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APPLICATION OF A HYBRID COMPUTER TO SWEEP FREQUENCY DATA PROCESSING

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16. Abstract

Presented is a hybrid computer program which can process as many as 10 channels of sweep frequency data simultaneously. The program needs only the sine sweep signal used to drive the system, and its corresponding quadrature component, to process the data. It can handle a maximum frequency range of 0.5 to 500 hertz. Magnitude and phase are calculated at logarithmically spaced points covering the frequency range of interest. When the sweep is completed, these results are stored in digital form. Thus, a tabular listing and/or a plot of any processed data channel or the transfer function relating any two of them is immediately available.

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SUMMARY

The hybrid computer is ideally suited for processing large amounts of sweep frequency data. The hybrid contains an analog computer, a digital computer, and interface equipment between them combined into one integrated unit. This not only allows data processing in both analog and digital form, but also makes information from one computer accessible to the other. Thus, final results are available in digital form without the need for an external digitizing process.

This report presents a hybrid computer program which can process as many as 10 channels of sweep frequency data simultaneously on each pass through the computer. The program needs only the sine sweep signal used to drive the system, and its corresponding quadrature component, to obtain frequency, magnitude, and phase of each data signal. It will handle a maximum frequency range of 0.5 to 500 hertz. Magnitude and phase will be calculated at logarithmically spaced points to cover the frequency range of interest. When the sweep is completed, a tabular listing and/or plot of any processed data channel or the transfer function relating any two of them is immediately available.

INTRODUCTION

Frequency response techniques are fundamental to system dynamic analysis. However, the determination of the frequency responses for a complex system can be a time-consuming task. In the past, frequency response testing was largely performed through sinusoidal testing at discrete frequencies. More recent approaches, however, have used sweep frequency inputs in place of discrete frequency testing (ref. 1). Although the sweep techniques significantly reduce the required test time, they do not substantially ease the problems associated with the reduction of the test data. This report presents hybrid computer techniques which greatly facilitate the data processing associated with sweep frequency testing.

In the past, raw recorded data from a test were processed on an analog computer to obtain signals corresponding to the real and imaginary parts of the data signals. If digital plots were desired, these signals had to be digitized before the plots could be generated. The handling and checking of several data tapes in the digitizing process can make that procedure tedious and time consuming, especially for the reduction of large amounts of data.

This procedure can be accomplished very quickly with a hybrid computer because this computer is ideally suited for processing large amounts of sweep frequency data. The hybrid contains an analog computer, a digital computer, and interface equipment combined into one integrated unit. Such a system allows data processing in both analog and digital form and also allows the rapid transfer of data from one form to the other. The analog computer is used to condition the data signals and, by filtering and manipulating, obtains for the digital computer that portion of each data signal which varies only with the slowly changing frequency used to drive the system. Meanwhile, the digital computer is used to obtain the magnitude and phase of the data, thus eliminating the need for an external digitizing process.

This report describes a program for processing sweep frequency data using a hybrid computer. The basic analog circuits presented are not new and have been in use for some time. However, the use of these techniques in conjunction with a hybrid computer is new. Reference 1 presents some of the theory of sweep frequency testing. The hybrid computer stores frequency, magnitude, and phase for each data signal. Hence, when the sweep is completed, any processed data channel or the transfer function relating any two of them may be examined. This program will process as many as 10 data channels simultaneously, with a maximum frequency range of 0.5 to 500 hertz. The user may request as many as 100 logarithmically spaced points to cover the frequency range of interest.

With this program, the hybrid computer could also be used to process as many as 10 selected data signals directly as a test is being conducted. While all the data signals of interest are being recorded, the hybrid computer can be calculating and storing frequency, magnitude, and phase of the selected signals. Thus, tabular listings and/or analog plots of the desired transfer functions can be available minutes after the completion of the test.

The program was implemented on an Electronic Associates, Inc., 690 Hybrid Computer using two analog consoles. The FORTRAN IV source listings of the program are provided in appendix A. The only statements that are specific to the computer that was used are the calls to the hybrid linkage subroutines. These subroutines have names beginning with the letter "Q" and are used to transmit information between the analog and digital portions of the computer. In general, the hybrid linkage subroutines are specific to the equipment being used. The analog circuits and digital subroutines that are presented can be used with any hybrid computer system provided the proper hybrid linkage calls are used.

Calculations for the magnitude and phase of the sweep frequency data are presented in appendix B. All symbols are defined in appendix C.

APPLICATION AND DISCUSSION

In the past, the processing of sweep frequency test data has been a time-consuming task. One technique would be to use only an analog computer to process the data. The engineer would obtain on-line plots of the frequency responses needed. This approach can be perfectly satisfactory, especially if only a small amount of data is to be processed. However, the raw data tape must be played through the recorder for each frequency response plot. Also, an operator is required at all times to operate the necessary equipment.

A second technique available to the engineer would be to process his data on an analog computer obtaining the frequency and the real and imaginary parts of the data signals. These results could be digitized and a digital computer and plotter used to obtain transfer functions.

This is a better approach if there is much data to process and many transfer functions are desired. Once the digital plotter is set up, it will operate by itself. Thus, the engineer or operator does not have to be present the whole time the plots are being made. But, a tedious part to this approach may be the digitizing procedure. It may require several operations involving the handling and checking of tapes.

Using the hybrid computer eliminates the digitizing requirement for it is done automatically at the computer interface. Hence, the raw data can be processed very quickly. Analog plots and tabular listings of transfer functions may be obtained on line. Also the engineer can take advantage of using a digital plotter to obtain the frequency responses he desires.

PROGRAM DESCRIPTION

The hybrid program is discussed in two parts. Analog circuit diagrams and explanation of their use constitute the first part. The second part consists of a brief description of each digital subroutine. Finally, an example using the entire hybrid program is presented.

Analog Portion of Program

The hybrid computer program is capable of processing 10 channels of sinusoidal sweep frequency data. For a system perturbed by a sine wave driver, $A' \sin \theta(t) + \delta_a$, each output signal will be of the form $A'C \sin [\theta(t) + \psi] + \delta_{ac}$. (All symbols are defined in appendix C.) To obtain the magnitude and phase of the sweep frequency data, both the sine wave driver and its corresponding quadrature signal, $B' \cos [\theta(t) + \varphi] + \delta_b$, must be recorded along with the data of interest. A later section of this report (Period determination) describes a technique for determining frequency from the sine wave driver.

Signal conditioning. - Each data signal, the sine wave driver, and its quadrature signal undergo conditioning in the analog computer prior to entering the digital computer by way of analog-to-digital converters (ADC's). This conditioning, shown schematically in figure 1, consists of amplifying, filtering, and multiplying signals.

Each signal is passed through a first-order, high-pass filter to remove any bias (δ_a, δ_b, δ_{ac}) that might exist. Figure 1(a) shows the analog circuit used to accomplish this filtering. A time constant τ of 5 seconds was used for the processing of jet engine test data at the Lewis Research Center. This time constant provides a cutoff frequency ω_c of 0.2 radian per second. An amplification factor K_1 is included in the filter to provide reasonable signal levels in the computer. Hence, the outputs of the sine, cosine, and data filters are $A \sin \theta(t)$, $B \cos [\theta(t) + \varphi]$, and $AC \sin [\theta(t) + \psi]$, respectively, where $A = K_1 A'$ and $B = K_1 B'$.

Appendix B presents a detailed account of the calculations for the magnitude and phase of the sweep frequency data. Those calculations require forming the following product signals:

$$\text{DRIVER}*\text{DRIVER} = [A \sin \theta(t)]^2 \tag{1}$$

$$\text{QUAD}*\text{QUAD} = \{B \cos [\theta(t) + \varphi]\}^2 \tag{2}$$

$$\text{DRIVER}*\text{QUAD} = A \sin \theta(t) * B \cos [\theta(t) + \varphi] \tag{3}$$

$$\text{DRIVER}*\text{OUTPUT} = A \sin \theta(t) * AC \sin [\theta(t) + \psi] \tag{4}$$

$$\text{QUAD}*\text{OUTPUT} = B \cos [\theta(t) + \varphi] * AC \sin [\theta(t) + \psi] \tag{5}$$

The analog circuit for forming the DRIVER*OUTPUT and QUAD*OUTPUT signals is shown in figure 1(b). It consists simply of two multipliers. The data signal comes out of the high-pass filter and is mixed - that is, multiplied - by the filtered driver and quadrature signals to yield the desired products.

The product signals given by equations (1) to (5) are each passed through a second-order, low-pass filter, as shown in figure 1(c). The filter attenuates the second harmonic terms associated with equations (1) to (5), in addition to removing any unwanted frequencies (such as 60 Hz) that might be present. The natural frequency ω_n of the filter used was 1.414 radians per second. The damping ratio ζ was 0.707. An amplification factor K_2 of 2 was included in the filter to remove the attenuation of 1/2 resulting from the previous mixing of signals.

The resulting two signals, $A^2C \cos \psi$ and $ABC \sin (\psi - \varphi)$, correspond to the real and imaginary parts, respectively, of the data signal. These signals are converted in the ADC's and are transferred to the digital computer for further processing.

The sine wave driver and its quadrature signal are each squared and multiplied together, as shown in figure 1(d), to form the products given in equations (1) to (3). These signals are also passed through second-order, low-pass filters similar to that shown in figure 1(c). For these filters, the natural frequency ω_n was 0.3535 radian per second and the damping ratio ζ was 0.707. The amplification factor K_2 was equal to 2 for the squared signals but was increased to 20 for the sine-cosine product signal since this product is always very small. The resultant outputs from the filters are A^2, $10AB \sin \varphi$, and B^2. These signals are transmitted through ADC's to the digital computer, where they are used to obtain the magnitude C and phase ψ of each data signal being processed.

Period determination. - In order to determine the actual frequency of the driving function, the analog computer is used to calculate a scaled representation of the driving function's period. This scaled period signal is then converted in an ADC to a digital value which the digital computer unscales and inverts to form the frequency. The first step in the period calculation is to form a square wave using the filtered sine wave driver $A \sin \theta(t)$. The square wave is essential for sharp zero crossings. In some cases, depending on the signal quality and the computer used, it may be necessary to precede the square wave generating circuit by a low-pass filter to attenuate any noise or spikes which might otherwise result in erroneous zero crossings.

The circuits used for period determination are shown in figure 2. The circuit used to form the square wave consists of two high-gain limiting amplifiers in series; it is illustrated, along with a first-order, low-pass filter, in figure 2(a).

As shown in figure 2(b), the square wave output of the second limiting amplifier is connected to an analog comparator where the square wave is compared to signal ground. The comparator is thus used to form a logic-level square wave. In turn, the comparator output is connected to a logic differentiator which outputs a pulse on each positive-going change of the comparator - that is, a change from 0 to 1.

The differentiator output is connected to the trigger input of a flip-flop. The flip-flop will change state on each pulse from the differentiator. In other words, the flip-flop will be low (logic 0) for odd-numbered cycles of the driving function and high (logic 1) for even-numbered cycles.

As shown in figure 2(c), the normal and complimentary outputs of the flip-flop are used to cycle repetitively an integrator and a pair of track-and-store amplifiers (T/S amplifiers). The integrator is used to generate a ramp on the odd cycles of the driving function. And the integrator is reset to its initial value on the even cycles.

The control of the integrator modes - both initial condition (IC) and operate - comes from one of the flip-flop outputs. It is important to note at this point that the lower the frequency of the driving function, the longer the flip-flop remains in a given state, and the longer the integrator will generate a ramp. Hence, the length of the ramp is proportional to the period of the driving function.

The integrator output is connected to the input of one of the T/S amplifiers, which, in turn, is connected to the second T/S amplifier. These amplifiers function as a memory pair, storing the final value of each cycle of the integrator ramp. As shown in figure 2(c), the control of the T/S amplifiers is from the flip-flop outputs. Whenever the integrator is in the operate mode, the first T/S amplifier is in the track mode tracking the integrator output. The second T/S amplifier is in the store mode holding the previous cycle's final ramp value. When the integrator switches to the IC mode, the first T/S amplifier switches to the store mode to store the current ramp's final value. At this point the second T/S amplifier is in the track mode, where it updates its value to correspond to the final value of the just completed ramp. The output of T/S amplifier 2 is a voltage proportional to P, the period of the driving function, but delayed by one cycle of the driver.

Depending on the frequency - or in this case, the period - of the driving function, some scaling of the circuit presented in figure 2(c) would be required. As shown, the lowest frequency that could be handled without overloads is 1 hertz. If the test data started at 0.5 hertz, a potentiometer set at 0.5 could be inserted before the integrator. The integrator rate would then be 0.5 times reference volts per second. The output of the circuit would then be proportional to P/2.

But to take advantage of the full voltage range of the analog computer, the recommended way to obtain the period for data starting at 0.5 hertz is to set the integrator initial condition at (-) reference and integrate at reference volts per second. The output of the circuit, T/S amplifier 2, would then be a voltage proportional to P - 1. Depending on the upper limit of the frequency - or, the lower limit of the period - this signal could be fed directly to an ADC or could be automatically rescaled to obtain a larger amplitude signal to connect to the ADC. If this is done, the digital computer must be made aware of the scale change, since it manipulates the period signal to obtain the driving function frequency.

Digital Portion of Program

The digital part of the program consists of a main program plus seven subroutines: SETUP, PROCES, MONITR, PUNCH, TYPIN, TYPOUT, and PNCH. A description of each of these routines and its function in the whole program follows. Particular attention is paid to answering the questions the user is asked at the teletype.

A flow diagram of the digital program is presented in figure 3.

MAIN program. - This program controls the complete digital part of the program. MAIN calls the various subroutines used in the program. After the execution of a subroutine is completed, control is returned to MAIN. Through the MAIN program, the user also informs the computer how he wants the final results: in tabular form, as an on-line plot, and/or output on paper tape.

Subroutine SETUP. - This subroutine obtains information needed to process the raw data on the recorder tapes. The user supplies the computer with the needed information by following a series of directions given him at the teletype. The teletype will wait for user's response.

ALL numeric answers must include a decimal point.

The directions are as follows:

(1) TYPE 3 LINES FOR DATA IDENTIFICATION.

The user types three lines of identification of his choice to be associated with the channels of data about to be processed. Each line can consist of up to 68 characters and is fed into the computer by pressing the RETURN key. If the user does not wish to use all three lines allotted him, pressing just the RETURN key will enter a blank line. Three lines must be used, however, even though some may be blank.

(2) NO. OF CHANNELS (MAX. 10).

The user types the number of data channels he wishes to process. As many as 10 data channels may be processed at a time.

(3) LIST TAPE RECORDER CHANNELS IN ORDER.

(a) CH. 1. The user types the recorder channel number of his raw data tape that he wishes to be associated with hybrid computer channel 1. (Assumed to be channel X in direction 3(b).)

(b) MAX. GAIN OF CH. X. The user types an upper bound for the maximum gain of raw data tape channel X (maximum resonance value/dc value). The value of this upper bound, though not critical, should be reasonably accurate if analog plots are desired. If the number entered is too small and it is not an upper bound, any resonance in the data will be clipped off at the value entered as an upper bound. If the number is much too large, the accuracy the analog computer is capable of will not be used to its fullest extent.

The computer will continue cycling through 3(a) and 3(b) until it has received the information for each channel the user wishes to process.

(4) CHECK THAT EACH RECORDER CHANNEL IS CONNECTED TO THE PROPER TRUNK. WHEN CHECKED, R-S-R.

The user is to make sure that the proper connections have been made between the tape recorder and the hybrid computer. The computer is now at a pause waiting for the trunk connections to be checked. The computer will not continue until it is physically restarted by the user. It is restarted by an operation symbolized by R-S-R. The notation R-S-R signifies the following operations at the digital control panel: (1) release EXECUTE RUN, (2) press EXECUTE SINGLE, and (3) depress EXECUTE RUN. For the remainder of this report, R-S-R (run-single-run) will signify this operation.

(5) STARTING FREQUENCY IN HERTZ? (>=0.5).

The user types the lowest frequency (in hertz) on his data tape that he is interested in. The number entered must be at least 0.5.

(6) MAXIMUM FREQUENCY IN HERTZ? (<=500).

The user types the highest frequency (in hertz) on his data tape that he is interested in. The number entered must be no larger than 500.

(7) NO. OF POINTS? (10<=PTS<=100).

The user types the number of points he wants per plot. The computer automatically selects frequencies equally spaced on a log scale. The number of points per plot must be at least 10, but not more than 100.

(8) THE PROGRAM IS INITIALIZED. WHEN THE SWEEP IS COMPLETED, SET SSW(A). NOW START SWEEP, THEN R-S-R.

The user now feeds the recorded raw data into the hybrid computer. This is done as follows:

(a) Start the recorder having the sweep data. The starting frequency on this tape should be held constant (no sweep) for about 5 seconds before the sweep begins. This will allow the filters on the analog computer to settle out and the period signal to become established.

(b) Place the analog computer in the operate mode.

(c) R-S-R on the digital control panel (see direction 4 above).

(d) When the tape data has all been read in, depress sense switch A (SSW A) on the digital control panel (with the tape recorder still running).

(e) Place the analog computer in the pot set mode.

(f) Turn off the tape recorder.

Subroutine PROCES. - This subroutine computes the frequency, magnitude, and phase of the raw data specified in subroutine SETUP. First, it calculates the frequency values needed for the Bodé plot to have equal log spacing. Subroutine PROCES continuously samples the period signal coming from the analog portion of the computer. Once the frequency determined from the period signal exceeds the desired frequency value, the frequency, magnitude, and phase of all the data channels are calculated and stored. (The details of these calculations are presented in appendix B.) The desired frequency value is then updated. This whole process is repeated, with the sweep frequency being compared with the new desired frequency. This process continues until the entire frequency range has been covered.

When PROCES returns control to MAIN, tables of frequency, magnitude, and phase of all the data channels to be processed have been stored in digital form.

Subroutine MONITR. - This subroutine allows the user to obtain the digital values of any processed data channel or the transfer function relating any two of them. The output of this subroutine is either an on-line plot or a digital tabular listing.

The user is given the following directions at the teletype:

(1) FOR AN AMPLITUDE RATIO, SET SSW(B). R-S-R.

If the user wishes to form an amplitude ratio of two channels of data which have been processed, he must depress sense switch B (SSW B) located on the digital control console. Then he must R-S-R. If the user does not desire an amplitude ratio, he need only R-S-R. If sense switch B is not set, the following message appears next.

(2) RECORDER CHANNEL TO MONITOR?

The user types the tape channel number of the data he is interested in seeing. If sense switch B was set in item 1 above, the following two messages will appear.

(3) CALCULATE (RECORDER CHANNEL B)/(RECORDER CHANNEL A). ENTER B.

The user types the tape channel number of the data to be used as the numerator in the amplitude ratio.

(4) ENTER A.

The user types the tape channel number of the data to be used as the denominator in the amplitude ratio.

(5) PLOT: SET SSW(C).
LISTING: SET SSW(D).
R-S-R.

If the user desires an on-line plot, he should depress sense switch C. If the user desires a tabular listing of the data, he should depress sense switch D. Then R-S-R. The tabular listing will consist of

(a) The user-supplied data identification printed on top of the page

(b) A listing of the tape channel number (channel B/channel A if an amplitude ratio is being printed), the data point number, the frequency, the magnitude, and the phase for the frequency range of interest

Phase angle is in degrees, and for frequencies greater than 1 hertz it is forced to be between -360° and 0°. Also, if an amplitude ratio was called for, the magnitude will be normalized to have a value of unity at the lowest frequency. The normalizing factor will be printed at the teletype when the tabular listing or on-line plot is completed.

Both sense switches C and D may be depressed if the user desires both an on-line plot and a tabular listing.

Subroutine PUNCH. - This subroutine punches a channel of processed data on paper tape. The data punched on tape are the same data that would have been printed at the teletype had a tabular listing been requested.

First, the data identification the user supplied is punched. The remaining tape consists of the proper number of data sets - that is, data channel number, data point number, frequency, magnitude, and phase - for the frequency range the user had specified in subroutine SETUP.

Subroutine PUNCH prints the following directions at the teletype:

(1) TURN ON THE HSPT PUNCH. THEN R-S-R.

Upon receiving this message, the user is to turn on the high-speed paper-tape punch. After the punch is turned on, he is to R-S-R.

(2) RECORDER CHANNEL TO PUNCH?

The user types the tape channel number of the processed data he wishes to punch.

Subroutines TYPIN, TYPOUT, and PNCH. - In subroutine SETUP the user is asked to supply three lines of identification to be associated with the processed data. These subroutines accept at the teletype, write at the teletype, and punch on paper tape, respectively, that identification information.

Source listings of all these subroutines as used at Lewis are presented in appendix A.

PROGRAM OPERATION

Assume the sweep frequency tests have been conducted and the data of interest have been recorded on tape, along with both the sine signal used to drive the system and its corresponding cosine signal. The analog portion of the computer has been patched so that the period signal and the filtered product signals needed to obtain the magnitude and phase of the data are wired to ADC's. And finally, the digital program has been loaded into the computer and the directions given at the teletype have been followed.

A message at the teletype will inform the user that the program has been initialized and that he should start the tape recorder and switch the analog part of the computer to OPERATE.

While the raw data tape is being played, the analog computer is updating the period (1/frequency) every other cycle of the sine wave driver.

At the same time, the digital computer is comparing the frequency value from the analog computer with the next frequency point at which magnitude and phase are desired. As soon as the analog signal exceeds this value, the digital computer reads all the ADC's. Using these ADC values, it calculates magnitude and phase for each data channel and stores them away, along with the corresponding frequency value.

The digital computer then updates to the next test frequency point and compares it with the values coming from the analog part of the computer. This process continues until the sweep has been completed. Depressing sense switch A on the digital control panel signals the digital computer that the sweep is completed and to continue with the program. The tape recorder may now be stopped and the analog computer may be returned to POT SET. The frequency, magnitude, and phase of each data channel are now stored in the digital computer.

When sense switch A is depressed at the end of the sweep, a message at the teletype will ask the user whether he wishes to examine the magnitude and phase of a particular tape recorder channel or to obtain a frequency response. Through answering similar questions at the teletype, the user may choose an on-line plot and/or a tabular listing of the results. Or, he may choose to punch out the results for future use.

EXAMPLE

The following example illustrates the use of this program to obtain frequency response curves from typical experimental data and shows how the various parts of the program work together.

Figure 4 displays a teletype listing for a known second-order system having natural frequency f_n of 40 hertz and damping ratio ζ of 0.4. The frequency range of interest is 2 to 200 hertz, and 30 points are desired to cover this range. The system was driven by a sine wave whose frequency varied logarithmically with time. A sweep rate of 1 decade per minute was used.

The maximum amplitude of this system is

$$C_r = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} = \frac{1}{0.8\sqrt{0.84}} = 1.364$$

The frequency at which C_r occurs is

$$f_r = f_n\sqrt{1 - 2\zeta^2} = 40\sqrt{0.68} = 32.98 \text{ Hz}$$

The phase at this frequency is

$$\psi_r = \tan^{-1}\left[\frac{-2\zeta\left(\frac{f_r}{f_n}\right)}{1 - \left(\frac{f_r}{f_n}\right)^2}\right] = \tan^{-1}(-2.0616) = -64.12^{\circ}$$

Moreover, we can calculate the second-order system's magnitude and phase at any frequency from the complex form,

$$G(jf) = \frac{1}{\left[1 - \left(\frac{f}{f_n}\right)^2\right] + j2\zeta\left(\frac{f}{f_n}\right)} = \frac{1}{\left[1 - \left(\frac{f}{40}\right)^2\right] + j0.8\left(\frac{f}{40}\right)}$$

Table I lists calculated values of magnitude and phase at selected frequencies. Comparing these values with the listing shown in figure 4, we see that the computer results are in good agreement with the calculated values. Finer resolution could be obtained in the computer results by requesting a larger number of points to cover the frequency range 2 to 200 hertz or by choosing a smaller frequency range of interest.

It is important to emphasize that to obtain valid results the starting frequency must be held for about 5 seconds before starting the sweep. If the period signal is not given time to become established and the filters are not given time to settle out, the first point or two stored by the computer will not have the correct magnitude and phase. The magnitude of the first point is important because it is used to obtain the factor for normalizing an amplitude ratio.

CONCLUDING REMARKS

A technique which gives the engineer the ability to quickly and easily process large amounts of sweep data has been presented. A hybrid computer will allow the user to go rapidly from the raw data on tape to an on-line tabular listing and/or analog plots of the frequency responses desired. If many transfer functions are required, a digital plotter may be used to obtain them. Once the plotter is set up, the engineer or operator does not have to be present during the entire plotting procedure.

Since on-line tabular listings or plots are readily available, this technique could be used to obtain needed results even while a test cell was running. It would be possible to take the data from one test condition; then obtain necessary transfer functions from it to determine the next test condition. This could be accomplished in only a matter of minutes, thus helping to minimize the amount of time needed in the test cell to gather the required data.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 1, 1972,
501-24.

APPENDIX A

FORTRAN IV SOURCE LISTINGS OF DIGITAL PORTION OF PROGRAM[1]

MAIN PROGRAM

```
      DIMENSION JRC(10),ARMAX(10),FREQ(102),XMAG(10,102),PHASE(10,102)
      LOGICAL SENSW
      CALL QSHYIN (IERR,680)
      TYPE 10
   10 FORMAT (/3X,29HRELEASE ALL SSW.  THEN R-S-R./)
      PAUSE 1
      CALL SETUP (NCHAN,STRTF,FRQMAX,POINTS,JRC,ARMAX)
      CALL PROCES (NCHAN,STRTF,FRQMAX,POINTS,LAST,FREQ,XMAG,PHASE)
   20 TYPE 30
   30 FORMAT (/3X,28HPLOT OR LISTING:  SET SSW(H)/13X,18HPUNCH:  SET SSW
     1(E)/3X,6HR-S-R./)
      PAUSE 2
      IF (SENSW(8)) CALL MONITR (NCHAN,FRQMAX,JRC,LAST,FREQ,ARMAX,XMAG,
     1                           PHASE)
      IF (SENSW(5)) CALL PUNCH (NCHAN,LAST,JRC,FREQ,XMAG,PHASE)
      TYPE 10
      PAUSE 3
      GO TO 20
      END
```

[1]Calls to subroutines starting with the letter "Q" are Electronic Associates, Inc., Hybrid Linkage Subroutines.

SUBROUTINE SETUP

```
      SUBROUTINE SETUP (NCHAN,STRTF,FRQMAX,POINTS,JRC,ARMAX)
C
C.....SUBROUTINE SETUP OBTAINS INFORMATION NEEDED TO PROCESS THE RAW
C.....DATA.
C
C.....NCHAN     NO. OF CHANNELS TO ANALYZE
C.....STRTF     SWEEP STARTING FREQUENCY
C.....FRQMAX    SWEEP ENDING FREQUENCY
C.....POINTS    NO. OF POINTS FOR BODE PLOT
C.....JRC       TAPE RECORDER CHANNEL NUMBERS
C.....ARMAX     MAXIMUM GAIN (RESONANCE/DC) OF EACH RECORDER CH.
C
      DIMENSION JRC(10),ARMAX(10)
C
      TYPE 10
   10 FORMAT (/3X,37HTYPE 3 LINES FOR DATA IDENTIFICATION./)
C.....ACCEPT USER IDENTIFICATION INFORMATION
      CALL TYPIN
      TYPE 20
   20 FORMAT (/3X,44HINCLUDE DECIMAL POINT WITH ALL DATA ENTRIES./)
   30 TYPE 40
   40 FORMAT (/3X,25HNO. OF CHANNELS (MAX. 10)/)
      ACCEPT 50, XNCHAN
   50 FORMAT (F10.4)
      NCHAN=XNCHAN+.1
      IF ((NCHAN.GT.0).AND.(NCHAN.LT.11)) GO TO 70
      TYPE 60
   60 FORMAT (/3X,27HPLEASE REREAD INSTRUCTIONS!//)
      GO TO 30
   70 TYPE 80
   80 FORMAT (/3X,37HLIST TAPE RECORDER CHANNELS IN ORDER./)
      DO 120 I=1,NCHAN
      TYPE 90, I
   90 FORMAT (/3X,4HCH. ,I2,2H =/)
      ACCEPT 50, RECCH
      JRC(I)=RECCH+.1
      IF ((JRC(I).GT.0).AND.(JRC(I).LT.43)) GO TO 100
      TYPE 60
      GO TO 70
  100 TYPE 110, JRC(I)
  110 FORMAT (/3X,17HMAX. GAIN OF CH. ,I2,2H =/)
      ACCEPT 50, ARMAX(I)
      IF (ARMAX(I).GT.0.) GO TO 120
      TYPE 60
      GO TO 100
  120 CONTINUE
      TYPE 130
  130 FORMAT (/3X,59HCHECK THAT EACH RECORDER CHANNEL IS CONNECTED TO TH

     1E PROPER/3X,28HTRUNK.  WHEN CHECKED, R-S-R./)
      PAUSE 100
  140 TYPE 150
  150 FORMAT (/3X,32HSTARTING FREQUENCY IN HZ? (>=.5)/)
      ACCEPT 50, STRTF
      IF (STRTF.GT.0.49) GO TO 160
      TYPE 60
      GO TO 140
  160 TYPE 170
  170 FORMAT (/3X,32HMAXIMUM FREQUENCY IN HZ? (<=500)/)
      ACCEPT 50, FRQMAX
      IF ((FRQMAX.GT.STRTF).AND.(FRQMAX.LT.500.01)) GO TO 180
      TYPE 60
      GO TO 160
  180 TYPE 190
  190 FORMAT (/3X,29HNO. OF POINTS? (10<=PTS<=100)/)
      ACCEPT 50, POINTS
      IF ((POINTS.GT.9.99999).AND.(POINTS.LT.100.001)) GO TO 200
      TYPE 60
      GO TO 180
  200 TYPE 210
  210 FORMAT (/3X,27HTHE PROGRAM IS INITIALIZED./3X,40HWHEN THE SWEEP IS

     1 COMPLETED, SET SSW(A)./3X,29HNOW START SWEEP.  THEN R-S-R./)
      PAUSE 110
      RETURN
      END
```

SUBROUTINE PROCES

```
      SUBROUTINE PROCES (NCHAN,FRQSRT,FRQMAX,POINTS,LAST,FREQ,XMAG,
     1                     PHASE)
C
C
C.....SUBROUTINE PROCES OBTAINS THE FREQUENCY, MAGNITUDE, AND PHASE OF
C.....THE RAW DATA.
C
C
C INPUT QUANTITIES:
C       NCHAN - NUMBER OF CHANNELS TO BE ANALYZED
C      FRQSRT - STARTING FREQUENCY IN HZ
C      FRQMAX - ENDING FREQUENCY IN HZ
C      POINTS - NUMBER OF POINTS DESIRED FOR BODE PLOT
C
C OUTPUT QUANTITIES:
C        LAST - THE NUMBER OF POINTS THE COMPUTER WAS ABLE TO OBTAIN
C        FREQ - MATRIX CONTAINING FREQUENCY (HZ) OF ANALYZED DATA
C        XMAG - MATRIX CONTAINING AMPLITUDE OF ANALYZED DATA
C       PHASE - MATRIX CONTAINING PHASE ANGLE OF ANALYZED DATA
C
C
      DIMENSION FREQ(102),ADC(24),XMAG(10,102),PHASE(10,102)
      LOGICAL SENSW,VAL9,VAL39,VAL69
      SCALED FRACTION SADC(24)
      RAD=57.29578
C.....ARBITRARY NUMBER GREATER THAN POINTS
      ILAST=POINTS+10.
      SCTFR=FRQSRT/FRQMAX
C.....NUMBER OF ADC'S USED
      N=2*NCHAN+4
      ALG=ALOG(FRQSRT)
      DELTA=(ALOG(.99*FRQMAX)-ALG)/(POINTS-1.)
      J=0
C.....THE EQUIVALENT OF:  DO 100 J=1,ILAST
C.....COULD NOT USE A DO LOOP BECAUSE IT WILL NEVER BE COMPLETED
   10 J=J+1
   20 IF (SENSW(1)) GO TO 110
C.....SEE IF FRQMAX HAS BEEN SURPASSED.
C.....IF SO, WAIT FOR SSW(A) TO BE SET.
      IF (SCTFR.GT..9999) GO TO 20
C.....DETERMINE CONSTANT RELATING SCALED FREQUENCY AND (1/ADC(1))
      CALL QRCPL (9,VAL9,IERROR)
      IF (VAL9) GO TO 30
      CALL QWCLL (1,.TRUE.,IERROR)
      CALL QRCPL (39,VAL39,IERROR)
      IF (VAL39) GO TO 40
      CALL QWCLL (2,.TRUE.,IERROR)
      CALL QRCPL (69,VAL69,IERROR)
      IF (VAL69) GO TO 50
      CALL QWCLL (3,.TRUE.,IERROR)
      GAIN=62.5/FRQMAX
      GO TO 60
   30 GAIN=0.5/FRQMAX
      GO TO 60
   40 GAIN=2.5/FRQMAX
      GO TO 60
   50 GAIN=12.5/FRQMAX
C.....READ SCALED PERIOD AND CONVERT TO SCALED FREQUENCY
   60 CALL QRBADS (SADC,0,1,IERROR)
```

```
      ADC(1)=SADC(1)
      SCFREQ=GAIN/ADC(1)
C.....COMPARE SCALED FREQUENCY WITH SCALED TEST FREQUENCY TO SEE IF WE
C.....WANT TO STORE THIS POINT
      IF (SCFREQ.LT.SCTFR) GO TO 20
C.....READ ALL ADC'S
      CALL QRBADS (SADC,0,N,IERROR)
      DO 70 I=1,N
      ADC(I)=SADC(I)
   70 CONTINUE
      FREQ(J)=(GAIN/ADC(1))*FRQMAX
C.....CALCULATE NEW SCALED TEST FREQUENCY
      XLN=ALG+DELTA*FLOAT(J)
      TESTF=EXP(XLN)
      SCTFR=TESTF/FRQMAX
C
C.....DRIVER = A*SIN(THETA)
C.....OUTPUT = AC *SIN(THETA+PSI)  , N>=1
C             N    N
C
C.....  QUAD = B*COS(THETA+PHI)
C
C
C.....   ADC(1) = PERIOD/2
C.....   ADC(2) = DRIVER*DRIVER
C.....   ADC(3) = QUAD*QUAD
C.....   ADC(4) = DRIVER*QUAD
C.....ADC(2N+3) = QUAD*(OUTPUT)   , N>=1
C                             N
C
C.....ADC(2N+4) = DRIVER*(OUTPUT)   , N>=1
C                               N
C
      DENOM=SQRT(ADC(2)*ADC(3)-.01*ADC(4)**2)
      DO 100 I=1,NCHAN
      ARCOS=ADC(2*I+4)/ADC(2)
      ARSIN=(ADC(2*I+3)-.1*ADC(4)*ARCOS)/DENOM
C.....CALCULATE MAGNITUDE
      XMAG(I,J)=SQRT(ARCOS**2+ARSIN**2)
C.....CALCULATE PHASE ANGLE IN DEGREES.  MAKE SURE IT IS FINITE.
      IF (ARCOS.NE.0.) GO TO 80
      IF (ARSIN.LT.0.) PHASE(I,J)=-90.
      IF (ARSIN.EQ.0.) PHASE(I,J)=UUUU
      IF (ARSIN.GT.0.) PHASE(I,J)=-270.
      GO TO 100
   80 PHASE(I,J)=RAD*ATAN(ARSIN/ARCOS)
C.....MAKE SURE PHASE ANGLE IS BETWEEN 0. AND (-360.) DEGREES
      IF (ARCOS.GT.0.) GO TO 90
      PHASE(I,J)=PHASE(I,J)-180.
      GO TO 100
C.....ALLOW POSITIVE PHASE ANGLE FOR FREQUENCIES LESS THAN 1.
   90 IF (ADC(1).GT.0.5) GO TO 100
      IF (ARSIN.GT.0.) PHASE(I,J)=PHASE(I,J)-360.
  100 CONTINUE
      IF (J.LE.ILAST) GO TO 10
C.....THE FOLLOWING STATEMENT SHOULD NOT BE REACHED ON A GOOD RUN
      STOP 31
  110 LAST=J-1
      TYPE 120
  120 FORMAT (/3X,37HTHE DATA IS NOW PROCESSED AND STORED./)
      RETURN
      END
```

SUBROUTINE MONITR

```
      SUBROUTINE MONITR (NCHAN,FRQMAX,IRC,LAST,FREQ,ARMAX,XMAG,PHASE)
C
C
C.....SUBROUTINE MONITR FORMS THE AMPLITUDE RATIO OF ANY TWO PROCESSED
C.....DATA CHANNELS.  ALSO, THE USER MAY MONITOR ANY PROCESSED DATA
C.....CHANNEL.  THE OUTPUT OF THIS SUBROUTINE IS EITHER AN ANALOG
C.....X-Y PLOT OR A DIGITAL LISTING.
C
C
C INPUT QUANTITIES:
C       NCHAN - NUMBER OF CHANNELS ANALYZED
C      FRQMAX - ENDING FREQUENCY IN HZ
C         IRC - MATRIX CONTAINING TAPE RECORDER CHANNEL NUMBERS
C        LAST - THE NUMBER OF POINTS THE COMPUTER WAS ABLE TO OBTAIN
C        FREQ - MATRIX CONTAINING FREQUENCY (HZ) OF ANALYZED DATA
C       ARMAX - MATRIX CONTAINING MAXIMIM GAIN (RESONANCE/DC) OF EACH
C               RECORDER CHANNEL
C        XMAG - MATRIX CONTAINING AMPLITUDE OF ANALYZED DATA
C       PHASE - MATRIX CONTAINING PHASE ANGLE OF ANALYZED DATA
C
C
      REAL NORMAL
      LOGICAL SENSW
      DIMENSION IRC(10),FREQ(102),ARMAX(10),XMAG(10,102),PHASE(10,102)
      SCALED FRACTION SDAC(3)
      TYPE 10
   10 FORMAT (/3X,43HFOR AN AMPLITUDE RATIO, SET SSW(B).  R-S-R./)
      PAUSE 41
      IF (SENSW(2)) GO TO 150
   20 TYPE 30
   30 FORMAT (/3X,28HRECORDER CHANNEL TO MONITOR?/)
      ACCEPT 40, RECCH
   40 FORMAT (F10.4)
      IRECCH=RECCH+.1
C.....CHECK TO SEE IF THIS RECORDER CHANNEL WAS PROCESSED
      IOKAY=0
      DO 50 I=1,NCHAN
      IF (IRECCH.EQ.IRC(I)) IOKAY=I
   50 CONTINUE
      IF (IOKAY.NE.0) GO TO 70
      TYPE 60
   60 FORMAT (/3X,51HYOU HAVE SELECTED A CHANNEL THAT WAS NOT PROCESSED!

     1/)
      GO TO 20
   70 TYPE 80
   80 FORMAT (/6X,17HPLOT:  SET SSW(C)/3X,20HLISTING:  SET SSW(D)/3X,
     16HR-S-R./)
      PAUSE 42
      IF (.NOT.(SENSW(3).OR.SENSW(4))) RETURN
      IF (.NOT.SENSW(4)) GO TO 100
C.....TYPE USER IDENTIFICATION INFORMATION
      CALL TYPOUT
      TYPE 90
   90 FORMAT (///10X,7HCHANNEL,2X,5HPOINT,2X,9HFREQUENCY,2X,9HMAGNITUDE,

     12X,5HPHASE//)
  100 DO 130 J=1,LAST
      IF (.NOT.SENSW(3)) GO TO 110
C.....SCALE DATA FOR AN ANALOG X-Y PLOT
      SDAC(1)=FREQ(J)/FRQMAX
      SDAC(2)=XMAG(IOKAY,J)/ARMAX(IOKAY)
      SDAC(3)=PHASE(IOKAY,J)/500.
      CALL QWBDAS (SDAC,1,3,IERROR)
      CALL QSTDA
C.....ALLOW PLOTTER PEN TO INITIALIZE
      IF (J.EQ.1) CALL QSDLY (2000)
C.....FOR A PLOT ONLY, DELAY BETWEEN POINTS SO THE PLOTTER PEN HAS A
C.....CHANCE TO MOVE
      IF (.NOT.SENSW(4)) CALL QSDLY (500)
  110 IF (SENSW(4)) TYPE 120, IRECCH,J,FREQ(J),XMAG(IOKAY,J),
     1                         PHASE(IOKAY,J)
  120 FORMAT (10X,I4,I8,4X,F6.2,5X,F6.3,3X,F6.1)
  130 CONTINUE
      TYPE 140, IRECCH
```

```
140 FORMAT (//3X,22HMONITORING OF CHANNEL ,I2,13H IS COMPLETE./)
    RETURN
C.....AN AMPLITUDE RATIO IS DESIRED
150 TYPE 160
160 FORMAT (/3X,62HCALCULATE (RECORDER CHANNEL B)/(RECORDER CHANNEL A)

   1.  ENTER B./)
    ACCEPT 40, B
    IB=B+.1
    TYPE 170
170 FORMAT (/)
C.....CHECK TO SEE IF THIS RECORDER CHANNEL WAS PROCESSED
    IOKAYB=0
    DO 180 I=1,NCHAN
    IF (IB.EQ.IRC(I)) IOKAYB=I
180 CONTINUE
    IF (IOKAYB.NE.0) GO TO 190
    TYPE 60
    GO TO 150
190 TYPE 200
200 FORMAT (/3X,8HENTER A./)
    ACCEPT 40, A
    IA=A+.1
    TYPE 170
C.....CHECK TO SEE IF THIS RECORDER CHANNEL WAS PROCESSED
    IOKAYA=0
    DO 210 I=1,NCHAN
    IF (IA.EQ.IRC(I)) IOKAYA=I
210 CONTINUE
    IF (IOKAYA.NE.0) GO TO 220
    TYPE 60
    GO TO 190
220 TYPE 80
    PAUSE 43
    IF (.NOT.(SENSW(3).OR.SENSW(4))) RETURN
    IF (.NOT.SENSW(4)) GO TO 240
C.....TYPE USER IDENTIFICATION INFORMATION
    CALL TYPOUT
    TYPE 230
230 FORMAT (//9X,9HAMPLITUDE/11X,5HRATIO,3X,5HPOINT,2X,9HFREQUENCY,2X,

   19HMAGNITUDE,2X,5HPHASE//)
240 DO 290 J=2,LAST
C.....CALCULATE AND NORMALIZE AMPLITUDE RATIO
    AMPRAT=XMAG(IOKAYB,J)/XMAG(IOKAYA,J)
    IF (J.EQ.2) NORMAL=AMPRAT
    AMPRAT=AMPRAT/NORMAL
C.....CALCULATE PHASE ANGLE IN DEGREES
    ANGLE=PHASE(IOKAYB,J)-PHASE(IOKAYA,J)
C.....MAKE SURE PHASE ANGLE IS BETWEEN 0. AND (-360.) DEGREES
250 IF (ANGLE.GE.(-360.)) GO TO 260
    ANGLE=ANGLE+360.
    GO TO 250
260 IF (ANGLE.LE.0.) GO TO 270
C.....ALLOW POSITIVE PHASE ANGLE FOR FREQUENCIES LESS THAN 1.
    IF (FREQ(J).LT.1.) GO TO 270
    ANGLE=ANGLE-360.
    GO TO 260
270 IF (.NOT.SENSW(3)) GO TO 280
C.....SCALE DATA FOR AN ANALOG X-Y PLOT
    SDAC(1)=FREQ(J)/FRQMAX
    SDAC(2)=AMPRAT/5.
    SDAC(3)=ANGLE/500.
    CALL QWBDAS (SDAC,1,3,IERROR)
    CALL QSTDA
C.....ALLOW PLOTTER PEN TO INITIALIZE
    IF (J.EQ.2) CALL QSDLY (2000)
C.....FOR A PLOT ONLY, DELAY BETWEEN POINTS SO THE PLOTTER PEN HAS A
C.....CHANCE TO MOVE
    IF (SENSW(3).AND.(.NOT.SENSW(4))) CALL QSDLY (500)
280 IF (SENSW(4)) TYPE 300, IB,IA,J,FREQ(J),AMPRAT,ANGLE
290 CONTINUE
300 FORMAT (11X,I2,1H/,I2,3X,I3,4X,F6.2,5X,F6.3,3X,F6.1)
    TYPE 310, IB,IA
310 FORMAT (//3X,30HMONITORING OF AMPLITUDE RATIO ,I2,1H/,I2,
   113H IS COMPLETE./)
    TYPE 320, NORMAL
320 FORMAT (/3X,54HTHE FACTOR USED TO NORMALIZE THE AMPLITUDE RATIO WA

   1S: ,F8.3,1H./)
    RETURN
    END
```

SUBROUTINE PUNCH

```
      SUBROUTINE PUNCH (NCHAN,M,JRC,FREQ,XMAG,PHASE)
C
C.....SUBROUTINE PUNCH OUTPUTS ON PAPER TAPE THE FREQUENCIES,
C.....MAGNITUDES, AND PHASE ANGLES OBTAINED FROM A PROCESSED DATA
C.....CHANNEL
C
C.....NCHAN      NO. OF CHANNELS ANALYZED
C.....M          NO. OF POINTS TAKEN
C.....JRC        TAPE RECORDER CHANNEL NUMBERS
C.....FREQ       NAME OF FREQUENCY ARRAY
C.....XMAG       NAME OF MAGNITUDE ARRAY
C.....PHASE      NAME OF PHASE ARRAY
C
      DIMENSION JRC(10),FREQ(102),XMAG(10,102),PHASE(10,102)
      LOGICAL SENSW
      TYPE 10
   10 FORMAT (/3X,36HTURN ON THE HSPT PUNCH.  THEN R-S-R./)
      PAUSE 400
   20 TYPE 30
   30 FORMAT (/3X,26HRECORDER CHANNEL TO PUNCH?/)
      ACCEPT 40, RECCH
   40 FORMAT (F10.4)
      K=RECCH+.1
C.....CHECK TO SEE IF THIS RECORDER CHANNEL WAS PROCESSED
      N=0
      DO 50 I=1,NCHAN
      IF (K.EQ.JRC(I)) N=I
   50 CONTINUE
      IF (N.EQ.0) GO TO 80
C.....PUNCH USER IDENTIFICATION INFORMATION
      CALL PNCH
      WRITE (5,60) (K,L,FREQ(L),XMAG(N,L),PHASE(N,L),L=1,M)
   60 FORMAT ((3(I2,I3,F6.2,F6.3,F7.1)))
      IF (.NOT.SENSW(6)) TYPE 70, K
   70 FORMAT (//3X,16HPUNCHING OF CH. ,I2,18H DATA IS COMPLETE./3X,
     137HTO PUNCH ADDITIONAL DATA, JUST R-S-R./3X,53HIF NO MORE DATA IS

     2TO BE PUNCHED, TURN OFF THE PUNCH,/3X,26HRELEASE SSW(E), AND R-S-R

     3./)
      PAUSE 410
      IF (.NOT.SENSW(5)) GO TO 100
      GO TO 20
   80 TYPE 90
   90 FORMAT (/3X,51HYOU HAVE SELECTED A CHANNEL THAT WAS NOT PROCESSED!

     1/)
      GO TO 20
  100 RETURN
      END
```

SUBROUTINE TYPIN

```
      SUBROUTINE TYPIN
C
C
C.....SUBROUTINE TYPIN ACCEPTS THREE LINES OF USER IDENTIFICATION
C.....INFORMATION FROM THE TELETYPE.
C
C
      COMMON/LABEL/TITLE(3,18),LAST(3)
      DATA  SPACE/4H    /
      DO 40 LINE=1,3
      LAST(LINE)=1
      ACCEPT 10, (TITLE(LINE,J),J=1,18)
   10 FORMAT (18A4)
      TYPE 20
   20 FORMAT (/)
      J=0
   30 J=J+1
      ILAST=19-J
      IF (ILAST.LT.1) GO TO 40
      IF (TITLE(LINE,ILAST).EQ.SPACE) GO TO 30
      LAST(LINE)=ILAST
   40 CONTINUE
      RETURN
      END
```

SUBROUTINE TYPOUT

```
      SUBROUTINE TYPOUT
C
C
C.....SUBROUTINE TYPOUT WRITES THREE LINES OF USER IDENTIFICATION
C.....INFORMATION AT THE TELETYPE.
C
C
      COMMON/LABEL/TITLE(3,18),LAST(3)
      DO 10 LINE=1,3
      ILAST=LAST(LINE)
      TYPE 20, (TITLE(LINE,J),J=1,ILAST)
   10 CONTINUE
   20 FORMAT (18A4)
      RETURN
      END
```

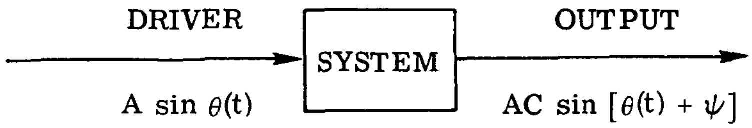
SUBROUTINE PNCH

```
      SUBROUTINE PNCH
C
C
C.....SUBROUTINE PNCH PUNCHES ON PAPER TAPE THREE LINES OF USER
C.....IDENTIFICATION INFORMATION.
C
C
      COMMON/LABEL/TITLE(3,18),LAST(3)
      DO 10 LINE=1,3
      WRITE (5,20) (TITLE(LINE,J),J=1,18)
   10 CONTINUE
   20 FORMAT (18A4)
      RETURN
      END
```

APPENDIX B

CALCULATIONS FOR MAGNITUDE AND PHASE OF SWEEP FREQUENCY DATA

Consider a system being perturbed by an unbiased sine wave driver:



Let

$$\text{DRIVER} = A \sin \theta(t)$$

$$\text{OUTPUT} = AC \sin [\theta(t) + \psi]$$

$$\text{QUAD} = B \cos [\theta(t) + \varphi]$$

where OUTPUT is the signal resulting from the sine wave perturbation, DRIVER; and QUAD is the quadrature component of DRIVER.

From the simple trigonometric identities

$$\sin^2 \theta(t) = \frac{1}{2} [1 - \cos 2\theta(t)]$$

$$\cos^2 \theta(t) = \frac{1}{2} [1 + \cos 2\theta(t)]$$

$$\sin \theta(t) * \cos \theta(t) = \frac{1}{2} \sin 2\theta(t)$$

it follows that

$$\text{DRIVER} * \text{DRIVER} = A \sin \theta(t) * A \sin \theta(t) = A^2 \sin^2 \theta(t) = \frac{A^2}{2} [1 - \cos 2\theta(t)]$$

$$\text{QUAD}*\text{QUAD} = B\cos[\theta(t)+\varphi]*B\cos[\theta(t)+\varphi] = B^2\cos^2[\theta(t)+\varphi]$$

$$= B^2[\cos^2\varphi\cos^2\theta(t) - 2\sin\varphi\cos\varphi\sin\theta(t)\cos\theta(t) + \sin^2\varphi\sin^2\theta(t)]$$

$$= B^2\left\{\frac{1}{2}\cos^2\varphi[1+\cos 2\theta(t)] - \sin\varphi\cos\varphi\sin 2\theta(t) + \frac{1}{2}\sin^2\varphi[1-\cos 2\theta(t)]\right\}$$

$$= B^2\left[\frac{1}{2} + \left(\frac{1}{2} - \sin^2\varphi\right)\cos 2\theta(t) - \sin\varphi\cos\varphi\sin 2\theta(t)\right]$$

$$= \frac{B^2}{2}[1 + \cos 2\varphi\cos 2\theta(t) - \sin 2\varphi\sin 2\theta(t)]$$

$$\text{DRIVER}*\text{QUAD} = A\sin\theta(t)*B\cos[\theta(t)+\varphi]$$

$$= AB[\cos\varphi\sin\theta(t)\cos\theta(t) - \sin\varphi\sin^2\theta(t)]$$

$$= \frac{AB}{2}\{\cos\varphi\sin 2\theta(t) - \sin\varphi[1-\cos 2\theta(t)]\}$$

$$= \frac{AB}{2}[\cos\varphi\sin 2\theta(t) + \sin\varphi\cos 2\theta(t) - \sin\varphi]$$

$$\text{DRIVER}*\text{OUTPUT} = A\sin\theta(t)*AC\sin[\theta(t)+\psi]$$

$$= A^2C[\cos\psi\sin^2\theta(t) + \sin\psi\sin\theta(t)\cos\theta(t)]$$

$$= \frac{A^2C}{2}\{\cos\psi[1-\cos 2\theta(t)] + \sin\psi\sin 2\theta(t)\}$$

$$= \frac{A^2C}{2}[\cos\psi - \cos\psi\cos 2\theta(t) + \sin\psi\sin 2\theta(t)]$$

$$\text{QUAD}*\text{OUTPUT} = B\cos[\theta(t)+\varphi]*AC\sin[\theta(t)+\psi]$$

$$= ABC[\cos\varphi\cos\theta(t) - \sin\varphi\sin\theta(t)]*[\cos\psi\sin\theta(t) + \sin\psi\cos\theta(t)]$$

$$= \frac{ABC}{2}\{\cos\varphi\cos\psi\sin 2\theta(t) + \cos\varphi\sin\psi[1+\cos 2\theta(t)] -$$

$$\sin\varphi\cos\psi[1-\cos 2\theta(t)] - \sin\varphi\sin\psi\sin 2\theta(t)\}$$

$$= \frac{ABC}{2}[\cos(\psi+\varphi)\sin 2\theta(t) + \sin(\psi+\varphi)\cos 2\theta(t) +$$

$$(\cos\varphi\sin\psi - \sin\varphi\cos\psi)]$$

When filters are used to attenuate terms involving 2θ, these products become

$$\text{DRIVER}*\text{DRIVER} = \frac{A^2}{2} \tag{B1}$$

$$\text{QUAD}*\text{QUAD} = \frac{B^2}{2} \tag{B2}$$

$$\text{DRIVER}*\text{QUAD} = -\frac{AB}{2} \sin \varphi \tag{B3}$$

$$\text{DRIVER}*\text{OUTPUT} = \frac{A^2C}{2} \cos \psi \tag{B4}$$

$$\text{QUAD}*\text{OUTPUT} = \frac{ABC}{2} (\cos \varphi \sin \psi - \sin \varphi \cos \psi) \tag{B5}$$

Dividing equation (B4) by equation (B1),

$$C \cos \psi = \frac{\frac{A^2C}{2} \cos \psi}{\frac{A^2}{2}} = \frac{\text{DRIVER}*\text{OUTPUT}}{\text{DRIVER}*\text{DRIVER}} \tag{B6}$$

From equation (B5) we have

$$\begin{aligned} \text{QUAD}*\text{OUTPUT} &= \frac{ABC}{2} (\cos \varphi \sin \psi - \sin \varphi \cos \psi) \\ &= \left(\frac{AB}{2} \cos \varphi\right)(C \sin \psi) + \left(-\frac{AB}{2} \sin \varphi\right)(C \cos \psi) \\ &= \left(\frac{AB}{2} \cos \varphi\right)(C \sin \psi) + (\text{DRIVER}*\text{QUAD})(C \cos \psi) \end{aligned}$$

Solving for $C \sin \psi$ and simplifying,

$$C \sin \psi = \frac{(\text{QUAD*OUTPUT}) - (\text{DRIVER*QUAD})(C \cos \psi)}{\frac{AB}{2} \cos \varphi}$$

$$= \frac{(\text{QUAD*OUTPUT}) - (\text{DRIVER*QUAD})(C \cos \psi)}{\sqrt{\left(\frac{A^2}{2}\right)\left(\frac{B^2}{2}\right) - \left(- \frac{AB}{2} \sin \varphi\right)^2}}$$

$$= \frac{(\text{QUAD*OUTPUT}) - (\text{DRIVER*QUAD})(C \cos \psi)}{\sqrt{(\text{DRIVER*DRIVER})(\text{QUAD*QUAD}) - (\text{DRIVER*QUAD})^2}} \tag{B7}$$

Hence, the five filtered signals, equations (B1) to (B5), allow us to evaluate equations (B6) and (B7). Using the two identities

$$C = \sqrt{(C \cos \psi)^2 + (C \sin \psi)^2}$$

and

$$\psi = \tan^{-1}\left(\frac{C \sin \psi}{C \cos \psi}\right)$$

we obtain the magnitude C and phase ψ desired.

APPENDIX C

SYMBOLS

A	scaled amplitude of sine wave driver
A'	amplitude of sine wave driver
B	scaled amplitude of quadrature component of sine wave driver
B'	amplitude of quadrature component of sine wave driver
C	system gain
e	voltage, V
f	frequency, Hz
G	system transfer function
j	imaginary operator, $\sqrt{-1}$
K	scale factor
P	voltage proportional to period signal, V
s	LaPlace operator
t	time
δ	signal bias
ζ	damping ratio
θ	angle, radians
τ	time constant of first-order filter, sec
φ	phase shift between sine wave driver and its quadrature component, radians
ψ	phase shift between sine wave driver and system, radians
ω	frequency, radians/sec

Subscripts:

a	sine wave driver signal
ac	output signal
b	quadrature signal
c	cutoff
in	input

n	natural
o	output
r	resonant
1, 2	identification for constants

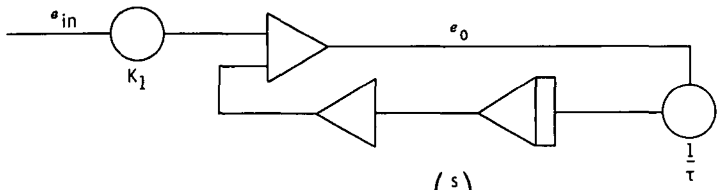
REFERENCE

1. Drain, Daniel I.; Bruton, William M.; and Paulovich, Francis J.: Airbreathing Propulsion System Testing Using Sweep Frequency Techniques. NASA TN D-5485, 1969.

TABLE I. - CALCULATED VALUES OF MAGNITUDE AND PHASE AT SELECTED FREQUENCIES FOR A KNOWN SECOND-ORDER SYSTEM HAVING NATURAL FREQUENCY f_n OF 40 HERTZ AND DAMPING RATIO ζ OF 0.4

$$\left[G(jf) = \frac{1}{\left[1 - \left(\frac{f}{f_n}\right)^2\right] + j2\zeta\left(\frac{f}{f_n}\right)} = \frac{1}{\left[1 - \left(\frac{f}{40}\right)^2\right] + j0.8\left(\frac{f}{40}\right)}\right]$$

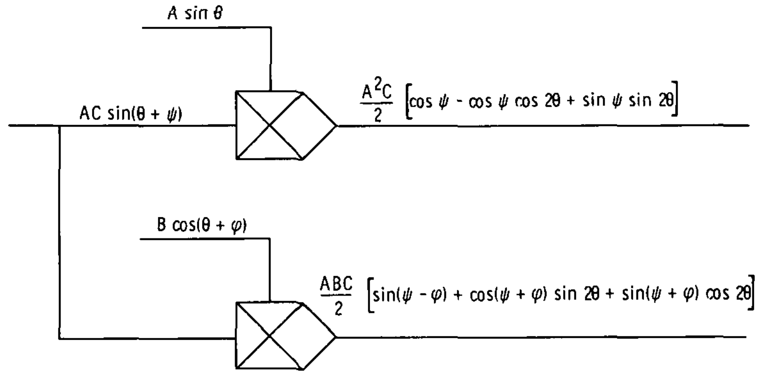
Frequency, Hz	Magnitude	Phase, deg
4.46	1.008	-5.2
7.15	1.022	-8.4
18.45	1.150	-25.1
32.98	1.364	-64.1
75.85	.333	-149.7
119.62	.121	-163.2
166.66	.060	-168.5



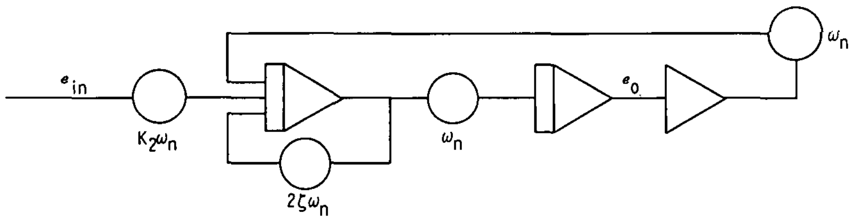
$$\frac{e_o}{e_{in}} = -K_1 \frac{\tau s}{1 + \tau s} = -K_1 \frac{\left(\frac{s}{\omega_c}\right)}{1 + \left(\frac{s}{\omega_c}\right)}$$

$$\tau = \frac{1}{\omega_c} = 5 \text{ sec}$$

(a) High-pass filter to remove bias.



(b) Mixing (multiplying) circuit for data signals.



$e_{in}(1) = \left[A \sin\theta\right] \circ \left[AC \sin(\theta + \psi)\right]$

$e_o(1) = A^2C \cos\psi$

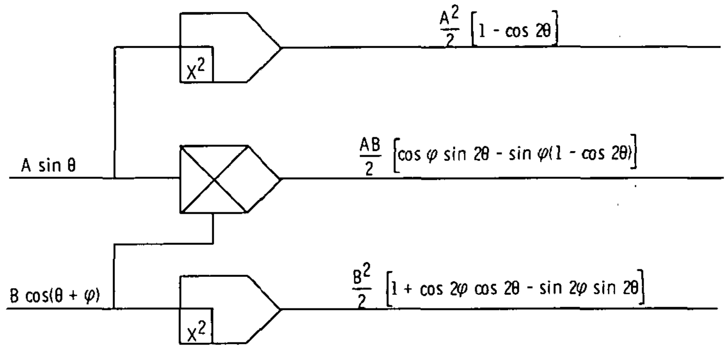
$e_{in}(2) = \left[B \cos(\theta + \varphi)\right] \circ \left[AC \sin(\theta + \psi)\right]$

$e_o(2) = ABC \sin(\psi - \varphi)$

$$\frac{e_o}{e_{in}} = K_2 \frac{\omega_n^2}{\omega_n^2 + 2\zeta\omega_n s + s^2} = K_2 \frac{1}{1 + 2\zeta\left(\frac{s}{\omega_n}\right) + \left(\frac{s}{\omega_n}\right)^2}$$

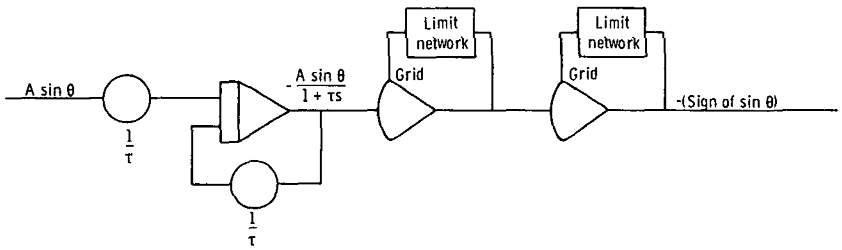
where $K_2 = 2$, $\omega_n = 1.414$, and $\zeta = 0.707$

(c) Low-pass filter to attenuate unwanted frequency content in mixed signals.

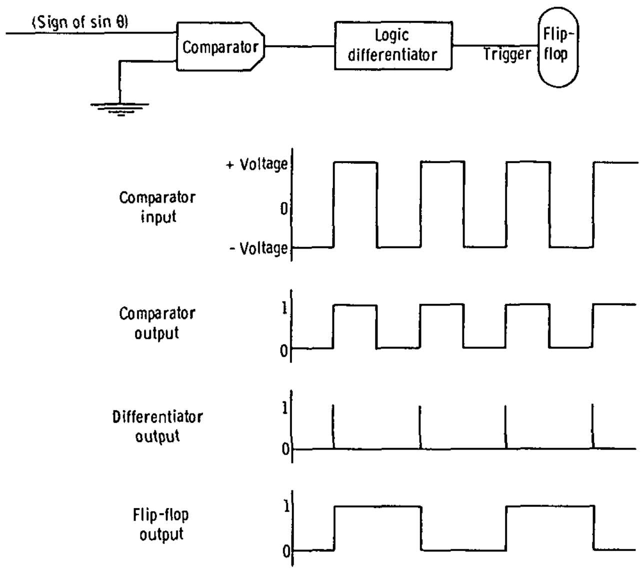


(d) Multiplying and squaring of sine and cosine signals.

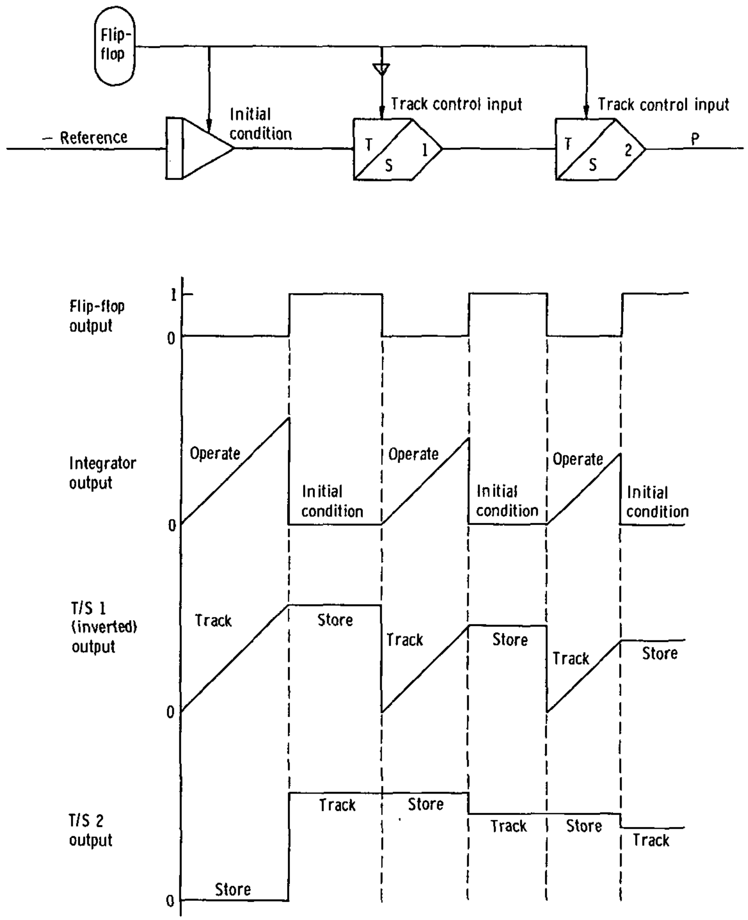
Figure 1. - Signal conditioning.



(a) Generation of a square wave from filtered sine wave driver. Time constant of first-order filter τ, 0.01 second.



(b) Zero-crossing detector.



(c) Determination of period of driver signal from zero-crossing detector output.

Figure 2. - Period determination.

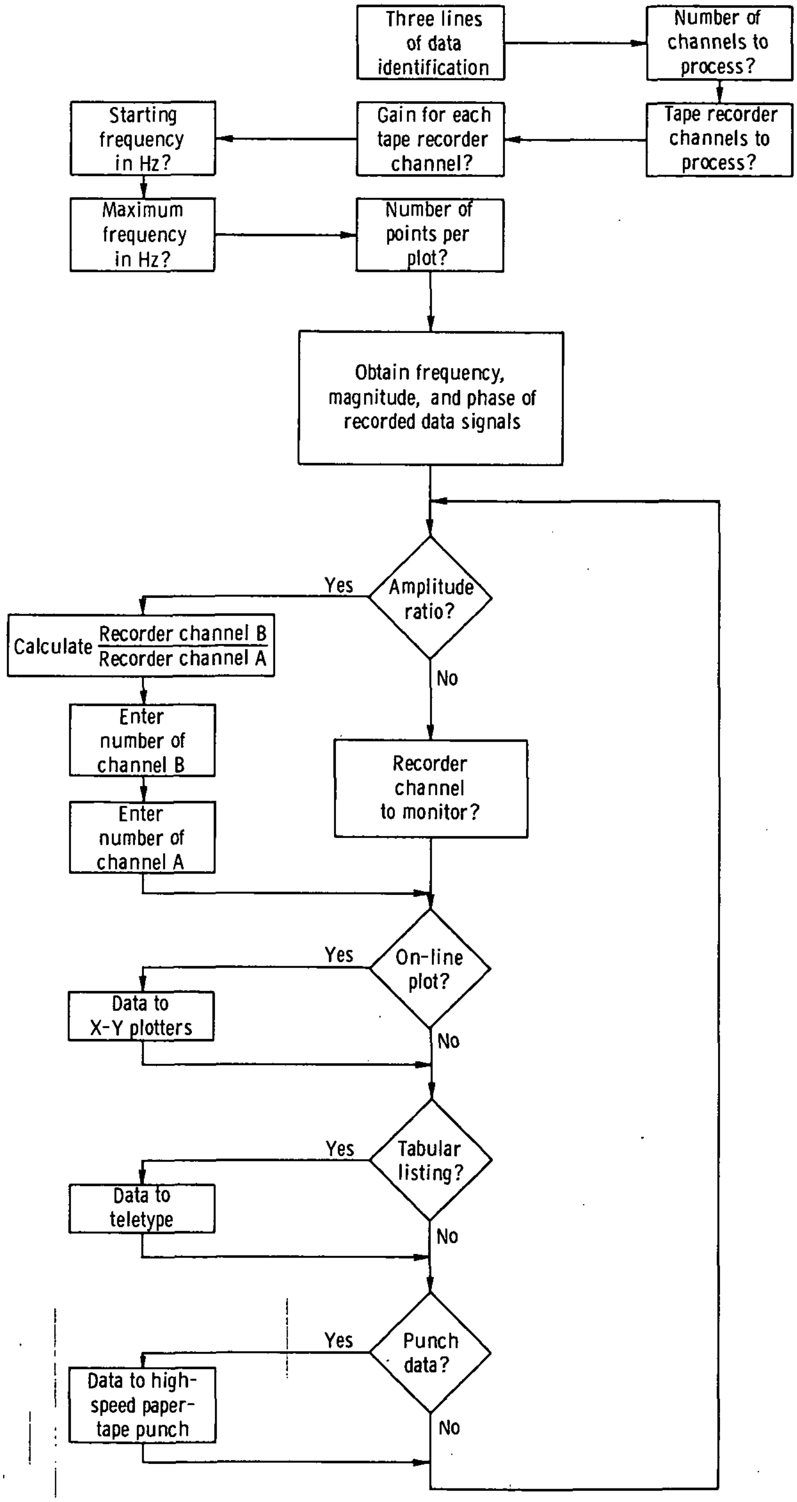


Figure 3. - Flow diagram of digital computer program.

```
RELEASE ALL SSW.  THEN R-S-R.

PAUSE  00001

TYPE 3 LINES FOR DATA IDENTIFICATION.
SAMPLE LISTING USING A SECOND-ORDER SYSTEM WITH A

NATURAL FREQUENCY OF 40 HZ AND A DAMPING RATIO OF 0.4

FREQUENCY RANGE:  2 HZ TO 200 HZ        30 POINTS DESIRED


INCLUDE DECIMAL POINT WITH ALL DATA ENTRIES.

NO. OF CHANNELS (MAX. 10)
2.

LIST TAPE RECORDER CHANNELS IN ORDER.

CH.  1 =
7.

MAX. GAIN OF CH.  7 =
5.

CH.  2 =
3.

MAX. GAIN OF CH.  3 =
2.

CHECK THAT EACH RECORDER CHANNEL IS CONNECTED TO THE PROPER
TRUNK.  WHEN CHECKED, R-S-R.

PAUSE  00100

STARTING FREQUENCY IN HZ? (>=.5)
2.

MAXIMUM FREQUENCY IN HZ? (<=500)
200.

NO. OF POINTS? (10<=PTS<=100)
30.

THE PROGRAM IS INITIALIZED.
WHEN THE SWEEP IS COMPLETED, SET SSW(A).
NOW START SWEEP.  THEN R-S-R.

PAUSE  00110

THE DATA IS NOW PROCESSED AND STORED.

PLOT OR LISTING:  SET SSW(H)
         PUNCH:  SET SSW(E)
R-S-R.

PAUSE  00002
```

SSW(H) was set.

```
FOR AN AMPLITUDE RATIO, SET SSW(B).  R-S-R.

PAUSE  00041
```

SSW(B) was not set.

```
RECORDER CHANNEL TO MONITOR?
3.

   PLOT:  SET SSW(C)
LISTING:  SET SSW(D)
R-S-R.
```

SSW(D) was set.

```
PAUSE  00042

SAMPLE LISTING USING A SECOND-ORDER SYSTEM WITH A
NATURAL FREQUENCY OF 40 HZ AND A DAMPING RATIO OF 0.4
FREQUENCY RANGE:  2 HZ TO 200 HZ        30 POINTS DESIRED
```

CHANNEL	POINT	FREQUENCY	MAGNITUDE	PHASE
3	1	2.04	1.001	-1.7
3	2	2.38	1.001	-2.3
3	3	2.77	1.001	-2.9
3	4	3.28	1.003	-3.6
3	5	3.83	1.001	-4.3
3	6	4.46	1.005	-5.2
3	7	5.23	1.008	-6.0
3	8	6.06	1.010	-7.0
3	9	7.15	1.015	-8.3
3	10	8.32	1.023	-9.6
3	11	9.79	1.034	-11.3
3	12	11.44	1.045	-13.5
3	13	13.40	1.064	-16.2
3	14	15.76	1.088	-19.7
3	15	18.45	1.121	-24.0
3	16	21.55	1.165	-29.5
3	17	25.26	1.226	-37.2
3	18	29.53	1.295	-48.3
3	19	34.66	1.329	-64.3
3	20	40.50	1.266	-84.3
3	21	47.76	1.047	-106.8
3	22	55.92	.774	-125.2
3	23	65.18	.541	-138.7
3	24	75.85	.376	-148.0
3	25	89.27	.264	-154.8
3	26	106.11	.186	-159.9
3	27	119.62	.135	-163.2
3	28	144.18	.096	-166.7
3	29	166.66	.071	-168.4
3	30	198.52	.053	-170.6

```
MONITORING OF CHANNEL  3 IS COMPLETE.

RELEASE ALL SSW.  THEN R-S-R.

PAUSE  00003
```

Figure 4. - Computer tabular listing of example second-order system.

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