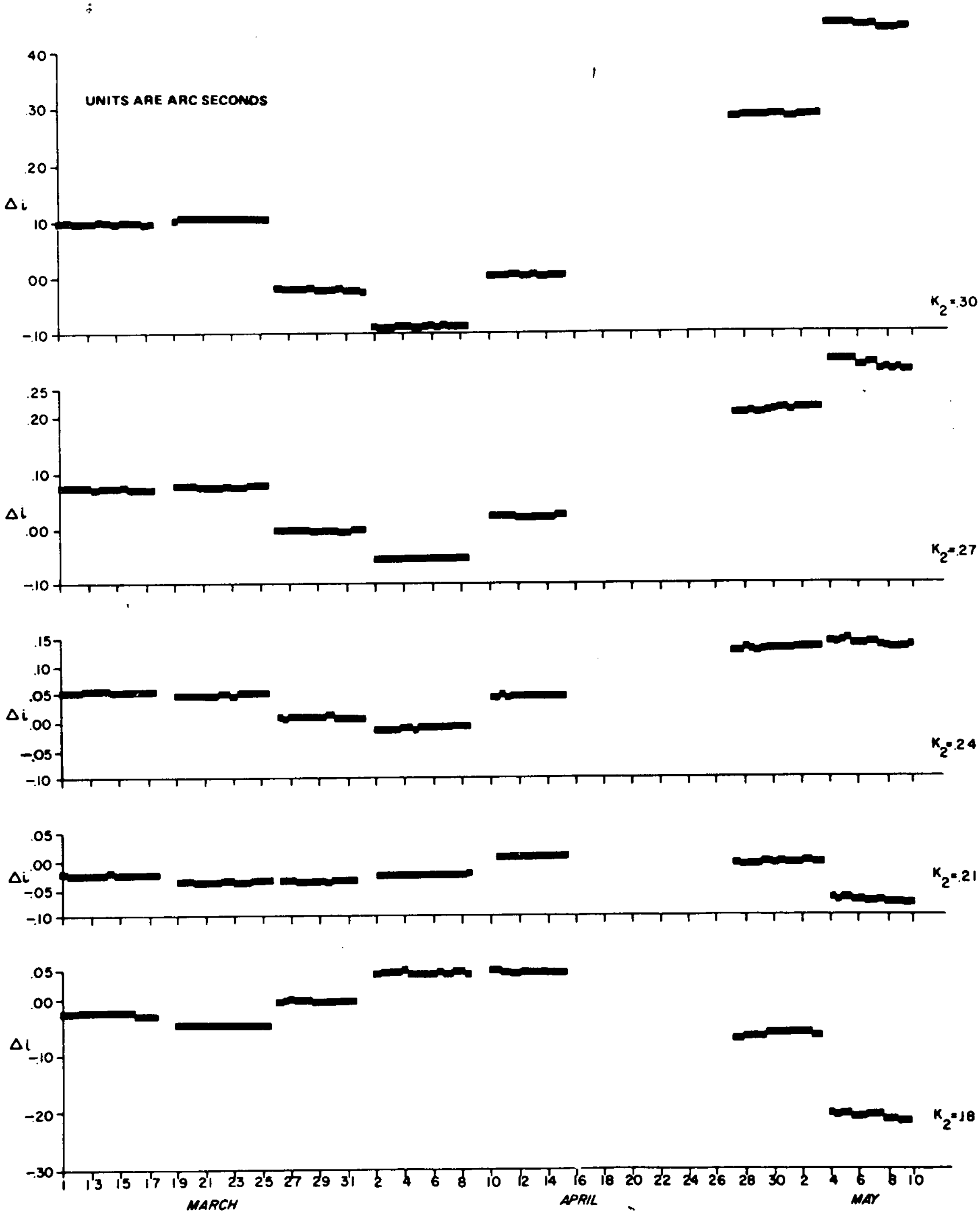

Figure 2.2. Differences Between Long (65^{d}) and Short ($6^{d}.5$) Arc Inclination for Various Values of k_2

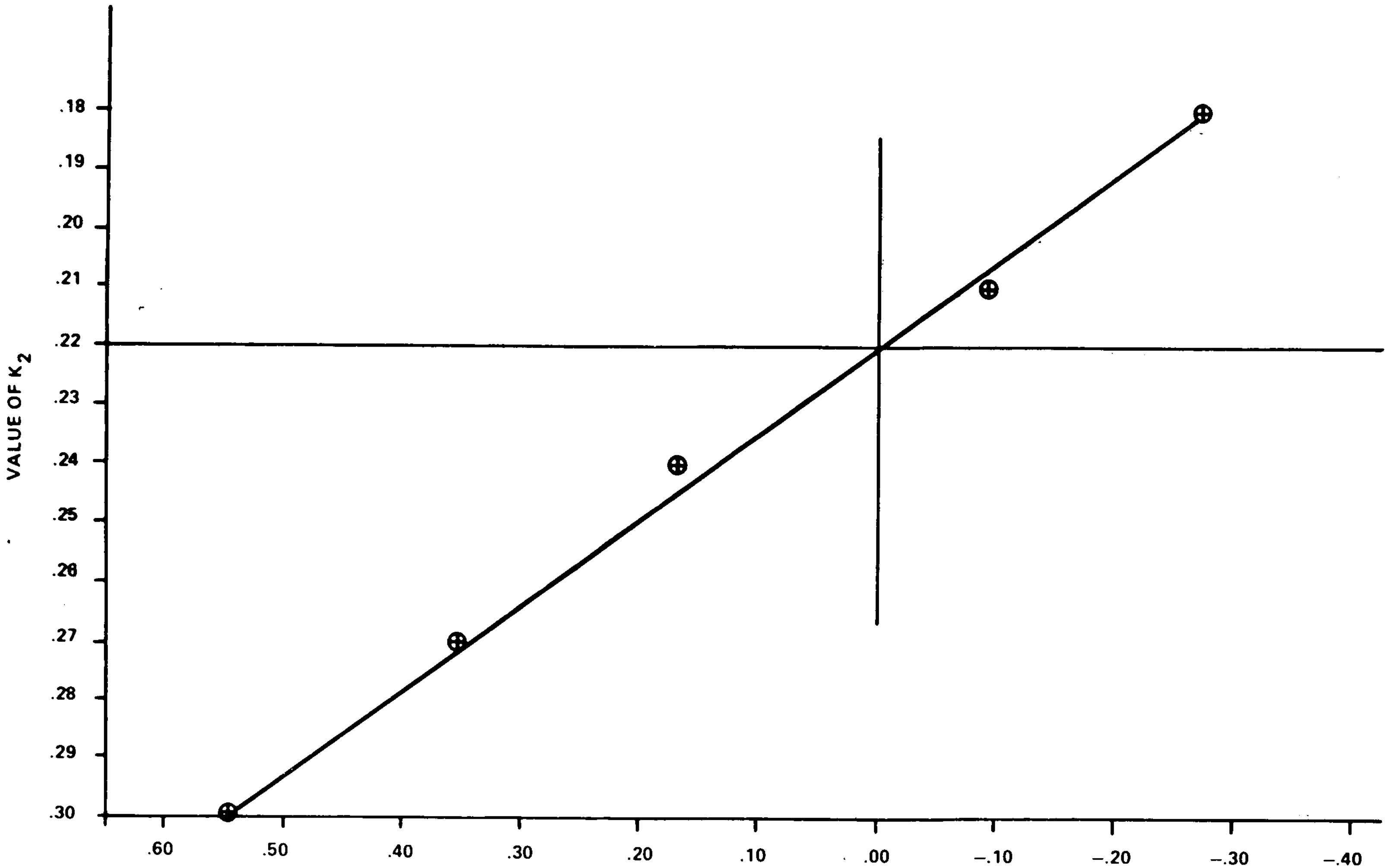

Figure 2.3. Unmodeled Inclination Variation as a Function of k_2

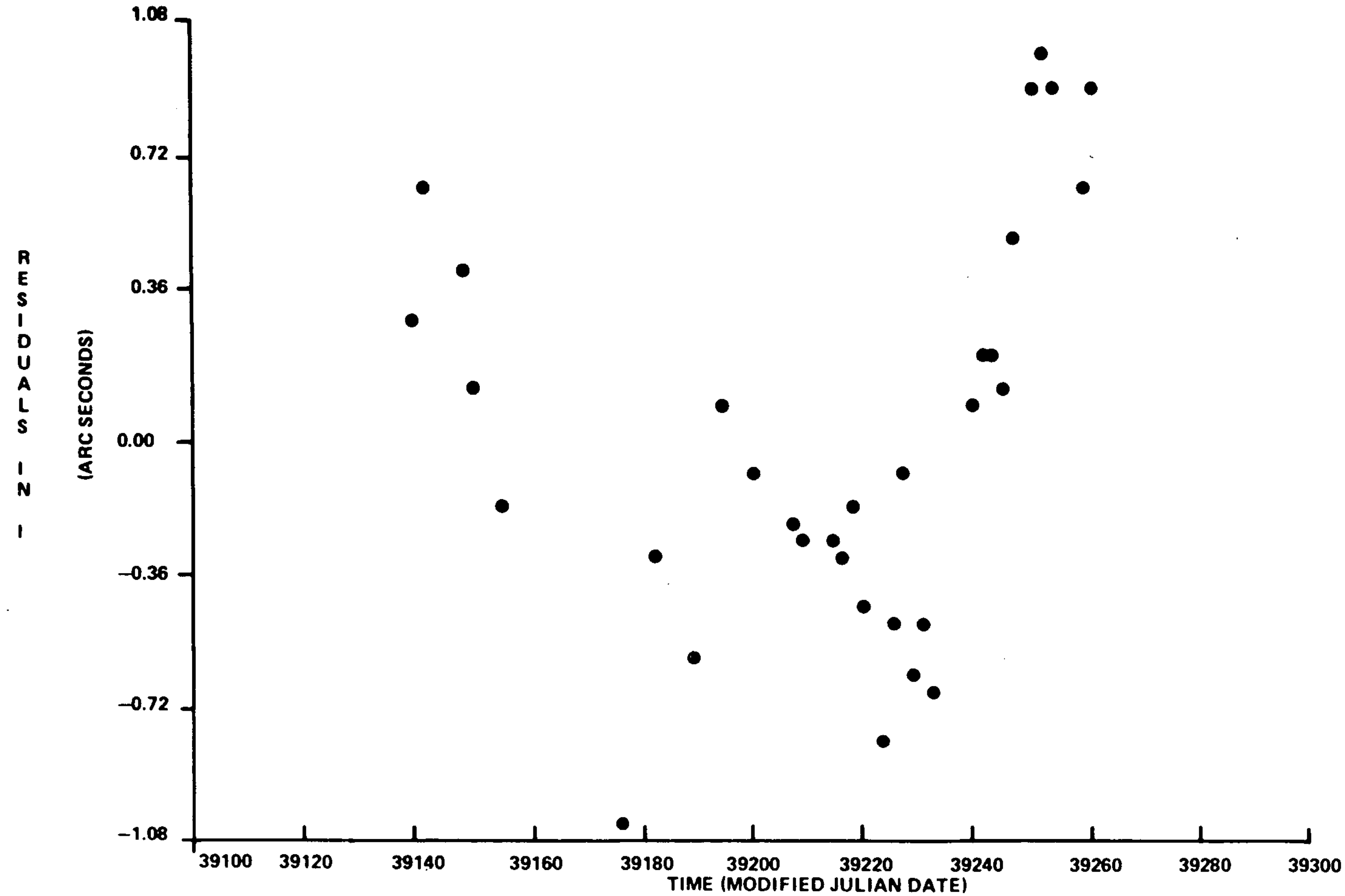

Figure 3.1. GEOS-1 Inclination Residuals Before Solution for k_2 (122 Day Arc)

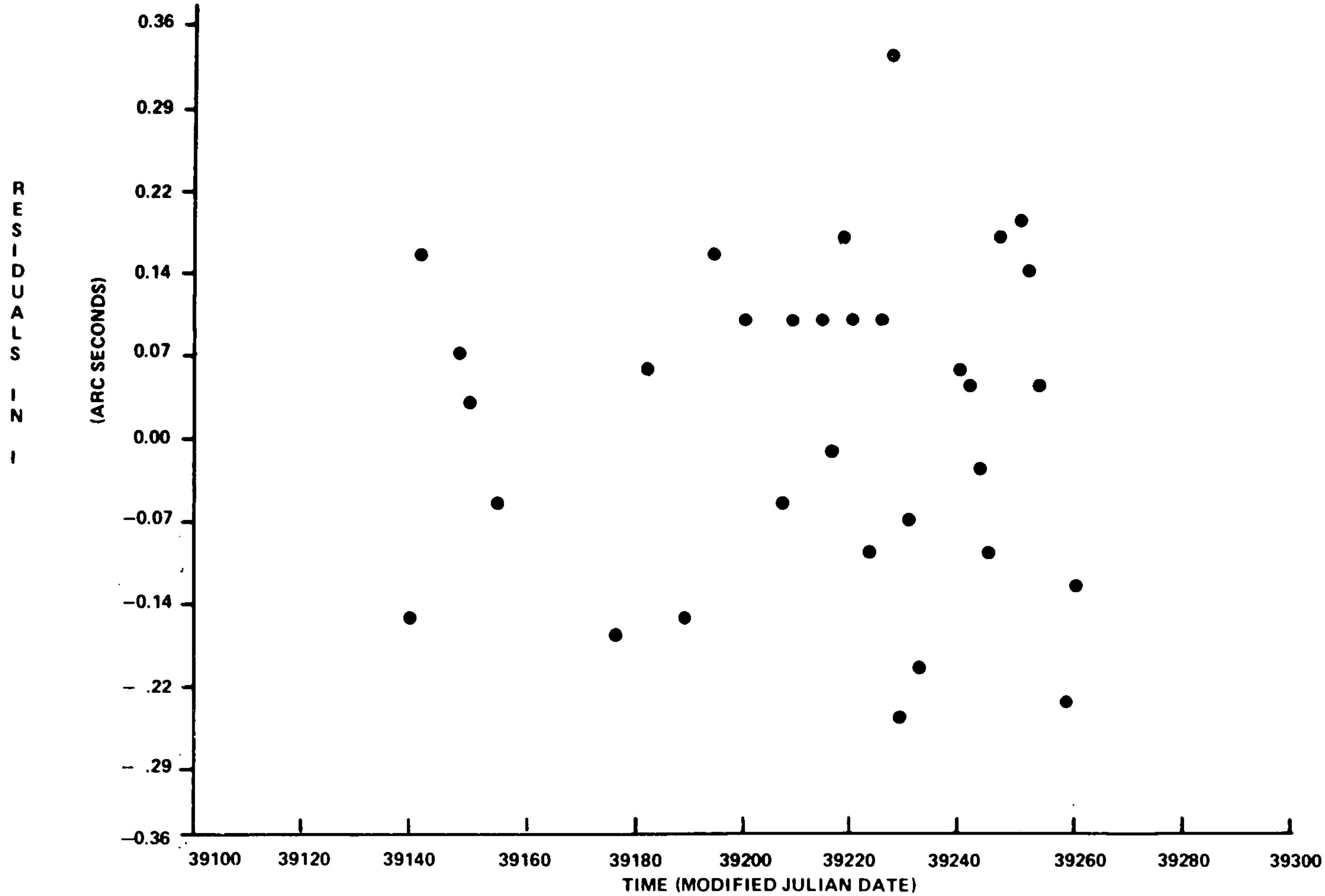

Figure 3.2. GEOS-1 Inclination Residuals with $k_2 = 0.22$

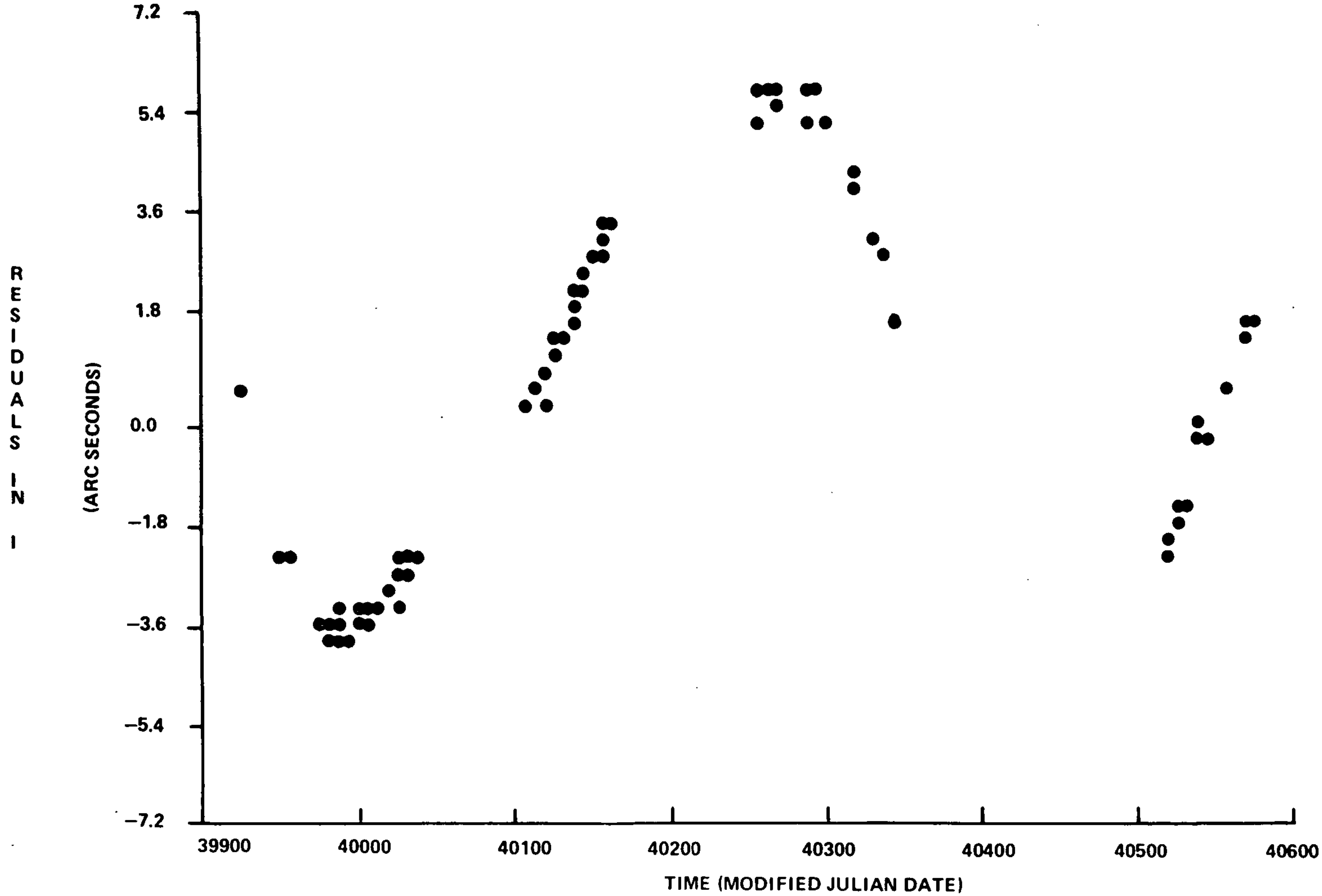

Figure 3.3. GEOS-2 Inclination Residuals Before Solution for k_2

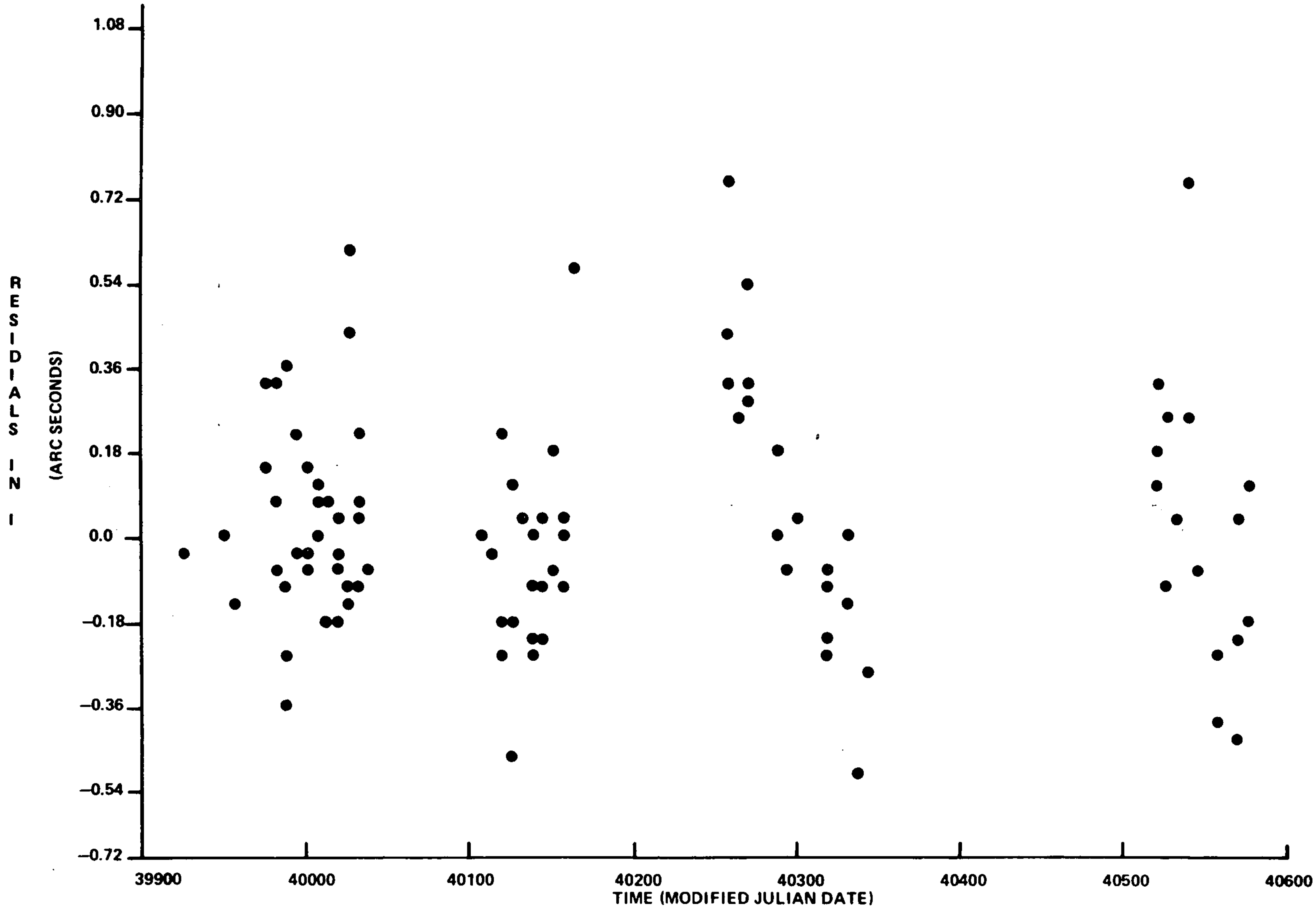

Figure 3.4. GEOS-2 Inclination Residuals with $k_2 = 0.31$

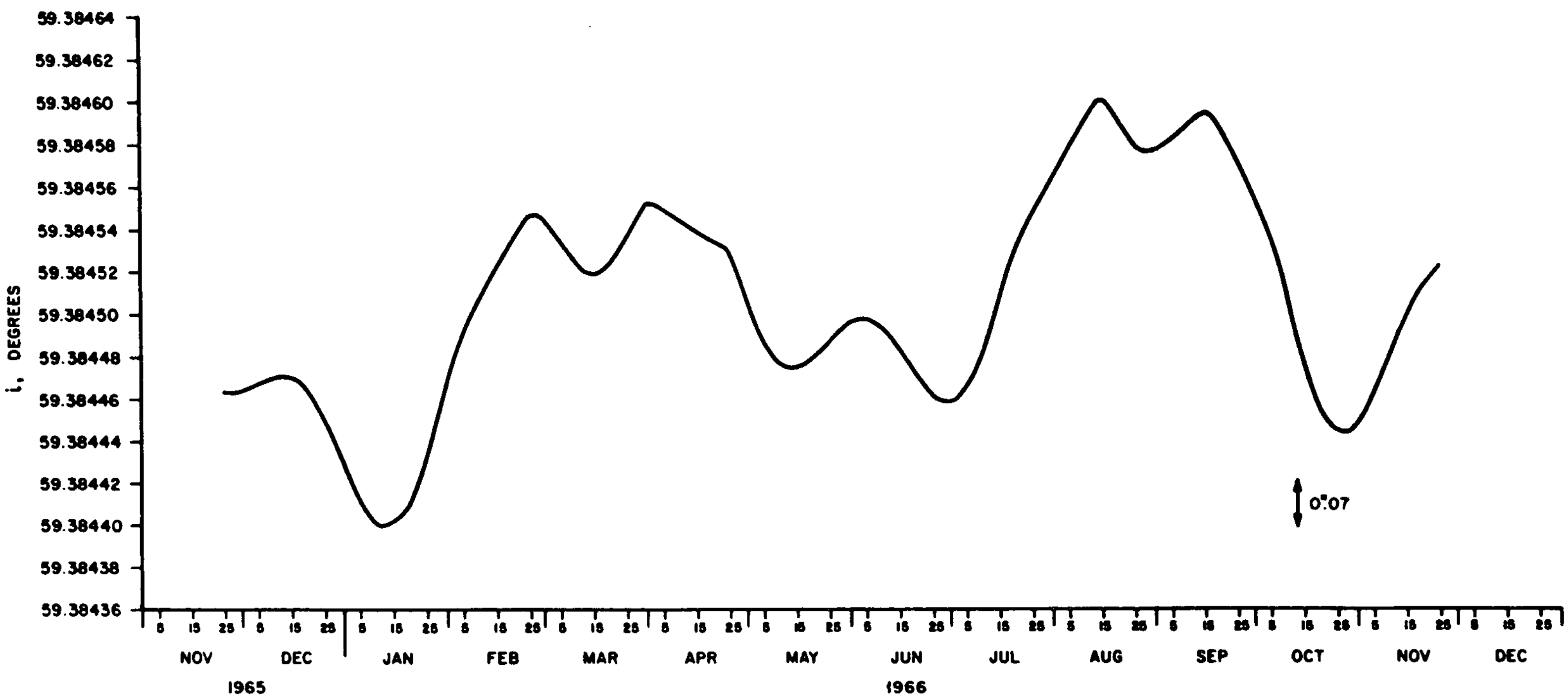

Figure 4.1. Effect of Solar Radiation Pressure on the Inclination of GEOS-1

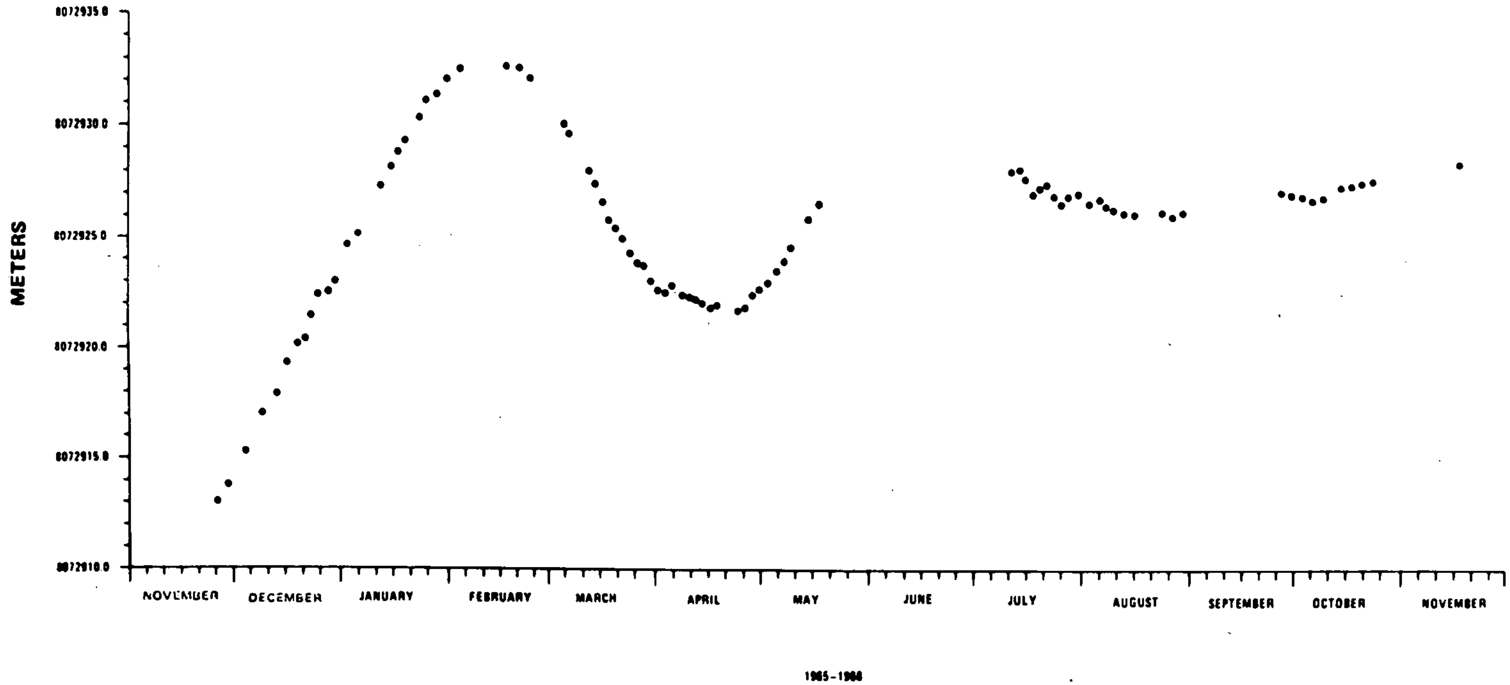

Figure 4.2. GEOS-1 Mean Semi-Major Axis from Two-Day Optical Data Arcs

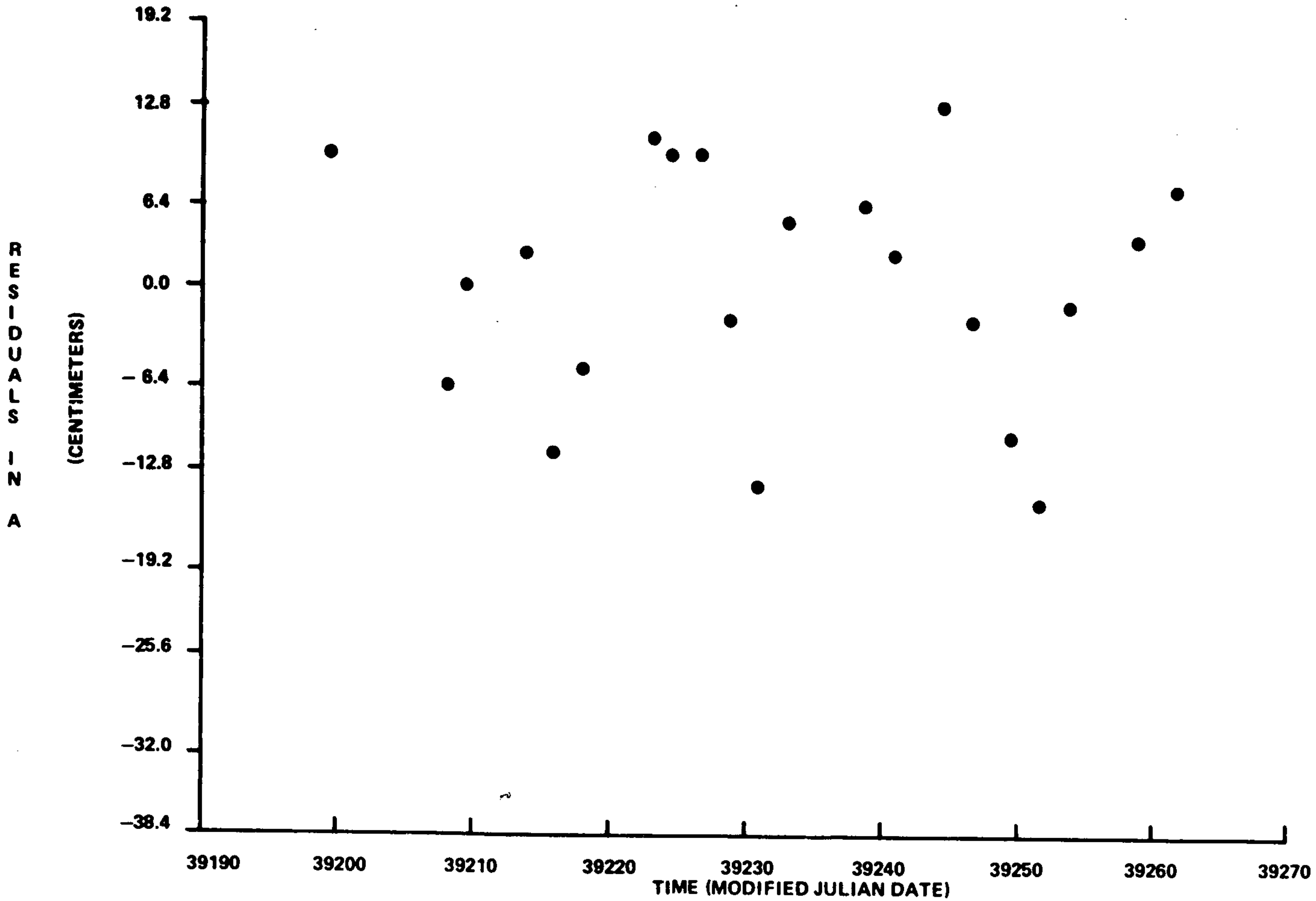

Figure 4.3. Mean Semi-Major Axis Residuals for GEOS-1