NASA CONTRACTOR REPORT

NASA CR-2204

NASA CR-2204

AN ANALYTICAL STUDY OF THE RESPONSE OF A CONSTANT-ATTITUDE AIRCRAFT TO ATMOSPHERIC TURBULENCE

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1973

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-2204		

4. Title and Subtitle	5. Report Date
AN ANALYTICAL STUDY OF THE RESPONSE OF A CONSTANT-ATTITUDE AIRCRAFT TO ATMOSPHERIC TURBULENCE	March 1973
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Frederick O. Smetana and Ronald K. Carden	
	10. Work Unit No. 736-05-10-02
9. Performing Organization Name and Address North Carolina State University Raleigh, N.C. 27607	11. Contract or Grant No. NAS1-9603
	13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A light airplane equipped with an automatic control system which drives large wing flaps and the stabilator so as to produce a constant pitch attitude in all flight modes is analyzed for its response to a specific gust. The aircraft is also equipped with a bank-angle steering, zero sideslip automatic control system which is studied for its effectiveness in suppressing a specific lateral gust. The gusts are assumed to be comprised of 200 lateral and 400 vertical sinusoids. Each is used to excite the controlled aircraft and the time response to the sum of all sinusoids is plotted. The assumption is that the gust may be treated as stationary in space but variable in time rather than the reverse as is usually the case. Results indicate that such a control system can suppress vertical gusts up to the limit of control authority. Either the lateral accelerations or the yawing velocity response to lateral gusts can be suppressed with this system but not both simultaneously.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Gust response Automatic control system Light aircraft	

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	50	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

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NOMENCLATURE

A_ω	Fourier Cosine Coefficients
B_ω	Fourier Sine Coefficients
S	variable in Laplace domain
T	period of fundamental frequency
U_o	aircraft initial forward speed
u	forward speed
V'_g	lateral gust velocity input
v	sideslip velocity
W'_g	longitudinal gust velocity input
Y_α	α loop feedback function
Y_β	β loop feedback element
Y_ϕ	ϕ loop feedback element
Y_θ	θ loop feedback function
α	angle of attack
α_g	vertical gust angle of attack input
α_T	total angle of attack
β	sideslip velocity
β_g	sidegust angle input
β_T	total sideslip angle
δ	general input
δ_A	aileron input
δ_F	flap input
δ_R	rudder input
δ_S	stabilator input

ϕ roll angle

ϕ_T total roll angle

θ pitch angle

θ_T total pitch angle

ψ yaw angle

ω frequency

compensation, an artificial modification of a control surface deflection command such that the apparent vehicle dynamics are altered; a device in a control system by which one can remove--or compensate for--undesired dynamic characteristics in the vehicle. The command may be modified by filtering and/or by adding signal components proportional to vehicle position, velocity, and acceleration to the command. Simple feedback of position is not considered compensation.

shaping network, a device for automatically altering a command signal such that some aspects of a vehicle's dynamic characteristics will be more heavily excited than others.

INTRODUCTION

A recent design study (Ref. 1) took as its problem the development of a simple-to-fly, constant-attitude light aircraft. The concept which evolved employs fast-acting, full-span, Fowler-type wing flaps--operated together for lift modulation at all flight conditions and differentially for roll control--a completely separate fuselage leveler driving the elevator, and a turn coordinator controlling the rudder and differential flap motions. A flap-power interconnect is also provided to permit ascent at constant forward speed and airspeed changes at constant rate-of-climb. The pilot has only two controls: a wheel and a throttle. Changes in a throttle position change speed only. Sustained rotation of the wheel produces a coordinated turn. Forward motion of the wheel causes descent at constant forward speed and rearward positioning of the wheel results in a rate-of-climb at constant forward speed.

The effort to provide the aircraft with satisfactory handling characteristics led to the specification of powerful, fast-acting servo-controls. Such controls, however, can also form the basis of a gust alleviation system. With safety of flight in turbulent weather an area of growing national concern it was felt that an investigation of the applicability of the constant attitude concept to gust alleviation service was indicated.

The availability of a digital computer program (Ref. 2) to calculate and plot aircraft time histories given involved transfer functions made possible its frequent use in visualizing the response of an aircraft to a variety of control imputs. The ease with which such results are interpreted when contrasted to the transfer function representation led to a search for ways in which analytical gust responses could be presented in the time domain. (It is, of course, conventional practice to represent gust response as the product of the appropriate gust transfer function amplitude ratio squared and the power spectral density of the gust.) This approach would also facilitate direct comparison with flight records.

It is generally assumed that the air motions associated with a gust are <u>stationary</u> in time but vary from point to point in space. Thus the aircraft will encounter angles of attack and sideslip due to gusts which vary across the dimensions of the aircraft and with changes in flight velocity. One common simplification to this picture is to assume that a vertical gust, for example, does not vary in the span-wise direction. It is therefore a one-dimensional gust. This gust is the one most frequently used in analysis because of its simplicity. It has been stated (Ref. 3) that the effects of encountering different portions of the gust wave with different portions of the aircraft--so called penetration effects--can usually be treated adequately by including the effect in the stability derivatives involving $\dot{\alpha}$ or $\dot{w}$. If this is the case, it would seem reasonable to conclude that one could obtain similar results by assuming the gust to be stationary in space and variable in time. One would merely need to include a time delay term--proportional to flight velocity--in these derivatives. The stationary-in-space, variable-in-time model could be used in any event for gust wave lengths much larger than the distance between the wing quarter-chord and the tail quarter-chord. For most light aircraft a gust frequency of less than 1.0 to 1.5 radians/sec will insure that this condition is satisfied.

The time response of a linear, uncoupled aircraft to a sinusoidal disturbance of a control surface is usually fairly easy to determine. Since the system is linear, response to complex control surface inputs can be determined by superposition. Then if a gust can be considered to represent a disturbance similar to that induced by a control surface deflection, the time response of the aircraft to a gust input can be computed in a similar fashion. This is the viewpoint adopted for the present work.

METHOD

Flight records of aircraft response to gusts have the unfortunate aspect that the gust exciting that particular response is almost never a "standard" gust. The principal differences will be in mean square amplitude and in harmonic content at various frequencies. However, unless the aircraft has highly resonant stability modes, the departure in harmonic content with frequency from that of theoretical models or standards is a second order effect compared with the variation in amplitude so far as aircraft response is concerned. This will be shown aposteriori. Amplitude variations of course are easy to standardize so long as the system is linear. Thus it would seem that any well-documented sample of atmospheric turbulence would provide a suitable forcing function.

A recent report (Ref. 4) contained measurements of vertical and lateral gust velocities taken at relatively high altitudes during a storm in the Rocky Mountain area. Since the gust velocities were rather severe the records seemed to be well-suited for determining the harmonic content of the gusts. Accordingly, the period of time between about 15 seconds and 90 seconds (see figures 1 and 2) was selected as the fundamental period. The curves were photographically enlarged and the gust amplitude read at 638 evenly-spaced time increments. The lateral gust amplitude was read at 572 evenly-spaced time increments. Each of these data sets was then expanded by a factor of 4 by interpolating linearly between successive data points. The 2549 data points comprising the vertical gust and the 2285 data points comprising the lateral gust were processed by a Fourier analysis routine to obtain cosine amplitude, A_ω, and sine amplitude, B_ω, at each frequency. Approximately 1100 harmonics of $2\pi/75$ were computed. f(t) was computed from the series and compared with the original photograph. It was found that 200 harmonics were sufficient to reproduce all except the very sharpest peaks of V_g' while 400 harmonics were necessary to obtain a good fit to W_g'. Table I presents A_ω, and B_ω for the 200 frequencies needed to form V_g'. Table II presents the same results for W_g'. These tables give the amplitudes and frequencies of the sine and cosine forcing functions used as the imputs to the aircraft transfer functions. The quantity

$$\frac{T}{4\pi}\,(A_\omega{}^2 + B_\omega{}^2)$$

can be shown to approximate closely the one-dimensional power spectral density of the gust. Figures 3 and 4 show

$$\frac{T}{4\pi}\,(A_\omega{}^2 + B_\omega{}^2)$$

plotted against log frequency. Except for the scatter the plots are somewhat similar to those for the Dryden turbulence model.

This model predicts that the power spectral density will fall off as frequency to the -2 power above a certain frequency. The present data show a slope of about -3. Note also that in the present case the vertical and lateral gust amplitudes and harmonic content are somewhat different, indicating that the turbulence was not isotropic.

Measurements made during flights in clear air turbulence over the Eastern Seaboard (Ref. 8) show a slope of about -2 at the higher frequencies. Scatter in these data ranged from about ±15% at high frequencies to about ±40% at low frequencies. The data in Ref. 8 covered a frequency spectrum from about half the minimum value used here to about 10 times the maximum value used here. The low frequency amplitude of the gust analyzed here was about an order of magnitude larger than that found in Ref. 8. At the middle frequencies, $\sim$ 1 Hz, the amplitudes were approximately the same while at the higher frequencies the amplitudes in the present case were less than those reported in Ref. 8. The fact that the gust time histories of Ref. 4 were inferred from aircraft responses and plotted in such a way as to obscure very high frequency information is probably responsible for the more rapid attenuation in gust amplitude found at frequencies above 0.13 Hz by the Fourier analysis.

It may be noted that for the present calculation the frequencies of the aircraft stability modes (except for the rolling mode) are all less than 1 Hz. Thus, this more rapid attenuation in gust amplitude with frequency has no significant effect on the displacement of the airframe center of gravity but rather will be felt primarily as a somewhat reduced "shaking" of the airframe.

The time response of the system to the function

$$\sqrt{A_\omega^2 + B_\omega^2}\ \sin\left(\omega t + \tan^{-1}\frac{A_\omega}{B_\omega}\right) \quad \text{where } \omega = \frac{2\ n\pi}{T} \tag{1}$$

was found by multiplying the aircraft transfer functions by the Laplace transform of equation (1) and then solving for the time response by the method of residues. The time responses of the system for each value of n, n = 0,1,2,...200, were then summed to find the total system response to a lateral gust. A similar procedure was used for the vertical gust. The results are shown in figures 5 and 6 . It may be seen that within the assumptions used to develop the aircraft transfer functions (see next section for development) the vertical gust alleviation produced by the constant attitude control system is significant. The side gust alleviation appears to be outstanding. The system does in fact reduce the β and ϕ errors to very small values. But because the air mass is moving relative to the aircraft, the gust alleviation capability is actually somewhat more limited than it appears as will be explained in a later section.

AIRCRAFT TRANSFER FUNCTIONS

Lateral-Directional

The analysis of the response of an aircraft with rudder and aileron control loops to a lateral gust input has been treated in a very general fashion by Onstott and Salmon (Ref. 5). In terms of the present problem, the output variables chosen for minimization are ϕ and β. In their treatment both β and ϕ must be measured in an inertial reference system. This characterization of β is unfortunate because for simplicity and reliability one would like to use a vane or other aerodynamic device to sense β. A vane, of course, measures the orientation of the aircraft body axis relative to the instantaneous air mass velocity over the aircraft and thus does not give β in an inertial reference system. This is easily seen from the following example.

Consider an aircraft with stable lateral-directional dynamics to be flying along a straight line at constant speed in still air. Suddenly, a step gust approaches from the right. Before the aircraft can respond, it is subject to a positive β_g which will be sensed by a vane. β as given by Onstott and Salmon is zero because the aircraft has not yet turned with respect to the original heading. After a while the aircraft will relieve the side force imposed on it by the gust in two ways: by accelerating its c.g. to the gust velocity and/or by rotating so as to align itself with the resultant air mass velocity. When all the system dynamics have decayed, the vane will indicate zero sideslip, but the sum of the ground track angle and the heading will now correspond to the vector sum of the aircraft velocity and the gust velocity. In the inertial reference frame there will be a steady state final value of β equal to $-\beta_g$. This is what the equations of Onstott and Salmon predict. Thus if one wishes to use a vane as the feedback control element, he must rewrite the equations to account for its behavior.

The need for this rewriting makes it attractive to use a somewhat simpler derivation than that presented by Onstott and Salmon to obtain the required result. Since it is assumed that the system is linear, a sum of elementary solutions is also a solution. It is a familiar school exercise to obtain ϕ, β, and ψ in response to either a rudder or aileron deflection. (See for example Ref. 6 for elaboration.) The response of a system to both inputs simultaneously is simply a sum of the response of the system to the inputs separately:

$$\phi_T = \left(\frac{\phi}{\delta_A}\right) \delta_A + \left(\frac{\phi}{\delta_R}\right) \delta_R \qquad (2)$$

$$\beta_T = \left(\frac{\beta}{\delta_A}\right) \delta_A + \left(\frac{\beta}{\delta_R}\right) \delta_R \tag{3}$$

The quantities in parentheses represent the indicated open loop transfer functions. (See standard texts or Ref. 6 for development.) The various open loop transfer functions appear in Table III with numerical values substituted. ϕ_T and β_T are the total response to combined rudder and aileron inputs. The derivation of the transfer functions in Ref. 6 assumes the air mass to be stationary. Hence, ϕ_T and β_T in equations (2) and (3) are measured in an <u>inertial</u> reference system, even though β is usually thought of as an aerodynamic quantity. The response to a gust input may be included in the total response simply by adding the term

$$\left(\frac{\phi}{\beta_g}\right) \beta_g$$

to the bank angle equation and the term

$$\left[\left(\frac{\beta}{\beta_g}\right) + 1\right] \beta_g$$

to the sideslip equation. Since ϕ is always measured in an inertial reference system the open loop expression developed by Onstott and Salmon for

$$\left(\frac{\phi}{\beta_g}\right)$$

may be used unaltered. However, in order to describe the value of β_T which would be sensed by a vane mounted on the aircraft, it is necessary to add the sidewise air mass velocity to the sidewise aircraft velocity in inertial space as computed by Onstott and Salmon; hence β_g is added to the right hand side of the equation for β_T in addition to the term

$$\left(\frac{\beta}{\beta_g}\right) \beta_g.$$

For the purposes of this analysis it is assumed the pilot makes no inputs to the control system during a gust encounter. The lateral-directional control system used for the study is shown in figure 7. The basic aircraft is shown in three views in figure 8 and its stability derivatives are given in Table IV. Because no pilot inputs are assumed the feedback elements in the aileron and rudder loops can be represented by

$$Y_\phi = (0.117)(S + 3)\left(\frac{71.4}{S + 50}\right) \tag{4}$$

$$Y_\beta = (3.2)(S + 3.4)(S + 3.4)\left(\frac{71.4}{S + 50}\right)\left(\frac{100}{S + 100}\right) \tag{5}$$

respectively. Since the signals processed through these elements are fed into the aircraft negatively, one can represent the aileron and rudder inputs to the aircraft as

$$\delta_A = -Y_\phi \cdot \phi_T \tag{6}$$

$$\delta_R = -Y_\beta \cdot \beta_T \ . \tag{7}$$

With these substitutions, the equations for ϕ_T and β_T may be solved simultaneously to yield

$$\left(\frac{\beta_T}{\beta_g}\right) = \frac{\left[1 + Y_\phi\left(\frac{\phi}{\delta_A}\right)\right]\left[\left(\frac{\beta}{\beta_g}\right) + 1\right] - Y_\phi\left(\frac{\beta}{\delta_A}\right)\left(\frac{\phi}{\beta_g}\right)}{\left[1 + Y_\phi\left(\frac{\phi}{\delta_A}\right)\right]\left[1 + Y_\beta\left(\frac{\beta}{\delta_R}\right)\right] - Y_\phi Y_\beta\left(\frac{\beta}{\delta_A}\right)\left(\frac{\phi}{\delta_R}\right)} \tag{8}$$

$$\left(\frac{\phi_T}{\beta_g}\right) = \frac{\left[1 + Y_\beta\left(\frac{\beta}{\delta_R}\right)\right]\left(\frac{\phi}{\beta_g}\right) - Y_\beta\left(\frac{\phi}{\delta_R}\right)\left[\left(\frac{\beta}{\beta_g}\right) + 1\right]}{\left[1 + Y_\phi\left(\frac{\phi}{\delta_A}\right)\right]\left[1 + Y_\beta\left(\frac{\beta}{\delta_R}\right)\right] - Y_\phi Y_\beta\left(\frac{\beta}{\delta_A}\right)\left(\frac{\phi}{\delta_R}\right)} \ . \tag{9}$$

It can be shown that the denominators of these equations are identical to those one would obtain using the procedure of Onstott and Salmon. The β_T numerator, however, contains two additional terms,

$$1 + Y_\phi\left(\frac{\phi}{\delta_A}\right)$$

and the ϕ_T numerator has the additional term

$$- Y_\beta\left(\frac{\phi}{\delta_R}\right)$$

to account for the fact that an aerodynamic β is sensed rather than an inertial.

The responses β_T and ϕ_T are computed by evaluating

$$\beta_T = \mathcal{L}^{-1} \sum_{n=0}^{200} \left[\left(\frac{A_\omega \omega + B_\omega}{S^2 + \omega^2}\right)\left(\frac{\beta_T(S)}{\beta_g(S)}\right)\right] \frac{1}{U_o} \tag{10}$$

$$\phi_T = \mathcal{L}^{-1} \sum_{n=0}^{200} \left[\left(\frac{A_\omega \omega + B_\omega}{S^2 + \omega^2}\right)\left(\frac{\phi_T(S)}{\beta_g(S)}\right)\right] \frac{1}{U_o} \tag{11}$$

by the method of residues. $\omega = \frac{2\pi n}{T}$ and U_o is aircraft flight velocity.

Responses for both open and closed loop cases are shown in figure 5.

To accomplish the gust response attenuation noted in figure 5, the aircraft is yawed sufficiently to develop a side force equal and opposite to that imposed on the aircraft by the gust. In turbulent air where the gust approaches the aircraft first from one side and then the other in fairly rapid succession, the oscillations of the aircraft will be much larger than if the control system were not operational. However, the oscillations are phased so that the lateral acceleration of the aircraft c.g. is almost zero and ground track remains straight.

If one wished instead to exchange oscillations about a c.g. moving along a straight line for lateral accelerations of the whole airplane and an altered ground track this is easily done by measuring inertial β instead of aerodynamic β. The control system will then deflect the rudder only enough to develop sufficient yawing moment to keep the aircraft from rotating (but not sufficient to rotate the fuselage so that it develops a balancing side force.) While the heading remains essentially constant, the fuselage is accelerated first to one side and then to the other. Because of the aircraft inertia, however, the fuselage will not move as rapidly as the impinging gust.

The construction of an inertial β sensor is somewhat more complex than is that of a simple aerodynamic vane. From the equations of motion one sees that

$$\beta = \frac{v}{U_o} = \frac{\int_o^t (a_y - g\phi)dt}{U_o} \quad . \qquad (12)$$

Hence one requires a lateral accelerometer, a measurement of airspeed, and an indication of bank angle. The bank angle signal multiplied by a constant is subtracted from the accelerometer signal in a summing amplifier. The resulting signal is then integrated and fed to one terminal of a divider. The other terminal is fed with a signal proportional to airspeed. The output of the divider is then the inertial β signal.

There is no clear-cut human factors evidence to support a choice of which aircraft motion is inherently preferable in response to a gust. Probably the magnitudes of the two motions will be the determining factor. If the sidewise displacement of the c.g. is fairly small it would seem to be preferable to the aggravated rotation necessary to neutralize the aerodynamic side force. During approach, however, one would probably prefer a constant ground track.

It may be noted that the airframe dynamics are the same with either aerodynamic or inertial β input. The same compensation circuit may also be used. (Compensation is defined on page viii.)

There is the problem, however, that the indication of an inertial β sensor constructed in the manner described above will drift over long periods of time. To use this sensor successfully, it will probably be necessary to pass the output signal through a high pass filter and sum the result with the output of a low pass filter fed by the position indication of an aerodynamic vane. The filter cross-over frequency should be well below the natural frequency of any airframe oscillatory mode and in fact as low as the drift level permits. Rigorously, the use of a vane to provide D.C. error correction is possible of course only under the assumption that the gust has no components with frequencies below the cross-over frequency. Otherwise, the vane will force the aircraft to track these low-frequency components.

There is also the possibility of sensing a_y directly rather than having to calculate inertial β. The compensation required with acceleration input has not been investigated, however.

Longitudinal

The longitudinal control system as designed modulates thrust so as to keep forward speed constant during ascent and descent unless throttle position is changed. For the purpose of this analysis, however, the thrust will be assumed to be constant. The flap motion is then controlled entirely by the α sensor and stabilator motion by the θ sensor. This simplified longitudinal control system is depicted in figure 9. In a manner analogous to the analysis of lateral motions

$$\theta_T = \left(\frac{\theta}{\delta_S}\right) \delta_s + \left(\frac{\theta}{\delta_F}\right) \delta_F + \left(\frac{\theta}{\alpha_g}\right) \alpha_g \tag{13}$$

$$\alpha_T = \left(\frac{\alpha}{\delta_S}\right) \delta_s + \left(\frac{\alpha}{\delta_F}\right) \delta_F + \left(\frac{\alpha}{\alpha_g}\right) \alpha_g + \alpha_g \tag{14}$$

and

$$\left(\frac{\alpha_T}{\alpha_g}\right) = \frac{\left[1 + Y_\theta \left(\frac{\theta}{\delta_s}\right)\right]\left[\left(\frac{\alpha}{\alpha_g}\right) + 1\right] - Y_\theta \left(\frac{\alpha}{\delta_s}\right)\left(\frac{\theta}{\alpha_g}\right)}{\left[1 + Y_\theta \left(\frac{\theta}{\delta_s}\right)\right]\left[1 + Y_\alpha \left(\frac{\alpha}{\delta_F}\right)\right] - Y_\theta Y_\alpha \left(\frac{\alpha}{\delta_s}\right)\left(\frac{\theta}{\delta_F}\right)} \tag{15}$$

$$\left(\frac{\theta_T}{\alpha_g}\right) = \frac{\left[1 + Y_\alpha \left(\frac{\alpha}{\delta_F}\right)\right]\left(\frac{\theta}{\alpha_g}\right) - Y_\alpha \left(\frac{\theta}{\delta_F}\right)\left[\left(\frac{\alpha}{\alpha_g}\right) + 1\right]}{\left[1 + Y_\theta \left(\frac{\theta}{\delta_s}\right)\right]\left[1 + Y_\alpha \left(\frac{\alpha}{\delta_F}\right)\right] - Y_\theta Y_\alpha \left(\frac{\alpha}{\delta_s}\right)\left(\frac{\theta}{\delta_F}\right)} \tag{16}$$

where

$$Y_\theta = \frac{52\ (S + 7)(S + 8)}{(S + 50)} \left(\frac{S + 5.8}{S + 0.58}\right)\left(\frac{100}{S + 100}\right) \tag{17}$$

$$Y_\alpha = \frac{5\ (S + 3)(S + 4)}{S\ (S + 50)} \tag{18}$$

The six transfer functions needed for the evaluation of α_T/α_g and θ_T/α_g are obtained from the equations* of motion in the frequency domain.

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* The notation used in these equations follows that of references 2 and 6.

$$(S - X_u)u - (SX_{\dot{w}} + X_w)\alpha/U_o - (SX_q - g)\theta = X_{\delta_S}\delta_S + X_{\delta_F}\delta_F + X_{\alpha_g}\alpha_g \quad (19)$$

$$-(Z_u)u + \left[S(1 - Z_{\dot{w}}) - Z_w\right]\alpha/U_o - S(U_o + Z_q)\theta = Z_{\delta_S}\delta_S + Z_{\delta_F}\delta_F + Z_{\alpha_g}\alpha_g \quad (20)$$

$$-(M_u)u - (SM_{\dot{w}} + M_w)\alpha/U_o + (S^2 - M_qS)\theta = M_{\delta_S}\delta_S + M_{\delta_F}\delta_F + M_{\alpha_g}\alpha_g \quad (21)$$

From these equations two general transfer functions may be derived:

$$\left(\frac{\alpha}{\delta}\right) = \frac{A\alpha S^3 + B\alpha S^2 + C\alpha S + D\alpha}{D_1} \quad (22)$$

$$\left(\frac{\theta}{\delta}\right) = \frac{A\theta S^2 + B\theta S + C\theta}{D_1}, \quad (23)$$

where

$$A\alpha = Z\delta\ /U_o$$

$$B\alpha = \left[-Z\delta(M_q + X_u) + M_\delta U_o\right]/U_o$$

$$C\alpha = \left[X_u(Z_\delta M_q - M_\delta U_o)\right]/U_o$$

$$D\alpha = \left[g(Z_\delta M_u - M_\delta Z_u)\right]/U_o$$

$$A\theta = (Z_\delta M_{\dot{w}} + M_\delta)$$

$$B\theta = Z_\delta(M_w - M_{\dot{w}}X_u) - M_\delta(X_u + Z_w)$$

$$C\theta = Z_\delta(M_uX_w - M_wX_u) + M_\delta(X_uZ_w - X_wZ_u)$$

$$D_1 = AS^4 + BS^3 + CS^2 + DS + E$$

with $A = 1$

$$B = -(M_q + X_u + Z_w + U_o M_{\dot{w}})$$

$$C = M_q Z_w - U_o M_w + X_u(M_q + Z_w + U_o M_{\dot{w}}) - X_w Z_u$$

$$D = -X_u(M_q Z_w - U_o M_w) - M_u U_o X_w + M_q X_w Z_u + g(M_{\dot{w}} Z_u + M_u)$$

$$E = g(M_w Z_u - M_u Z_w)$$

The six individual transfer functions needed in the analysis are obtained from these two general functions as follows:

(1) For $(\frac{\alpha}{\delta_F})$ substitute X_{δ_F}, Z_{δ_F}, etc., for X_δ, Z_δ, etc., in the numerator of $(\frac{\alpha}{\delta})$

(2) For $(\frac{\alpha}{\delta_S})$ substitute X_{δ_S}, Z_{δ_S}, etc., for X_δ, Z_δ, etc.

(3) For $(\frac{\alpha}{\alpha_g})$ substitute X_{α_g}, Z_{α_g}, etc., for X_δ, Z_δ, etc.

A similar procedure is employed to find the θ transfer functions.

The values of the longitudinal stability derivatives, X_{δ_F}, Z_{δ_F}, etc., are given in table V. Table VI presents numerical values of the transfer functions $(\frac{\alpha}{\delta_F})$ etc., as a ratio of polynomials.

The reader unfamil'iar with this notation, the correspondance between the dimensional stability derivatives given here and the more widely used non-dimensional stability derivatives, and the equations of motion in the frequency domain rather than in the time domain is directed to Reference 6 for a very lucid, comprehensive explanation. Those finding Reference 6 difficult to locate may find NASA Contractor Report 1975 more accessible. This work treats many of the same questions in a similar fashion.

DISCUSSION

It has been pointed out in the literature (Reference 3) that effective gust suppression in the longitudinal mode requires control of both plunging and pitching motions. In the control system discussed herein pitching motion is restrained by stabilator operation and plunging motion by modulation of flap position. Unfortunately, in the lateral-directional mode there is no simple means to modulate the side force on the airframe without simultaneously yawing the fuselage. One can therefore suppress either the sidewise velocity or the yawing motion, but not both. Experience with vertical gust alleviators has shown that suppression of only one of the two motions involved is not very satisfactory. It would appear, therefore, that completely satisfactory lateral gust suppression using this concept will require a substantial increase in vertical tail and rudder area, a decrease in effective dihedral (so as to develop a yawing moment due to rolling counter to the yawing moment developed by the rudder as it produces a side force to oppose the gust), and the use of β inertial, or perhaps a_y, as the control system input. As now constituted, that is, with a vane input the lateral control system provides improved capability to fly a given ground track in exchange for somewhat greater oscillatory response to a side gust.

Another possible means of lateral gust suppression which has not been investigated in detail is the use of combined rolling and pitching motions to develop the necessary side force. There are some indications that aircrew members might find such motions preferable to either yawing motions or lateral accelerations. A control system to provide the necessary motions would have to couple the lateral-directional system with the forward speed-rate of climb system and would be useful only during gusts. For this reason its cost is likely to be fairly high and therefore its employment on general aviation craft is less likely in the near future.

A third possibility is the use of movable vertical surfaces mounted on the wings to modulate the side force on the aircraft. Two surfaces, each with a 2' chord and a 3' height, will provide the necessary side force generation capability for a typical 4-place general aviation craft. The surfaces would be actuated by the control system upon receipt of an $(a_y - g\phi)$ indication. The vertical tail would then be responsible only for control of yawing motions. An aircraft employing side force surfaces is discussed in reference 7.

The suppression of vertical gust responses as shown in figure 5 is, in some respects, very satisfactory and in others less than one would wish. Note that the pitching response has been completely eliminated while the vertical motion has been reduced about 50%. This can be explained by the fact that the pitch loop (fuselage leveler) has a relatively high gain and the stabilator a very high aerodynamic effectiveness. The plunge channel, however, has the aerodynamic ability to compensate for changes in angle of attack of only about ± 10°. Now, when a vertical gust of 30 ft/sec encounters an airplane moving at 180 ft/sec it is equivalent, so far as the airplane is concerned, to a 9.4 degree change in angle of attack. If the vertical gust velocity is greater or the aircraft speed slower, the aircraft has insufficient flap effectiveness to alter the wing lift enough to compensate for the change in lift due to change in angle of attack. The aircraft will then seek to plunge (up or down) to relieve the excess lift force.

In the example used, the vertical gust velocities reached 120 ft/sec which at an aircraft speed of 200 ft/sec, corresponds to a change in angle of attack of 31°. As the aircraft accelerates in the direction of the gust velocity, the effective forcing function is reduced so that after a while the vertical velocity of the aircraft equals that of the gust. For this very severe example it is seen that the aircraft's inertia tends to limit the vertical velocity in response to a gust while the control system will negate about 10° of the induced angle. The remaining vertical velocity divided by the forward flight speed is the response quantity plotted against time (expressed as degrees) in figure 5. Because of the need to scale gains for commands up to 10° and because of the presence of a well damped high frequency oscillation at the gain level chosen, it is not practical to choose a higher gain for the control system. Thus, the level of gust attenuation shown represents about the practical limit for this system.

The dominant time constant in the control system is about 0.6 secs. Since the gust components begin to diminish in strength as their periods fall below 1 second, it is evident that most of the gust energy can be absorbed by this system if the gust amplitude is not too large. While a somewhat shorter time constant (say 0.1 secs) would be desirable, there is not a simple way this can be achieved because of the location of the open loop zeros in the α/δ_F transfer function, the system gain limits, and the presence of other poles and zeros along the negative real axis to the left of -5.0. Further study of this problem would be desirable.

A word about the possible mechanization of the compensation circuits is probably in order since those familiar with control systems will probably recognize some unconventional practices here. The longitudinal control system involves two compensators:

$$\theta: \qquad (S + 7)(S + 8)\left(\frac{S + 5.8}{S + 0.58}\right)$$

$$\alpha: \qquad \frac{(S + 3)(S + 4)}{S}$$

The second of these presents few problems if an accelerometer is used to measure $\dot{w}$. It is seen that in addition to $\dot{w}$ itself, a first and second integral are required. Since angular accelerometers are available as well as rate gyros and position gyros, the function in the θ loop (S + 7)(S + 8) is easily mechanized. (S + 5.8)/(S + 0.58) can be built entirely with passive elements.

The lateral control system also contains two compensators:

ϕ: (S + 3)

β: (S + 3.4)(S + 3.4)

the first of these requires only a rate gyro and a position gyro. No instrument presently exists which can measure $\ddot{v}$. Thus it will be necessary to develop this signal electrically either by differentiating an accelerometer output or by multiplying the signal (S + 3.4) by itself. It is felt that recent advances in solid state device technology will permit this to be done successfully.

Finally, it may be mentioned that because the pitch attitude with the control system operating is always near zero, the vertical acceleration, a quantity of interest when evaluating vehicle riding qualities, is just $U_o\dot{\alpha}$. Examination of Figure 5 shows that while the control system does not reduce the peak accelerations a great deal it is effective in suppressing the low frequency components of the vertical acceleration. As a result, the acceleration exceeds a given value for a much shorter period of time with the control system on than with it off.

CONCLUSIONS

1. The control system for the simple-to-fly, constant-attitude aircraft can provide significant alleviation of both vertical and lateral gusts, provided aircraft velocity is sensed inertially. Such a control system seems capable of very beneficial application to low wing loading STOL machines for gust alleviation.

2. The vertical gust alleviation is limited by the aerodynamic capability of the flaps to modulate aircraft C_L and by the difficulty one has in compensating electrically for inherently slow airframe dynamics. Because of a very aerodynamically effective stabilator and the use of high gain in the feedback and servo systems, pitching due to gusts is very heavily suppressed.

3. Because of the inability of the aircraft and its control system to generate significant sideforce without yawing, one cannot suppress both sidewise motion and yawing simultaneously. Either can be suppressed separately, however, the choice being determined by which of the motions is less objectionable.

4. The suppression of linear motions requires inertial sensing of aircraft velocity. These signals then surplant those of aerodynamic sensors in the feedback loops.

5. The representation of a gust by a specific series of sine and cosine disturbances in angle of attack (or angle of sideslip) is an effective means of developing a suitable forcing function with which to excite the aircraft transfer function and thereby determine the time response of the aircraft to a specific gust. The method is easily modified to treat (a) lags in the build up of aerodynamic forces and (b) "penetration" effects. By accepting additional complexity the method can be extended to permit spatial variations in gust amplitude.

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ω	A_ω	B_ω	ω	A_ω	B_ω
0.0	-15.1565	0.0	4.2267	0.3386	0.3717
0.0845	-2.5819	-4.2308	4.3112	-0.1575	0.4943
0.1691	-6.5174	-12.4664	4.3957	-0.9078	0.4530
0.2536	12.3449	-3.9885	4.4803	0.8067	0.1196
0.3381	9.4475	5.1918	4.5648	-0.3080	0.1378
0.4227	1.3761	12.5217	4.6493	0.5884	-0.2818
0.5072	-3.0421	-1.2528	4.7339	0.5109	0.7909
0.5917	3.2976	-1.2151	4.8184	0.6725	-0.2755
0.6763	-0.3553	2.1634	4.9029	0.4375	0.1987
0.7608	-2.4624	2.9969	4.9875	-0.1472	-0.4292
0.8453	-1.3731	-7.1903	5.0720	0.4112	0.0762
0.9299	5.5350	-0.4211	5.1565	0.0202	-0.0208
1.0144	1.5373	4.1692	5.2411	-0.1143	-0.5613
1.0989	-0.3137	-1.2628	5.3256	-0.3239	-0.2835
1.1835	1.2980	-0.5713	5.4101	0.1297	1.1923
1.2680	1.9351	3.0186	5.4947	-0.0837	0.1850
1.3525	-2.6106	3.2557	5.5792	0.8295	-0.4156
1.4371	-4.0396	-4.1908	5.6637	-0.3365	-0.4631
1.5216	-2.9728	-3.6991	5.7483	0.0755	0.3484
1.6061	-1.3616	2.8891	5.8328	0.2777	-1.0973
1.6907	-1.7784	-1.1184	5.9174	0.5779	0.3218
1.7752	0.2725	-1.9029	6.0019	0.8584	-0.3924
1.8597	-1.1677	-0.4809	6.0864	-0.3408	0.4593
1.9443	-0.8639	3.2249	6.1710	-0.3657	-0.7167
2.0288	-2.4017	-0.4452	6.2555	0.6823	0.6419
2.1133	-0.3786	0.3934	6.3400	0.2888	-0.5659
2.1979	-0.1954	1.9286	6.4246	-0.7789	0.3238
2.2824	-2.3833	3.8770	6.5091	0.8700	-0.5334
2.3669	-0.7154	-3.4317	6.5936	0.2818	-0.2353
2.4515	2.0021	-0.2202	6.6782	0.1441	-0.1896
2.5360	0.6312	1.3912	6.7627	-0.4934	0.0654
2.6205	0.0642	0.0738	6.8472	0.3742	-0.2762
2.7051	1.9433	1.7554	6.9318	-0.2321	0.2955
2.7896	-2.9708	-0.5784	7.0163	0.0711	-0.1593
2.8741	1.5060	-0.5970	7.1008	0.0154	-0.3987
2.9587	2.1156	0.2218	7.1854	0.0109	0.1146
3.0432	0.8345	-2.3328	7.2699	-0.0386	0.5237
3.1277	1.1434	2.6238	7.3544	-0.0379	0.1632
3.2123	0.5756	0.0393	7.4390	-0.0429	-0.1507
3.2968	0.9146	-0.6781	7.5235	0.0296	-0.2757
3.3813	0.2298	0.1711	7.6080	0.1797	0.1184
3.4659	0.0283	1.0041	7.6926	-0.2110	-0.0705
3.5504	0.3928	-0.9769	7.7771	0.2651	-0.1465
3.6349	-0.6965	0.7672	7.8616	-0.3535	0.0150
3.7195	0.7298	0.9424	7.9462	-0.1629	0.2214
3.8040	0.1666	-0.6853	8.0307	0.0968	-0.2467
3.8885	0.3001	-0.2777	8.1152	-0.4904	0.1068
3.9731	-1.1733	0.3837	8.1998	-0.3836	-0.2976
4.0576	-0.3552	-0.0865	8.2843	0.1951	-0.0233
4.1421	1.0456	-2.1478	8.3688	0.3110	-0.3442

TABLE I. FOURIER COEFFICIENTS FOR LATERAL GUST

ω	A_ω	B_ω	ω	A_ω	B_ω
8.4534	0.1123	0.2810	12.6800	0.1633	0.1711
8.5379	0.0680	-0.2543	12.7646	-0.0471	-0.0230
8.6224	-0.0170	0.1616	12.8491	-0.0649	0.1598
8.7070	0.1466	-0.1422	12.9336	-0.2313	0.0990
8.7915	-0.1111	0.0137	13.0182	0.3022	-0.0936
8.8760	0.1300	-0.3063	13.1027	0.1525	-0.0623
8.9606	0.0894	-0.0170	13.1872	-0.1619	0.0598
9.0451	-0.0602	0.0700	13.2718	0.0684	-0.0312
9.1296	-0.0802	0.1246	13.3563	0.2655	0.0459
9.2142	0.2637	-0.1302	13.4408	-0.1050	0.1098
9.2987	0.0792	0.2924	13.5254	-0.1159	-0.0501
9.3832	-0.1685	-0.2801	13.6099	0.0078	-0.0305
9.4678	0.1096	-0.1027	13.6944	-0.0624	0.0859
9.5523	-0.0171	0.0208	13.7790	-0.1477	0.0428
9.6368	0.0615	0.1630	13.8635	0.0847	-0.0570
9.7214	0.1618	0.0083	13.9480	0.0787	-0.0674
9.8059	-0.1239	0.0283	14.0326	-0.1858	0.1225
9.8904	-0.0438	0.1382	14.1171	0.0837	0.0165
9.9750	-0.2970	-0.0641	14.2016	0.0900	-0.1216
10.0595	0.1898	-0.1847	14.2862	-0.0132	-0.0084
10.1440	0.1581	0.0725	14.3707	-0.0003	0.0149
10.2286	0.1637	0.2440	14.4552	0.2087	0.1488
10.3131	-0.0642	-0.1018	14.5398	-0.0718	-0.2374
10.3976	0.0947	0.1230	14.6243	-0.0784	0.2459
10.4822	-0.2088	0.2404	14.7088	-0.1194	0.0113
10.5667	-0.0926	0.0190	14.7934	0.0202	-0.0441
10.6512	0.1334	-0.2662	14.8779	0.0752	-0.0530
10.7358	0.0500	0.1795	14.9624	0.0432	0.0010
10.8203	0.1113	-0.0161	15.0470	0.1829	-0.0545
10.9048	-0.0017	-0.1415	15.1315	0.0371	0.3337
10.9894	0.0567	-0.0732	15.2160	0.0573	0.1452
11.0739	-0.0297	0.4089	15.3006	-0.0633	-0.1089
11.1584	-0.1331	-0.0928	15.3851	-0.0453	-0.0533
11.2430	-0.1268	-0.2419	15.4696	-0.0058	0.0910
11.3275	0.1328	0.0994	15.5542	0.1816	-0.1035
11.4120	0.0172	0.0394	15.6387	0.0670	-0.0028
11.4966	0.0289	-0.0406	15.7232	0.0296	0.0737
11.5811	0.0563	0.0106	15.8078	-0.0154	0.1916
11.6656	0.1291	0.0235	15.8923	-0.0705	0.0202
11.7502	-0.0767	-0.1255	15.9768	0.0654	-0.0764
11.8347	0.1379	-0.3008	16.0614	-0.0561	0.0071
11.9192	0.2256	0.2923	16.1459	-0.0312	0.0790
12.0038	-0.2230	0.0244	16.2304	0.0774	0.0110
12.0883	-0.0422	0.0979	16.3150	0.1533	0.0893
12.1728	0.1175	-0.0982	16.3995	-0.1744	-0.0410
12.2574	-0.1708	0.1864	16.4840	-0.1131	0.0733
12.3419	0.0966	-0.2551	16.5686	0.0186	-0.0298
12.4264	0.1748	0.1176	16.6531	0.1733	0.1128
12.5110	0.1753	-0.1335	16.7376	0.0308	-0.1264
12.5955	-0.0210	0.0739	16.8222	-0.0042	0.1121
			16.9067	0.0838	0.0608

TABLE I. (CONTINUED)

ω	A_ω	B_ω	ω	A_ω	B_ω
0.0	7.7858	0.0	4.2414	-0.2462	-1.1699
0.0848	-34.1993	18.7712	4.3262	0.2169	-0.5909
0.1697	-5.2141	-10.7245	4.4111	1.2776	0.5261
0.2545	2.9693	-12.7898	4.4959	-0.1348	-0.4732
0.3393	-10.5732	-2.8275	4.5807	-0.3215	-0.8833
0.4241	7.8371	-0.4548	4.6655	0.6830	0.4089
0.5090	7.1061	-3.8237	4.7504	0.7358	0.4164
0.5938	-0.6898	-8.4897	4.8352	-0.8098	-0.9820
0.6786	8.5946	12.8641	4.9200	0.1380	-1.2568
0.7635	-1.1093	-0.1530	5.0048	0.3387	0.3130
0.8483	-2.4229	-3.0862	5.0897	0.5009	-0.6876
0.9331	-1.6441	-5.1240	5.1745	-0.3815	0.4766
1.0179	7.8943	7.9156	5.2593	1.2539	-1.4600
1.1028	-6.7736	-7.5299	5.3442	0.0690	1.2013
1.1876	5.4831	1.4475	5.4290	0.2494	0.4642
1.2724	-3.8009	1.0806	5.5133	0.1034	0.2503
1.3572	-1.0464	-1.4885	5.5986	-0.8498	-0.6973
1.4421	2.2141	-1.6628	5.6835	-0.0082	0.8246
1.5269	-3.7380	3.2265	5.7683	-0.2348	-0.5466
1.6117	3.8694	-5.9981	5.8531	0.2209	-0.1294
1.6966	-0.4377	4.0549	5.9380	0.2811	-0.0314
1.7814	0.5657	0.5747	6.0228	-1.0496	-0.0081
1.8662	0.6529	1.2636	6.1076	0.9677	-0.3510
1.9510	-0.5005	-1.0339	6.1924	-0.2347	1.0612
2.0359	4.5345	-0.5081	6.2773	0.2001	-0.4200
2.1207	1.7748	0.7765	6.3621	0.0220	-0.1934
2.2055	-0.3763	-0.0108	6.4469	-0.2902	-0.1630
2.2904	0.7077	1.0971	6.5317	1.0936	-0.0096
2.3752	1.0257	0.3822	6.6166	-0.9015	-0.4389
2.4600	-1.2794	2.9730	6.7014	-0.2122	0.1562
2.5448	-0.8425	-2.4258	6.7862	0.1332	0.0077
2.6297	-2.8902	-0.5938	6.8711	-0.7177	-0.2797
2.7145	1.3177	-0.7458	6.9559	0.0586	-1.0540
2.7993	-0.1706	2.4496	7.0407	-0.1223	0.5658
2.8841	1.9009	-1.9004	7.1255	-0.1811	-0.6216
2.9690	-3.4657	0.1033	7.2104	0.2377	-0.4776
3.0538	0.4444	-0.1166	7.2952	0.6708	-0.2838
3.1386	0.4679	-0.6550	7.3800	0.3215	0.6238
3.2235	0.6639	-1.3458	7.4649	-0.8054	-0.6317
3.3083	-0.0616	1.6818	7.5497	0.5269	0.4487
3.3931	0.1912	-0.6179	7.6345	0.0708	-0.8338
3.4779	2.3286	-0.8667	7.7193	0.0372	0.6919
3.5628	-1.9586	1.4549	7.8042	-0.1744	-0.1672
3.6476	-1.7425	0.0279	7.8890	0.6364	-0.5375
3.7324	-0.5729	0.4989	7.9738	0.1330	0.0108
3.8173	-0.4941	-1.7447	8.0587	-0.1141	-0.0347
3.9021	0.8561	1.0155	8.1435	0.3094	-0.2434
3.9869	-1.0799	-2.2123	8.2283	0.3315	0.3288
4.0717	1.7318	1.2144	8.3131	-0.0546	-0.2155
4.1566	0.8359	0.1410	8.3980	0.0260	0.3637

TABLE II. FOURIER COEFFICIENTS FOR LONGITUDINAL GUST

ω	A_ω	B_ω	ω	A_ω	B_ω
8.4828	0.1261	-0.3549	12.7242	-0.1536	0.0265
8.5676	0.3037	0.1686	12.8090	0.1790	-0.0711
8.6524	0.0210	0.1191	12.8938	-0.0698	0.0972
8.7373	-0.0172	-0.4689	12.9787	-0.0421	-0.1893
8.8221	-0.0850	0.3815	13.0635	0.0775	-0.1261
8.9069	0.0203	-0.4228	13.1483	0.0113	-0.0360
8.9918	-0.1638	-0.5626	13.2332	0.0722	-0.0251
9.0766	0.1330	0.2957	13.3180	-0.1362	-0.1367
9.1614	0.0503	-0.0560	13.4028	0.3608	-0.1353
9.2462	-0.0166	-0.1549	13.4876	-0.0464	0.0003
9.3311	0.1423	-0.0607	13.5725	0.0075	0.1353
9.4159	0.2510	0.1272	13.6573	-0.0622	0.0543
9.5007	-0.3599	-0.1000	13.7421	0.0407	-0.1395
9.5856	0.0626	-0.0466	13.8270	-0.0918	0.1905
9.6704	-0.3849	-0.0926	13.9118	0.0118	-0.1789
9.7552	0.3031	-0.1759	13.9966	0.0813	-0.2117
9.8400	-0.4082	-0.0478	14.0814	0.0550	0.0432
9.9249	0.2622	0.1276	14.1663	-0.0017	-0.0837
10.0097	-0.1007	-0.1678	14.2511	0.2274	0.1381
10.0945	-0.2421	-0.2754	14.3359	-0.0157	-0.0205
10.1793	0.3957	-0.0321	14.4207	0.0218	0.0738
10.2642	0.1676	0.0149	14.5056	0.0446	-0.0873
10.3490	-0.2029	-0.1435	14.5904	-0.0609	-0.0606
10.4338	0.3094	-0.0576	14.6752	0.1477	-0.0132
10.5187	0.1199	-0.1139	14.7601	-0.0275	0.0235
10.6035	0.0637	0.3280	14.8449	0.1030	-0.0955
10.6883	-0.3474	0.0117	14.9297	0.0705	0.1042
10.7731	0.1329	-0.3025	15.0145	-0.0621	-0.0931
10.8580	0.2299	-0.1611	15.0994	0.0846	-0.0399
10.9428	0.0631	-0.0489	15.1842	0.0954	-0.0933
11.0276	0.2003	-0.1269	15.2690	-0.0309	0.0503
11.1125	-0.0501	-0.4010	15.3539	0.1433	-0.1737
11.1973	0.0042	0.2810	15.4387	0.0878	-0.0046
11.2821	0.1289	-0.1605	15.5235	0.1538	0.0040
11.3669	-0.0272	-0.0294	15.6083	-0.1142	-0.0001
11.4518	0.0503	0.0013	15.6932	-0.0959	0.0745
11.5366	0.0485	-0.1015	15.7780	-0.0028	-0.0789
11.6214	0.1251	-0.0627	15.8628	-0.0531	-0.0710
11.7063	0.0241	0.1840	15.9476	-0.0264	0.0103
11.7911	-0.1481	-0.0284	16.0325	0.1068	-0.1576
11.8759	0.0189	-0.0813	16.1173	0.0782	0.0673
11.9607	0.1391	-0.2560	16.2021	-0.0211	-0.1823
12.0456	-0.0722	0.3051	16.2870	0.1134	0.1241
12.1304	0.0254	-0.4272	16.3718	0.0156	-0.0558
12.2152	0.1526	0.0525	16.4566	0.2049	-0.0202
12.3000	0.2830	0.0549	16.5414	-0.1290	-0.0435
12.3849	0.1176	0.0311	16.6263	0.1564	-0.0332
12.4697	-0.1680	0.0391	16.7111	-0.0211	0.0385
12.5545	-0.0082	-0.0948	16.7959	0.0557	0.0117
12.6394	0.0645	-0.1480	16.8808	0.0086	-0.0474

TABLE II. (CONTINUED)

ω	A_ω	B_ω	ω	A_ω	B_ω
16.9656	-0.0331	-0.0804	21.2070	0.1544	-0.0266
17.0504	-0.0053	0.0126	21.2918	-0.0934	0.0003
17.1352	-0.0697	-0.0868	21.3766	0.0003	-0.0595
17.2201	0.1032	-0.1626	21.4615	0.0900	-0.0586
17.3049	0.0505	0.0808	21.5463	0.0322	0.0269
17.3897	0.1491	-0.0231	21.6311	0.0374	0.0210
17.4745	0.0543	-0.0073	21.7159	0.0282	-0.0231
17.5594	0.0660	-0.0546	21.8008	0.0054	0.0581
17.6442	-0.0756	0.0051	21.8856	0.0963	0.0999
17.7290	0.1347	0.0365	21.9704	-0.0791	-0.0374
17.8139	-0.0602	-0.0069	22.0553	0.0266	-0.0651
17.8987	-0.0394	-0.0534	22.1401	0.0133	0.0359
17.9835	0.0875	-0.0647	22.2249	-0.0387	0.0732
18.0683	0.0169	0.0792	22.3097	0.0795	-0.0881
18.1532	-0.0814	0.0045	22.3946	-0.0061	-0.0045
18.2380	0.0760	-0.0580	22.4794	0.0568	0.0732
18.3228	0.0732	0.0082	22.5642	0.0414	0.0098
18.4077	0.0153	-0.0184	22.6490	0.0054	0.0005
18.4925	0.1527	0.0143	22.7339	0.0099	-0.0348
18.5773	-0.0227	-0.0503	22.8187	-0.0174	0.0315
18.6621	0.0779	0.0017	22.9035	0.0845	-0.0344
18.7470	0.0028	0.0714	22.9884	0.0900	-0.0052
18.8318	0.0914	0.0104	23.0732	-0.0845	-0.0408
18.9166	-0.0259	-0.0466	23.1580	0.0242	-0.0150
19.0014	-0.0129	-0.0049	23.2428	0.0198	-0.0670
19.0863	-0.0935	-0.1278	23.3277	0.0811	-0.0219
19.1711	0.1190	-0.0137	23.4125	-0.0506	-0.0662
19.2559	-0.0310	-0.0433	23.4973	0.0587	-0.0018
19.3408	0.0859	0.0415	23.5822	0.0617	-0.0109
19.4256	-0.0378	-0.0624	23.6670	0.0263	-0.0327
19.5104	0.1110	0.0204	23.7518	-0.0074	0.0035
19.5952	0.0161	-0.0825	23.8366	-0.0002	0.0048
19.6801	0.0696	0.0080	23.9215	0.0648	0.0945
19.7649	-0.0245	0.0187	24.0063	-0.0091	-0.1039
19.8497	0.0878	0.0153	24.0911	0.0383	-0.1079
19.9346	-0.0223	-0.0540	24.1759	0.0281	0.0742
20.0194	-0.0102	-0.0615	24.2608	0.0578	-0.0140
20.1042	0.0263	0.0576	24.3456	0.0423	-0.0254
20.1890	-0.0915	-0.0712	24.4304	0.0253	0.0312
20.2739	0.1294	-0.0518	24.5153	-0.0154	0.0165
20.3587	-0.0234	-0.0308	24.6001	0.0947	0.0294
20.4435	0.0447	-0.0384	24.6849	0.0472	-0.0513
20.5284	0.1147	0.0453	24.7697	-0.0017	0.0834
20.6132	0.0913	-0.0304	24.8546	0.0149	0.0729
20.6980	0.0143	0.0161	24.9394	0.0310	-0.0566
20.7828	-0.0272	0.0589	25.0242	0.0803	0.0331
20.8677	0.1353	-0.0813	25.1091	-0.0449	-0.0374
20.9525	0.0824	0.0436	25.1939	-0.0541	0.0305
21.0373	-0.0234	-0.0423	25.2787	0.0798	-0.0655
21.1221	-0.0404	0.0045	25.3635	0.0938	-0.0253

TABLE II. (CONTINUED)

ω	A_ω	B_ω	ω	A_ω	B_ω
25.4484	-0.0307	-0.0006	29.6898	0.0460	0.0106
25.5332	0.0212	0.0295	29.7746	0.0330	-0.0046
25.6180	0.0319	-0.0197	29.8594	-0.0109	-0.0156
25.7029	0.0209	0.0268	29.9442	0.0762	0.0621
25.7877	-0.0322	-0.0488	30.0291	0.0244	0.0440
25.8725	-0.0249	0.1244	30.1139	-0.0246	-0.0695
25.9573	0.0218	0.0092	30.1987	0.0030	0.0244
26.0422	-0.0133	-0.0680	30.2836	0.0082	0.0169
26.1270	0.0129	-0.0366	30.3684	0.0384	0.0116
26.2118	0.0171	0.0788	30.4532	0.0340	-0.0097
26.2966	0.0160	-0.0027	30.5380	0.0228	0.0022
26.3815	0.0797	-0.0740	30.6229	0.0088	0.0613
26.4663	0.0241	-0.0332	30.7077	0.0429	-0.0202
26.5511	0.0887	0.0708	30.7925	0.0227	0.0303
26.6360	0.0548	-0.0070	30.8773	0.0010	0.0034
26.7208	-0.0095	-0.0343	30.9622	-0.0170	0.0431
26.8056	0.0291	0.0281	31.0470	0.0416	-0.0445
26.8904	0.0148	0.0408	31.1318	0.0289	-0.0104
26.9753	0.0612	0.0013	31.2167	-0.0319	0.0054
27.0601	0.0253	-0.0565	31.3015	0.0149	0.0131
27.1449	-0.0240	-0.0311	31.3863	-0.0026	-0.0054
27.2298	0.0134	-0.0200	31.4711	0.0081	-0.0199
27.3146	0.0312	0.0340	31.5560	-0.0069	-0.0321
27.3994	0.0464	-0.0429	31.6408	0.0073	0.0244
27.4842	0.0156	-0.0791	31.7256	0.0493	0.0110
27.5691	0.0454	0.0445	31.8105	-0.0513	-0.0052
27.6539	0.0776	0.0122	31.8953	0.0044	-0.0029
27.7387	-0.0260	0.0011	31.9801	0.0327	0.0253
27.8235	-0.0449	-0.0340	32.0649	-0.0173	0.0276
27.9084	0.0346	0.0481	32.1498	0.0043	-0.0703
27.9932	0.0178	0.0510	32.2346	-0.0053	-0.0000
28.0780	-0.0174	-0.0285	32.3194	0.0301	0.0123
28.1629	-0.0813	-0.0397	32.4043	0.0130	-0.0074
28.2477	0.0392	0.0614	32.4891	0.0150	-0.0009
28.3325	0.0149	-0.0294	32.5739	0.0508	0.0118
28.4173	0.0406	0.0013	32.6587	0.0080	0.0333
28.5022	0.0022	-0.0156	32.7436	-0.0221	0.0249
28.5870	0.0081	0.0259	32.8284	0.0592	-0.0363
28.6718	0.0828	-0.0299	32.9132	-0.0183	0.0234
28.7567	0.0555	0.0119	32.9980	0.0148	0.0148
28.8415	-0.0620	0.0245	33.0829	0.0219	-0.0376
28.9263	0.0108	0.0312	33.1677	0.0549	-0.0027
29.0111	0.0344	-0.0669	33.2525	-0.0344	0.0020
29.0960	0.0351	0.0168	33.3374	0.0115	-0.0002
29.1808	-0.0382	-0.0459	33.4222	0.0392	-0.0086
29.2656	-0.0184	-0.0237	33.5070	0.0156	-0.0226
29.3504	0.0715	0.0141	33.5918	0.0173	0.0266
29.4353	0.0279	-0.0447	33.6767	0.0009	0.0295
29.5201	-0.0138	0.0236	33.7615	0.0395	-0.0034
29.6049	0.0151	-0.0099	33.8463	-0.0196	0.0522
			33.9312	-0.0073	-0.0178

TABLE II. (CONTINUED)

$$\frac{\beta}{\delta_R} = \frac{.079\ S^3 + 11.0\ S^2 + 60.0\ S - 2.04}{D_2}$$

$$\frac{\beta}{\delta_A} = \frac{-3.48\ S^2 + 80.0\ S + 33.1}{D_2}$$

$$\frac{\beta}{\beta_g} = \frac{-0.168\ S^3 - 10.37\ S^2 - 54.64\ S + 0.868}{D_2}$$

$$\frac{\phi}{\delta_R} = \frac{1.35\ S^2 - 12.8\ S - 58.9}{D_2}$$

$$\frac{\phi}{\delta_A} = \frac{204.0\ S^2 + 237.0\ S + 1883.0}{D_2}$$

$$\frac{\phi}{\beta_g} = \frac{7.07\ S^2 - 5.34\ S}{D_2}$$

where:

$$D_2 = 0.998\ S^4 + 6.70\ S^3 + 15.8\ S^2 + 53.9\ S - 0.868$$

$$= 0.998\ (S + 5.6)(S - 0.016)(S + 0.564 + j\ 3.062)(S + 0.564 - j\ 3.062)$$

TABLE III. OPEN LOOP LATERAL TRANSFER FUNCTIONS

U_o = 198 ft/sec	N_r = - 1.01
g = 32.2 ft/sec^2	N_β = 9.27
θ_o = 0.0	$Y^*_{\delta_A}$ = 0.0
Y_v = - 0.167	$Y^*_{\delta_R}$ = 0.079
L_p = - 5.54	L_{δ_A} = 204.0
L_r = 1.35	L_{δ_R} = 1.95
L_β = - 7.06	N_{δ_A} = - 2.99
N_p = - 0.156	N_{δ_R} = -10.7

TABLE IV. LATERAL EQUATIONS OF MOTION PARAMETERS

$U_o = 198$ ft/sec	$M_w = -0.175$
$g = 32.2$ ft/sec^2	$M_q = -5.56$
$X_u = -0.0466$	$X_{\delta_S} = 0.0$
$X_{\dot{w}} = 0.0$	$X_{\delta_F} = -8.84$
$X_w = 0.0793$	$X_{\alpha_g} = 15.7$
$X_q = 0.0$	$Z_{\delta_S} = 89.3$
$Z_u = -0.325$	$Z_{\delta_F} = -283.1$
$Z_{\dot{w}} = -0.0154$	$Z_{\alpha_g} = -3.05$
$Z_w = -3.045$	$M_{\delta_S} = 61.2$
$Z_q = -8.12$	$M_{\delta_F} = -17.8$
$M_u = 0.00079$	$M_{\dot{\alpha}_g} = 0.156$
$M_{\dot{w}} = -0.0106$	

TABLE V. LONGITUDINAL EQUATIONS OF MOTION PARAMETERS

$$\frac{\alpha}{\delta_F} = \frac{-1.433\ S^3 - 25.2\ S^2 - 3.814\ S - 1.082}{D_1}$$

$$\frac{\alpha}{\delta_S} = \frac{0.452\ S^3 + 61.29\ S^2 + 9.49\ S + 3.354}{D_1}$$

$$\frac{\alpha}{\alpha_g} = \frac{-3.045\ S^3 - 50.62\ S^2 - 7.876\ S - 2.131}{D_1}$$

$$\frac{\theta}{\delta_F} = \frac{-15.03\ S^2 - 6.851\ S - 1.805}{D_1}$$

$$\frac{\theta}{\delta_S} = \frac{61.22\ S^2 + 180.3\ S + 28.1}{D_1}$$

$$\frac{\theta}{\alpha_g} = \frac{-28.78\ S^2 - 4.364\ S}{D_1}$$

where:

$$D_1 = 1.015\ S^4 + 10.85\ S^3 + 51.80\ S^2 + 8.075\ S + 2.131$$

$$= 1.015\ (S + 0.076 + j\ 0.1916)(S + 0.076 - j\ 0.1916)$$

$$\times\ (S + 5.268 + j\ 4.65)(S + 5.268 - j\ 4.65)$$

TABLE VI. OPEN LOOP LONGITUDINAL TRANSFER FUNCTIONS

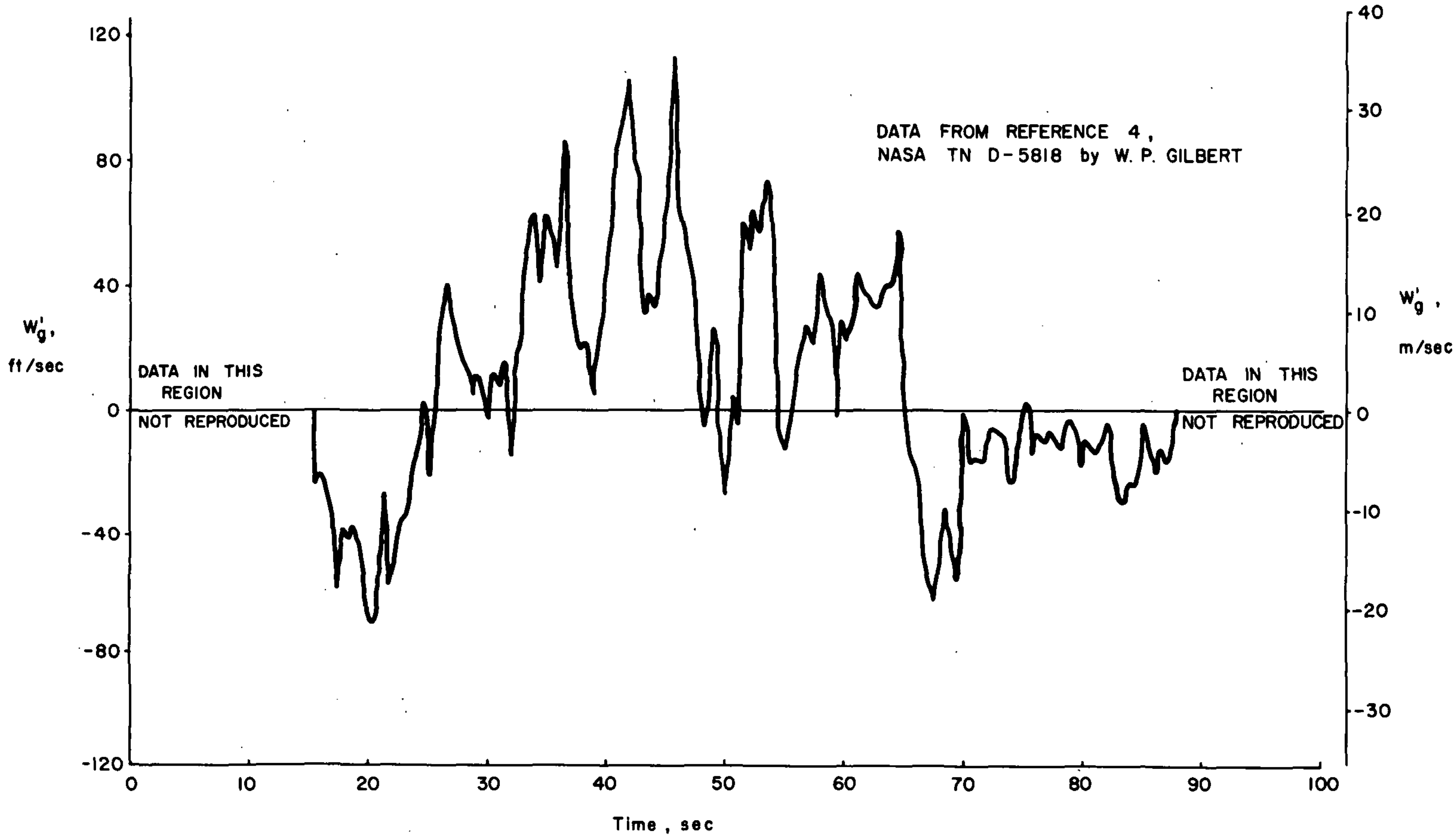

FIGURE I. TIME HISTORY OF VERTICAL GUST USED TO EXCITE AIRCRAFT

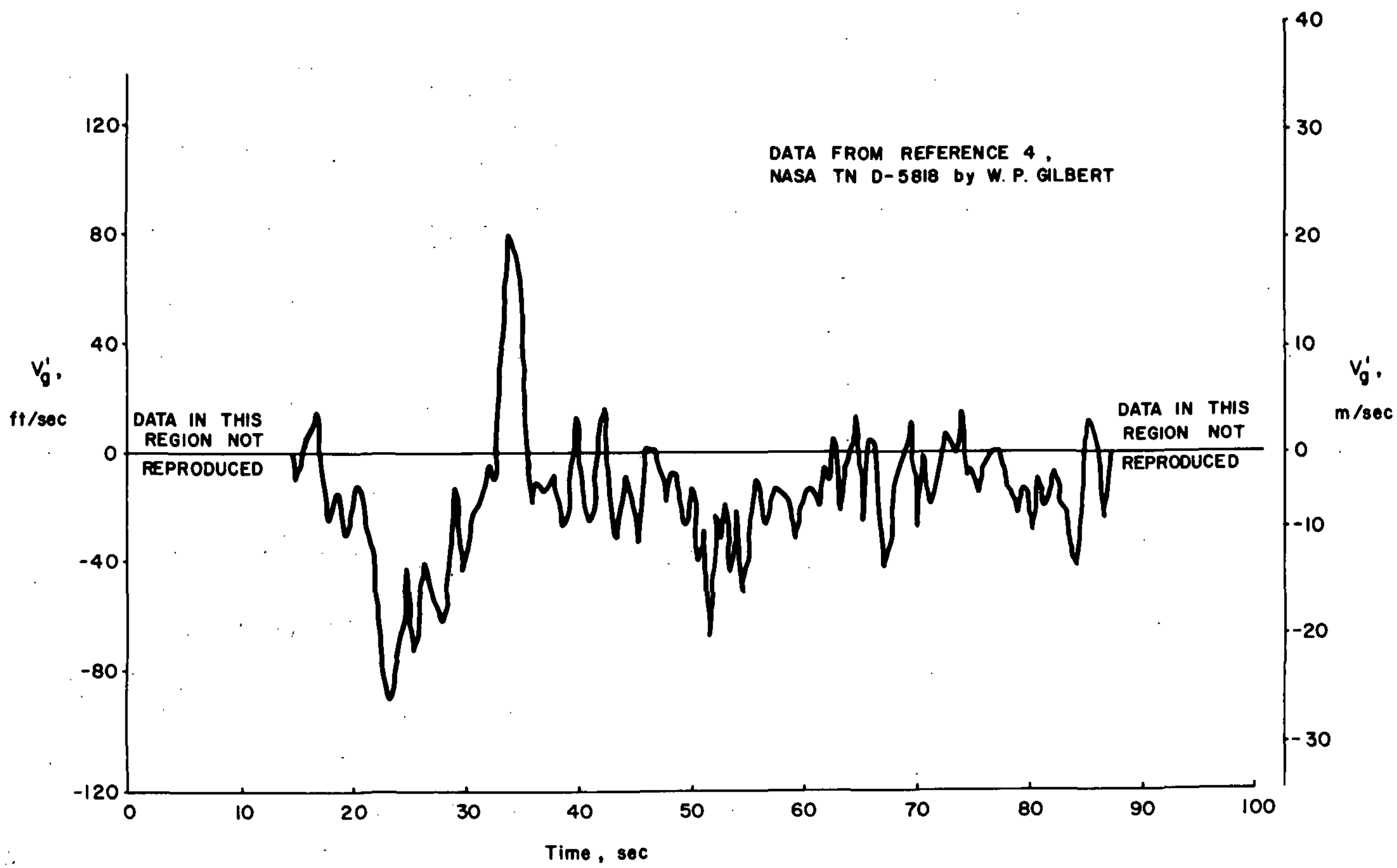

FIGURE 2. TIME HISTORY OF LATERAL GUST USED TO EXCITE AIRCRAFT

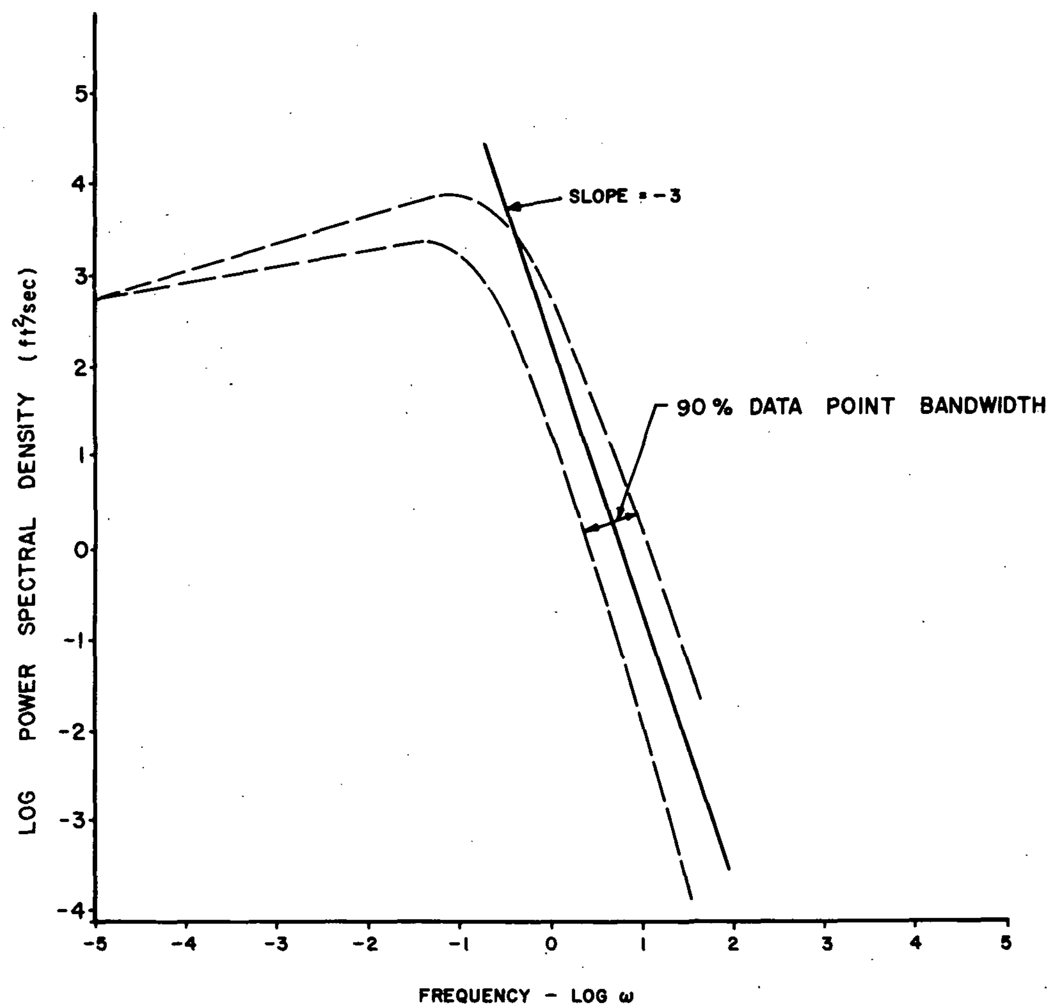

FIGURE 3. APPROXIMATE ONE-DIMENSIONAL POWER SPECTRAL DENSITY OF VERTICAL GUST

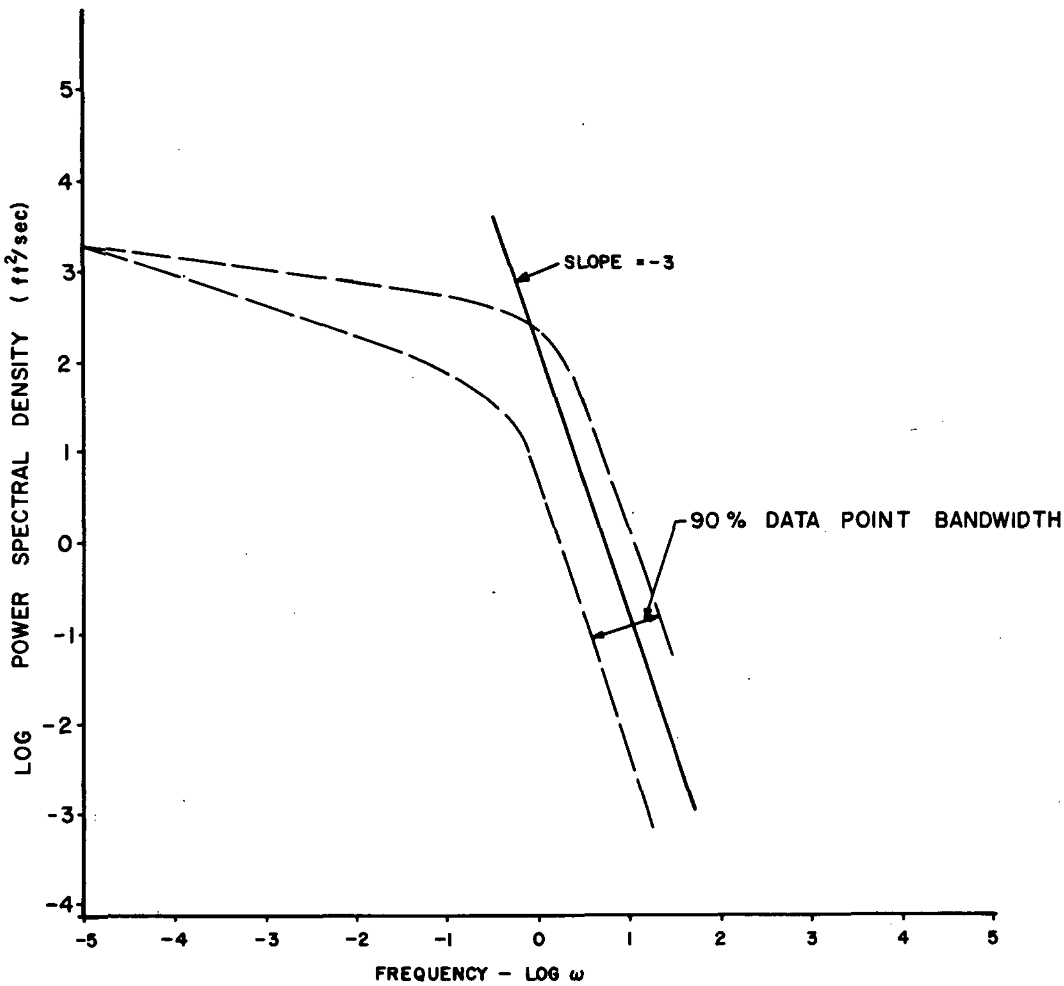

FIGURE 4. APPROXIMATE ONE-DIMENSIONAL POWER SPECTRAL DENSITY OF LATERAL GUST

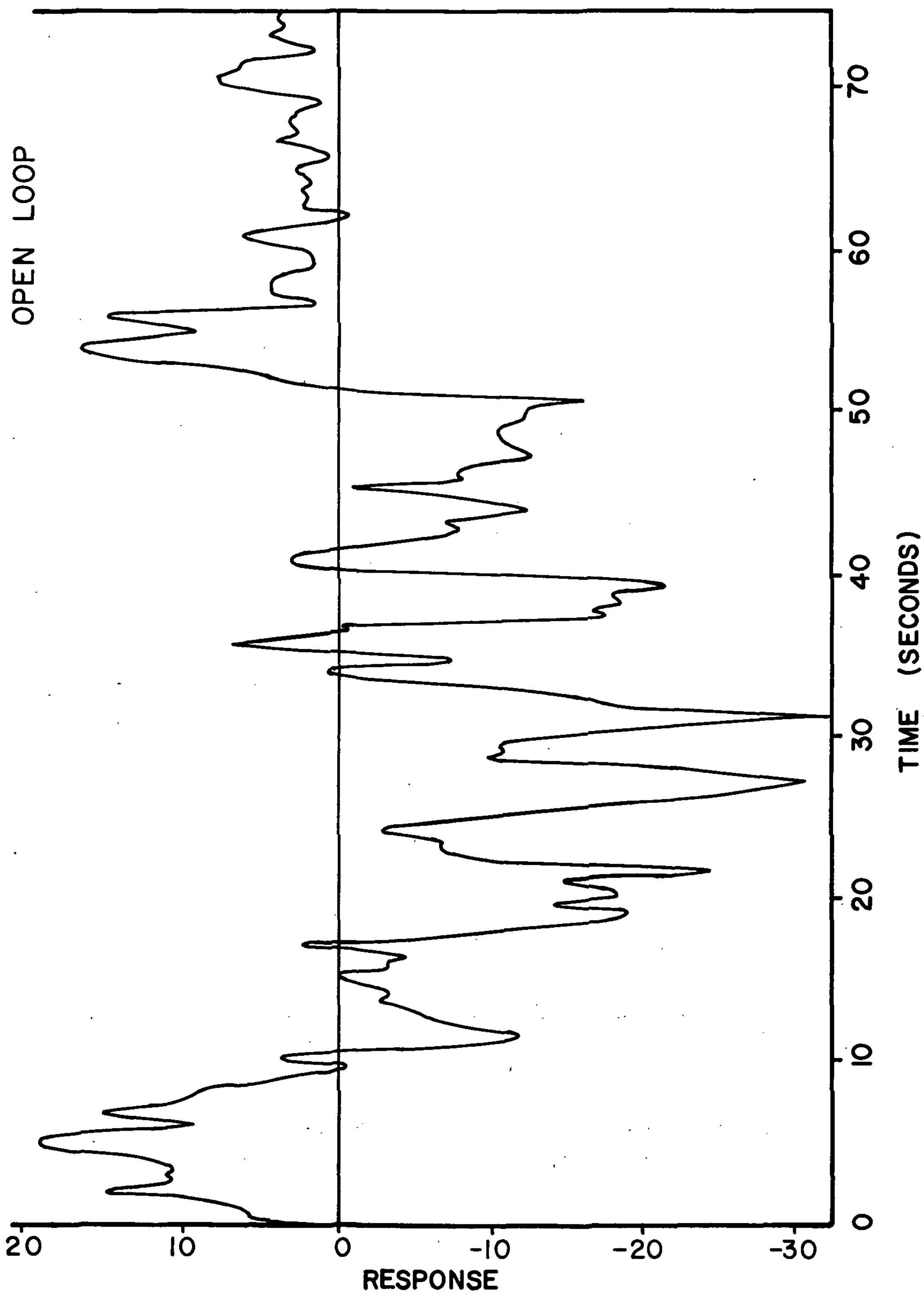

FIGURE 5A. ANGLE OF ATTACK RESPONSE TO VERTICAL GUST INPUT

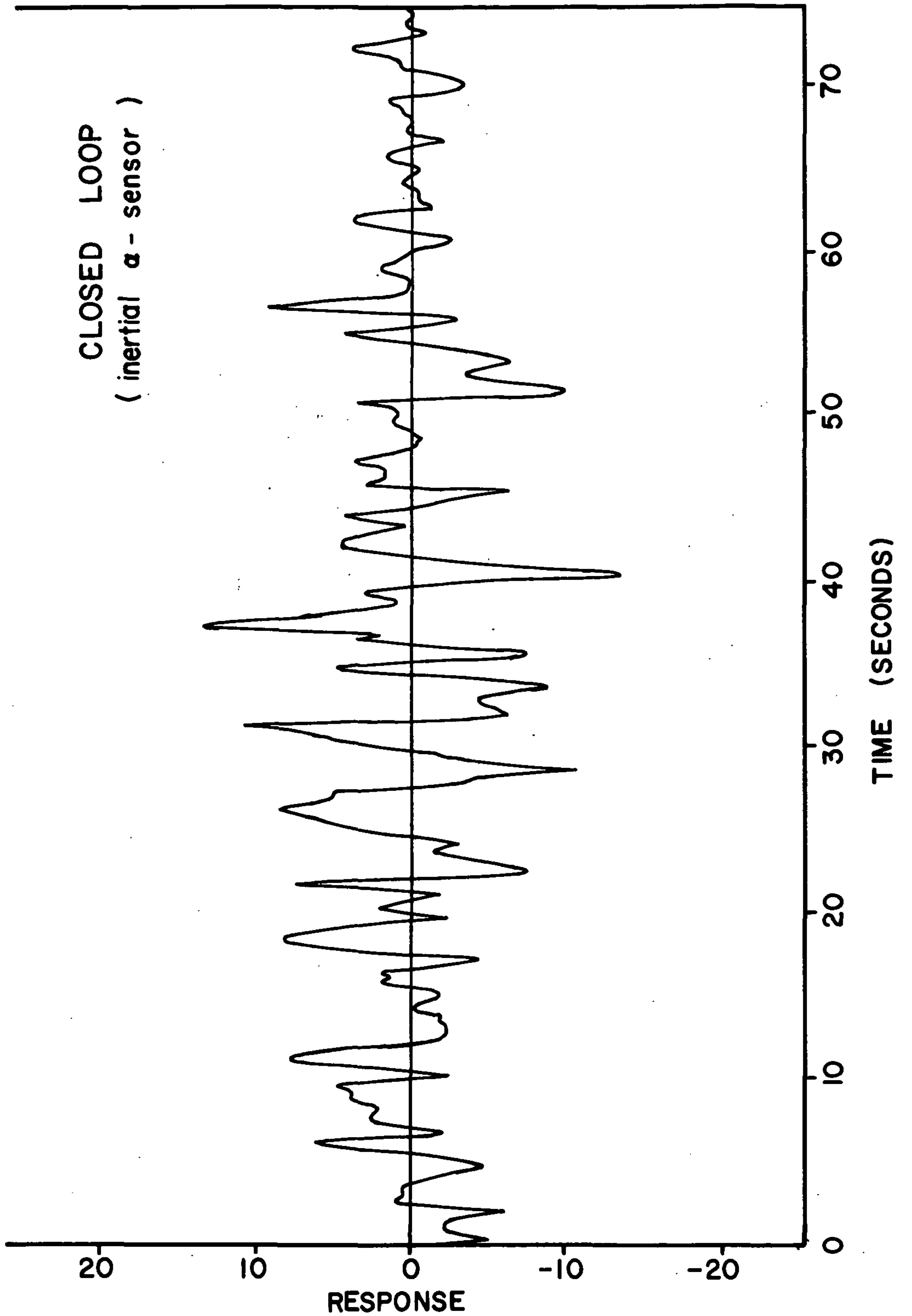

FIGURE 5B. ANGLE OF ATTACK RESPONSE TO VERTICAL GUST INPUT

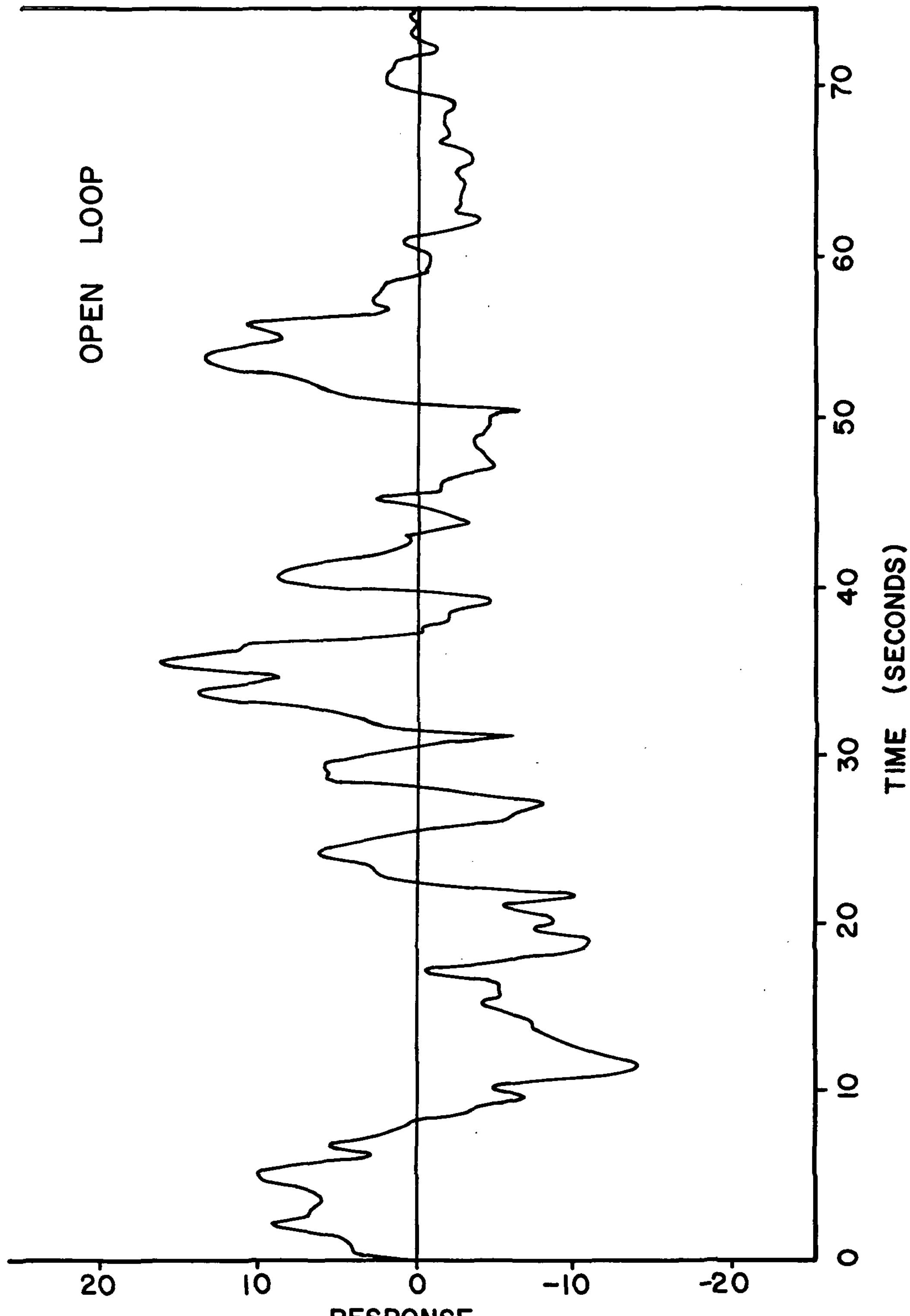

FIGURE 5C. PITCH ANGLE RESPONSE TO VERTICAL GUST INPUT

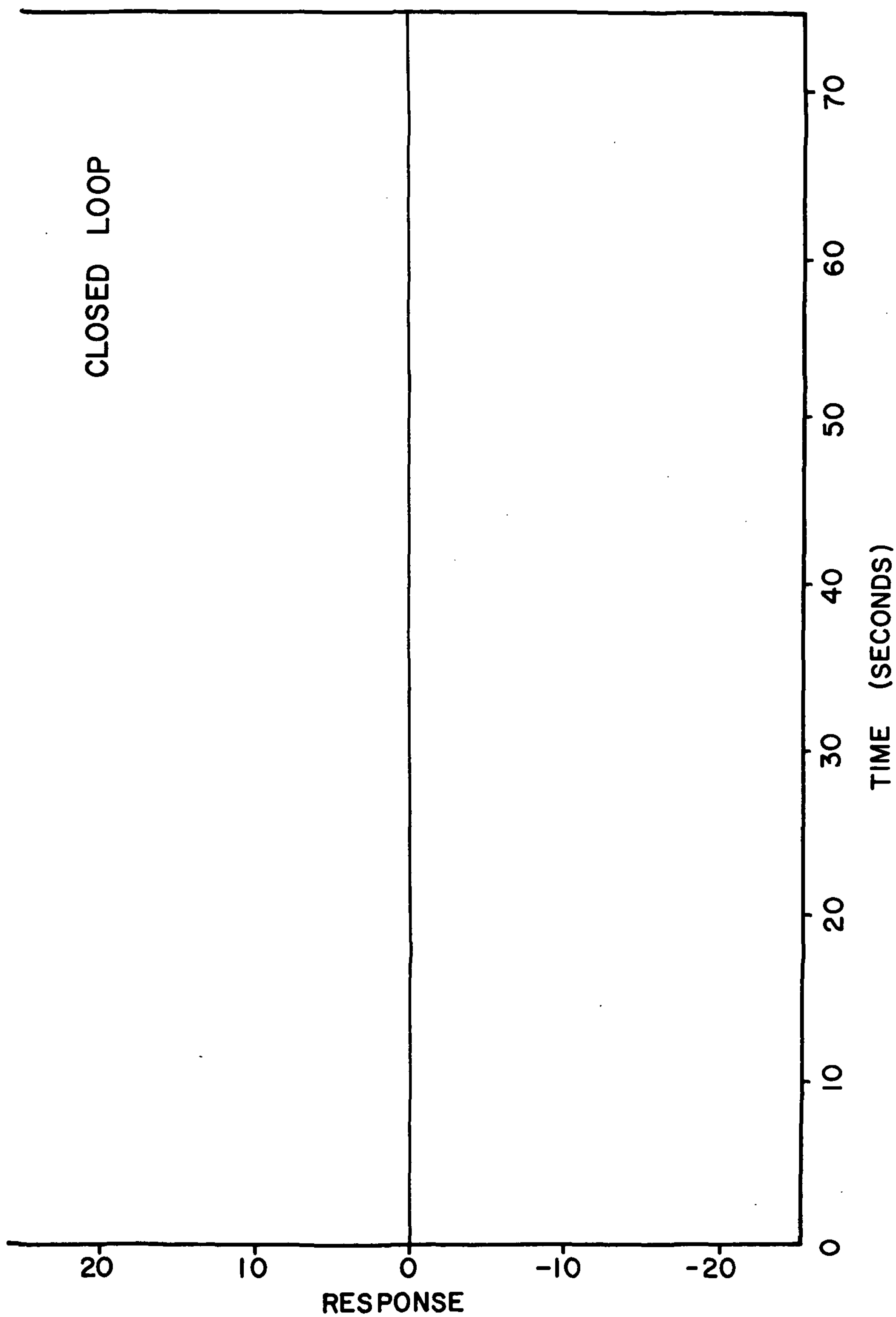

FIGURE 5D. PITCH ANGLE RESPONSE TO VERTICAL GUST INPUT

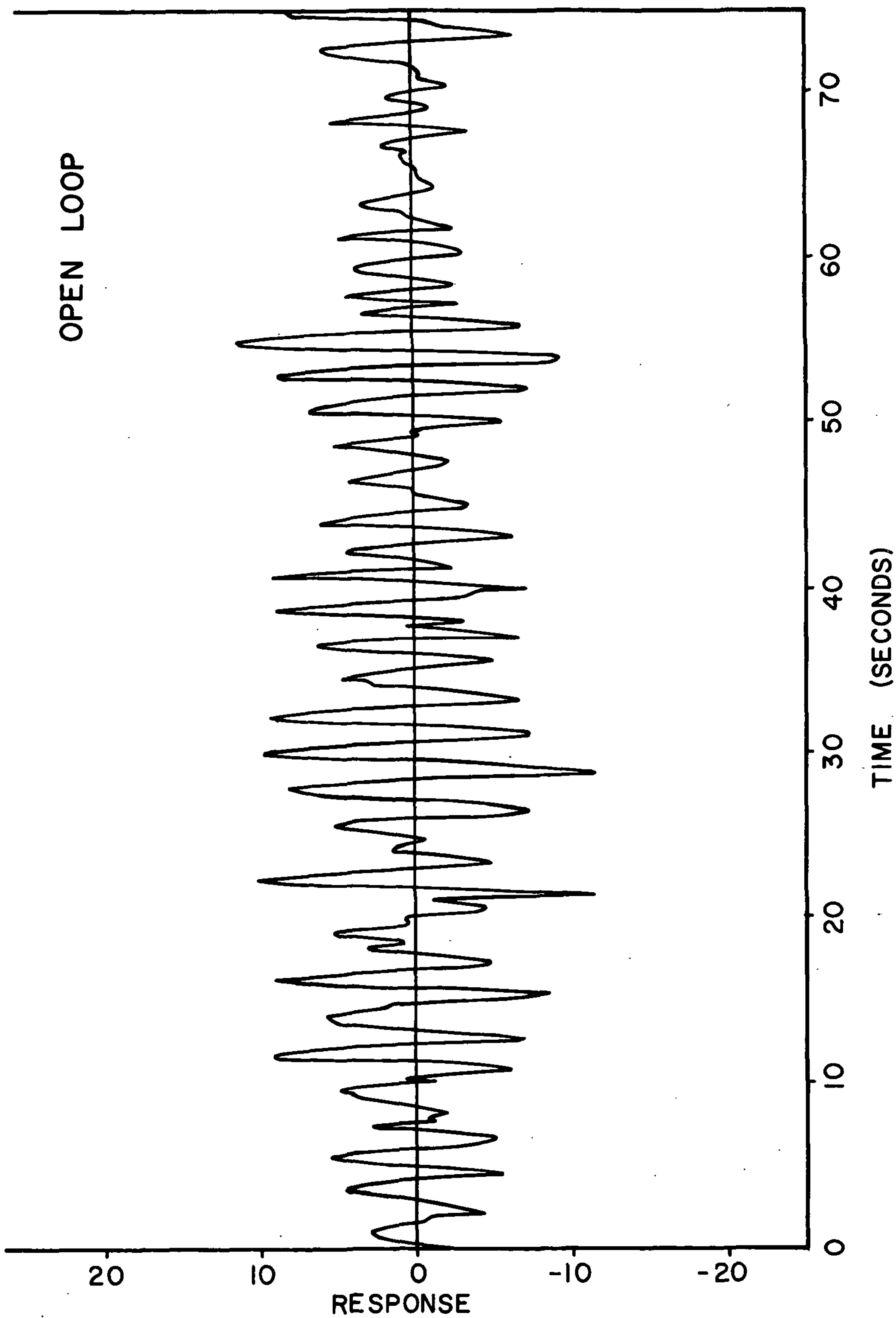

FIGURE 6A. AERODYNAMIC SIDESLIP ANGLE RESPONSE TO LATERAL GUST INPUT

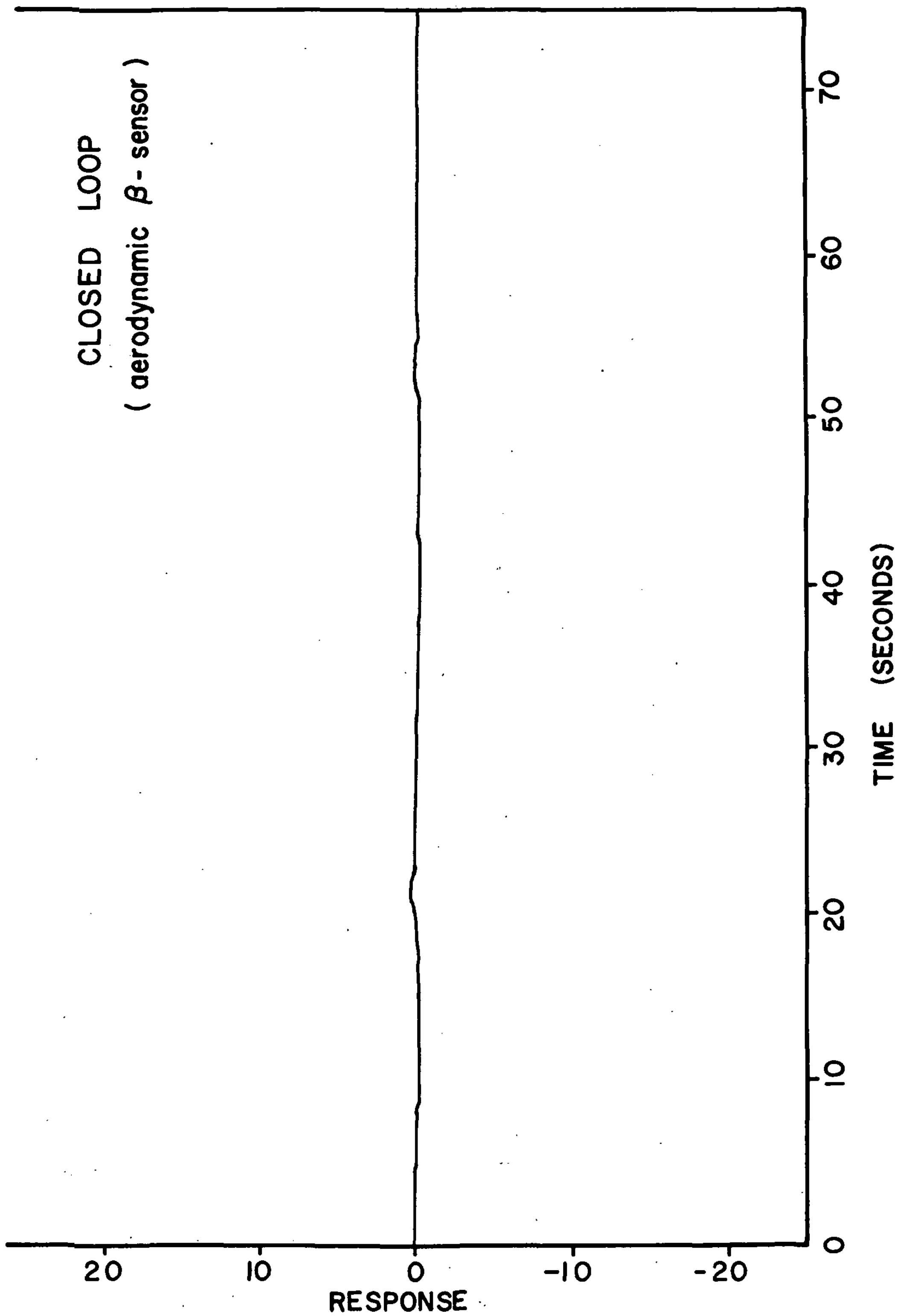

FIGURE 6B. AERODYNAMIC SIDESLIP ANGLE
RESPONSE TO LATERAL GUST INPUT

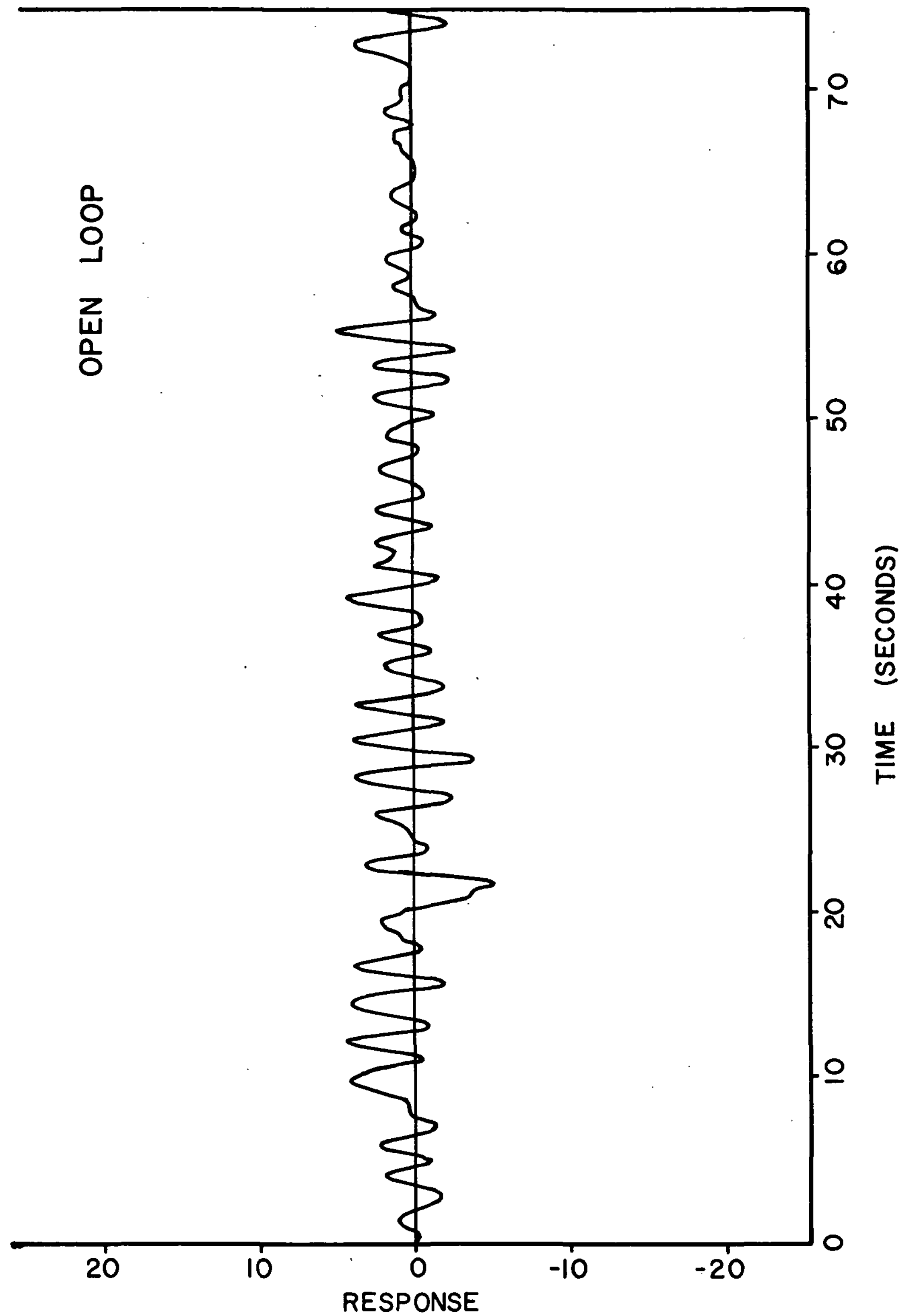

FIGURE 6C. ROLL ANGLE RESPONSE TO
LATERAL GUST INPUT

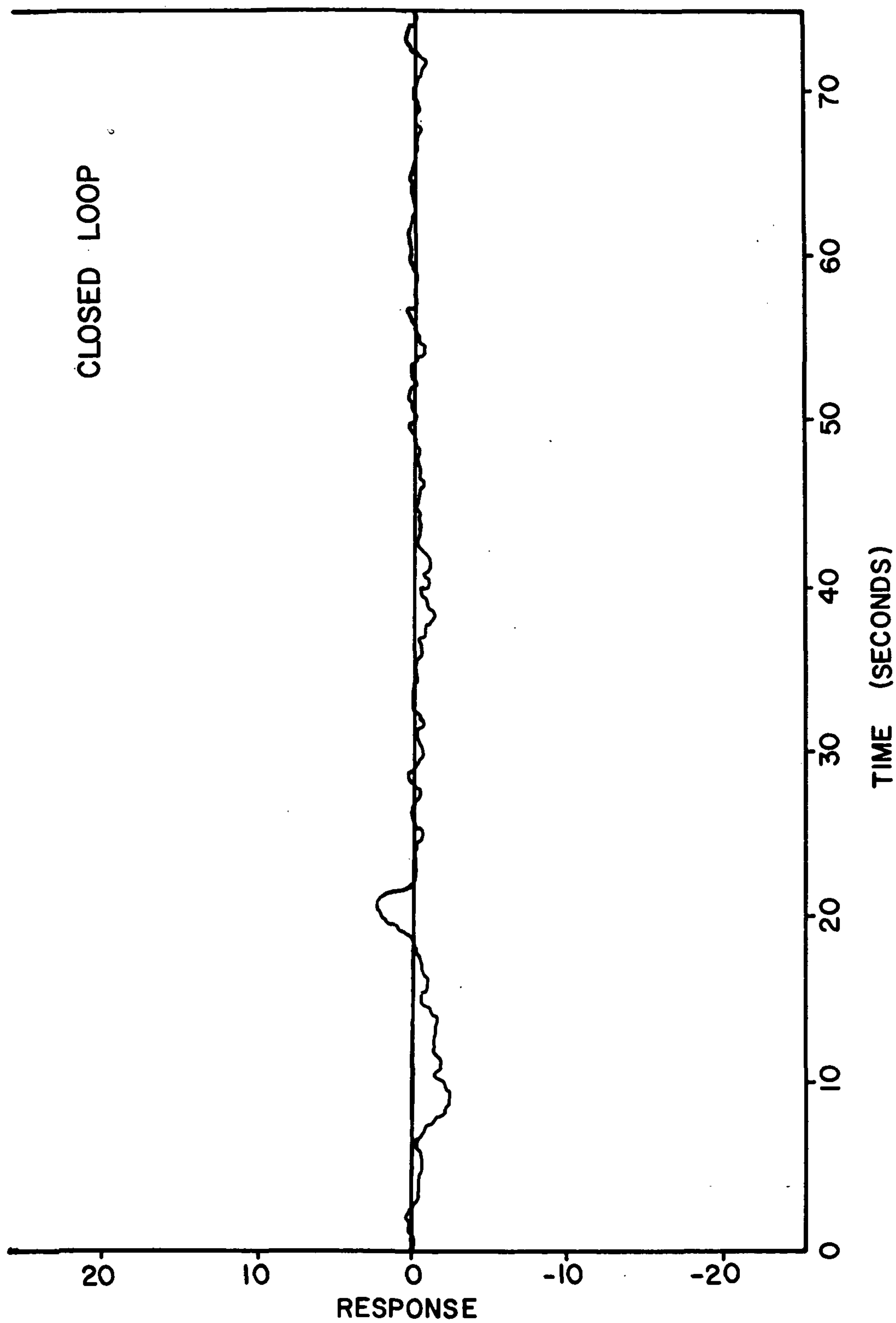

FIGURE 6D. ROLL ANGLE RESPONSE TO LATERAL GUST INPUT

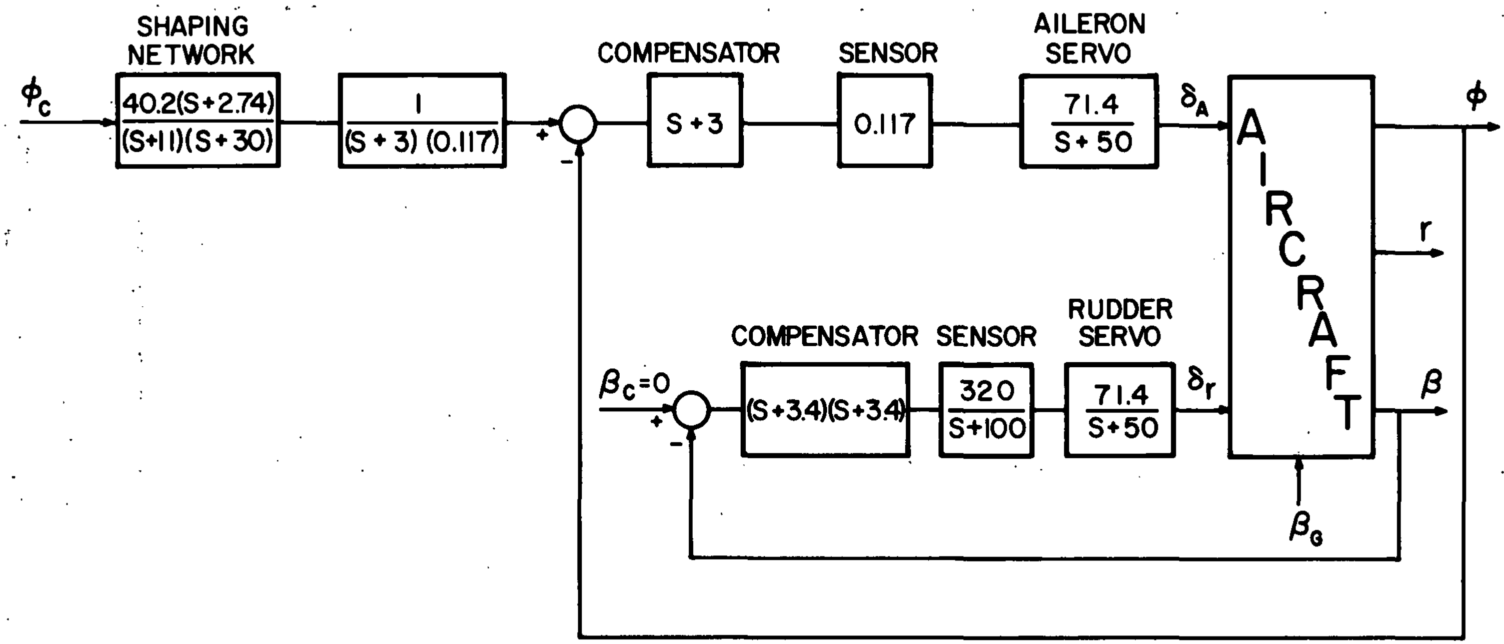

FIGURE 7. UNITY FEEDBACK REPRESENTATION OF LATERAL CONTROL SYSTEM

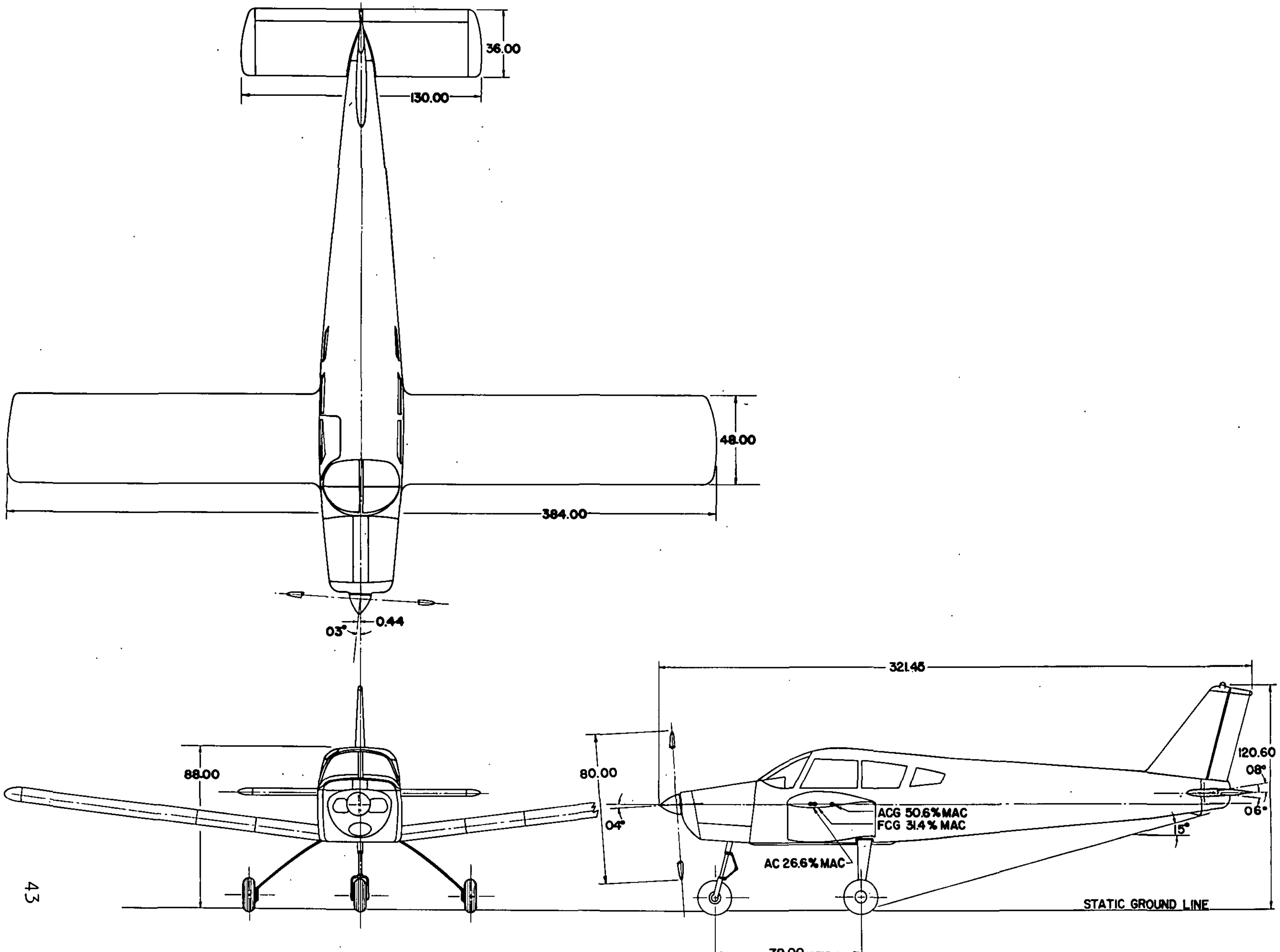

FIGURE 8 MODIFIED AIRCRAFT

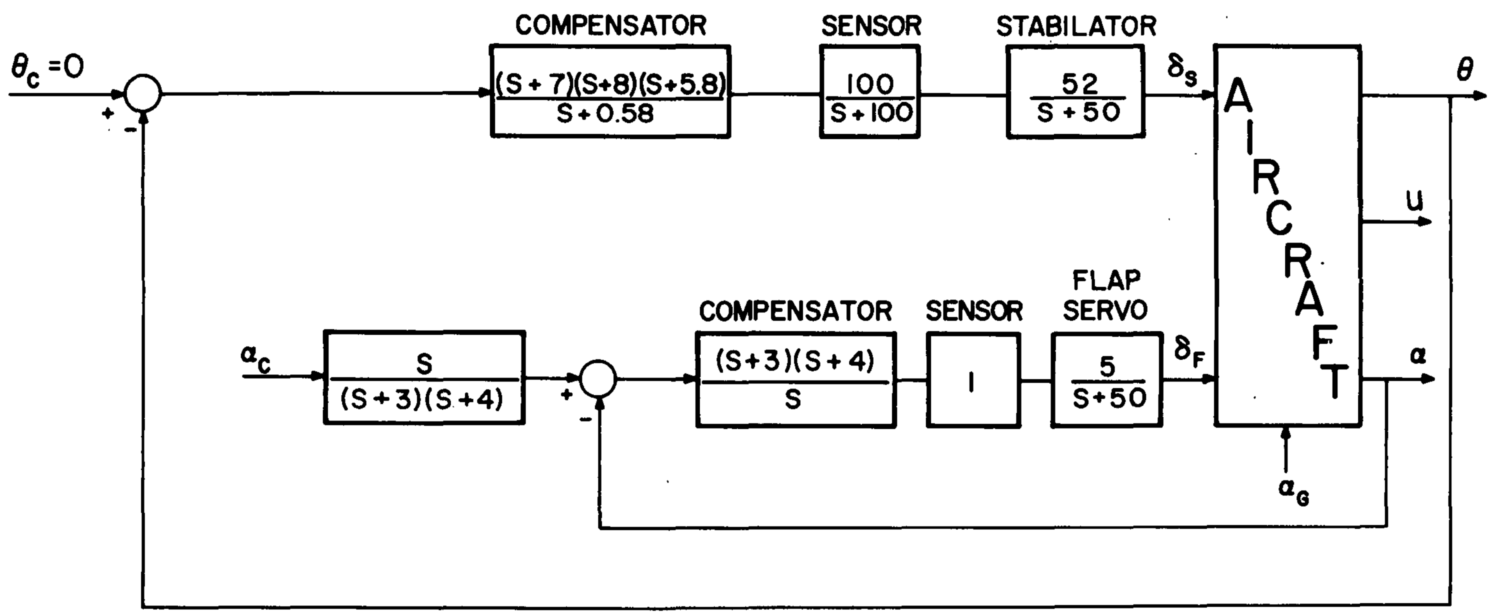

FIGURE 9. UNITY FEEDBACK REPRESENTATION OF LONGITUDINAL CONTROL SYSTEM

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