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POOLMS - A COMPUTER PROGRAM FOR FITTING AND MODEL SELECTION FOR TWO-LEVEL FACTORIAL REPLICATION-FREE EXPERIMENTS

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16. Abstract

A computer program is described that performs a statistical multiple-decision procedure called chain pooling. It uses a number of mean squares assigned to error variance that is conditioned on the relative magnitudes of the mean squares. The model selection is done according to user-specified levels of type 1 or type 2 error probabilities.

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SUMMARY

A statistical, multiple-decision procedure called chain pooling has been described in an earlier report. The procedure's application is model fitting to two-level, factorial, fixed-effects, replication-free experiments. It uses a conditional number of mean squares for estimating error variance and for testing for real effects, where the number is conditioned on the relative magnitudes of the mean squares. POOLMS is a computer program that performs the operations of the procedure in accordance with user-specified levels of type 1 or type 2 error probabilities.

INTRODUCTION

Some of the most efficient experiments that can be designed are the two-level, fixed-effects, crossed-classification, full or fractional, factorial experiments without replication. Some methods for the design of such experiments, particularly when allowing for sequential experimenting in the presence of block effects, have been described in references 1 to 5. The economy achieved by not replicating carries the penalty that the experiment might not provide a valid mean square for estimating error variance. Such an estimate is needed, if parameter estimates are to be judged significant or insignificant according to well-established statistical procedures.

The absence of an obviously valid estimate of error variance has led to the development of several alternative procedures, all possessing some degree of heuristic appeal, with most of them being accompanied by some amount of Monte Carlo evaluation (refs. 6 to 11).

The present report gives a qualitative comparison of these procedures, which leads to the conclusion that chain pooling (refs. 10 and 11) is the procedure that should be most generally preferred for model selection applied to data from the type of experiment already described.

Following the qualitative comparison of the alternative procedures, a computer program, POOLMS, is described. It is a program designed specifically to do the computations required by the model-selection procedure referred to as chain pooling. The complete program listing is given and described, together with detailed descriptions of the input and the output. The use of the program is illustrated by an example, for which a detailed statistical discussion is given in references 10 and 11. The reader should consult reference 10 for an understanding of the statistical basis of POOLMS and, therefore, for an understanding of the capabilities and limitations of the decision procedure provided by POOLMS. Whereas reference 10 deals only with the analysis of 2^4 design experiments, reference 11 deals also with the analysis of 2^5 and 2^6 design experiments. Computations for any of them are provided by POOLMS.

SYMBOLS

Mathematical symbol	FORTRAN name	Description
g		Number of factors in two-level experiment
h		Fractional replicate contains $(1/2)^h$ observations of full factorial experiment
i	I	Subscript denoting order of computing mean squares according to yates' algorithm: $i = 0, 1, 2, \ldots, 2^\ell - 1$, I = 1, . . ., 2**L
j	J	Subscript denoting the j smallest mean square (exclusive of grand mean): $j = 1, 2, \ldots, 2^\ell - 1$
k	K	Subscript
ℓ	L	$g - h$: Experiment contains 2^ℓ treatment combinations and produces that many observation sets
m	M	Number of mean squares pooled before testing begins

Mathematical symbol	FORTRAN name	Description
R_1		Risk associated with type 1 errors
R_2		Risk associated with type 2 errors
U_j	TB	Test statistic
x		Levels of independent variables
y	Y(I,K)	Response variate: I = 1, . . ., 2**L; K = 1, . . ., NY
Z	Z(...)	Mean square
α		Test size
α_f		Nominal size of final significance test
	NY	Number of observations averaged for each treatment (maximum of six)
	KF	Final test size index, KF = 1, . . ., 10 (see section on INPUT)
	KP	Preliminary test size index, KP = 1, . . ., 11 (see section on INPUT)
α_p		Nominal size of preliminary pooling test
η		Number of mean squares having noncentrality parameter of zero
$\hat{\eta}$	JETA	Estimate of η
μ_i		Coefficients of model equation that are estimated in Yates' order with Yates' contrasts
$\hat{\mu}_i$	B(I)	Estimate of μ_i
ρ		Number of real effects
$\hat{\rho}$	IRHO	Estimate of ρ
σ		Standard deviation
$\hat{\sigma}$		Estimate of σ

ALTERNATIVE MODEL-SELECTION PROCEDURES

For two-level, fixed-effects, factorial experiments without replication, a suitable practice consists, according to Davies (ref. 12), of pooling some arbitrary number of the highest order interaction mean squares into an estimate of error variance. When this is done, any of these high-order interactions that are not actually small or any unknown block effects (major changes in experimental conditions not accounted for by the model) could inflate some of the pooled interactions and thereby give too large an estimate of error variance. Too large an error estimate reduces the sensitivity of subsequent tests.

The preservation of sensitivity, when pooling mean squares into the estimate of error variance, has been an object of the procedure of Daniel (ref. 7) and of Wilk, Gnanadesikan, and Freeny (ref. 9).

Daniel (ref. 7) uses the absolute values of the effect estimates as order statistics. They are plotted on probability paper, and the result is called a half-normal plot. In addition to conditional structuring of the ANOVA model, Daniel's objectives included the determination of "bad values, heteroscedasticity, dependence of variance on mean, and some types of defective randomization, . . ." The half-normal plot, combined with a background of experience, might provide a method by which a skillful user could pass judgement on the results of an experiment. Daniel concluded that such a plot can be used to make judgements about the reality of the largest effects observed only if a small proportion of the effect estimates represent real effects. Birnbaum (ref. 8) investigated procedures related to half-normal plotting. His results on ". . . the probabilities of the various possible sorts of errors . . ." are limited to the single largest order statistic. He surmised that if only a small number of effect estimates have nonzero means, the power and sensitivity properties will tend to hold approximately.

The procedure of Wilk, Gnanadesikan, and Freeny (ref. 9) if used with 2^{ℓ} treatments is benefited if some subjective or prior knowledge is used to decide that η of the $2^{\ell} - 1$ mean squares do not contain real effects or that $\rho = 2^{\ell} - \eta - 1$ mean squares do contain real effects. As was shown in reference 9, their procedure is not robust against errors in guessing the value of η, and η must be guessed if the prior knowledge is lacking. Chain pooling does not require a prior knowledge of η. The procedure gives an estimate of η.

Zahn (ref. 6) presented results of a Monte Carlo study of Daniel's original version of the half-normal plot and two modifications of it when as many as six real contrasts of size 1σ to 8σ are present in a 2^{p-q}, $p - q = 4$, factorial experiment. Zahn considered the power, false positive behavior, and variance estimation of these versions of the half-normal plot when they are applied to the general problem in the case $n = 15$, since there are 15 contrasts of interest, ignoring the grand mean, in a 2^4 factorial experiment. He then pointed out that these versions, or obvious modifications of them, can be used for any n.

Zahn, Birnbaum, and Daniel (refs. 6 to 8) have limited their results to experiments where only a small proportion (at most 6 out of 15) of the effects are anticipated to be significant. On the other hand, situations can exist where the experimenter might use a two-level, fractional, factorial experiment designed such that a large proportion of the effects are significant. These situations are met by the procedures of references 10 and 11.

As stated by Birnbaum (ref. 8) the optimal decision procedure when $\rho \leq 1$ uses a test developed by Cochran (ref. 13). The Z_i (originally in Yates' order) are ordered (omitting the mean square for the grand mean) according to non-decreasing magnitude and renamed Z_j:

$$Z_1 \leq Z_2 \leq \ldots \leq Z_j \leq \ldots \leq Z_n$$

Cochran's statistic is:

$$C_n = \frac{Z_n}{Z_1 + \ldots + Z_n}$$

and the null hypothesis is rejected with test size α if C_n exceeds the upper 100 α-percent point of Cochran's distribution, which has been additionally tabulated (ref. 14).

As pointed out by Zahn (ref. 6) the operating characteristics of the decision procedures that have been intended for $\rho \leq 1$ deteriorate rapidly with $\rho > 1$. But in addition to dealing with the situation of $\rho > 1$, the model-selection procedure must also deal with the possibility that not all the nonzero effects are of equal magnitude. As shown in table 6.2 of reference 6, the detection rate for two means of size 4σ, 6σ, or 8σ is usually reduced in the presence of two other means of size 2σ, 4σ, or 6σ, respectively, from the value it would have been in the presence of two other means of the same size, namely 4σ, 6σ, or 8σ, respectively. Because inequality of the means is clearly a disadvantage, the question arises as to what is the least favorable distribution of these means. As developed in reference 10, a normal distribution of these means would be highly unfavorable.

Accordingly, the procedures of references 10 and 11 were developed empirically to be optimal when the means do have an approximate normal distribution. Furthermore, the prior assumption of a normal distribution for these means in the general experimental situation does not seem to be unreasonable. Thus, as pointed out in reference 11, the Monte Carlo founded procedures of references 10 and 11 can be regarded as being empirically optimized against both what is highly likely to occur and what would be highly unfavorable if it did occur. Therefore these procedures should be regarded as being approximately both Bayes and minimax optimal.

OUTLINE OF COMPUTING PROBLEM

This description of the computing problem is divided into four steps: (1) read the input data and provide some elementary conditioning if required, (2) compute parameter estimates and associated mean squares, (3) perform the model selection, and (4) write the output.

Input Data

Two-level, full or fractional experiments are sometimes performed in a manner that provides superficial rather than true replication. For example, if the treatments are each applied to different specimens but the dependent variable is observed through duplicate measurements on each of the specimens, such replication does not measure the specimen-to-specimen variability. A conservative procedure would be to compute the means of the repeated measurements and then proceed with these mean values as if they were observations from an experiment with no replication. The first step of the program computes the mean values in such cases of superficial replication.

A logarithmic transformation of the dependent variates is also provided.

Parameter Estimates and Mean Squares

The second step of the program computes parameter estimates (effects) and mean squares according to Yates' method (ref. 12). (Because these estimates are estimates of coefficients in an equation where the levels of the independent variables are represented by +1 and -1, the coefficients have one-half the absolute values of the "effect" estimates of ref. 12.)

Model Selection

The third step of the program is to invoke the model-selection procedure described in references 10 and 11. This is done by ordering the computed mean squares in the ascending order of their magnitudes and simultaneously setting up a pointer function (IND(J)) that will relate the ordered mean squares to the model parameter estimates that they contain. The test statistic is then computed. With the statistician having specified m, the statistic consists of a denominator containing the m + 1 smallest mean squares and a numerator containing the m + 1 smallest mean square. The program then performs a table look-up to compare the test statistic with the tabular entry for nominal significance level α_p and m + 1 degrees of freedom. The computation of test statistics and table look-up at significance level α_p or α_f with j degrees of freedom then continues as required by the sequential procedure and branching of references 10 and 11.

Output

On completion of the decision procedure, the fourth step of the program is writing both the input and output data. The input data include (1) the number of treatment combinations, namely, 2^{ℓ}; (2) the value of KODE to show whether a logarithmic transformation was used; (3) the strategy (m, α_p, and α_f); and (4) the input observations. The output data include (1) the smallest mean square selected as being significant and the associated values of $\hat{\eta}$ and $\hat{\rho}$, (2) the treatment identifications (if any) and associated transformed observations; (3) the regression coefficient estimates; (4) the ordered mean squares; and (5) the pointers that match the ordered mean squares with the regression coefficients.

PROGRAM DESCRIPTION

The working part of the program is described in this section. The input and output are described subsequent to the description of an example illustrating the use of the program.

The program is concerned with the 2^{ℓ} sets of observations corresponding to the 2^{ℓ} treatments of the experiment. These sets must be in the correct order for Yates' method. If there is more than one observation in a set for any treatment, the observations in the set are first averaged to form one "observation" per treatment.

The "observations" are then operated on according to Yates' method (ref. 12) to compute the parameter estimates. The algorithm for Yates' method is described as follows.

The "observations" $y_{i,j}$ may be visualized as a column (j = 1) with row index $i = 1, \ldots, 2^{\ell}$. The column is then operated on according to Yates' method to produce a succession of columns $j = 2, \ldots, \ell$. The successive columns for any k^{th} row are computed as follows:

$$y_{k,j} = y_{i+1,j-1} + y_{i,j-1} \begin{cases} i = 1, 3, 5, \ldots, 2^{\ell} - 1 \\ k = 1, 2, 3, \ldots, 2^{\ell}/2 \end{cases}$$

$$y_{k,j} = y_{i+1,j-1} - y_{i,j-1} \begin{cases} i = 1, 3, 5, \ldots, 2^{\ell} - 1 \\ k = (2^{\ell}/2) + 1, (2^{\ell}/2) + 2, \ldots, 2^{\ell} \end{cases}$$

New columns are computed according to the two preceding equations for $j = 2, \ldots, \ell$ (to create ℓ columns). The program performs these operations (appendix A) following the comments card "CALCULATE MEAN SQUARES BY YATES METHOD." Furthermore, an array of pointers to the B(I) array is created by the statement IND(I) = I. This array will serve to identify the coefficients in the B(I) array after the process of ordering mean squares according to rank. The ordering is done in the sequence of statements 10 to 13.

Operations thus far have created a column of mean squares Z(J) with mean squares indexed on J in the order of increasing rank, together with a column of integers IND(J) indexed on J. Thus, any address J will lead to a mean square Z(J) and also to the integer IND(J). This integer is the index I that the associated regression coefficient has in the original Yates' order.

Now, ordered according to rank, the mean squares are subjected to the hypothesis testing as given by the statements following the comment "DO HYPOTHESIS TESTING." These operations require that the program contain the critical values against which the test statistic is to be compared. The table of 620 critical values consisting of the test statistic U_j (table I) plus 20 dummy values is stored internally in the program as TB(64,10). Also stored internally are the 11 values of nominal test size ALPHA, ranging from 0.001 to 1.0, with the first 10 being the column headings for U_j in table I. (The values of $\alpha_p = 1.0$ and associated KP = 11 are simply a code for the strategy of terminating the pooling with the prior chosen number of m smallest mean squares.)

The hypothesis testing operations are displayed by figure 1. (The initial pooling of m mean squares is performed in the loop "DO 15 J = 1, M".)

Statement 17 is the final test of significance. If significance is observed, it has occurred when J = JA, as recorded at statement 19. The significant mean squares are that particular one, Z(JA), and all that are as large or larger. Thus, the number of mean squares decided to be insignificant is $\hat{\eta}$ = JETA; and the number (aside from the grand mean) decided to be significant is $\hat{\rho} = 2^{\ell} - \hat{\eta} - 1$ = IRHO, as given in the statement following 20.

ILLUSTRATIVE EXAMPLE - COBALT-BASE-ALLOY DEVELOPMENT, ONE-HALF RELICATE WITH FIVE FACTORS

The statistical implications of the example to be described were discussed at length in references 10 and 11. In those references the analysis was presented from two points of view. The first, or significance point of view, assumed that the statistician was mainly interested in controlling the probabilities of making type 1 errors. The second, or screening point of view, assumed that all effects that might be real should be so identified and that therefore the statistician was mainly interested in controlling the probabilities of type 2 errors. In either case, the model selection proceeded in two stages. The first stage began with the strategy m = 1, which led to an estimate $\hat{\eta}$ of the number of null mean squares. Based on this estimate, a second iteration of the procedure was performed with m > 1.

In the present discussion, the use of the computer program is illustrated only for the significance point of view and only for the iteration with m = 1.

The responses y_i (table II) are logarithms of stress-rupture times to failure of cast specimens ($\ell = 4$). The parameters to be estimated (table II) include one grand mean, five main effects, and 10 interactions.

The strategy (m, α_p, α_f) is chosen according to information on the operating characteristics given for m = 1 by figure 2. The use of these curves requires that a prior estimate be made for η. For this example, the number of null mean squares is assumed to be equal to the number of highest order interactions; namely, the assumption is $\eta = 10$.

In addition to making a prior estimate of η, the use of the curves of figure 2 requires that some assumptions be made about the value of the type 2 risk, $\bar{R}_2(\lambda)$. Because the operating characteristics with $m = 1$ are not sensitive to the value of λ, the curves of figure 2 are closely spaced with respect to $\bar{R}_2(\lambda)$, and therefore a single arbitrary value (the middle curve for $\bar{R}_2(\lambda) = 0.10$) is used.

Because the type 2 risks should always be minimized, the largest α_f value that will not result in too large a value of $\bar{R}_1(\eta)$ should be chosen. Since the operating characteristics with $m = 1$ are inferior to those with preferred $m > 1$ (fig. 3), the statistician should be less stringent about $\bar{R}_1(\eta)$ when using $m = 1$ than when using preferred $m > 1$. Under the previously stated assumptions concerning $\bar{R}_2(\lambda)$, the strategy (m, α_p, α_f) will be chosen from figure 2 according to the resulting value of $\bar{R}_1(\eta)$.

With $m = 1$:

(1) Assume that $\eta = 10$ and $\bar{R}_2(\lambda) = 0.10$.

(2) In figure 2(a), $(\ell = 4)$ with $\eta = 10$ and $\bar{R}_2(\lambda) = 0.10$, choose the highest α_f that yields an acceptable $\bar{R}_1(\eta)$ and note the required α_p. Figure 2(a) shows that if $\alpha_f = 0.01$, $\bar{R}_1(\lambda)$ is 0.14 and α_p is required to be 0.25.

(3) The strategy to be used is $(m, \alpha_p, \alpha_f) = (1, 0.25, 0.01)$, which is used as input.

INPUT

Card type 1. One card with two values read with FORMAT (3I5) containing:

KODE	(In first field)	Either 0 or 1. If KODE = 0, Y is used. If KODE = 1, log Y is used.
L	(In second field)	Power of 2 to denote number of treatments.

Card type 2. - As many as 64 cards (2**L cards), with two to eight values on each card, read with FORMAT(A6, I6, 6F6.0) containing:

DATAID	(In first field)	Treatment identification (may be omitted).
NY	(In second field)	Number of responses (Y values) to be averaged
Y	(In subsequent fields)	Response variates.

Card type 3. One card for each (m, α_p, α_f) strategy, with three values read with FORMAT (3I5) containing:

M	(In first field)	Number of mean squares pooled before testing begins.
KP	(In second field)	Column number (table I) of α_p.
KF	(In third field)	Column number (table I) of α_f.

Example. Use α_p to get corresponding value of KP; for example, if $\alpha_p = 0.25$, KP = 8. Use α_f to get corresponding value of KF; for example, if $\alpha_f = 0.01$, KF = 4. If several strategies (m, α_p, α_f) are to be tried, card type 3 is the only card that needs to be changed. The card input pertaining to the illustrative example is shown by figure 4.

OUTPUT

The manner in which output is printed is illustrated in appendix B. The value of ℓ (which is determined by the number of observations) is given by the heading "2**L EXPERIMENT" and in the present example L = 4. Next in the headings are the input values of m, α_p, and α_f. The raw data or observations are listed under the column headings Y(1), Y(2), . . ., Y(6).

The decision procedure is illustrated by figure 1 and is described in the section PROGRAM DESCRIPTION. The decision procedure ends (statement 17 of fig. 1) when the value of U_j computed from the mean squares exceeds the critical value of U_j as tabulated for the stated value of α_f. At this point, J = JA and the values of $\hat{\eta}$ and $\hat{\rho}$ are computed from JA and printed, as indicated, by IRHO = 7 and JETA = 8 (appendix B). The particular mean square that tested as significant at J = JA (statement 17) is printed with the label Z(JETA + 1) = 2.8594 E-02 (appendix B).

The subsequent columns are as follows: DATAID is the physical label given to the treatments producing the observations. Y(I) is the transformed input value resulting from the observations and is the actual value used in Yates' algorithm. I is the subscript identifying the regression coefficients (parameter estimates) as listed in Yates' order under the heading B(I), and J is the index of the mean squares as ordered in increasing order in the column headed Z(J). (The mean square associated with B(0), that is, the mean square associated with the grand mean, does not participate in the decision procedure and is not listed in the Z(J) column.)

The last column is the pointer array IND. It gives the subscript I of the regression coefficient that corresponds to the tabulated mean square. Thus, for the smallest mean square IND = 10 and so B(10) = 2.2307E-03 is the parameter estimate with smallest absolute magnitude. The number of mean squares concluded to be insignificant was $\hat{\eta}$ = JETA = 8. The smallest significant mean square was Z(J) = Z(JETA + 1) = 2.8594E-02, for which j = 9 and IND = 12. Thus, the smallest significant coefficient is B(12) = 4.2274E-02. All the other significant coefficients are found by continuing downward under the IND column, and their subscripts (as read from the IND column) are IND = 11, 14, 3, 4, 8, 2, in increasing order of significance.

The output as illustrated by appendix B has been summarized in table II.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 1, 1972,
501-21.

APPENDIX A

PROGRAM LISTING

```
C***********************************************************************
C     POGLMS      2**L EXPERIMENTS
C***********************************************************************
C     KODE = 0   ACTUAL Y VALUE USED          KODE = 1    LOG OF Y USED
C***********************************************************************
      DIMENSION DATAID(64), Y(64,6), NI(64), IND(64), RN(65), YA(64), B(
     164), S(64), Z(64), TB(64,10), ALPHA(11), YI(64,6)
      DATA (ALPHA(I),I=1,11)/0.001,0.002,0.005,0.01,0.025,0.05,0.10,
     10.25,0.50,0.75,1.0/
      DATA ((TB(I,J),J=1,10),I=1,24)/0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
     10.0,2.0000,1.99999,1.99997,1.99986,1.99917,1.99687,1.9877,1.923,1
     22.706,1.382,2.9976,2.9960,2.9904,2.9809,2.951,2.904,2.806,2.527,2.0
     3386,1.688,3.976,3.962,3.925,3.870,3.760,3.625,3.412,2.949,2.395,1.9
     4461,4.887,4.845,4.758,4.65,4.44,4.21,3.89,3.287,2.658,2.184,5.74,5.
     5503,5.46,5.31,4.99,4.68,4.26,3.57,2.893,2.371,6.51,6.33,6.12,5.87,5
     66.46,5.09,4.61,3.83,3.11,2.54,7.20,6.90,6.65,6.35,5.88,5.44,4.91,4.
     7706,3.79,2.69,7.81,7.52,7.10,6.76,6.26,5.75,5.17,4.27,3.45,2.82,8.3
     884,8.01,7.53,7.17,6.59,6.03,5.41,4.45,3.60,2.95,8.82,8.44,7.95,7.53
     99,6.89,6.28,5.61,4.62,3.74,3.07,9.20,8.84,8.33,7.87,7.13,6.50,5.81,
     A4.77,3.87,3.17,9.67,9.21,8.68,8.18,7.37,6.71,5.99,4.92,3.99,3.27,1
     B0.05,9.55,8.95,8.42,7.59,6.91,6.15,5.05,4.10,3.37,10.43,9.86,9.20,
     C8.66,7.79,7.07,6.30,5.17,4.20,3.46,10.72,10.14,9.43,8.83,7.96,7.23
     D6.44,5.29,4.30,3.55,11.01,10.40,9.64,9.00,8.12,7.38,6.57,5.40,4.3
     E9,3.63,11.28,10.64,9.84,9.17,8.28,7.52,6.69,5.50,4.48,3.70,11.53,1
     F0.86,10.03,9.34,8.43,7.65,6.81,5.60,4.56,3.77,11.76,11.07,10.22,9.
     G51,8.58,7.78,6.92,5.69,4.64,3.84,11.98,11.28,10.40,9.67,8.72,7.90,
     H7.03,5.78,4.71,3.90,12.19,11.46,10.56,9.83,8.86,8.02,7.13,5.87,4.7
     I8,3.96,12.39,11.66,10.70,9.99,8.99,8.13,7.23,5.95,4.85,4.02,12.58,
     J11.87,10.95,10.14,9.12,8.24,7.33,6.03,4.92,4.08/
      DATA ((TB(I,J),J=1,10),I=25,47)/12.76,12.05,11.10,10.29,9.23,8.34,
     17.42,6.11,4.98,4.14,12.93,12.22,11.26,10.43,9.34,8.44,7.51,6.18,5.
     204,4.19,13.09,12.38,11.41,10.56,9.44,8.54,7.60,6.25,5.10,4.24,13.2
```

```
     34,12.53,11.55,10.68,9.54,8.63,7.68,6.32,5.16,4.30,13.39,12.68,11.0
     48,10.78,9.64,8.72,7.70,6.38,5.22,4.35,13.53,12.62,11.80,10.88,9.74
     58.81,7.83,6.44,5.28,4.40,13.67,12.96,11.91,10.98,9.83,8.89,7.90,6
     6.50,5.33,4.45,13.80,13.09,12.01,11.07,9.91,8.97,7.97,6.56,5.38,4.5
     70,13.93,13.21,12.16,11.16,9.99,9.04,8.04,6.62,5.43,4.54,14.05,13.3
     82,12.19,11.25,10.07,9.11,8.11,6.68,5.48,4.58,14.17,13.43,12.27,11.
     934,10.15,9.18,8.17,6.74,5.53,4.62,14.29,13.53,12.35,11.43,10.22,9.
     A25,8.23,6.80,5.58,4.66,14.41,13.63,12.43,11.51,10.29,9.31,8.29,6.8
     B5,5.63,4.70,14.53,13.73,12.51,11.59,10.36,9.37,8.35,6.90,5.67,4.74
     C,14.64,13.82,12.59,11.67,10.43,9.43,8.41,6.95,5.71,4.78,14.75,13.9
     D1,12.67,11.75,10.50,9.49,8.46,6.99,5.75,4.82,14.85,14.00,12.75,11.
     E83,10.57,9.55,8.51,7.03,5.79,4.86,14.95,14.09,12.83,11.90,10.64,9.
     F61,8.56,7.07,5.83,4.90,15.05,14.17,12.90,11.97,10.70,9.67,8.61,7.1
     G1,5.87,4.94,15.15,14.25,12.97,12.04,10.76,9.72,8.66,7.15,5.91,4.98
     H,15.24,14.33,13.05,12.11,10.82,9.77,8.71,7.19,5.95,5.01,15.33,14.4
     I0,13.12,12.18,10.88,9.82,8.76,7.23,5.99,5.04,15.42,14.47,13.19,12.
     J25,10.94,9.87,8.81,7.27,6.03,5.07/
      DATA((TB(I,J),J=1,10),I=48,64)/15.50,14.54,13.26,12.32,11.00,9.92,
     18.85,7.31,6.07,5.10,15.58,14.60,13.32,12.38,11.06,9.97,8.89,7.35,6
     2.11,5.13,15.66,14.66,13.38,12.44,11.11,10.02,8.93,7.39,6.14,5.16,1
     35.73,14.72,13.44,12.50,11.16,10.07,8.97,7.43,6.17,5.19,15.80,14.79
     4,13.50,12.56,11.21,10.12,9.01,7.47,6.20,5.22,15.87,14.85,13.56,12.
     562,11.26,10.17,9.05,7.51,6.23,5.25,15.93,14.91,13.62,12.68,11.31,1
     60.21,9.09,7.55,6.26,5.28,15.99,14.97,13.67,12.73,11.36,10.25,9.13,
     77.59,6.29,5.31,16.05,15.03,13.72,12.78,11.40,10.29,9.17,7.63,6.32,
     85.34,16.11,15.10,13.77,12.83,11.44,10.33,9.21,7.67,6.35,5.37,16.17
     9,15.16,13.82,12.88,11.48,10.37,9.25,7.70,6.38,5.40,16.23,15.22,13.
     A87,12.93,11.52,10.41,9.29,7.73,6.41,5.43,16.29,15.28,13.92,12.97,1
     B1.56,10.45,9.33,7.76,6.44,5.46,16.34,15.34,13.97,13.01,11.60,10.49
     C,9.37,7.79,6.47,5.48,16.39,15.40,14.02,13.05,11.64,10.53,9.41,7.82
     D,6.50,5.50,16.44,15.46,14.06,13.09,11.67,10.57,9.45,7.85,6.53,5.52
     E,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0/
C***********************************************************************
C       READ INPUT
C***********************************************************************
        READ (5,24) KODE,L
        KT=2**L
        DO 3 I=1,KT
        READ (5,27) DATAID(I),NY,(Y(I,K),K=1,NY)
        DO 1 K=1,NY
1       YI(I,K)=Y(I,K)
        NI(I)=NY
        YA(I)=0.0
        DO 2 K=1,NY
        XNY=NY
```

```
      IF (KODE.LT.1) GO TO 2
      Y(I,K)=ALOG10(Y(I,K))
2     YA(I)=Y(I,K)/XNY+YA(I)
3     CONTINUE
4     READ (5,24) M,KP,KF
      KTM1=KT-1
      KTM2=KT-2
      FKT=KT
      MP1=M+1
      DO 5 I=1,KT
5     RN(I)=YA(I)
      WRITE (6,26) L,KODE,M,ALPHA(KP),ALPHA(KF)
      WRITE (6,33)
      DO 6 I=1,KT
      NY=NI(I)
      WRITE (6,28) (YI(I,K),K=1,NY)
6     CONTINUE
C***********************************************************************
C     CALCULATE MEAN SQUARES BY YATES METHOD
C***********************************************************************
      NN=KT/2
      DO 8 J=1,L
      DO 7 I=1,KT,2
      IP1D2=(I+1)/2
      S(IP1D2)=RN(I+1)+RN(I)
      LL=IP1D2+NN
7     S(LL)=RN(I+1)-RN(I)
      DO 8 K=1,KT
8     RN(K)=S(K)
      DO 9 K=1,KT
9     B(K)=RN(K)/FKT
      DO 10 I=1,KTM1
      IND(I)=I
10    Z(I)=S(I+1)*S(I+1)/FKT
C***********************************************************************
C     ORDER IN INCREASING MAGNITUDE
C***********************************************************************
      DO 13 J=1,KTM2
      TEST=Z(KTM1)
      IN=KTM1
      DO 12 N=J,KTM2
      IF (TEST-Z(N)) 12,12,11
11    TEST=Z(N)
      IN=N
```

```
12    CONTINUE
      ITEM=IND(IN)
      TEM=Z(IN)
      IND(IN)=IND(J)
      Z(IN)=Z(J)
      IND(J)=ITEM
13    Z(J)=TEM
C*********************************************************************
C     DO HYPOTHESIS TESTING
C*********************************************************************
      N=MP1
      TEM=0
      DO 15 J=1.M
15    TEM=TEM+Z(J)
      DO 18 J=MP1.KTM1
      FN=N
      TEST=FN*Z(J)/(TEM+Z(J))
      IF(KP.EQ.11) GO TO 17
      IF(TEST-TB(N.KP)) 16.16.17
16    TEM=TEM+Z(J)
      N=N+1
      GO TO 18
17    IF (TEST-TB(N.KP)) 18.18.19
18    CONTINUE
      JA=KT
      GO TO 20
19    JA=J
20    JETA=JA-1
      IRHO=KT-JETA-1
C*********************************************************************
C     PRINT OUTPUT
C*********************************************************************
      WRITE (6.32) IRHO.JETA.Z(JA)
21    WRITE (6.29)
22    IM1=0
      WRITE (6.31) DATAID(1).YA(1).IM1.B(1)
      DO 23 I=2.KT
      IM1=I-1
      WRITE (6.31) DATAID(I).YA(I).IM1.B(I).Z(IM1).IND(IM1)
23    CONTINUE
      GO TO 4
C*********************************************************************
C     FORMAT STATEMENTS
C*********************************************************************
```

```
24     FORMAT (3I5)
25     FORMAT (10F5.0)
26     FORMAT (1H139X,27H         CALCULATIONS  -- 2**L/1HL9X,3H2**I1,11H EX
      1PERIMENT,10X,6HKODE =I2//10X,3HM =I2,15X,9HALPHA,P =F6.3,7X,9HALPH
      2A,F =F6.3//)
27     FORMAT (A6,I6,6F6.0)
28     FORMAT (1X,1P10E13.4)
29     FORMAT (1H0,6HDATAID,7X,4HY(I),7X,3HI,J,7X,4HB(I),11X,4HZ(J),10X,6
      1HIND(J))
31     FORMAT (A6,1PE15.4,I6,1PE15.4,1PE15.4,I10)
32     FORMAT (1H0,6X,6HIRHO= ,I3,9X,6HJETA= ,I5,9X,11HZ(JETA+1)= ,1PE13.
      14)
33     FORMAT (1H0,4X,4HY(1),9X,4HY(2),9X,4HY(3),9X,4HY(4),9X,4HY(5),9X,4
      1HY(6))
       END
```

APPENDIX B
ILLUSTRATIVE OUTPUT

CALCULATIONS -- 2**L

2**4 EXPERIMENT KODE = 1

M = 1 ALPHA.P = 0.250 ALPHA.F = 0.010

Y(1)	Y(2)	Y(3)	Y(4)	Y(5)	Y(6)
1.7510E+02	1.9940E+02				
8.3200E+01	1.6650E+02				
2.2900E+01	2.4500E+01				
1.4700E+01	2.1100E+01				
1.5350E+02	2.3760E+02				
1.1950E+02	1.2960E+02				
2.8200E+01	3.9000E+01				
3.0000E+01	3.6100E+01				
5.5100E+01	7.9200E+01				
2.9200E+01	4.7000E+01				
3.5000E+00	1.1100E+01				
1.7700E+01	1.5600E+01				
1.3210E+02	1.5070E+02				
9.4100E+01	9.5100E+01				
1.2700E+01	1.9100E+01				
1.6700E+01	1.6800E+01				

IRHO= 7 JETA= 8 Z(JETA+1)= 2.8594E-02

LATAIC	Y(I)	I.J	B(I)	Z(J)	IND(J)
(1)	2.2715E+00	0	1.6522E+00		
AE	2.0706E+00	1	-2.9749E-02	7.9617E-05	10
BE	1.3745E+00	2	-3.6526E-01	1.0263E-04	6
AB	1.2458E+00	3	7.6084E-02	4.4075E-03	5
CE	2.2810E+00	4	1.0016E-01	7.5673E-03	7
AC	2.0950E+00	5	-1.6597E-02	1.4160E-02	1
BC	1.5207E+00	6	-2.5326E-03	1.8099E-02	9
ABCE	1.5290E+00	7	-2.1747E-02	2.0246E-02	13
DE	1.8199E+00	8	-1.4636E-01	2.1949E-02	15
AD	1.5687E+00	9	3.3654E-02	2.8594E-02	12
BD	7.5470E-01	10	-2.7307E-03	3.2091E-02	11
ABDE	1.2701E+00	11	4.4785E-02	4.3254E-02	14
CD	2.2006E+00	12	4.2274E-02	5.7554E-02	3
ACDE	1.9759E+00	13	-3.5574E-02	1.6051E-01	4
BCDE	1.1924E+00	14	-5.1994E-02	3.4275E-01	8
ABCD	1.2240E+00	15	-3.7038E-02	2.3502E+00	2

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TABLE I. - UPPER 100α PERCENT POINTS OF TEST STATISTIC U_j

Number of denominator mean squares, j	Nominal test size, α									
	0.001	0.002	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75
	Nominal test size index, KP or KF									
	1	2	3	4	5	6	7	8	9	10
2	2.00000	1.99999	1.99997	1.99986	1.99917	1.99687	1.9877	1.923	1.706	1.382
3	2.9976	2.9960	2.9904	2.9809	2.951	2.904	2.806	2.527	2.086	1.688
4	3.976	3.962	3.925	3.870	3.760	3.625	3.412	2.949	2.395	1.961
5	4.887	4.845	4.758	4.65	4.44	4.21	3.89	3.287	2.658	2.184
6	5.74	5.63	5.46	5.31	4.99	4.68	4.28	3.57	2.893	2.371
7	6.51	6.33	6.11	5.87	5.46	5.09	4.61	3.83	3.11	2.54
8	7.20	6.96	6.65	6.35	5.88	5.44	4.91	4.06	3.29	2.69
9	7.81	7.52	7.10	6.78	6.26	5.75	5.17	4.27	3.45	2.82
10	8.34	8.01	7.53	7.17	6.59	6.03	5.41	4.45	3.60	2.95
11	8.82	8.44	7.95	7.53	6.89	6.28	5.61	4.62	3.74	3.07
12	9.26	8.84	8.33	7.87	7.13	6.50	5.81	4.77	3.87	3.17
13	9.67	9.21	8.68	8.16	7.37	6.71	5.99	4.92	3.99	3.27
14	10.05	9.55	8.95	8.42	7.59	6.91	6.15	5.05	4.10	3.37
15	10.40	9.86	9.20	8.66	7.79	7.07	6.30	5.17	4.20	3.46
16	10.72	10.14	9.43	8.83	7.96	7.23	6.44	5.29	4.30	3.55
17	11.01	10.40	9.64	9.00	8.12	7.38	6.57	5.40	4.39	3.63
18	11.28	10.64	9.84	9.17	8.28	7.52	6.69	5.50	4.48	3.70
19	11.53	10.86	10.03	9.34	8.43	7.65	6.81	5.60	4.56	3.77
20	11.76	11.07	10.22	9.51	8.58	7.78	6.92	5.69	4.64	3.84
21	11.98	11.28	10.40	9.67	8.72	7.90	7.03	5.78	4.71	3.90
22	12.19	11.48	10.58	9.83	8.86	8.02	7.13	5.87	4.78	3.96
23	12.39	11.68	10.76	9.99	8.99	8.13	7.23	5.95	4.85	4.02
24	12.58	11.87	10.93	10.14	9.12	8.24	7.33	6.03	4.92	4.08
25	12.76	12.05	11.10	10.29	9.23	8.34	7.42	6.11	4.98	4.14
26	12.93	12.22	11.26	10.43	9.34	8.44	7.51	6.18	5.04	4.19
27	13.09	12.38	11.41	10.56	9.44	8.54	7.60	6.25	5.10	4.24
28	13.24	12.53	11.55	10.68	9.54	8.63	7.68	6.32	5.16	4.30
29	13.39	12.68	11.68	10.78	9.64	8.72	7.76	6.38	5.22	4.35
30	13.53	12.82	11.80	10.88	9.74	8.81	7.83	6.44	5.28	4.40
31	13.67	12.96	11.91	10.98	9.83	8.89	7.90	6.50	5.33	4.45
32	13.80	13.09	12.01	11.07	9.91	8.97	7.97	6.56	5.38	4.50
33	13.93	13.21	12.10	11.16	9.99	9.04	8.04	6.62	5.43	4.54
34	14.05	13.32	12.19	11.25	10.07	9.11	8.11	6.68	5.48	4.58
35	14.17	13.43	12.27	11.34	10.15	9.18	8.17	6.74	5.53	4.62

TABLE I. - Concluded. UPPER 100α PERCENT POINTS OF TEST STATISTIC U_j

Number of denominator mean squares, j	Nominal test size, α									
	0.001	0.002	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75
	Nominal test size index, KP or KF									
	1	2	3	4	5	6	7	8	9	10
36	14.29	13.53	12.35	11.43	10.22	9.25	8.23	6.80	5.58	4.66
37	14.41	13.63	12.43	11.51	10.29	9.31	8.29	6.85	5.63	4.70
38	14.53	13.73	12.51	11.59	10.36	9.37	8.35	6.90	5.67	4.74
39	14.64	13.82	12.59	11.67	10.43	9.43	8.41	6.95	5.71	4.78
40	14.75	13.91	12.67	11.75	10.50	9.49	8.46	6.99	5.75	4.82
41	14.85	14.00	12.75	11.83	10.57	9.55	8.51	7.03	5.79	4.86
42	14.95	14.09	12.83	11.90	10.64	9.61	8.56	7.07	5.83	4.90
43	15.05	14.17	12.90	11.97	10.70	9.67	8.61	7.11	5.87	4.94
44	15.15	14.25	12.97	12.04	10.76	9.72	8.66	7.15	5.91	4.98
45	15.24	14.33	13.05	12.11	10.82	9.77	8.71	7.19	5.95	5.01
46	15.33	14.40	13.12	12.18	10.88	9.82	8.76	7.23	5.99	5.04
47	15.42	14.47	13.19	12.25	10.94	9.87	8.81	7.27	6.03	5.07
48	15.50	14.54	13.26	12.32	11.00	9.92	8.85	7.31	6.07	5.10
49	15.58	14.60	13.32	12.38	11.06	9.97	8.89	7.35	6.11	5.13
50	15.66	14.66	13.38	12.44	11.11	10.02	8.93	7.39	6.14	5.16
51	15.73	14.72	13.44	12.50	11.16	10.07	8.97	7.43	6.17	5.19
52	15.80	14.79	13.50	12.56	11.21	10.12	9.01	7.47	6.20	5.22
53	15.87	14.85	13.56	12.62	11.26	10.17	9.05	7.51	6.23	5.25
54	15.93	14.91	13.62	12.68	11.31	10.21	9.09	7.55	6.26	5.28
55	15.99	14.97	13.67	12.73	11.36	10.25	9.13	7.59	6.29	5.31
56	16.05	15.03	13.72	12.78	11.40	10.29	9.17	7.63	6.32	5.34
57	16.11	15.10	13.77	12.83	11.44	10.33	9.21	7.67	6.35	5.37
58	16.17	15.16	13.82	12.88	11.48	10.37	9.25	7.70	6.38	5.40
59	16.23	15.22	13.87	12.93	11.52	10.41	9.29	7.73	6.41	5.43
60	16.29	15.28	13.92	12.97	11.56	10.45	9.33	7.76	6.44	5.46
61	16.34	15.34	13.97	13.01	11.60	10.49	9.37	7.79	6.47	5.48
62	16.39	15.40	14.02	13.05	11.64	10.53	9.41	7.82	6.50	5.50
63	16.44	15.46	14.06	13.09	11.67	10.57	9.45	7.85	6.53	5.52

TABLE II. - ILLUSTRATIVE EXAMPLE OF ONE-HALF 2^5 DESIGN EXPERIMENT

i, j	Treatment levels	Responses, y_i	Parameters	Parameter estimates,[a] $\hat{\mu}_i$	Mean squares, Z_j	Significance point of view					
						First analysis with $(m, \alpha_p, \alpha_f) = 1, 0.25, 0.01$			Second analysis with $(m, \alpha_p, \alpha_f) = 5, 1.0, 0.01$		
						U_j(exper.)	$U_j(\alpha_p)$	$U_j(\alpha_f)$	U_j(exper.)	$U_j(\alpha_p)$	$U_j(\alpha_f)$
0	(1)	2.2715	μ_0	1.6522							
1	ae	2.0708	μ_1	-.0297	0.000079						
2	be	1.3745	μ_2	-.3833	.000103	1.1319	1.923	1.99986			
3	ab	1.2458	μ_{12}	.0781	.004408	[b]2.8810	2.527	2.9809			
4	ce	2.2810	μ_3	.1002	.007567	2.9295					
5	ac	2.0950	μ_{13}	-.0166	.014160	2.9619					
6	bc	1.5207	μ_{23}	-.0025	.018099	2.9701			2.4449		5.31
7	abce	1.5290	$-\mu_{45}$	-.0217	.020248	2.9733			2.6090		
8	de	1.8199	μ_4	-.1464	.021949	2.9753			2.7285		
9	ad	1.5687	μ_{14}	.0336	.028594	[c]2.9810			3.1244		
10	bd	.7947	μ_{24}	-.0022	.032091				3.2966		
11	abde	1.2701	$-\mu_{35}$	.0448	.043254				3.7303		
12	cd	2.2006	μ_{34}	.0423	.097554				4.7253		
13	acde	1.9759	$-\mu_{25}$	-.0356	.160510				5.1548		
14	bcde	1.1924	$-\mu_{15}$	-.0520	.342750				[c]5.5722		
15	abcd	1.2240	$-\mu_5$	-.0370	2.350200						

[a] $\hat{\mu}_i$ are contrasts divided by 2^{ℓ}.
[b] Significant at level α_p.
[c] Significant at level α_f.

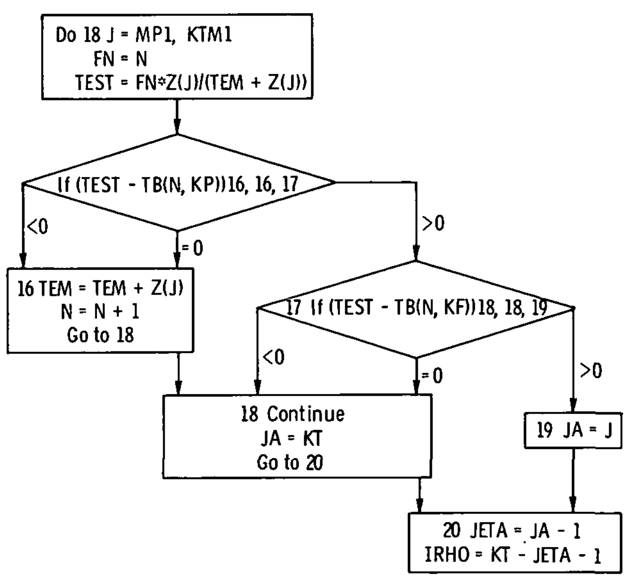
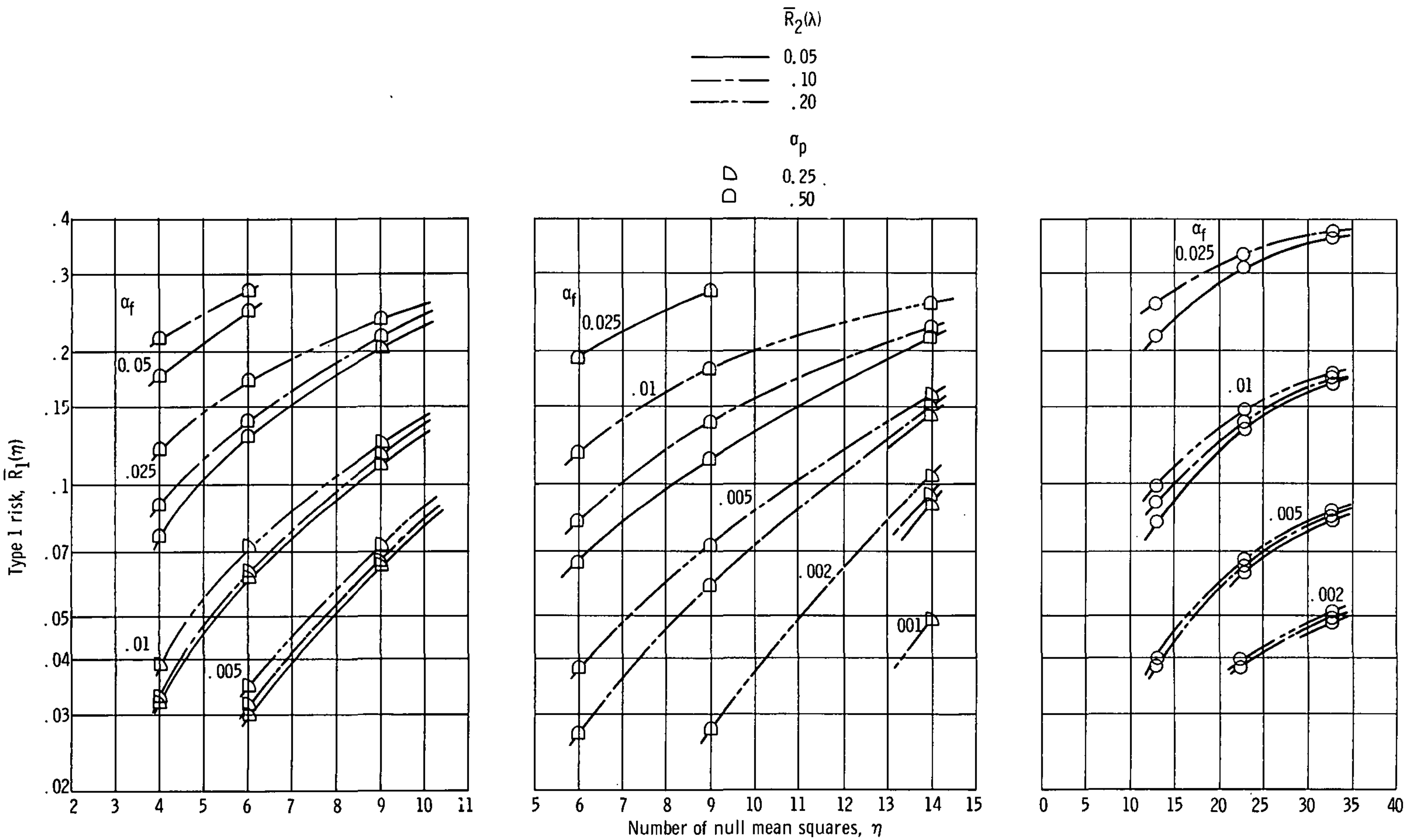


Figure 1. - Flow chart for tests of hypotheses.

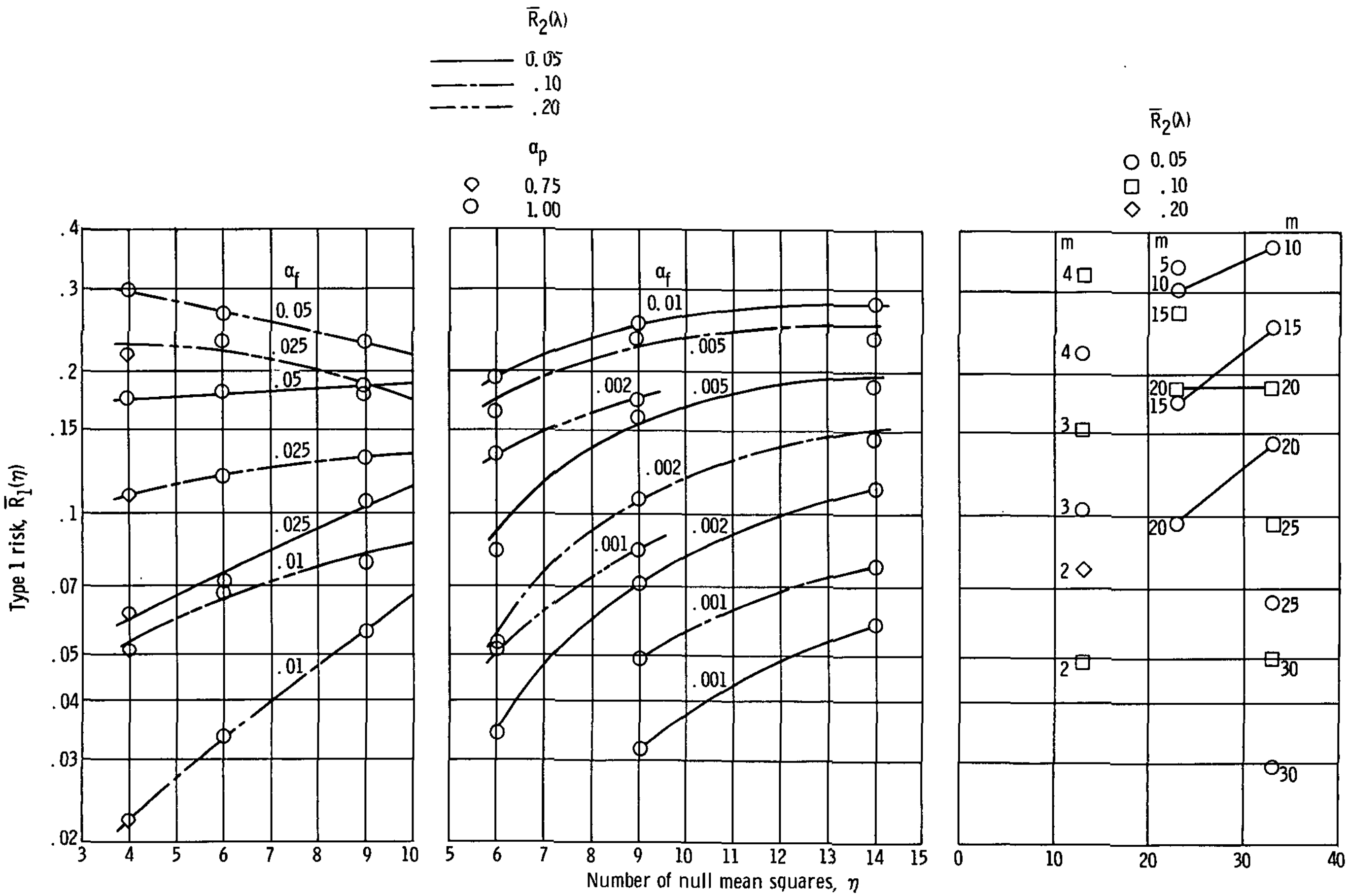


(a) $\ell = 4$.

(b) $\ell = 5$.

(c) $\ell = 6$, $\alpha_p = 1$.

Figure 2. - Curves for control of type 1 risks with $m = 1$ and preferred α_p.



(a) ℓ = 4.

(b) ℓ = 5, α_p = 1.0.

(c) ℓ = 6, α_p = 1.0, α_f = 0.001.

Figure 3. - Type 1 risks with preferred m and α_p.

1–6	7–12	13–18	19–24	Card type	FORMAT
1	4			1	(3I5)
(1)	2	175.10	199.40	2	(A6, I6, 6F6.0)
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BE	2	22.90	24.50		
AB	2	14.70	21.10		
CE	2	153.50	237.60		
AC	2	119.50	129.60		
BC	2	28.20	39.00		
ABCE	2	30.00	38.10		
DE	2	55.10	79.20		
AD	2	29.20	47.00		
BD	2	3.50	11.10		
ABDE	2	17.70	19.60		
CD	2	132.10	190.70		
ACDE	2	94.10	95.10		
BCDE	2	12.70	19.10		
ABCD	2	16.70	16.80		
1	8	4		3	3I5

Figure 4. - Input format - POOLMS.

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