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ABSTRACT

An algorithm has been developed to solve the following problem: given the characteristics of the engine to be used to make a finite burn maneuver and given the desired orbit, when must the engine be ignited and what must be the orientation of the thrust vector so as to obtain the desired orbit?

The desired orbit is characterized by classical elements and functions of these elements whereas the control parameters are characterized by the time to initiate the maneuver and three direction cosines which locate the thrust vector.

The algorithm has been built with a Monte Carlo capability whereby samples are taken from the distribution of errors associated with the estimate of the state and from the distribution of errors associated with the engine to be used to make the maneuver.

CONTENTS

GLOSSARY OF SYMBOLS

$\{x, y, z\}$ – inertial coordinates of the spacecraft

μ – gravitational parameter of the central body

$\dot{m}$ – mass flow rate of the engine

V_e – exhaust velocity of the engine

m – mass of the spacecraft

t_0 – time to start the engine

t_f – time of engine shutdown (terminal time)

$\bar{\Theta}$ – vector whose components are the parameters being targeted to (terminal conditions)

$\hat{e}$ – unit vector whose components locate the direction of thrust

$\bar{x}(t)$ – the spacecraft state vector at time t

a – semi-major axis

e – eccentricity

ι – inclination

RCA – radius of closest approach

V_P – orbital velocity of spacecraft at RCA

α – pitch angle

β – yaw angle

AN ALGORITHM FOR TARGETING FINITE BURN MANEUVERS

INTRODUCTION

During orbiting and interplanetary missions it is usually necessary to perform maneuvers so as to attain prescribed boundary conditions. Such boundary conditions can be given in terms of a particular state to be reached or, more frequently, in terms of an orbit or trajectory which the spacecraft must be on at the termination of the maneuver. For example, during a planetary orbit insertion maneuver the spacecraft is initially on a hyperbolic trajectory; it is required that after the maneuver the spacecraft be in an elliptical orbit about the planet and that this elliptic orbit have prescribed characteristics.

In most instances the targeting of a spacecraft during a maneuver is accomplished using the impulsive algorithm which does not involve any numerical integration. However, as advanced missions become more complex, the impulsive algorithm may not be sufficient to calculate fuel requirements for a given maneuver. A recent mission which underscores this point is that of the Mariner Mars 1971 spacecraft where the orbit insertion maneuver lasts for roughly 15 minutes.

According to Robbins [1] the impulsive algorithm can fail to produce the same results as a finite burn algorithm for two reasons: one is the gravity gradient effect and the other is the effect due to non-constant thrust vector orientation during a maneuver. The gravity gradient effect is the contribution, to the motion of the spacecraft during a maneuver, of the time and position dependence of the gravitational acceleration.

Consequently, algorithms which simulate the motion of a spacecraft during a maneuver are needed. This need can be satisfied in two ways depending on the mission:

(i) integrate the linearized equations of motion through the burn

(ii) integrate the nonlinear equations of motion through the burn

In both procedures the problem of how to treat the thrust vector arises; here, there are three options:

(iii) keep the thrust vector orientation fixed throughout the burn interval

(iv) allow the thrust vector to pitch at a fixed rate, or,

(v) allow the thrust vector to have three degrees of freedom

This report will present some results obtained from our development of a program incorporating (ii) and (iii).

The problem of reaching prescribed boundary conditions is a matter of determining the correct thrust vector orientation and the best time or place to begin the maneuver. The problem is statistical in nature because the actual trajectory of the spacecraft is never completely known; the filtering of tracking data yields an estimate of the actual location of the spacecraft and associated with this estimate is a covariance matrix which yields information about the quality of the estimate.

The thrust vector will be determined by using the best estimate of the trajectory and a guess at the thrust vector orientation as the initial conditions in the non-linear differential equations describing the motion of a thrusting spacecraft. An algorithm which minimizes a given performance index is then used to iterate on the initial guess of the thrust vector orientation.

The Monte Carlo option of the algorithm is carried out in the standard way. Explicitly the covariance matrix associated with an estimate of the state of the spacecraft is assumed to be available and furthermore it is assumed that the standard deviations associated with the thrust magnitude error and two pointing errors are known. The algorithm first targets the spacecraft to the constraint parameters; this yields an initial time to begin the maneuver and the direction of the thrust vector. Next the algorithm diagonalizes the state covariance matrix and then samples from this matrix and the diagonal covariance matrix associated with the engine. The errors obtained from this sampling procedure are then added to the spacecraft state and to the model which is being used to simulate powered flight.

It must be emphasized, however, that during the sampling procedure, the thrust vector orientation obtained during the targeting phase is associated with each member of the sample. This means that no member of the sample is targeted to the constraint parameters. In developing the algorithm in this way, the time consuming iteration procedure is used only once, when solving for the time to begin the maneuver and the thrust vector orientation.

The rationale for this procedure is twofold: (i) the engine to be used for making the maneuver is assumed to have been sized so that fuel loading optimization is not needed and, (ii) with (i) in view, any maneuver is always performed using the best estimate of the state of the spacecraft as a reference so that it becomes an esoteric study to determine the effect of varying the initial orientation of the thrust vector for each member of the sample.

CONTROL PARAMETERS

The equations governing the motion of a thrusting vehicle are well known to be

$$\ddot{x} = -\frac{\mu}{r^3} x + \frac{\dot{m} V_e}{m(t)} \cos\psi \cos\theta$$

$$\ddot{y} = -\frac{\mu}{r^3} y + \frac{\dot{m} V_e}{m(t)} \cos\psi \sin\theta$$

$$\ddot{z} = -\frac{\mu}{r^3} z + \frac{\dot{m} V_e}{m(t)} \sin\psi$$

where x, y, z are inertial Cartesian coordinates, μ is the gravitational parameter of the control body, $\dot{m}$ is the mass flow rate of the engine, V_e is the exhaust velocity and m is the mass of the vehicle. The angles ψ and θ, shown in Figure I serve to define the thrust vector orientation.

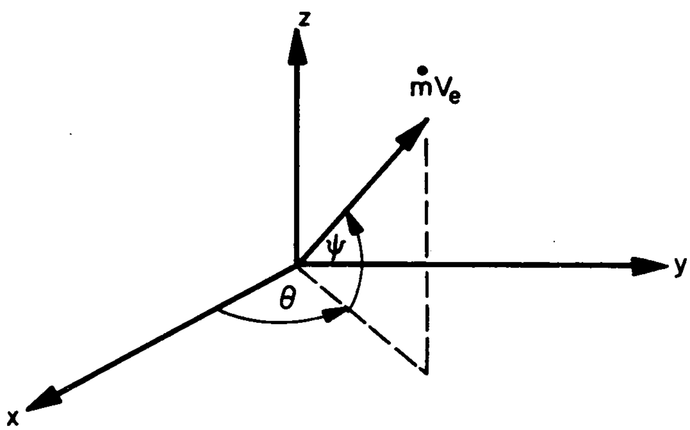

Figure I

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The magnitude of the radius vector of the spacecraft with respect to the origin of the coordinate system is given by

$$r^2 = x^2 + y^2 + z^2$$

The mass flow rate is allowed to vary in the program; however, since the engine has been sized the burn time is known. Consequently there are four parameters, called control parameters, which characterize the maneuver and must be solved for. Specifically, the time to begin the maneuver is unknown as is the orientation of the thrust vector at the beginning of the burn.

The orientation of the thrust vector is given by the three quantities, $E_1 = \cos\psi \cos\theta$ $E_2 = \cos\psi \sin\theta$ and $E_3 = \sin\psi$. The initial orbit is known and consequently the state of the spacecraft on this orbit can be associated with the time to begin the maneuver. Once the orientation of the thrust vector is determined at the beginning of the maneuver it is held fixed throughout the burn interval.

CONSTRAINT PARAMETERS

The constraint conditions are characterized by the classical orbit parameters and functions of these parameters; they represent terminal boundary conditions which must be satisfied by the vehicle at the end of the burn interval.

In particular the set of constraint conditions is composed of the following parameters:

- semi-major axis
- inclination
- true anomaly
- long. ascending node
- velocity of perigee
- angular momentum
- eccentricity
- arg. of perigee
- radius of perigee
- arrival energy
- time from perigee

During any particular run, a subset of these conditions is usually used.

A direct solution of the problem is not possible since the constraint conditions are non-linear functions of the control parameters, that is, the final state attained by the spacecraft is quite critically dependent upon where the maneuver is initiated and upon the orientation of the thrust vector.

METHOD OF SOLUTION

The overall problem falls into the general category of two point boundary value problems. The technique used in this program to find a solution is to iterate on the initial conditions until the given terminal conditions are satisfied. In particular, let t_f be the terminal time, $\overline{x}(t)$ be the state at time t and $\overline{\Theta}\ [\overline{x}(t_f)]$ the vector of terminal conditions which must be satisfied. In order to satisfy this terminal vector, the initial time t_0 and the initial thrust vector orientation, $\hat{e}(t_0) = (E_1, E_2, E_3)$, must be corrected and recorrected until the terminal conditions are satisfied. To construct this iterative procedure it is realized that $\hat{e}(t_0)$ completely determines a solution because $\overline{x}(t_0)$ is fixed once t_0 is chosen. Thus $\overline{x}(t_f)$ depends on $\hat{e}(t_0)$ and this implies that the terminal conditions are representable in the form $\overline{\Theta}\ [\hat{e}(t_0), t_0]$.

In order to formulate the orbit transfer strategy a performance index is employed as a criteria to determine a solution. In this program the index is the weighted squared distance, with constraints, in the parameter space between the final orbit and the desired orbit; the objective is to find the time t_0 and orientation $\hat{e}(t_0)$ which minimizes this performance index.

The algorithm which performs the minimization is known as MINMAX [2] to which the interested reader is referred. A very brief overall description of the algorithm is presented below.

Denote the desired terminal conditions by $\overline{\Theta}_D\ [\hat{e}(t_0), t_0]$ and define the vector $\overline{y}(t) = [\hat{e}(t), t]$. Let t_{oi} be the ith value of the ignition time and $\hat{e}(t_{oi})$ the ith value of the orientation of the thrust vector. A first order Taylor series expansion about t_{oi}, $\hat{e}(t_{oi})$ yields

$$\overline{\Theta}_D\ [\overline{y}(t_{oi+1})] \approx \overline{\Theta}[\overline{y}(t_{oi})] + [\nabla_{\overline{y}(t_{oi})}\overline{\Theta}]\ [\overline{y}(t_{oi+1}) - \overline{y}(t_{oi})]$$

where

$$\nabla_{\overline{y}(t_{oi})}\overline{\Theta} = \left[\frac{\partial\overline{\Theta}}{\partial\hat{e}}\ \frac{\partial\overline{\Theta}}{\partial t}\right]_{t_{oi}} = \left[\frac{\partial\overline{\Theta}}{\partial E_1}\ \frac{\partial\overline{\Theta}}{\partial E_2}\ \frac{\partial\overline{\Theta}}{\partial E_3}\ \frac{\partial\overline{\Theta}}{\partial t}\right]_{t=t_{oi}}$$

is an $m \times 4$ matrix with m being the dimension of the vector of terminal conditions. Thus the original nonlinear problem has been replaced with a linear one whose solution is direct; however this solution does not satisfy the nonlinear problem. It is for this reason that a sequential procedure must be utilized.

The sequential process will now be constructed. A scalar $\Psi\,[\bar{y}(t_{oi})]$ is defined as follows:

$$\Psi\,[\bar{y}(t_{oi})] = [\bar{\Theta}_D - \bar{\Theta} - \nabla_{\bar{y}(t_{oi})}\bar{\Theta}\,\Delta\bar{y}(t_{oi})]^T\, W_\alpha\, [\bar{\Theta}_D - \bar{\Theta} - \nabla_{\bar{y}(t_{oi})}\bar{\Theta}\,\Delta\bar{y}(t_{oi})]$$

where W_α is a weighting matrix introduced so as to make the components of the vector $\bar{\Theta}_D - \bar{\Theta} - (\nabla_{\bar{y}(t_{oi})}\bar{\Theta})\Delta\bar{y}$ compatible among themselves and superscript T denotes the transpose.

It may happen that the minimization of $\Psi\,[\bar{y}(t_{oi})]$ will yield a $\Delta\bar{y}$ which is quite large; this can occur when $\bar{\Theta}_D - \bar{\Theta}$ is large or when $\nabla_{\bar{y}(t_{oi})}\bar{\Theta}$ is ill-conditioned. Large values of $\Delta\bar{y}$ can easily take us out of the region of linearity resulting in wild fluctuations of the residual vector $\bar{\Theta}_D(t_{oi}) - \bar{\Theta}(t_{oi})$. To prevent this from happening a constraint is imposed in the form

$$(\Delta\bar{y}(t_{oi}))^T\, W_\beta\, \Delta\bar{y}(t_{oi}) \le S_0$$

where W_β is a square matrix of scale factors and S_0 is a constant chosen in such a way that at no point during the iteration is the linearization (the Taylor series at each step) violated.

The performance index may now be defined to take the form

$$J\,[\bar{y}(t_{oi})] = \Psi\,[\bar{y}(t_{oi})] + \lambda(\Delta\bar{y}(t_{oi}))^T\, W_\beta\, \Delta\bar{y}(t_{oi})$$

The problem now is to find a set of $\bar{y}(t_{oi})$ $i = 1, 2, \ldots$ such that

$$J\,[\bar{y}(t_{oi+1})] < J\,[\bar{y}(t_{oi})]$$

and such that the sequence $\{\bar{y}(t_{oi})\}$ converges to a point $\bar{y}(t_0)$ which minimizes $J\,[\bar{y}]$.

Minimization of the performance index is accomplished by taking the derivative of J with respect to $(\Delta\bar{y})$ and equating this derivative to zero. This procedure yields

$$\frac{\partial J\,[\overline{y}(t_{oi})]}{\partial\,[\Delta\overline{y}(t_{oi})]} = -\,[\nabla_{\overline{y}(t_{oi})}\,\overline{\Theta}]^T\, W_\alpha\,(\overline{\Theta}_D - \overline{\Theta})$$

$$+\,[\nabla_{\overline{y}(t_{oi})}\,\overline{\Theta}]^T\, W_\alpha\,[\nabla_{\overline{y}(t_{oi})}\,\overline{\Theta}]\,\Delta\overline{y}(t_{oi})$$

$$+\,\lambda W_\beta\,\Delta\overline{y}(t_{oi}) = 0,$$

and solving for $\Delta\overline{y}(t_{oi})$ yields

$$\Delta\overline{y}(t_{oi}) = \left\{\left[\nabla_{\overline{y}(t_{oi})}\,\overline{\Theta}\right]^T W_\alpha \left[\nabla_{\overline{y}(t_{oi})}\,\overline{\Theta}\right] + \lambda W_\beta\right\}^{-1} \left[\nabla_{\overline{y}(t_{oi})}\,\overline{\Theta}\right]^T W_\alpha\,(\overline{\Theta}_D - \overline{\Theta})$$

Thus

$$\overline{y}(t_{oi+1}) = \overline{y}(t_{oi}) + \Delta\overline{y}(t_{oi})$$

and the next iteration can begin.

NUMERICAL INTEGRATION

The nonlinear equations of motion are integrated using FNOL2, [3]. This routine uses a fourth order Runge-Kutta or a fourth order Adams-Moulton method to solve a system of coupled ordinary differential equations. As currently modified, the routine uses a double precision arithmetic throughout. Furthermore, the truncation error can be held within input bounds by giving the user an option of automatically varying the step size.

ILLUSTRATIVE EXAMPLE

A lunar orbit insertion maneuver has been used to illustrate the targeting procedure.

The algorithm was used to target the spacecraft using a set of burn durations ranging from 5 seconds to 20 minutes. In each case the initial spacecraft mass was taken to be 317.37 kg and the exhaust velocity was taken to be 2.84 km/sec. The mass flow rate in each case had to be adjusted before making a computer run so that the thrust magnitude would be sufficient to attain a lunar orbit.

To obtain results the following procedure was implemented: a nominal earth-moon trajectory was generated and the spacecraft was targeted off of this trajectory to the constraint parameters, $i = 88\overset{\circ}{.}7$, RCA = 2428.88 km. In addition, for each burn interval considered, a minimum eccentricity for the lunar orbit was sought. For each case this targeting procedure yields a thrust vector orientation and time to begin the burn.

The thrust vector orientation, although computed in terms of inertial coordinates as mentioned earlier, is printed out in terms of the geometrically more meaningful pitch and yaw angles shown in Figure II.

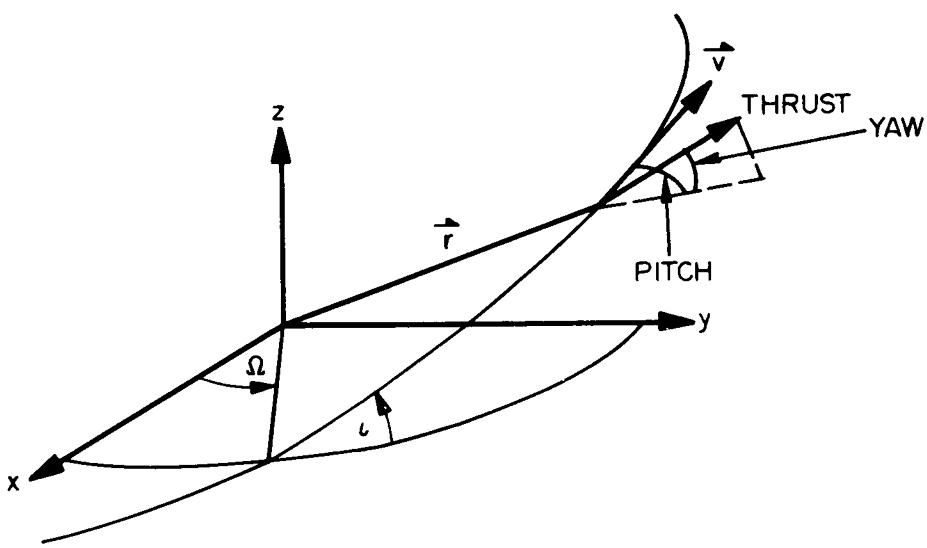

Figure II

The pitch angle is the angle measured from the spacecraft velocity vector $\overline{V}$ to the projection of the thrust vector onto the orbit plane. The yaw angle is the angle measured from the projection of the thrust vector onto the orbit plane to the thrust vector.

Results from several cases are shown in Table I.

Table I

Burn Duration (secs)	Values of Constraint Parameter	Values Obtained from the Algorithm	Pitch Angle (Deg)	Yaw Angle (Deg)	No. of Itera-tions
5	$i = 88.^{\circ}76$ RCA = 2428.88 km e = 0	$i = 88.^{\circ}76$ RCA = 2428.88 km e = .0006	-178.5	-.002	8
20	Same as above	$i = 88.^{\circ}76$ RCA = 2428.88 km e = .0004	-179.1	-.002	7
60	Same as above	$i = 88.^{\circ}76$ RCA = 2428.83 km e = .0009	+179.3	-.003	4

Although the time to initiate the burn was also one of the control parameters, it was found in all cases, the time t_0 never varied by more than 0.4 seconds from the initial guess. The reason for this behavior is that the mass flow rate had been adjusted properly beforehand to insure orbit insertion.

The circular orbit velocity at the point of insertion into lunar orbit is 1.4207 km/sec. Although the velocity at perigee was not one of the constraint parameters, this velocity was calculated for each case; for the three cases shown in Table I, the velocity attained is 1.421 km/sec.

Confidence in the program has been enhanced by a very favorable comparison with a program which integrates the linearized equations of motion. The comparisons were carried out for burn durations of 5 seconds and 30 seconds; such burn intervals are short enough that the linearization of the equations of motion does seriously affect the final nominal (targeted) parameters. Such comparisons have shown that the targeted parameters (e, i, RCA) agree to within less than half percent.

SUMMARY

A program has been developed which integrates the equations of motion of a thrusting spacecraft in order to optimally attain prescribed constraints. The algorithm is optimal in the sense that it minimizes a performance criteria which is the weighted squared distance between the final constraints and the desired constraints.

The program has been constructed so that a full error analysis of a maneuver can be carried out; this capability is available in the Monte Carlo mode of operation.

There are no restrictions concerning the class of orbital maneuvers the program will handle so that orbit to orbit transfers as well as orbit insertion maneuvers can be investigated.

ACKNOWLEDGEMENTS

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